Optimal and coherent economic-capital structures: Evidence from long and short-sales trading positions under illiquid market perspectives

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Optimal and coherent economic-capital structures: evidence from long and short-sales trading positions under illiquid market perspectives

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Abstract This paper broadens research literature associated with the assessment of modern portfolio risk management techniques by presenting a thorough modeling of nonlinear dynamic asset allocation and management under the supposition of illiquid and adverse market settings. Specifically, the paper proposes a re-engineered and robust approach to optimal economic capital allocation, in a Liquidity-Adjusted Value at Risk (L-VaR) framework, and particularly from the perspective of trading portfolios that have both long and short-sales trading positions. This paper expands previous approaches by explicitly modeling the liquidation of trading portfolios, over the holding period, with the aid of an appropriate scaling of the multiple-assets' L-VaR matrix along with GARCH-M technique to forecast conditional volatility and expected return. Moreover, in this paper, the authors develop a dynamic nonlinear portfolio selection model and an optimization algorithm which allocates both economic capital and trading assets subject to some selected financial and operational rational constraints. The empirical results strongly confirm the importance of enforcing financially and operationally meaningful nonlinear and dynamic constraints, when they are available, on economic capital optimization procedure. The empirical results are interesting in terms of theory as well as practical applications and can aid in developing robust portfolio management algorithms that financial entities could consider in light of the aftermath of the latest financial crisis.

Keywords Economic capital · Emerging markets · Financial engineering · Financial risk management · GARCH · GCC financial markets · Liquidity risk · Portfolio management · Liquidity-adjusted value at risk

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1 Introduction and outline

The significance of assessing the market risk of portfolios of financial securities has long been acknowledged by academics and practitioners. In recent years, the growth of trading activities and instances of financial market upheavals has prompted new research underlining the necessity for market participants to develop reliable dynamic portfolio management and risk assessment methods and algorithms. In measuring market risk of trading portfolios, one technique advanced in the literature involves the use of Value at Risk (VaR) models that ascertain how much the value of a trading portfolio would plunge, in monetary terms, over a given period of time with a given probability as a result of changes in market prices. Nowadays, VaR is by far the most popular and most accepted risk measure among financial institutions, however, whether or not there is a best way to estimate VaR is still debatable. Although VaR is a very popular measure of market risk of financial trading portfolios, it is not a panacea for all risk assessments and has several drawbacks, limitations and undesirable properties. From a portfolio market risk point of view, VaR faces some major difficulties. Three of the most researched and discussed issues are the non-normal behavior of market returns, volatility clustering and the impact of illiquid securities. The effect of the latter on portfolio risk management and dynamic economic capital allocation under market liquidity constraints is the main focus of this paper.

Indeed, methods for measuring market (or trading) risk have been well developed and standardized in the academic as well as the banking world. Asset liquidity trading risk, on the other hand, has received less attention from researchers, perhaps because it is less significant in developed countries where most of the market risk methodologies were originated. In all but the most simple of circumstances, comprehensive metrics of liquidity trading risk management do not exist explicitly within modern portfolio theory. Nonetheless, the combination of the recent rapid expansion of emerging markets' trading activities and the recurring turbulence in those markets, in light of the aftermaths of the sub-prime financial crisis, has propelled asset liquidity trading risk to the forefront of market risk management research and development (Al Janabi 2008). Likewise, the recent financial crisis shows that asset liquidity risk played a major role in, for example, the bankruptcy of Lehman Brothers and Bear Stearns; and also played a big role in the demise of Long-Term-Capital-Management (LTCM). As such, asset liquidity risk became in recent times of particular concern in developed markets as well.

In effect, the conventional VaR approach to computing market risk of a portfolio does not explicitly consider asset liquidity risk. Typical VaR models are based on modern portfolio management theory and assess the worst change in the mark-to-market portfolio value over a given time horizon but do not account for the actual trading risk of liquidation. In general, customary fine-tunings are made on an ad hoc basis. At most, the holding period (or liquidation horizon) over which the VaR number is calculated is adjusted to ensure the inclusion of liquidity risk. As a result, liquidity trading risk can be imprecisely factored into VaR assessments by assuring that the liquidation horizon is as a minimum larger than an orderly liquidation interval. Moreover, the same liquidation horizon is employed to all trading asset classes, albeit some assets may be more liquid than others.

In this backdrop and to address the above deficiencies, in this paper we characterize trading risk for emerging equity markets by using a multivariate Liquidity-Adjusted Value at Risk (L-VaR) approach that focuses on the modeling of optimum L-VaR under the notion of illiquid and adverse market conditions and by exercising different correlation factors and liquidity horizon periods. As such, the overall objective of this paper is to construct different equity portfolios, which include several stock markets indices of the Gulf Cooperation

Council (GCC) zone, and to evaluate the risk characteristics of such a portfolio besides examining an optimization algorithm process for assessing economic capital's[1] efficient and coherent[2] market portfolios. To this end, we propose a general liquidity trading risk model that accounts for the characteristics of the series of equity price returns—for example, fat tails (leptokurtosis), skewness, correlation factors, and liquidity horizons—and adequately forecasts market risk within a short time horizon.

In spite of the increasing importance of the GCC financial markets, there is very little published research in this respect and particularly within the context of a comprehensive testing of the six GCC stock markets risk-return characteristics and dynamic economic capital allocation with advanced risk measures. The literature on testing volatility, expected returns and risk measurement of the GCC equity markets has been relatively meager, inconclusive and providing mixed results. As such, and in contrast to all existing published literatures pertaining to the application of advanced risk-return analysis and dynamic economic capital allocation, this article intents to make the following main contributions to the academic literature in this specific risk management field. Firstly, it represents one of the limited numbers of research papers that empirically examines equity trading risk management using actual data of the six GCC financial markets. Secondly, a database of the six GCC states indices is utilized whose behavior is presumably more diverse than if equity assets of any particular stock market had been employed, as other authors have done heretofore. The basic argument is that specific country indices may have, compared to individual stocks, a more predictable structure due to aggregation. Thirdly, unlike most empirical studies in this field, this study employs a thorough and credible liquidity trading risk management model that considers risk analysis under normal, severe (crisis) and illiquid market conditions. The principal advantage of employing such a model is the ability to capture a full picture of possible loss scenarios of actual equity trading portfolios. Fourthly, this paper proposes a new approach to optimal economic capital allocation besides a robust optimization algorithm for the selection of efficient and coherent economic capital portfolios within an L-VaR framework.

In a nutshell, this paper provides portfolio risk management techniques and strategies (drawn from financially meaningful investment considerations) that can be applied to equity trading portfolios in emerging markets. More specifically, the intent of this paper is to propose a simple approach for including of liquidation trading risk in standard VaR analysis and to offer an optimization algorithm for the selection of optimal and coherent economic capital portfolios in a L-VaR framework. In this paper, we attempt to integrate and estimate the impact of liquidity trading risk into VaR models by explicitly incorporating the impact of the time-volatility dimension of liquidity risk instead of the movements in the bid-ask spread. The approach to assessing liquidity-adjusted VaR for distinctive equity portfolio has been illustrated with the help of a modified closed-form parametric model, where

[1] Economic capital (or risk capital) can be defined as the minimum amount of equity capital a financial entity needs to set aside to absorb worst losses over a certain time horizon with a certain confidence level. This is with the objectives of sustaining its trading operations activities and without subjecting itself to insolvency matters. Economic capital can be assessed with an internal method and modeling techniques such as L-VaR. Economic capital differs somehow from regulatory capital, which is necessary to comply with the requirements of Basel II committee on capital adequacy. However, building an internal market risk modeling techniques to assess economic capital can significantly aid the financial entity in complying with Basel II capital adequacy requirements.

[2] In this paper, the concept of coherent market portfolios refers to rational portfolios that are contingent on meaningful financial and operational constraints. In this sense, coherent market portfolios do not lie on the efficient frontiers as defined by Markowitz (1959), and instead have logical and well-structured long/short asset allocation proportions.

conditional volatility and expected returns are estimated with the aid of a generalized autoregressive conditional heteroscedasticity in mean (GARCH-M) model. We then demonstrate, by applying the liquidity risk measures to the GCC financial markets, to what extent the quantified liquidity trading risk effects can impact traditional measurement of market risk under different correlation assumption: empirical, zero, negative and positive unity.

2 Trading risk management in GCC markets and the Basel II accord

In the last two decades, financial institutions in emerging markets (such as in the case of the six GCC financial markets) have greatly increased their holdings of trading assets, such as bonds, equities, interest rate and equity derivatives, foreign exchange and commodity positions. Their intention in this has been to earn trading profits and to hedge exposures elsewhere in their trading portfolios. Nevertheless, the lack of adequate market risk measurement, management, and control tools are some of the contributing factors that have led to major financial losses among national/multinational corporations in emerging economies.

To quantify the risks involved in their trading operations, major financial and non-financial institutions are increasingly exploiting VaR internal models. Since these institutions differ in their individual characteristics, tailor-made internal risk models are more appropriate. Moreover, the increase in the relative importance of trading risk has obliged regulators to reconsider the system of capital requirements as outlined in previous Basel committee capital accords. Fortunately, and in accordance with the latest Basel II capital accord, trading institutions are permitted to develop their own internal risk models for the purposes of providing for adequate risk measures. Furthermore, internal risk models can be used in the determination of capital that entities must hold to endorse their trading of securities. The benefit of such an approach is that it takes into account the relationship between various asset types and can accurately assess the overall risk for a whole combination of trading assets.

The Basel accord (so-called Basel II), for the establishment of adequate internal models of risk management, has motivated several emerging countries to be part of the agreement at different implementation levels. This is aggravated by the fact that emerging markets financial institutions face a substantial competitive disadvantage if they are enforced to continue using the standardized approach. As such, several emerging markets, in the Asian and Latin American continents, would like to be Basel II-compliant and hence are already in advanced steps to implement internal models and to comply with the Basel agreement. Basel II overall intention is to endorse adequate capitalization of banks, and encourage improvements in risk measurement, management and control, thereby strengthening stability in the whole financial system. Basel II accord does so by implementing three complementary pillars: one concerning capital adequacy methodology and calculation, another on supervisory review, and a third setting disclosure terms to enable market discipline.

A number of Arab countries are also voluntarily joining the implementation of modified versions of the Basel II accord. In fact, the Gulf Cooperation Council (GCC) financial markets, in general, are in progressive stages of implementing advanced risk management regulations and techniques. Furthermore, in recent years outstanding progress has been done in cultivating the culture of risk management among local financial entities and regulatory institutions. In the Middle East the majority of banking assets is expected to be covered by Basel II regulations during 2008–2012. Generally speaking, capital ratios are fairly strong in the GCC, though they have fallen lately as banks have expanded their products and operations. Within the GCC, there have been negotiations for common application of the Basel II rules, though with different timeframes. This is due to the fact that some GCC countries are more diverse, for instance, in terms of the presence of foreign banks than others.

The financial industry in GCC countries is generally sound, and the six countries continue to develop their financial system to attract more foreign portfolio investors, and to expand the opening of its financial system to the exterior world. Consequently, several local financial institutions are in a consolidation route; and some others have already followed a process of convergence of their financial operations and have started the procedure of modernizing their internal risk management capabilities. By the standards of emerging market countries, the quality of banking supervision in the six states of the GCC is well above average. Despite the latest progress in the GCC financial markets to become Basel-compliant countries, recently in the midst of the latest market turbulence it has been deemed necessary (by local regulatory authorities) to adapt proper internal risk models, rules and procedures that financial entities, regulators and policymakers should consider in setting-up their daily trading risk management objectives and to determine optimal economic capital allocations effectively.

Set against this background and as a result of the previous discussion, trading risk management has become an important theme in emerging and illiquid markets, such as in the case of the GCC financial markets. Accordingly, the goals of this work are to demonstrate the necessary analytical steps and internal risk management processes that a market's participant (a market-maker or a fund manager) will need in his day-to-day positions' taking. This paper provides real-world risk management techniques and optimization strategies that can be applied to equity trading portfolios in emerging markets, such as the GCC financial markets. This is with the objective of setting up the basis of a proactive financial modeling technique for the management and control of risk exposures in the day-to-day trading operations. Analytical procedures that are discussed in this work will aid financial markets' participants, regulators and policymakers in founding sound and up to date internal risk management modeling methods to handle equity trading risk exposures.

3 Underlying principle of current research and literature review

Portfolio optimization and dynamic asset allocation has come a long way from Markowitz (1959) seminal work which pioneers the return/variance risk management framework. The developments in portfolio optimization techniques are stimulated by two basic requirements: a proper modeling of utility functions, risk measures, and budget constraints and the second requirement is related to the efficiency of the algorithms in handling a large number of securities and asset allocation scenarios. In fact, the simplicity and the intuitive appeal of portfolio construction using modern portfolio theory have attracted significant attention both in academia and in practice. Yet, despite considerable effort it took many years until portfolio managers started using modern portfolio theory for managing real money. Unfortunately, in real world applications there are many problems associated with it, and portfolio optimization is still considered by many practitioners to be difficult to apply (Fabozzi et al. 2006).

Indeed, since the commencement of modern finance theory there is a constant debate on the concept of risk, and a rising awareness in ways to measure it and to manage it properly. This controversy has gone along with a growing investment in portfolio models, based on sophisticated quantitative methods, which require an enormous computing power. One should not be surprised by this reality given that financial markets are now much more volatile and the use of derivative securities such as options and futures contracts, for hedging and speculation purposes require continuous developments in finance and investment modeling. In particular, the development of new methods of portfolio risk management (such as, advanced VaR algorithms) is now an overriding issue in the financial and academic communities.

Despite many criticisms and limitations of the VaR method, it has proven to be a very useful measure of market risk, and is widely used in financial and non-financial markets. Evidently, the overwhelming emphasis in VaR techniques has come from the finance literature, mostly as it pertains to the need of entities to satisfy regulatory requirements. Based on studies to date, there is little agreement as to the best method for developing VaR risk measures. However, literature related to VaR is continually growing as researchers attempt to reconcile several pending issues. The prior literature on VaR and portfolio risk management has been focused on two distinct lines of research. The first category focuses mainly on the use of different VaR models for market and credit risk management and for selecting optimal portfolios within VaR framework, whereas the second category emphasizes the development of asset liquidity risk as an integral part of market risk and, therefore, leads to several approaches for the estimation of L-VaR. Below we discuss some of the relevant literature classified according to the above two categories.

3.1 Literature review on optimal portfolio selection within a Value at Risk (VaR) structure

The literature on measuring financial risks and volatility using VaR models is extensive, yet Jorion (2007) and Dowd et al. (2004) should be pointed out for their integrated approach to the topic. The general recognition and use of large scale VaR models has initiated a considerable literature including statistical descriptions of VaR and assessments of different modeling techniques. For a comprehensive survey, and the different VaR analysis and techniques, one can refer to Jorion (2007).

On another front, other authors have investigated the use of VaR for the selection of optimum portfolios and for active portfolio management.[3] For instance, Campbell et al. (2001) develop a an optimal portfolio selection model which maximizes expected return subject to a downside risk constraint rather than standard deviation alone. The suggested model allocates financial assets by maximizing expected return conditional on the constraint that the expected maximum loss should be within the VaR limits set by the risk manager.

In another study, Yiu (2004) examines the optimal portfolio problem by imposing VaR as a dynamic constraint. This approach provides a path to control risks in the optimal portfolio and to satisfy the requirement of regulators on the assessment of market risks. Furthermore, the VaR constraint is derived for some risky assets plus a risk-free asset and is imposed continuously over time and the problem is formulated as a constrained utility maximization problem over a period of time. To this end, a dynamic programming technique is applied to derive the Hamilton-Jacobi-Bellman equation (HJB) and the method of Lagrange multiplier has been applied to handle the constraint. Moreover, a numerical method is proposed to solve the HJB-equation and hence the constrained optimal portfolio.[4]

In their paper, Alexander and Baptista (2004) analyze the portfolio selection implications arising from imposing a VaR constraint as a risk management tool on the mean-variance model, and compare them with those arising from the imposition of a conditional Value-at-Risk (CVaR) constraint. The authors find that under certain conditions, the presence of

[3]For an excellent survey of recent contributions to robust portfolio strategies from operations research and finance to the theory of portfolio selection, one can refer to Fabozzi et al. (2010).

[4]In a related approach, Elliott and Siu (2010) provide a verification theorem for the Markovian regime-switching HJB solution of the stochastic differential game corresponding to the risk-minimizing problem.

a VaR constraint causes a slightly risk-averse agent to select a portfolio that has a smaller standard deviation than the one that would have been selected in its absence. Furthermore, the authors show that for a given confidence level; a CVaR constraint is tighter than a VaR constraint if the CVaR and VaR bounds coincide.

Finally, in a relatively recent study, Alexander and Baptista (2008) look at the impact of adding a VaR constraint to the problem of an active manager who seeks to outperform a benchmark by a given percentage. In doing so, the authors minimize the tracking error variance (TEV) by using the model of Roll (1992). As such, the authors obtain three main results. First, portfolios on the constrained mean-TEV boundary still display three-fund separation; however the weights of the three funds when the constraint binds differ from those in Roll's model. Second, the VaR constraint mitigates the problem that when a manager seeks to outperform a benchmark using the mean-TEV model, he or she selects a portfolio that is mean-variance inefficient. Finally, when short-sales are not permitted, the extent to which the constraint decreases the optimal portfolio's efficiency loss can still be noteworthy but is less significant than when short-sales are permitted.

3.2 Literature related to Liquidity-adjusted Value at Risk (L-VaR) modeling

Asset liquidity risk becomes for the most part important to financial market participants who are about to hold or currently holding an asset, since it affects their ability to trade or unwind the trading position. Insolvencies often occur because financial entities cannot get out or unwind their holdings effectively and hence the liquidation value of assets may differ significantly from their current mark-to-market values.

As such, the combination of the latest swift expansion of emerging markets' trading activities and the persistent turbulence in those markets has impelled liquidity trading risk to the vanguard of market risk management research and development. To this end, within the VaR framework, Jarrow and Subramanian (1997) provide a market impact model of liquidity by considering the optimal liquidation of an investment portfolio over a fixed horizon. They derive the optimal execution strategy by determining the sales schedule that will maximize the expected total sales values, assuming that the period until liquidation is given as an exogenous factor. Although the model is simple and intuitively appealing, it suffers from practical difficulties for its implementation.

Bangia et al. (1999) approach liquidity risk from another angle and provide a model of VaR adjusted for what they call exogenous liquidity—defined as common to all market players and unaffected by the actions of any one participant. It comprises such execution costs as order processing costs and adverse selection costs resulting in a given bid-ask spread faced by investors in the market. On the contrary, endogenous liquidity is specific to one's position in the market and depends on one's actions and varies across market participants. Their model consists of measuring exogenous liquidity risk, computed using the distribution of observed bid-ask spreads and then integrating it into a standard VaR framework.

On another front, Berkowitz (2000) argues that unless the likely loss arising from liquidity risk is quantified, the models of VaR would lack the power to explicate the embedded risk. In practice, operational definitions vary from volume-related measures to bid-ask spreads and to the elasticity of demand. The author asserts that elasticity based measures are of most relevance since they incorporate the impact of the seller actions on prices. Moreover, under certain conditions the additional variance arising from seller impact can easily be quantified given observations on portfolio prices and net flows; and that it is possible to estimate the entire distribution of portfolio risk through standard numerical methods.

Lately, in his research paper, Al Janabi (2008) establishes a practical framework for the measurement, management and control of trading risk. The effects of illiquid assets, that are dominant characteristics of emerging markets, are also incorporated in the risk models. The key methodological contribution is a different and less conservative liquidity scaling factor than the conventional root-t multiplier. The proposed add-on is a function of a predetermined liquidity threshold defined as the maximum position which can be unwound without disturbing market prices during one trading day.

3.3 Foundations and explicit objectives of current research

As indicated above, equity prices are exposed to a variety of volatile market risk factors that can be and have been examined in a portfolio context. However, despite the rising interest in emerging markets, as in the case of the six GCC financial markets, earlier research does not provide any broad methods for handling trading risk and coherent assessment of economic capital allocation under illiquid and adverse market settings and particularly within emerging markets equity trading portfolios. Considering the recent interest in L-VaR and the variability of the market risk factors of different emerging markets, the overall aim of this paper is to examine L-VaR and economic capital measures in the context of equity trading portfolios (of varied combinations of long/short trading positions) and under the notion of different correlation factors and liquidity horizons. In particular, this paper develops and tests L-VaR and economic capital measures, using several alternative strategies in predicting large losses, with the aid of different liquidation horizons and under a pre-determined confidence level. Thus, equity trading portfolios provide a practical case for testing L-VaR and economic capital methodologies in the prospect of equity prices, helping to establish the appropriateness of L-VaR as a viable and important risk management tool for equity risk managers and portfolio managers.

This paper shows that the performance of efficient and coherent economic capital portfolios depends on the expected return, individual L-VaR positions, liquidity horizons of each trading asset, and the set of portfolio weights. The empirical findings indicate that the risk tolerance in the L-VaR framework is time-varying and closely related to the selection of the unwinding liquidity horizons, expected returns in addition to the impact of the assumed correlation factors of the portfolio. Moreover, in this work, the relative performance of the L-VaR and economic capital selection model is compared in a dynamic asset allocation framework. The objective of the dynamic asset allocation is to find the optimum equity asset allocation mix by minimizing L-VaR and economic capital subject to the imposition of operational and financial constraints based on fundamental asset management considerations.

The rest of the paper proceeds as follows. Section 4 lays out the salient features and derives the necessary quantitative infrastructure of L-VaR, and its limitations. Section 5 analyses the overall results of the different empirical tests and discusses the process and infrastructure that support large-scale quantitative-based investing and the role of an optimization engine in this process. This section also reflects on construction of efficient and coherent economic capital portfolios. Section 6 remarks on conclusions with a brief summary and recommendations for future research. Full set of all relevant tables and figures of empirical testing, equity trading risk management reports, and the fund-manager's coherent economic capital portfolios are included in Tables 1–9.

4 Theoretical underpinning of L-Var and economic capital models for dynamic portfolio risk management

4.1 General parametric L-VaR approach for trading risk management

One of the most significant advances in the past two decades in the field of measuring and managing financial risks is the development and the ever-growing use of VaR methodology. VaR has become the standard measure that financial analysts use to quantify financial risks including equity risk. VaR represents the potential loss in the market-value of a portfolio of equities with a given probability over a certain time horizon. The main advantage of VaR over other risk measures is that it is theoretically simple. As such, VaR can be used to summarize the risk of an individual equity position or the risk of large portfolios of equity assets. Thus, VaR reduces the risk associated with any portfolio of equities (or other assets) to just one number—the expected loss associated with a given probability over a defined holding period.

To calculate VaR using the parametric (also known as the variance/covariance, analytical and delta-neutral) method, the volatility of each risk factor is extracted from a pre-defined historical observation period and can be estimated using GARCH-M model. The potential effect of each component of the portfolio on the overall portfolio value is then worked out. These effects are then aggregated across the whole portfolio using the correlations between the risk factors (which are, again, extracted from the historical observation period) to give the overall VaR value of the portfolio with a given confidence level. As such, for a single trading position the absolute value of VaR can be defined in monetary terms as follows:

$$\mathrm{VaR}_i = \left|(\mu_i - \alpha \cdot \sigma_i)(\text{Mark-to-Market Value of Asset}_i \cdot Fx_i)\right| \tag{1}$$

where μ_i is the expected return of the asset, α is the confidence level (or in other words, the standard normal variant at confidence level α) and σ_i is the conditional volatility of the return of the security that constitutes the single position and can be estimated using a GARCH-M model. While the Mark-to-Market Value of Asset$_i$ indicates the amount of investment in asset i, Fx_i denotes the unit foreign exchange rate of asset i. If the expected return of the asset, μ_i, is very small, then (1) can be reduced to:

$$\mathrm{VaR}_i = |\alpha \cdot \sigma_i \cdot \text{Mark-to-Market Value of Asset}_i \cdot Fx_i| \tag{2}$$

Indeed, (2) includes some simplifying assumptions, yet it is routinely used by researchers and practitioners in the financial markets for the estimation of VaR for a single trading position.

Trading risk in the presence of multiple risk factors is determined by the combined effect of individual risks. The extent of the total risk is determined not only by the magnitudes of the individual risks but also by their correlations. Portfolio effects are crucial in risk management not only for large diversified portfolios but also for individual instruments that depends on several risk factors. For multiple assets or portfolio of assets, VaR is a function of each individual security's risk and the correlation factor $[\rho_{i,j}]$ between the returns on the individual securities as follows:

$$\mathrm{VaR}_P = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{VaR}_i \mathrm{VaR}_j \rho_{i,j}} = \sqrt{[\mathrm{VaR}]^T[\rho][\mathrm{VaR}]} \tag{3}$$

This formula is a general one for the calculation of VaR for any portfolio regardless of the number of securities. It should be noted that the second term of the above formula is rewritten in terms of matrix-algebra—a useful form to avoid mathematical complexity, as more and more securities are added. This approach can simplify the programming process and permits easy incorporation of short positions in market risk management process. This means, in order to calculate portfolios' VaR we need first to construct a transpose vector $[\mathrm{VaR}]^T$ of individual VaR positions—an $(1 \cdot n)$ vector, and hence the superscript "T" indicates transpose of the vector; secondly a vector [VaR] of individual VaR positions—explicitly n rows and one column $(n \cdot 1)$ vector; and finally a matrix $[\rho]$ of all correlation factors (ρ)—an $(n \cdot n)$ matrix.

In effect, the VaR method is only one approach of measuring market risk and is mainly concerned with maximum expected losses under normal market conditions. It is not an absolute measure, as the actual amount of loss may be greater than the given VaR amounts under severe circumstances. In extreme situations, VaR models do not function very well. As a result, for prudent risk management and as an extra management tool, firms should augment VaR analysis with stress-testing and scenario procedures. In this paper, risk management procedure is developed to assess potential loss exposure due to an event risk (severe crisis) that is associated with large movements of the GCC stock markets indices, under the assumption that certain GCC markets have typical 3%–12% leap during periods of financial turmoil. The task here is to measure the potential trading risk exposure that is associated with a pre-defined leap and under the notion of several correlation factors and liquidation horizons.

4.2 Incorporating asset liquidity risk into L-VaR models

Liquidity is a key risk factor, which until lately, has not been appropriately dealt with by risk models. Illiquid trading positions can add considerably to losses and can give negative signals to traders due to the higher expected returns they entail. The concept of liquidity trading risk is immensely important for using VaR accurately and recent upheavals in financial markets confirm the need for laborious treatment and assimilation of liquidity trading risk into VaR models.

The simplest way to account for liquidity trading risk is to extend the holding period of illiquid positions to reflect a suitable liquidation period. An adjustment can be made by adding a multiplier to the VaR measure of each trading asset type, which at the end depends on the liquidity of each individual security. Nonetheless, the weakness of this method is that it allows for subjective assessment of the liquidation period. Furthermore, the typical assumption of a one-day horizon (or any inflexible time horizon) within VaR framework, neglects any calculation of trading risk related to liquidity effect (that is, when and whether a trading position can be sold out and at what price). A broad VaR model should incorporate a liquidity premium (or liquidity risk factor). This can be worked out by formulating a method by which one can unwind a position, not at some ad hoc rate, but at the rate that market conditions is optimal, so that one can effectively set a risk value for the liquidity effects. In general, this will raise significantly the VaR, or the amount of economic capital to support the trading position.

In fact, if returns are independent and they can have any elliptical multivariate distribution, then it is possible to convert the VaR horizon parameter from daily to any $t-$day horizon. The variance of a $t-$day return should be t times the variance of a $1-$day return or $\sigma^2 = f(t)$. Thus, in terms of standard deviation (or volatility), $\sigma = f(\sqrt{t})$ and the daily or overnight VaR number [VaR$(1-\text{day})$] can be adjusted for any $t-$ day horizon as:

$$\text{VaR}(t-\text{day}) = \text{VaR}(1-\text{day})\sqrt{t} \tag{4}$$

The above formula was proposed and used by *J.P. Morgan* in their earlier *RiskMetrics^{TM}* method (1994). This methodology implicitly assumes that liquidation occurs in one block sale at the end of the holding period and that there is one holding period for all assets, regardless of their inherent trading liquidity structure. Unfortunately, the latter approach does not consider real-life trading situations, where traders can liquidate (or re-balance) small portions of their trading portfolios on a daily basis. The assumption of a given holding period for orderly liquidation inevitably implies that assets' liquidation occurs during the holding period. Accordingly, scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding period.

In this work we present a re-engineered approach for calculating a closed-form parametric L-VaR with explicit treatment of liquidity trading risk and coherent assessment of economic capital. The proposed model and liquidity scaling factor is more realistic and less conservative than the conventional root-t multiplier. In essence the suggested multiplier is a function of a predetermined liquidity threshold(s) defined as the maximum position which can be unwound without disturbing market prices during one trading day. The essence of the model relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate overall trading risk assessment during market stress periods when liquidity dries up. To this end, a practical framework of a methodology (within a simplified mathematical approach) is proposed below with the purpose of incorporating and calculating illiquid assets' horizon L-VaR, detailed along these lines.

The market risk of an illiquid equity trading position is larger than the risk of an otherwise identical liquid position. This is because unwinding the illiquid position takes longer than unwinding the liquid position, and, as a result, the illiquid position is more exposed to the volatility of the market for a longer period of time. In this approach, an equity trading position will be thought of illiquid if its size surpasses a certain liquidity threshold. The threshold (which is determined by traders for different equities and/or financial markets) and defined as the maximum position which can be unwound, without disrupting market prices, in normal market conditions and during one trading day. Consequently, the size of the equity trading position relative to the threshold plays an important role in determining the number of days that are required to close the entire position. This effect can be translated into a liquidity increment (or an additional liquidity risk factor) that can be incorporated into VaR analysis. If for instance, the par value of an equity position is \$50,000 and the liquidity threshold is \$25,000, then it will take two days to sell out the entire trading position. Therefore, the initial position will be exposed to market variation for one day, and the rest of the position (that is, \$25,000) is subject to market variation for an additional day. If it assumed that daily changes of market values follow a stationary stochastic process, the risk exposure due to illiquidity effects is given by the following illustration, detailed as follows.

In order to take into account the full illiquidity of equity assets (that is, the required unwinding period to liquidate an asset) we define the following:

$t =$ number of liquidation days ($t-$ days to liquidate the entire equity asset fully)
$\sigma_{\text{adj}}^2 =$ variance of the illiquid equity trading position; and
$\sigma_{\text{adj}} =$ liquidity risk factor or standard deviation of the illiquid equity trading position.

The proposed approach assumes that the trading position is closed out linearly over $t-$ days and hence it uses the logical assumption that the losses due to illiquid trading positions over $t-$ days are the sum of losses over the individual trading days. Moreover, we can assume with reasonable accuracy that asset returns and losses due to illiquid trading positions are independent and identically distributed (iid) and serially uncorrelated day-to-day along the liquidation horizon and that the variance of losses due to liquidity risk over $t-$ days is the sum of the variance (σ_i^2, for all $i = 1, 2, \ldots, t$) of losses on the individual days, thus:

$$\sigma_{\text{adj}}^2 = \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots + \sigma_{t-2}^2 + \sigma_{t-1}^2 + \sigma_t^2\right) \tag{5}$$

In fact, the square root-t approach (4) is a simplified special case of (5) under the assumption that the daily variances of losses throughout the holding period are all the same as first day variance, σ_1^2, thus $\sigma_{\text{adj}}^2 = (\sigma_1^2 + \sigma_1^2 + \sigma_1^2 + \cdots + \sigma_1^2) = t\sigma_1^2$. As discussed above the square root-t equation overestimates asset liquidity risk since it does not consider that traders can liquidate small portions of their trading portfolios on a daily basis and then the whole trading position can be sold completely on the last trading day. Indeed, in real financial markets operations, liquidation occurs during the holding period and thus scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding period. As such, for this special linear liquidation case and under the assumption that the variance of losses of the first trading day decreases linearly each day (as a function of t) we can derive from (5) the following:

$$\sigma_{\text{adj}}^2 = \left(\left(\frac{t}{t}\right)^2 \sigma_1^2 + \left(\frac{t-1}{t}\right)^2 \sigma_1^2 + \left(\frac{t-2}{t}\right)^2 \sigma_1^2 + \cdots + \left(\frac{3}{t}\right)^2 \sigma_1^2 + \left(\frac{2}{t}\right)^2 \sigma_1^2 + \left(\frac{1}{t}\right)^2 \sigma_1^2\right) \tag{6}$$

Evidently, the additional liquidity risk factor depends only on the number of days needed to sell an illiquid equity position linearly. In the general case of $t-$ days, the variance of the liquidity risk factor is given by the following mathematical functional expression of t:

$$\sigma_{\text{adj}}^2 = \sigma_1^2 \left(\left(\frac{t}{t}\right)^2 + \left(\frac{t-1}{t}\right)^2 + \left(\frac{t-2}{t}\right)^2 + \cdots + \left(\frac{3}{t}\right)^2 + \left(\frac{2}{t}\right)^2 + \left(\frac{1}{t}\right)^2\right) \tag{7}$$

To calculate the sum of the squares, it is convenient to use a short-cut approach. From mathematical finite series the following relationship can be obtained:

$$(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2 = \frac{t(t+1)(2t+1)}{6} \tag{8}$$

Hence, after substituting equation (8) into (7), the following can be achieved:

$$\begin{aligned} \sigma_{\text{adj}}^2 &= \sigma_1^2 \left[\frac{1}{t^2}\left\{(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2\right\}\right] \quad \text{or} \\ \sigma_{\text{adj}}^2 &= \sigma_1^2 \left(\frac{(2t+1)(t+1)}{6t}\right) \end{aligned} \tag{9}$$

Accordingly, from (9) the liquidity risk factor can be expressed in terms of volatility (or standard deviation) as:

$$\begin{aligned} \sigma_{\text{adj}} &= \sigma_1 \left\{ \sqrt{\frac{1}{t^2}\left[(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2\right]} \right\} \quad \text{or} \\ \sigma_{\text{adj}} &= \sigma_1 \left\{ \sqrt{\frac{(2t+1)(t+1)}{6t}} \right\} \end{aligned} \tag{10}$$

The final result of (10) is of course a function of time and not the square root of time as employed by some financial market's participants based on the *RiskMetrics™* methodologies. The above approach can also be used to calculate L-VaR for any time horizon. Likewise, in order to perform the calculation of L-VaR under illiquid market conditions, it is possible to use the liquidity factor of (10) and define the following:

$$\text{L-VaR}_{\text{adj}} = \text{VaR}\sqrt{\frac{(2t+1)(t+1)}{6t}} \tag{11}$$

where VaR = Value at Risk under liquid market conditions and; $\text{L-VaR}_{\text{adj}}$ = Value at Risk under illiquid market conditions. The latter equation indicates that $\text{L-VaR}_{\text{adj}} > \text{VaR}$, and for the special case when the number of days to liquidate the entire equity assets is one trading day, then $\text{L-VaR}_{\text{adj}} = \text{VaR}$. Consequently, the difference between $\text{L-VaR}_{\text{adj}}$-VaR should be equal to the residual market risk due to the illiquidity of any equity asset under illiquid markets conditions. As a matter of fact, the number of liquidation days (t) necessary to liquidate the entire equity assets fully is related to the choice of the liquidity threshold; however the size of this threshold is likely to change under severe market conditions. Indeed, the choice of the liquidation horizon can be estimated from the total trading position size and the daily trading volume that can be unwound into the market without significantly disrupting equity market prices; and in actual practices it is generally estimated as:

$$t = \frac{\text{Total Trading Position Size of Asset}_i}{\text{Daily Trading Volume of Asset}_i} \tag{12}$$

In practice the daily trading volume of any trading equity asset is estimated as the average volume over some period of time, generally a month of trading activities. In effect, the daily trading volume of assets can be regarded as the average daily volume or the volume that can be unwound in a severe crisis period. The trading volume in a crisis period can be roughly approximated as the average daily trading volume less a number of standard deviations. Albeit this alternative approach is quite simple, it is still relatively objective. Moreover, it is reasonably easy to gather the required data to perform the necessary liquidation scenarios.

In essence, the above liquidity scaling factor (or multiplier) is more realistic and less conservative than the conventional root-t multiplier and can aid financial entities in allocating reasonable and liquidity market-driven regulatory and economic capital requirements. Furthermore, the above mathematical formulas can be applied for the calculation of L-VaR for every equity trading position and for the entire portfolio of equities. In order to calculate the L-VaR for the full trading portfolio under illiquid market conditions ($\text{L-VaR}_{P_{\text{adj}}}$), the above mathematical formulation can be extended, with the aid of (3), into a matrix-algebra form to yield the following:

$$\text{L-VaR}_{P_{\text{adj}}} = \sqrt{[\text{L-VaR}_{\text{adj}}]^T[\rho][\text{L-VaR}_{\text{adj}}]} \tag{13}$$

The above mathematical structure (in the form of two vectors and a matrix, $[\text{L-VaR}_{\text{adj}}]^T$, $[\text{L-VaR}_{\text{adj}}]$ and $[\rho]$) can facilitate the programming process so that the trading risk manger can specify different liquidation days for the whole portfolio and/or for each individual trading security according to the necessary number of days to liquidate the entire assets fully. The latter can be achieved by specifying an overall benchmark liquidation horizon to liquidate the entire constituents of the portfolio fully. The number of days required to liquidate a position (of course, depending on the type of equity) can be obtained from the various publications of equity markets and can be compared with the assessments' of individual traders of each trading unit. As a result, it is possible to create simple statistics of the equity volume that can be liquidated and the necessary time horizon to unwind the whole volume.

4.3 Assessment of economic capital with different liquidation horizons and correlation factors

On another front, the annual economic capital necessary to support trading activities under illiquid normal and severe market settings is examined in this paper. When calculating economic capital, we want to use the same time horizon and confidence level for all asset risk exposures. The time horizon is usually one year ahead (assuming 260 active business trading days in the year) and the confidence level is often chosen as 99.97% (or 3.43 quantile) for an AA-rated financial institution. The assumption behind this approach is that the probability distribution of profits and losses ($P\&L$) for each day during the next year will be the same as that estimated for the first current trading day and that the distribution of $P\&L$ are independent. Moreover, financial entities are involved in several large businesses besides the business of taking equity risk. The variety of trading activities provides a diversification benefit that will significantly reduce the risk of firm-wide default from a movement in equity prices. To take the diversification effect into account in our estimation of the contribution of the equity trading unit to firm-wide $P\&L$, we multiply the stand alone value for the conditional liquidity-adjusted volatility by the square root of the correlation between business units ($\sqrt{\rho_{\text{BU}}}$). As such the 99.97% worst-case loss is then 3.43 times the conditional overall portfolio liquidity-adjusted volatility of the one-year profit/loss, or more formally from (3) and (13) we can define:

$$
\begin{aligned}
&\text{Economic Capital (EC)}\\
&\quad = \left(\frac{\alpha_{\text{EC}}}{\alpha}\right)\sqrt{H}\sqrt{\rho_{\text{BU}}}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\text{L-VaR}_i\,\text{L-VaR}_j\,\rho_{i,j}}\\
&\quad = \left(\frac{\alpha_{\text{EC}}}{\alpha}\right)\sqrt{H}\sqrt{\rho_{\text{BU}}}\sqrt{[\text{L-VaR}_{\text{adj}}]^T[\rho][\text{L-VaR}_{\text{adj}}]}
\end{aligned}
\tag{14}
$$

where α_{EC} is the economic capital quantile of 3.43, α is the daily VaR quantile as illustrated in (1), H is the number of active trading days in the year, ρ_{BU} is the correlation factor required to account for the diversification benefit provided by having the equity trading risk unit as one of a number of diversified financial businesses. Furthermore, $\text{L-VaR}_{P_{\text{adj}}}$ is defined in (13), so that:

$$
\text{Economic Capital (EC)} = \left(\frac{\alpha_{\text{EC}}}{\alpha}\right)\sqrt{H}\sqrt{\rho_{\text{BU}}}\,\text{L-VaR}_{P_{\text{adj}}} \tag{15}
$$

The elements of the vectors of (14), i.e. $\text{L-VaR}_{i_{\text{adj}}}$, for each trading asset can now be calculated with the aid of (2) and (11) in this manner:

$$\text{L-VaR}_{i_{\text{adj}}} = \left| \alpha \cdot \sigma_i \cdot \text{Mark-to-Market Value of Asset}_i \cdot Fx_i \sqrt{\frac{(2t_i + 1)(t_i + 1)}{6t_i}} \right| \tag{16}$$

Now, we can define the ultimate two vectors $[\text{L-VaR}_{\text{adj}}]^T$ and $[\text{L-VaR}_{\text{adj}}]$ as follows:

$$[\text{L-VaR}_{\text{adj}}]^T = [\text{L-VaR}_{1_{\text{adj}}} \quad \text{L-VaR}_{2_{\text{adj}}} \quad \cdots \quad \text{L-VaR}_{n_{\text{adj}}}] \tag{17}$$

$$[\text{L-VaR}_{\text{adj}}] = \begin{bmatrix} \text{L-VaR}_{1_{\text{adj}}} \\ \text{L-VaR}_{2_{\text{adj}}} \\ \cdots \\ \text{L-VaR}_{n_{\text{adj}}} \end{bmatrix} \tag{18}$$

5 Examination of price risk exposure of dynamic economic capital portfolios

In this work, database of daily return of six GCC stock markets' main indicators (indices) are gathered, filtered and adequately adapted for the creation of relevant inputs for the calculation of all risk factors. Historical database (of more than six years) of daily closing index levels, for the period 17/10/2004–22/05/2009, are assembled for the purpose of carrying out this study and further for the construction of market and liquidity risk management parameters. The historical database of daily indices levels is drawn from Reuters 3000 Xtra Hosted Terminal Platform and Thomson's Datastream datasets. The total numbers of indices that are considered in this work are nine indices; seven local indices for the six GCC stock markets (including two indices for the UAE markets) and two benchmark indices, detailed as follows:

DFM General Index (United Arab Emirates, Dubai Financial Market General Index)
ADSM Index (United Arab Emirates, Abu Dhabi Stock Market Index)
BA All Share Index (Bahrain, All Share Stock Market Index)
KSE General Index (Kuwait, Stock Exchange General Index)
MSM30 Index (Oman, Muscat Stock Market Index)
DSM20 Index (Qatar, Doha Stock Market General Index)
SE All Share Index (Saudi Arabia, All Share Stock Market Index)
Shuaa GCC Index (Shuaa Capital, GCC Stock Markets Benchmark Index)
Shuaa Arab Index (Shuaa Capital, Arab Stock Markets Benchmark Index)

Moreover, in this work index returns are defined as $R_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1})$ where $R_{i,t}$ is the daily return of index i, ln is the natural logarithm; $P_{i,t}$ is the current level of index i, and $P_{i,t-1}$ is the previous day index level. Furthermore, for this particular study we have chosen a confidence interval of 95% (or 97.5% with "one-tailed" loss side) and several liquidation time horizons to compute L-VaR. Furthermore, an iterative optimization-algorithm software package is programmed for the purpose of creating trading portfolios of the above indices and consequently for carrying out L-VaR and economic capital scenario-analysis under extreme illiquid market conditions and by implementing different correlation factors. The analysis of data and discussions of relevant findings and results of this study are organized and explained as follows.

Table 1 Risk analysis dataset: daily and annual volatility and sensitivity factor

Stock market indices	Daily volatility (normal market)[a]	Daily volatility (severe market)[a]	Annual volatility (normal market)[a]	Annual volatility (severe market)[a]	Sensitivity factor
DFM general index	1.81%	12.2%	29.2%	196.0%	0.58
ADSM index	1.32%	7.1%	21.4%	114.1%	0.40
BA all share index	0.58%	3.8%	9.4%	60.8%	0.06
KSE general index	0.71%	3.7%	11.5%	60.2%	0.14
MSM30 index	0.79%	8.7%	12.8%	140.3%	0.10
DSM20 index	1.48%	8.1%	23.9%	130.2%	0.31
SE all share index	1.86%	11.0%	30.0%	177.9%	0.98
Shuaa GCC index	1.30%	8.1%	20.9%	130.6%	1.05
Shuaa Arab index	1.15%	7.6%	18.5%	122.1%	1.00

[a]Denotes estimation of conditional volatility using GARCH-M model

5.1 Statistical analysis of conditional volatility, correlation patterns and testing for asymmetric distribution of returns

To examine the relationship between stock market indices' expected returns and volatility, we implement a conditional volatility approach to determine the risk parameters that are needed for the L-VaR's engine and thereafter for the estimation of daily asset market liquidity risk exposure and economic capital requirements. Indeed, the time-varying pattern of stock market volatility has been widely recognized and modeled as a conditional variance within the GARCH framework, as originally developed by Engle (1982, 1995). Engle (1982) introduced a likelihood ratio test to ARCH effects and a maximum likelihood method to estimate the parameters in the ARCH model. This approach generalized by Bollerslev (1986) and Engle and Kroner (1995). The following generalized autoregressive conditional heteroskedasticity in mean, GARCH-M (1,1) model, is used for the estimation of expected return and conditional volatility for each of the time series variables:[5]

$$R_{it} = a_i + b_i \sigma_{it} + \varepsilon_{it}, \tag{19}$$

$$\sigma_{it}^2 = c_i + \beta_{i1} \sigma_{it-1}^2 + \beta_{i2} \varepsilon_{it-1}^2, \tag{20}$$

where R_{it} is the continuous compounding return of time series i, σ_{it} is the conditional standard deviation as a measure of volatility, and ε_{it} is the error term return for time series i. The denotations a_i, b_i, c_i, β_{i1}, and β_{i2} represent parameters to be estimated. The parameters representing variance are assumed to undertake a positive value.

Table 1 illustrates the daily conditional volatility of each of the sample indices under normal market and severe[6] (crisis) market conditions and by implementing the GARCH-M

[5]In this class of models (that is, GARCH-M), the conditional variance enters into the conditional mean equation as well as the usual error variance part. As such, when the return of a security is dependent on its volatility, one can use the GARCH-M model formulation. Indeed, the GARCH-M model implies that firstly there exists serial correlation in the return series and secondly these serial correlations are introduced by the volatility process due to a risk-premium.

[6]In this paper, severe or crisis market conditions refer to unexpected extreme adverse market situations at which losses could be several-fold larger than losses under normal market situation. Stress-testing technique is usually used to estimate the impact of unusual and severe events.

Table 2 Risk analysis dataset: descriptive statistics of daily returns, Skewness, Kurtosis and Jarque-Bera test of normality

Stock market indices	Maximum	Minimum	Arithmetic mean	Expected return[a]	Skewness	Kurtosis	Jarque–Bera (JB) test
DFM general index	9.9%	−12.2%	0.12%	0.14%	0.01	7.86	955[b]
ADSM index	6.6%	−7.1%	0.07%	0.07%	0.12	7.26	734[b]
BA all share index	3.6%	−3.8%	0.05%	0.04%	0.43	10.24	2142[b]
KSE general index	5.0%	−3.7%	0.09%	0.08%	−0.18	8.38	1173[b]
MSM30 index	5.2%	−8.7%	0.12%	0.10%	−0.57	18.40	9617[b]
DSM20 index	6.2%	−8.1%	0.06%	0.07%	−0.11	5.59	273[b]
SE all share index	9.4%	−11.0%	0.03%	0.01%	−0.97	8.47	1361[b]
Shuaa GCC index	11.1%	−8.1%	0.06%	0.08%	−0.66	14.00	4949[b]
Shuaa Arab index	9.4%	−7.6%	0.07%	0.10%	−0.61	13.79	4758[b]

[a]Denotes estimation of expected return using GARCH-M model

[b]Denotes statistical significance at the 0.01 levels

model. Severe market conditional volatilities are calculated by implementing an empirical distribution of past returns for all stock market indices' time series and, hence, the maximum negative returns (losses), which are witnessed in the expected return time series, are selected for this purpose. This downside risk approach can aid in overcoming some of the limitations of the normality assumption and can provide a better analysis of L-VaR and coherent assessment of economic capital allocation, especially under severe and illiquid market settings.

From Table 1 it is apparent that the index with the highest volatility is the SE All Share Index (under normal market condition) whereas the DFM General Index demonstrates the highest volatility under severe market conditions. Annualized volatilities are depicted in Table 1, and this is performed by adjusting (multiplication) the daily conditional volatilities with the square root of 260—assuming there are 260 trading days in the calendar year. An interesting outcome of the study of sensitivity factors (beta factors for systematic risk) is the manner in which the results are varied across the sample indices as indicated in Table 1. SE All Share Index appears to have the highest sensitivity factor (0.98) vis-à-vis the Shuaa Arab Index (that is the highest systematic risk) and the BA All Share Index seems to have the lowest beta factor (0.06). Moreover, and in accordance with general belief, Shuaa GCC Index (with a sensitivity factor of 1.05) is the best candidate of the entire sample indices that appears to move very closely with respect to the benchmark Shuaa Arab Index (with a beta factor of 1.0).

In another study, descriptive statistical analysis and tests of non-normality (asymmetry) are performed on the sample indices. To take into account the distributional anomalies of asset returns, tests of non-normality are performed on the sample equity indices using the Jarque–Bera (JB) test. In the first study, the measurements of skewness and kurtosis are achieved on the sample equity indices. The results are depicted in Table 2. It is seen that all indices show asymmetric behavior (between both positive and negative values). Moreover, kurtosis studies show similar patterns of abnormality (i.e. peaked distributions). At the upper extreme, MSM30 Index shows a big negative skewness (−0.57) which is combined with a very high Kurtosis—peakedness of (18.40). Some indices, such as in the case of DSM20 Index, show less abnormality pattern (Skewness of −0.11 and kurtosis of 5.59). As evidenced in Table 2, the above results of general departure from normality are also confirmed with the

Table 3 Risk analysis data: correlation matrix of stock market indices

	DFM general index	ADSM index	BA all share index	KSE general index	MSM30 index	DSM20 index	SE all share index	Shuaa GCC index	Shuaa Arab index
DFM general index	100%								
ADSM index	56%	100%							
BA all share index	12%	8%	100%						
KSE general index	17%	16%	12%	100%					
MSM30 index	12%	17%	11%	11%	100%				
DSM20 index	18%	23%	12%	12%	20%	100%			
SE all share index	20%	20%	7%	16%	11%	10%	100%		
Shuaa GCC index	37%	35%	13%	19%	13%	26%	62%	100%	
Shuaa Arab index	39%	36%	12%	24%	15%	26%	60%	93%	100%

JB test. The JB statistic is calculated in this manner:

$$\mathrm{JB} = n/6\big[S^2 + (K-3)^2/4\big] \approx \chi^2(2) \tag{21}$$

where S is the skewness, K is the kurtosis, and n is the number of observations. The JB statistic reassembles an approximately a Chi-squared distribution [$\chi^2(2)$] with 2 degrees of freedom. The 95% and 99% percentage points of the Chi-squared distribution with 2 degrees of freedom are 5.99 and 9.21 respectively, thus, the lower the JB statistic, the more likely a distribution is normal. Nonetheless, the JB test shows an obvious general deviation from normality and, thus, rejects the hypothesis that the GCC stock markets' time series returns are normally distributed. The interesting outcome of this study suggests the necessity of combining L-VaR and economic capital appraisals—which assumes normal distributions of returns—with other methods such as stress testing and scenario analysis to get a detailed picture of other remaining risks (fat-tails in the probability distribution) that cannot be captured with the simple assumption of normality.

With the aim of assessing L-VaR and economic capital, four matrices of correlations are used in this study, namely $\rho = 1, -1, 0$, and empirical correlations. The objectives here are to establish the necessary quantitative infrastructures for advanced risk management analysis that will follow shortly. The assembled correlation matrix is depicted in Table 3, for the empirical correlation case, of all nine indices. The latter correlation matrix is an essential aspect along with conditional volatility matrices for the calculations of L-VaR, stress-testing and economic capital's efficient and coherent portfolios. The results of Table 3 are integrated into the L-VaR's engine to estimate L-VaR parameters and economic capital requirements under the notion of different unwinding horizon periods and for varied combinations of long/short trading positions.

5.2 Economic capital assessment with L-VaR modeling algorithm and simulation results

In order to illustrate the linkage between the theoretical foundations of L-VaR and its practical application and value as a tool for equity trading risk management, the following hypothetical simulation trading portfolios with full case studies are presented for the assessment of economic capital. These case studies also help in understanding the methods used in determining the performance of alternative economic capital estimation procedures in the context of equity trading risk management.

Table 4 Economic capital analysis under different market conditions, full case study

Asset Allocation and Economic Capital Analysis

Stock Market Indices	Market Value in AED	Asset Allocation Percentage	Liquidity Holding Horizon	Daily Volatility (Normal)	Daily Volatility (severe)	Sensitivity Factor
DFM General Index	$ 6,000,000	60.0%	1	1.81%	12.16%	0.58
ADSM Index	$ 4,000,000	40.0%	1	1.32%	7.08%	0.40
BA All Share Index	$ 1,000,000	10.0%	1	0.58%	3.77%	0.06
KSE General Index	$ (2,000,000)	-20.0%	1	0.71%	3.74%	0.14
MSM30 Index	$ 2,000,000	20.0%	1	0.79%	8.70%	0.10
DSM20 Index	$ 3,000,000	30.0%	1	1.48%	8.07%	0.31
SE All Share Index	$ (4,000,000)	-40.0%	1	1.86%	11.03%	0.98
Shuaa GCC Index	$ -	0.0%	1	1.30%	8.10%	1.05
Shuaa Arab Index	$ -	0.0%	1	1.15%	7.57%	1.00
Total Portfolio Value in AED	$ 10,000,000	100%				

Expected Return and Risk-Adjusted Return

Trading Portfolio Annual Expected Return	48.40%
Risk-Adjusted Expected Return (Normal)	53.47%
Risk-Adjusted Expected Return (Severe)	8.48%

Annual Economic Capital (EC) in Million AED [Normal Market Conditions]

ρ = *Empirical*	$\rho = 1$	$\rho = 0$	$\rho = -1$
9.05	7.71	8.32	8.89
5.6%	4.8%	5.2%	5.5%

Diversification Benefits in EC	
$ (1.35)	-14.88%

Annual Economic Capital (EC) in Million AED [Severe (Crisis) Market Conditions]

ρ = *Empirical*	$\rho = 1$	$\rho = 0$	$\rho = -1$
57.08	52.56	52.55	52.55
35.4%	32.6%	32.6%	32.6%

Diversification Benefits in EC	
$ (4.53)	-7.93%

Overall Sensitivity Factor: Portfolio of Stock Indices
0.209

Table 5 Economic capital analysis under different market conditions, full case study

Asset Allocation and Economic Capital Analysis

Stock Market Indices	Market Value in AED	Asset Allocation Percentage	Liquidity Holding Horizon	Daily Volatility (Normal)	Daily Volatility (severe)	Sensitivity Factor
DFM General Index	$ 6,000,000	60.0%	10	1.81%	12.16%	0.58
ADSM Index	$ 4,000,000	40.0%	10	1.32%	7.08%	0.40
BA All Share Index	$ 1,000,000	10.0%	10	0.58%	3.77%	0.06
KSE General Index	$ (2,000,000)	-20.0%	10	0.71%	3.74%	0.14
MSM30 Index	$ 2,000,000	20.0%	10	0.79%	8.70%	0.10
DSM20 Index	$ 3,000,000	30.0%	10	1.48%	8.07%	0.31
SE All Share Index	$ (4,000,000)	-40.0%	10	1.86%	11.03%	0.98
Shuaa GCC Index	$ -	0.0%	10	1.30%	8.10%	1.05
Shuaa Arab Index	$ -	0.0%	10	1.15%	7.57%	1.00
Total Portfolio Value in AED	$ 10,000,000	100%				

Expected Return and Risk-Adjusted Return

Trading Portfolio Annual Expected Return	48.40%
Risk-Adjusted Expected Return (Normal)	27.25%
Risk-Adjusted Expected Return (Severe)	4.32%

Annual Economic Capital (EC) in Million AED [Normal Market Conditions]

ρ = *Empirical*	$\rho = 1$	$\rho = 0$	$\rho = -1$
17.76	15.12	16.33	17.45
11.0%	9.4%	10.1%	10.8%

Diversification Benefits in EC	
$ (2.64)	-14.88%

Annual Economic Capital (EC) in Million AED [Severe (Crisis) Market Conditions]

ρ = *Empirical*	$\rho = 1$	$\rho = 0$	$\rho = -1$
112.01	103.13	103.12	103.11
69.5%	64.0%	63.9%	63.9%

Diversification Benefits in EC	
$ (8.88)	-7.93%

Overall Sensitivity Factor: Portfolio of Stock Indices
0.209

Using the definition of economic capital in Sect. 4 and under the assumption that a given equity portfolio has both long and short-sales trading positions, Tables 4 and 5 illustrate practical simulation risk reports for the coverage of equity trading risk management activities of a hypothetical equity portfolio consisting of several indices of the GCC stock markets. Asset allocation and economic capital analysis are performed under the assumption that local indices represent exact replicas of diversified portfolios of local stocks for each GCC stock market respectively. Furthermore, all risk analyses are performed at the one-tailed 97.5% level of confidence over different liquidation periods.

In these two full case analysis studies the total portfolio value is AED10 millions (UAE Dirham) with different asset allocation percentage. The analysis is carried out with one-day and ten-day liquidity horizon—that is, one or ten days to unwind all equity trading positions fully. Furthermore, Tables 4 and 5 illustrate the effects of stress testing (that is, economic capital under severe market conditions) and different correlation factors on annual economic capital calculations. The economic capital engine's report depicts also the overnight (daily) conditional volatilities using GARCH-M model, in addition to their respective sensitivity factors (or beta factors) vis-à-vis the benchmark index. Crisis market daily volatilities (or downside-risk) are calculated and illustrated in the report too. These daily severe downside-risk volatilities represent the maximum negative returns (losses), which are perceived in the historical time series, for all stock market indices. In essence, this approach can aid in overcoming some of the limitations of the normality assumption and can provide a better analysis of economic capital especially under severe and illiquid market settings. The effects of short-sales (albeit short-sales are currently not permitted in the GCC stock markets) are depicted in Tables 4 and 5. One of the interesting results of this study is the way in which economic capital figures have decreased. This behavior might be explained by the way in which the overall portfolio is funded—in other words, long positions have been funded with short-sales of other stocks (or indices) and consequently have led to reduction in the overall risk exposure. In fact, one of the principal advantages of calculating L-VaR and economic capital with matrix-algebra framework is the ability in which one can incorporate the effects of short-sales without tedious mathematical analysis.

The economic capital modeling results are calculated under normal and severe market conditions by taking into account different correlation factors (empirical, zero and negative/positive unity correlations between the various risk factors). Under correlation $+1$, the assumption is for 100% positive relationship between all risk factors (risk positions) all the time, whereas for the zero-correlation case, there is no relationship between all positions. While the -1 correlation case assumes 100% negative relationship, the empirical correlation case considers the actual empirical correlation factors between all positions and is calculated via a variance/covariance matrix. Therefore, with 97.5% confidence, the actual equity trading portfolio should expect to realize no greater than AED 9.05 millions reduction in the value of annual economic capital over a one-day time frame. In other words, the loss of AED 9.05 millions in annual economic capital is one that an equity portfolio should realize only 2.5% of the time. If the actual loss exceeds the economic capital estimate, then this would be considered a violation of the assessment. From a risk management perspective, the economic capital estimation of AED 9.05 millions is a valuable piece of information. Since every equity trading business has different characteristics, limited economic capital and tolerances toward risk, the trading risk manager must examine the economic capital estimate relative to the overall position of the entire business. Simply put, can the firm tolerate or even survive such a rare event—a loss of AED 9.05 millions in its annual economic capital (or a 5.6% of total portfolio value)? This question is not only important to the equity trading unit, but also to financial institutions (or other funding units such as the treasury unit within the same hierarchy and organizational structure of the trading unit) who lend money to these trading units. The inability of a trading unit to absorb large losses may jeopardize their ability to make principal and interest payments. Therefore, various risk management strategies could be examined in the context of how they might affect economic capital assessment. Presumably, risk management strategies, such as the use of futures and options contracts in hedging possible fluctuation in equity prices, should reduce the assessment of economic capital to set aside to absorb abnormal shocks. In other words, those extreme losses in equity trading, that would normally occur only 2.5% of the time, should be smaller with the incorporation of some type of risk management strategy.

Furthermore, the analysis of economic capital under illiquid market conditions is performed with four different correlation factors: empirical, zero and positive/negative unity respectively and for long and short-sales trading positions. Indeed, it is essential to include different correlation factors in any economic capital and stress-testing exercises. This is because existing trends in correlation factors may break down (or change signs) under adverse and severe market movements, caused by unforeseen financial or political crises. In theory, the case with correlation $+1$ should provide the maximum economic capital numbers (AED 7.71 millions and AED 15.12 millions) as a result of the fact that under these circumstances the total economic capital of actual trading portfolio is the weighted average of the individual economic capital of each equity trading position. Furthermore, the degree of risk-diversification (namely, the effects of diversification benefits in economic capital) of this hypothetical equity trading portfolio can also be deduced simply as the difference in the values of the two greatest economic capitals—that is the economic capital of unity correlation case versus the economic capital of empirical correlation case (AED -1.35 millions or -14.88% for the normal market condition case). However, it is appealing to note here that for this particular case, economic capital under correlation $+1$ is less than the economic capital under empirical correlation case due to the impact of short-sales of some equity trading positions. In addition, the overall sensitivity factor (beta factor) of this long/short equity portfolio is indicated in this report as 0.209, or in other words, the total equity portfolio value, with actual asset allocation ratios, has little positive sensitivity with the benchmark index (Shuaa Arab Index). Moreover, expected returns and risk-adjusted expected returns (under normal and severe market conditions) are also included in the economic capital risk analysis report.

Finally, since the variations in economic capital are mainly related to the ways in which the assets are allocated in addition to the liquidation horizon, it is instructive to examine the way in which economic capital figures are influenced by changes in such parameters. All else equal, and under the assumption of normal and severe market conditions, Table 5 illustrates the non-linear alterations to economic capital figures when the liquidation periods is increased in line and across the board to 10 days unwinding period for all indices within the equity trading portfolio.

5.3 Constrained optimization of economic capital and assessment of efficient & coherent portfolios

The portfolio mean-variance analysis approach, pioneered by Markowitz (1959), is one of the cornerstones of modern portfolio management and has served as the standard procedure for constructing portfolios. Albeit Markowitz's mean-variance portfolio optimization methodology is a landmark in the development of modern investment theory, there are no risk measures universally adopted in financial applications.

One of the basic problems of applied finance is the optimal selection of assets, with the aim of maximizing future returns and constraining risk by appropriate measures. To this end, Markowitz (1959) illustrated that, for a given levels of risk, one can identify certain groups of equity securities that maximize expected return. He considered these optimum portfolios as 'efficient' and referred to a continuum of such portfolios in dimensions of expected return and standard deviation as the efficient frontier. Accordingly, for asset allocation purposes, fund managers should choose portfolios located along the efficient frontier. Optimized portfolios do normally not perform as well in practice as one would expect from theory. For example, they are often outperformed by simple allocation strategies such as the equally weighted portfolio (Jobson and Korkie 1981) or the global minimum variance portfolio (Jorion 1991). Simply put, the "optimized" portfolio is not optimal at all. Portfolio weights are often not stable over time but change significantly each time the portfolio is re-optimized, leading to unnecessary turnover and increased transaction costs. Moreover, these portfolios typically present extreme holdings ("corner solutions") in a few securities while other securities have close to zero weight. It is well documented (Michaud 1989) that mean-variance optimizers, if left to their own devices, can sometimes lead to unintuitive portfolios with extreme positions in asset classes. Consequently, these "optimized" portfolios are not necessarily well diversified and exposed to unnecessary ex-post risk (Michaud 1989). The reason for these phenomena is not a sign that mean-variance optimization does not work but rather that the modern portfolio theory framework is very sensitive to small changes in the inputs. In a portfolio optimization context, securities with large expected returns and low standard deviations will be overweighted and conversely, securities with low expected returns and high standard deviations will be underweighted. Therefore, large estimation errors in expected returns and/or variances/covariances will introduce errors in the optimized portfolio weights (Fabozzi et al. 2006).

Consequently, for more than four decades a wide body of knowledge has been accumulated about the performance, strengths, and weaknesses of this approach when applied to equity portfolios. However, much less is known about portfolio optimization techniques in emerging equity markets and particularly under illiquid and adverse market conditions. As such, in this paper we look at the optimization problem from a different realistic operational angle. In view of that, the enigma is formulated by finding the set of portfolios that minimize economic capital, with expected return, trading volume and liquidation horizons are constrained according to the requirements of the fund manager. As such, the focus in this work is on the forecast of risk measure, rather than on expected returns for two reasons: first, several studies have analyzed the forecasts of expected returns in the context of mean-variance optimization (see for instance Best and Grauer 1991). The common opinion is that expected returns are not easy to forecast, and that the optimization process is very sensitive to these variations. Second, there exists a general notion that L-VaR and economic capital, in a wide sense, are simpler to assess than expected returns from historical data.

In this work we develop a model for optimizing portfolio risk-return with economic capital constraints using realistic operational and financial scenarios and conduct a case study on optimizing equity portfolios of the six GCC stock markets. The case study shows that the optimization algorithm, which is based on linear programming techniques, is very stable and efficient in handling different liquidity horizons and correlation factors. Moreover, the approach can tackle large number of equity securities and rational fund management scenarios. Indeed, the economic capital's risk management constraints (reduced to linear constraints) can be used in various applications to bound percentiles of loss distributions.

Essentially, our approach is a straightforward extension of the classic Markowitz mean-variance approach, where the original risk measure, variance, is replaced by L-VaR and economic capital algorithms. The task is attained here by minimizing economic capital, while requiring a minimum expected return subject to several financially meaningful operational constraints. Thus, by considering different expected returns, we can generate an efficient economic capital frontier. Alternatively, we can also maximize expected returns while not allowing for large risks. For the purpose of this study, the optimization problem is formulated as follows.

It is possible to compute from (14) the minimum optimal amount of economic capital necessary to serve as a cushion to support current trading operation by solving for the following non-linear quadratic programming formulation:

$$\text{Minimize: Economic Capital (EC)} = \left(\frac{\alpha_{\text{EC}}}{\alpha}\right)\sqrt{H}\sqrt{\rho_{\text{BU}}}\sqrt{[\text{L-VaR}_{\text{adj}}]^T[\rho][\text{L-VaR}_{\text{adj}}]} \quad (22)$$

Subject to the following operational and financial budget constraints as specified by the risk manager:

$$\sum_{i=1}^{n} R_i x_i = R_P; \quad l_i \le x_i \le u_i,\ i = 1, 2, \dots, n \quad (23)$$

$$\sum_{i=1}^{n} x_i = 1.0; \quad l_i \le x_i \le u_i,\ i = 1, 2, \dots, n \quad (24)$$

$$\sum_{i=1}^{n} V_i = V_P \quad i = 1, 2, \dots, n \quad (25)$$

$$[\text{LHF}] \ge 1.0; \quad \forall_i\ i = 1, 2, \dots, n \quad (26)$$

Here R_P and V_P denote the target portfolio mean expected return and total portfolio volume respectively, and x_i the weight or percentage asset allocation for each asset. The values l_i and $u_i,\ i = 1, 2, \dots, n$, denote the lower and upper constraints for the portfolio weights x_i. If we choose $l_i = 0,\ i = 1, 2, \dots, n$, then we have the situation where no short-sales are allowed. Moreover, [LHF] indicates an $(n \cdot 1)$ vector of the individual liquidity horizon of each asset for all $i = 1, 2, \dots, n$. Where LHF_i is defined, with the aid of (11), for each trading asset in this way:

$$\text{LHF}_i = \sqrt{\frac{(2t_i + 1)(t_i + 1)}{6t_i}} \ge 1.0; \quad \forall_i\ i = 1, 2, \dots, n \quad (27)$$

Now the risk manager can specify different liquidity horizon and correlation factors and calculate the necessary amount of annual economic capital to sustain the trading operation of the financial entity without subjecting the entity to insolvency matters. The rationality behind imposing the above constraints is to comply with current regulations which enforce capital requirements on investment companies, proportional to their L-VaR and economic capital, besides other operational limits (for instance, volume trading limits).

In this backdrop, the empirical optimization process is based on the definition of economic capital as the minimum possible loss over a specified time horizon within a given confidence level. The iterative-optimization modeling algorithm solves the problem by finding the market positions that minimize the loss, subject to the fact that all constraints are satisfied within their boundary values. Further, in all cases the liquidation horizons as indicated in Tables 6, 7, 8 and 9 are assumed constant throughout the optimization process.

Table 6 Fund manager economic capital coherent market portfolio [1] (case analysis of long and short trading positions)

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM general index	3.0	$ (3,000,000)	−30%
ADSM index	3.0	$ 8,000,000	80%
BA all share index	4.0	$ 6,000,000	60%
KSE general index	3.0	$ (2,000,000)	−20%
MSM30 index	5.0	$ 6,000,000	60%
DSM20 index	6.0	$ (1,000,000)	−10%
SE all share index	3.0	$ (4,000,000)	−40%
ρ	EC (normal)	EC/volume	Annual expected return
Empirical	10,076,454	6.2%	9.72%
1.0	2,200,724	1.4%	Sensitivity Factor
0.0	12,340,256	7.7%	−0.215

Table 7 Fund manager economic capital coherent market portfolio [2] (case analysis of long and short trading positions)

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM general index	3.0	$ (2,000,000)	−20%
ADSM index	3.0	$ 2,000,000	20%
BA all share index	4.0	$ 6,000,000	60%
KLSE general index	3.0	$ 3,000,000	30%
MSM30 index	5.0	$ (2,000,000)	−20%
DSM20 index	6.0	$ (3,000,000)	−30%
SE all share index	3.0	$ 6,000,000	60%
ρ	EC (normal)	EC/volume	Annual expected return
Empirical	10,740,249	6.7%	16.20%
1.0	6,961,365	4.3%	Sensitivity factor
0.0	11,061,832	6.9%	0.469

For the sake of simplifying the optimization algorithm and thereafter its analysis, a volume trading position limit of 10 million AED is assumed as a constraint—that is the equity trading entity must keep a maximum overall market value of different equities of no more than 10 million AED (between long and short-sales positions). As such, Figs. 1 and 2 provide evidence of the empirical economic capital efficient frontiers (under 1-day and 10-days liquidation horizons respectively) defined using a 97.5% confidence level. As mentioned above, the optimal portfolio selection is performed by relaxing the short sale constraint, for the different equity assets. On the other hand, efficient portfolios cannot always be attained (e.g. short-sales without realistic lower boundaries on x_i) in the day-to-day real-world portfolio management operations and, hence, the fund manager should establish proactive coherent portfolios under realistic and restricted dynamic budget constraints, detailed as follows:

Table 8 Fund manager economic capital coherent market portfolio [3] (case analysis of long and short trading positions)

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM general index	3.0	$ 3,000,000	30%
ADSM index	3.0	$ 2,000,000	20%
BA all share index	4.0	$ 3,000,000	30%
KSE general index	3.0	$ (2,000,000)	−20%
MSM30 index	5.0	$ 1,000,000	10%
DSM20 index	6.0	$ (1,000,000)	−10%
SE all share index	3.0	$ 4,000,000	40%
ρ	EC (normal)	EC/volume	Annual expected return
Empirical	9,050,784	5.6%	32.76%
1.0	11,921,747	7.4%	Sensitivity factor
0.0	8,034,212	5.0%	0.513

Table 9 Fund manager economic capital coherent market portfolio [4] (case analysis of long and short trading positions)

Market index	Liquidation period (in days)	Market value in AED	Asset allocation per market
DFM general index	3.0	$ 10,000,000	100%
ADSM index	3.0	$ (6,000,000)	−60%
BA all share index	4.0	$ 6,000,000	60%
KSE general index	3.0	$ (2,000,000)	−20%
MSM30 index	5.0	$ (6,000,000)	−60%
DSM20 index	6.0	$ 3,000,000	30%
SE all share index	3.0	$ 5,000,000	50%
ρ	EC (normal)	EC/volume	Annual expected return
Empirical	16,375,682	10.2%	47.52%
1.0	17,736,375	11.0%	Sensitivity factor
0.0	18,746,835	11.6%	0.746

- Total trading volume (between long and short-sales equity trading positions) is 10 million AED.
- Asset allocation for long equity trading position varies from 10% to 100%.
- Asset allocation for short-sales equity trading position varies from −10% to −60%.
- All liquidity horizons for all equities are kept constant according to the specified values as indicated in Tables 6–9.

Now the weights are allowed to take negative or positive values, however, since arbitrarily high or low percentages have no financial sense, we determined to introduce lower and upper boundaries for the weights and in accordance with reasonable trading practices. Furthermore, for comparison purposes and since the endeavor in this work is to minimize economic capital subject to specific expected returns, we decide to plot economic capital

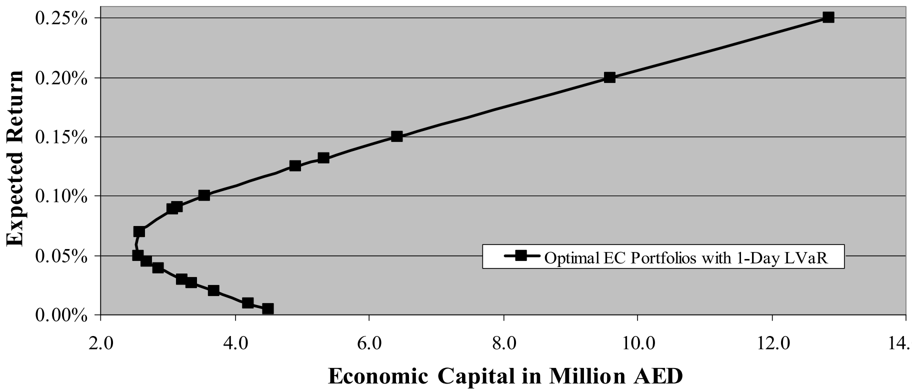

Fig. 1 Optimal economic capital portfolios with 1-day L-VaR horizon (case of long and short trading positions)

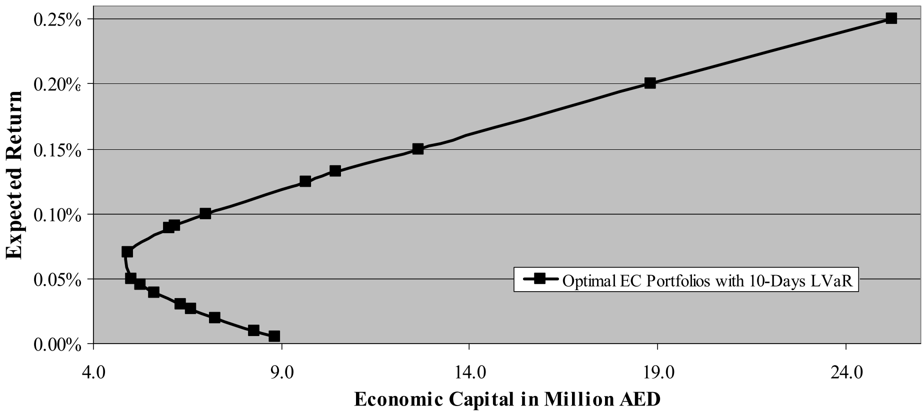

Fig. 2 Optimal economic capital portfolios with 10-days L-VaR horizon (case of long and short trading positions)

versus expected returns and not the reverse, as is commonly done in the various portfolio management literature. Accordingly, it is worthy of note that the four benchmark portfolios (coherent portfolios [1, 2, 3] and [4]) are noticeably located way off from the efficient frontiers as indicated in Fig. 3. This is because financially and operationally real-world investment considerations make it unlikely that a trading portfolio will behave exactly as theory predicts. Imperfections such as restriction on long and short-sales trading positions, total trading volume and liquidation horizons make it unlikely to create an efficient equity trading portfolio. Thus, the fund manager should apply active strategies in order to earn excess returns. These considerations are especially relevant for individual fund managers who may spread their trading positions across a few securities. Nevertheless, the elegance and compelling logic of the theory prompt attempts to apply the theory even though practitioners recognize the variance between the simplifying assumptions of the theory and the realities of the world.

In order to illustrate the composition of coherent portfolios [1, 2, 3] and [4], Tables 6–9 point out the asset allocation weights for all equity assets in all the liquidation periods under

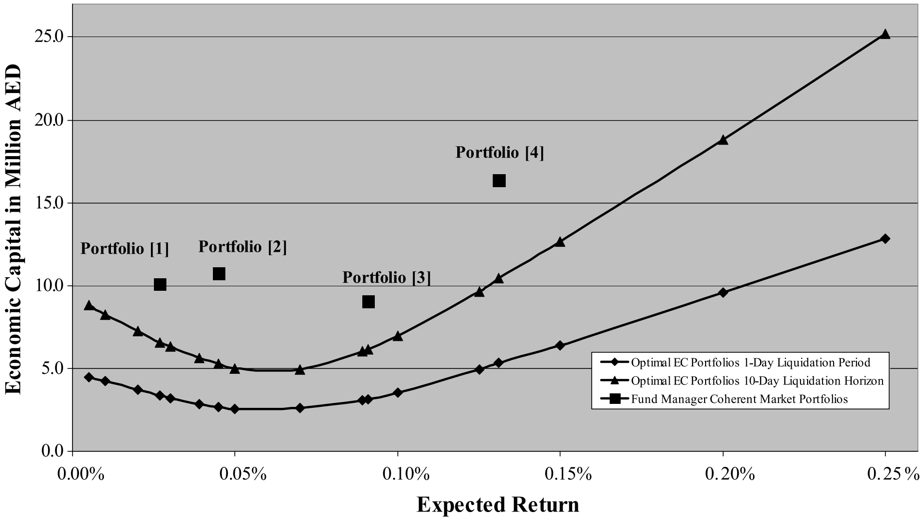

Fig. 3 Optimal and coherent economic capital portfolios with different L-VaR horizons (case of long and short trading positions)

consideration in addition to their expected return and sensitivity factor. Similarly, the four tables depict the minimum economic capital to sustain the operations of these particular portfolios and the Economic Capital/Volume ratio under normal market settings and with the assumption of three different correlation factors. In this way, fund managers should employ risk measures which allow them to take decisions which would produce a risk budget lower than a specific target. Thus, this analysis is substantially a generalization of the Markowitz analysis that permits one to determine the asymmetric aspect of risk. In any case, the benefit of portfolio optimization critically depends on how accurately the implemented economic capital risk measure is forecasted.

6 Summary and discussion

Given the fact that mean-variance optimizers have serious financial deficiencies, which could often lead to financially meaningless 'optimal' portfolios, in this paper we examine how to determine the optimal economic capital portfolio choice for an equity fund manager under the assumption of different liquidation horizons and by implementing different combinations of long/short equity trading strategies. In this paper, we develop an optimal and coherent economic capital portfolio selection model which implements a downside risk constraint rather than standard deviation alone. In our approach, downside risk is written in terms of portfolio L-VaR, so that additional risk resulting from any non-normality and illiquid assets may be used to estimate the portfolio L-VaR and economic capital. This enables a much more generalized framework to be developed, with the distributional assumption most appropriate to the type of financial assets to be employed, and which can be of crucial value for more accurate market risk assessment during market stress periods and particularly when liquidity dries up. We then provide a robust portfolio optimization technique using L-VaR as a risk measure subject to the imposition of financially and operationally meaningful constraints. In the final section of this paper, we describe the selection process for economic capital coherent portfolios, of various combinations of long/short trading positions, and provide the composition of each trading portfolio. In addition to the standard economic capital optimization algorithm that are adjusted for liquidity risk, related topics such as multi-horizon portfolio selection models and robust L-VaR and economic capital optimization approaches with various correlation factors are discussed. Admittedly, the liquidity modeling framework presented in this paper does not incorporate all the aspects of liquidity trading risk. However, it is effective as a tool for evaluating trading risk and economic capital appraisal when the impact of illiquidity of specified financial products is significant.

The empirical analyses are provided using nine equity indices for the GCC zone. Several case studies for the assessment of L-VaR and economic capital are performed to demonstrate how the new optimization techniques can be implemented in the real-world equity markets. The results indicate that the fund manager's L-VaR and economic capital depend on the minimum expected return and conditional volatility as measured by GARCH-M model, degree of correlation factors under adverse market setting, individual L-VaR positions, liquidity horizons of each asset, and the set of portfolio weights.[7]

In order to overcome the shortcomings of asymmetrical distribution of asset returns, in this work we implement the empirical distribution of past returns for all equity assets' time series. This approach has aid in providing a better analysis of L-VaR and a proper assessment of economic capital especially under severe and illiquid market settings. The results we report in this paper suggest that the introduction of L-VaR and economic capital into an asset allocation model allows the fund manager to focus attention on downside risk. In particular, the developed optimization algorithm allows the trading assets that must enter the optimal portfolio in order to meet a shortfall constraint (defined as the L-VaR or economic capital limit) to be determined. As such, the L-VaR and economic capital estimations have been obtained for various equity trading portfolios in the GCC stock markets through the implementation of a modified closed-form parametric L-VaR approach, where conditional volatilities and expected returns are estimated via GARCH-M model. The empirical testing results are then used to draw conclusions about the relative liquidity of the different equity indices and the importance of liquidity risk in L-VaR and economic capital assessment. The results indicate that our approach performs better than the standard mean-variance VaR model in terms of the optimal portfolio's selection process as well as in determining the fund manager's economic capital coherent portfolios. The empirical findings are persistent over the entire sample period (2004–2009) and robust across alternative investment horizons.

Furthermore, the obtained coherent benchmark portfolios are noticeably located way off from the efficient frontiers. This is due to the fact that financially and operationally real-world investment considerations make it unlikely that a trading portfolio will behave exactly as theory predicts. Imperfections such as restriction on long and short-sales trading positions, total trading volume and liquidation horizons make it unlikely to create an effi-

[7] An interesting issue for further research would be the implementation of advanced asymmetric GARCH models that allow for asymmetry in both the conditional mean and the variance equations. As such, it is suggested that future research on the topic could focus on the modified models developed by Glosten et al. (1993) and Gonzalez-Rivera (1998), the sign and volatility-switching ARCH (SVSARCH) model by Fornari and Mele (1997), the Markov switching volatility ARCH (MSVARCH) model by Hamilton and Susmel (1994) and the asymmetric non-linear smooth-transition generalized autoregressive conditional heteroskedasticity (ANST-GARCH) model (Anderson et al. 1999; Nam et al. 2001). Furthermore, there are new members to the extended GARCH models family which can be focused on as well in future research, such as, the dynamic conditional correlation (DCC) GARCH model of Engle (2002) and the asymmetric generalized dynamic conditional correlation (AG-DCC) model (Cappiello et al. 2006), which permits conditional asymmetries in correlation dynamics and accounts for heteroskedasticity directly by estimating correlation coefficients using standardized residuals.

cient equity trading portfolio. As such, the fund manager should apply active strategies in order to earn excess returns.

In a nutshell, the empirical results are interesting in terms of theory as well as practical applications and provide an incentive for further research in the area of L-VaR, economic capital and equity price risk management. Moreover, the different case analysis studies and discussions are widely applicable to any equity end-user, providing potential applications to practitioners and research ideas to academics. Indeed, the theoretical foundations and empirical results can aid in developing robust portfolio management algorithms that financial entities could consider in light of the aftermath of the latest financial crisis. Moreover, our optimization-algorithms are useful in developing enterprise-wide portfolio-management models that financial entities could consider in improving economic capital allocation across various equity asset-classes and particularly under illiquid and severe market circumstances.

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