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Parameter constraints on closed-form soilwater relationships

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ABSTRACT

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The constraints on the different fitting parameters used in the water retention equation, $h(\theta)$, and hydraulic conductivity, $K(\theta)$, are analyzed using the infiltration equation as a testing tool. The following characteristic equations are considered: those of Gardner; Brooks and Corey; Brutsaert; Van Genuchten subject to both Mualem's and Burdine's condition; Van Genuchten combined with Brooks and Corey; and Fujita. It is shown that most combinations of $h(\theta)$ and $K(\theta)$ or $K(h)$ break down, when tested over the large range of soil types encountered in field situations. For clay soils, especially, the best-fit parameter values often become inconsistent with the infiltration theory. The best combination is the Van Genuchten equation for $h(\theta)$ with the Burdine condition $m = 1 - 2/n$ and the Brooks and Corey equation for $K(\theta)$. This combination satisfies the infiltration condition for all soil types, even when applied to the two extreme cases used by Green and Ampt and Talsma and Parlange. The interdependence of $h(\theta)$ and $K(\theta)$ parameters is discussed.

INTRODUCTION

Knowledge of hydraulic soil properties, expressing water pressure head, h(cm), as a function of volumetric water content, $\theta(\mathrm{cm^3\,cm^{-3}})$ and hydraulic conductivity, $K(\mathrm{cm\,hr^{-1}})$ as a function of θ, is of prime importance in many field studies dealing with water transport in the unsaturated zone. Laboratory and/or field measurements of the $h(\theta)$ and $K(\theta)$ values give scattered experimental data points. As model studies require continuous relations, closed-form expressions are used to fit the experimental data points. The many relations proposed in the literature, can be divided into four groups based on their dependent variables:

(1) $\theta(h)$ (e.g. Brooks and Corey, 1964; King, 1965; Brutsaert, 1966; Farrel

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and Larson, 1972; Van Genuchten, 1980; Haverkamp and Vauclin, 1981; Haverkamp and Parlange, 1986);

(2) $K(\theta)$ (e.g. Averyanov, 1950; Irmay, 1954; Brooks and Corey, 1964; Simmons et al., 1979; Van Genuchten, 1980; Broadbridge and White, 1988; Sander et al., 1988);

(3) $K(h)$ (e.g. Wind, 1955; Rijtema, 1965; Gardner, 1958); and

(4) $D(\theta)$, where D is the diffusivity defined by $D(\theta) = K(\theta)\,\mathrm{d}h/\mathrm{d}\theta$ (e.g. Fujita, 1952; Gardner and Mayhugh, 1958).

The solution of the transport equation (e.g. Fokker Planck's equation) requires only two functional relations, but the number of fitting parameters involved can be of the order of four to five depending on the relations chosen. To overcome this problem some authors (e.g. Brooks and Corey, 1964; Brutsaert, 1967; Van Genuchten, 1980) developed relationships between two of the parameters used in $h(\theta)$ and $K(\theta)$, based on the use of ad hoc capillary models (e.g. Childs and Collis-George, 1950; Burdine, 1953; Millington and Quirk, 1961; Mualem, 1976a).

Most parameters are pure fitting parameters without any physical meaning. In the past, little attention has been paid by authors to the constraints on the fitting parameters when used in the transport equations. Only Brutsaert (1974) studied the problem for the very early stages of infiltration by considering the constraint of a finite sorptivity.

The aim of this paper is to analyze the constraints on the different fitting parameters which enter the closed-form relations, when using the infiltration equation over its complete time range as a criterion. In this paper only six expressions are considered; including the most frequently used expressions like those of Fujita (1952), Gardner (1958), Brooks and Corey (1964), Brutsaert (1966), and Van Genuchten (1980).

THEORY

The solution of Fokker Planck's equation for vertical infiltration:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial\theta}{\partial z} - K(\theta)\right] \tag{1}$$

can be expanded in powers of $t^{1/2}$, when

$$\theta = \theta_s \quad \text{at } z = 0 \tag{2}$$

$$\theta = \theta_0 = \theta_r \quad \text{at } z = \infty \tag{3}$$

$$\theta = \theta_0 = \theta_r \quad \text{at } t = 0 \tag{4}$$

where θ_s is water content at natural saturation, θ_0 is initial water content

considered for convenience as equal to residual water content θ_r, z (cm) is depth positive downwards, and t (s) is time.

In particular the cumulative infiltration I (cm) can be written as (Philip, 1957):

$$I(t) = S\sqrt{t} + At + 0(t^{3/2}) \tag{5}$$

where S (cm/$\sqrt{\mathrm{s}}$) is the usual sorptivity given by:

$$S(\theta_s, \theta_0) = \int_{\theta_0}^{\theta_s} \phi(\theta)\,\mathrm{d}\theta \tag{6}$$

and

$$A(\theta_s, \theta_0) = \int_{\theta_0}^{\theta_s} \chi(\theta)\,\mathrm{d}\theta \tag{7}$$

where ϕ is the Boltzmann (1894) transformation and χ is the second term of the time series of Philip (1957).

The shape-functions $f(\theta, \theta_0)$ and $g(\theta, \theta_0)$, are defined by:

$$f(\theta, \theta_0) = \frac{\int_{\theta_0}^{\theta} \phi(\bar{\theta})\,\mathrm{d}\bar{\theta}}{\int_{\theta_0}^{\theta_s} \phi(\theta)\,\mathrm{d}\theta} \tag{8}$$

and

$$g(\theta, \theta_0) = \frac{\int_{\theta_0}^{\theta} \chi(\bar{\theta})\,\mathrm{d}\bar{\theta}}{\int_{\theta_0}^{\theta_s} \chi(\theta)\,\mathrm{d}\theta} \tag{9}$$

Since ϕ and χ are near constant over most of the range for $\bar{\theta}$, f and g are nearly equal to $(\theta - \theta_0)/(\theta_s - \theta_0)$.

The parameters S and A can then be written as

$$S^2(\theta_s, \theta_0) = 2\int_{\theta_0}^{\theta_s} \frac{(\theta - \theta_0)}{f(\theta)} D(\theta)\,\mathrm{d}\theta \tag{10}$$

and

$$A(\theta_s, \theta_0) = \tfrac{1}{3}(1 + \mu)(K_s - K_0) \tag{11}$$

with

$$\mu = \frac{\int_0^1 \left\{ 2\frac{K^*(\Theta)}{f(\Theta)} - \frac{1}{3}\left[1 + 2\frac{g(\Theta)}{f(\Theta)}\right]\right\} \frac{\Theta}{f(\Theta)} D(\Theta)\,\mathrm{d}\Theta}{\int_0^1 \frac{1}{3}\left[1 + 2\frac{g(\Theta)}{f(\Theta)}\right] \frac{\Theta}{f(\Theta)} D(\Theta)\,\mathrm{d}\Theta} \tag{12}$$

Θ and K^* stand, respectively, for the degree of saturation

$$\Theta = \frac{\theta - \theta_0}{\theta_s - \theta_0} \tag{13}$$

and the relative conductivity

$$K^* = \frac{K - K_0}{K_s - K_0} \tag{14}$$

where K_s is the hydraulic conductivity at natural saturation θ_s and $K_0 = K(\theta_0)$.

An extensive analysis of the parameter μ carried out by Fuentes et al. (1991), shows that μ varies over the interval $0 \leqslant \mu \leqslant 1$ depending on the type of soil tested. The two extreme values $\mu = 0$ and $\mu = 1$ correspond, respectively, to the limiting soil described by Talsma and Parlange (1972) for which $D(\theta)$ and $\mathrm{d}K/\mathrm{d}\theta$ behave like a Dirac delta function (Parlange, 1977), and the soil used by Green and Ampt (1911) for which only $D(\theta)$ behaves like a delta function. The behaviour of the parameter μ is illustrated in Fig. 1.

Substitution of the extreme μ-values into eqn. (11) gives the experimentally verified interval of variation for A:

$$1/3\, K_2 \leqslant A \leqslant 2/3\, K_s \tag{15}$$

as suggested by Youngs (1968), Philip (1969) and Talsma and Parlange (1972).

Since μ is not easily accessible for routine handling, it can be estimated by using the approximation of $f(\Theta) = g(\Theta) = \Theta$ in eqn. (12) (Parlange, 1975), yielding the estimator $\hat{\mu}$:

$$\hat{\mu} = \frac{\lambda_s}{\lambda_d} \tag{16}$$

where λ_s and λ_d are two scaling variables characterizing the soil behavior under infiltration. The scaling variables are expressed by:

$$\lambda_d = \frac{\theta_s - \theta_r}{K_s} \int_0^1 D(\Theta)\,\mathrm{d}\Theta \left[= \frac{1}{K_s} \int_{-\infty}^0 K(h)\,\mathrm{d}h \right] \tag{17}$$

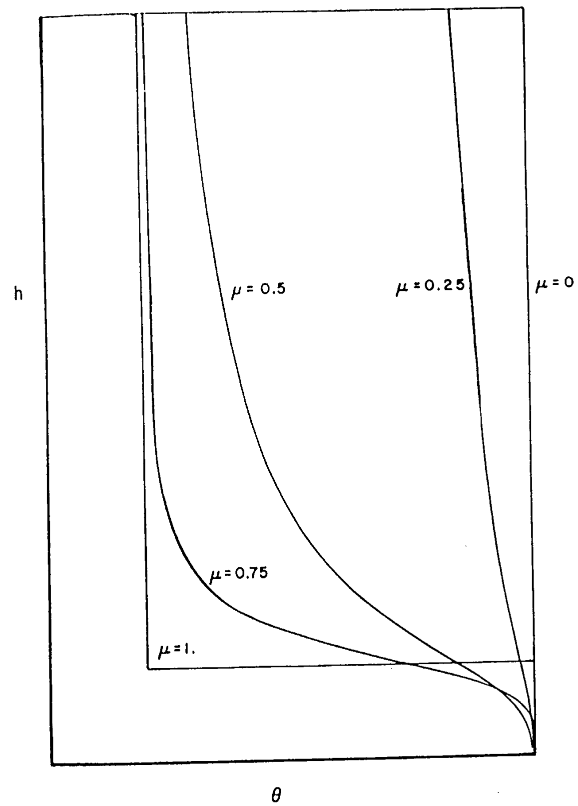

Fig. 1. Water retention diagram showing the influence of parameter μ on the shape of $h(\theta)$.

used earlier by Bouwer (1964, 1966), and:

$$\lambda_s = \frac{\theta_s - \theta_r}{K_s} \int_0^1 \left[2\frac{K^*(\Theta)}{\Theta} - 1 \right] D(\Theta)\,\mathrm{d}\Theta$$

$$\left[= \frac{1}{K_s} \int_{-\infty}^0 \left[2\frac{K^*(h)}{\Theta(h)} - 1 \right] K(h)\,\mathrm{d}h \right] \tag{18}$$

where θ_0 is taken equal to θ_r and $K(\theta_0) = K(\theta_r) = 0$ in order to cover the whole range of θ-values, independently of the initial conditions chosen.

Comparison between estimator $\hat{\mu}$-values and their theoretical μ-values calculated for 20 soils ranging from heavy clay soils to coarse sands $0 \leqslant \mu \leqslant 1$ are shown in Fig. 2. The behavior of $\hat{\mu}$ is correct over the range $0 \leqslant \mu \leqslant 1$ nevertheless, the main importance for this study lies in the fact that

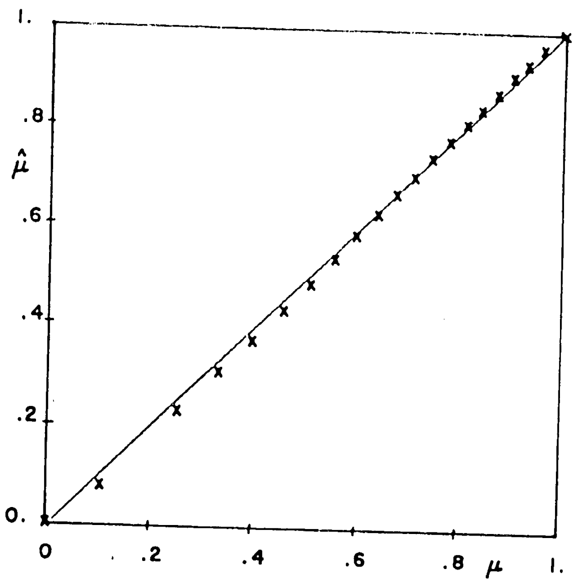

Fig. 2. Estimator $\hat{\mu}$ plotted as function of μ generated for 20 theoretical soils ranging from heavy clay to coarse sands ($0 \leqslant \mu \leqslant 1$); $r = 0.96$.

estimator $\hat{\mu}$ approaches, in an exact way, the limiting values: $\mu = \hat{\mu} = 0$ and $\mu = \hat{\mu} = 1$, as these limits impose the restrictive conditions for the parameter constraints.

Condition $\hat{\mu} \geqslant 0$ leads to:

$$\int_0^1 \left[2\frac{K^*(\Theta)}{\Theta} - 1\right] D(\Theta)\,\mathrm{d}\Theta \geqslant 0 \tag{19}$$

or expressed in terms of water pressure (h):

$$\int_{-\infty}^0 \left[2\frac{K^*(h)}{\Theta(h)} - 1\right] K^*(h)\,\mathrm{d}h \geqslant 0 \tag{20}$$

The other limiting value $\hat{\mu} \leqslant 1$ is automatically conserved as it implies $K^*(\Theta) \leqslant \Theta$.

The latter conditions will be used in the following to test the parameters defining the different soil characteristic relations mentioned above.

APPLICATIONS

Gardner's equation

Gardner (1958) proposed a relation between K and h of the form:

$$K(h) = K_s \exp(\alpha h) \text{ with } \alpha > 0 \tag{21}$$

Substitution of eqn. (21) into condition (20) with $\Theta = 1$, yields:

$$\int_{-\infty}^0 \left[2\frac{K(h)}{K_s} - 1\right] \frac{K(h)}{K_s}\,\mathrm{d}h = 0 \tag{22}$$

showing that Gardner's equation (eqn. (21)) satisfies the infiltration condition (20), since $1/\Theta = (\theta_s - \theta_r)/(\theta - \theta_r)$ by definition is greater than 1. However, it shows at the same time that eqn. (21) cannot be applied over the whole range of possible soil types when λ_s is near zero, since the left-hand side of eqn. (20) cannot be zero for any α, i.e. eqn. (21) cannot be applied for sandy soils.

An alternative form of eqn. (21) also frequently used in literature, is written in the form:

$$K(h) = K_s \exp[\alpha(h - h_0)] \text{ for } h < h_0 \tag{23}$$

and

$$K(h) = K_s \text{ for } h_0 \leqslant h \leqslant 0 \tag{24}$$

where h_0 is a constant negative pressure value for which $\theta = \theta_s$.

This expression proposed by Gardner (1958) is more flexible to describe characteristics of field soils. Note that the introduction of parameter h_0 in the conductivity equation imposes a composed diffusivity function of the form:

$$D(\theta) = D_c(\theta) + D_\delta(\theta) \tag{25}$$

where $D_C(\theta)$ refers to the finite part of $D(\theta)$ given by eqn. (23) and valid over the interval $[\theta_r, \theta_s[$; and $D_\delta(\theta)$ is the infinite part of $D(\theta)$ for $\theta = \theta_s$, where the 'subscript' δ refers to it being approached by the use of a standard Dirac delta function (Haverkamp et al., 1990) of the form:

$$D_\delta(\theta) = K_s |h_0| \delta(\theta - \theta_s) \tag{26}$$

Substitution of eqns. (23) and (24) into condition (20) with $\Theta = 1$, yields:

$$\int_{-\infty}^0 \left[2\frac{K(h)}{K_s} - 1\right] \frac{K(h)}{K_s}\,\mathrm{d}h = |h_0| \tag{27}$$

showing similar shortcomings in the use of eqns. (23) and (24) as was noted for the earlier conductivity equation of Gardner (eqn. (21)): for heavy clay soils expressions (23) and (24) should not be used, as condition (20) cannot be zero for any value of α.

Brooks and Corey's equations

Brooks and Corey (1964) assumed that:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\frac{h_{cr}}{h}\right)^{\lambda} \text{ for } h < h_{cr} \quad (28)$$

and

$$\theta = \theta_s \text{ for } h_{cr} \leqslant h \leqslant 0 \quad (29)$$

where h_{cr} is a parameter commonly termed the 'air entry pressure' and λ is a positive soil index, being small for clay soils and large for sandy soils.

For the hydraulic conductivity function Brooks and Corey (1964) proposed:

$$\frac{K(\theta)}{K_s} = \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{\eta} \text{ for } h < h_{cr} \quad (30)$$

and

$$K(\theta) = K_s \text{ for } h_{cr} \leqslant h \leqslant 0 \quad (31)$$

with

$$\eta = \frac{2}{\lambda} + 3 \quad (32)$$

Note that several authors obtained other values of parameter η unrelated to λ, i.e. Yuster (1951) suggested $\eta = 2$; Irmay (1954) $\eta = 3$ and Averyanov (1950) $\eta = 3.5$.

Substitution of eqns. (28)–(32) into condition (20) gives at once:

$$(-h_{cr})(15\lambda^2 + 15\lambda + 2) \geqslant 0 \quad (33)$$

which is always satisfied, since $\lambda > 0$ and $h_{cr} < 0$. However, eqn. (33) shows again that the Brooks and Corey model does not cover the entire range of $\hat{\mu}$ and is inconsistent for heavy clay soils.

Brooks and Corey chose the power value $\eta = 2/\lambda + 3$ of $K(\theta)$ from the capillary model of Burdine (1953), who derived the $\eta(\lambda)$ relationship:

$$\eta = \frac{2}{\lambda} + 2 + p \quad (34)$$

with $p = 1$. Other authors proposed different values of p, i.e. Childs and Collis-George (1950) $p = 0$; Mualem (1976a) $p = 1/2$; and Millington and Quirk (1961) $p = 4/3$. For any p-value in the Brooks and Corey model, condition (20) is satisfied if:

$$[3 + (3 + 2p)\lambda][1 + (2 + p)\lambda] + \lambda - 1 \geqslant 0 \quad (35)$$

Only for small values of λ, i.e. $\lambda \leqslant 1$, as is the case for clay soils and negative values of p (i.e. $p \leqslant -3$) condition (35) will become zero or negative. However, such a combination is rather unlikely for field soils, as it implies values of η close to 1 (eqn. (34)), which is rarely met for clay soils. As a result the modified Brooks and Corey model with η expressed as a function of p only covers a limited range of $\hat{\mu}$ ($\hat{\mu} > 0$, eqn. (16)) and should, therefore, not be used for heavy clay soils.

Brutsaert's equation

Brutsaert (1966) proposed the following $h(\theta)$ relation:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{1 + \left(\frac{h}{h_b}\right)^b} \quad (36)$$

with b ($b > 0$) and h_b being curve fitting parameters. Since Brutsaert did not suggest a special $K(\theta)$ relation, eqn. (36) is tested in combination with Brooks and Corey's equation (eqn. (30)) as used by Ahuja and Swartzendruber (1972).

Substitution of eqns. (30), (34) and (36) into condition (20) gives:

$$\frac{2\Gamma\left(3 + 2p + \frac{3}{b}\right)}{\Gamma\left(3 + 2p + \frac{4}{b}\right)} - \frac{\Gamma\left(2 + p + \frac{1}{b}\right)}{\Gamma\left(2 + p + \frac{2}{b}\right)} \geqslant 0 \quad (37)$$

where Γ stands for the gamma function.

This condition is satisfied only for $b \geqslant 1$, independently of the value of p chosen, which imposes a constraint on the use of Brutsaert's equation (eqn. (36)).

Van Genuchten's equations

Van Genuchten (1980) suggested the use of:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + \left(\frac{h}{h_g}\right)^n\right]^m} \qquad (38)$$

with $h_g \leqslant 0$ and

$$\frac{K(\theta)}{K_s} = \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{1/2} \left\{1 - \left[1 - \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{1/m}\right]^m\right\}^2 \qquad (39)$$

This conductivity was obtained through the capillary model of Mualem (1976a) imposing

$$m = 1 - \frac{1}{n} \text{ with } n > 1 \qquad (40)$$

Substitution of eqns. (38) and (39) into condition (20) gives:

$$2\int_0^1 x^{m-2}\,[1 - (1 - x)^m]^4\,(1 - x)^{-m}\,dx \geqslant \int_0^1 x^{(3m/2)-2}\,[1 - (1 - x)^m]^2\,(1 - x)^{-m}\,dx \qquad (41)$$

where x stands for $[\theta - \theta_r/\theta_s - \theta_r]^{1/m}$
Numerical calculations of the integrals in eqn. (41) indicate that the condition holds only for:

$$m \geqslant 0.4669 \qquad (42)$$

which imposes a constraint on Van Genuchten's equation (eqn. (38)) when used together with the conductivity equation (eqn. (39)) for clay soils.

If the conductivity equation is derived through the capillary theory of Burdine (1953)

$$\frac{K(\theta)}{K_s} = \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^2 \left\{1 - \left[1 - \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{1/m}\right]^m\right\} \qquad (43)$$

with

$$m = 1 - \frac{2}{n} \text{ and } n > 2 \qquad (44)$$

a constraint of the same type is obtained:

$$\int_0^1 x^{1/2(5m-1)-1}\,[1 - (1 - x)^m]\{2x^m[1 - (1 - x)^m] - 1\}(1 - x)^{1/2(1-m)-1}\,dx \geqslant 0 \qquad (45)$$

where x stands again for $(\theta - \theta_r/\theta_s - \theta_r)^{1/m}$. Numerical calculations show that condition (45) holds if:

$$m \geqslant 0.2466 \qquad (46)$$

limiting the use of Van Genuchten's equation together with Burdine's equation for clay soils.

Note that the limiting m-value of eqn. (46) is smaller than that calculated by eqn. (42). This follows directly from different definitions of m: eqn. (40) or eqn. (44).

Combination of Van Genuchten's h(θ) *and Brooks and Corey's* K(θ) *equations*

As an alternative to eqns. (39) or (43), Van Genuchten's $h(\theta)$ equation (eqn. (38)) combined with the Brooks and Corey relation for $K(\theta)$, (eqn. (30)), is analyzed with m independent of n. One can take in general

$$m = 1 - \frac{c}{n} \qquad (47)$$

The two cases of the previous section corresponded, respectively, to $c = 1$ and $c = 2$. Here c is kept arbitrary as long as $c \leqslant n$.

Van Genuchten's and Brooks and Corey's $h(\theta)$ equations behave similarly at low water content and by analogy with eqn. (34) the power η can be written in the form:

$$\eta = \frac{2}{mn} + 2 + p \qquad (48)$$

Substitution of eqns. (30), (38), (47) and (48) into condition (20) then gives

$$2B\left[\frac{3}{c} + \left(3 - \frac{3}{c} + 2p\right)m, \frac{1 - m}{c}\right] \geqslant B\left[\frac{1}{c} + \left(2 - \frac{1}{c} + p\right)m, \frac{1 - m}{c}\right] \qquad (49)$$

where B is the usual beta function.

Analyzing relation (49) for a value of $c = 1$, shows that the condition is not satisfied for values of $m < 0.15$, independently of the value of p chosen (shaded area of Fig. 3). Adding moreover the condition imposed by the power

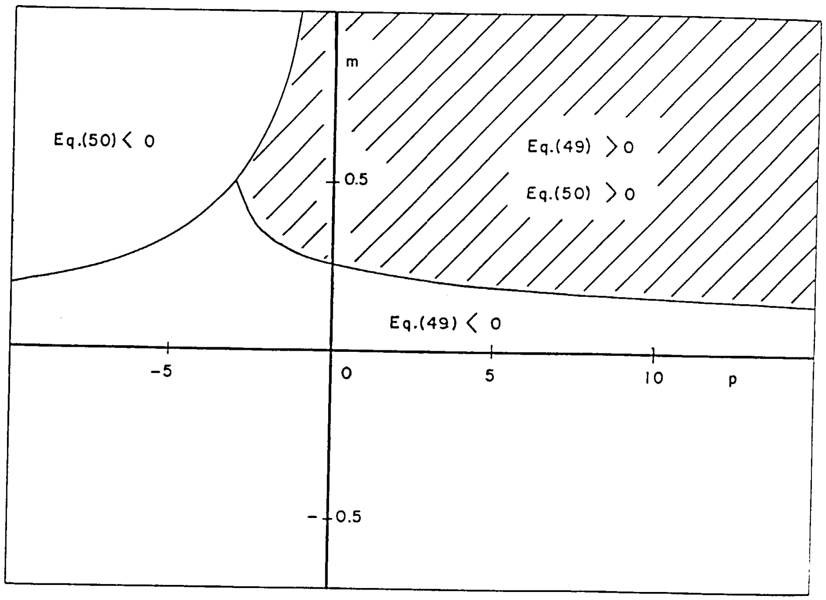

Fig. 3. Domain of validity (shaded area) of the Van Genuchten–Brooks and Corey model with $c = 1$, plotted as a function of m and p.

term $\eta_{\cdot} \geqslant 1$ of the conductivity equation (eqn. (30)):

$$\frac{2}{m} + p - 1 \geqslant 0 \tag{50}$$

it follows that p should be greater than -3 (Fig. 3). In consequence the use of eqns. (30) and (38) with $c = 1$ does not permit a full description of the soil characteristics for all types of soils.

Considering now the case of $m = 0$, which is the most constraining, eqn. (49) is satisfied if:

$$2B\left(\frac{3}{c}, \frac{1}{c}\right) \geqslant B\left(\frac{1}{c}, \frac{1}{c}\right) \tag{51}$$

which imposes the condition on parameter c independently of p:

$$c \geqslant 2. \tag{52}$$

The value of $c = 2$ is optimal as it is the only value which allows description of the limiting soil $\hat{\mu} = 0$ (Talsma and Parlange, 1972) with condition (20) equal to zero.

As a result the combination of Brooks and Corey's $K(\theta)$ equation (eqn. (30)) and Van Genuchten's $h(\theta)$ equation (eqn. (38)) together with Burdine's

condition $m = 1 - \frac{2}{n}$, can be applied to the full range of field soils without being limited by the infiltration condition (eqn. (20)). The only remaining constraint is:

$$p \geqslant -\frac{1}{m} \tag{53}$$

for the conductivity function $K(\theta)$ to have a positive second derivative.

Combined Fujita and Parlange's equation

The last equation tested is the Fujita (1952) diffusivity equation:

$$D(\Theta) = \frac{K_s|h_c|(1-\beta)}{(\theta_s - \theta_r)} \frac{1}{(1-\beta\Theta)^2} \tag{54}$$

where β is a parameter between 0 and 1, and h_c is a constant negative pressure used as a fitting parameter.

In a recent article by Sander et al. (1988) the Fujita equation (eqn. (54)) was used in combination with an integral $K(D)$ relation initially introduced by Parlange et al. (1982) and later refined by Parlange et al. (1985) and Haverkamp et al. (1990) of the form:

$$\frac{K(\theta)}{K_s} = \Theta\left[1 - \gamma \frac{\int_{\Theta}^{1} D(\bar{\Theta})\,\mathrm{d}\bar{\Theta}}{\int_{0}^{1} D(\Theta)\,\mathrm{d}\Theta}\right] \tag{55}$$

γ is a shape parameter defined over $0 \leqslant \gamma \leqslant 1$. The analysis of Sander et al. (1988) showed that their results are identical to those of Rogers et al. (1983) with only a change in notations. Broadbridge and White (1988) rederived the same results independently of Rogers et al. (1983).

The Fujita equation (eqn. (54)) is used for the finite part $D_c(\theta)$, to which a delta function is added:

$$D_\delta(\theta) = K_s|h_s|\delta(\theta - \theta_s) \tag{56}$$

Similarly to the value h_0 of the Gardner conductivity equation (eqn. (23)), the parameter h_s is such that $\theta = \theta_s$ for $h > h_s$. Haverkamp et al. (1990) calculated h_s-values of the order of $-10\,\text{cm} < h_s < -5\,\text{cm}$ for uniformly structured soils like coarse sands, and smaller h_s-values for less uniformly structured soils like silt and clay soils.

Combination of eqns. (54), (55) and (56) yields the following $h(\theta)$ function for $h < h_s$ (Sander et al., 1988)

$$h(\Theta) = h_s + h_c \left\{ \frac{\gamma - \beta}{\gamma(1-\gamma)} \ln \left[\frac{1 - \gamma + (\gamma - \beta)\Theta}{(1-\beta)\Theta} \right] + \frac{\beta}{\gamma} \ln \left[\frac{1 - \beta\Theta}{(1-\beta)\Theta} \right] \right\} \quad (57)$$

Note that h_s is not the fitting parameter h_{cr} of Brooks and Corey (eqn. (28)), as h_s can be equal to zero, when $d\theta/dh$ is non-zero for $\theta = \theta_s$.

In a similar way combination of eqns. (54) and (55) yields the $K(\theta)$ function (Sander et al., 1988),

$$\frac{K(\theta)}{K_s} = \Theta \left[1 - \gamma \frac{(1-\Theta)}{(1-\beta\Theta)} \right] \quad (58)$$

It is interesting to note that for $\beta = \gamma$, eqn. (58) reduces to Gardner's conductivity in the form:

$$K(h) = K_s \exp\left[-\frac{1}{h_c}(h - h_s) \right] \quad (59)$$

Substitution of eqns. (54), (55) and (56) into condition (20) gives:

$$|h_c|(1-\gamma) + |h_s| \geqslant 0 \quad (60)$$

which is always satisfied since $0 \leqslant \gamma \leqslant 1$. Moreover, condition (60) clearly shows that for heavy clay soils the value of h_s tends to zero. The limiting soil of Talsma and Parlange (1972) corresponds to the case $\gamma = 1$ and $h_s = 0$.

In consequence the Fujita diffusivity equation (eqn. (54)) together with the conductivity given by eqn. (55) can be used to describe the soil characteristics over the full range of field soils.

ILLUSTRATIONS

In order to illustrate the practical problem of curve fitting $h(\theta)$ and $K(\theta)$, nine different soils are chosen from the literature, hygiene sandstone, touched silt loam, Guelph loam (main drying curve), silt loam (GE3) and Beit Netofa clay soil (Van Genuchten, 1980); Sellingen loam (Beuving, 1984); Yolo light clay (Moore, 1939); Columbia silt (main wetting curve) (Mualem, 1976b); and Grenoble sand (Parlange et al., 1985).

The different parameters are optimized over the pressure and conductivity functions using least-square analyses with the standard deviation as criterion; e.g. for the retention curve $\sigma_\theta(h)$ is expressed by:

$$\sigma_\theta(h) = \sqrt{\frac{\sum_{i=1}^{N} [\theta_i - \theta(h_i)]^2}{N-1}} \quad (61)$$

where θ_i and $\theta(h_i)$ are, respectively, the measured and calculated water content values and N is the total number of data.

For all soils the values of θ_s and K_s are known experimentally. For the first five soils (Van Genuchten, 1980) the conductivity data were given as a function of pressure (h), while for the other soils conductivity was given as a function of θ.

Brooks and Corey's equations

The results of Brooks and Corey's equations are shown in Table 1. For the first five soils with conductivity given by $K(h)$, a value of $p = 1$ was chosen in order to optimize the parameters θ_r, h_{cr} and λ. For the other soils h_{cr}, λ and p were optimized with θ_r equal to zero.

The results for μ clearly show the main drawback of the Brooks and Corey model; it does not permit values of μ close to zero (even for clay soils, i.e. soils 5 and 7, μ is greater than 0.3), explaining the poor fit of the model to heavy clay soils.

The value of p optimized for soils 6–9 illustrate the ambiguity of taking an a priori value for p based on some ad hoc capillary model.

Brutsaert's equation

The results related to Brutsaert's equation are given in Table 2. For soils 1–5 parameters θ_r, h_b and b are optimized with $p = 1$, while for soils 6–9 parameters h_b, b and p are calculated with θ_r equal to zero.

The results for μ clearly show the flaw of Brutsaert's equation: for values of b smaller than 1 (soils 6, 7 and 8), μ becomes negative, which is physically unacceptable. Moreover, for the same soils the fitting of the $h(\theta)$ relation is poor, as the optimization criterion $\sigma_\theta(h)$ is ten times greater than for the other soils (i.e. 2×10^{-2} instead of $3 \times 10^{-3}\,cm^3\,cm^{-3}$).

Note the great variation in θ_r for soils 1–5, as compared with the Brooks and Corey model especially for heavy soils. This emphasizes that θ_r should be considered as a fitting parameter without any physical meaning and depends on the model chosen.

Van Genuchten's equations

Van Genuchten's equations are tested separately for the two capillary models of Mualem ($c = 1$) and Burdine ($c = 2$).

The results for $m = 1 - \frac{1}{n}$ are shown in Table 3. For soils 1–5 the values of parameters θ_s, θ_r, K_s, h_g and m are those given by Van Genuchten (1980). For soils 6–9, h_g and m are optimized with θ_r equal to zero.

TABLE 1

Fitting parameters for Brooks and Corey's equations

No.	Soil type	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_{cr}$	λ	p	μ
1	Hygiene sandstone	0.2500	0.1326	4.5	105.35	2.9483	1.0	0.9179
2	Touched silt loam (G.E.3)	0.4690	0.1207	12.625	148.80	1.7429	1.0	0.8704
3	Guelph loam (main drying)	0.5200	0.1708	1.3167	45.82	0.5205	1.0	0.6952
4	Silt loam (G.E.3)	0.3960	0.0127	0.2067	128.48	0.3854	1.0	0.6436
5	Beit Netofa clay	0.4460	0.0887	0.0034	208.04	0.1305	1.0	0.4823
6	Sellingen loam	0.4554	0.0	0.4455	16.05	0.1827	−0.248	0.4671
7	Yolo light clay	0.4950	0.0	0.0443	16.56	0.2074	−2.5	0.3110
8	Columbia silt (main wetting)	0.4010	0.0	0.21	32.81	0.2745	0.102	0.5394
9	Grenoble sand	0.3120	0.0	15.37	11.43	1.2876	1.288	0.7272

$h(\theta)$, eqn. (28); $K(\theta)$, eqn. (30).

TABLE 2

Fitting parameters for Brutsaert's equation

Soil no.	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_b$(cm)	b	p	μ
1	0.2500	0.1536	4.5	127.84	10.112	1.0	0.8847
2	0.4690	0.1923	12.625	204.05	6.846	1.0	0.8302
3	0.5200	0.2447	1.3167	127.51	1.819	1.0	0.3938
4	0.3960	0.1606	0.2067	342.86	1.873	1.0	0.4096
5	0.4460	0.3100	0.0034	1093.26	1.289	1.0	0.1833
6	0.4554	0.0	0.4455	1020.99	0.434	6.093	−0.6220
7	0.4950	0.0	0.0443	392.62	0.695	4.266	−0.2773
8	0.4010	0.0	0.21	406.22	0.805	4.904	−0.1621
9	0.3120	0.0	15.37	40.07	1.621	3.494	0.3176

$h(\theta)$, eqn. (36); $K(\theta)$, eqn. (30).

As stated earlier the model breaks down for values of m smaller than $m = 0.4669$ (eqn. (42)); this is clearly illustrated for soils 5–8 with negative μ-values. The problem occurs especially for heavy soils. However, when comparing the different values of the optimization criterion $\sigma_\theta(h)$, it is obvious that the $h(\theta)$ equation (eqn. (38)) is accurate. It is the conductivity function which is erratic in its behavior.

This point is illustrated by comparing the results of soil 5 (Table 3) with those obtained with Brutsaert's $h(\theta)$ equation (eqn. (36)) (which is a particular case of Van Genuchten's equation (eqn. (38)) with $m = 1$) combined with Brooks and Corey's conductivity function, eqn. (30) (Table 2). The use of eqn. (39) is far less physical ($\mu < 0$) than Brooks and Corey's $K(\theta)$ relation ($\mu > 0$).

The results of the second form of Van Genuchten's equations with $m = 1 - \frac{2}{n}$ (eqn. (44)) are shown in Table 4.

As in the previous case the model breaks down for soils 4–8 ($\mu < 0$), when the value of m drops under the threshold value $m = 0.2466$ (eqn. (46)). Once more this is due to the conductivity function (eqn. (43)).

In view of these results it seems preferable to use a $K(\theta)$ function not connected to the chosen $h(\theta)$.

Van Genuchten–Brooks and Corey's equations

The results of the Van Genuchten–Brooks and Corey's equations are shown in Tables 5 and 6, with $c = 1$ and 2, respectively.

Considering first $c = 1$ (Table 5), p had to be chosen for the first five soils (Van Genuchten, 1980) since the conductivity data were given by $K(h)$. The value $p = 0.5$ is chosen, as suggested by Mualem (1976a). For soils 6–9, p is

TABLE 3

Fitting parameters for the Van Genuchten–Mualem model with $c = 1$

Soil no.	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_g$(cm)	m	λ_s(cm)	μ	$\sigma_0(h)$(cm^3 cm^{-3})
1	0.2500	0.1531	4.5	126.13	0.9035	97.34	0.8768	2.9×10^{-3}
2	0.4690	0.1903	12.625	198.04	0.8690	132.48	0.8096	8.9×10^{-3}
3	0.5200	0.2183	1.3167	86.96	0.5089	6.59	0.1722	7.3×10^{-3}
4	0.3960	0.1312	0.2067	236.25	0.5142	11.36	0.1140	2.4×10^{-3}
5	0.4460	0.2859	0.0034	495.81	0.3725	−30.85	−0.2360	2.8×10^{-3}
6	0.4554	0.0	0.4455	21.61	0.1629	−1.11	−0.7592	6.9×10^{-3}
7	0.4950	0.0	0.0443	30.82	0.2080	−2.07	−0.6525	9.9×10^{-3}
8	0.4010	0.0	0.21	56.95	0.2560	−4.42	−0.5332	6.2×10^{-3}
9	0.3120	0.0	15.37	23.16	0.5096	0.99	0.1031	6.2×10^{-3}

$h(\theta)$, eqn. (38); $K(\theta)$, eqn. (39).

TABLE 4

Fitting parameters for the Van Genuchten–Burdine model with $c = 2$

Soil no.	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_g$(cm)	m	λ_s(cm)	μ	$\sigma_0(h)$(cm^3 cm^{-3})
1	0.2500	0.1526	4.5	124.41	0.8123	95.14	0.8738	2.9×10^{-3}
2	0.4690	0.1878	12.625	192.01	0.7297	127.52	0.8038	8.4×10^{-3}
3	0.5200	0.2125	1.3167	62.50	0.2888	3.65	0.1138	5.7×10^{-3}
4	0.3960	0.0412	0.2067	155.91	0.1942	−9.97	−0.1598	3.6×10^{-3}
5	0.4460	0.2237	0.0034	282.13	0.1198	−34.32	−0.4298	1.8×10^{-3}
6	0.4554	0.0	0.4455	16.57	0.0842	−2.06	−0.5802	4.5×10^{-3}
7	0.4950	0.0	0.0443	19.31	0.0995	−2.43	−0.5137	5.2×10^{-3}
8	0.4010	0.0	0.21	36.06	0.1248	−4.31	−0.4099	5.0×10^{-3}
9	0.3120	0.0	15.37	16.39	0.2838	0.84	0.1010	6.3×10^{-3}

$h(\theta)$, eqn. (38); $K(\theta)$, eqn. (43).

TABLE 5

Fitting parameters for the Van Genuchten–Brooks and Corey model with $c = 1$

Soil no.	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_g$(cm)	m	p	μ	$\sigma_\theta(h)$(cm^3 cm^{-3})
1	0.2500	0.1531	4.5	126.13	0.9035	0.5	0.8897	2.9×10^{-3}
2	0.4690	0.1903	12.625	198.04	0.8690	0.5	0.8366	8.9×10^{-3}
3	0.5200	0.2183	1.3167	86.96	0.5089	0.5	0.3657	7.3×10^{-3}
4	0.3960	0.1312	0.2067	236.25	0.5142	0.5	0.3854	2.4×10^{-3}
5	0.4460	0.2859	0.0034	495.81	0.3725	0.5	0.1865	2.8×10^{-3}
6	0.4554	0.0	0.4455	21.61	0.1629	0.4215	−0.1119	6.9×10^{-3}
7	0.4950	0.0	0.0443	30.82	0.2080	−0.4721	−0.0764	9.9×10^{-3}
8	0.4010	0.0	0.21	56.95	0.2560	1.5751	0.0468	6.2×10^{-3}
9	0.3120	0.0	15.37	23.16	0.5096	2.8036	0.3962	6.2×10^{-3}

$h(\theta)$, eqn. (38); $K(\theta)$, eqn. (30).

TABLE 6

Fitting parameters for the Van Genuchten–Brooks and Corey model with $c = 2$

Soil no.	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_g$(cm)	m	p	μ	$\sigma_\theta(h)$(cm^3 cm^{-3})
1	0.2500	0.1526	4.5	124.41	0.8123	1.0	0.8868	2.9×10^{-3}
2	0.4690	0.1878	12.625	192.01	0.7297	1.0	0.8339	8.4×10^{-3}
3	0.5200	0.2125	1.3167	62.50	0.2888	1.0	0.4744	5.7×10^{-3}
4	0.3960	0.0412	0.2067	155.91	0.1942	1.0	0.3598	3.6×10^{-3}
5	0.4460	0.2237	0.0034	282.13	0.1198	1.0	0.2496	1.8×10^{-3}
6	0.4554	0.0	0.4455	16.57	0.0842	−0.1775	0.1492	4.5×10^{-3}
7	0.4950	0.0	0.0443	19.31	0.0995	−1.9070	0.0888	5.2×10^{-3}
8	0.4010	0.0	0.21	36.06	0.1248	0.3748	0.2362	5.0×10^{-3}
9	0.3120	0.0	15.37	16.39	0.2838	2.2046	0.4887	6.3×10^{-3}

$h(\theta)$, eqn. (38); $K(\theta)$, eqn. (30).

fitted using eqn. (48) for $K(\theta)$. For all soils the $h(\theta)$ parameters θ_r, h_g and m are those calculated before in Table 3.

Although the use of Brooks and Corey's conductivity function (eqn. (30)) is a slight improvement on Van Genuchten's conductivity function (eqn. (39)), the results are still physically unacceptable for soils 6 and 7, the small values of m resulting in negative values for μ (see Fig. 3). Even for fixed p-values, e.g. $p = 0.5$, μ is negative for these soils.

The $\sigma_\theta(h)$-values are identical to those given in Table 3.

The second form of the Van Genuchten–Brooks and Corey model consists in taking $c = 2$, following Burdine's (1953) hypothesis. The results are shown in Table 6. Once more the value of $p = 1$ is chosen for the first five soils (as suggested by Burdine, 1953), while for soils 6–9, p is obtained by curve-fitting. For all soils, the $h(\theta)$ parameters: θ_r, h_g and m are those given in Table 4.

As shown in Table 6 the infiltration criterion ($0 \leqslant \mu \leqslant 1$) is satisfied for all nine soils. This confirms the result found theoretically before: the combination of Van Genuchten's $h(\theta)$ equation (eqn. (38)) and Brooks and Corey's $K_{\frac{2}{n}}(\theta)$ equation (eqn. (30)) together with Burdines' condition $m = 1$ — is one of only two acceptable possibilities among the eight models tested. That is, it can be used without any restriction to describe soil characteristics of all soil types encountered in field situations.

The model can be applied for two purposes:

(1) for simulation: parameters θ_r, h_g and m are optimized from pressure data, while η (or p) is calculated independently from $K(\theta)$;

(2) for prediction: only θ_r, h_g and m are optimized from $h(\theta)$ data and the power term η of the $K(\theta)$ expression is directly calculated from m and p with p equal to 0, 1/2, 1 or 4/3 depending on the chosen capillary model.

Fujita and Parlange's equation

The results of the Fujita-based equations eqn. (57) for $h(\theta)$, and eqn. (58) for $K(\theta)$ are reported in Table 7. Only the last four soils are analyzed, since their conductivity was readily available in $K(\theta)$ form.

The parameters to optimize are: θ_r, h_c, h_s, β and γ, where θ_r, β and γ enter in both soil characteristics $h(\theta)$ and $K(\theta)$. Simultaneous optimizations need an objective function taking into account weighting between pressure and conductivity data. The objective function $O(v)$ proposed by Wösten and Van Genuchten (1988) is used:

$$O(v) = \sum_{i=1}^{M} [\theta_i - \theta_i(v)]^2 + \sum_{i=M+1}^{N} \{w[\ln(K_i) - \ln(K_i(v))]\}^2 \qquad (62)$$

where θ_i and K_i are the measured water contents and hydraulic conductivities,

TABLE 7

Fitting parameters for the Fujita model

Soil no.	θ_s(cm^3 cm^{-3})	θ_r(cm^3 cm^{-3})	K_s(cm h^{-1})	$-h_s$(cm)	$-h_c$(cm)	β	γ	μ	$\sigma_\theta(h)$(cm^3 cm^{-3})
6	0.4554	0.2351	0.4455	0.21	26.54	0.8559	1.0	0.008	5.63×10^{-3}
7	0.4950	0.2285	0.0443	1.61	27.05	0.8912	1.0	0.036	6.95×10^{-3}
8	0.4010	0.1738	0.21	0.41	37.15	0.8834	1.0	0.0107	2.54×10^{-3}
9	0.3120	0.0438	15.37	3.97	9.20	0.8882	1.0	0.3014	6.34×10^{-3}

$h(\theta)$, eqn. (57); $K(\theta)$, eqn. (58).

respectively; $\theta_i(v)$ and $K_i(v)$ are the predicted values for given parameter vector $\{\boldsymbol{v}\}$; and w is the weighting coefficient, chosen in such a way that water content data have roughly the same weight as the $\ln(K)$ data.

The choice of different weighting coefficients w, leads to different parameter vectors $\{\boldsymbol{v}\} = \{\theta_r, h_c, h_s, \beta, \gamma\}$, which makes the use of these Fujita-based characteristic functions rather subjective.

Table 7 clearly shows that the infiltration criterion ($0 \leqslant \mu \leqslant 1$) is satisfied for all soils tested, confirming the theoretical result. In general the Fujita-based $h(\theta)$ expressions are less accurate for θ_r-values close to zero, which explains the relatively large values of θ_r for soils 6 and 7. Moreover, the values of $\gamma = 1$, confirm the choice $\gamma = 1$ stated earlier by Haverkamp et al. (1990).

Although these characteristic forms of $h(\theta)$ and $K(\theta)$ permit a description of all types of soil, the great number of fitting parameters involved, combined with the delicate optimization procedure, makes the Fujita–Parlange model useful primarily, for theoretical case studies. For practical applications Van Genuchten–Brooks and Corey's equations are the most convenient.

CONCLUSIONS

Of all eight combinations of $h(\theta)$ and $K(\theta)$ characteristic equations tested, six models, i.e. (1) Gardner's (1958) equations; (2) Brooks and Corey's (1964) equations; (3) Brutsaert's (1966) equation; (4) and (5) Van Genuchten's (1980) equations subjected to either Mualem's (1976a) or Burdine's (1953) condition; and (6) Van Genuchten's $h(\theta)$ equation subjected to Mualem's condition, together with Brooks and Corey's $K(\theta)$ equation, may not describe heavy clay soils properly, since for these soils the values of the fitting parameters may become physically inconsistent with infiltration theory.

Two combinations, i.e. (1) Van Genuchten's equation for $h(\theta)$ subject to Burdine's condition, $m = 1 - \frac{2}{n}$, together with Brooks and Corey's equation for $K(\theta)$; and (2) the combined Fujita (1952) and Parlange et al. (1982) equations, can be applied to the full range of field soils without any restrictions for the fitting parameters.

The former combination (1) has the advantage of using one parameter less than the latter model, resulting in a straightforward optimization procedure. Thus, this combination is particularly useful for practical application. The second combination (2) of characteristic equations is more convenient for theoretical case studies, as it can yield the solution of certain infiltration and/or evaporation problems in analytical form.

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[4]

Dominant events in extreme rainfall records

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ABSTRACT

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A maximum rainfall over a short interval will sometimes be greater than the maximum persisting over a longer interval. Data from this situation are usually taken as point values in the analysis of extremes but these maxima are interpretable as upper bounds for the longer interval. Physically meaningful parameters of a model of rainfall extremes can still be estimated from data including upper bounds, by the method of maximum likelihood, with little loss of precision compared with the analysis of point valued data only. The benefits of recognizing these upper bounds are an improved ordering of the estimated average number of rainfall events with increasing event duration.

INTRODUCTION

For hydrological engineering design purposes it is necessary to obtain estimates of the maximum water flows that a culvert or catchment outlet is expected to accommodate over the design lifetime of the engineering works. In the absence of flow records of streams and rivers there are well-established methods for estimating extreme flows from catchment characteristics. These flows are obtained from extreme rainfall accumulations over particular lead times which characterize the catchment response (World Meteorological Organization (WMO), 1974).

Traditional methods of analysing extreme values involve the estimation of empirical scale and location parameters of a statistical distribution derived from asymptotic theory. A plausible small sample theory leads to a distribution with physically meaningful parameters (Revfeim, 1983) which gives predictors of long-term extremes similar to, but simpler than, traditional methods. However, if qualifications on the data are ignored, interpretation of the estimated physically meaningful parameters exposes anomalies in the parameter sequence for increasing interval lengths (duration). In particular it may expose an increase in a parameter sequence where a decrease was expected.

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