MINOR: COMPUTATIONAL MATHEMATICS

Transcendental Equations

Q.1) Finding Real Root using Bisection Method in C Programming.

AIM: To write a C program to find the real root of an equation using the Bisection Method.

Formula:

$$x_r = \frac{a + b}{2}$$

Programming Formula:

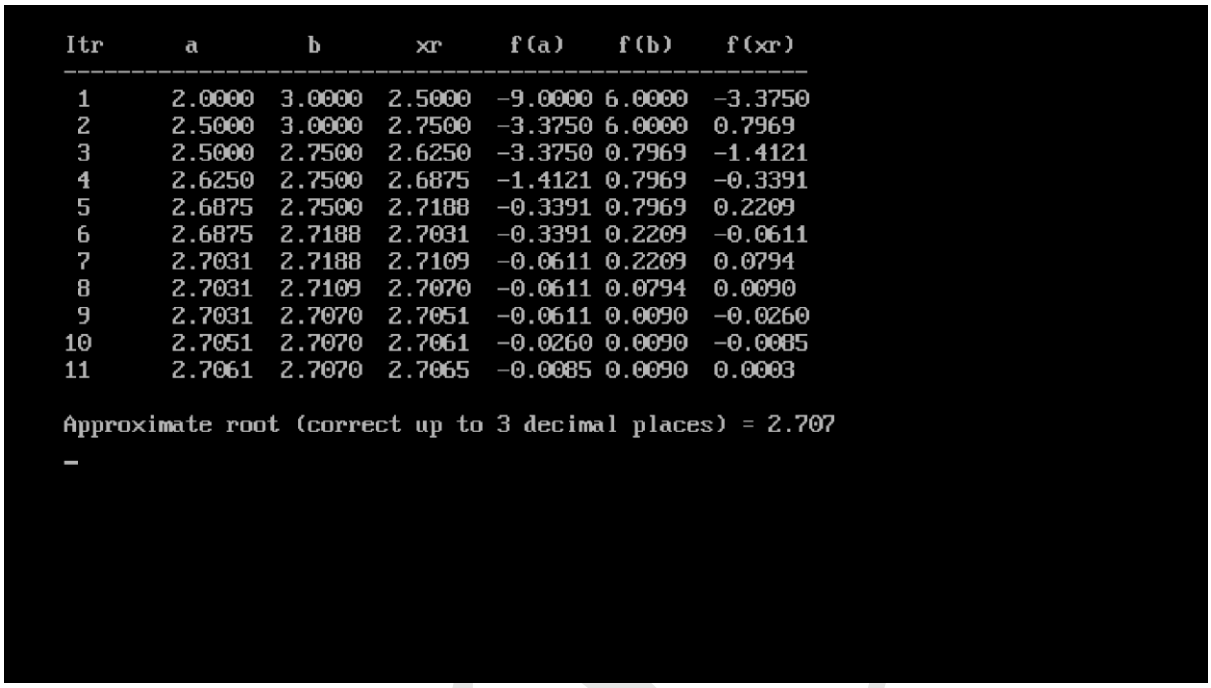
```
xr = (a + b) / 2;
```

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
float f(float x) {
   return (x * x * x - 4 * x - 9);
}
int main() {
   float a = 2, b = 3, xr, fx, fb, fxr;
   int intr = 0;
   float eps = 0.001;
   float prev1 = 0.0, prev2 = 0.0;
   int same_count = 0;
   clrscr();
   fx = f(a);
   fb = f(b);
   if (fx * fb > 0) {
         printf("Root not found in the given interval.\n");
         return 0;
```

```
    }
    printf("\nItr\t a\t  b\t  xr\t f(a)\t f(b)\t f(xr)\n");
    printf("-------------------------------------------------------\n");
    do {
        intr++;
        xr = (a + b) / 2;
        fxr = f(xr);
        fx = f(a);
        fb = f(b);
        printf("%2d\t%.4f\t%.4f\t%.4f\t%.4f\t%.4f\t%.4f\n",
            intr, a, b, xr, fx, fb, fxr);
        if (fx * fxr < 0)
           b = xr;
        else
           a = xr;
        float xr3 = floor(xr * 1000 + 0.5) / 1000;
        if (fabs(xr3 - prev1) < 0.0001 && fabs(prev1 - prev2) < 0.0001)
           same_count++;
        else
           same_count = 0;
        prev2 = prev1;
        prev1 = xr3;
        if (same_count >= 1)
           break;
    } while (fabs(fxr) > eps);
    printf("\nApproximate root (correct up to 3 decimal places) = %.3f\n", prev1);
    getch();
    return 0;
}
```

Output:



```
Itr      a        b       xr     f(a)    f(b)    f(xr)
-------------------------------------------------------
 1      2.0000  3.0000  2.5000  -9.0000 6.0000  -3.3750
 2      2.5000  3.0000  2.7500  -3.3750 6.0000  0.7969
 3      2.5000  2.7500  2.6250  -3.3750 0.7969  -1.4121
 4      2.6250  2.7500  2.6875  -1.4121 0.7969  -0.3391
 5      2.6875  2.7500  2.7188  -0.3391 0.7969  0.2209
 6      2.6875  2.7188  2.7031  -0.3391 0.2209  -0.0611
 7      2.7031  2.7188  2.7109  -0.0611 0.2209  0.0794
 8      2.7031  2.7109  2.7070  -0.0611 0.0794  0.0090
 9      2.7031  2.7070  2.7051  -0.0611 0.0090  -0.0260
10      2.7051  2.7070  2.7061  -0.0260 0.0090  -0.0085
11      2.7061  2.7070  2.7065  -0.0085 0.0090  0.0003

Approximate root (correct up to 3 decimal places) = 2.707
_
```

Conclusion: The program was executed successfully.

Q.2) Finding Real Root using Regular Falsi Method in C Programming.

Aim: To write a C program to find the real root of an equation using the Regular Falsi Method.

Formula:

$$x_r = \frac{a \times f(b) - b \times f(a)}{f(b) - f(a)}$$

Programming Formula:

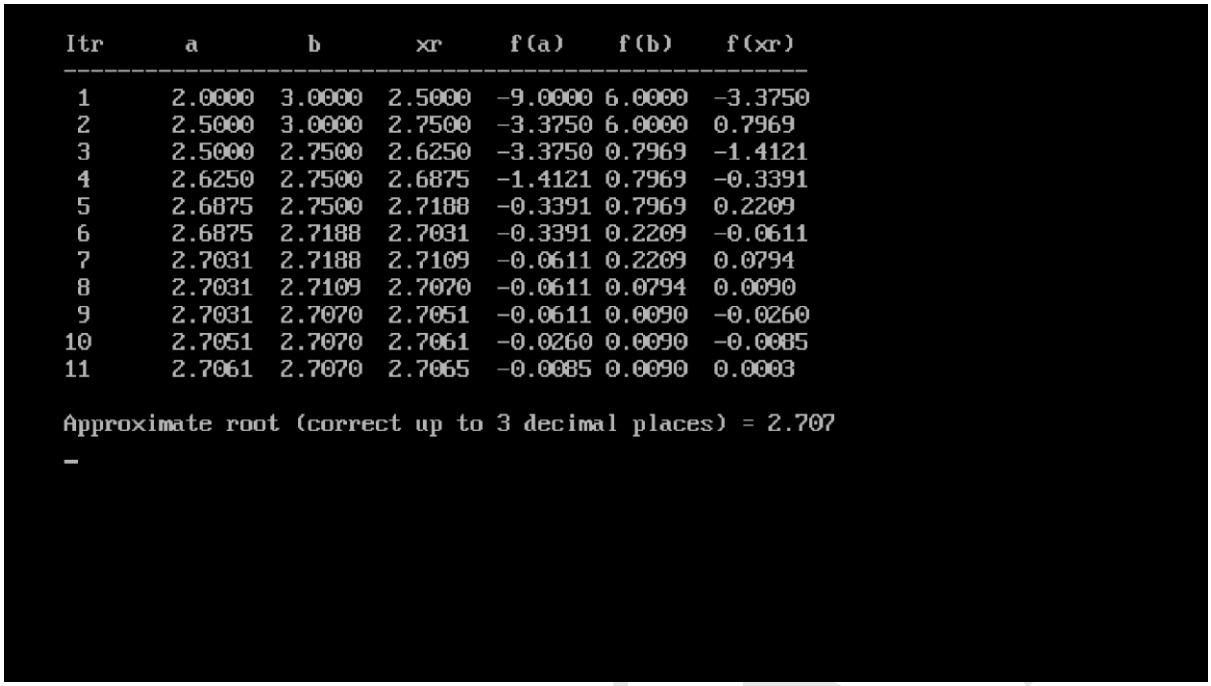
xr = (a * f(b) - b * f(a)) / (f(b) - f(a));

Program:

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
float f(float x) {
   return (x * x * x - 4 * x - 9);
}
int main() {
   float a = 2, b = 3, xr, fx, fb, fxr;
   int i = 0;
   float eps = 0.001;
   float prev1 = 0.0, prev2 = 0.0;
   int same_count = 0;
   clrscr();
   fx = f(a);
   fb = f(b);
   if (fx * fb > 0) {
      printf("Root not found in the given interval.\n");
      getch();
      return 0;
   }
   printf("\nIter\t a\t\t b\t\t xr\t\t f(a)\t\t f(b)\t\t f(xr)\n");
   printf("--------------------------------------------------------------------------\n");
```

```
    do {
        i++;
        xr = (a * fb - b * fx) / (fb - fx);
        fxr = f(xr);
        fx = f(a);
        fb = f(b);
       printf("%d\t %.6f\t %.6f\t %.6f\t %.6f\t %.6f\t %.6f\n", i, a, b, xr, fx, fb, fxr);
        if (fx * fxr < 0)
            b = xr;
        else
            a = xr;
        float xr3 = floor(xr * 1000 + 0.5) / 1000;
        if (fabs(xr3 - prev1) < 0.0001 && fabs(prev1 - prev2) < 0.0001)
            same_count++;
        else
            same_count = 0;
        prev2 = prev1;
        prev1 = xr3;
        if (same_count >= 1)
            break;
    } while (fabs(fxr) > eps);
    printf("\nApproximate root (correct up to 3 decimal places) = %.3f\n", prev1);
    getch();
    return 0;
}
```

Output:



```
Itr      a        b       xr     f(a)    f(b)    f(xr)
-------------------------------------------------------
 1      2.0000  3.0000  2.5000  -9.0000 6.0000  -3.3750
 2      2.5000  3.0000  2.7500  -3.3750 6.0000  0.7969
 3      2.5000  2.7500  2.6250  -3.3750 0.7969  -1.4121
 4      2.6250  2.7500  2.6875  -1.4121 0.7969  -0.3391
 5      2.6875  2.7500  2.7188  -0.3391 0.7969  0.2209
 6      2.6875  2.7188  2.7031  -0.3391 0.2209  -0.0611
 7      2.7031  2.7188  2.7109  -0.0611 0.2209  0.0794
 8      2.7031  2.7109  2.7070  -0.0611 0.0794  0.0090
 9      2.7031  2.7070  2.7051  -0.0611 0.0090  -0.0260
10      2.7051  2.7070  2.7061  -0.0260 0.0090  -0.0085
11      2.7061  2.7070  2.7065  -0.0085 0.0090  0.0003

Approximate root (correct up to 3 decimal places) = 2.707
_
```

Conclusion: The program was executed successfully.

Q.3) Finding Real Root using Newton–Raphson Method in C Programming

Aim: To write a C program to find the real root of an equation using the Newton–Raphson Method.

Formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Programming Formula:

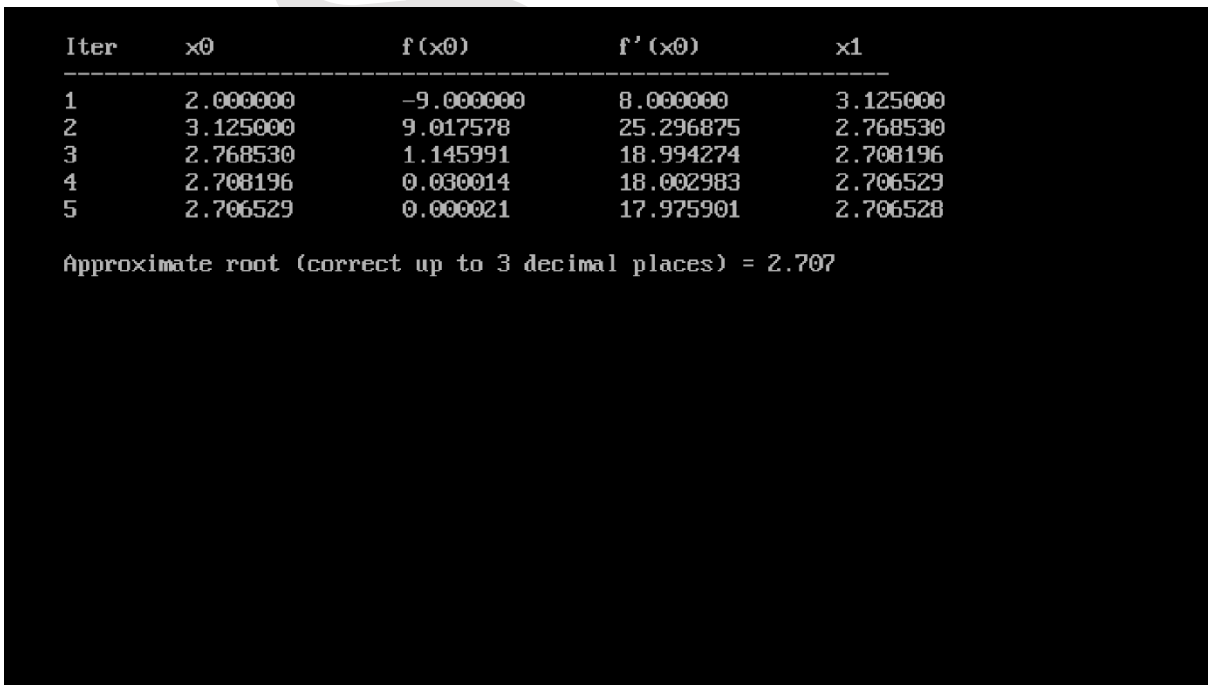
x1 = x0 - (f(x0) / f1(x0));

Program:

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
float f(float x) {
   return (x * x * x - 4 * x - 9);
}
float f1(float x) {
   return (3 * x * x - 4);
}
int main() {
   float x0 = 2, x1, f0, f1x;
   int i = 0;
   float eps = 0.001;
   float prev1 = 0.0, prev2 = 0.0;
   int same_count = 0;
   clrscr();
   printf("\nIter\t x0\t\t f(x0)\t\t f'(x0)\t\t x1\n");
   printf("-------------------------------------------------------------\n");
   do {
      i++;
      f0 = f(x0);
      f1x = f1(x0);
```

```
        x1 = x0 - (f0 / f1x);
        printf("%d\t %.6f\t %.6f\t %.6f\t %.6f\n", i, x0, f0, f1x, x1);
        float x13 = floor(x1 * 1000 + 0.5) / 1000;
        if (fabs(x13 - prev1) < 0.0001 && fabs(prev1 - prev2) < 0.0001)
            same_count++;
        else
            same_count = 0;
        prev2 = prev1;
        prev1 = x13;
        if (same_count >= 1)
            break;
        x0 = x1;
    } while (fabs(f0) > eps);
    printf("\nApproximate root (correct up to 3 decimal places) = %.3f\n", prev1);
    getch();
    return 0;
}
```

Output:



Conclusion: The program was executed successfully.

Interpolation

Q.1) Newton Forward Interpolation Table in C Programming

Aim: To create a Newton Forward Difference Table and interpolate a value using Newton's Forward Interpolation Formula.

Formula:

$$y(x) = y_0 + p\Delta y_0 + \frac{i(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$p = \frac{x - x_0}{h}, h = x_0 - x_1$$

Programming Formula:

```
y_int = y_int + (p_term * diff[0][i]) / fact(i);
p_term = p_term * (p - i);
```

Program:

```
#include <stdio.h>
#include <conio.h>
float fact(int num)
{
   float f = 1;
   int i;
   for(i = 2; i <= num; i++)
      f = f * i;
   return f;
}
int main()
{
   int n = 5;
   float x[5] = {0, 1, 2, 3, 4};
   float y[5][5] = {
      {1}, {8}, {27}, {64}, {125}
```

```
    };
    float value = 2.5, h, p, y_int, p_term;
    int i, j;
    clrscr();
    for(j = 1; j < n; j++)
        for(i = 0; i < n-j; i++)
            y[i][j] = y[i+1][j-1] - y[i][j-1];
    printf("\nNewton Forward Difference Table:\n");
    printf("x\tf(x)\ty\tDeltay\tDelta2y\tDelta3y\n");
    for(i = 0; i < n; i++)
    {
        printf("%.1f", x[i]);
        for(j = 0; j < n-i; j++)
            printf("\t%.2f", y[i][j]);
        printf("\n");
    }
    h = x[1] - x[0];
    p = (value - x[0]) / h;
    y_int = y[0][0];
    p_term = 1;
    for(i = 1; i < n; i++)
    {
        p_term = p_term * (p - i + 1);
        y_int = y_int + (p_term * y[0][i]) / fact(i);
    }
    printf("\nValue at x = %.2f is %.4f\n", value, y_int);
    getch();
    return 0;
}
```

Output:

```
Newton Forward Difference Table:
x       f(x)    y       Deltay  Delta2y Delta3y
0.0     1.00    7.00    12.00   6.00    0.00
1.0     8.00    19.00   18.00   6.00
2.0     27.00   37.00   24.00
3.0     64.00   61.00
4.0     125.00

Value at x = 2.50 is 42.8750
_
```

Conclusion: The program was executed successfully.

Q.2) Newton Backward Interpolation Table in C Programming

Aim: To write a C program to create a Newton Backward Difference Table and interpolate a value using Newton's Backward Interpolation Method.

Formula:

$$y(x) = y_n + p\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \cdots$$

$$p = \frac{x - x_0}{h}, h = x_0 - x_1$$

Programming Formula:

y_int = y_int + (p_term * diff[n-i-1][i]) / fact(i);

p_term = p_term * (p + i);

Program:

```
#include <stdio.h>
#include <conio.h>
float fact(int num)
{
   float f = 1;
   int i;
   for(i = 2; i <= num; i++)
      f = f * i;
   return f;
}
int main()
{
   int n = 5;
   float x[5] = {0, 1, 2, 3, 4};
   float y[5][5] = {
      {1}, {8}, {27}, {64}, {125}
   };
   float value = 3.5, h, p, y_int, p_term;
   int i, j;
```

```
    clrscr();
    for(j = 1; j < n; j++)
        for(i = n-1; i >= j; i--)
            y[i][j] = y[i][j-1] - y[i-1][j-1];
    printf("\nNewton Backward Difference Table:\n");
    printf("x\tf(x)\tDelta y\tDelta^2 y\tDelta^3 y\tDelta^4 y\n");
    for(i = 0; i < n; i++)
    {
        printf("%.1f", x[i]);
        for(j = 0; j <= i; j++)
            printf("\t%.2f", y[i][j]);
        printf("\n");
    }
    h = x[1] - x[0];
    p = (value - x[n-1]) / h;
    y_int = y[n-1][0];
    p_term = 1;
    for(i = 1; i < n; i++)
    {
        p_term = p_term * (p + i - 1);
        y_int = y_int + (p_term * y[n-1][i]) / fact(i);
    }
    printf("\nValue at x = %.2f is %.4f\n", value, y_int);
    getch();
    return 0;
}
```

Output:

```
Newton Backward Difference Table:
x       f(x)    Deltay  Delta2y Delta3y Delta4y
0.0     1.00
1.0     8.00    7.00
2.0     27.00   19.00   12.00
3.0     64.00   37.00   18.00   6.00
4.0     125.00  61.00   24.00   6.00    0.00

Value at x = 3.50 is 91.1250
```

Conclusion: The program was executed successfully.

Numerical Integration

Q.1) To find the approximate value of a definite integral using the Trapezoidal Rule.

Aim: To find the approximate value of a definite integral using the Trapezoidal Rule.

Formula:

$$\int_a^b f(x)\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1}) + y_n]$$

Programming Formula :

```
sum = y0 + yn;
for(i = 1; i < n; i++)
sum += 2 * yi;
area = (h / 2) * sum;
```

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
float f(float x) {
   return x*x;
}
int main() {
   int n = 4;
   float a = 0.0, b = 2.0;
   float h, area, sum = 0.0;
   int i;
clrscr();
   h = (b - a) / n;
   sum = f(a) + f(b);
   for(i = 1; i < n; i++) {
      sum += 2 * f(a + i*h);
   }
```

```
    area = (h / 2) * sum;
    printf("Approximate value of the integral: %.4f\n", area);
getch()
    return 0;
}
```

Output:

Conclusion: The program was executed successfully.

Q.2):To find the approximate value of a definite integral using Simpson's 1/3 Rule.

Aim: To calculate the area under the curve using Simpson's 1/3 Rule.

Formula:

$$\int_a^b f(x)dx \approx \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2}) + y_n]$$

Programming Formula:

```
sum = y0 + yn;
for(i = 1; i < n; i++) {
if(i % 2 == 0)
sum += 2 * yi;
   else
sum += 4 * yi;
}
integral = (h / 3) * sum;
```

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
float f(float x) {
   return x*x;
}
int main() {
   int n = 4;
   float a = 0.0, b = 2.0;
   float h, area, sum = 0.0;
   int i;
clrscr();
   h = (b - a) / n;
   sum = f(a) + f(b);
   for(i = 1; i < n; i++) {
```

```
        if(i % 2 == 0)
            sum += 2 * f(a + i*h);
        else
            sum += 4 * f(a + i*h);
    }
    area = (h / 3) * sum;
    printf("Approximate value of the integral (Simpson 1/3): %.4f\n", area);
    getch();
    return 0;
}
```

Output:

Conclusion: The program was executed successfully.

Q.3) To find the approximate value of a definite integral using Simpson's 3/8 Rule.

Aim: To calculate the area under the curve using Simpson's 3/8 Rule.

Formula:

$$\int_a^b f(x)dx \approx \frac{3h}{8}[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots) + 2(y_3 + y_6 + \cdots) + y_n]$$

where

- $n$is a **multiple of 3**
- $h = \frac{b-a}{n}$

Programming Formula:

```
sum = y0 + yn;
for(i = 1; i < n; i++) {
   if(i % 3 == 0)
      sum += 2 * yi;
   else
      sum += 3 * yi;
}
integral = (3*h/8) * sum;
```

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
float f(float x) {
   return x*x;
}
int main() {
   int n = 6;
   float a = 0.0, b = 2.0;
   float h, area, sum = 0.0;
   int i;
clrscr();
```

```
    h = (b - a) / n;
    sum = f(a) + f(b);
    for(i = 1; i < n; i++) {
        if(i % 3 == 0)
            sum += 2 * f(a + i*h);
        else
            sum += 3 * f(a + i*h);
    }
    area = (3*h / 8) * sum;
    printf("Approximate value of the integral (Simpson 3/8): %.4f\n", area);
getch();
    return 0;
}
```

Output:

Conclusion: The program was executed successfully.

Curve fitting

Q.1) To find the best-fitting curve for a set of data points using the method of least squares.

Aim: To determine the quadratic curve $y = a + bx + cx^2$ that best fits the given data points.

Formulas (Least Squares)

$$\sum y_i = na + b\sum x_i + c\sum x_i^2$$
$$\{\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$$
$$\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$$

Programming Formula :

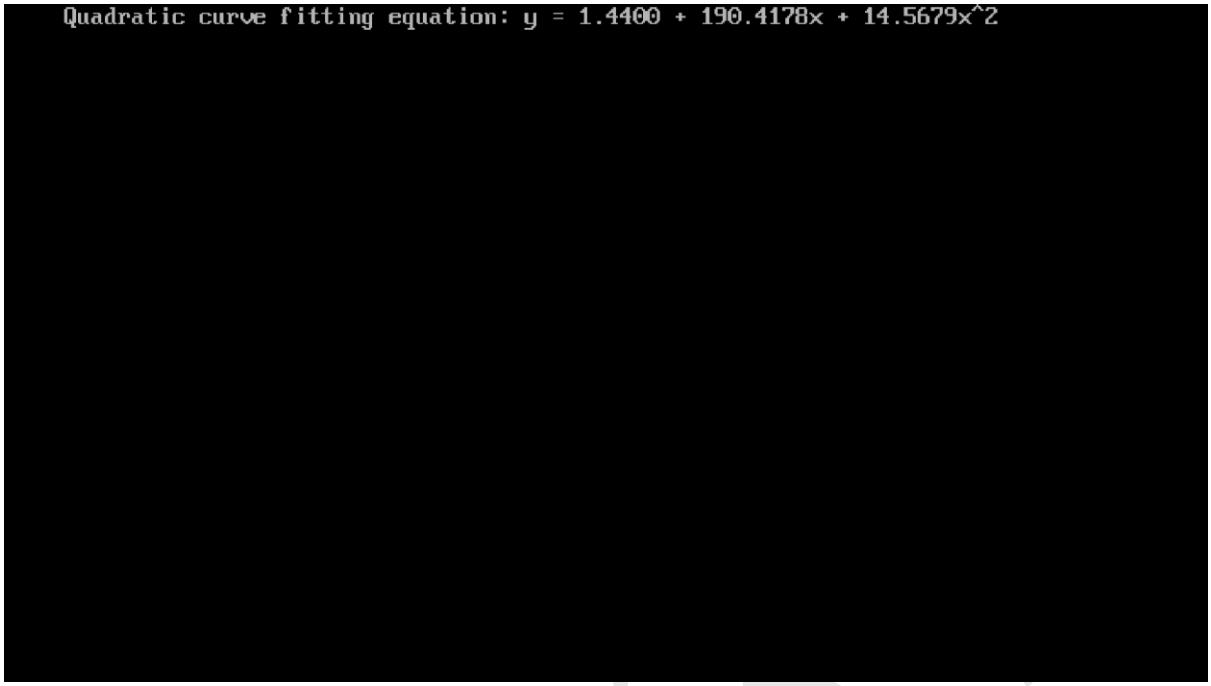
```
sum_x = sum_x2 = sum_x3 = sum_x4 = 0;
sum_y = sum_xy = sum_x2y = 0;
for(i=0; i<n; i++){
   sum_x += x[i];
   sum_x2 += x[i]*x[i];
   sum_x3 += x[i]*x[i]*x[i];
   sum_x4 += x[i]*x[i]*x[i]*x[i];
   sum_y += y[i];
   sum_xy += x[i]*y[i];
   sum_x2y += x[i]*x[i]*y[i];
}
a = Da / D;
b = Db / D;
c = Dc / D;
```

Program:

```
#include <stdio.h>
#include <conio.h>
int main() {
   clrscr();
   int n = 5;
```

```
    float x[] = {1, 2, 3, 4, 5};
    float y[] = {2.2, 2.8, 3.6, 4.5, 5.1};
    float sum_x=0, sum_x2=0, sum_x3=0, sum_x4=0;
    float sum_y=0, sum_xy=0, sum_x2y=0;
    int i;
    for(i=0; i<n; i++){
        sum_x += x[i];
        sum_x2 += x[i]*x[i];
        sum_x3 += x[i]*x[i]*x[i];
        sum_x4 += x[i]*x[i]*x[i]*x[i];
        sum_y += y[i];
        sum_xy += x[i]*y[i];
        sum_x2y += x[i]*x[i]*y[i];
    }
    float D = n*(sum_x2*sum_x4 - sum_x3*sum_x3) - sum_x*(sum_x*sum_x4 -
sum_x2*sum_x3) + sum_x2*(sum_x*sum_x3 - sum_x2*sum_x2);
    float Da = sum_y*(sum_x2*sum_x4 - sum_x3*sum_x3) - sum_x*(sum_xy*sum_x4 -
sum_x2y*sum_x3) + sum_x2*(sum_xy*sum_x3 - sum_x2y*sum_x2);
    float Db = n*(sum_xy*sum_x4 - sum_x2y*sum_x3) - sum_y*(sum_x*sum_x4 -
sum_x2*sum_x3) + sum_x2*(sum_x*sum_x2y - sum_y*sum_x2);
    float Dc = n*(sum_x2*sum_x2y - sum_x3*sum_xy) - sum_x*(sum_x*sum_x2y -
sum_x3*sum_y) + sum_y*(sum_x*sum_x3 - sum_x2*sum_x2);
    float a = Da / D;
    float b = Db / D;
    float c = Dc / D;
    printf("Quadratic curve fitting equation: y = %.4f + %.4fx + %.4fx^2\n", a, b, c);
    getch();
    return 0;
}
```

Output:



Conclusion: The program was executed successfully.

Q.1) Runge–Kutta 4th Order Method in C Programming.

Aim: To solve the differential equation $y' = x + y$using the Fourth Order Runge–Kutta Method and compute the values of $y$for the given step size.

Formula:

$$k_1 = h\,f(x_0, y_0)$$
$$k_2 = h\,f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$$
$$k_3 = h\,f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$$
$$k_4 = h\,f(x_0 + h, y_0 + k_3)$$

Weighted average slope:

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

Final update formula:

$$y_1 = y_0 + k$$
$$x_1 = x_0 + h$$

Programming Formula:

k1 = H * f(x, y);

k2 = H * f(x + H/2, y + k1/2);

k3 = H * f(x + H/2, y + k2/2);

k4 = H * f(x + H, y + k3);

kavg = (k1 + 2*k2 + 2*k3 + k4) / 6;

y = y + kavg;

x = x + H;

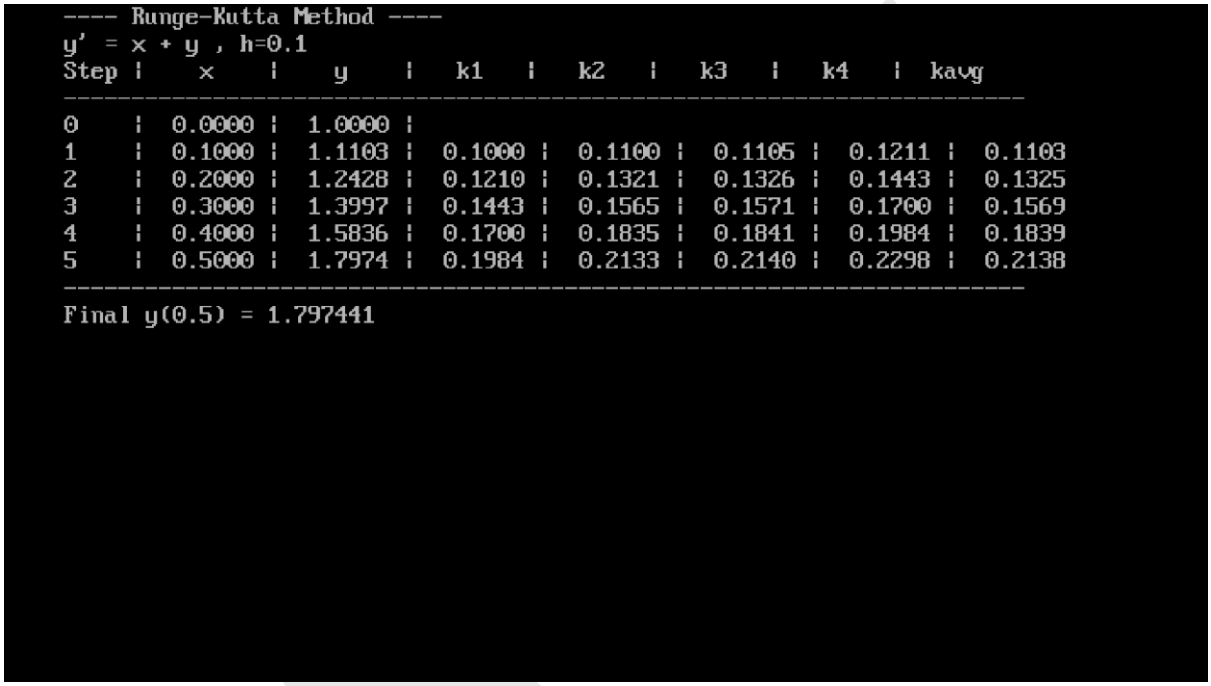
Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
double f(double x, double y) {
```

```
    return x + y;
}
void runge_kutta_4th_order_solve() {
    const double X0 = 0.0;
    const double Y0 = 1.0;
    const double H = 0.1;
    const int N = 5;
    double x = X0, y = Y0;
    double k1, k2, k3, k4;
    int i;
    clrscr();
    printf("---- RK4 Method ----\n");
    printf("y' = x + y , h=%.1f\n", H);
    printf("Step |    x    |    y    |   k1   |   k2   |   k3   |   k4   | kavg\n");
    printf("-------------------------------------------------------------------\n");
    printf("0    | %7.4f | %7.4f |\n", x, y);
    for (i = 1; i <= N; i++) {
        k1 = H * f(x, y);
        k2 = H * f(x + H/2, y + k1/2);
        k3 = H * f(x + H/2, y + k2/2);
        k4 = H * f(x + H, y + k3);
        double kavg = (k1 + 2*k2 + 2*k3 + k4) / 6.0;
        y = y + kavg;
        x = x + H;
        printf("%-4d | %7.4f | %7.4f | %7.4f | %7.4f | %7.4f | %7.4f\n",
               i, x, y, k1, k2, k3, k4, kavg);
    }

    printf("-------------------------------------------------------------------\n");
    printf("Final y(%.1f) = %.6f", x, y);
```

```
}
int main() {
   runge_kutta_4th_order_solve();
   getch();
   return 0;
}
```

Code:



Conclusion: The program was executed successfully.

Solution Of Simultaneous Algebraic Equations

Q.1) Gauss Elimination Method.

Aim: To write a C program to find the solution of **simultaneous algebraic equations** using the **Gauss Elimination Method**.

Formula:

For a system of 3 equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

Row transformation used:

$$R_j \rightarrow R_j - (\frac{a_{ji}}{a_{ii}})R_i$$

Programming formula:

ratio = a[j][i] / a[i][i];

a[j][k] = a[j][k] - ratio * a[i][k];

Program:

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
int main()
{
   float a[10][11] = {
      {  5, -3,  2,  12 },
      { -4,  6,  1,  -7 },
      {  3,  2, -5,  10 }
   };
   float x[10], ratio, temp;
   int n = 3;
```

```
int i, j, k, max_row;
clrscr();
printf("\nSolving the augmented matrix:\n");
for(i = 0; i < n; i++)
{
   for(j = 0; j <= n; j++)
   {
      printf("%0.0f ", a[i][j]);
   }
   printf("\n");
}
for(i = 0; i < n; i++)
{
   max_row = i;
   for(k = i + 1; k < n; k++)
   {
      if(fabs(a[k][i]) > fabs(a[max_row][i]))
         max_row = k;
   }
   for(k = 0; k <= n; k++)
   {
      temp = a[i][k];
      a[i][k] = a[max_row][k];
      a[max_row][k] = temp;
   }
   if(a[i][i] == 0.0)
   {
      printf("\nMathematical Error: No unique solution\n");
      getch();
      return 0;
```

```
        }
        for(j = 0; j < n; j++)
        {
           if(i != j)
           {
              ratio = a[j][i] / a[i][i];
              for(k = 0; k <= n; k++)
              {
                 a[j][k] = a[j][k] - ratio * a[i][k];
              }
           }
        }
     }
     for(i = 0; i < n; i++)
     {
        x[i] = a[i][n] / a[i][i];
     }
     printf("\nThe solution is:\n");
     for(i = 0; i < n; i++)
     {
        printf("x%d = %0.3f\n", i+1, x[i]);
     }

     getch();
     return 0;
}
```

Output:

Conclusion: The program was executed successfully.

Numerical solutions of first and second order differential equation

Q.1) Euler's Method in C Programming.

Aim: To solve the differential equation $y' = x + y$ using Standard Euler's Method and compute the approximate values of $y$for a given step size.

Formula:

$$yn+1=yn+h\cdot f(xn,yn)$$

Programming Formula:

```
slope = f(x, y);
y = y + STEP_SIZE * slope;
x = x + STEP_SIZE;
```

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
double f(double x, double y) {
   return x + y;
}
void eulers_method_solve() {
   const double X_INITIAL = 0.0;
   const double Y_INITIAL = 1.0;
   const double STEP_SIZE = 0.1;
   const int N_STEPS = 5;
   double x = X_INITIAL;
   double y = Y_INITIAL;
   double slope;
   int i;
clrscr();
   printf("--- Standard Euler's Method (y' = x + y) ---\n");
   printf("Initial Condition: y(%.1f) = %.1f, Step Size h = %.1f\n", X_INITIAL, Y_INITIAL,
STEP_SIZE);
```

```
    printf("Step |      x      |      y      |    f(x, y)\n");
    printf("-----------------------------------------------\n");
    printf(" 0   | %10.6f | %10.6f |\n", x, y);
    for (i = 1; i <= N_STEPS; i++) {
        slope = f(x, y);
        y = y + STEP_SIZE * slope;
        x = x + STEP_SIZE;
        printf(" %d   | %10.6f | %10.6f | %10.6f\n", i, x, y, slope);
    }
    printf("-----------------------------------------------\n");
    printf("Final Approximation at x=%.1f: y = %.6f\n", x, y);
}
int main() {
    eulers_method_solve();
getch();
    return 0;
}
```

Output:

```
--- Standard Euler's Method (y' = x + y) ---
Initial Condition: y(0.0) = 1.0, Step Size h = 0.1
Step |      x      |      y      |    f(x, y)
-----------------------------------------------
 0   |   0.000000 |   1.000000 |
 1   |   0.100000 |   1.100000 |   1.000000
 2   |   0.200000 |   1.220000 |   1.200000
 3   |   0.300000 |   1.362000 |   1.420000
 4   |   0.400000 |   1.528200 |   1.662000
 5   |   0.500000 |   1.721020 |   1.928200
-----------------------------------------------
Final Approximation at x=0.5: y = 1.721020
_
```

Conclusion: The program was executed successfully.

Q.2) Modified Euler's Method in C Programming

Aim: To solve the differential equation $y' = x + y$using the Modified Euler's Method and compute the approximate values of $y$for each step.

Formula:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_p)]$$

$$x_{n+1} = x_n + h$$

Programming Formula:

k1 = f(x, y);

y_predictor = y + H * k1;

k2 = f(x_next, y_predictor);

y = y + (H / 2.0) * (k1 + k2);

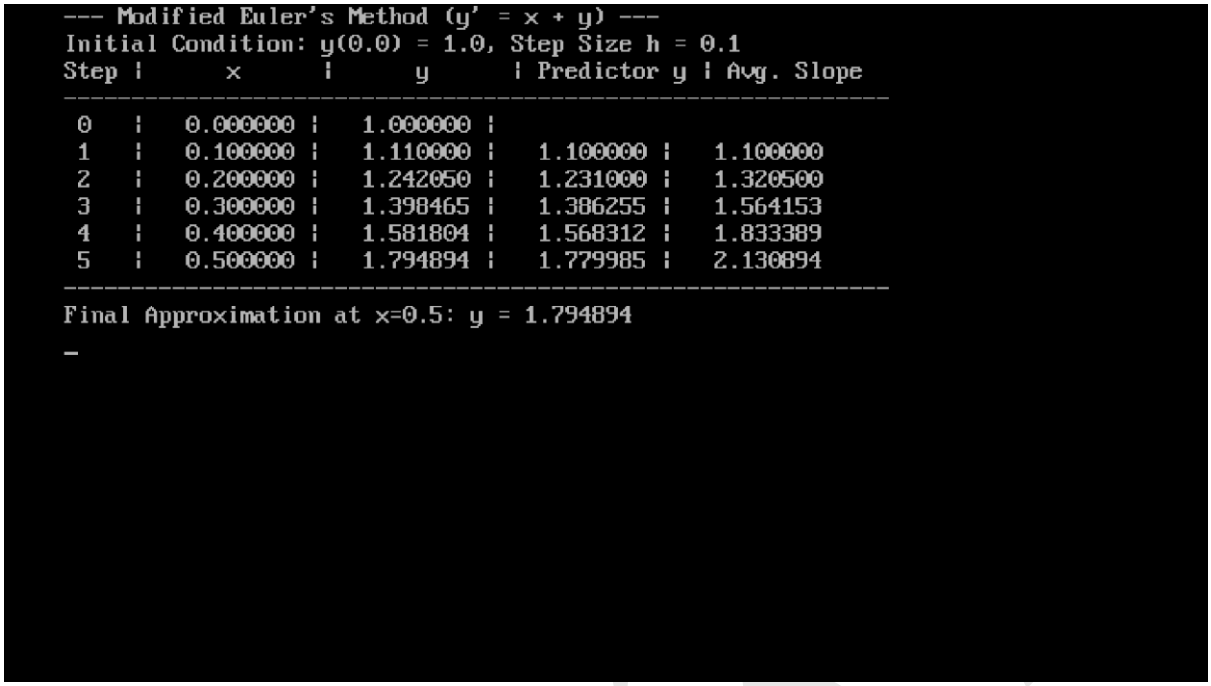
x = x_next;

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
double f(double x, double y) {
   return x + y;
}
void modified_eulers_method_solve() {
   const double X_INITIAL = 0.0;
   const double Y_INITIAL = 1.0;
   const double H = 0.1;
   const int N_STEPS = 5;
   double x = X_INITIAL;
   double y = Y_INITIAL;
```

```
    double x_next;
    double y_predictor;
    double k1, k2;
    int i;
clrscr();
    printf("--- Modified Euler's Method (y' = x + y) ---\n");
    printf("Initial Condition: y(%.1f) = %.1f, Step Size h = %.1f\n", X_INITIAL, Y_INITIAL,
H);
    printf("Step |      x      |      y      | Predictor y | Avg. Slope\n");
    printf("-------------------------------------------------------------\n");
    printf(" 0   | %10.6f | %10.6f |\n", x, y);
    for (i = 1; i <= N_STEPS; i++) {
        k1 = f(x, y);
        x_next = x + H;
        y_predictor = y + H * k1;
        k2 = f(x_next, y_predictor);
        y = y + (H / 2.0) * (k1 + k2);
        x = x_next;
        printf(" %d   | %10.6f | %10.6f | %10.6f | %10.6f\n",
              i, x, y, y_predictor, (k1 + k2) / 2.0);
    }
    printf("-------------------------------------------------------------\n");
    printf("Final Approximation at x=%.1f: y = %.6f\n", x, y);
}
int main() {
    modified_eulers_method_solve();
getch();
    return 0;
}
```

Output:



```
--- Modified Euler's Method (y' = x + y) ---
Initial Condition: y(0.0) = 1.0, Step Size h = 0.1
Step |      x      |      y      | Predictor y | Avg. Slope
-------------------------------------------------------------
 0   |   0.000000 |   1.000000 |
 1   |   0.100000 |   1.110000 |   1.100000 |   1.100000
 2   |   0.200000 |   1.242050 |   1.231000 |   1.320500
 3   |   0.300000 |   1.398465 |   1.386255 |   1.564153
 4   |   0.400000 |   1.581804 |   1.568312 |   1.833389
 5   |   0.500000 |   1.794894 |   1.779985 |   2.130894
-------------------------------------------------------------
Final Approximation at x=0.5: y = 1.794894
_
```

Conclusion: The program was executed successfully.

Q. 3) Taylor Series Method

Aim: To solve the differential equation $y' = x + y$using the Taylor Series Method with given initial conditions.

Formula:

$$y_{i+1} = y_i + h\,y_i' + \frac{h^2}{2}\,y_i''$$
$$y_i' = f(x_i, y_i) = x_i + y_i$$
$$y_i'' = f'(x_i, y_i) = 1 + x_i + y_i$$

Programming Formula:

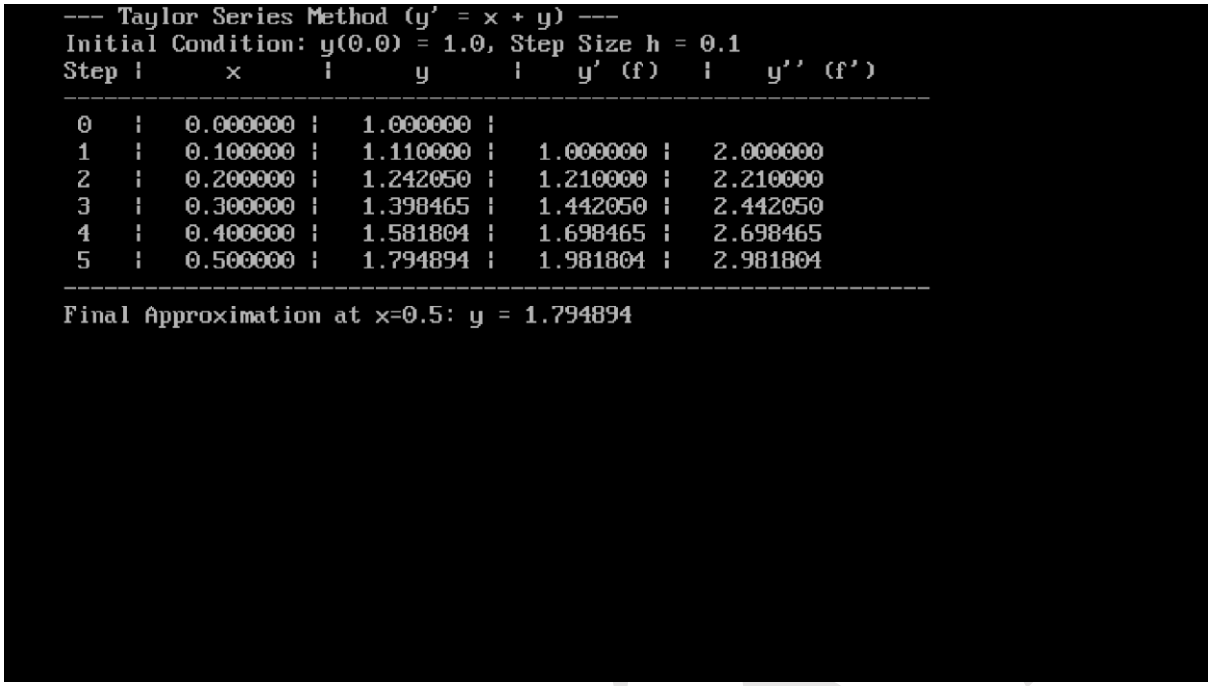
```
y_prime = f(x, y);
y_double_prime = f_prime(x, y);
y = y + h * y_prime + (h * h / 2) * y_double_prime;
```

Program:

```
#include <stdio.h>
#include <math.h>
#include <conio.h>
double f(double x, double y) {
   return x + y;
}
double f_prime(double x, double y) {
   return 1.0 + x + y;
}
void taylor_series_solve() {
   const double X_INITIAL = 0.0;
   const double Y_INITIAL = 1.0;
   const double H = 0.1;
   const int N_STEPS = 5;
   double x = X_INITIAL;
   double y = Y_INITIAL;
   double y_prime;
```

```
    double y_double_prime;
    int i;
clrscr();
    printf("--- Taylor Series Method (y' = x + y) ---\n");
    printf("Initial Condition: y(%.1f) = %.1f, Step Size h = %.1f\n", X_INITIAL, Y_INITIAL,
H);
    printf("Step |      x      |      y      |    y' (f)   |    y'' (f') \n");
    printf("-----------------------------------------------------------------\n");
    printf(" 0   | %10.6f | %10.6f | \n", x, y);
    for (i = 1; i <= N_STEPS; i++) {
        y_prime = f(x, y);
        y_double_prime = f_prime(x, y);
        y = y + H * y_prime + (H * H / 2.0) * y_double_prime;
        x = x + H;
        printf(" %d   | %10.6f | %10.6f | %10.6f | %10.6f\n", i, x, y, y_prime, y_double_prime);
    }
    printf("-----------------------------------------------------------------\n");
    printf("Final Approximation at x=%.1f: y = %.6f\n", x, y);
}
int main() {
    taylor_series_solve();
    getch();
    return 0;
}
```

Output:



```
--- Taylor Series Method (y' = x + y) ---
Initial Condition: y(0.0) = 1.0, Step Size h = 0.1
Step |      x      |      y      |    y' (f)   |    y'' (f')
-------------------------------------------------------------
 0   |   0.000000 |   1.000000 |
 1   |   0.100000 |   1.110000 |   1.000000 |   2.000000
 2   |   0.200000 |   1.242050 |   1.210000 |   2.210000
 3   |   0.300000 |   1.398465 |   1.442050 |   2.442050
 4   |   0.400000 |   1.581804 |   1.698465 |   2.698465
 5   |   0.500000 |   1.794894 |   1.981804 |   2.981804
-------------------------------------------------------------
Final Approximation at x=0.5: y = 1.794894
```

Conclusion: The program was executed successfully.

Transportation problem

Q.1) Transportation Problem using the **Least Cost Method**.

Aim:

To write a C program to find the **initial feasible solution** of a **Transportation Problem** using the **Least Cost Method**, and to compute the **minimum transportation cost**.

Formula:

1. Least Cost Selection Rule

Choose the cell with:

$$\text{Minimum Cost} = \min(c_{ij})$$

Allocation Rule

$$x_{ij} = \min(\text{Supply}_i, \text{Demand}_j)$$

Update Supply & Demand

$$\text{Supply}_i = \text{Supply}_i - x_{ij}$$
$$\text{Demand}_j = \text{Demand}_j - x_{ij}$$

Total Transportation Cost

$$\text{Total Cost} = \sum(x_{ij} \times c_{ij})$$

Programming Formula:

min_cost = temp_cost[i][j];

alloc = (supply_left[min_i] < demand_left[min_j]) ? supply_left[min_i] : demand_left[min_j];

supply_left[min_i] -= alloc;

demand_left[min_j] -= alloc;

total_cost += alloc * cost[min_i][min_j];

Program:

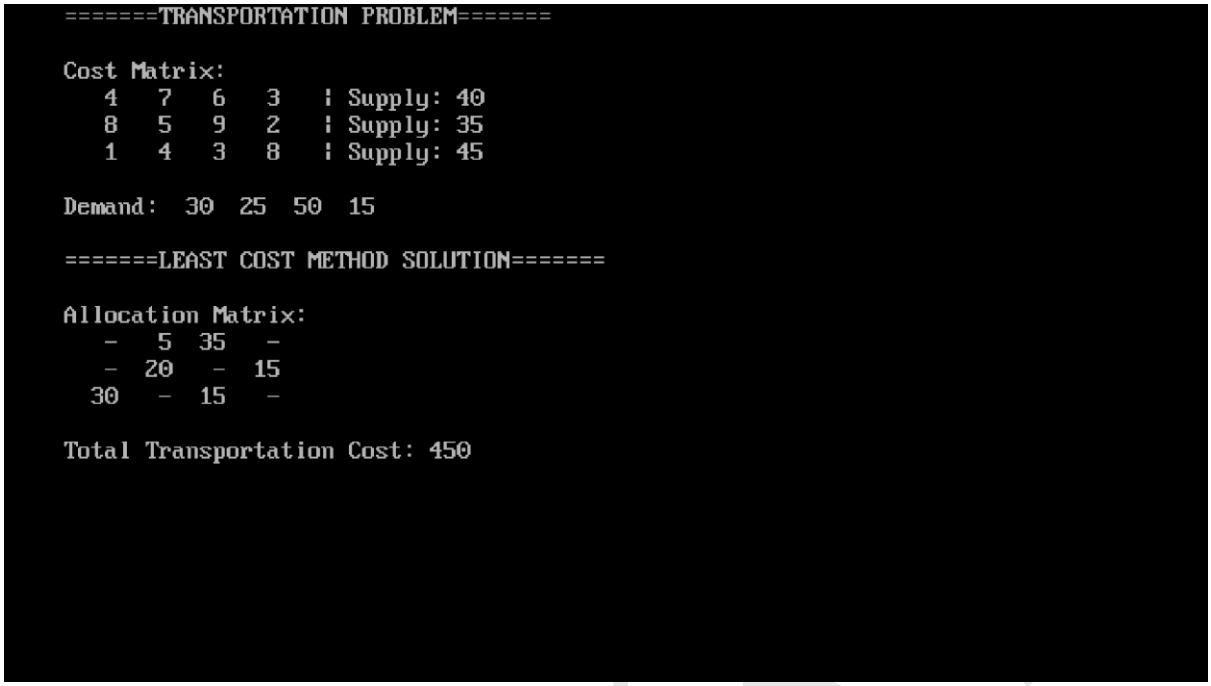
```
#include <stdio.h>

#include <conio.h>

int main() {

   int cost[3][4] = {
```

```
    { 4, 7, 6, 3 },
    { 8, 5, 9, 2 },
    { 1, 4, 3, 8 }
};
int supply[3] = { 40, 35, 45 };
int demand[4] = { 30, 25, 50, 15 };
int allocation[3][4];
int i, j;
int min_cost, min_i, min_j, alloc;
long total_cost = 0;
int items_to_alloc = 0;
int supply_left[3];
int demand_left[4];
int temp_cost[3][4];
clrscr();
printf("=======TRANSPORTATION PROBLEM========\n\n");
printf("Cost Matrix:\n");
for (i = 0; i < 3; i++) {
    for (j = 0; j < 4; j++) {
        printf("%4d", cost[i][j]);
    }
    printf("   | Supply: %d\n", supply[i]);
}
printf("\nDemand:");
for (j = 0; j < 4; j++) {
    printf("%4d", demand[j]);
}
printf("\n\n");
for (i = 0; i < 3; i++) {
    supply_left[i] = supply[i];
```

```
    items_to_alloc += supply[i];
    for (j = 0; j < 4; j++) {
        allocation[i][j] = 0;
        temp_cost[i][j] = cost[i][j];
    }
}
for (j = 0; j < 4; j++) {
    demand_left[j] = demand[j];
}
while (items_to_alloc > 0) {
    min_cost = 9999;
    min_i = -1;
    min_j = -1;
    for (i = 0; i < 3; i++) {
        for (j = 0; j < 4; j++) {
            if (temp_cost[i][j] < min_cost) {
                min_cost = temp_cost[i][j];
                min_i = i;
                min_j = j;
            }
        }
    }
    if (supply_left[min_i] < demand_left[min_j]) {
        alloc = supply_left[min_i];
    } else {
        alloc = demand_left[min_j];
    }
    allocation[min_i][min_j] = alloc;
    total_cost += (long)alloc * cost[min_i][min_j];
    items_to_alloc -= alloc;
```

```
        supply_left[min_i] -= alloc;
        demand_left[min_j] -= alloc;
        if (supply_left[min_i] == 0) {
            for (j = 0; j < 4; j++) {
                temp_cost[min_i][j] = 9999;
            }
        }
        if (demand_left[min_j] == 0) {
            for (i = 0; i < 3; i++) {
                temp_cost[i][min_j] = 9999;
            }
        }
    }
    printf("=======LEAST COST METHOD SOLUTION=======\n\n");
    printf("Allocation Matrix:\n");
    for (i = 0; i < 3; i++) {
        for (j = 0; j < 4; j++) {
            if (allocation[i][j] > 0) {
                printf("%4d", allocation[i][j]);
            } else {
                printf("%4s", "-");
            }
        }
        printf("\n");
    }
    printf("\nTotal Transportation Cost: %ld", total_cost);
    getch();
return 0;
}
```

Output:



Conclusion: The program was executed successfully.