3rd Semester Python Notes (AG mam)

Integration by Simpson's Rule

$$I = \int_a^b f(x)dx = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

For Simpson's rule, we divide the interval [a,b] into an even number of sub-intervals.

$$I = \frac{h}{3}[y_0 + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2}) + y_n]$$

In [1]:
```
import numpy as np
```

In [2]:
```
def f1(x):
    return 1/(1 + x**2)
a = -10**4  # change it and see the results
b = 10**4   # change it and see the results
n = 10**6   # change it and see the results
```

In [3]:
```
h = float(b-a)/n
x0 = np.arange(a+h,b,2*h)
xe = np.arange(a+2*h,b,2*h)
val = h/3*(f1(a) + 4*sum(f1(x0)) + 2*sum(f1(xe)) + f1(b))
print('value of the integration is ', val)
```

```
value of the integration is  3.1413926535219634
```

Verification

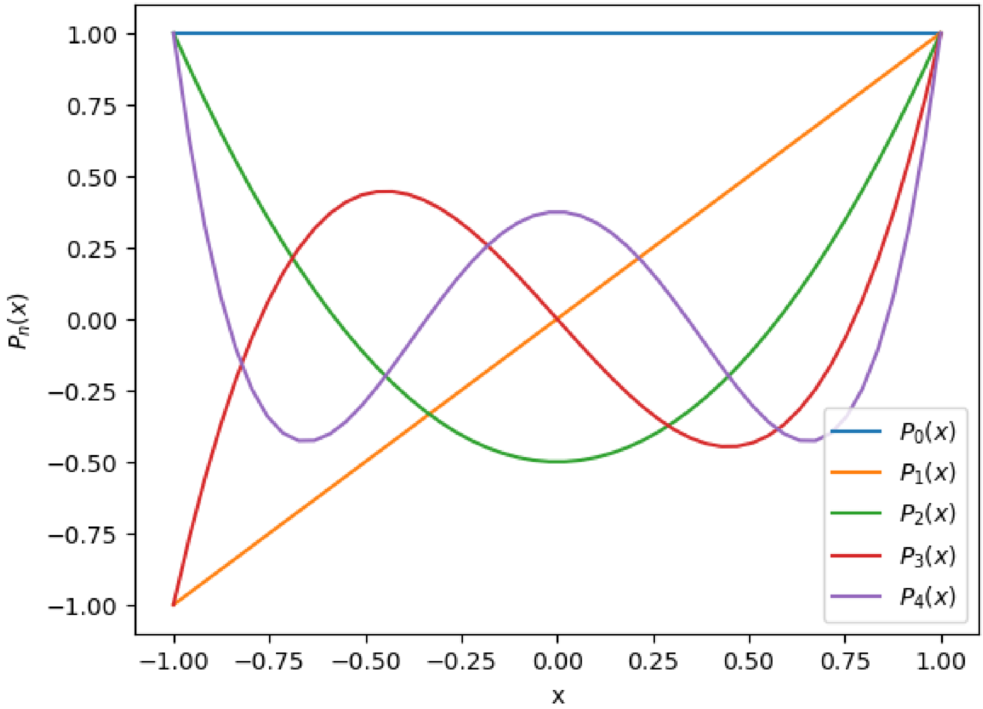
In [4]:
```
import sympy as smp
x = smp.symbols('x')
y = 1/(1 + x**2)
smp.integrate(y, [x,-smp.oo, smp.oo])
```

Out[4]: π

Legendre Polinomials

In [5]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import legendre as P
```

In [6]:
```
x = np.linspace(-1,1,50)
for i in range(5):
    plt.plot(x,P(i)(x))
    plt.legend(['$ P_0(x) $', '$ P_1(x) $', '$ P_2(x) $', '$ P_3(x) $', '$ P_4(x) $'])
    plt.xlabel('x')
    plt.ylabel('$ P_n(x) $')
```



Orthogonality of Legendre Polynomial

$$\int_{-1}^{+1} P_m(x)P_n(x)dx = \frac{2}{2n+1}\delta_{mn}$$

In [7]:
```
import numpy as np
from scipy.special import legendre
```

Define $P_m(x)P_n(x)$:

In [8]:
```
def pmpn(m,n):
    pm = legendre(m)
    pn = legendre(n)
    return pm*pn
```

Integration by **Simpson's 1/3rd Rule**: We want to perform $\int_a^b f(x)dx$.

In [9]:
```
def simp13x(f,a,b,n):
    h = float(b-a)/n
    x0 = np.arange(a+h,b,2*h)
    xe = np.arange(a+2*h,b,2*h)
    val = h/3*(f2(a) + 4*sum(f2(x0)) + 2*sum(f2(xe)) + f2(b))
    return val
```

Check Orthogonality for different values of m and n.

In [10]:
```
for m in range(4):
    for n in range(4):
        f2 = pmpn(m,n)
        intg = simp13x(f2,-1,1,1000)  # integration

        if m==n:
            dmn = 1
        else:
            dmn = 0
        res = (2/(2*n + 1))* dmn  # result
        print(m, n, '\t', intg, '\t', res)  # compare the values of intg and res
```

```
0 0      2.0     2.0
0 1      1.633064054355297e-15   0.0
0 2      1.6703675479827022e-15          0.0
0 3      1.7248424910576432e-15          0.0
1 0      1.633064054355297e-15   0.0
1 1      0.6666666666666675      0.6666666666666666
1 2      1.6838382540148207e-15          0.0
1 3      1.0668277224586594e-11          0.0
2 0      1.6703675479827022e-15          0.0
2 1      1.6838382540148207e-15          0.0
2 2      0.40000000000960123     0.4
2 3      1.785978772280335e-15   0.0
3 0      1.7248424910576432e-15          0.0
3 1      1.0668277224586594e-11          0.0
3 2      1.785978772280335e-15   0.0
3 3      0.28571428581562036     0.2857142857142857

C:\ProgramData\Anaconda3\lib\site-packages\numpy\lib\polynomial.py:1329: FutureWarning: In
the future extra properties will not be copied across when constructing one poly1d from an
other
  other = poly1d(other)
```

Bessel Functions

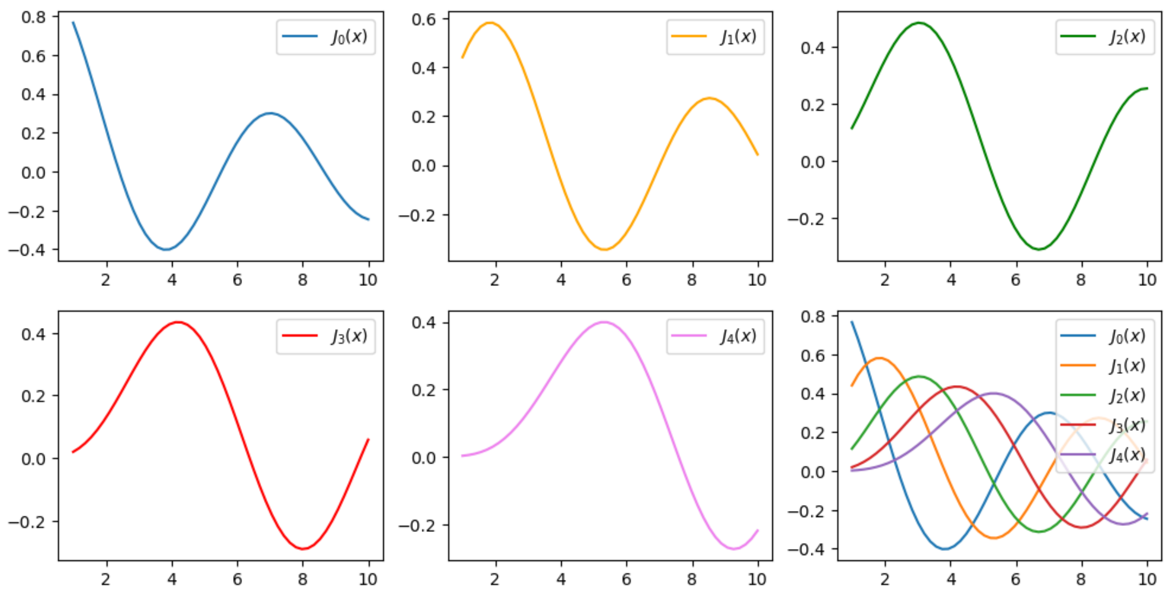
$$x^2y'' + xy' + (x^2 - n^2)y = 0$$

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m+n+1)} (\frac{x}{2})^{(2m+n)}$$

$$J_{-n}(x) = (-1)^n J_n(x)$$

In [11]:
```
import numpy as np
import scipy.special as sps
import matplotlib.pyplot as plt
```

In [12]:
```python
x = np.linspace(1,10,50)
j0 = sps.jn(0,x)
j1 = sps.jn(1,x)
j2 = sps.jn(2,x)
j3 = sps.jn(3,x)
j4 = sps.jn(4,x)
fig, axes = plt.subplots(2,3, figsize=(12,6))
ax1 = axes[0][0]
ax1.plot(x,j0, label='$J_0(x)$')
ax1.legend()
ax2 = axes[0][1]
ax2.plot(x,j1, 'orange', label='$J_1(x)$')
ax2.legend()
ax3 = axes[0][2]
ax3.plot(x,j2, 'green', label='$J_2(x)$')
ax3.legend()
ax4 = axes[1][0]
ax4.plot(x,j3, 'red', label='$J_3(x)$')
ax4.legend()
ax5 = axes[1][1]
ax5.plot(x,j4, 'violet', label='$J_4(x)$')
ax5.legend()
ax6 = axes[1][2]
ax6.plot(x,j0, label='$J_0(x)$')
ax6.plot(x,j1, label='$J_1(x)$')
ax6.plot(x,j2, label='$J_2(x)$')
ax6.plot(x,j3, label='$J_3(x)$')
ax6.plot(x,j4, label='$J_4(x)$')
ax6.legend()
plt.show()
```



$$\sin(x) = 2\sum_{n=0}^{\infty} J_{2n+1}(x)$$

In [13]:
```python
x = np.pi/3
n = 20
s = 0
for i in range(n):
    s += ((-1)**i)*(sps.jn(2*i+1, x))
s = 2*s
print(s)
```

```
0.8660254037844388
```

Square Root of Complex Numbers

$$\sqrt{x+iy} = \sqrt{r}\cos(\theta/2) + i\sqrt{r}\sin(\theta/2)$$

where $r = \sqrt{x^2+y^2}$ and $\tan(\theta) = \frac{y}{x}$

In [14]:
```
import numpy as np
```

Put values of x and y:

In [15]:
```
x = 3
y = 4
```

In [16]:
```
a = float(x)
b = float(y)
z2 = a**2 + b**2
r = (z2)**0.5
tn1 = np.arctan(b/a)
tn2 = tn1/2
rtr = (r)**0.5
sn = np.sin(tn2)
cs = np.cos(tn2)
```

In [17]:
```
rl = rtr * cs   # real part
img = rtr * sn  # imaginary part
print(rl, '+ i', img)
```

```
2.0 + i 1.0
```

n^{th} Root of Unity

$$x^n = 1 = e^{i(2\pi k)}$$

$$x = e^{i(\frac{2\pi k}{n})} = \cos(\frac{2\pi k}{n}) + i\sin(\frac{2\pi k}{n})$$

In [18]:
```
import numpy as np
```

In [19]:
```
def croot(k,n):
    if n<=0:
        return None
    return np.exp((2*np.pi*1j*k)/n)
n = 4   # put an integer
for k in range(n):
    print(croot(k,n))
```

```
(1+0j)
(6.123233995736766e-17+1j)
(-1+1.2246467991473532e-16j)
(-1.8369701987210297e-16-1j)
```

Least Square Method

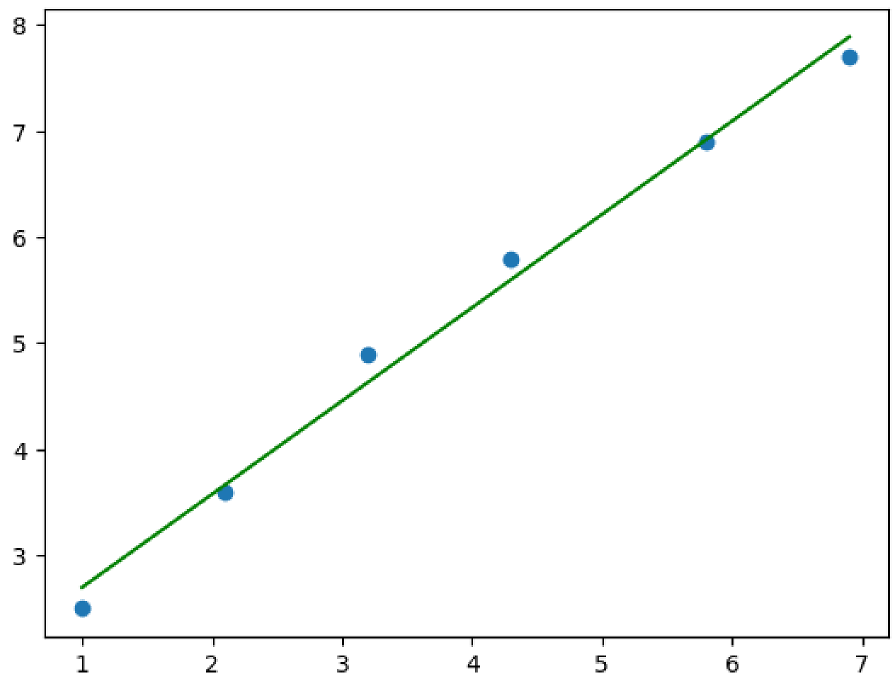
We want to obtain a straight line $y = a_0 + a_1 x$ from a given datset of points (x_i, y_i).

$$a_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{S_{xy}}{S_{xx}}$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

In [20]:
```
x = [1, 2.1, 3.2, 4.3, 5.8, 6.9]
y = [2.5, 3.6, 4.9, 5.8, 6.9, 7.7]
yfit = []
n = len(x)
xav = sum(x)/n
yav = sum(y)/n
Sxy = sum((i-xav)*(j-yav) for i,j in zip(x,y))
Sxx = sum((i-xav)**2 for i in x)
a1 = Sxy/Sxx
a0 = yav - a1*xav
yfit = [a0 + a1*i for i in x]
plt.scatter(x,y)
plt.plot(x, yfit, color='green')
```

Out[20]:
```
[<matplotlib.lines.Line2D at 0x1609174d5b0>]
```



Gauss Elimination

To solve system of linear equations we use Gauss Elimination.

$$AX = B$$

Here, we make the *Echelon* form of the matrix A. After that, we can solve the equations by *Back Substitution*. In this we make the matrix A a identity matrix in $[A|B]$ by *Row Operations* and get the solution as matrix B after the whole process.

In [21]:
```
def GaussElim(A):
    n = len(A)
    a = [[A[i][j] for j in range(len(A[0]))] for i in range(n)]
    for i in range(n-1):
        mxa = a[i][i]
        m = i
        for j in range(i+1,n):
            if abs(a[j][i])>mxa:
                mxa = abs(a[j][i])  # finding the maximum element
                m = j
        ta = a[i]
        a[i] = a[m]
        a[m] = ta

        for j in range(i+1,n):
            cf = a[j][i]/a[i][i]
            for k in range(n+1):
                a[j][k] = a[j][k] - cf*a[i][k]

# Back substitution
    X = [0.0 for i in range(n)]
    X[n-1] = a[n-1][n]/a[n-1][n-1]
    for i in range(n-2,-1,-1):
        sm = 0.0
        for j in range(i+1,n):
            sm += a[i][j]*X[j]
        X[i] = 1.0/a[i][i]*(a[i][n]-sm)
    return X
```

Put the matrix $[A|B]$.

In [22]:
```
AB = [[-5,16,-4,0],[10,-5,0,12],[0,-4,11,0]]
X = GaussElim(AB)
print('X =', X)
```

```
X = [1.449056603773585, 0.49811320754716976, 0.1811320754716981]
```

Question: Solve the equations:

$$3x + 2y + 4z = 7$$

$$2x + y + z = 4$$

$$x + 3y + 5z = 2$$

Solution: First, define the Gauss Elimination function. Then, write the matrix $[A|B]$.

In [23]:
```
AB = [[3,2,4,7],[2,1,1,4],[1,3,5,2]]
X = GaussElim(AB)
print ('[x y z] =', X)
```

```
[x y z] = [2.25, -1.1249999999999, 0.6249999999999994]
```

Verification

In [24]:
```
import numpy as np
A = np.array([[3,2,4],[2,1,1],[1,3,5]])
B = np.array([7,4,2])
np.linalg.solve(A,B)
```

Out[24]:
```
array([ 2.25 , -1.125,  0.625])
```

Alternate?

In [25]:
```
def GElim(A):
    n = len(A)
    a = [[A[i][j] for j in range(len(A[0]))] for i in range(n)]
    for i in range(n):
        for j in range(i+1,n):
            c = a[j][i]/a[i][i]
            for k in range(n+1):
                a[j][k] = a[j][k] - c*a[i][k]

# Back substitution
    X = [0.0 for i in range(n)]
    X[n-1] = a[n-1][n]/a[n-1][n-1]
    for i in range(n-2,-1,-1):
        sm = 0.0
        for j in range(i+1,n):
            sm += a[i][j]*X[j]
        X[i] = 1.0/a[i][i]*(a[i][n]-sm)
    return X
```

In [26]:
```
AB = [[3,2,4,7],[2,1,1,4],[1,3,5,2]]
X = GElim(AB)
print ('[x y z] =', X)
```

```
[x y z] = [2.25, -1.1249999999999996, 0.6249999999999996]
```

In [27]:
```
AB = [[-5,16,-4,0],[10,-5,0,12],[0,-4,11,0]]
X = GElim(AB)
print('X =', X)
```

```
X = [1.4490566037735848, 0.49811320754716976, 0.1811320754716981]
```

In [ ]:
```
```

Inverse of a Matrix by Gauss Elimination Method

Here by Gauss Elimination we convert $[A|I]$ to $[I|A']$ where $A' = A^{-1}$.

Create the function:

In [28]:
```
def InvGaussElim(A):
    n = len(A)
    a = [[A[i][j] for j in range(n)] for i in range(n)]
    b = [[1.0 if i==j else 0.0 for j in range(n)] for i in range(n)]  # identity matrix
    for i in range(n):
        for j in range(n):
            if j!=i:    # operations on non-diagonal terms
                r = a[j][i]/a[i][i]
                for k in range(n):
                    a[j][k] = a[j][k] - r*a[i][k]
                    b[j][k] = b[j][k] - r*b[i][k]
    for i in range(n):
        for j in range(n):
            b[i][j] = b[i][j]/a[i][i]
    return b
```

Write the Matrix:

In [29]:
```python
A = [[2,5,6,8],[7,10,5,4],[1,3,4,8],[4,8,9,12]]
invA = InvGaussElim(A)
print(" \t Inverse of the Matrix A ")
for i in invA:
    for ii in i:
        print("%0.3f,"%ii,end=" ")   # all ii in same line
    print("")
```

```
         Inverse of the Matrix A 
-2.500, -0.167, -0.167, 1.833, 
2.000, 0.333, 0.333, -1.667, 
-0.250, -0.250, -0.750, 0.750, 
-0.312, 0.021, 0.396, 0.021, 
```

Verification

In [30]:
```python
import sympy as smp
A2 = smp.Matrix([[2,5,6,8],[7,10,5,4],[1,3,4,8],[4,8,9,12]])
A2**(-1)
```

Out[30]:

$$\begin{bmatrix} -\frac{5}{2} & -\frac{1}{6} & -\frac{1}{6} & \frac{11}{6} \\ 2 & \frac{1}{3} & \frac{1}{3} & -\frac{5}{3} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{3}{4} & \frac{3}{4} \\ -\frac{5}{16} & \frac{1}{48} & \frac{19}{48} & \frac{1}{48} \end{bmatrix}$$

In [ ]: