4th Semester Exam Questions

Question 1

Solve by using Euler's method:

$$\frac{d^2y}{dx^2} + y = 0$$

Given: $y(0) = 1$ and $y'(0) = 0$.

Solution:

$$\frac{dy}{dx} = f(x, y)$$

For n intervals between the integration limits (x_0, x),

$$x_n = x_0 + nh \quad ; (n = 1, 2, 3, \dots)$$

By Euler's Formula,

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Algorithm:

1. Define the function $f(x, y)$.
2. Set interval and initial values of x and y.
3. Update,

$$y = y + hf(x, y)$$
$$x = x + h$$

   Iterate this in a loop.
4. Collect the (x,y) data.
5. Plot the graph.

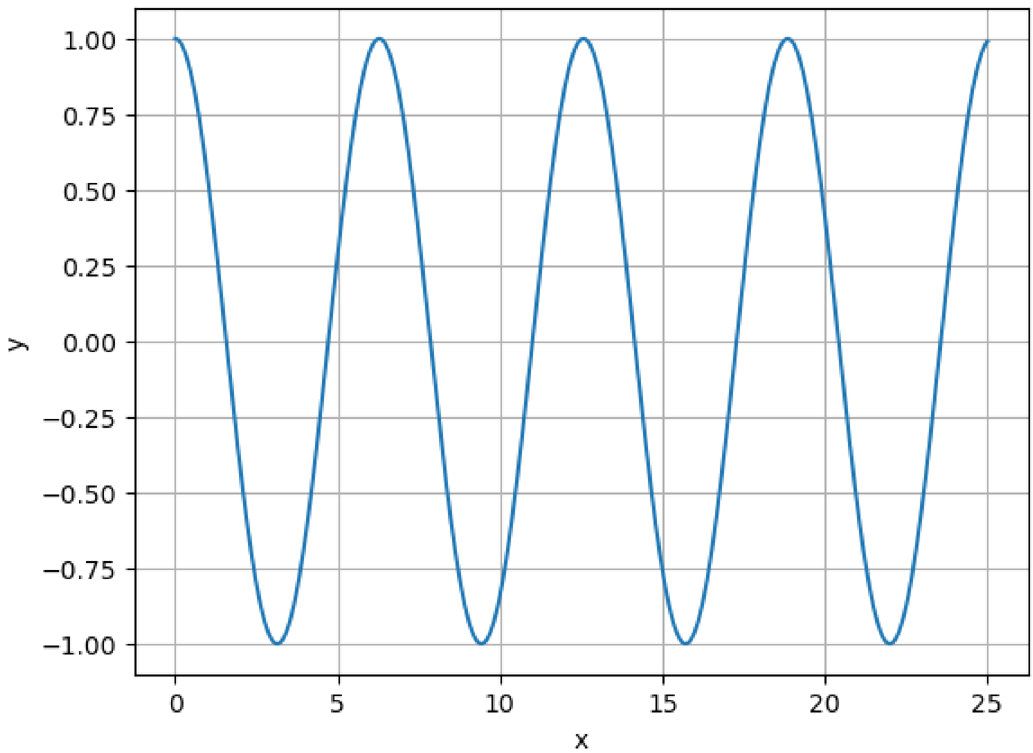
Let,

$$\frac{dy}{dx} = z$$

So,

$$\frac{dz}{dx} = -y$$

In [1]:
```
def dydx(x,y,z):
    return z
def dzdx(x,y,z):
    return -y

import matplotlib.pyplot as plt

x, y, z = 0, 1, 0  # given condition
xm = 25
dx = 0.001
xx, yy, zz = [], [], []

while abs(x) < abs(xm):
    x = x + dx
    y = y + dx * dydx(x,y,z)
    z = z + dx * dzdx(x,y,z)
    xx.append(x)
    yy.append(y)
    zz.append(z)

plt.plot(xx,yy)
plt.xlabel('x')
plt.ylabel('y')
plt.grid()
plt.show()
```



Question 2

Solve by using Runge Kutta - 4 method:

$$\frac{dy}{dx} = e^{-x}$$

Given: $y(0) = 0$.

Solution:

Here, the change of y is further modified. Let h and k be the changes in x and y.

$$k_1 = hf(x, y)$$

$$k_2 = hf(x + \frac{h}{2}, y + \frac{k_1}{2})$$

$$k_3 = hf(x + \frac{h}{2}, y + \frac{k_2}{2})$$

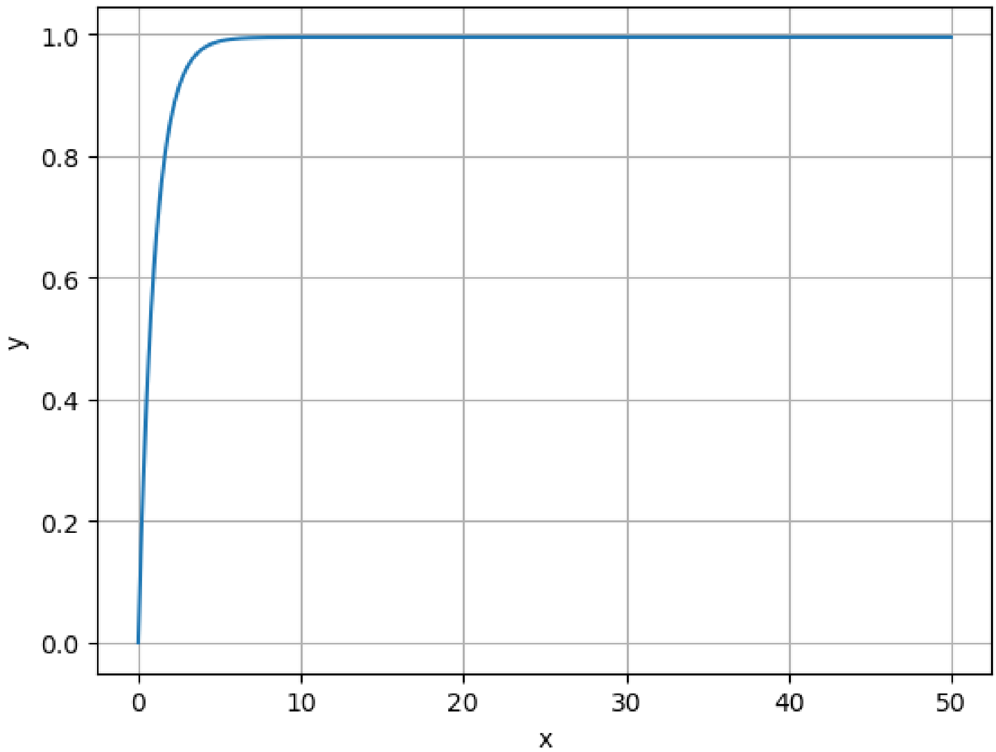
$$k_4 = hf(x + h, y + k_3)$$

At last, y should be,

$$y = y + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$$

In [2]:

```
import numpy as np
import matplotlib.pyplot as plt

def dydx(x,y):
    return np.exp(-x)

xm = 50
x, y = 0, 0 # given condition
h = 0.005
xx, yy = [], []

while abs(x) < abs(xm):
    xx.append(x), yy.append(y)
    x += h
    k1 = h * dydx(x,y)
    k2 = h * dydx(x + (h/2), y + (k1/2))
    k3 = h * dydx(x + (h/2), y + (k2/2))
    k4 = h * dydx(x + h, y + k3)
    y += (1/6)*(k1 + 2*(k2 + k3) + k4)

plt.plot(xx,yy)
plt.xlabel('x')
plt.ylabel('y')
plt.grid()
plt.show()
```



Question-3

$\frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty e^{-\frac{(2-x)^2}{2\sigma^2}}(x+3)dx$; for $\sigma = 1, 0.1, 0.01$ and show that the value tends to 5.

In [3]:
```
import numpy as np
import scipy as sp
from scipy.integrate import simps

sigmas = [1, 0.1, 0.01]  # given values of sigma
for sigma in sigmas:
    xmax = 100  # set the upper limit
    x = np.linspace(0, xmax, int(xmax/sigma))  # set no. of points
    f1 = (1/np.sqrt(2*np.pi*sigma**2))*np.exp(-(2-x)**2/(2*sigma**2))*(x+3)
    f2 = simps(f1,x)
    print(f'sigma = {sigma} \t value = {f2} \t error = {5-f2}')
```

```
sigma = 1        value = 4.978300045718836        error = 0.021699954281164224
sigma = 0.1      value = 5.000000028048047        error = -2.8048047440165647e-08
sigma = 0.01     value = 5.000000026689211        error = -2.668921084136855e-08
```

So, the value tends to 5.

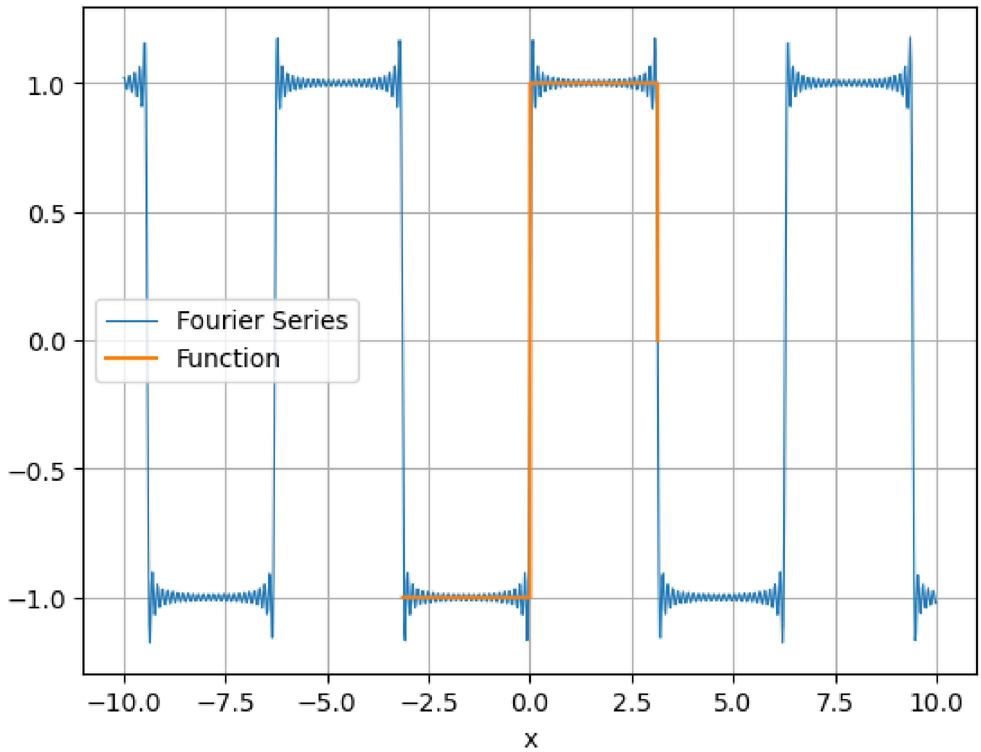
Question 4

$$\begin{aligned} f(x) &= -1 \quad ; (-\pi \le x < 0) \\ &= 1 \quad ; (0 \le x < \pi) \end{aligned}$$

Find the Fourier coefficients.

In [4]:
```python
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from scipy.integrate import simps

# function
def f(x):
    if -np.pi<=x<0:
        return -1
    elif 0<=x<np.pi:
        return 1
    else:
        return 0

M1, M2 = -np.pi, np.pi         # period of x

x = np.linspace(M1, M2, 1000)
fx = np.array([f(i) for i in x])

a0 = 2/(M2-M1) * simps(fx, x)
an = lambda n: (2/(M2-M1)) * simps(fx*np.cos(n*np.pi*x*2/(M2-M1)), x)
bn = lambda n: (2/(M2-M1)) * simps(fx*np.sin(n*np.pi*x*2/(M2-M1)), x)

L1, L2 = -10, 10      # Length of the signal
N = 50      # no. of terms in Fourier Series
xp = np.linspace(L1,L2,1000)
S = a0 * 0.5 + sum([an(n)* np.cos(n*np.pi*xp*2/(M2-M1)) +
                    bn(n)*np.sin(n*np.pi*xp*2/(M2-M1)) for n in range (1,N)])

print('Fourier Coefficients:')
print(f'a0 = {a0}')
for n1 in range(11):
    print(f'n = {n1}, \t a{n1} = {an(n1)}, \t b{n1} = {bn(n1)}')

import matplotlib.pyplot as plt
plt.plot(xp, S, lw=0.8, label='Fourier Series')
plt.plot(x, fx, label='Function')
plt.xlabel('x')
plt.legend()
plt.grid()
plt.show()
```

```
Fourier Coefficients:
a0 = -0.0008341675008340569
n = 0,    a0 = -0.0008341675008340569,    b0 = 0.0
n = 1,    a1 = 0.0008341675008343749,     b1 = 1.2732405940162461
n = 2,    a2 = -0.0008341675008340755,    b2 = 2.098721623656208e-06
n = 3,    a3 = 0.0008341675008343928,     b3 = 0.4244163290854335
n = 4,    a4 = -0.0008341675008340712,    b4 = 4.1982318658349e-06
n = 5,    a5 = 0.0008341675008343441,     b5 = 0.2546531536714814
n = 6,    a6 = -0.0008341675008340659,    b6 = 6.299319249466849e-06
n = 7,    a7 = 0.0008341675008343574,     b7 = 0.18189870379468842
n = 8,    a8 = -0.0008341675008340868,    b8 = 8.402772107272308e-06
n = 9,    a9 = 0.0008341675008343597,     b9 = 0.14148049397117393
n = 10,         a10 = -0.0008341675008341066,    b10 = 1.0509378486630784e-05
```

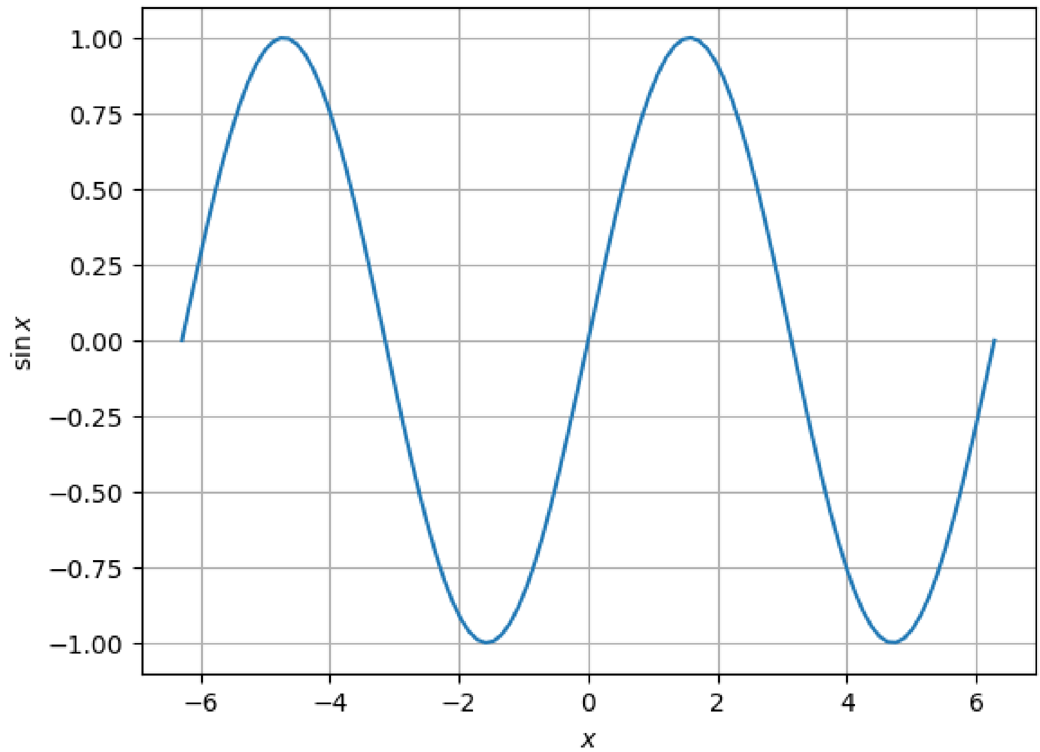


In [ ]:

Question 6

Sine series computed from recurrence reation.

In [5]:
```python
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(-2.0*np.pi, 2.0*np.pi, 100)
a = 1.0
n = 100
sum = 1.0
for i in range(1,n+1):
    a = (-1)*(x**2)*a/(2*i*(2*i+1))
    sum += a
sum = sum*x

plt.plot(x,sum)
plt.xlabel('$x$')
plt.ylabel('$\sin x$')
plt.grid()
plt.show()
```



Computation of $\sin(6)$:

In [6]:
```python
import numpy as np

x = 6
a = 1.0
nmax = 100
sum = 1.0
for n in range(1,nmax+1):
    a = (-1)*(x**2)*a/(2*n*(2*n+1))
    # a is the (n+1)th term in lhs nth term in rhs
    sum += a
sum = sum*x

print(f'Value of sin6 by using recurrence formula: {sum}. ({nmax} no. of terms used)'
print(f'Value of sin6 by using numpy function: {np.sin(6)}')
```

```
Value of sin6 by using recurrence formula: -0.27941549819892436. (100 no. of terms
used)
Value of sin6 by using numpy function: -0.27941549819892586
```

Question 7

Plot first 6 Legendre Polynomials from $x = -1$ to $x = 1$.

Solution:

The recurrence relation is,

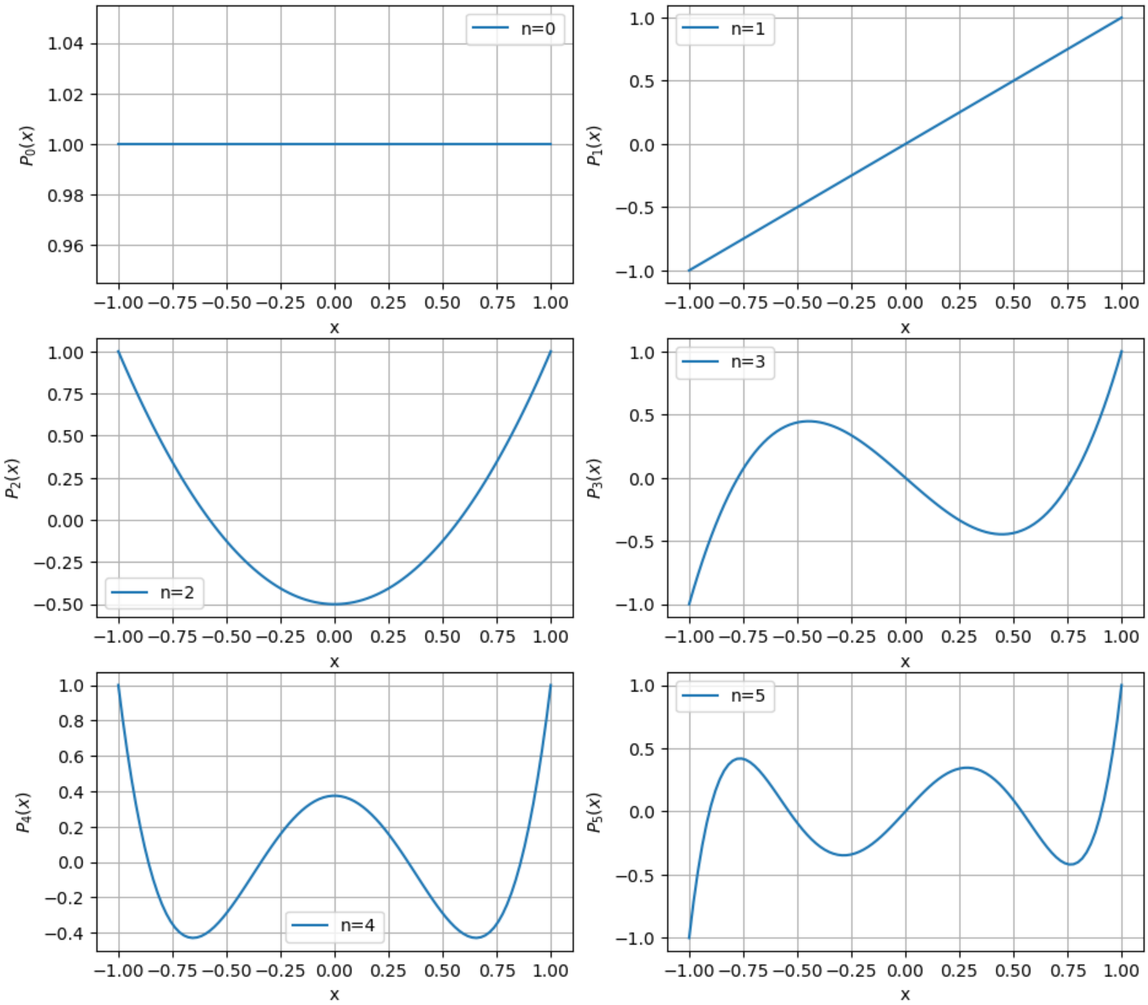
$$P_n(x) = \frac{2n-1}{n} x P_{n-1}(x) - \frac{n-1}{n} P_{n-2}(x)$$

Given, $P_0(x) = 1$ and $P_1(x) = x$.

In [7]:

```
def recPn(n,x):
    P0 = 1
    P1 = x

    if n == 0:
        Pn = P0
    elif n == 1:
        Pn = P1
    else:
        Pn_1, Pn_2 = P1, P0

    for i in range(2, n+1):
        Pn = (2*i - 1)/i*x*Pn_1 - (i - 1)/i*Pn_2
        Pn_1, Pn_2 = Pn, Pn_1
    return Pn
```

In [8]:
```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(-1,1,100)

plt.figure(figsize=(11,10))
for n in range(6):
    recPnplot = [recPn(n,x[i]) for i in range(len(x))]
    plt.subplot(3,2,n+1)
    plt.plot(x, recPnplot, label=f'n={n}')
    plt.legend()
    plt.xlabel('x')
    plt.ylabel(f'$P_{n}(x)$')
    plt.grid()
plt.show()
```



Question 9

Program to sum numerically:

$$\sum_{n=1}^{\infty}(0.2)^n$$

In [9]:
```
nmax = 1000   # the upper limit of sum
a = 0
for i in range(1,nmax+1):
    a = a + 0.2**i

print(f'Sum of the series = {a}. (Using {nmax} no. of terms)')
```

```
Sum of the series = 0.2500000000000001. (Using 1000 no. of terms)
```

Question 10

n^{th} Root of Unity:

$$x^n = 1 = e^{i(2\pi k)}$$

$$x = e^{\frac{2\pi i k}{n}} = \cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right)$$

In [10]:
```
import numpy as np

def general_root(k,n):
    if n<=0:
        return None
    return np.exp(1j*(2*np.pi*k)/n)

print(' roots for n=2:')
n = 2   # put an integer
for k in range(n):
    print(general_root(k,n))

print('\n roots for n=3:')
n = 3   # put an integer
for k in range(n):
    print(general_root(k,n))

print('\n roots for n=4:')
n = 4   # put an integer
for k in range(n):
    print(general_root(k,n))
```

```
 roots for n=2:
(1+0j)
(-1+1.2246467991473532e-16j)

 roots for n=3:
(1+0j)
(-0.4999999999999998+0.8660254037844387j)
(-0.5000000000000004-0.8660254037844385j)

 roots for n=4:
(1+0j)
(6.123233995736766e-17+1j)
(-1+1.2246467991473532e-16j)
(-1.8369701987210297e-16-1j)
```

Question 11

Square Root of $(-5 + 12i)$

$$\sqrt{x + iy} = \sqrt{r}\cos(\theta/2) + i\sqrt{r}\sin(\theta/2) = \sqrt{r}e^{i\theta/2}$$

where $r = \sqrt{x^2 + y^2}$ and $\tan \theta = y$

In [11]:
```python
import numpy as np

# Values of x and y
x, y = -5, 12

r = (x**2 + y**2)**0.5
th = np.arctan(y/x)
root = r**0.5 * np.exp(1j*th/2)
print(root)
```

```
(2.9999999999999996-2j)
```

In [ ]: