# SciPy (Mr. P Solver)

Video Link: https://youtu.be/jmX4FOUEfgU (https://youtu.be/jmX4FOUEfgU)

Codes: https://www.youtube.com/redirect?
event=video_description&redir_token=QUFFLUhqbS01X0lmZVhJbGpVdVVVblZEb2lWdWx1Q0NEQXxB
(https://www.youtube.com/redirect?
event=video_description&redir_token=QUFFLUhqbS01X0lmZVhJbGpVdVVVblZEb2lWdWx1Q0NEQXxB

SciPy means SCIENTIFIC PYTHON.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
```

## Basics

**Optimization**

In [2]:
```
from scipy.optimize import minimize
```

In [3]:
```
minimize
```

Out[3]:
```
<function scipy.optimize._minimize.minimize(fun, x0, args=(), method=None, jac=None, hess=None, hessp=None, bounds=None, constraints=(), tol=None, callback=None, options=None)>
```

run 'minimize?' to know all about it.

Minimize $f(x) = (x-3)^2$

In [4]:
```
def f(x):
    return (x-3)**2
ans= minimize(f,5)
```

In [5]:
```
ans.x
```

Out[5]:
```
array([2.99999998])
```

Minimize $f(x,y) = (x-1)^2 + (y-2.5)^2$ subject to

$$
\begin{gathered}
x - 2y + 2 \geq 0 \\
-x - 2y + 6 \geq 0 \\
-x + 2y + 2 \geq 0 \\
x \geq 0, y \geq 0
\end{gathered}
$$

2D function takes in vector $x$

Constraints must be specified as $g_i(x) \geq 0$

Bounds specified as rectangular

In [6]:
```
f= lambda x: (x[0]-1)**2 + (x[1]-2.5)**2
# constraints
cons= ({'type': 'ineq', 'fun': lambda x: x[0] - 2 * x[1] + 2},
       {'type': 'ineq', 'fun': lambda x: -x[0] - 2 * x[1] + 6},
       {'type': 'ineq', 'fun': lambda x: -x[0] +2*x[1] +2})
# bounds: x is in (0, inf) and y is in (0, inf)
bnds= ((0,None), (0, None))
ans= minimize(f, (7,5), bounds=bnds, constraints=cons)
```

In [7]:
```
f([5,4])
```

Out[7]:
```
18.25
```

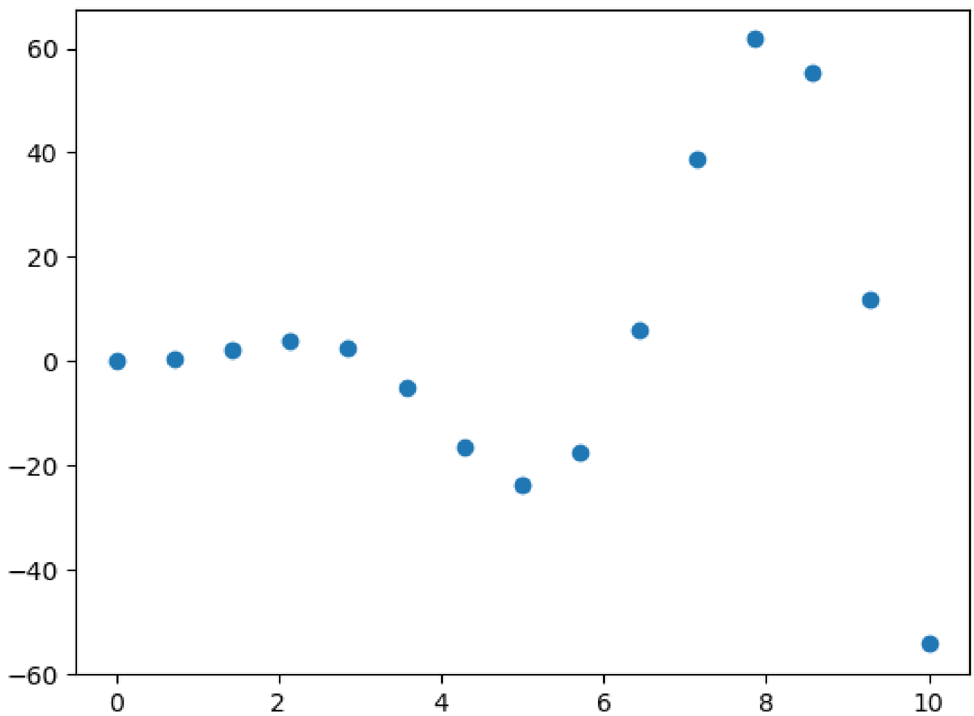
In [8]:
```
ans.x
```

Out[8]:
```
array([1.39999999, 1.7       ])
```

## Interpolation

In [9]:
```
x= np.linspace(0,10,15)
y= x**2 * np.sin(x) + 0.02
plt.scatter(x,y)
```

Out[9]:
```
<matplotlib.collections.PathCollection at 0x27ae56bef40>
```

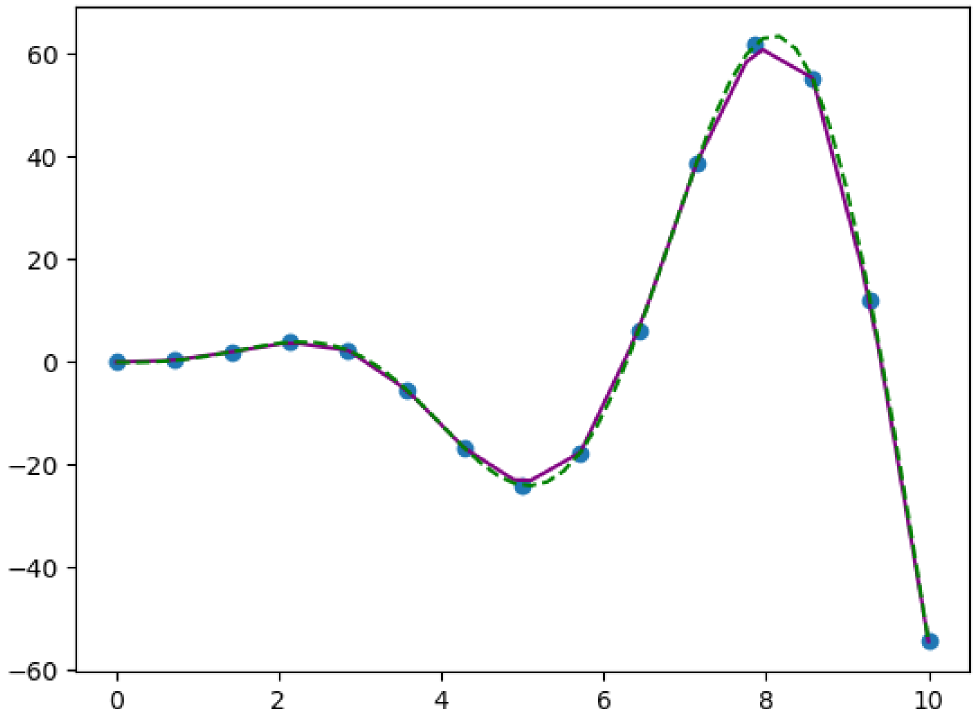



We want to know the values in between

In [10]:
```
from scipy.interpolate import interp1d
```

In [11]:
```
intr1 = interp1d(x,y, kind='linear')
x1= np.linspace(0,10,50)
y1 = intr1(x1)
intr2 = interp1d(x,y, kind='cubic')
x2= np.linspace(0,10,50)
y2 = intr2(x2)
```

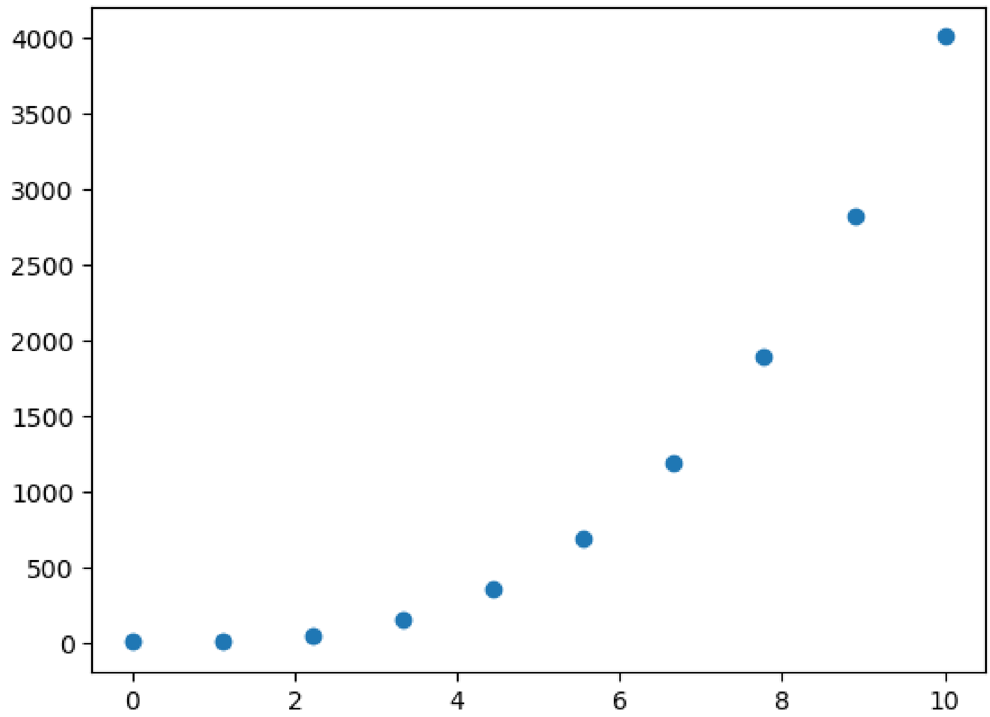
In [12]:
```
plt.scatter(x,y)
plt.plot(x1,y1, color='purple')
plt.plot(x2,y2, '--', color='green')
plt.show()
```



### Curve Fitting

In [13]:
```
xdata= np.linspace(0,10,10)
ydata= 4*xdata**3 + 6
plt.scatter(xdata, ydata)
```

Out[13]: <matplotlib.collections.PathCollection at 0x27ae58f49d0>




We want to fit the data to the curve $y = ax^2 + b$, and determine the values of a and b.

In [14]:
```
from scipy.optimize import curve_fit
```

In [15]:
```
def fn(x,a,b):
    return a*x**2 +b
# optimal parameters i.e. values of a and b and covariance parameters
optp, covp = curve_fit(fn, xdata, ydata, p0= (3,3))
```

In [16]:
```
optp
```

Out[16]:
```
array([  38.90160182, -251.648953  ])
```

In [17]:
```
covp # what is it ?
```

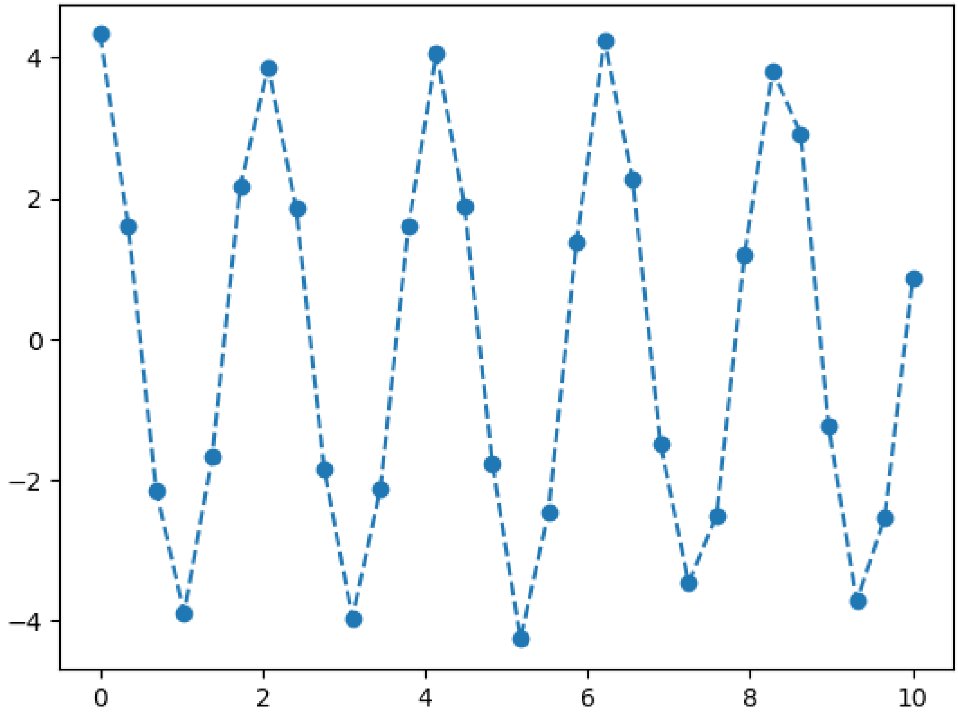
Out[17]:
```
array([[ 5.45418498e+00, -1.91906509e+02],
       [-1.91906509e+02,  1.27463829e+04]])
```

The equation for spring motion is $y(t) = A\cos(\omega t + \phi)$. Want to find the natural frequency of oscillation $\omega$ for the spring. You collect the following data.

In [18]:
```python
tdata = np.array([ 0.        ,  0.34482759,  0.68965517,  1.03448276,  1.37931034,
        1.72413793,  2.06896552,  2.4137931 ,  2.75862069,  3.10344828,
        3.44827586,  3.79310345,  4.13793103,  4.48275862,  4.82758621,
        5.17241379,  5.51724138,  5.86206897,  6.20689655,  6.55172414,
        6.89655172,  7.24137931,  7.5862069 ,  7.93103448,  8.27586207,
        8.62068966,  8.96551724,  9.31034483,  9.65517241, 10.        ])
ydata = np.array([ 4.3303953 ,  1.61137995, -2.15418696, -3.90137249, -1.67259042,
        2.16884383,  3.86635998,  1.85194506, -1.8489224 , -3.96560495,
       -2.13385255,  1.59425817,  4.06145238,  1.89300594, -1.76870297,
       -4.26791226, -2.46874133,  1.37019912,  4.24945607,  2.27038039,
       -1.50299303, -3.46774049, -2.50845488,  1.20022052,  3.81633703,
        2.91511556, -1.24569189, -3.72716214, -2.54549857,  0.87262548])
```

In [19]:
```python
plt.plot(tdata, ydata, 'o--')
```

Out[19]: [<matplotlib.lines.Line2D at 0x27ae596a7c0>]



$\omega = 2\pi f, f = 1/T and T \approx 2s$. Thus good initial guess is

$$\omega = 2\pi(1/2) = \pi, A = 4 \; and \; \phi = 0$$

In [20]:
```python
def f(t,A,w, phi):
    return A*np.cos(w*t +phi)
optp, covp = curve_fit(f, tdata, ydata, p0=(4, np.pi, 0))
```

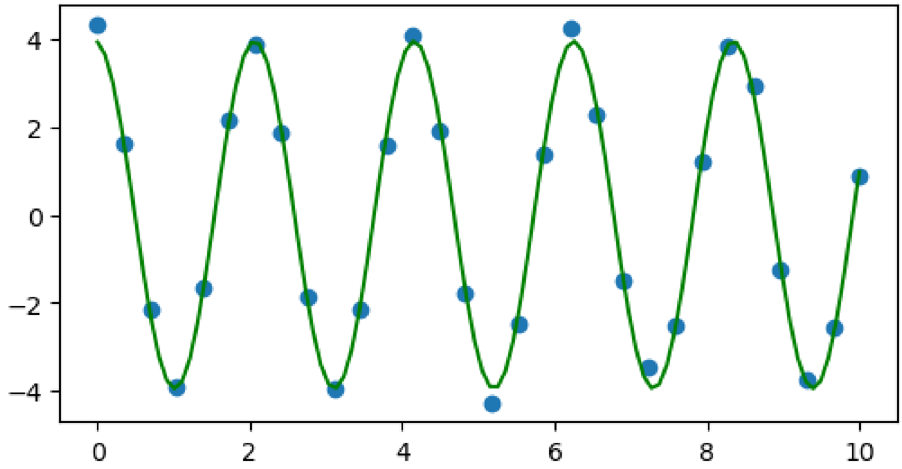
In [21]:
```python
optp
```

Out[21]: array([3.94836219, 2.99899521, 0.10411352])

In [22]:
```python
A, w, phi = optp
```

In [23]:
```python
t= np.linspace(0,10,100)
y= f(t,A,w, phi)
```

In [24]:
```
plt.figure(figsize=(6,3))
plt.scatter(tdata,ydata)
plt.plot(t,y,color='g')
plt.show()
```




In [25]:
```
w
```

Out[25]: 2.998995205451252

In [26]:
```
error = np.sqrt(np.diag(covp))  # std deviation
errA, errw, errphi = error
```

In [27]:
```
error
```

Out[27]: array([0.05117448, 0.00430857, 0.02575701])

In [28]:
```
errw
```

Out[28]: 0.004308565132396146

# Special Functions

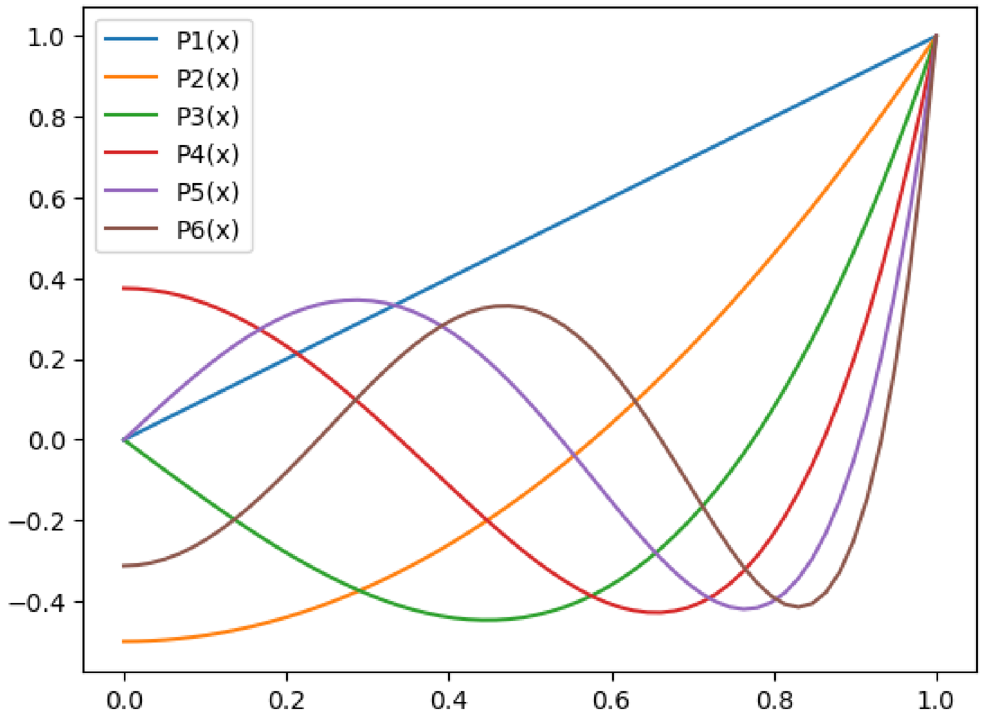
## Legendre Polynomials, $P_l(x)$

Satisfy $(1-x^2)y'' - 2xy' + l(l+1)y = 0$**

In [29]:
```
from scipy.special import legendre
```

In [30]:
```
x= np.linspace(0,1,60)
P1= legendre(1)(x)
P2= legendre(2)(x)
P3= legendre(3)(x)
P4= legendre(4)(x)
P5= legendre(5)(x)
P6= legendre(6)(x)
```

In [31]:
```
plt.plot(x, P1, label='P1(x)')
plt.plot(x, P2, label='P2(x)')
plt.plot(x, P3, label='P3(x)')
plt.plot(x, P4, label='P4(x)')
plt.plot(x, P5, label='P5(x)')
plt.plot(x, P6, label='P6(x)')
plt.legend()
plt.show()
```



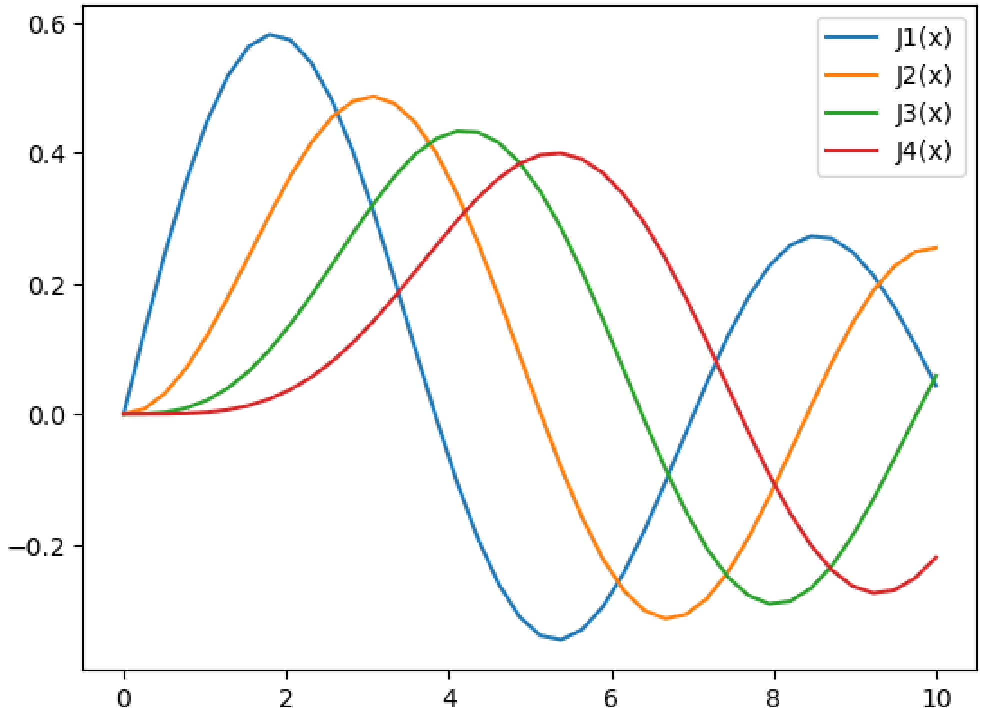

## Bessel Functions, $J_\alpha(x)$

Satisfy $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$

In [32]:
```
from scipy.special import jv
```

In [33]:
```
x= np.linspace(0,10,40)
J1= jv(1,x)
J2= jv(2,x)
J3= jv(3,x)
J4= jv(4,x)
```

In [34]:
```
plt.plot(x, J1, label='J1(x)')
plt.plot(x, J2, label='J2(x)')
plt.plot(x, J3, label='J3(x)')
plt.plot(x, J4, label='J4(x)')
plt.legend()
plt.show()
```



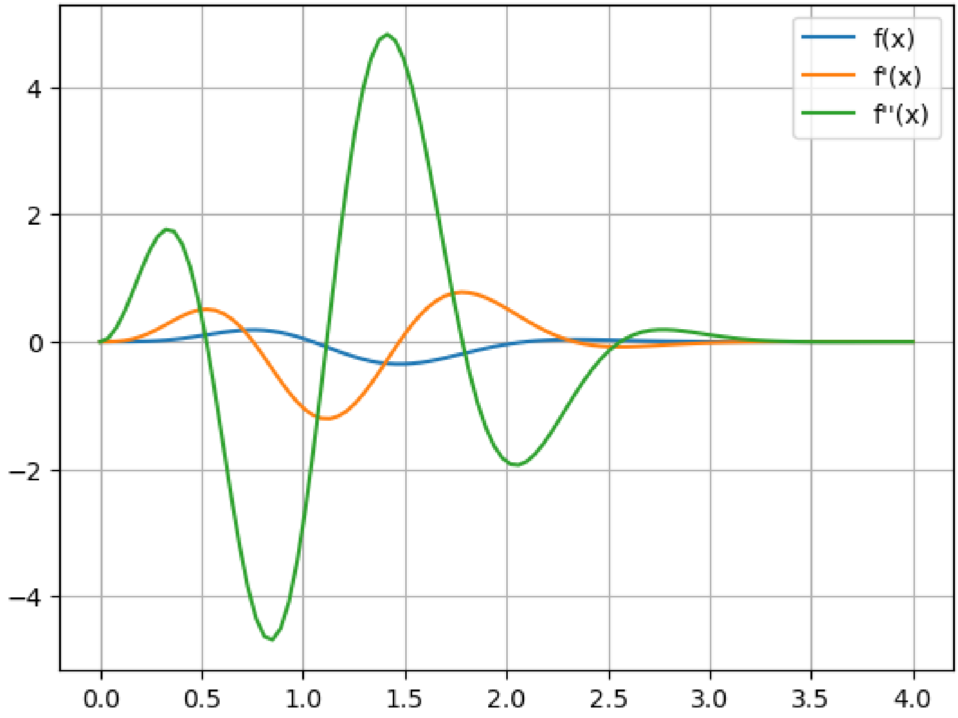

# CALCULUS

## Differentiation

In [35]:
```
from scipy.misc import derivative
```

In [36]:
```
def f(x):
    return x**3 * np.sin(3*x) * np.exp(-x**2)
x= np.linspace(0,4,100)
f1 = derivative(f, x, dx= 1e-3)
f2 = derivative(f, x, dx= 1e-3, n=2)
```

In [37]:
```
plt.plot(x, f(x), label='f(x)')
plt.plot(x, f1, label='f\'(x)')
plt.plot(x, f2, label='f\'\'(x)')
plt.legend()
plt.grid()
plt.show()
```




## Integration

### Single integrals

$$\int_0^1 x^2 \sin(2x) e^{-x} dx$$

In [38]:
```
from scipy.integrate import quad
```

In [39]:
```
intg= lambda x: x**2 * np.sin(x) * np.exp(-x)
integral, integral_error = quad(intg, 0, 1)
```

In [40]:
```
integral
```

Out[40]: 0.10246777930717413

In [41]:
```
integral_error
```

Out[41]: 1.1376208786903388e-15

### Double integrals

$$\int_0^1 \int_{-x}^{x^2} \sin(x + y^2) dy dx$$

In [42]:
```
from scipy.integrate import dblquad
```

In [43]:
```
int2 = lambda y,x: np.sin(x + y**2)
llim_y = lambda x: -x
ulim_y = lambda x: x**2
integral, integral_error = dblquad(int2, 0, 1, llim_y, ulim_y)
```

In [44]:
```
integral
```

Out[44]:
```
0.590090324408853
```

In [45]:
```
integral_error
```

Out[45]:
```
2.0545915475811425e-14
```

**nth order integrals**

In [46]:
```
from scipy.integrate import nquad
```

run "nquad?" to know all about this.

try an triple integral by nquad

# Differetial Equations

## First Order ODEs

Air friction while falling considered here.

$$v' - \alpha v^2 + \beta = 0 \qquad v(0) = 0$$

In [47]:
```
from scipy.integrate import odeint
```

defining the differential equation and the boundary conditions.

In [48]:
```
def dvdt(v,t):
    return 4*v**2 - 10
v0 = 0
```

solving

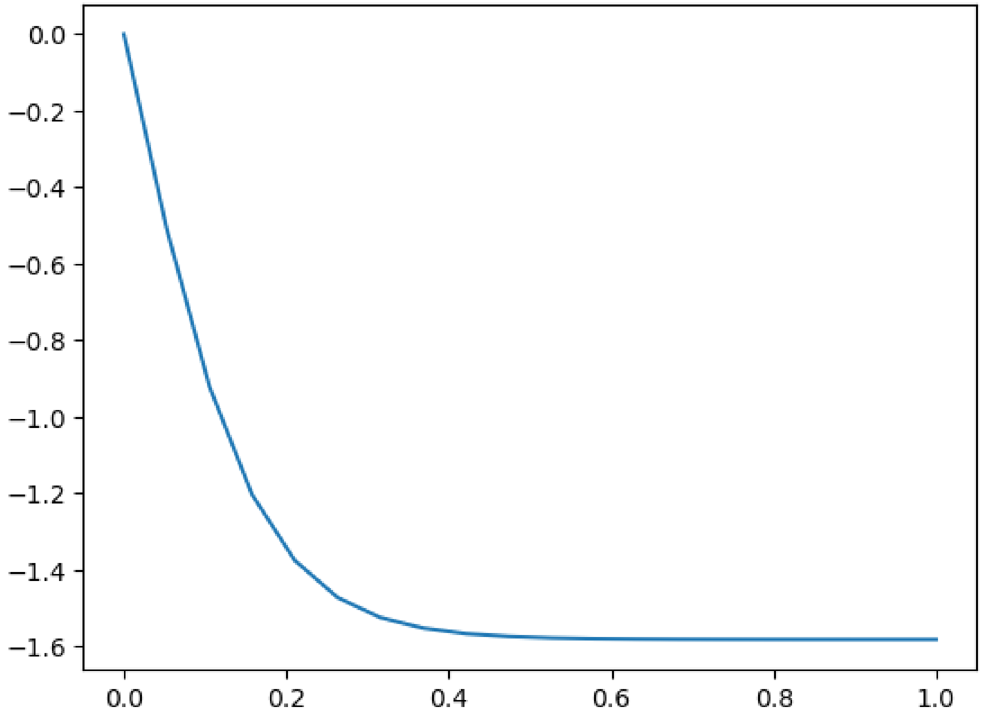
In [49]:
```
t= np.linspace(0,1,20)
soln = odeint(dvdt, v0, t)
```

converting solution to a suitable array

In [50]:
```
v_t = soln.T[0]
```

```
In [51]: plt.plot(t, v_t)
```

```
Out[51]: [<matplotlib.lines.Line2D at 0x27ae70d08e0>]
```




### Coupled first order ODEs

$$
\begin{aligned}
y_1' &= y_1 + y_2^2 + 3x \qquad & y_1(0) = 0 \\
y_2' &= 3y_1 + y_2^3 - \cos(x) \qquad & y_2(0) = 0
\end{aligned}
$$

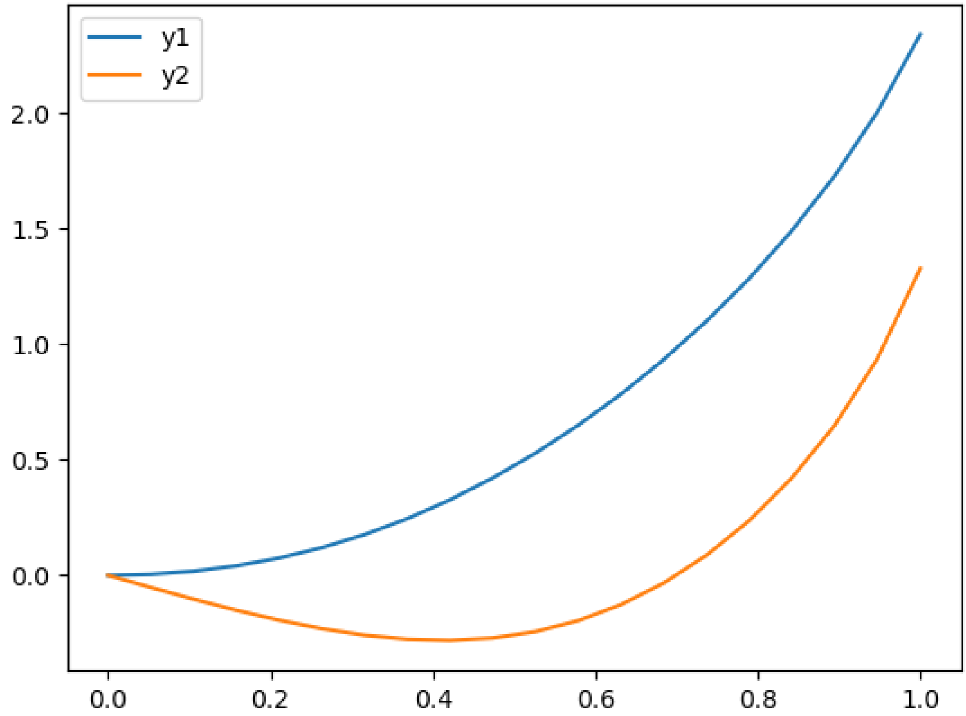
Letting $S = (y_1, y_2)$ we need to write a function that returns $dS/dx(dy_1/dx, dy_2/dx)$. The function $dS/dx$ can take in $S = (y_1, y_2)$ and $x$.

```
In [52]: def dSdx(S,x):
             y1, y2 = S
             return [y1 + y2**2 + 3*x, 3*y1 + y2**3 - np.cos(x)]
         y1_0, y2_0 = 0,0
         S_0 = (y1_0, y2_0)
```

```
In [53]: x = np.linspace(0,1,20)
         soln = odeint(dSdx, S_0, x)
```

```
In [54]: y1_x = soln.T[0]
         y2_x = soln.T[1]
```

In [55]:
```
plt.plot(x, y1_x, label='y1')
plt.plot(x, y2_x, label='y2')
plt.legend()
plt.show()
```




### Second Order ODEs

Equation for a pendulum -

$$\theta'' - \sin(\theta) = 0$$

Scipy can *only* solve coupled first order ODEs, but *any second order ODE can be turned into two coupled first order ODEs*. The same thing goes for higher order ODEs.

Define $\omega = d\theta/dt$ so that one has the following coupled ODEs

$$d\theta/dt = \omega$$
$$d\omega/dt = \sin(\theta)$$

Let $S = (\theta, \omega)$

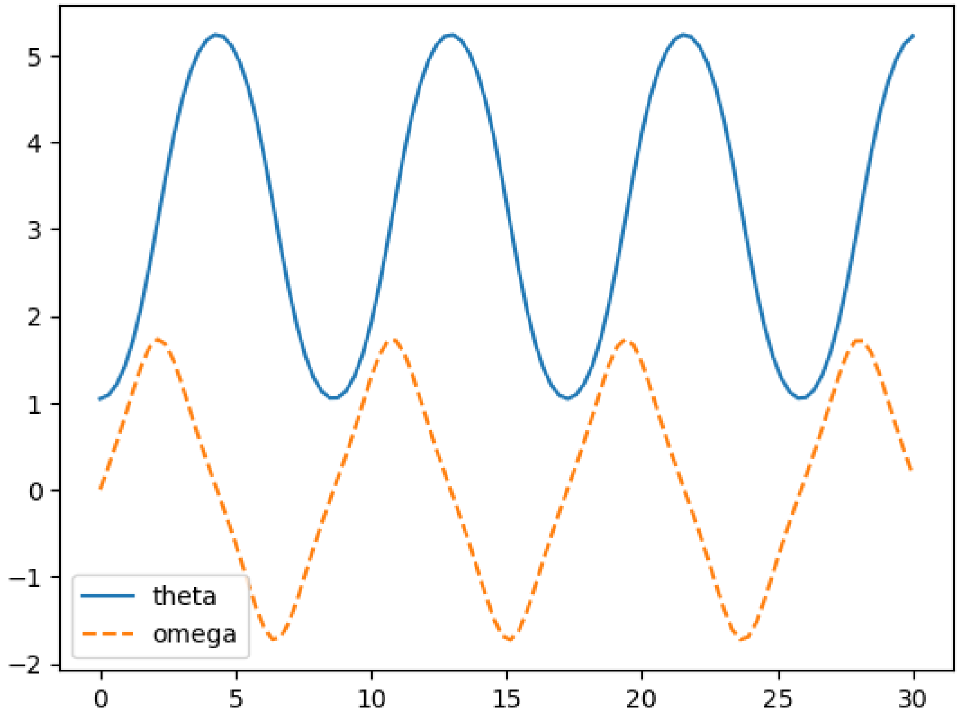
In [56]:
```
def dSdt(S,t):
    theta, omega = S
    return [omega, np.sin(theta)]
theta0 = np.pi/3   # initial condition
omega0 = 0
S0 = (theta0, omega0)
```

In [57]:
```
t= np.linspace(0, 30, 100)
soln = odeint(dSdt, S0, t)
```

In [58]:
```
theta, omega = soln.T
```

In [59]:
```
plt.plot(t, theta, label='theta')
plt.plot(t, omega, '--', label='omega')
plt.legend()
```

Out[59]: <matplotlib.legend.Legend at 0x27ae716ce20>




**Fourier Transform - do it later**

# Examples

**Example 1**

In [60]:
```
def func(A):
    intg= lambda t: 5* (1 + (np.pi*A/10)**2 * (np.cos(np.pi*t/10))**2)**0.5 + 2/(A*np
    return quad(intg,0,10)[0]
minimize(func,0.001).x
```

Out[60]: array([1.02735846e-05])

In [61]:
```
E = quad(intg,0,10)[0]
E
```

Out[61]: 0.5035272509544617

**Example 2**

Newton's law of cooling is

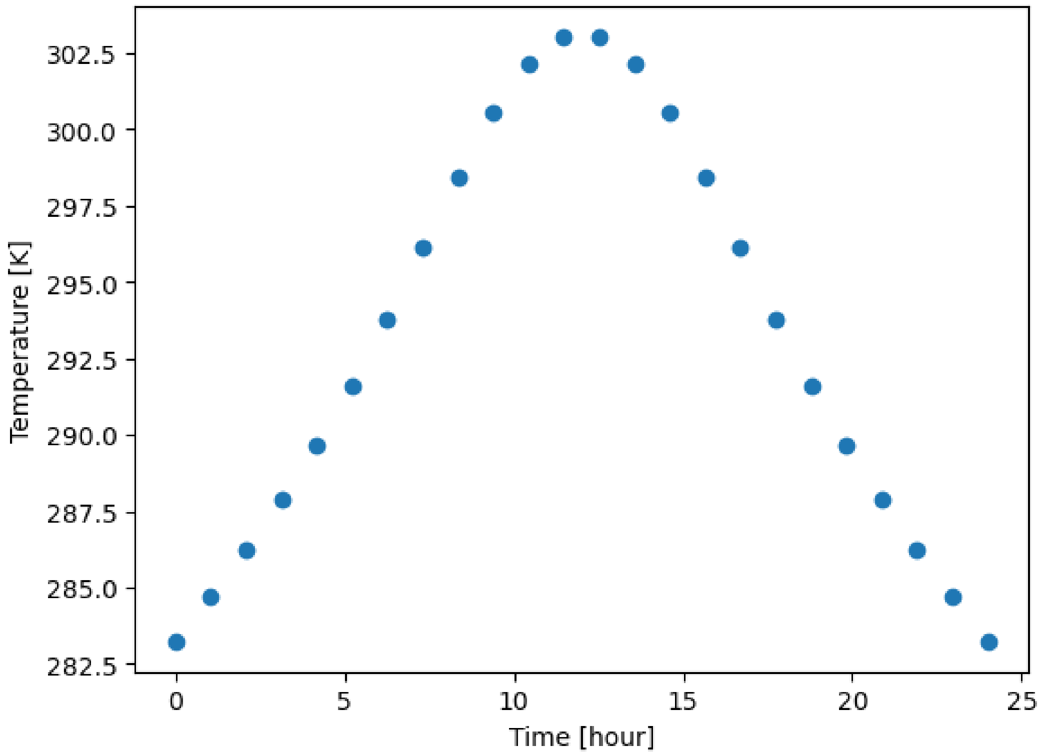
$$\frac{dT}{dt} = -k(T - T_s(t))$$

In [62]:
```
tdata = np.array([ 0.,  1.04347826,  2.08695652,  3.13043478,  4.17391304,
        5.2173913 ,  6.26086957,  7.30434783,  8.34782609,  9.39130435,
       10.43478261, 11.47826087, 12.52173913, 13.56521739, 14.60869565,
       15.65217391, 16.69565217, 17.73913043, 18.7826087 , 19.82608696,
       20.86956522, 21.91304348, 22.95652174, 24.        ])  # in hours

Tdata = np.array([283.2322975, 284.6945461, 286.2259041, 287.8603625, 289.6440635,
       291.6187583, 293.7939994, 296.1148895, 298.4395788, 300.5430675,
       302.1566609, 303.0363609, 303.0363609, 302.1566609, 300.5430675,
       298.4395788, 296.1148895, 293.7939994, 291.6187583, 289.6440635,
       287.8603625, 286.2259041, 284.6945461, 283.2322975])
```

In [63]:
```
plt.scatter(tdata, Tdata)
plt.xlabel('Time [hour]')
plt.ylabel('Temperature [K]')
```

Out[63]:
```
Text(0, 0.5, 'Temperature [K]')
```




In [64]:
```
Ts= interp1d(tdata,Tdata, kind='cubic')
```

In [65]:
```
Tdata, Ts(tdata)  # creating the function Ts(t) by interpolation
```

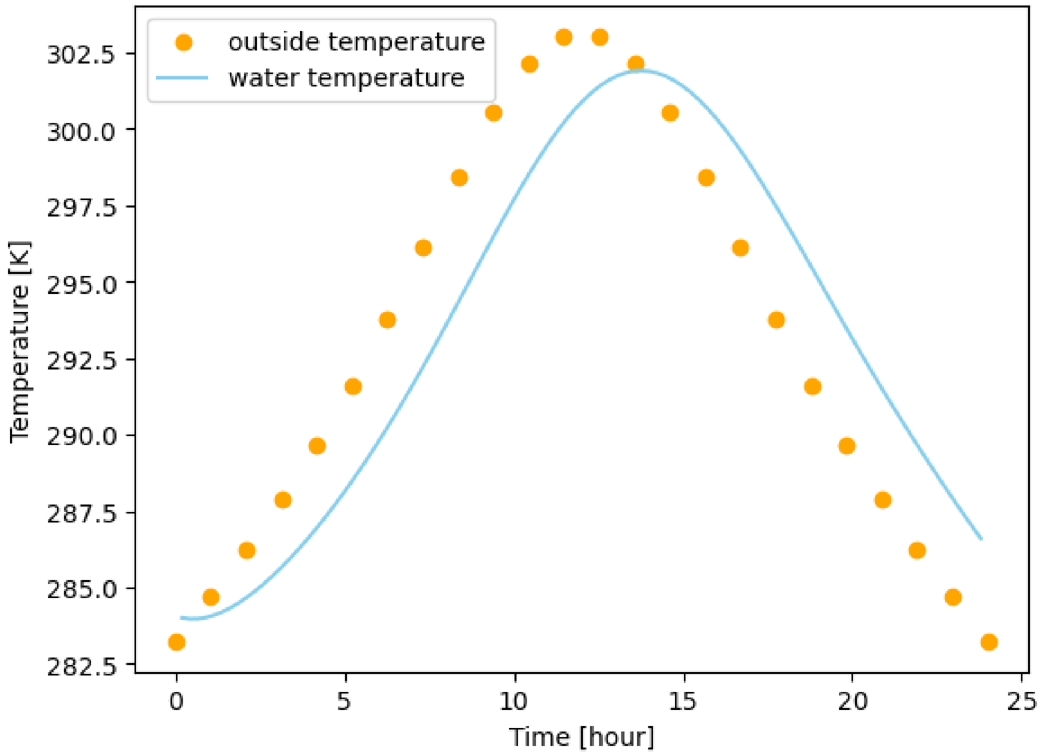
Out[65]:
```
(array([283.2322975, 284.6945461, 286.2259041, 287.8603625, 289.6440635,
        291.6187583, 293.7939994, 296.1148895, 298.4395788, 300.5430675,
        302.1566609, 303.0363609, 303.0363609, 302.1566609, 300.5430675,
        298.4395788, 296.1148895, 293.7939994, 291.6187583, 289.6440635,
        287.8603625, 286.2259041, 284.6945461, 283.2322975]),
 array([283.2322975, 284.6945461, 286.2259041, 287.8603625, 289.6440635,
        291.6187583, 293.7939994, 296.1148895, 298.4395788, 300.5430675,
        302.1566609, 303.0363609, 303.0363609, 302.1566609, 300.5430675,
        298.4395788, 296.1148895, 293.7939994, 291.6187583, 289.6440635,
        287.8603625, 286.2259041, 284.6945461, 283.2322975]))
```

In [66]:
```
def dTdt(T,t):
    return -0.5*(T-Ts(t))

ts = np.linspace(0.2,23.8,100)
T0= 284
```

In [67]:
```
Tsoln = odeint(dTdt, T0, ts).T[0]
```

In [68]:
```
plt.scatter(tdata, Tdata, color='orange', label='outside temperature')
plt.xlabel('Time [hour]')
plt.ylabel('Temperature [K]')
plt.plot(ts, Tsoln, color='skyblue', label='water temperature')
plt.legend()
plt.show()
```




# Linear Algebra

## Basics

Triangular matrices

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}$$

In [69]:
```
from scipy.linalg import solve_triangular
A = np.array([[3,0,0,0],[2,1,0,0],[1,0,1,0],[1,1,1,1]])
B = np.array([4,2,4,2])
X = solve_triangular(A, B, lower=True)
X
```

Out[69]:
```
array([ 1.33333333, -0.66666667,  2.66666667, -1.33333333])
```

Toeplitz Matrices (matrices with constant diagonals)

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 3 & 1 & -1 & 2 \\ 6 & 3 & 1 & -1 \\ 10 & 6 & 3 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 5 \end{bmatrix}$$

In [70]:
```
from scipy.linalg import toeplitz, solve_toeplitz
cl = np.array([1,3,6,10])  # 1st column
rw = np.array([1,-1,2,3])  # 1st row
B = np.array([1,2,2,5])
X = solve_toeplitz((cl,rw),B)
X
```

Out[70]:
```
array([ 0.17741935,  0.48387097, -0.0483871 ,  0.46774194])
```

## Eigenvalue Problems

Eigenvalue problems can be solved using numpy, so here we focus on particular cases for optimization

In [71]:
```
from scipy.linalg import eigh_tridiagonal
```

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \vec{x} = \lambda x$$

In [72]:
```
d = 3* np.ones(4)
e = -1* np.ones(3)
w, v = eigh_tridiagonal(d,e)
```

In [73]:
```
A = np.diag(d) + np.diag(e, k=1) + np.diag(e, k=-1)
A  # making of the matrix
```

Out[73]:
```
array([[ 3., -1.,  0.,  0.],
       [-1.,  3., -1.,  0.],
       [ 0., -1.,  3., -1.],
       [ 0.,  0., -1.,  3.]])
```

In [74]:
```
w, v.T  # eigrnvalues and corresponding eigenvectors
```

Out[74]:
```
(array([1.38196601, 2.38196601, 3.61803399, 4.61803399]),
 array([[ 0.37174803,  0.60150096,  0.60150096,  0.37174803],
        [ 0.60150096,  0.37174803, -0.37174803, -0.60150096],
        [ 0.60150096, -0.37174803, -0.37174803,  0.60150096],
        [-0.37174803,  0.60150096, -0.60150096,  0.37174803]]))
```

In [75]:
```
A @ v.T[0], w[0] * v.T[0]  # verification of eigenvalue equation
```

Out[75]:
```
(array([0.51374315, 0.83125388, 0.83125388, 0.51374315]),
 array([0.51374315, 0.83125388, 0.83125388, 0.51374315]))
```

## Special Matrices

Fiedler matrix $A_{ij} = |a_i - a_j|$ where $a_i$ is some sequence of numbers.

In [76]:
```
from scipy.linalg import fiedler
fiedler([5,3,1,4])
```

Out[76]:
```
array([[0, 2, 4, 1],
       [2, 0, 2, 1],
       [4, 2, 0, 3],
       [1, 1, 3, 0]])
```

Toeplitz matrices (matrices with constant diagonals)

In [77]:
```
from scipy.linalg import toeplitz
toeplitz([4,3,8],[4,8,0]) # (columns, rows)
```

Out[77]:
```
array([[4, 8, 0],
       [3, 4, 8],
       [8, 3, 4]])
```

See other types of matrices [https://docs.scipy.org/doc/scipy/reference/linalg.html#special-matrices (https://docs.scipy.org/doc/scipy/reference/linalg.html#special-matrices)](https://docs.scipy.org/doc/scipy/reference/linalg.html#special-matrices)

## Decompositions

**LU** decomposition: $A = PLU$ where $P$ is a permutation matrix, $L$ is a lower triangular matrix and $U$ is an upper triangular matrix.

In [78]:
```
from scipy.linalg import lu
A = np.array([[69,3,0],[6,4,1],[3,0,9]])
```

In [79]:
```
P, L, U = lu(A)
```

In [80]:
```
P, L, U
```

Out[80]:
```
(array([[1., 0., 0.],
        [0., 1., 0.],
        [0., 0., 1.]]),
 array([[ 1.        ,  0.        ,  0.        ],
        [ 0.08695652,  1.        ,  0.        ],
        [ 0.04347826, -0.03488372,  1.        ]]),
 array([[69.        ,  3.        ,  0.        ],
        [ 0.        ,  3.73913043,  1.        ],
        [ 0.        ,  0.        ,  9.03488372]]))
```

Choleski decomposition: find matrix $C$ such that $A = CC^T$.

In [81]:
```
from scipy.linalg import cholesky
A = np.array([[69,3,0],[6,4,1],[3,0,9]])
C = cholesky(A, lower=True)
```

In [82]:
```
C
```

Out[82]:
```
array([[ 8.30662386,  0.        ,  0.        ],
       [ 0.72231512,  1.86500962,  0.        ],
       [ 0.36115756, -0.13987572,  2.97489496]])
```

In [83]:
```
C @ C.T  # verification
```

Out[83]:
```
array([[6.90000000e+01, 6.00000000e+00, 3.00000000e+00],
       [6.00000000e+00, 4.00000000e+00, 1.17319166e-17],
       [3.00000000e+00, 1.17319166e-17, 9.00000000e+00]])
```

### Sparse Matrices

Matrices that contain lots of zeros (so lots of space can be reduced)

**Do it later when needed.**

## Statistics

**$\beta$ distribution**

$$f(x; a, b) = \frac{\Gamma(a+b)x^{a-1}(1-x)^{b-1}}{\Gamma(a)\Gamma(b)} \qquad\qquad 0 \le x \le 1$$

In [84]:
```
from scipy.stats import beta
```

Basic Statistics

In [85]:
```
a, b = 10, 12
```

In [86]:
```
mean, var, skew, kurt = beta.stats(a,b, moments='mvsk')
```

In [87]:
```
mean
```

Out[87]:
```
array(0.45454545)
```

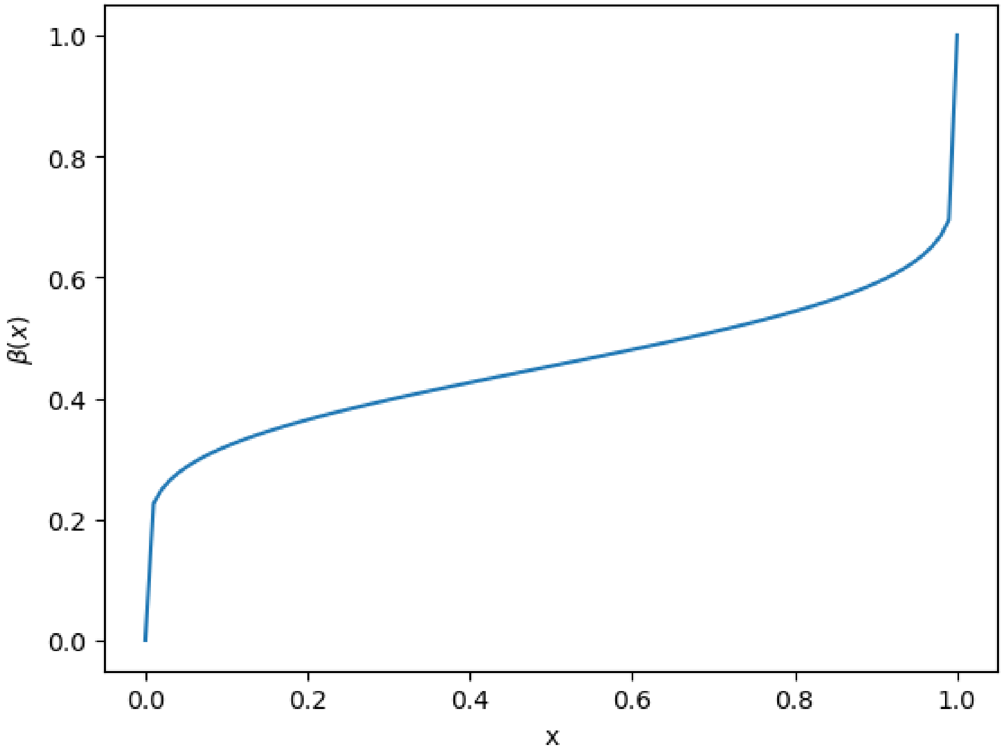
Probability Distribution Plotting:

In [88]:
```
x= np.linspace(beta.ppf(0,a,b), beta.ppf(1,a,b), 100)
```

In beta.pdf the 1st arguement is input fraction of area and the out put is value of x between (a,b) separating the fractiin of area. To know more input 'beta.ppf?'.

In [89]:
```
plt.plot(x,beta.ppf(x,a,b))
plt.xlabel('x')
plt.ylabel(r'$\beta(x)$')
```

Out[89]:
```
Text(0, 0.5, '$\\beta(x)$')
```




*Graph not matched !!*

Generating Random Variables:

In [90]:
```
r = beta.rvs(a,b, size=10)
r
```

Out[90]:
```
array([0.32212614, 0.67361288, 0.54366034, 0.45015316, 0.46085457,
       0.55773435, 0.49493856, 0.42023212, 0.54077363, 0.38851176])
```

**Gaussian Distribution**

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{\sigma^2}\right) \qquad -\infty < x \leq \infty$$

In [91]:
```
from scipy.stats import norm
```

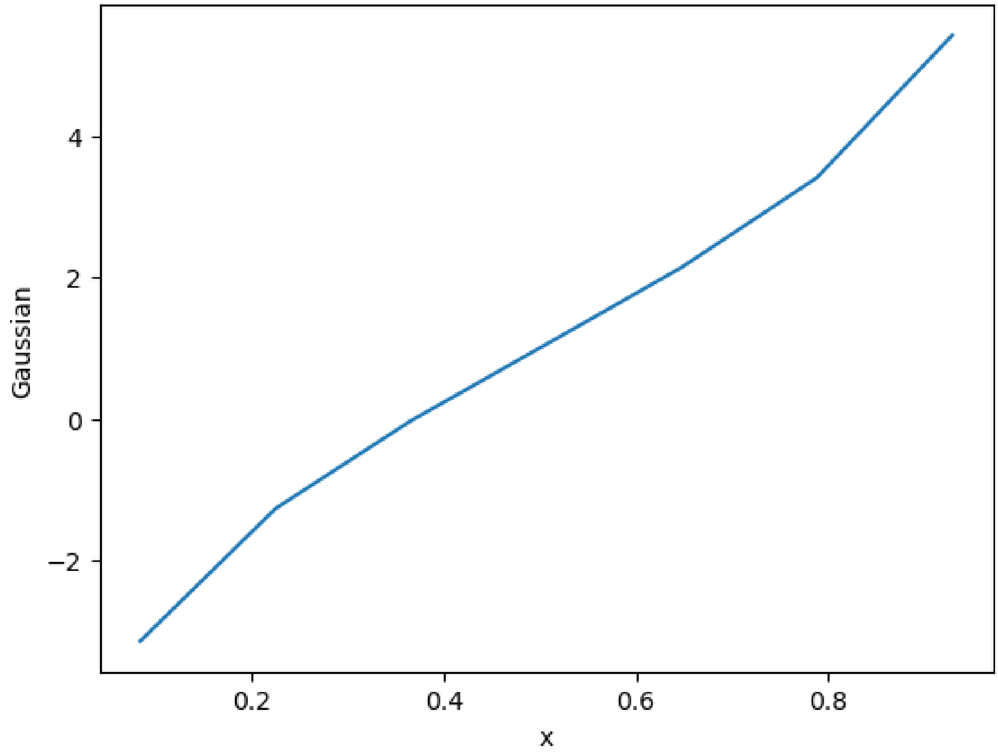
In [92]:
```
mu = 1
sigma = 3
mean, var = norm.stats(loc=mu, scale=sigma, moments='mv')
```

In [93]:
```
mean, var
```

Out[93]:
```
(array(1.), array(9.))
```

In [94]:
```
x= np.linspace(norm.ppf(0.01, mu, sigma), norm.ppf(0.99, mu, sigma), 100)
plt.plot(x,norm.ppf(x, mu, sigma))
plt.xlabel('x')
plt.ylabel('Gaussian')
```

Out[94]:
```
Text(0, 0.5, 'Gaussian')
```




*Graph not matched !!*

**Multinomial Distribution**

$$f(x; a_1, a_2, b_1, b_2) = \frac{1}{2(a_1 b_1 + a_2 b_2)}\left(b_1 \exp\left(-\sqrt{\frac{x}{a_1}}\right) + b_2 \exp\left(-\sqrt{\frac{x}{a_2}}\right)\right) \qquad 0 \leq x$$

Rolling a dice

In [95]:
```
from scipy.stats import multinomial
```

In [96]:
```
p = np.ones(6)/6
multinomial.pmf([6,0,0,0,0,0], n=6, p=p)
```

Out[96]:
```
2.143347050754453e-05
```

In [97]:
```
multinomial.rvs(n=50, p=p, size=5)  # 5 trials, each trial containing throwing dice 5
```

Out[97]:
```
array([[10,  7, 10,  7,  7,  9],
       [11,  6,  9,  5, 13,  6],
       [ 7,  9,  9, 10,  9,  6],
       [ 9, 12,  8,  7,  6,  8],
       [ 9,  9,  6, 10,  7,  9]])
```

Generating Random Numbers from your own distribution

```
In [98]: import scipy.stats as st
```

making of a new kind distribution. SKIPPED

```
In [ ]:
```