Convert cylindrical to spherical.

$r = \rho\sin\phi$, $z = \rho\cos\phi$, $\theta = \theta$. where $\rho = \sqrt{r^2+z^2}$,

$\theta = \tan^{-1}(y/x)$, $\phi = \cos^{-1}(z/\rho)$.

partial derivatives of above equs are:

$$\partial/\partial r = \frac{\partial\rho}{\partial r}\cdot\partial/\partial\rho + \frac{\partial\phi}{\partial r}\cdot\partial/\partial\phi = \sin\phi\,\partial/\partial\rho + \frac{r^2}{\sqrt{r^2-z^2}\,\rho^{3/2}}\,\partial/\partial\phi$$

$$\partial/\partial z = \partial\rho/\partial z\cdot\partial/\partial\rho + \frac{\partial\phi}{\partial z}\cdot\partial/\partial\phi = \cos\phi\,\partial/\partial\rho + \frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}}\,\partial/\partial\phi$$

Now,

$$u_r = u_\rho\sin\phi + u_\phi\frac{r^2}{\sqrt{r^2-z^2}\,\rho^{3/2}}, \quad u_z = u_\rho\cos\phi + u_\phi\frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}}$$

$$u_\theta = u_\theta.$$

$$\Rightarrow \hat{e}_\rho = \frac{\partial u_r}{\partial r} = \sin\phi\left[\partial/\partial\rho\left(u_\rho\sin\phi + u_\phi\frac{r^2}{\sqrt{r^2-z^2}\,\rho^{3/2}}\right)\right] + \frac{r^2}{\sqrt{r^2-z^2}}$$
$$\frac{1}{\rho^{3/2}}\,\partial/\partial\phi\left[u_\rho\sin\phi + u_\phi\frac{r^2}{\sqrt{r^2-z^2}\,\rho^{3/2}}\right].$$

$$= \frac{\partial u_\rho}{\partial\rho}\sin^2\phi + \left(\frac{\partial u_\phi}{\partial\rho}\cdot\frac{1}{\rho^{3/2}} + \frac{u_\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial\phi}\cdot\frac{1}{\rho^{3/2}}\right)$$
$$\frac{r^2\sin\phi}{\sqrt{r^2-z^2}} + \left(u_\rho\cos\phi + \frac{\partial u_\phi}{\partial\phi}\cdot\frac{1}{\rho^3}\right)\frac{r^2}{\sqrt{r^2-z^2}}\Bigg]\frac{r^2}{\sqrt{r^2-z^2}}$$

$$\Rightarrow \hat{e}_\phi = \frac{\partial u_z}{\partial z} = \cos\phi\,\partial/\partial\rho\left[u_\rho\cos\phi + u_\phi\frac{rz}{\rho^{3/2}\sqrt{r^2-z^2}}\right] + \frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}}$$
$$\partial/\partial\phi\left[u_\rho\cos\phi + u_\phi\frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}}\right]$$

$$e_\phi = \frac{\partial u_\rho}{\partial\rho}\cos^2\phi + \left(\frac{\partial u_\phi}{\partial\rho}\cdot\frac{1}{\rho^{3/2}} + \frac{u_\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial\phi}\cdot\frac{1}{\rho^{3/2}}\right)$$
$$\frac{\cos\phi\, rz}{\sqrt{r^2-z^2}} + \left(\frac{\partial u_\phi}{\partial\phi}\frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}} - \frac{u_\rho\sin\phi}{\rho^{3/2}}\right)\frac{rz}{\sqrt{r^2-z^2}}$$

therefore strain displacement relation becomes

$$e_\rho = \frac{\partial u_r}{\partial r}, \quad e_\phi = \frac{1}{\rho}\left(u_r + \frac{\partial u_z}{\partial \phi}\right)$$

$$e_\theta = \frac{1}{\rho \sin\phi}\left(\frac{\partial u_\theta}{\partial \theta} + \sin\phi\, u_r + \cos\phi\, u_z\right)$$

$$e_{\rho\phi} = \frac{1}{2}\left(\frac{1}{\rho}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_z}{\partial \rho} - \frac{u_z}{\rho}\right).$$

$$e_{\phi\theta} = \frac{1}{\partial\rho}\left(\frac{1}{\sin\phi} \cdot \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial \phi} - \cot\phi\, u_\theta\right).$$

$$e_{\theta\rho} = \frac{1}{2}\left(\frac{1}{\rho \sin\phi} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial \rho} - \frac{u_\theta}{\partial \rho}\right)$$

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