Statistical Inference Course Project, Part 1: Simulation Exercise

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Project for the "Statistical Inference" course Part - 1

Comparing the simulated mean and variance with the theoretical values

The purpose of this data analysis is to investigate the exponential distribution and compare it to the Central Limit Theorem. For this analysis, the lambda will be set to 0.2 for all of the simulations. This investigation will compare the distribution of averages of 40 exponentials over 1000 simulations with $\lambda = 0.2$, using a fixed seed, and compare the distribution of the simulated mean and variance with the theoretical value of $1/\lambda$:

```r
library(pander)
```

```
## Warning: package 'pander' was built under R version 3.1.3
```

```r
nsim <- 1000
nvals <- 40
lambda <- 0.2
set.seed(567)
simdata <- t(replicate(nsim, rexp(nvals, lambda)))
df <- data.frame(Mean=c(mean(rowMeans(simdata)), 1/lambda),
                 Variance=c(mean(apply(simdata, 1, var)), 1/lambda^2))
rownames(df) <- c("Simulated", "Theoretical")
pander(df, round=2)
```

	Mean	Variance
Simulated	4.99	24.78
Theoretical	5	25

The simulated and theoretical values are very close, as expected by the CLT.

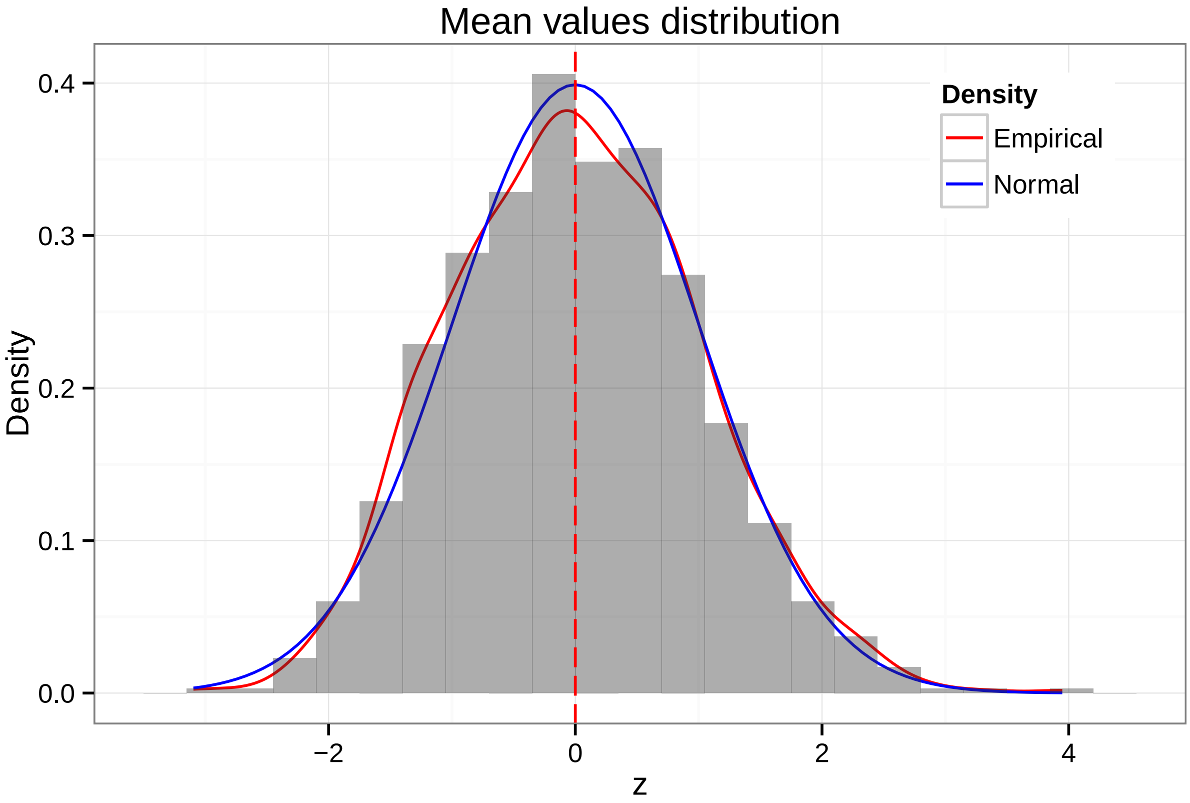
Assessing if the simulated values are approximately normal

Also, according to the CLT, the distribution of the simulated means should be approximately normal. To illustrate this we will normalize the vectors and compare it to a $N(0, 1)$ distribution.

```r
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.1.2
```

```r
meanvals <- rowMeans(simdata)
zmean <- (meanvals - mean(meanvals)) / sd(meanvals)
qplot(zmean, geom = "blank") +
    geom_line(aes(y = ..density.., colour = 'Empirical'), stat = 'density') +
    stat_function(fun = dnorm, aes(colour = 'Normal')) +
    geom_histogram(aes(y = ..density..), alpha = 0.4, binwidth=.35) +
    geom_vline(xintercept=0, colour="red", linetype="longdash") +
    scale_colour_manual(name = 'Density', values = c('red', 'blue')) +
    ylab("Density") + xlab("z") + ggtitle("Mean values distribution") +
    theme_bw() + theme(legend.position = c(0.85, 0.85))
```



Evaluating the coverage of the confidence interval

Theoretically, a 95% confidence interval should contain, if we simulate a big number of them, the mean value for the exponential distribution ($1/\lambda$) 95% of the time.

```
set.seed(567)
lambda <- 0.2
# checks for each simulation if the mean is in the confidence interval
inconfint <- function(lambda) {
                ehats <- rexp(1000, lambda)
                se <- sd(ehats)/sqrt(1000)
                ll <- mean(ehats) - 1.96 * se
                ul <- mean(ehats) + 1.96 * se
                (ll < 1/lambda & ul > 1/lambda)
        }
# estimate the coverage in each round of simulations
coverage <- function(lambda) {
    covvals <- replicate(100, inconfint(lambda))
    mean(covvals)
}
```

```r
# perform the simulation
simres <- replicate(100, coverage(lambda))
mean(simres)
```

```
## [1] 0.9484
```

As expected, the confidence interval contains the theoretical value 94.84% of the time (close to the expected 95%).

Conclusion

In this analysis we have shown that the sampling distribution of the mean of an exponential distribution with $n = 40$ observations and $\lambda = 0.2$ is approximately $N(\frac{1}{0.2}, \frac{\frac{1}{0.2}}{\sqrt{40}})$ distributed.