CS 380
ALGORITHM DESIGN AND ANALYSIS

Lecture 16: String Matching and Searching
Brute Force, Horspool

References:

Levitin, Introduction to the Design & Analysis of Algorithms,3rd Edition, Pearson, 2012

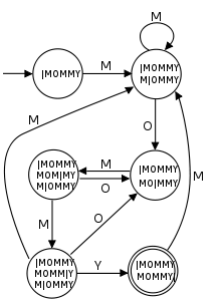
https://en.wikipedia.org/wiki/String_searching_algorithm

String Matching / Searching

- Naive
- **Horspool[1]**
- Boyer-Moore[1]
- Rabin-Karp[2] (Hash)
- **Knuth Morris Pratt[2] (DFA!)**

Not Dynamic Programming because there are not subproblems.

But precompute a table to help you solve the problem.



[1] Levitin, Introduction to The Design and Analysis of Algorithms, 3rd edition, Pearson Addison Wesley, p 259

[2] CLRS, p 990 &1002

Naïve: Example

- text: Asymptotic pattern: tic

http://whocouldthat.be/visualizing-string-matching/

Naïve Search

ALGORITHM *BruteForceStringMatch*$(T[0..n-1], P[0..m-1])$

```
//Implements brute-force string matching
//Input: An array T[0..n − 1] of n characters representing a text and
//       an array P[0..m − 1] of m characters representing a pattern
//Output: The index of the first character in the text that starts a
//        matching substring or −1 if the search is unsuccessful
for i ← 0 to n − m do
    j ← 0
    while j < m and P[j] = T[i + j] do
        j ← j + 1
    if j = m return i
return −1
```

Naïve Search: Analysis

Suppose text.length = n, pattern.length = m

- Worse case?
  - O(m*(n-m+1))=O(mn) comparisons
- Best case:
  - Ω(n) comparisons

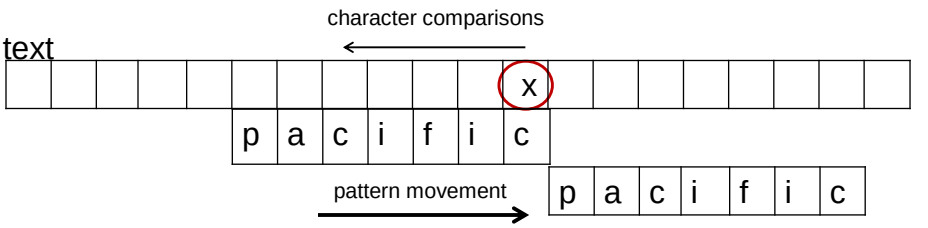
Horspool

- Match the pattern right (last character) to left (first character)
- On **mismatch**, shift the pattern as far as you can
  - By 1+ character(s)
- Preprocess string to determine shifting
  - build a table for shifts for each valid character

Four Possibilities: Case 1

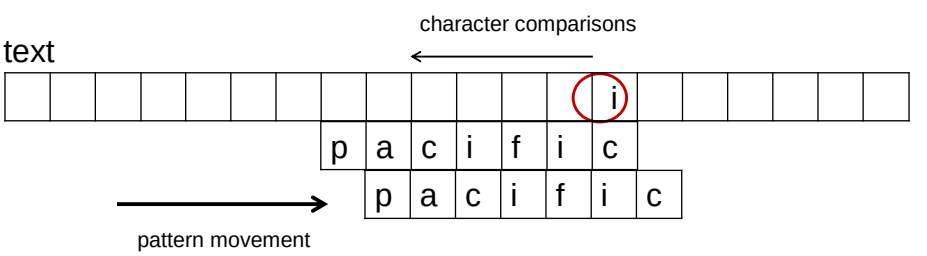
Assume on a **particular character c** in text

1) If character c (x in diagram below) is not in the **pattern**, can shift pattern by its entire length to the right!



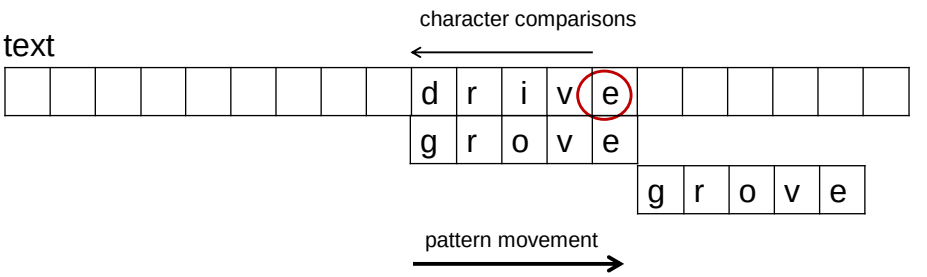
Four Possibilities: Case 2

2) If c (i below) is in the pattern **but not in last position**, shift pattern so character aligns with right-most occurrence of c in pattern



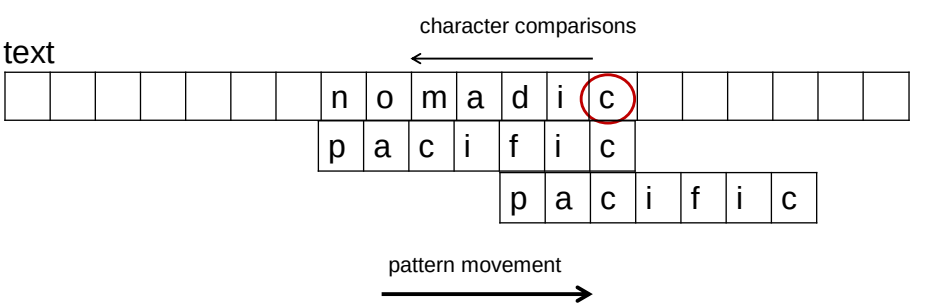
Four Possibilities: Case 3

3) If character c (e below) is **in the last position** and there are **no other** c's in rest of pattern, shift pattern by its entire length to right



Four Possibilities: Case 4

4) If c (c below) is **in the last position** but there are other c's in pattern, align with the right most occurrence of c in first m-1 characters



Moral from Cases

- Always align character with its right most occurrence in first m-1 characters of pattern. If there is no such occurrence, shift pattern by its entire length to the right.

Shifting

0 array indexing!

$$t(c) = \begin{cases} \text{the pattern's length } m, \\ \text{if } c \text{ is not among the first } m-1 \text{ characters of the pattern;} \\ \text{the distance from the rightmost } c \text{ among the first } m-1 \text{ characters} \\ \text{of the pattern to its last character, otherwise.} \end{cases} \quad \textbf{(7.1)}$$

p	a	c	i	f	i	c

a	b	c	…	f	…	i	...	p	...	x	y	z
			…		…		…		…	7		

ShiftTable

ALGORITHM *ShiftTable*$(P[0..m-1])$

//Fills the shift table used by Horspool's and Boyer-Moore algorithms
//Input: Pattern $P[0..m-1]$ and an alphabet of possible characters
//Output: $Table[0..size-1]$ indexed by the alphabet's characters and
// filled with shift sizes computed by formula (7.1)
for $i \leftarrow 0$ **to** $size-1$ **do** $Table[i] \leftarrow m$
for $j \leftarrow 0$ **to** $m-2$ **do** $Table[P[j]] \leftarrow m-1-j$
return *Table*

p	a	c	i	f	i	c

Pattern (m=7)

After being a 3!

a	b	c	…	f	…	i	…	p	…	x	y	z
7	7	7	…	7	…	7	…	7	…	7	7	7

Table

a	b	c	…	f	…	i	…	p	…	x	y	z
5	7	4	…	2	…	1	…	6	…	7	7	7

Horspool Algorithm

0 array indexing!

ALGORITHM *HorspoolMatching*$(P[0..m-1], T[0..n-1])$

//Implements Horspool's algorithm for string matching
//Input: Pattern $P[0..m-1]$ and text $T[0..n-1]$
//Output: The index of the left end of the first matching substring
// or -1 if there are no matches
ShiftTable$(P[0..m-1])$ //generate *Table* of shifts
$i \leftarrow m-1$ //position of the pattern's right end
while $i \leq n-1$ **do**
  $k \leftarrow 0$ //number of matched characters
  while $k \leq m-1$ **and** $P[m-1-k] = T[i-k]$ **do**
    $k \leftarrow k+1$
  if $k = m$
    return $i-m+1$
  else $i \leftarrow i + Table[T[i]]$
return -1

Horspool: Summary (p. 261)

1. Given text and pattern, construct the shift table
2. Align left side of pattern with left side of beginning of text.
3. Start with last character in pattern, start matching characters until all characters in pattern are matched OR you encounter a mismatch
4. If c is text character aligned with last character of pattern, shift pattern by t(c)
5. Repeat steps 3+4 until match or end of text.

Horspool analysis

m=pattern length, n=text length, k=alphabet length

- Preprocessing time: $\theta(m+k)$

- Searching time: Best: $\Omega(n/m)$
  Worst: $O(nm)$
  Average: $\theta(n)$

Trades space (construct ShiftTable) for time

Example

- String
  - G T A C T A G A G G A C G T A T G T A C T G
- Pattern:
  - A T G T A

- Generate the shift table
- Show the steps of the algorithm