Homework 11
due 25.01.16

Exercise 11.1:

This exercise will demonstrate that the method of Gaussian elimination for solving systems of linear equations $Ax = b$ can be made safer by "pivoting" and the numerical accuracy can thereby be increased.

The starting point is the program `gaussel.m`, which you can download from `http://csis.uni-wuppertal.de/courses/ics16.html`. In order to warm up, determine the numerical error of the Gaussian elimination method **without** pivoting for random matrices of size $n = 2, \ldots, 100$. To this end, generate a random "solution" x, compute $b = Ax$ and solve $Ay = b$ numerically for y. The error is defined as $\delta = \max_i |x_i - y_i|$. Plot δ vs. n on a logarithmic scale. Afterwards implement the **partial pivoting**, i.e. permutation of the equations, which are the rows of the matrix $B = [A\ b]$. This should be done as discussed in the lecture and without scaling.

Provide a printout of the program with partial pivoting and a plot that shows the numerical errors for Gaussian elimination with and without pivoting compared to MATLAB's built-in solver `A\b`. Try to reduce the error in both versions by using the iterative improvement discussed in the lecture. How many iterations are meaningful?

(15 points)

Exercise 11.2:

Using the program `gaussel.m` investigate the computational cost of the Gaussian elimination method as a function of the problem size. In the problem $Ax = b$, A is a $n \times n$ matrix, x and b are of size $n \times m$. Derive formulae for the number of multiplications and divisions as a function of n and m. Give separate formulae for the elimination step and for the back-substitution. Check your formulae by inserting appropriate counters in the program.

Where is most of computer time needed for the case $n \gg m$? Why is it more efficient for the same matrix A to solve for several right hand sides b *simultaneously*? Explain the results by considering for example $n = 50$ and comparing $m = 1$ with $m = 2$.

(10 points)