2. **Using pumping lemma prove that A = {yy/y ∈ {0, 1}* is not regular.**

Solution:

According to pumping lemma, 'if a language is regular then it must have string $|S| \geq P$ where P is pumping length and can be divided into xyz such that:

(a) $xy^iz \in A$ for some i ... ... (i)

(b) $|y| > 0$ ... ... (ii)

(c) $|xy| \leq p$ ... ... (iii)

To prove: Language A = {yy//y ∈ (0, 1)*} is not regular

Proof: Let us consider language A is regular then it must follow above 3 conditions. Then it must have pumping length P = 7 (say).

Let, the strings be accepted by finite state machine

$$S = 0^p10^p1$$
$$= 0^7 1 0^7 1$$
$$= 0000000100000001$$

Dividing S into x, y and z we get,

$$S = \underbrace{00}_{x} \quad \underbrace{00000}_{y} \quad \underbrace{100000001}_{z}$$

Here, the above condition (ii) and (iii) are satisfied as

$|y| \geq 0$

$|xy| = 7 \leq 7$

Changing for condition 1

$$S = xy^iz \qquad \text{let, } i = 2$$
$$= xy^2z$$
$$= 00\ 0000000000\ 100000001$$

Here, , on pumping i.e. (xy^iz) the language doesn't follow pattern (O^p10^p1) (i.e. first half is equal to the second half). So none of the cases follow all 3 conditions of regular language as stated by pumping lemma. Therefore language A is not regular language.

State the pumping lemma for the regular languages. Show that the languages

$L = \{0^{n^2} \mid n >= 1\}$ not regular example,

if x = 1, w = 0, n = 2, w = 0000, n = 3, w = 000000000 [2074, Chaitra]

Pumping lemma states that if a language A is regular then it must have string $|s| \geq P$ where P is pumping length and can be divided into xyz such that

(a) $xy^iz \in A$ for some i

(b) $|y| > 0$

(c) $|xy| \leq P$

Here, given language is $L = \{0^{n^2}\} / n >= 1\}$

To prove: L is not regular

Proof: Using pumping lemma

Let's assume language L is regular.

Then, if must have pumping length P.

Let P = 3 (say)

Then, let s be the string accepted by finite automaton.

$$\text{i.e. } s = 0^{P^2}$$
$$= 0^{3^2}$$
$$= 0^9$$
$$= 000000000$$

Dividing s into x, y and z as

$$\underbrace{0}_{x} \quad \underbrace{00}_{y} \quad \underbrace{000000}_{z}$$

Checking for condition 1

$$s = xy^iz = xy^2z \qquad (e = 2 \text{ (say)})$$
$$= 0\ 0000\ 000000$$

Now, $|s| = 11$

Here, no value of P(or n) would give (s) = 1 in other words, length of string s on being pumped is not perfect square which means it doesn't belong to language L so, condition 1 fails. Though, condition 2 and 3 are satisfied.

$$|y| = 2 > 0$$
$$|xy|\ \ 3 \geq P$$

Since, condition 1 is failed, our assumption contradicts. That's why language L is not regular.

11. State pumping lemma for regular language. Use pumping lemma and prove that language L = {w | w ∈ {0, 1}* and w has an equal number of 0's and 1's} is not regular. **[2073 Chaitra]**

For statement of pumping lemma for regular language, please see theory part.

Proof: Let, assume that language L is regular. Now, since it is regular, it must follow these conditions:

(a) $xy^iz \in L$ for every $i \geq 0$

(b) $|y| > 0$

(c) $|xy| \leq P$ where P is pumping length

Lent, $S \in L = 01010101$ and $P = 3$

Now, dividing S into x, y and z, we get

$$\underbrace{0}_{x} \quad \underbrace{10}_{y} \quad \underbrace{10101}_{z}$$

Here, the above condition (ii) and (iii) are satisfied as

$|y| = 2 > 0$

$|xy| = 2 \leq 3$

Now, checking for condition 1,

xy^iz

Let, $l = 2$

xy^2z

$= 01010\ 10101$

Then, no of 0's is not equal to number of 1's so, it doesn't belong to L and condition 1 fails. If any of the above condition fails, then our assumption is also false which means language L is not regular.

16. State pumping lemma, for regular language and use the theorem to prove that $L = \{a^n b^{2n} : n \geq 1\}$ is not regular. [2073, Shrawan]

If a language L is a regular language, then it must have a pumping length 'P' such that any string $S \in L$ having length $|S| \geq P$ can be divided into 3 parts such that following conditions are true.

(a) xy^iz for every $i \geq 0$

(b) $|y| > 0$

(c) $|xy| \leq p$

Given, $L = \{a^n b^{2n} : n \geq 1\}$

To prove: L is not regular

Proof: We prove it by contradiction using pumping lemma.

Let's assume language L is regular having pumping length 'P'.

Let, $S \in L = a^p b^{2p}$

Let, P = 4 (say)

Then, $S = a^4 b^8$

= aaaabbbbbbbb

Dividing S into x, y and z, we get

Case - I: When y contains m' y 'a'

$$S = \underbrace{aa}_{x}\ \underbrace{aa}_{y}\ \underbrace{bbbbbbbb}_{z}$$

Checking for condition I

Let, i = 2

$S = xy^2z$

= aa aaaa bbbbbbbb

$= a^6b^8 \notin L$

Here, condition 1 fails as 'b' is not as twice as 'a'.

Case – II: Why y contains both kind of symbols 'a' and 'b' i.e.

$$S = \underbrace{aa}_{x}\ \underbrace{aaa}_{y}\ \underbrace{bbbbbbb}_{z}$$

Checking for condition I

Let, $i = 2$

$S = xy^2z$

$= aa\ aab\ aab\ bbbbbbb \notin L$

Again, condition 1 fails as it doesn't follow pattern

Case – III: When y contains only b's i.e.

$$S = \underbrace{aaaa}_{x}\ \underbrace{bbbb}_{y}\ \underbrace{bbbb}_{z}$$

Checking for condition I

Let, $i = 2$

$S = xy^2z$

$= aaaa\ bbbbbbbbbbbb$

$= a^4 b^{12} \notin L$

Again, condition 1 fails

Here, in pumping (i.e. xy^iz) doesn't follow condition 1. So, none of the cases follow all three condition of regular language as stated by pumping lemma. Therefore, language L is not regular language.

18. Define pumping lemma for regular languages. Use pumping lemma for regular language to show L [a^nba^n for n = 0, 1, 2, ...] is not regular. **[2072, Chaitra]**

Pumping lemma states that if a language L is regular then it must have $|s| \geq p$ where p is pumping length and s is the string, $S \in L$ which can be divided into x, y and z parts such that following conditions are true.

(1) $xy^iz \in L$ for every $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$

To prove: $L = \{a^n b a^n$ for $n = 0, 1, 2, ...\}$ is not regular.

Proof: Let's assume language L is regular. Then it must have pumping length 'p'.

Let, $S \in L = q^p b a^p$

Let, $p = 3$

$S = a^3 b a^3$

aaabaaa

Dividing S into x, y and z, we get

Case – I: When 'y' contains only 'a'

$$S = \underbrace{a}_{x}\ \underbrace{aa}_{y}\ \underbrace{baaa}_{z}$$

Checking for condition I

Let, $i = 2$

$S = xy^2z$

$= a\ aaaa\ bbaaa$

$= a^5ba^3 \notin L$

So, condition 1 fails as the power of a's at the start and end of the string are not equal.

Case – II: When y contains both symbol 'a' and 'b'

$$S = \underbrace{aa}_{x}\ \underbrace{ab}_{y}\ \underbrace{aaa}_{z}$$

Checking for condition I

Let, $i = 2$

$S = xy^2z$

$= aa\ abab\ aaa \notin L$

So, condition 1 fails as it doesn't follow pattern $a^n ba^n$.

Hence, on pumping (i.e. xy^iz), none of the cases follow all three conditions of regular language as stated by pumping lemma. Therefore, language L is not regular.

21. State pumping lemma for regular language. Use this lemma to prove language L : $\{a^{n^2} : n \geq 0\}$ is not regular. **[2071 Chaitra]**

For statement of pumping lemma, see theory part.

Here, given language $L = \{a^{n^2}, n \geq 0\}$

To prove: L is not regular

Proof: Let's assume language L is regular.

Then, it must have pumping length P.

Let, P = 3 (say)

Then, let S be the string accepted by finite automata.

$$\text{i.e. } S = a^{P^2}$$
$$= a^{3^2}$$
$$= a^9$$

Dividing S into x, y and z, we get

$$S = \underbrace{a}_{x}\ \underbrace{aa}_{y}\ \underbrace{aaaaaa}_{z}$$

Now, checking for condition I

$$S = xy^iz$$

Let, $i = 2$

$$\therefore S = xy^2z$$
$$= a\ aaaa\ aaaaaa$$

Now, $|S| = 11$

Here, no value of P (or n) would give $|S| = 11$ in other words, length of string S an being pumped is not perfect square which means it doesn't belong to language L.

26. **State the pumping lemma for regular language show that the language $L = \{a^n : n \text{ is prime}\}$ is not regular using the pumping lemma.** **[2072 Kartik, Back]**

✍ For statement of pumping lemma, see theory part.

To prove: $L = \{a^n : x \text{ is a prime}\}$ is not a regular

Proof: Let's assume that language L is regular. Then, according to pumping lemma, it must follow these conditions.

(1) $xy^iz \in L$ for every $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq P$

Since, language L is regular, it must have pumping length P.

Let $P = 7$

Let, $S \in L = a^P$

$= a^7$

$= aaaaaaa$

Dividing S into x, y and z, we get

$$S = \underbrace{a}_{x}\ \underbrace{aa}_{y}\ \underbrace{aaaa}_{z}$$

Checking for condition I

Let $i = 2$

Then, $S = xy^2z$

$= a\ aaaa\ aaaa$

$= a^9 \notin L$

Since, 9 is not a prime number. Therefore, on pumping i.e. xy^iz, condition 1 fails. Though condition (2) and (3) are satisfied.

$|y| = 2 > 0$

$|xy| = 3 < 7$

Here, assuming language L as regular language doesn't follow all three conditions of pumping lemma to be regular language. So, it contradicts our assumption. Therefore, language L is not regular.