505 STAT MECH TAKE-HOME (MIDTERM 2)

BEGIN THU APRIL 17 2014 (You may consult anything but no person other than yourself)

SUBMIT ANSWERS BY SAT NIGHT (7 pm, April 19 2014, by emailing pdf to the instructor kenkre@unm.edu with copy to the TA aierides@unm.edu)

SOLVE ALL 3 QUESTIONS. PROVIDE PLOTS WHERE ASKED.

THE ANSWER HAS TO BE AN ELECTRONICALLY DELIVERED PDF.

QUESTION 1

Consider a 1-d ring, i.e., a lattice in 1-d with periodic boundary conditions. There are 8 sites in the ring, all equivalent. Initially ($t = 0$), a walker is straddling equally two neighboring sites. The walker executes a random walk in continuous time with nearest-neighbor rates F to jump left or right.

(i) Write a Master equation for the probabilities of occupation of the sites. (ii) Solve the equation analytically (NOT on mathematica or a similar program, NOT numerically either) for all probabilities at all times t, showing the analytic expression explicitly, and (iii) evaluate in particular the probability of occupation of the site next to one of the initially occupied sites (and not occupied initially) for all time.

QUESTION 2

Consider the Master equation for a hyperhedron, a system wherein each site of the system is connected via equal rates to EVERY ONE of the N sites of the system. The name hyperhedron comes from a generalization of a tetrahedron. Calculate the probability of occupation of an initially occupied site explicitly and plot for the case of $N = 10$ versus an appropriately defined dimensionless time. (The initial condition is that only one site is occupied initially.)

QUESTION 3

1.(a) Write down a Master equation to describe the motion of a random walker taking biased steps on the sites of a 1-dimensional infinite lattice, explaining all symbols you use. Biased means that the random walker tends to move to its left more often than to its right.

(b) Do the same as above for a situation in which the walker is attracted towards a specific single site, the attraction being more intense the farther the walker is from the site. In each of these cases use rates that will make it possible for you to solve the problem at least in principle. Comment here on what kind of rates would make it tough or impossible for you to solve the problem and why. **REMEMBER both a and b refer to DISCRETE lattices not a continuum** (what was discussed in class was a continuum.)

For the case a (but not b) calculate the propagator giving an explicit expression in the form of quadratures (that means as an integral) and attempt if you can (if you cannot it is fine) to express it in terms of special functions.