Introduction to Computer Simulation
WS 16/17
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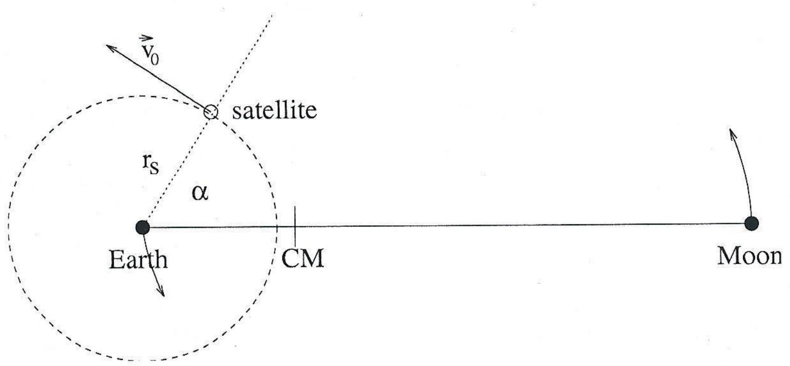
Exercise 9
due 11.01.17

Exercise 9.1:



The goal of this exercise is to make a satellite land on the Moon. Earth and Moon move (as an approximation) on circular orbits around the common center of mass (CM), which we take to be the origin of the coordinate system. See the above figure. The orbits of the Moon and the Earth are in a plane and they are given by $\vec{r}_{\text{Moon}} = R_M(\cos(\omega t), \sin(\omega t))$, $\vec{r}_{\text{Earth}} = -R_E(\cos(\omega t), \sin(\omega t))$. They are not computed from the differential equation. The satellite orbits around the Earth with a radius r_S: at time $t = 0$ the engines are turned on and the satellite leaves the orbit at the angle α. We approximate this event with an instantaneous tangential velocity v_0 in the center-of-mass frame. The satellite's trajectory is in the plane. Please maintain these conventions in order to be able to compare results.

Choice of units:
length: 1 EU (Earth Unit) = Earth radius (6400 km), time 1 h = 1 hour. $GM_{\text{Earth}} = 20\ \text{EU}^3/\text{h}^2$.

Constants:
The radius of the satellite's parking orbit is $r_S = 1.06$ EU. $M_{\text{Earth}}/M_{\text{Moon}} = 81.3$. The orbital period of the Moon is 27.322×24 h. All the remaining parameters can then be derived.

please turn the page

a) Use Kepler's 3rd law to compute $R_E + R_M$ and then determine individually R_E and R_M. Please work this out on paper and hand it in with your solution.

(6 points)

b) Solve the differential equation for the satellite's motion using **ode45**. Set the relative tolerance to 10^{-6} by calling **odeset** with parameter `'RelTol'`. Find initial conditions for (α, v_0) through trial and error such that the satellite "lands" on the Moon, that is it gets closer than 3500 km to the Moon's center. Hand in the values of (α, v_0) you found, your program and an interesting picture of the orbit. [hint for the order of magnitude: estimate v_0 for an orbit around the Earth whose semi-major axis reaches further than the Moon]

(14 points)

Hint for the organization of the program and for MATLAB:

- the satellite's position and velocity form a vector $\vec{y}$ and the differential equation is $\dot{\vec{y}}(t) = \vec{f}(t, \vec{y}(t))$, where $\vec{f}$ is the total force acting on the satellite.
- 1 routine for $\vec{f}(t, \vec{y})$, 1 routine for $\vec{r}_{\text{Earth}}(t), \vec{r}_{\text{Moon}}(t)$.
- Main loop: from day 1 to day 28, inside the loop an integration over 24 h,
  - computation of the distance between the satellite and the Moon for the satellite positions returned in $\vec{y}$ by the call to **ode45**
  - exit the loop with **break** if the satellite has "landed".
  - Plot of the current position of the Moon and of the satellite orbit during the day.

The command **hold** after the first plot command (for example to mark the origin of the coordinate system) guarantees that the remaining plots will be added. A **clf** deletes the present view. The command **drawnow** after the plot commands in the loop updates the plot while the loop is processed.