Differential Equations (Mr. P Solver)

Video Link: https://youtu.be/MM3cBamj1Ms (https://youtu.be/MM3cBamj1Ms)

Codes: https://www.youtube.com/redirect?event=video_description&redir_token=QUFFLUhqbUQ0LVI0VnlDNG1FeVhxSlZqcHFFdXBnVWlJd3xBQ3
(https://www.youtube.com/redirect?event=video_description&redir_token=QUFFLUhqbUQ0LVI0VnlDNG1FeVhxSlZqcHFFdXBnVWlJd3xBQ3

In [1]:
```
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
from scipy.integrate import odeint
from scipy.integrate import solve_ivp
```

There are 2 main solvers in scipy -

1. `odeint` : Uses a particular solver called lsoda from the FORTRAN library odepack.
2. `solve_ivp` : Can choose from a list of possible solvers.

In [2]:
```
odeint
```

Out[2]:
```
<function scipy.integrate._odepack_py.odeint(func, y0, t, args=(), Dfun=None, col_d
eriv=0, full_output=0, ml=None, mu=None, rtol=None, atol=None, tcrit=None, h0=0.0,
hmax=0.0, hmin=0.0, ixpr=0, mxstep=0, mxhnil=0, mxordn=12, mxords=5, printmessg=0,
tfirst=False)>
```

In [3]:
```
solve_ivp
```

Out[3]:
```
<function scipy.integrate._ivp.ivp.solve_ivp(fun, t_span, y0, method='RK45', t_eval
=None, dense_output=False, events=None, vectorized=False, args=None, **options)>
```

First Order ODEs

Example: Air Friction while falling:

$$\frac{dv}{dt} - \alpha v^2 + \beta = 0 \qquad v(0) = 0$$

Solution:

$$\frac{dv}{dt} = f(t, v)$$

or,

$$\frac{dv}{dt} = \alpha v^2 - \beta$$

In [4]:
```
def dvdt(t,v):
    return 3*v**2 -5
v0=0
```

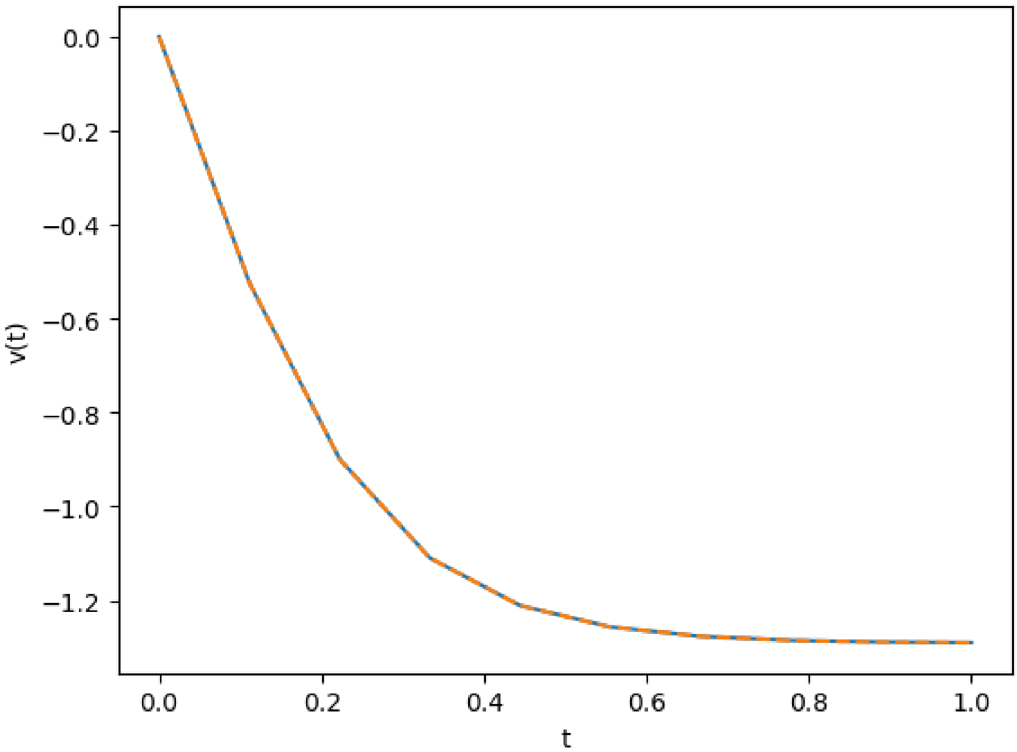
In [5]:
```python
t = np.linspace(0,1,10)
```

In [6]:
```python
sol1 = odeint(dvdt, y0=v0, t=t, tfirst=True)
sol2 = solve_ivp(dvdt, t_span=(0,max(t)), y0=[v0], t_eval=t)
```

In [7]:
```python
v1 = sol1.T[0]
v2 = sol2.y[0]
```

In [8]:
```python
plt.plot(t,v1)
plt.plot(t,v2, '--')
plt.xlabel('t')
plt.ylabel('v(t)')  # Same solution is obtained by the 2 methods.
```

Out[8]:
```
Text(0, 0.5, 'v(t)')
```



Coupled 1st Order ODEs

$$y_1' = y_1 + y_2^2 + 3x \qquad y_1(0) = 0$$

$$y_2' = 3y_1 + y_2^3 - \cos(x) \qquad y_2(0) = 0$$

Let,

$$\vec{S} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \Longrightarrow \qquad \frac{d\vec{S}}{dx} = \vec{f}(x, \vec{S}) = \vec{f}(x, y_1, y_2) = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_1 + y_2^2 + 3x \\ 3y_1 + y_2^3 - \cos(x) \end{bmatrix}$$

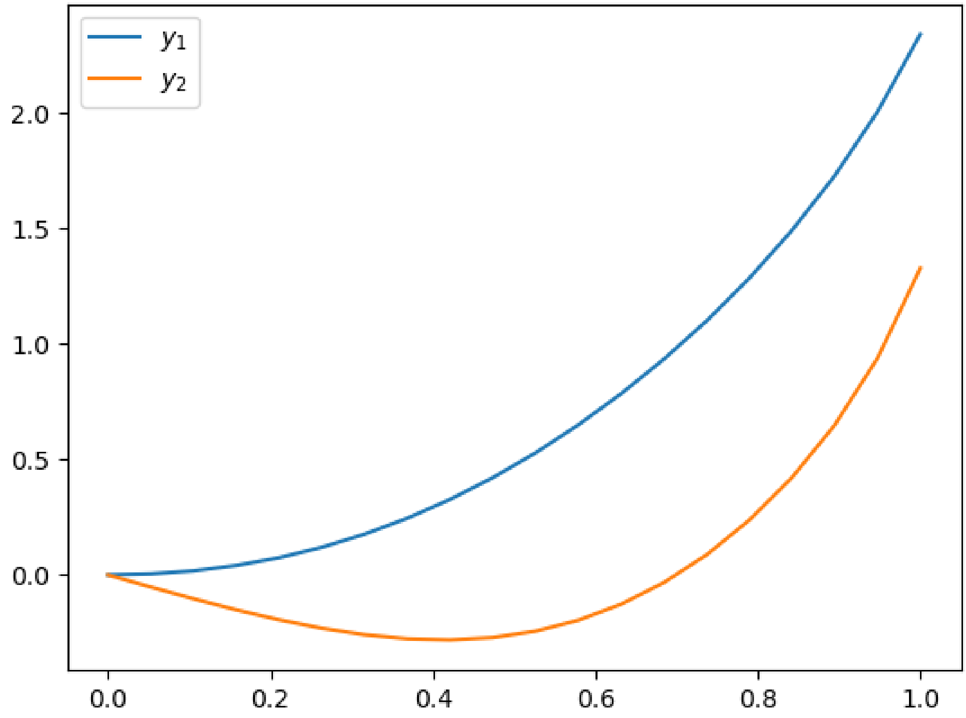
In [9]:
```
def dSdx(x,S):
    y1, y2 = S
    return [y1 + y2**2 + 3*x, 3*y1 + y2**3 - np.cos(x)]
y10 = 0
y20 = 0
S0 = (y10, y20)
```

In [10]:
```
x = np.linspace(0,1,20)
sol = odeint(dSdx, y0=S0, t=x, tfirst=True)
```

In [11]:
```
y1sol = sol.T[0]
y2sol = sol.T[1]
```

In [12]:
```
plt.plot(x, y1sol, label='$y_1$')
plt.plot(x, y2sol, label='$y_2$')
plt.legend()
```

Out[12]: <matplotlib.legend.Legend at 0x21d4c41aac0>



2nd Order ODEs

Can't solve 2nd order ODEs directly. We need to convert 2nd order ODE into 2 1st order ODEs and solve those.

Example: Consider,

$$\ddot{x} = -\dot{x}^2 + \sin(x)$$

Solution: To solve this our 1st order ODEs will be,

$$\dot{x} = v$$
$$\dot{v} = -v^2 + \sin(x)$$

These are 2 coupled 1st order ODEs. Let, the initial conditions for this problem are $\dot{x}_0 = 0$ and $x_0 = 1$.

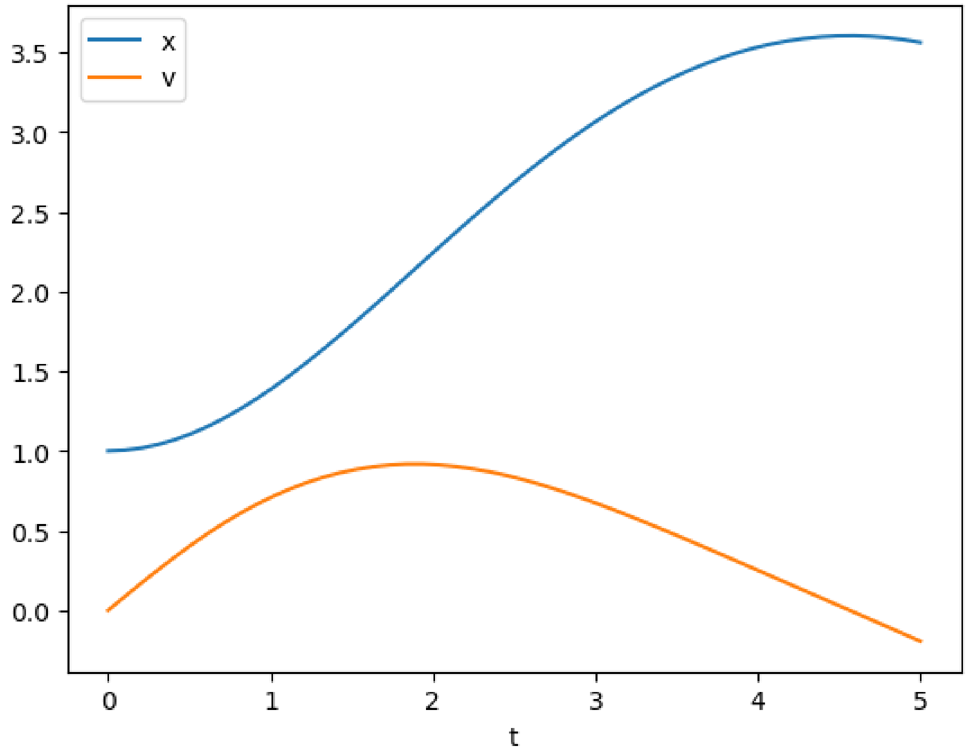
In [13]:
```
def dSdt(S,t):
    x, v = S
    return [v, -v**2 + np.sin(x)]
x0 = 1
v0 = 0
S0 = (x0, v0)
```

In [14]:
```
t = np.linspace(0,5,50)
sol = odeint(dSdt, y0=S0, t=t)
```

In [15]:
```
xsol = sol.T[0]
vsol = sol.T[1]
```

In [16]:
```
plt.plot(t, xsol, label='x')
plt.plot(t, vsol, label='v')
plt.xlabel('t')
plt.legend()
```

Out[16]: <matplotlib.legend.Legend at 0x21d4c5037f0>



Example:

$$\dddot{x}_1 = -2\dot{x}_2{}^2 + x_2$$

$$\ddot{x}_2 = -\ddot{x}_1{}^3 + \dot{x}_2 + x_1 + \sin(t)$$

Solution: Dependent variables are

$$x_1, x_2, v_1 = \dot{x}_1, v_2 = \dot{x}_2, a_1 = \ddot{x}_1 = \dot{v}_1, a_2 = \ddot{x}_2 = \dot{v}_2$$

So,

$$\dot{a}_1 = -2v_2^2 + x_2$$
$$\dot{a}_2 = -a_1^3 + v_2 + x_1 + \sin(t)$$

Then,

$$\vec{S} = \begin{bmatrix} x_1 \\ v_1 \\ a_1 \\ x_2 \end{bmatrix} \implies \frac{d\vec{S}}{dt} = \begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{a}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ a_1 \\ -2v_2^2 + x_2 \\ v_2 \end{bmatrix}$$

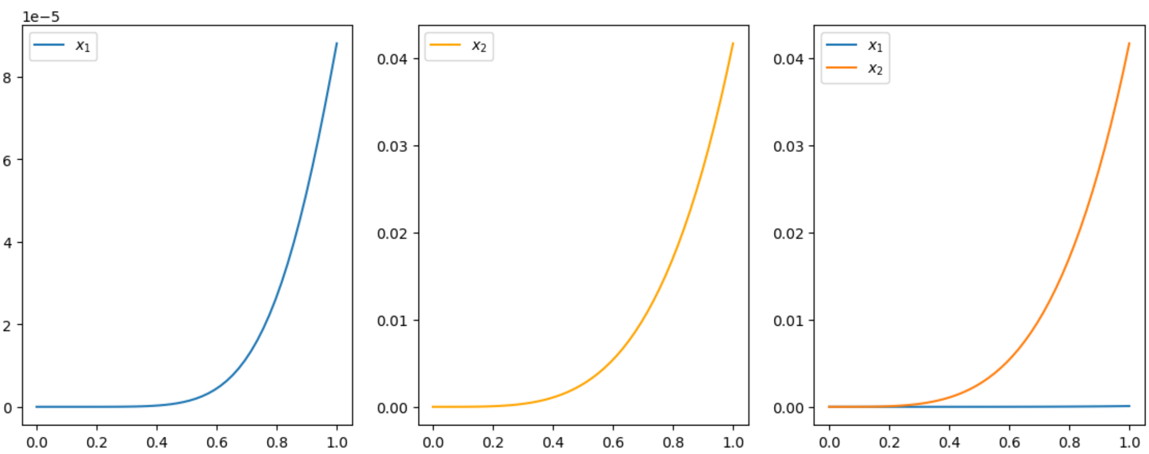
In [17]:
```
def dSdt(S, t):
    x1, v1, a1, x2, v2, a2 = S
    return [v1, a1, -2*v2**2 + x2, v2, a2, -a1**3 + v2 + x1 + np.sin(t)]

x10, v10, a10, x20, v20, a20 = 0, 0, 0, 0, 0, 0
S0 = (x10, v10, a10, x20, v20, a20)
```

In [18]:
```
t = np.linspace(0,1,50)
sol = odeint(dSdt, y0=S0, t=t)
```

In [19]:
```
x1sol = sol.T[0]
x2sol = sol.T[3]
```

In [20]:
```
fig, axes = plt.subplots(1,3, figsize=(14,5))
ax1 = axes[0]
ax1.plot(t, x1sol, label='$x_1$')
ax1.legend()
ax2 = axes[1]
ax2.plot(t, x2sol, 'orange', label='$x_2$')
ax2.legend()
plt.plot(t, x1sol, label='$x_1$')
plt.plot(t, x2sol, label='$x_2$')
plt.legend()
```

Out[20]: <matplotlib.legend.Legend at 0x21d4d7db5e0>



A Final Note:

Do it later

In [ ]: