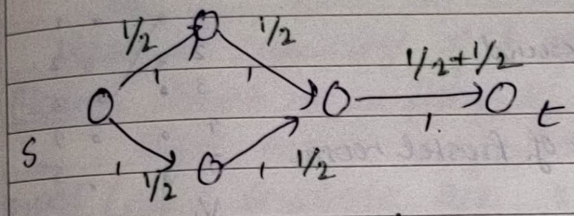
ii). If all capacities are integers, then the alg. always deals w/ integers ⇒ the max flow found by the alg. sends integral amount of flow on each edge. ↓



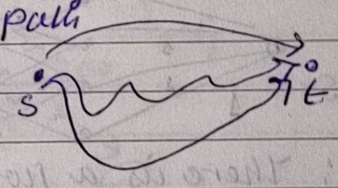
there is a max flow which only sends integer flow (Integrality of max flow).

Class 21. 01/Nov.

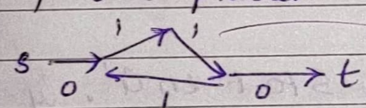
i) Max flow = Min cut Alg. for finding these

ii) Integrality of max flow: There is A max flow which is integral.

iii) how do we think of a flow: Can we think of flow a consisting of flow along path



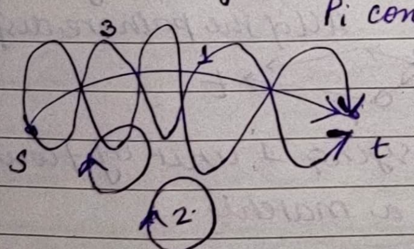
Yes w/ one problem.

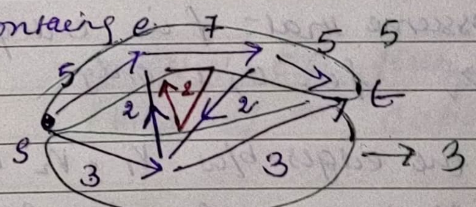


no useful flow

Thm [Path decomposition of flow] Let f be any flow value in G. then we can find s-t paths $P_1 \ldots P_l$ and values $v_1 \ldots v_l$ and cycles $C_1 \ldots C_n$ & values $w_1 \ldots w_n$ st for every edge e,

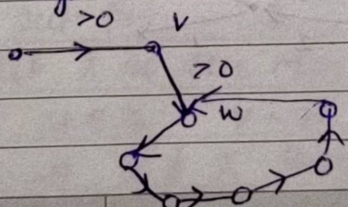
$$f_e = \sum_{P_i \text{ containing } e} v_i + \sum_{C_j \text{ containing } e} w_j$$



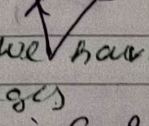


Pf: H: subgraph of G consisting of edges w/ $f_e > 0$

$e \in H$



(A)

H =


i) Repeat a vertex: we have a cycle w/ $f_e > 0$ on all edges

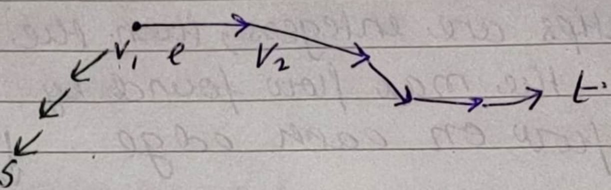
ii) Min f_e on an edge in C & subtract Δ Δ from each edge in C.

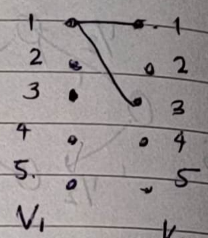
22 THURSDAY JULY

MESSAGES	PHONE CALLS

(B) could reach t:

i) find path to s.
↳ cycle (same)
↳ path to s found
↳ new updated flow is found.

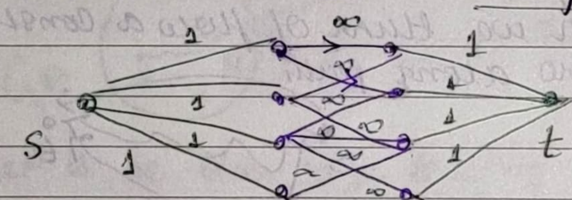




Applications: ① Bipertite Matching:
V_1: set of students. V_2 = set of hostel room.
edge: preference

Matching: subset of edes w/ do't share a common vertex.
- can we have a matching of size n?
- what is the max size of matching?

how do we use max flow to find max^m matching → (s, t, u_e, directed.)



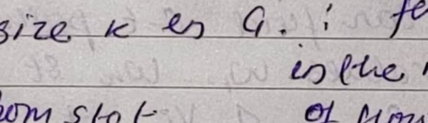
G' P matching = P' max flow

Thm: There is a flow of value k from s to t in G'. if & only if there is a matching of size k in G.

Pf: Suppose there a matching of size k in G.: for every edge e in the matching, send 1 unit of flow along P_e: s, u, v, t. All of the path are disjoint.

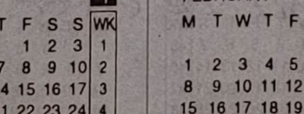
→ suppose there a flow f of value k from s to t
we can assume that f is integral

$s \xrightarrow{f_e} v$ 1 f_e: 0, 1 only

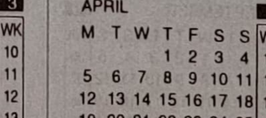


look at the edges b/w V_1, V_2 which are carrying 1 unit of flow
there will be k such edges & they will form a matching

i)



ii)



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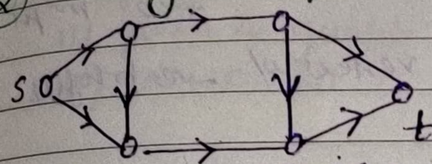
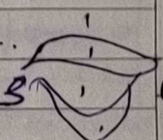
② Disjoint-paths: Given a directed graph G. we say that two paths from s to t are "edges" disjoint if they do not share a common edge.



place a capacity of 1 on every edge find max flow from s - t

Claim: there is a flow of value k. from s to t if and only if there are k edge disjoint paths from s to t.

proof.($\Leftarrow$) suppose are k-edge disjoint paths from s to t.



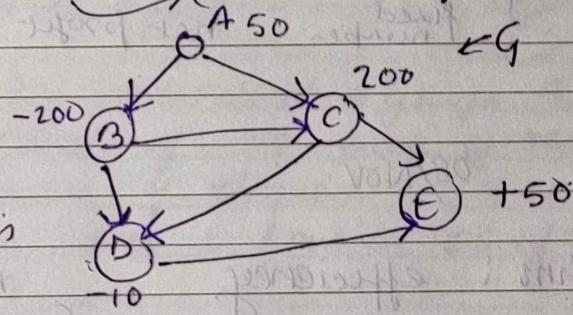
($\Rightarrow$) suppose there is a flow of value k to s to t. (follows fr path decomposition).

③ Application of Min-Cut-

project selection problem. DAG: $\to$ precidence constraint
(directed acyclic graph) tasks

every task either generate or cost money.

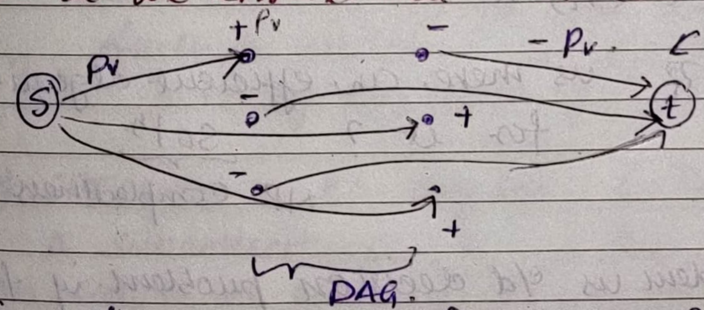
v: P_v



Q: we want to perform subset of tasks such that total profit is maximized.

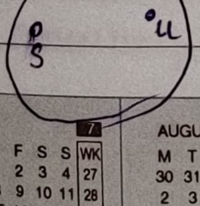
Idea: add a vertex s, t -- min cut --

how do we ensure that min-cut is valid.?? (just not doing 200 w/ 50 & -200)



if $(u,v) \in G$. then in G' we add. (v,u) w/ ∞ capacity.

Claim: find a min cut for s & t



let X be the min cut

$u \in X$ $v \to u$ in G

if $u \notin X$ then (u,v) has cap ∞ the cut X has ∞ capacity

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MESSAGES

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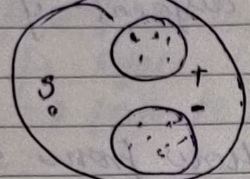
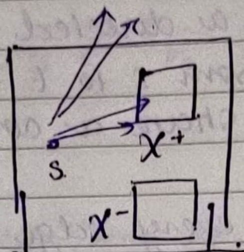
Cap of X = ?





V^+ : +ve profit.

V^- : vertex w/ −ve profit.

going fr s.

$$cap(X) = \sum_{v \in V^+ - X^+} P_v + \sum_{v \in X^-} (-P_v).$$ → going to t

$$= \sum_{v \in V^+} P_v - \sum_{v \in X^+} (P_v) - \sum_{v \in X^-} (-P_v)$$

min cap of (X) = max profit of +

$$= \Big[\sum_{v \in V^+} P_v\Big] - \Big[\sum_{v \in X} P_v\Big]$$

fixed number — net profit in X.

fixed a + b = c.

find this only to set c.

Class 22 08/Nov. NP completeness.

Algorithm : efficiency — n bits to specify the i/p.

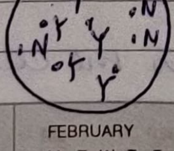
polynomial time $\begin{cases} O(n^c) & \text{where } c \text{ is a constant} \\ O(n) \quad n^2 \quad n^3 \end{cases}$

Q: given an algorithmic problem ?? is there an efficient algorithm for it ?

complexity theory

soln. NP-completeness

Defn: Decision problems: A problem is c/d decision problem if for every i/p the answer is either YES or NO.



All possible i/p.

1999

JANUARY 1

M	T	W	T	F	S	S	WK
				1	2	3	1
4	5	6	7	8	9	10	2
11	12	13	14	15	16	17	3
18	19	20	21	22	23	24	4
25	26	27	28	29	30	31	5

FEBRUARY 2

M	T	W	T	F	S	S	WK
1	2	3	4	5	6	7	6
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15	16	17	18	19	20	21	8
22	23	24	25	26	27	28	9

MARCH 3

M	T	W	T	F	S	S	WK
1	2	3	4	5	6	7	10
8	9	10	11	12	13	14	11
15	16	17	18	19	20	21	12
22	23	24	25	26	27	28	13
29	30	31					14

APRIL 4

M	T	W	T	F	S	S	WK
			1	2	3	4	14
5	6	7	8	9	10	11	15
12	13	14	15	16	17	18	16
19	20	21	22	23	24	25	17
26	27	28	29	30			18

MAY 5

M	T	W	T	F	S	S	WK
31					1	2	18/23
3	4	5	6	7	8	9	19
10	11	12	13	14	15	16	20
17	18	19	20	21	22	23	21
24	25	26	27	28	29	30	22

JUNE 6

M	T	W	T	F	S	S	WK
	1	2	3	4	5	6	23
7	8	9	10	11	12	13	24
14	15	16	17	18	19	20	25
21	22	23	24	25	26	27	26
28	29	30					27

Ex: I) given a number n, is n prime.

II) given a graph G w/ edge capacities u_e, does it have a max flow T_1

III) SATISFIABILITY: Boolean formula.

Q: boolean variable T/F

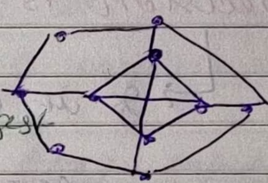
$x_1\ x_2\ x_3 \ldots x_n$ T/F

$Q = (x_1 \underset{AND}{\wedge} x_2 \wedge x_3) \underset{OR}{\vee} (\overset{NOT}{\bar{x}_1} \wedge \bar{x}_2 \wedge x_3)$

Prob: is there any setting of T/F values to the ~~expressio~~ variable $x_1 \ldots x_n$ such that Q is true. (T,T,T) ⇒ Yes.

IV) G is a graph (undirected). A subset of vertices are said to form a clique if there is an edge b/w every pair of them.

Given a graph G, $K \geq 0$ does graph have clique of size $\geq K$? find the biggest clique.

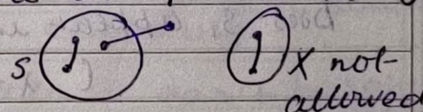


V) INDEPENDENT SET: G is an undirected graph. A subset of vertices is said to be independent if NO pair of vertices have an edge b/w them. (exact opposite of clique).

Given a graph G, $K \geq 0$; does G have an IS of size $\geq K$?

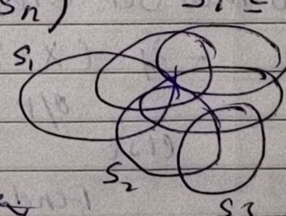
VI) VERTEX COVER: Given a graph G, a subset S of vertices is c/d vertex cover if every edge has at least one end points in S.

Given a graph G, $K \geq 0$: does G have a VC of size $\leq K$?



VII) SET COVER: X: any set $(X, S_1, S_2, \ldots S_n)$ $S_i \subseteq X$.

A subcollectn $S_{i_1}, S_{i_2}, \ldots S_{i_K}$ is c/d a set cover if $S_{i_1} \cup S_{i_2} \cup \ldots S_{i_K} = X$



Q: does $(X, S_1, S_2 \ldots S_n)$ have a set cover ~~consi~~ consisting of $\leq K$ sets

27 TUESDAY JULY

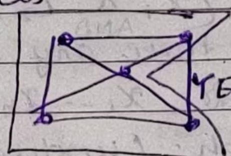
MESSAGES	PHONE CALLS

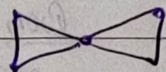
(+ve int)

VIII) Subset Sum: Given n numbers $X_1, X_2 .. X_n$ & a target T is there a subset of $X_1, .. X_n$ which add to exactly T?

IX) A graph G is said to be hamiltonian if there is a cycle which contain all of its vertices

Q: given a ~~pH~~ graph G, is it hamiltonian?


YES.


NO.

Decision vs optimization.
↳ more complex solution.
↳ Is it enough to solve them ?? Yes

SET COVER.

Decision version — optimization version. : find a subcollectn which is a min set cover.

(A)

[If there is an efficient algorithm to solve the decision version, then there is an efficient algo to solve the optimization version also.

By using A_2 we can find the min # sets in a set cover (k)

Does S_1 appear in the min set cover

$(X - S_1, S_2, .. S_n) \leftarrow (k-1)$

A	
YES	S_1 is in k
NO.	S_1 not in k

Find Min Set Cover $(X, S_1, .. S_n)k${
- if $(X - S_1\}, S_2, S_3 .. S_n)_{k-1}$ has a set cover of size $k-1$
  O/P S_1 & find min set cover $(X - S_1, S_2, S_3 .. S_n)$
else
  Find Min Set Cover $(X, S_2, .. S_n)k$

}

calls to findMin set Cover $= n$.

1999

MESSAGES

ark for this thing $D \to 0$ for problems.

PHONE CALLS

WEDNESDAY 28 JULY

- same approach for subset sum
  ($S - x_1, x_2, \dots x_n$) is possible?
- Satisfiability

$$\begin{array}{ccc} & x_1 & \\ T \swarrow & & \searrow F \\ Q_1 & & Q_1' \end{array}$$

only one branch should be checked. if either of them satisfiable?

in Q put $x_1 = T$

Reducibility : you use the solution to one problem to solve another problem.

A [problem] $\longrightarrow$ [machine for] B.

let A, B be two decision problems, we say that $A \leq_p B$ (A is reducible to B). if "a polynomial algorithm for B implies a polynomial time Algorithm for A.

efficient Algo for A. $\rightarrow$ multiple calls to B is allowed

Black box for solving B

x: input (A) $\xrightarrow{Q}$ $Q(x)$: input for B (B)

$\downarrow$

Yes/No. [ ] Algo for B.

(only one call to B is allowed)

i) Q can be implemented in polynomial time.

ii) Answer to x is YES $\Rightarrow$ Answer to $Q(x)$ is YES.

NO $\Rightarrow$ $Q(x)$ is NO.

$p \Rightarrow q$. $A \leq_p B$ if B can be solved efficiently, the A can be solved efficiently

$\neg q \Rightarrow \neg p$. if A can not be solved efficiently, then B can't be solved efficiently.

11/11/2021 Class 23

Ex Independent set $\leq_p$ clique. A is a polynomial time algo to solve clique.

IS $\longrightarrow$ CLIQUE

input G, k

Q: does G have an IS of size k.

input $\bar{G}$ $\to$ A' $\to$ yes/No

opposite of G.

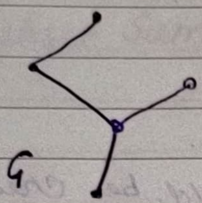
$\bar{G}(V, \bar{E}) = \{u, v : (u,v) \notin E\}$

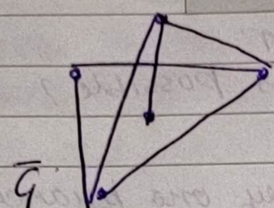
time : A + ~~time taken to~~ i/p $\to$ i/p'

1999

29 THURSDAY JULY

MESSAGES | PHONE CALLS





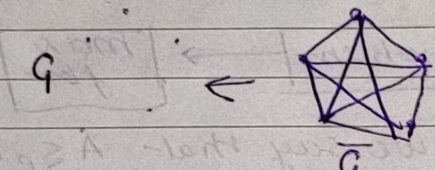
Answer for $\bar{G}$ $\equiv$ Ans for G

<u>To show</u>: $\bar{G}$ has a clique of size k. if & only if G has IS of size k

Important: there are two statements here

Pf: ① suppose $\bar{G}$ has a clique of size k

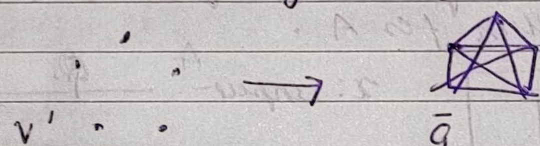
V' : set of k vertices in $\bar{G}$ which form a clique



in G, there is no edge b/w any pair of vertices in V'

$\Rightarrow$ V' is an IS in G.

② Suppose G has an IS of size k $\rightarrow$ $\bar{G}$ has a clique of size k



ex2 CLIQUE $\leq_P$ IS.

ex3 IS $\leq_P$ $\boxed{VC}$ A

IS $\longrightarrow$ VC

i/p i/p'

does G has IS of size k (G, k) $\longrightarrow$ (G', k') ; does G' has a VC of size k.

yes $\longleftarrow$ yes

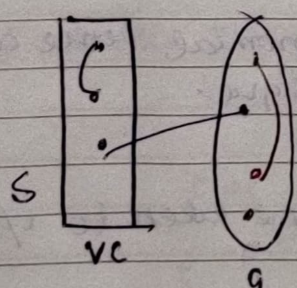
yes $\longrightarrow$ yes

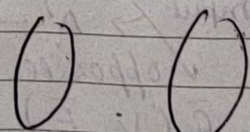
G has n vertices

IS $\longrightarrow$ VC

(G, K) (G, n−k)

If G has an IS of size k then G has A VC of size n−V





S: size of IS

if G has a VC of size n−k then G has IS of size k

1999

JANUARY

M	T	W	T	F	S	S	WK
				1	2	3	1
4	5	6	7	8	9	10	2
11	12	13	14	15	16	17	3
18	19	20	21	22	23	24	4
25	26	27	28	29	30	31	5

FEBRUARY

M	T	W	T	F	S	S	WK
1	2	3	4	5	6	7	6
8	9	10	11	12	13	14	7
15	16	17	18	19	20	21	8
22	23	24	25	26	27	28	9

MARCH

M	T	W	T	F	S	S	WK
1	2	3	4	5	6	7	10
8	9	10	11	12	13	14	11
15	16	17	18	19	20	21	12
22	23	24	25	26	27	28	13
29	30	31					14

APRIL

M	T	W	T	F	S	S	WK
			1	2	3	4	14
5	6	7	8	9	10	11	15
12	13	14	15	16	17	18	16
19	20	21	22	23	24	25	17
26	27	28	29	30			18

MAY

M	T	W	T	F	S	S	WK
31					1	2	18/23
3	4	5	6	7	8	9	19
10	11	12	13	14	15	16	20
17	18	19	20	21	22	23	21
24	25	26	27	28	29	30	22

JUNE

M	T	W	T	F	S	S	WK
	1	2	3	4	5	6	23
7	8	9	10	11	12	13	24
14	15	16	17	18	19	20	25
21	22	23	24	25	26	27	26
28	29	30					27

6-7 ques^n

1- Greed → follow the patten

PP

flow

NP: ↳ show reduction
 ↳ NP complete

1st decision then undecid^a.

03 TUESDAY AUGUST

MESSAGES

PHONE CALLS

9:00 email & Teams

11:30 stopwaily

two classes of problems: P & NP.

P: set of decision problems for which there is a polynomial time alg.

NP: non deterministic polynomial time

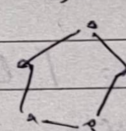
↳ A decision problem is in class NP. if there is an efficient algorithm A which can check solution of A.

I: i/p

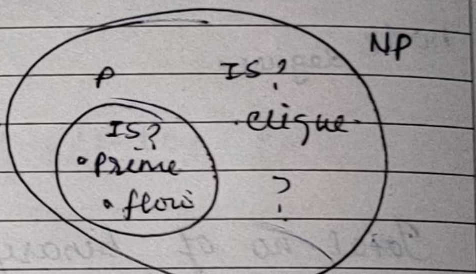
sol^n → $\boxed{A}$ → ok.

take a subset S of vertices of size k. & check whether it is an IS?

G, k IS.



n^k.

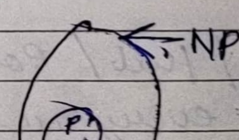


Def^n NP: A decision problem $A \in NP$ if there is an polynomial time algo A w/ following property.

1) suppose x is a i/p for which answer is YES, then there exist an string y such that A(x,y) o/p Yes
 ↳ check whether y is a valid sol^n for x

ii) if x is an i/p for which the answer is NO. then for all string y A(x,y) o/p NO.

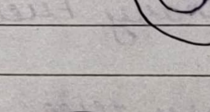
(1) $P \subseteq NP$?
 suppose $A \in P$ ← A.
 y = empty string



Yes → Yes
NO → No.

Problem: A
 what to know is $A \in P$ or not ?}
 suppose $B \notin P$. & $B \subseteq_p A$. $A \notin P$

$A \subseteq_p B$.
$B \subseteq_p A$.



flow $\subseteq_p$ IS.

Q: there any problem which is not in P?
 What if $P \neq NP$?! Yes.
(cook's theorem' 7).

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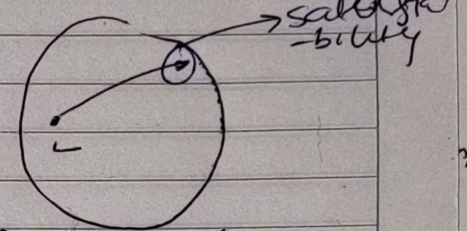
If L is a problem in NP. & $L \leq_p$ SATISFIABILITY

$\Rightarrow$ if there is a polynomial time alg for satisfiability then there is a polynomial Algo for any $L \in NP$.

$\Rightarrow$ if $P \neq NP$ then satisfiability $\notin P$



Def^n NP compleatness : A problem B is c/d NP compleate

$\left\{\begin{array}{l} \text{i) } B \in NP \\ \text{ii). } \forall \text{ problem } L \in NP,\ L \leq_p B. \end{array}\right.$

acc to cook's theorem satisfiability is NP complete.

$C \leq_p B$ & C is NP complete
$\Rightarrow$ B is also NP complete

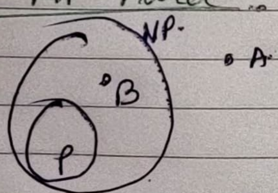
IS $\leq_p$ CLIQUE
IS $\leq_p$ VC
IS $\leq_p$ Ham.

$\Rightarrow$ clique, VC, Ham all are NP complete.

Thm 3SAT (satisfiability) : $\leq_p$ IS.
$\Rightarrow$ IS is also NP complete.

What is 3 SAT. 3 CNF Boolean Formula.
$C_1 \wedge \dots \wedge C_m$. m clauses. each clause has 3 variables.

Def^n NP hard : A problem is c/d NP hard. if
$\forall\ L \in NP \quad L \leq_p A.$



B; Np complete. A; NP hard.
$B \leq_p A$.

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JULY							7
M	T	W	T	F	S	S	WK
			1	2	3	4	27
5	6	7	8	9	10	11	28
12	13	14	15	16	17	18	29
19	20	21	22	23	24	25	30
26	27	28	29	30	31		31

AUGUST							8
M	T	W	T	F	S	S	WK
30	31					1	31/36
2	3	4	5	6	7	8	32
9	10	11	12	13	14	15	33
16	17	18	19	20	21	22	34
23	24	25	26	27	28	29	35

SEPTEMBER							9
M	T	W	T	F	S	S	WK
		1	2	3	4	5	36
6	7	8	9	10	11	12	37
13	14	15	16	17	18	19	38
20	21	22	23	24	25	26	39
27	28	29	30				40

OCTOBER							10
M	T	W	T	F	S	S	WK
				1	2	3	40
4	5	6	7	8	9	10	41
11	12	13	14	15	16	17	42
18	19	20	21	22	23	24	43
25	26	27	28	29	30	31	44

NOVEMBER							11
M	T	W	T	F	S	S	WK
1	2	3	4	5	6	7	45
8	9	10	11	12	13	14	46
15	16	17	18	19	20	21	47
22	23	24	25	26	27	28	48
29	30						49

DECEMBER							12
M	T	W	T	F	S	S	WK
		1	2	3	4	5	49
7	6	8	9	10	11	12	50
14	13	15	16	17	18	19	51
21	20	22	23	24	25	26	52
28	27	29	30	31			53