Journal of Mathematical Sociology
1998, Vol. 23(1), pp. 29–49
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STATISTICAL MECHANICS OF COLLECTIVE BEHAVIOR: MACRO-SOCIOLOGY

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The spatial and temporal evolution of collective behavior in large populations is simulated with a cellular automaton model and predicted with a statistical mechanical analytical theory of macro-sociological behavior. The numerical cellular automaton simulations show that the type of collective behavior observed in a group depends sensitively on the group's social temperature with consensus more likely at higher temperatures and fragmented pockets of majority and minority opinions at lower temperatures. An analytical derivation using a mean field approximation confirms this behavior and also identifies a critical social temperature ($T_s \approx 1$) above which organized collective behavior disappears. Using social forces as well as social temperatures, the statistical mechanical theory predicts existing macro-sociological data on collective behavior.

INTRODUCTION

Each member of a group or population is influenced, to varying degrees, by the other people with whom they interact. In many cases, as each person adjusts his or her opinion according to the influence of other members of a group, a consensus will emerge. In some cases the group opinion can fluctuate wildly, never settling on a specific collective behavior or decision. In still other cases, the group will fragment into stable subsets of majority and minority opinions. The following statistical mechanical theory outlines the emergence of these macro-sociological

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† E. Passerini was supported by National Science Foundation Grant CMS-9312647 at the University of Colorado, Boulder, USA.

collective behaviors from a set of micro-sociological rules, and then identifies the most important group characteristics which determine the overall type of collective behavior.

The basic collective behavior problem can be visualized as a set of interactions between individuals. Imagine, for example, that a theater full of people are attending a presentation. Each person can whisper to people in the adjacent seats as well as individuals one seat forward and one seat behind (called nearest neighbors) but cannot effectively communicate with anybody further away. Suppose every individual in the theater has an opinion about the presentation, either positive or negative. Each individual is influenced by the opinions of the nearest neighbors, and according to the micro-sociological analysis developed in Bahr and Passerini (1998), this leads to some probability that the individual will change opinions. As the evening progresses, each person can change opinions many times, in each case being influenced by a network of nearest neighbors. The question is whether or not all the theater attendants will eventually come to a consensus or if there will be pockets of majority and minority opinions.

The same scenario can be repeated in many different settings with different social networks for the neighbors. At a football game, the range of communication is roughly shouting distance, so interactions could occur with neighbors over some radius. Interactions would be strongest with nearest neighbors, and weaker with neighbors farther away. A jury, on the other hand, has a social network which is more connected; everyone in the group has the potential to interact with everyone else. The question is again, under what circumstances will each group come to a collective behavior (such as storming the field or reaching a verdict)? Can the group dynamics be predicted or studied in detail?

A number of previous analyses have described and attempted to predict this type of collective social behavior based on theories ranging from thresholds (Granovetter, 1978; Macy, 1990, 1991) and critical mass (Oliver *et al.*, 1985; Oliver and Marwell, 1988; Marwell and Oliver, 1993) to games (Glance and Huberman, 1993; Heckathorn, 1993) and consensus emergence (Johnson and Feinberg, 1977; Feinberg and Johnson, 1988, 1990). Each of these theories has made different assumptions regarding micro-sociological interactions between actors in a group. Our previous work (Bahr and Passerini, 1998) shows that micro-level interactions can be derived as a consequence of basic probability and statistical mechanical theory. The results are then consistent with both empirical data as well as the basic premises of threshold and critical mass theories. In fact, the probabilistic approach predicts the size of the critical mass and shows that actors have a distribution of thresholds centered around this critical mass. The most significant advantage of the probabilistic approach is its rigorous mathematical foundation with few assumptions, which makes analytical and numerical predictions of macro-level collective behavior more accurate and easier to test against existing data.

Using the statistical mechanical foundation, the following theory shows that the most probable collective behavior depends on a group's social temperature – a measure of the group's decision making volatility (for a definition of temperature see Bahr and Passerini, 1998). The extreme of zero temperature leads to stable, unchanging collective behavior with pockets of minority and majority opinions. As group temperatures increase, the model's collective behavior tends toward a uniform decision without clustering of minority opinions. When the social temperature exceeds a certain limit, the group will have a well defined average opinion, but individuals are no longer stable and vacillate in a nearly random manner between different possible opinions. All of these diverse behaviors describe real group dynamics under different situations.

This paper is divided into four sections. The first gives a brief review of the micro-sociological foundation. The second and third sections both make a link from the micro-level to a macro-level model of collective behavior, but each section takes a very different tact. The second section simulates large scale collective behavior by using a numerical cellular automaton implementation of each actor's social interactions. On the other hand, the third section makes the same link, but uses an analytical statistical mechanical derivation of the macro-level collective behavior rather than a numerical simulation. Both of these very different approaches give the same results, which bolsters our confidence in the general statistical mechanical technique and suggests that the cellular automaton model is a reasonable approach to simulating collective behavior. To demonstrate the potential applications of the new theory, the final section uses the statistical mechanics to briefly examine teen delinquency and frequency of church attendance, and shows that the theory agrees with general macroscopic sociological observations on delinquency and religion.

1. MICRO-SOCIOLOGICAL BACKGROUND

The following discussion briefly outlines the statistical mechanical micro-sociology of Bahr and Passerini (1998) which is fundamental to the macro-level theory developed later in this paper. Suppose that each member of a group is choosing between a set of opinions (or actions) represented as $\sigma_1, \sigma_2, \ldots, \sigma_m$. Each pair of individuals in the group has the potential to interact, so let p_{ij} be the "interaction strength" or "opinion strength" which gives the ability of actor j to persuade or support actor i. Then Bahr and Passerini (1998) show that the probability of an actor selecting opinion k is given by

$$P_k(i) = \frac{\sum_k p_{ij}}{\sum_{j=1}^N p_{ij}} = \frac{\sum_{j=1}^N p_{ij} \prod_{l=1, l \neq k}^m ((s_j - \sigma_l)/(\sigma_k - \sigma_l))}{\sum_{j=1}^N p_{ij}} \tag{1}$$

where s_j is the opinion of the jth group member, and N is the total number of members in the group. In essence equation (1) just says that the probability of selecting a given opinion (or taking a certain action) is proportional to the number of actors that already have that opinion, modified to account for the strength of interactions between each actor. In fact, if the opinion strengths are the same for all actors, then the p_{ij} factor out and

$$P_k(i) = n_k / N \tag{2}$$

where n_k is the number of individuals with opinion k. In this case the proportionality is clear.

Notice that a relationship to network theories is immediately apparent through the parameter p_{ij}. The opinion strength p_{ij} links actors and can be a strong or weak connection. If p_{ij} is zero, then actor j is not influencing actor i. The opposite link p_{ji} could also be zero (indicating that the actors i and j have no contact whatsoever), or p_{ji} could be non-zero so that actor i is influencing j, even though j is not influencing i. This asymmetry in influence is typical of many social network theories (e.g., Marwell *et al.*, 1988), and could be caused, for example, by political figures that impact the public but who are separated or sheltered from the direct influence of other community members. In general, the set of all p_{ij} can be specified to mimic the structure of any social network, potentially allowing the results of both the network and statistical mechanical approaches to be integrated and applied to each other.

In addition to the opinion strengths in equation (1), Bahr and Passerini introduce several other important statistical mechanical parameters which influence opinion formation: social temperature, social noise, and social forces. Intuitively social temperature is the volatility of a group, and it indicates how much of a change in resources are required for an average individual in the group to change opinions. Social noise is a measure of miscommunications, mistakes, misunderstandings, and random unpredictable factors that will influence opinion formation. Social forces are defined as external processes which bias opinions (e.g., Helbing, 1994). Examples include political advertisements, news reports, laws and sanctions, and even natural disasters, which can highlight the need for social change. Social forces have a "direction" which indicates the opinion favored by the resulting bias, although social forces do not have to be solely directed at a single opinion and can be divided into biases favoring fractional amounts of many different opinions.

Social temperature (T_s), social forces (h), and noise (ϕ) can be incorporated into the micro-sociology by defining the probability of choosing opinion σ_k as

$$P_k = \frac{1}{z} e^{h_k/T_s} (g(k, N))^{1/T_s} + \phi_k \tag{3}$$

where

$$z = \sum_{k=1}^{m} e^{h_k/T_s} (g(k, N))^{1/T_s} \tag{4}$$

and $g(k, N)$ is the right hand side of equation (1) (or equation (2) if the opinion strengths p_{ij} are all identical). The variable h_k is the amount of the social force in the direction of opinion k, and $0 \leqslant \phi_k \leqslant 1$ is a random number for the noise. Note that social forces and noise are subject to the constraints that $\sum_k h_k = 0$ and $\sum_k \phi_k = 0$ (so that the total probabilities sum to one ($\sum_k p_k = 1$), as required by probability theory). Details of the derivations of equations (3) and (4) are in Bahr and Passerini (1998).

Equations (1)–(4) summarize the basic rules of opinion formation as derived by Bahr and Passerini (1998) using basic probability and statistical mechanical arguments. In this previous analysis these rules were shown to imply basic critical mass and threshold behavior, and were also shown to be consistent with available empirical data on micro-level opinion formation. The following two sections illustrate how these rules can be used to predict macro-sociological collective behavior.

2. NUMERICAL SIMULATION: A CELLULAR AUTOMATON

Many different techniques can be used to derive the macro-sociological behavior which emerges as a consequence of the fundamental micro-sociological equations (1)–(4). Two main distinctions between possible techniques are numerical simulations versus analytical mathematical derivations. In most cases, due to the complexity of the equations, a numerical simulation will be easier and more intuitive. We start therefore, by describing a particularly simple but surprisingly robust and flexible computer model called a cellular automaton which uses any specified social network to examine the macro-level consequences of the micro-sociological equations. After gaining insights from the simpler cellular automaton model, the more complicated analytical method will be presented in Section 3.

Cellular automata are a class of models widely used in physics to study the complicated collective behavior of many individuals in an ensemble. A classic example is water molecules in a river. Each molecule of water interacts with other molecules according to a set of micro-level rules (for water the rules satisfy conservation of mass and momentum), and then the cellular automaton puts the molecules and rules together to form observed macroscopic river flow such as currents, eddies, etc. (details of many cellular automata models, such as the one with water, can be found in Wolfram, 1986). In this study, we are interested in the collective behavior of many people in a group. So, as in the physics models, each group member interacts with neighbors. However, in this case, the cellular automata rules are specified by the micro-sociological interactions outlined in the previous section, and the cellular automaton puts the people and micro-sociology together to form macro-level collective social behavior.

Cellular automata have been used in the past by several other authors examining group behavior (e.g., Schelling, 1971; Nowak *et al.*, 1990; Lewenstein *et al.*, 1992; Latané *et al.*, 1994). In those models, which were based on entirely different sets of micro-sociological assumptions, the authors observed that consensus was rare and that groups tend to cluster into subsets of majority and minority opinions. These results, however, assumed that each person makes a decision based on "majority rule" – if more neighbors have opinion one (two), then the individual will certainly choose opinion one (two). This behavior is a restrictive special case of equation (3) with no social temperature and no social forces. In particular, when $h_1=0$ and T_s approaches zero, equation (3) becomes a majority rule step function. i.e., for the case of two opinions,

$$P_1=\begin{cases}0, & \text{if } n_1/N<1/2;\\ 1, & \text{if } n_1/N\geqslant 1/2,\end{cases}$$

and $P_2=1-P_1$. The choice is no longer a probability. The model presented here, however, uses the more inclusive form of the micro-sociology (equations (1)–(4)) which generalizes to cases with probabilistic decision making and includes the impact of nonzero social temperature and social forces. With the more general micro-sociology, our cellular automaton simulations produce a wider and more realistic array of collective behaviors, including consensus under some conditions.

The construction of a cellular automaton is simple, so we refer to the example given in the introduction of a theater full of people. The generalization to other situations will be straightforward and obvious. As stated, each individual in the theater interacts with four nearest neighbors (the social network). The interactions result in an opinion selected according to the probability given in equation (3). So consider a square grid, with each grid cell representing one of the occupants seated in the theater. Each individual has opinion 1 or 2 which we indicate by coloring the cell white or black respectively; if an individual changes opinions, then the color in the cell changes. For simplicity, assume that $p_{ij}=1$ for all i and j, $T_s=1$, and $h_1=h_2=0$, so that equations (3) and (4) reduce to equation (2), the percent of neighbors with each opinion. For example, in Figure 1, the individual in the theater seat labeled i has opinion 1 (white), but all four neighbors have opinion 2 (black).

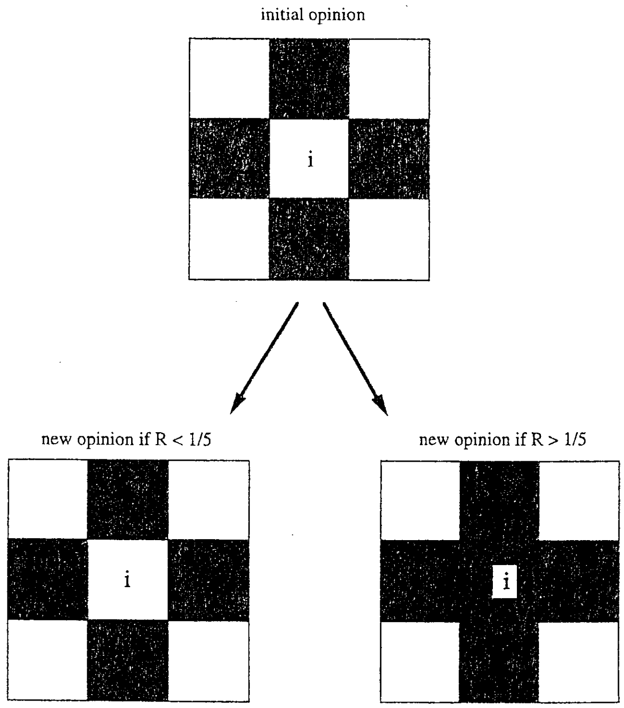

FIGURE 1 Nearest neighbor interactions for an individual at site *i*. White and black represent two different opinions at some initial time step. At the next time step, the individual changes opinion if a random number R is greater than the fractional number of neighbors with the same opinion (see text). Note that every site on the grid is simultaneously updated at each time step, although only site *i* is updated in this example.

Therefore, using equation (2) the *i*th individual's probability of choosing opinion 1 (white) at the next time step is $P_1 = 1/5$, and the probability of choosing opinion 2 (black) at the next time step is $P_2 = 4/5$ (because four out of the five are black). To determine which opinion is selected, a computer generates a random number between zero and one. If the random number is between 0 and 1/5, then individual *i* chooses opinion one, and if the random number is between 1/5 and 1, then the individual chooses opinion two. The same process is repeated for all the group members so that at each cellular automata model time step, everyone has the opportunity to change opinions or remain the same.

Note that in cellular automaton models there is no inherent restriction to the nearest neighbor geometry. The same type of interactions could occur with ties to distant individuals (as in Granovetter, 1978), and there could be "holes" where individuals are missing (in the above example, empty theater seats). Decisions could also be based on an even longer history of the neighbor's opinions (i.e., an even higher order Markov chain). Furthermore, rather than being fixed to their sites, the individuals could be allowed to move around according to some additional micro-sociological rules (see for example, Schelling, 1971). To illustrate only the basic principles, we keep the analysis simpler and do not include these generalizations. However, there is room for many interesting generalizations in future studies.

Note that another important feature of social cellular automaton models is that every individual in the simulation is allowed to interact with their neighbors simultaneously (rather than sequentially). For example, as in real life, many different conversations between different individuals can occur simultaneously in the theater; there is no restriction, as in some models (e.g., the learning theoretic model of Macy, 1991), that one "leader" formulates an opinion, and then other individuals in the theater follow with decisions in some sequence. Instead, many different leaders can be influencing many people simultaneously, and everyone is formulating opinions at the same time. Although, the following analysis focuses on simultaneous decision making processes, cellular automaton models are not inherently synchronous. A mix of sequential and asynchronous decision making is also possible. In a future analysis, for example, it would be interesting to allow interactions to occur with some frequency which depends on the distance of the neighbors. In other words, far away neighbors (like distant relatives) could interact infrequently, while nearest neighbors (the immediate family) could interact constantly.

The primary advantage of social cellular automata simulations is that the interactions of very large groups can be modeled efficiently, and the additional calculations associated with nonzero social forces, temperature and variable opinion strengths poses no difficulty for a computer simulation. Figure 2a, for example, illustrates an initially random assortment of opinions in the theater. $h_k = 0$, but p_{ij} are allowed to be different for each pair of individuals, and $T_s = 0.1$. After 500 time steps the opinions become stable and unchanging and are shown in

Figure 2b. Notice the clustering of minority and majority opinions. Clusters occurs around individuals with strong opinion strengths (i.e., large p_{ij}).

Cellular automata simulations with different networks lead to several general observations about the collective behavior of large groups choosing between two opinions or courses of behavior (call these σ_1 and σ_2). For $T_s \lesssim 1$, opinions fluctuate for a time and then settle on a consensus of all σ_1 or all σ_2. If $T_s \gtrsim 1$, then opinions fluctuate randomly with half having opinion σ_1 and half having σ_2 at any time. As T_s approaches 1 from below, the time it takes for the system to reach a consensus (or equilibrium) takes longer and longer and clusters of minority opinions take longer to disappear. As T_s approaches 0, the

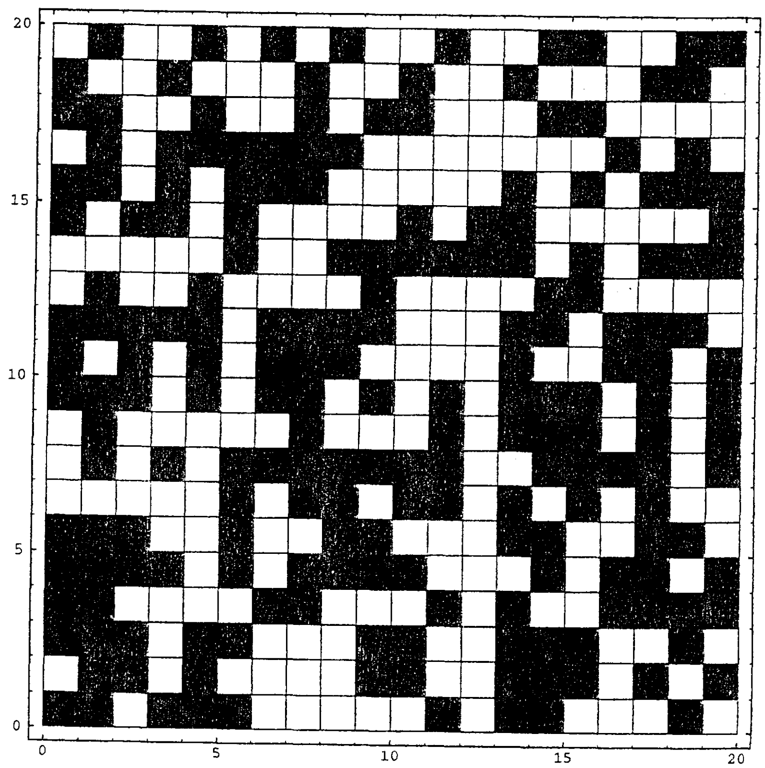

FIGURE 2(a)

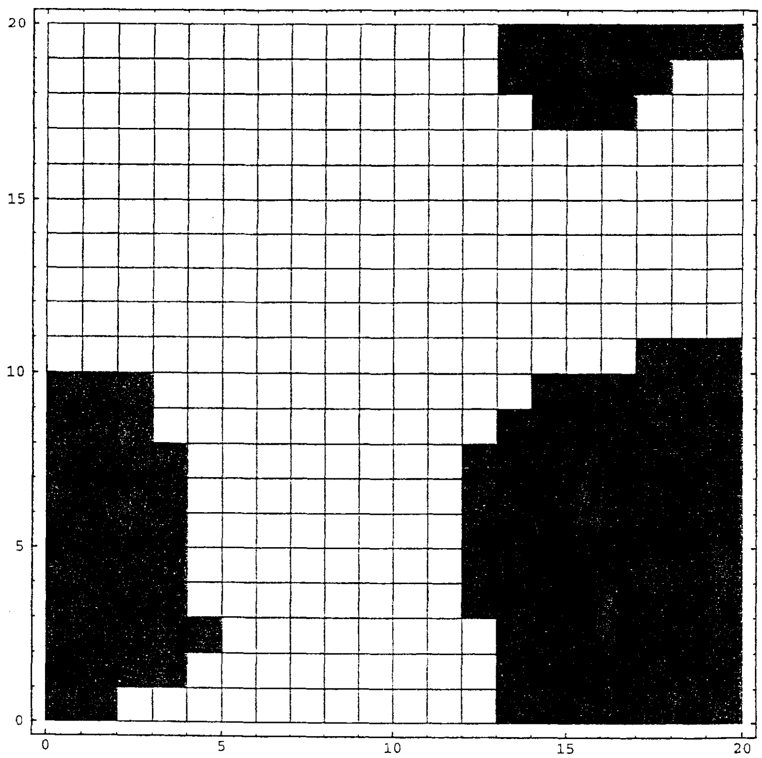

FIGURE 2(b)

FIGURE 2 (a) An initially random distribution of two opinions (white and black) in a group of 400 people with nearest neighbor interactions. (b) After 500 time steps of the cellular automata model, the opinions have clustered into majority (white) and minority (black) subsets. See text for model details.

time to consensus also increases dramatically, and clusters of minority opinions become essentially permanent (as in Figure 2b) because they are disappearing exceedingly slowly. If $h_k \neq 0$ and $T_s \lesssim 1$, then the consensus will be in the direction of the force's bias: e.g., if $\sigma_1 = 1$ and $\sigma_2 = -1$, then $h_1 > 0$ will lead to a consensus of 1, and $h_1 < 0$ will lead to a consensus of -1 (because $\sum_k h_k = 0$, $h_1 < 0$ is the same as $h_2 > 0$). As h_k gets larger, consensus is reached more rapidly. If $h_k \neq 0$ and $T_s \gtrsim 1$, then the opinions fluctuate randomly, but with the percent of individuals having a given opinion larger in the direction of the force's bias.

3. ANALYTICAL SIMULATION: MEAN FIELD APPROXIMATION

The simulated outcome of a group's behavior, numerically illustrated with the cellular automaton model, can be analytically predicted using the so-called mean field approximation from statistical mechanics (Plischke and Bergersen, 1989). The advantage of analytical solutions is that they are more accurate than numerical simulations (which always involve some level of numerical approximation), although this is often offset by insurmountably complicated derivations. Mean field approximations are exact only when everyone in a group is a neighbor with everyone else. However, the technique gives a reasonable estimate for other social networks, including the case of nearest neighbors (this generality is known from its application to different but analogous problems in the physical sciences), and the approximation to this simplified social network makes the extremely complicated derivations much more tractable. The technique is called mean field because it assumes that the opinion of any one individual can be estimated from the average (or mean) opinion of all other individuals. In other words, it assumes that the influence of an individual's neighbors is approximated by the average influence of the entire group.

For purposes of comparison with the cellular automaton examples given above, we restrict attention to groups choosing between two different opinions. (This also simplifies the derivations, although extensions to more opinions are possible). Without loss of generality, let $\sigma_1 = 1$ and $\sigma_2 = -1$. For ease of note keeping, let n_1 be the number of individuals with opinion "1" and n_{-1} be the number of individuals with opinion "-1". Also, let P_1 be the probability of choosing opinion "1" and P_{-1} be the probability of choosing opinion "-1". In the mean field approximation $p_{ij} \approx \langle p_{ij} \rangle$, so P_1 and P_{-1} are given by equations (2), (3) and (4) with n_{-1} substituted for n_2. Also define h_{-1} as the force in the "-1" direction, but recall $\sum_k h_k = 0$, so $h_{-1} = -h_1$.

Note that P_1 and P_{-1} are the same for all individuals because they must be functions of the same average influence of the entire group. Therefore, NP_1 is the number of -1's in the group, and NP_{-1} is the number of -1's in the group. $(NP_1 - NP_{-1})/N = P_1 - P_{-1} = \langle s_i \rangle$ is the average opinion of the group. In other words,

$$\langle s_i \rangle = \frac{e^{h_1/T_s}(n_1/N)^{1/T_s} - e^{-h_1/T_s}(n_{-1}/N)^{1/T_s}}{e^{h_1/T_s}(n_1/N)^{1/T_s} + e^{-h_1/T_s}(n_{-1}/N)^{1/T_s}}. \tag{5}$$

Now note that in the mean field approximation $s_j \approx \langle s_j \rangle$, so

$$n_1 = \sum_j \frac{s_j + 1}{2} \approx \sum_j \frac{\langle s_j \rangle + 1}{2} = \frac{1}{2} q(\langle s_j \rangle + 1) \tag{6}$$

and

$$n_{-1} = -\sum_j \frac{s_j - 1}{2} \approx -\sum_j \frac{\langle s_j \rangle - 1}{2} = -\frac{1}{2} q(\langle s_j \rangle - 1) \tag{7}$$

where the sum is over neighbors, and q is the number of neighbors. Substituting equations (6) and (7) into (5), and factoring out $1/2q$ and N, gives

$$\langle s_i \rangle = \frac{e^{h_1/T_s}(1 + \langle s_i \rangle)^{1/T_s} - e^{-h_1/T_s}(1 - \langle s_i \rangle)^{1/T_s}}{e^{h_1/T_s}(1 + \langle s_i \rangle)^{1/T_s} + e^{-h_1/T_s}(1 - \langle s_i \rangle)^{1/T_s}}. \tag{8}$$

Equation (8) can be solved for the average opinion $\langle s_i \rangle$ as a function of T_s and h_1. In general, this requires a numerical solution. Results are shown in Figures 3a and 3b. Note that as $h_1 \to 0$, there is a sudden transition from an average opinion of $\langle s_i \rangle = 1$ to an average opinion

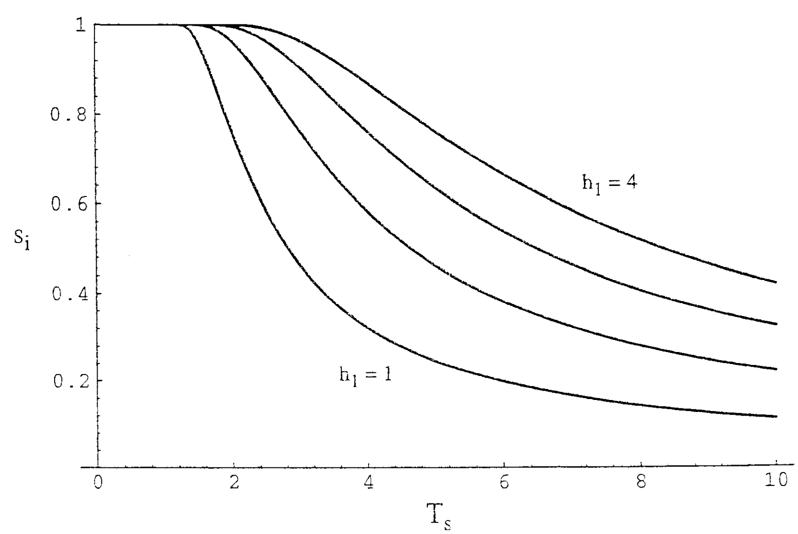

FIGURE 3(a)

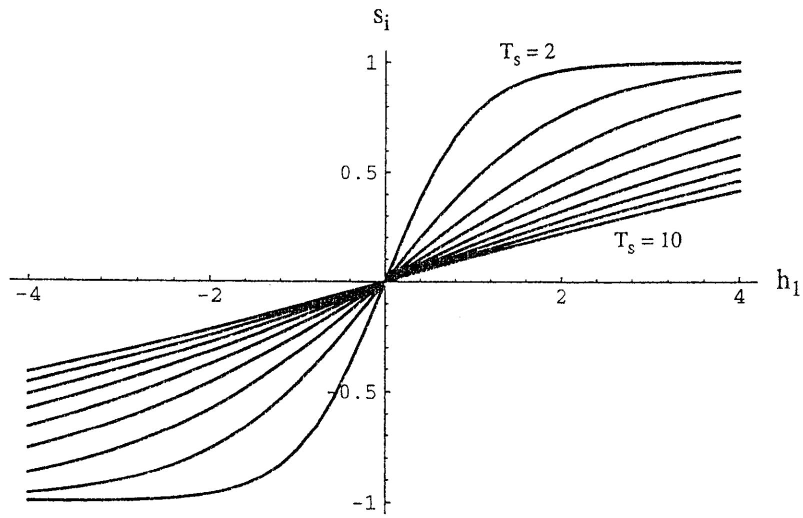

FIGURE 3(b)

FIGURE 3 (a) Average opinion of a group ($\langle s_i \rangle$) versus social temperature (T_s) with social forces $h_1 = 1, 2, 3$, and 4. Note that as h_1 approaches zero there is a step from $\langle s_i \rangle = 1$ to $\langle s_i \rangle = 0$ at the critical temperature T_c (see text). Note that there are two possible solutions to equation (38), $\pm\langle s_i \rangle$. Only the positive branch is shown here (these correspond to solutions with $h_1 > 0$). (b) Average opinion of a group versus social forces for temperatures $T_s = 2, 3, \ldots, 10$. As T_s approaches zero, $\langle s_i \rangle$ becomes a step function, and in this case, any amount of social forcing results in complete group consensus.

of $\langle s_i \rangle = 0$. This is the transition observed in the numerical simulations: from near consensus with well-ordered pockets of opinion at low temperatures (e.g., Figure 2b), to less-ordered, non-consensus at higher temperatures (e.g., Figure 2a) with average opinions determined by the magnitude of the social forces. The point of transition is called the critical temperature, T_c, and from the numerical solutions appears to be near $T_s = 1$.

From the plots (and from numerical simulations) as $T_s \to T_c$, the average opinion goes to zero. So consider equation (8) for $|\langle s_i \rangle| \ll 1$. This will describe group behavior near the critical temperature. Note that

$$\langle s_i \rangle = \frac{e^{h_1/T_s} e^{(1/T_s)\log(1+\langle s_i \rangle)} - e^{-h_1/T_s} e^{(1/T_s)\log(1-\langle s_i \rangle)}}{e^{h_1/T_s} e^{(1/T_s)\log(1+\langle s_i \rangle)} + e^{-h_1/T_s} e^{(1/T_s)\log(1-\langle s_i \rangle)}} \tag{9}$$

and $\log(1 \pm \langle s_i \rangle) \approx \pm \langle s_i \rangle$ for small s_i. Therefore,

$$\langle s_i \rangle \approx \frac{e^{(\langle s_i \rangle + h_1)/T_s} - e^{-(\langle s_i \rangle + h_1)/T_s}}{e^{(\langle s_i \rangle + h_1)/T_s} + e^{-(\langle s_i \rangle + h_1)/T_s}}$$

$$= \tanh\left(\frac{\langle s_i \rangle + h_1}{T_s}\right). \tag{10}$$

To find T_c, consider $h_1 = 0$. Then, by a low order Taylor expansion,

$$\langle s_i \rangle \approx \frac{\langle s_i \rangle}{T_s} - \frac{1}{3}\left(\frac{1}{T_s}\right)^3 \langle s_i \rangle^3 + \cdots \tag{11}$$

Therefore,

$$\langle s_i \rangle^2 = 3T_s^3\left(\frac{1}{T_s} - 1\right) \tag{12}$$

and

$$\langle s_i \rangle = \pm 3^{1/2} T_s^{3/2}\left(\frac{1}{T_s} - 1\right)^{1/2}. \tag{13}$$

As T_s approaches one, $\langle s_i \rangle$ approaches zero, and therefore $T_c = 1$.

In other words, when social forces are negligible, there is a critical temperature $T_s = 1$ at which groups will move from consensus opinions to disordered opinions with an average value of zero. If social forces are not negligible, then there will be an effective critical temperature $\tilde{T}_s > 1$ such that $T > \tilde{T}_s$ causes a transition from a consensus opinion to randomly arranged opinions with an average of zero. Small temperature perturbations of groups near the critical temperature could dramatically change their behavior. For social movement organizers, this means that if a large group is near the critical temperature, then infusing it with a much smaller group of very high or very low temperature can have a dramatic effect on the collective outcome. Infusing the large group with a smaller lower temperature group can have a stabilizing effect which keeps the group in consensus. Contrary to intuition, however, infusing with a higher temperature subgroup could actually be counter-productive by making everyone so volatile that consensus is lost to chaotic disorder.

By standard statistical mechanical arguments, the time to reach a consensus (or equilibrium) should depend on the square of the correlation length between opinions (the correlation length can be imagined as the average cluster size) (Ma, 1985). As $T_s \rightarrow T_c$, the correlation length goes to infinity. In the cellular automata simulations, the size of the group is always finite, so we just expect that the correlation length will reach a maximum at the size of the group. This means that the time to reach equilibrium is proportional to the size of the group. This has been confirmed with cellular automata simulations for the nearest neighbor social network.

In addition to confirming the behavior of the cellular automaton model, an important conclusion of the mean field approximation is that as the temperature approaches critical, the time to reach equilibrium or consensus becomes very long. Practically speaking, this suggests that groups near the critical temperature may be forced by time constraints to vote or choose an opinion long before reaching equilibrium. A notable policy implication is that to avoid misguided and premature actions, it would be wise to delay group decisions as long as possible when temperatures are high.

4. APPLICATION TO TEEN DELINQUENCY AND RELIGION

To demonstrate the preceding theory's applicability to sociological problems, we present a brief example using teenage delinquency and religion. The intent of this paper is primarily the derivation and brief exploration of a framework for studying social interactions and collective behavior, so a full exposition of this application is reserved for other papers. The presentation here is not intended to be exhaustive, and does not give a detailed analysis of the theories surrounding teen delinquency. What this example does show, however, is that the statistical mechanical theory and cellular automaton model are capable of predicting observed macro-sociological behavior. The example also shows that the theory and model are capable of suggesting new macro-level relationships which might be tested with additional empirical observations.

Stark *et al.* (1982) and many others (e.g., Higgens and Albrecht, 1977; Linden and Currie, 1977) have used empirical data to illustrate that teenage delinquency is negatively correlated with the frequency of church attendance. However, this correlation is stronger in regions where high percentages of a population attend church, and the correlation becomes negligible when the majority of a population does not attend church. A common explanation is that social interactions are responsible for helping frame the moral structure of teenagers; in regions with a visible and prevalent religious community, teenagers are more likely to frame issues in a religious (non-delinquent) context. Therefore, a strong religious community deters delinquent behavior.

In the context of a cellular automaton model, each member of a community of teenagers is faced with an issue which requires a choice between delinquent ($\sigma_{-1} = -1$) or non-delinquent ($\sigma_1 = 1$) behavior. The moral structure imposed by a religion can be treated as a force h_1 which is applied only to those individuals who attend church and is applied with a frequency which is related to the frequency of church attendance. Clearly, $h_1 > 0$ so that decisions made by teenagers attending church are biased toward non-delinquent behavior. Opinion strengths (p_{ij}) vary randomly from individual to individual, as expected in a real community.

In this example we assume that the social network is "next-nearest neighbor", so that each teenager interacts with only the other eight nearest teenagers on a square grid. In other words, the network is similar to the theater problem. This is a moderately restrictive network, but the results should still be qualitatively correct because different networks have the same fundamental behavior (Lewenstein *et al.*, 1992) which is also the reason that the mean field approximation works. A more detailed study of this problem could generate better results with a social network which more closely mimics the structure of a real teenage community. (Again, however, our intent is only to illustrate the general applicability of the statistical mechanical theory.)

To see roughly what the theory predicts about delinquent behavior, consider equation (3). With a little manipulation, the probability of a teenager choosing delinquency becomes

$$P_{-1} = \frac{1}{e^{2h_1/T_s}(n_1/(N-n_1))^{1/T_s}+1} \tag{14}$$

where n_1 is the number of neighbors who attend church (we assume momentarily that the opinion strengths are constant and not random). Then as h_1 increases, the probability of delinquency decreases. Similarly, as the number of neighbors attending church increases, P_{-1} decreases. In other words, increasing the frequency of church attendance in a community will decrease the probability of delinquency, and increasing the numbers of teens attending church will decrease delinquency. Our model corresponds to the empirical data (Stark *et al.*, 1982).

However, by running the cellular automaton at different social temperatures and different frequencies of church attendance, the outcome is invariably that teenage delinquency and church attendance are not correlated. Initially the correlations exist, but they disappear with time. Instead, over long periods of time, the most important factor becomes opinion strengths (ignored in equation (14)). If a strong opinion (i.e., a leader with high p_{ij}) behaves delinquently, then a cluster of surrounding individuals will also behave delinquently. If a strong opinion is not delinquent, then a cluster of surrounding individuals will also be non-delinquent. The lack of correlations, however, disagrees with empirical data.

What is missing? The correlations are restored if teenagers who attend church form stronger bonds with other teenagers who also attend church, and if teenagers who do not attend church form stronger bonds with other teenagers who do not attend church. In other words, let p_{ij} be large between two teenagers who both attend church and between two teenagers who both do not attend church; and let p_{ij} be small between a pair of teenagers with one attending church and the other not attending church. With this assumption, teenagers who attend church are more supportive and persuasive with each other than with teenagers who do not attend church. Now, in the cellular automaton simulations, clusters still form around the strongest personalities, but when the community of church attending teenagers is large there are also correlations between the frequency of church attendance and delinquency. Also, the correlations disappear when the community of teenagers attending church is small. These modeled results agree with the empirical data. It would be an interesting empirical study, therefore, to determine if moral interactions between teenagers attending church are indeed stronger (more persuasive, supportive, etc.) than interactions between two teenagers with one attending church and the other not.

CONCLUSIONS

By building on a set of micro-sociological rules, the collective behavior of large groups can be simulated with both analytical derivations from statistical mechanics and numerical cellular automaton models. The analytical and numerical approaches produce identical results and predict a sensitive dependence of collective behavior on social temperature. Higher social temperatures increase the likelihood of group consensus, while lower social temperatures tend to form stable majority and minority subgroups. However, temperatures which are too high make groups too volatile and increase the time it takes to reach a consensus. Above a critical temperature, the group becomes so volatile that opinions fluctuate randomly and organized collective behavior disappears.

The long times required to reach consensus near the critical social temperature have significant implications for social movement organizers and conflict managers. Forcing groups to select an option or take an action at higher temperatures can lead to decisions which would not otherwise be accepted at lower temperatures. At high temperatures, opinions fluctuate dramatically before settling on a consensus. Therefore, at high social temperatures small subgroups of a population can promote unpopular agendas by forcing a well-timed premature vote which coincides with a favorable fluctuation. If a more representative consensus is desired, however, then motivational rhetoric could assist by acting to center decisions around a strong leader (p_{ij} would be strong between the leader j and all other group members i). As expected, this will lead to a group decision which is most likely to agree with the strong leader's opinion. This is helpful for social movement organizers, but the biased outcome would be inappropriate for situations such as conflict resolution where the biases of the facilitator are not supposed to factor into the group's final decision. An alternative approach, in this case, is to lower the group's social temperature by adding new members as a subgroup with a lower temperature. The new subgroup's interactions will move the entire group away from the critical temperature and towards social temperatures which promote timely consensus.

The macro-sociological cellular automaton model and statistical mechanical theory of collective behavior have potential applications in many different contexts. Minorities, for example, are thought to be the instigators of many aspects of social change, and the ability of the model to couple minority and majority opinion formation may illuminate this process of social transformation. Using the concepts of social temperature and social force, the model can also be used to study the collective response of groups to natural disasters, political advertisements, news reports, laws, sanctions, and other social forces. By tying together micro-sociological interactions to macro-sociological phenomena and realizing that group behavior is predictable despite the ability of each individual to make personal decisions, the model and theory can also be used to understand and potentially mitigate and direct many other aspects of collective behaviors in committees, legislatures, clubs, sporting events, communities, nations, and other large groups.

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