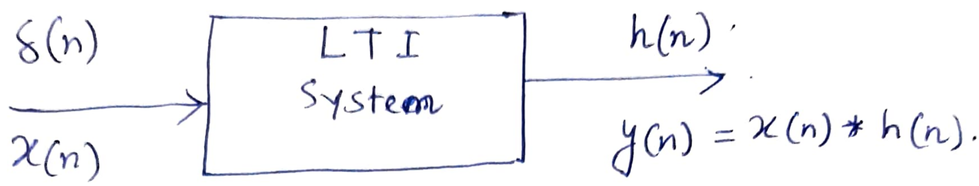
DSP

Recap ⇝ Discrete-time LTI System:

⇝ It's Response for any arbitary input Seqnence.
(Derivation).



$$\boxed{y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}$$

→ Convolution of two sequences: $x(n)$ & $h(n)$.

→ Convolution sum operation.

- **Fundamental result**:- The output of any discrete-time LTI Systeem is the Convolution of the input $x(n)$ with the impulse response $h(n)$ of the system.

- **Properties of the Convolution Sum**.

1. **Commutative** :- $\boxed{x(n) * h(n) = h(n) * x(n)}$

$$\sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

2. Associative :- $\{x(n) * h_1(n)\} * h_2(n) = x(n) * \{h_1(n) * h_2(n)\}$.

→ Cascade system.

3. Distributive :-

$$x(n) * \{h_1(n) + h_2(n)\} = x(n) * h_1(n) + x(n) * h_2(n).$$

→ parallel System.

These algebric properties ~~havein~~ have important implications on system interconnections.

Distributive → Impulse response of two parallel LTI systems. (equivalent system).

Associative & Commutative → Impulse of an equivalent system when we connect two LTI systems in Cascade.

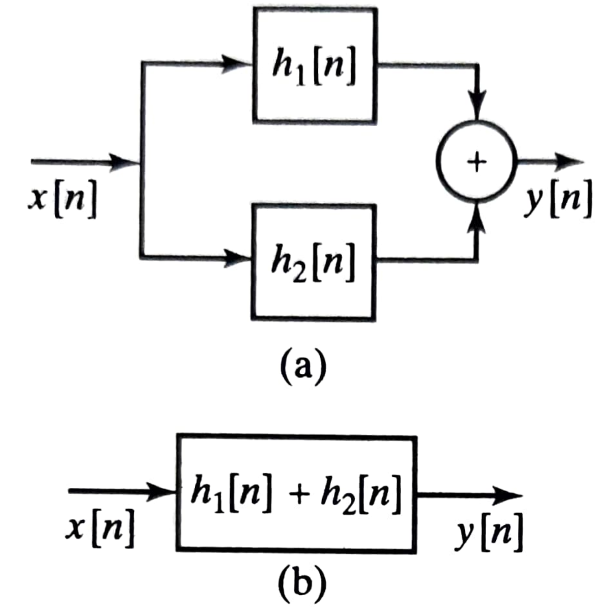


Figure 11 (a) Parallel combination of LTI systems. (b) An equivalent system.

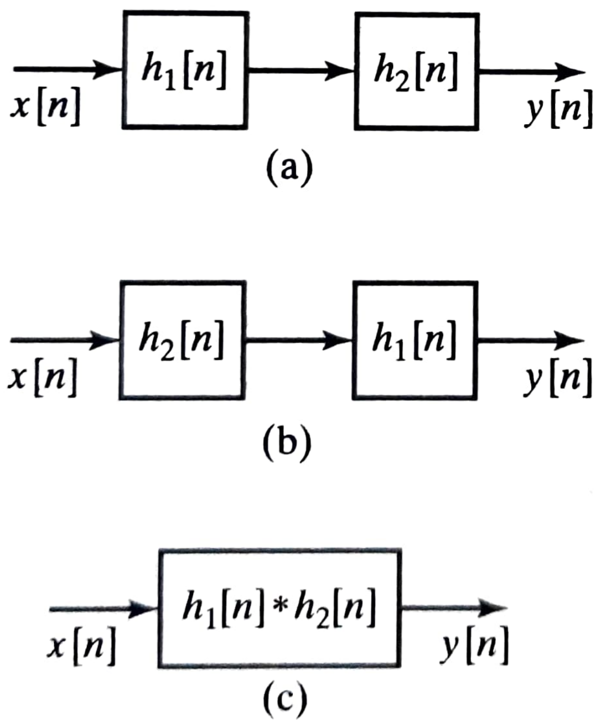


Figure 12 (a) Cascade combination of two LTI systems. (b) Equivalent cascade. (c) Single equivalent system.

- How to do the Linear Convolution? What are the methods?

  ⇒ Graphical method.

  ⇒ Matrix method.

<u>Graphical method.</u> → Follow the algebric equation of Convolution.

→ It plots $x(k)$ & $h(n-k)$. for a fixed n

→ Multiply two sequence samplewise & Sum them up.

<u>Matrix method.</u> → A simple & effective method.

→ Sort of a short-cut method.

• What would be the starting point of o/p sequence $y(n)$?

	Starting point	Length (sequence).
$x(n) \rightarrow$	n_1	L_1
$h(n) \rightarrow$	n_2.	L_2.
$y(n) \rightarrow$	(n_1+n_2).	(L_1+L_2-1).

Example :- $x(n) = \{1, 1, 1, 1\}$ $n_1 = 0$, $L_1 = 4$
$\uparrow$

$h(n) = \{6, 5, 4, 3, 2, 1\}$ $n_2 = -1$
$\uparrow$ $L_2 = 6$.

Starting point of o/p sequence $y(n)$

$y(n) \rightarrow (n_1+n_2) = -1$.

Length of $y(n) \rightarrow (L_1+L_2-1) = 9$.

Graphical method.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k).$$

Step-1. :- Plot $x(k)$.

Step-2 :- Folding $\rightarrow$ $h(k)$ is time-reversed. i.e. fold the $h(k)$ about the origin & obtain $h(-k)$

Step-3 :- Time-shifting $\rightarrow$ Select a value of 'n'. & then shift the $h(-k)$ by 'n' unit to right if n is positive & left if n is negative to obtain $h(n-k)$.
$= h[-(k-n)]$

Step-4 :- Multiplication → We need to multiply two sequences $x(k)$ and $h(n-k)$ for all values of k with ~~fixed~~ n fixed at some value.

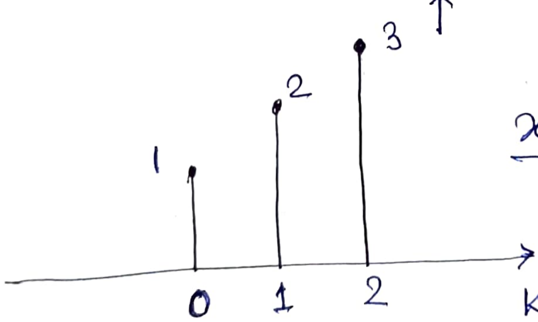
Step-5 :- Summation → The product $x(k)\,h(n-k)$ is summed over all k to produce a single output sample $y(n)$.

Step-6 :- ~~Step~~ Repeat step-3 to step-5 as n varies over $-\infty$ to ∞ to produce the o/p sequence.

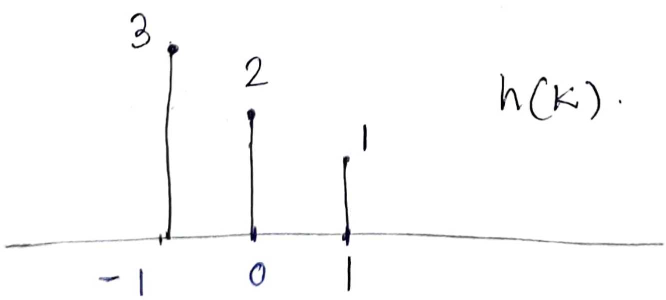
Example :-

$$x(n) = \{\underset{\uparrow}{1}, 2, 3\}.$$

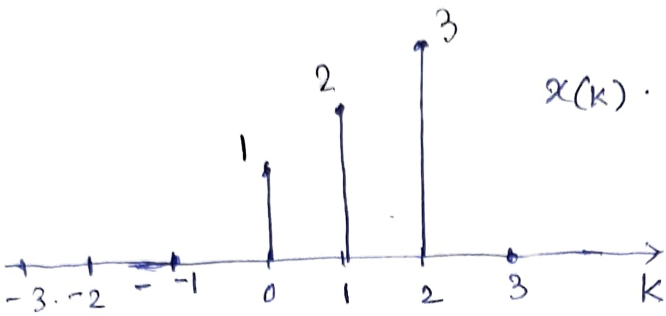
$$h(n) = \{3, \underset{\uparrow}{2}, 1\}.$$

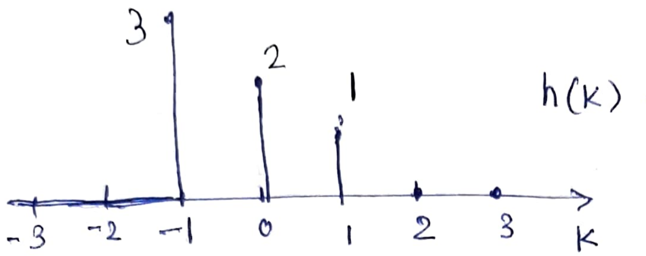


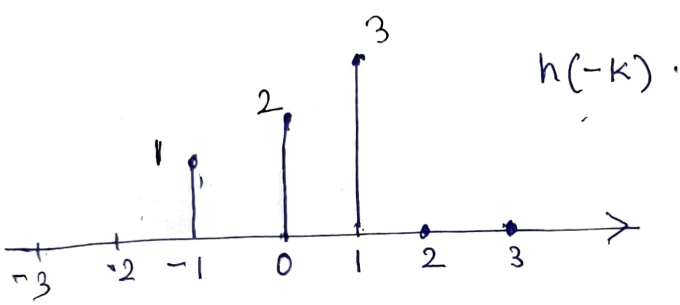
$x(k)$.



$h(k)$.





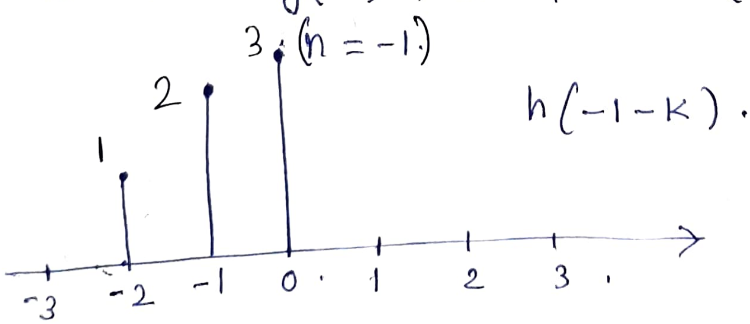


To obtain. $y[0] = 0 + 2 + 6 = 8$.

↳ Multiply & sum → $x(K)$ & $h(-K)$

$$y(0) = \sum x(K)\, h(-K).$$

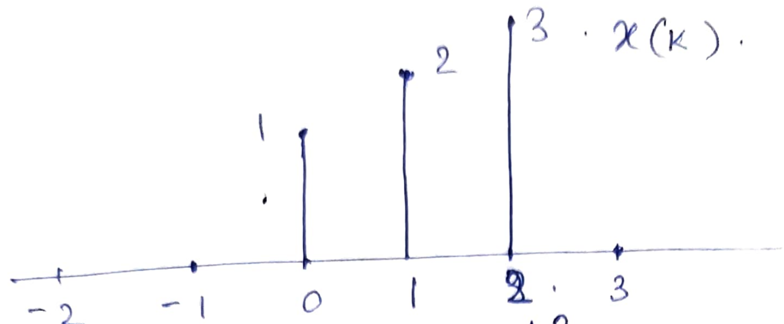
To obtain. $y(-1)$., let's plot $h(-1-K)$.

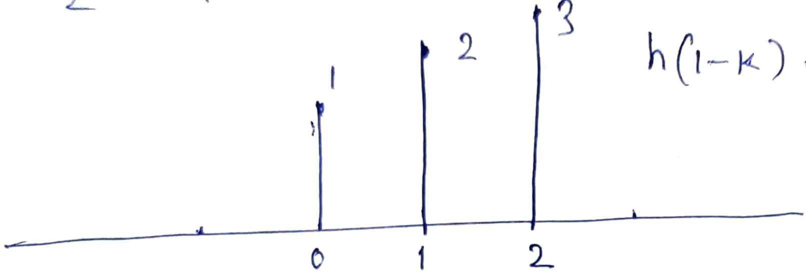


$$y(-1) = \sum x(K)\, h(-1-K)$$
$$= 3.$$

To obtain $y(1)$ i.e. $n = 1$.

let's plot $h(1-k)$.



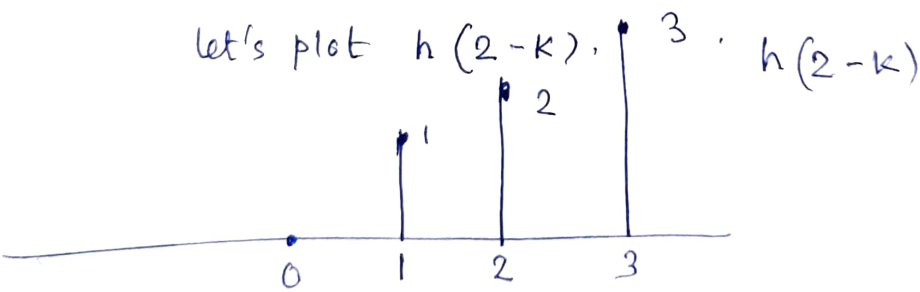


$$y(1) = \sum x(k)\, h(1-k)$$

$$= 1 + 4 + 9 = 14$$

To obtain $y(2)$ i.e. $n = 2$.

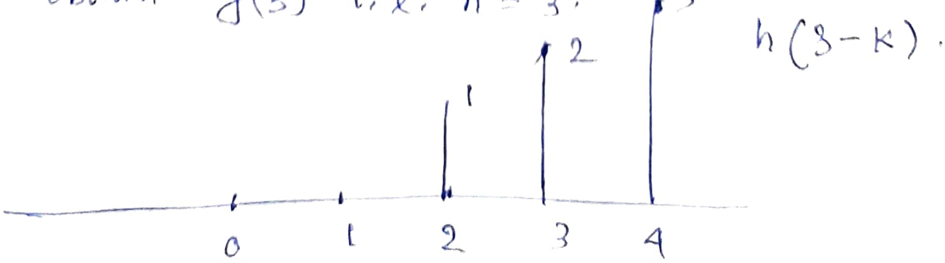
let's plot $h(2-k)$.



$$y(2) = \sum x(k)\, h(2-k)$$

$$= 2 + 6 = 8$$

To obtain $y(3)$ i.e. $n = 3$.



$$y(3) = \sum x(k)\, h(3-k) = 3$$

o/p Sequence. or the result of Convolution

$$y(n) = \{3, \underset{\uparrow}{8}, 14, 8, 3\}.$$

$$x(n) = \{\underset{\uparrow}{1}, 2, 3\}$$

$$h(n) = \{3, \underset{\uparrow}{2}, 1\}$$

Matrix method.

$$\begin{array}{c|ccc} & & x(n). & \\ & 1 & 2 & 3. \\ \hline 3 & 3 & 6 & 9 \\ h(n) \quad 2 & 2 & 4 & 6 \\ 1 & 1 & 2 & 3. \end{array}$$

Multiply. & diagonally addition.

Diagonally add the elements of the matrix,

$$y(n) = \{3, \underset{\uparrow}{8}, 14, 8, 3\}$$