Discrete Fourier Transform (DFT)

Discrete Fourier Transform (DFT):

$$F_m = \sum_{n=0}^{N-1} f_n e^{-2\pi jmn/N} \qquad ; (m = 0, 1, 2, 3, \ldots, N-1)$$

Discrete Inverse Fourier Transform (DIFT):

$$f_n = \sum_{m=0}^{N-1} F_m e^{2\pi jmn/N} \qquad ; (n = 0, 1, 2, 3, \ldots, N-1)$$

Algorithm:

1. Get the value of N.
2. Get f_n for $n = 0, 1, 2, \ldots, N-1$.
3. for $m = 0, 1, 2, \ldots, N-1:$ $\quad F_m \leftarrow 0$
4. for m in range(N): for n in range(N): $\quad F_m \leftarrow F_m + f_n e^{-2\pi jmn/N}$

In [1]:
```
import numpy as np
```

Defining **`dft`** function for both DFT and DIFT:

In [2]:
```
def dft(ft, isg):
    N = len(ft)
    Fs = []
    for m in range(N):
        Fk = 0
        for n in range(N):
            Fk += ft[n] * np.exp(-isg*2*np.pi*1j*m*n/N)
        if isg == 1:
            Fs.append(Fk)
        elif isg == -1:
            Fs.append(Fk/N)
    return Fs
```

Defining **`cntdft`** function for getting centered or two-sided transform instead of one-sided transform:

In [3]:
```
def cntdft(ft, isg):
    N = len(ft)
    a = (N-1)/2
    exft = [ft[i]*np.exp(2*np.pi*1j*a*i/N) for i in range(N)] # pre-transform
    Fs = dft(exft, isg)
    Fs = [Fs[i]*np.exp(2*np.pi*1j*a*(i-a)/N) for i in range(N)] # post-transform
    return Fs
```

Defining **`fourspc`** function for Fourier space co-ordinates:

$$\delta x \delta k = \frac{2\pi}{N} \; ; \quad k_{mx} = \left(1 - \frac{1}{N}\right) \frac{\pi}{\delta x}$$

In [4]:
```
def fourspc(x):
    N = len(x)
    dx = x[1]-x[0]
    dk = 2*np.pi/(N*dx)
    kmx = (1 -1/N)*np.pi/dx
    k = [-kmx + i*dk for i in range(N)]
    return k
```

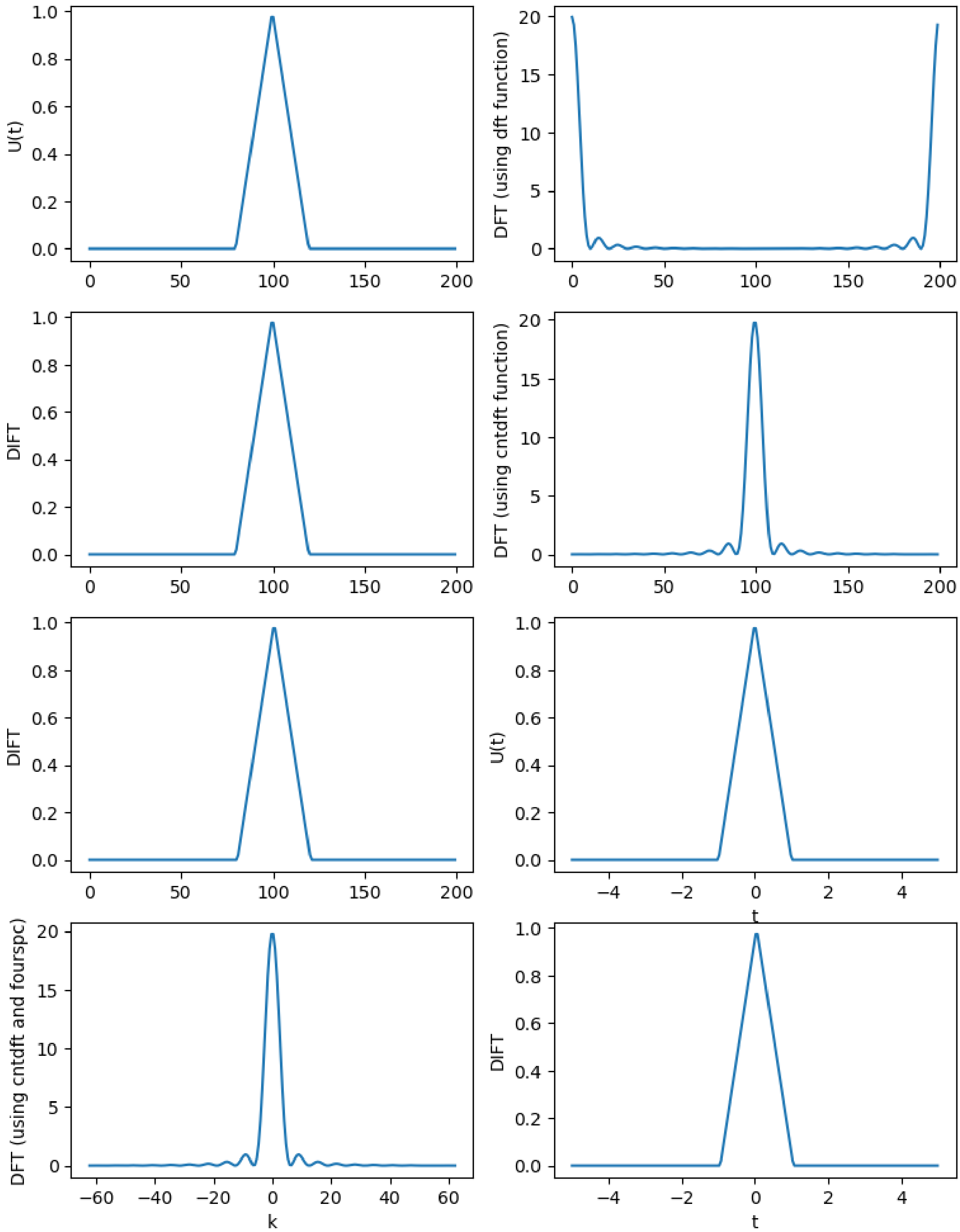
Example

Example 1

$$\begin{aligned} U(t) &= 1+t \quad ;(-1 \leq t < 0) \\ &= 1-t \quad ;(0 \leq t < 1) \\ &= 0 \quad otherwise \end{aligned}$$

In [5]:

```
import matplotlib.pyplot as plt

def u(t):
    if -1<=t<0:
        ut = 1+t
    elif 0<=t<1:
        ut = 1-t
    else:
        ut = 0
    return ut

tmn, tmx = -5, 5  # time bounds
N = 200    # no. of samples
dt = (tmx-tmn)/(N-1)

t = [tmn + i*dt for i in range(N)] # time samples
k = fourspc(t)
ut = [u(tt) for tt in t] # discrete signal

Fs1 = dft(ut, 1) # DFT
IFs1 = dft(Fs1, -1) # DIFT
Fs2 = cntdft(ut, 1)
IFs2 = cntdft(Fs2, -1)

plt.figure(figsize=(9,12))
plt.subplot(4,2,1)
plt.plot(ut)
plt.ylabel('U(t)')

plt.subplot(4,2,2)
plt.plot(np.abs(Fs1))
plt.ylabel('DFT (using dft function)')
plt.subplot(4,2,3)
plt.plot(np.abs(IFs1))
plt.ylabel('DIFT')

plt.subplot(4,2,4)
plt.plot(np.abs(Fs2))
plt.ylabel('DFT (using cntdft function)')
plt.subplot(4,2,5)
plt.plot(np.abs(IFs2))
plt.ylabel('DIFT')

plt.subplot(4,2,6)
plt.plot(t, ut)
plt.xlabel('t')
plt.ylabel('U(t)')
plt.subplot(4,2,7)
plt.plot(k, np.abs(Fs2))
plt.xlabel('k')
plt.ylabel('DFT (using cntdft and fourspc)')
plt.subplot(4,2,8)
plt.plot(t, np.abs(IFs2))
plt.xlabel('t')
plt.ylabel('DIFT')
plt.show()
```




```
err1 = np.array(ut) - np.array(IFs)
err2 = np.array(ut) - np.array(np.abs(IFs))
# plt.subplot(211)
plt.plot(err1, label='original value')
# plt.subplot(212)
plt.plot(err2, label='absolute value')
plt.legend()
plt.show()
```

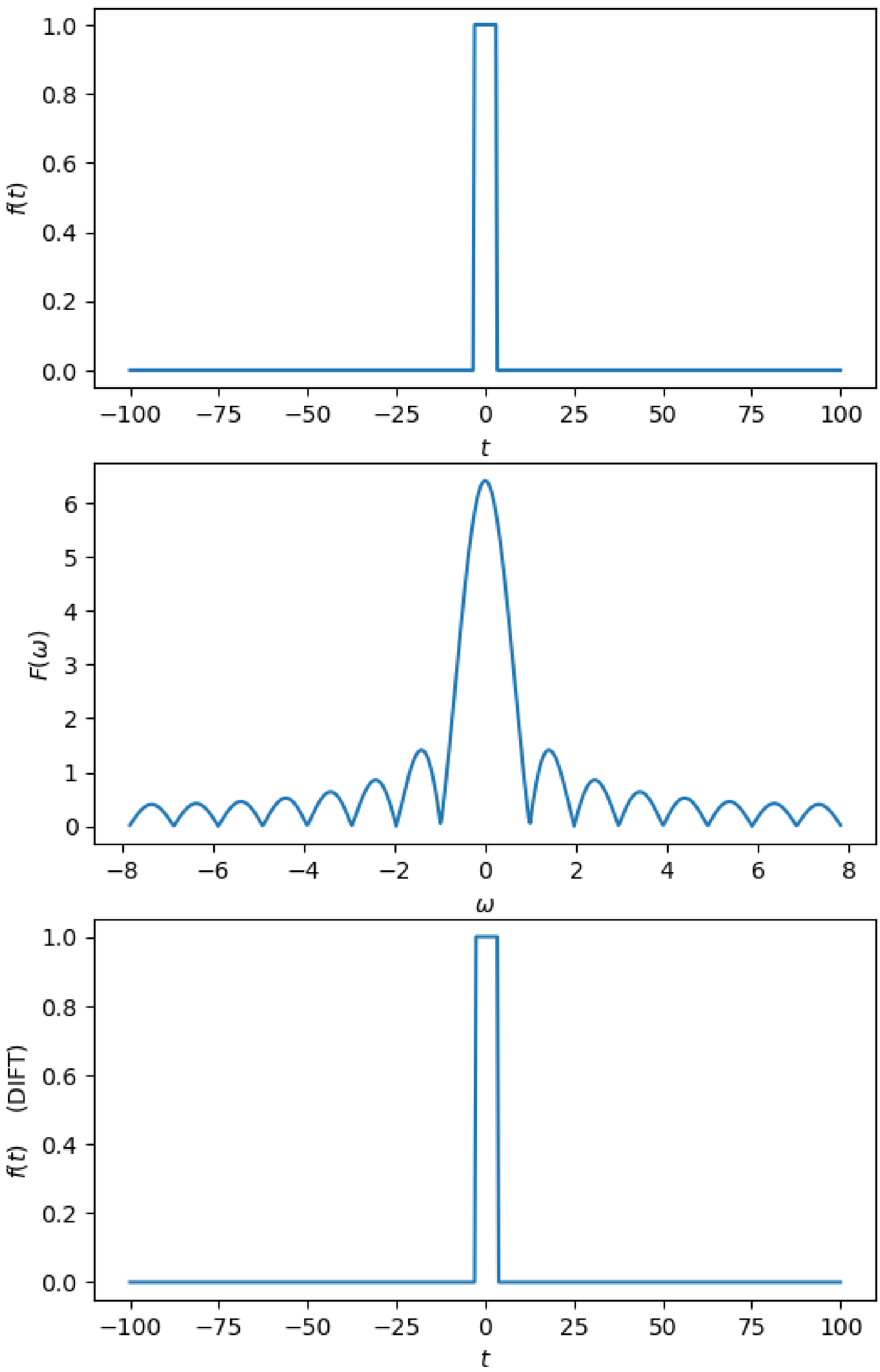
Example 2

Rectangular function:

$$
\begin{aligned}
f(t) &= 1 \quad ;(-\pi \leq t \leq \pi) \\
&= 0 \quad otherwise
\end{aligned}
$$

In [6]:

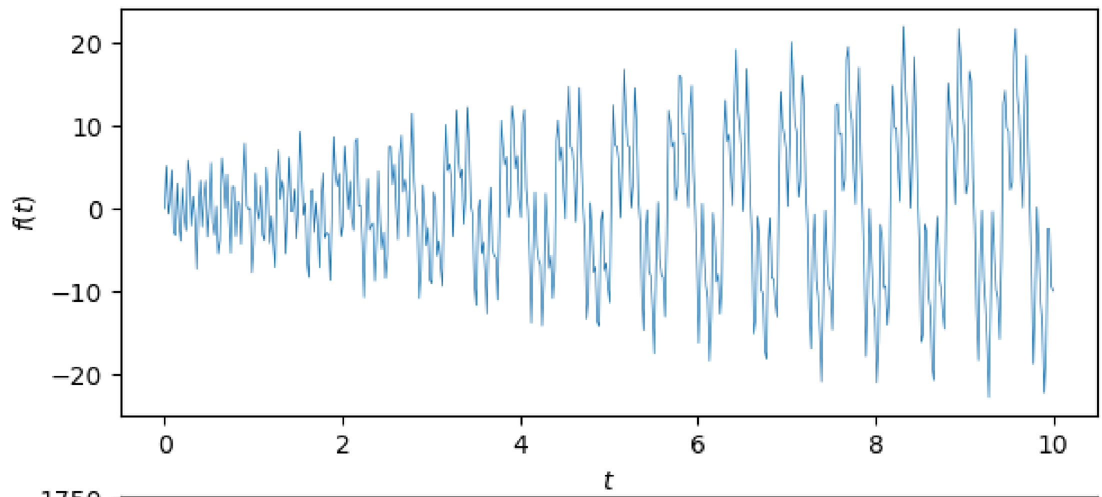
```python
def f(t):
    if -np.pi<=t<=np.pi:
        ft = 1
    else:
        ft = 0
    return ft

N = 500
tmn, tmx = -100, 100
dt = (tmx-tmn)/(N-1)
t = [tmn+i*dt for i in range (N)]
k = fourspc(t)
ft = [f(tt) for tt in t]
Fs = cntdft(ft, 1)
IFs = cntdft(Fs, -1)

plt.figure(figsize=(6, 10))
plt.subplot(3,1,1)
plt.plot(t, ft)
plt.xlabel('$t$')
plt.ylabel('$f(t)$')
plt.subplot(3,1,2)
plt.plot(k, np.abs(Fs)*dt)
plt.xlabel('$\omega$')
plt.ylabel('$F(\omega)$')
plt.subplot(3,1,3)
plt.plot(t, np.abs(IFs))
plt.xlabel('$t$')
plt.ylabel('$f(t) \quad$ (DIFT)')
plt.show()
```

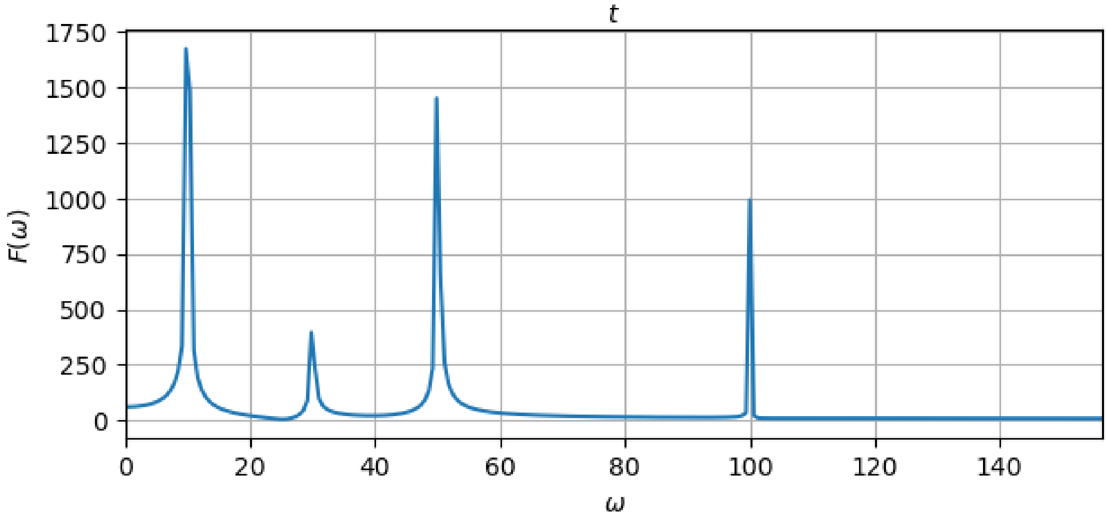


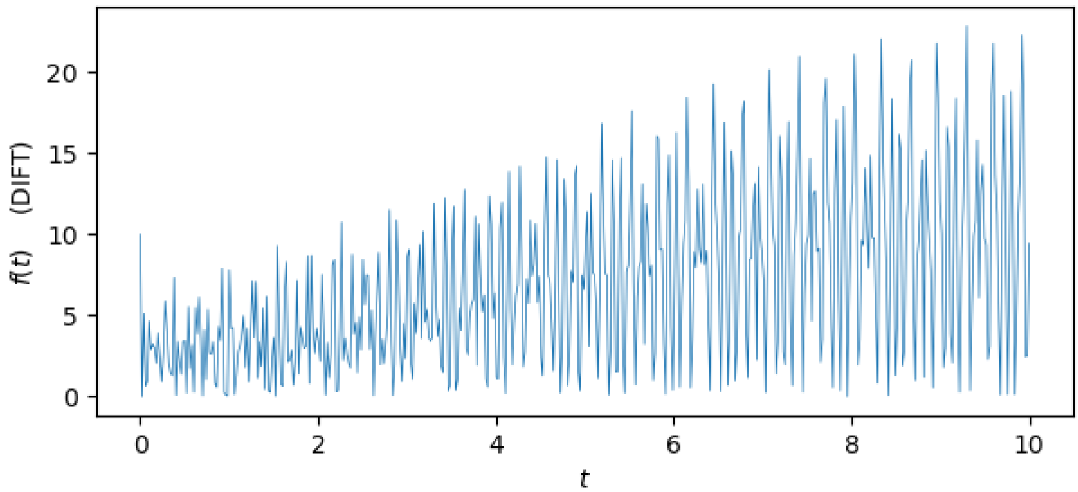
Example 3

Fourier transform of a signal which is superpositions of a number of signals with different frequencies and amplitudes.

In [7]:
```python
def f(t):
    freqs = [10, 30, 50, 100] # frequencies
    amps = [t**2*np.exp(-t/5), 2, 3*t**0.5, 4]  # amplitudes
    phis = [0, 0, 0, 0]  # initial phases
    ft = 0
    for i in range(len(freqs)):
        ft += amps[i]*np.sin(freqs[i]*t +phis[i])
    return ft

N = 500
tmn, tmx = 0, 10
dt = (tmx-tmn)/(N-1)
t = [tmn+i*dt for i in range(N)]
w = fourspc(t)
ft = [f(tt) for tt in t]
Fw = cntdft(ft, 1)
ift = cntdft(Fw, -1)

plt.figure(figsize=(7, 10))
plt.subplot(311)
plt.plot(t, ft, lw=0.5)
plt.xlabel('$t$')
plt.ylabel('$f(t)$')
plt.subplot(312)
plt.plot(w, np.abs(Fw))
plt.xlim(0, np.max(w))
plt.xlabel('$\omega$')
plt.ylabel('$F(\omega)$')
plt.grid()
plt.subplot(313)
plt.plot(t, np.abs(ift), lw=0.5)
plt.xlabel('$t$')
plt.ylabel('$f(t) \quad$ (DIFT)')
plt.show()
```







In [ ]:

Fast Fourier Transform (FFT)

In [8]:
```
import numpy as np
```

In [9]:
```
def fft2(ft, isg):
    N = len(ft)
    if N==1:
        F = ft # dft is length 1
    else:
        # divide the dft into 2 using radix-2 Cooley-Tukey
        Am = fft2(ft[::2], isg)
        Bm = fft2(ft[1::2], isg)
        # combine with appropriate weights
        m = np.arange(N/2)
        W = np.exp(-isg*2*np.pi*1j*m/N)
        F = np.concatenate([Am + W*Bm, Am - W*Bm])
    return F
```

In [10]:
```
def fft(ft, isg):
    N = len(ft)
    if isg==1:
        return fft2(ft, isg)
    elif isg==-1:
        return fft2(ft, isg)/N
```

In [11]:
```
def cntfft(ft, isg):
    N = len(ft)
    a = (N-1)/2
    exft = [ft[i]*np.exp(2*np.pi*1j*a*i/N) for i in range(N)] # pre-transform
    Fs = fft(exft, isg)
    Fs = [Fs[i]*np.exp(2*np.pi*1j*a*(i-a)/N) for i in range(N)] # post-transform
    return Fs
```

Example 1:

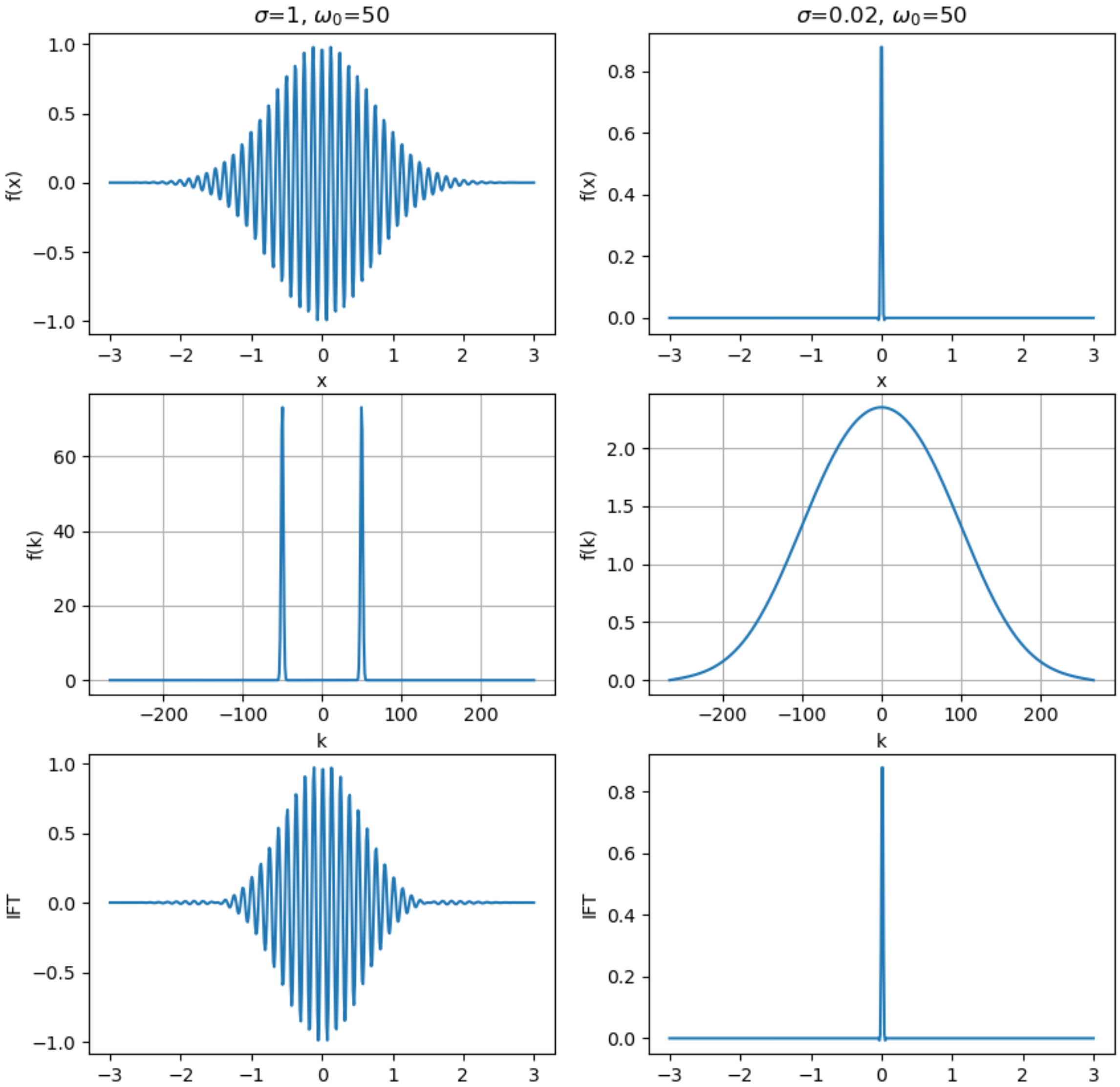
Real part of Gaussian function:

$$f(x) = e^{-\frac{x^2}{\sigma^2}} \cos \omega_0 x$$

In [12]:

```python
def f(pr, x):
    sig, w0 = pr
    return np.exp(-x**2/sig**2)*np.cos(w0*x)

N = 512  # N should be in the form 2**n
xmn, xmx = -3, 3
dt = (xmx-xmn)/(N-1)
x = [xmn+i*dt for i in range(N)]
k = fourspc(x)

sig1, w01 = 1, 50  # parameters
pr1 = [sig1, w01]
fx1 = [f(pr1, xx) for xx in x]
fk1 = cntfft(fx1, 1)
ifx1 = cntfft(fk1, -1)
sig2, w02 = 0.02, 50  # parameters
pr2 = [sig2, w02]
fx2 = [f(pr2, xx) for xx in x]
fk2 = cntfft(fx2, 1)
ifx2 = cntfft(fk2, -1)

plt.figure(figsize=(10, 10))
plt.subplot(321)
plt.title(f'$\sigma$={sig1}, $\omega_0$={w01}')
plt.plot(x, fx1)
plt.xlabel('x')
plt.ylabel('f(x)')
plt.subplot(323)
plt.plot(k, np.abs(fk1))
plt.xlabel('k')
plt.ylabel('f(k)')
plt.grid()
plt.subplot(325)
plt.plot(x, ifx1)
plt.ylabel('IFT')

plt.subplot(322)
plt.title(f'$\sigma$={sig2}, $\omega_0$={w02}')
plt.plot(x, fx2)
plt.xlabel('x')
plt.ylabel('f(x)')
plt.subplot(324)
plt.plot(k, np.abs(fk2))
plt.xlabel('k')
plt.ylabel('f(k)')
plt.grid()
plt.subplot(326)
plt.plot(x, ifx2)
plt.ylabel('IFT')
plt.show()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\matplotlib\cbook\__init__.py:1298: Compl
exWarning: Casting complex values to real discards the imaginary part
  return np.asarray(x, float)
```



In [ ]:

In [ ]: