CSIS
Computer Simulation in Science

Introduction to Computer Simulation
WS 16/17
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Homework 6
due 07.12.2016

Exercise 6.1: Image compression

In this exercise we study methods that are used in (lossy) image compression. Our test image `test_image.tif` as well as the MATLAB script `read_image.m` that reads it into a matrix and displays it on the screen are provided on `http://csis.uni-wuppertal.de/courses/ics16.html`.

a) Write functions that compute the two dimensional discrete cosine transform (DCT) and the inverse cosine transform (IDCT) of an input image. If $Y(n_1, n_2)$ contains the original image (size $N_1 \times N_2$), the transforms are

$$\tilde{Y}(k_1,k_2) = \sqrt{\frac{w(k_1)w(k_2)}{N_1\,N_2}} \sum_{n_1=1}^{N_1}\sum_{n_2=1}^{N_2} Y(n_1,n_2)\cos\left[\frac{\pi(2n_1-1)(k_1-1)}{2N_1}\right]\cos\left[\frac{\pi(2n_2-1)(k_2-1)}{2N_2}\right],$$

$$Y(n_1,n_2) = \sum_{k_1=1}^{N_1}\sum_{k_2=1}^{N_2} \sqrt{\frac{w(k_1)w(k_2)}{N_1\,N_2}}\tilde{Y}(k_1,k_2)\cos\left[\frac{\pi(2n_1-1)(k_1-1)}{2N_1}\right]\cos\left[\frac{\pi(2n_2-1)(k_2-1)}{2N_2}\right],$$

$$w(k) = \begin{cases} 1 & \text{if } k=1 \\ 2 & \text{else} \end{cases}$$

Demonstrate, that the transforms are inverse to each other. Hand in a plot of the transformed test image. (Hint: transform first along one dimension, then along the other)

(9 points)

b) Write a function `crop(Y, rho)` that sets pixels with $n_1^2 + n_2^2 > \rho^2(N_1^2 + N_2^2)$ to zero. Hand in a "cropped" test image (with $\rho = 0.3$).

(3 points)

c) DCT the test image, apply your cropping function to the transformed image (i.e. remove high frequencies $k_1^2 + k_2^2 > \rho^2(N_1^2 + N_2^2)$). This is a "compressed" image, since one would not need to store the zeros. The compression rate is given by the ratio of non-zero pixels to the total number of pixels.

To reconstruct the image IDCT the cropped result. Plot this for $\rho = 0.1, 0.3, 0.5$? What is the relative difference to the original in each case? What is the compression rate?

(3 points)

d) Divide the test image into blocks of size 8×8. Apply the compression algorithm to every block separately, i.e. DCT every 8×8 block and apply `crop` to every block ($\rho = 0.3$). What is the compression rate now? Reconstruct the original image by applying the IDCT to every cropped block. Is the quality of the compression, at a comparable compression rate, better or worse than in c)?

(3 points)