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Math-III

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IOE Engineering mathematics third semester (B.E.) very important and easy in overall mathematics of III semester. The main aim to prepare for 32 marks conform.

Tips and tricks Math-III 32 mark Conform

Cayley Hamilton: *Every square matrix satisfy its own characteristic equation*

For verification [5 mark]

- ⇨ Let given matrix be A
- ⇨ The characteristic equation is $|\boldsymbol{A-\lambda I}| = 0$
- ⇨ Expand the matrix you will get $(\lambda^3 + \lambda^2 + 2\lambda + 2) = 0 - - - - - - - - (1)$
- ⇨ By using Cayle Hamilton theorem $A^3 + A^2 + 2A + 2I = 0$; $(replace\ \lambda \rightarrow A\ and\ multiply\ \boldsymbol{I}\ with\ constant)$
- ⇨ find $A^3\ and\ A^2$ and put in above (1)equation and obtain RHS=0

#For A^{-1}

- ⇨ Multiply the expression $(A^3 + A^2 + 2A + 22I)$ by A^{-1}
- ⇨ and write $A^{-1} = (remaining\ expression)$ & solve it you will get A^{-1}

#To find MODEL Matrix

- ⇨ obtain Eigen vectors :- $a_1(x_1\ x_2\ )$ & $a_2(x_1\ x_2\ )$
- ⇨ so required model matrix is :- $C = \begin{bmatrix} x_1 & x_1 \\ x_2 & x_2 \end{bmatrix}$(*Arrange Eigen vectors elements column by column*)
- ⇨ For Diagonalization(D) of matrix A:- $D = C^{-1}AC$

#Eigen Vector and Eigen value [5 mark]

- ⇨ Let given matrix be A
- ⇨ The characteristic equation is $|\boldsymbol{A-\lambda I}| = 0$
  $expand\ and\ keep\ in\ factor\ to\ get\ value\ of\ \lambda\ = 1{,}2{,}3(factorial\ form)\lambda\ = 1{,}2{,}3\ are\ eigen\ values$
- ⇨ Let $x = (x_1\ x_2\ x_3)$ be the Eigen vector corresponding to Eigen value
- ⇨ the characteristic matrix is $(A - \lambda I)X = 0$
  - Obtain the equations: ($This\ is\ homogenous\ equation\ so\ for\ non\ trivalent\ solution\ (A - \lambda I) = 0\ which\ is\ characteristic\ equation\ so\ value\ of\ \boldsymbol{\lambda}\ must\ satisfy\ above\ equations.$
  - find the value of $x_1\ x_2\ x_3$ for when λ=1
  - find the value of $x_1\ x_2\ x_3$ for when λ=2
  - find the value of $x_1\ x_2\ x_3$ for when λ=3

  $for\ every\ value\ of\ x_1\ x_2\ x_3\ and\ \boldsymbol{\lambda}\ is\ eigen\ vector\ so\ multiple\ of\ this\ vector\ is\ also\ eigen\ vector\ so\ a_1(x_1\ x_2\ x_3)\ \ a_1! = 0\ is\ also\ eigen\ vector.$

  Arrange column by column values of Eigen vector to get $C -$vector which is model matrix and for diagninization find $C^{-1}AC$.

#Rank by minor of order[5 mark]

A matrix 'A' is said to have rank 'r' if it possess at least one non zero minor of order 'r' and all the minor of order 'r+1' is zero.

- ⇨ 3X3 को Matrix छ भने Determinant of 3X3 को हेर्ने zero हुन्छ किहुदैन।
- ⇨ यदि zero हुन्छ भने त्यो हुदैन ।
  Next Step
- ⇨ 2X2 को Matrix लाई check गर्ने । (सधै square matrix को check गर्ने -making several combination)
- ⇨ and so on...
- ⇨ $! = 0$ gives the order of rank (i.e. rank of matrix)

#Rank by triangular and echelon form[5 mark]

To find the rank by triangular form reduce the given matrix into upper triangular form by row or column operation. so the rank of matrix is equal to number of non zero row of triangular matrix.

$$\begin{bmatrix} a_{11} & & \\ 0 & a_{22} & \\ 0 & 0 & a_{33} \end{bmatrix}$$

- make a_{11} into 1 by row /column operation.
- make reaming element below a_{11} into zero with a_{11} by row operation.
- make $a_{22} = 1$
- and make below element zero by a_{22} & repeat same process
  (*when element below diagonal is zero than it is upper triangular & number of row element different than it is echelon form.*)
  बनाउने क्रम्मा exchange गर्नें भएन भने मात्र divide गर्नें ।

#Rank of matrix by normal and canonical form[5 mark]

The matrix in the form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ for eg: $-\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ where I_r is unit matrix of order r in normal canonical form of matrix.

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

2nd

1st make this zero

- make $a_{11} \to 1$ by row/column operation
- make a_{11} below element zero by row operation
- make a_{11} after element i.e. row elements(R_1) zero by column operation.
- make $a_{22} = 1$ & by the help of this element make below element zero by row operation
- make R_2 element after a_{22} zero by column operation..

  (repeat same process until reached to $[\mathbf{I}]$ matrix $I_r \to r =$ rank of marix

  बनाउने क्रम्मा exchange गर्नें भएन भने मात्र divide गर्नें ।

#Test of Consistency[5 mark]

$[A: B] \to$ Reduce it in upper triangular and echelon form by only row operation

- Case 1: if $\rho[\mathbf{A}:\mathbf{B}] = \rho[\mathbf{A}]$ =number of unknown(**x,y,z**) then solution of eq.1 is consistent and have unique finite solution.
- Case 2: if $\rho[\mathbf{A}:\mathbf{B}] != \rho[\mathbf{A}]$ then solution of eq.1 is **inconsistent** and has no solution.
- Case 3: if $\rho[\mathbf{A}:\mathbf{B}] = \rho[\mathbf{A}] <$ number of unknown(**x,y,z**) then solution of eq.1 is consistent and have infinite number of solution.(write equation in as function of available variable as x=f(z),y=f(z);z=z)

*For Homogenous equation (always **consistent**)*

- Case 1: if $\rho[\mathbf{A}]$ =number of unknown then ***consistent*** and have unique finite solution(x=y=z=0)
- Case 2: if $\rho[\mathbf{A}]$ < of unknown then ***consistent*** and have infinite number of solution

#Procedure of simplex method[10 mark]

- select the column with highest positive value
- If the highest positive value is repeated than select anyone.

- ⇨ To determine which variable will be replaced divide the value in the P_o column by corresponding coefficient in the optimum column i.e. $Q_o = \frac{P_o}{x_{o1}}$
- ⇨ select the row with the smallest positive ratio
- ⇨ If the smallest ratio is repeated choose anyone.
- ⇨ Computation of value for the replacing (new) row.