DTFT properties.

$$x(n) \longleftrightarrow X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}.$$

w = digital frequency (rad)

$\hookrightarrow$ Angle in polar form.

$$= X_R(e^{jw}) + jX_I(e^{jw}).$$

$\hookrightarrow$ Rectangular form

$$= \cancel{|X(w)| e}$$

$$= |X(e^{jw})| \, e^{j\angle X(e^{jw})}.$$

$\hookrightarrow$ polar form representation.

1. DTFT Visualization requires two plots:

$|X(e^{jw})|$ vs w $\longrightarrow$ Magnitude spectrum.

$\angle X(e^{jw})$ vs w $\longrightarrow$ phase spectrum.

2. Inverse DTFT.

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} \, dw.$$

$\hookrightarrow$ Try to prove this equation.

3. $X(e^{jw})$ is a periodic over w & period $= 2\pi$.

$\hookrightarrow$ periodic function of w.

$$\boxed{X(e^{jw}) = X(e^{j(w+2\pi)})}$$

LTI

$$\begin{array}{ccccc} x(n) & & & & y(n) \\ & \longrightarrow & \boxed{h(n)} & \longrightarrow & \\ X(e^{j\omega}) & & H(e^{j\omega}) & & Y(e^{j\omega}) \end{array}$$

$y(n) = x(n) * h(n)$ → in time domain.

$$y(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{j\omega})\, e^{j\omega n}\, d\omega \quad \to (1)$$

$$y(n) = T[x(n)]$$

$$= T\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{+j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, T[e^{j\omega n}]\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, H(e^{j\omega})\, e^{j\omega n}\, d\omega \quad \to (2)$$

$\cancel{T[e^{j\omega n}] = H(e^{j\omega})}$

as we know,

$$\underbrace{T[e^{j\omega n}]}_{\text{eigen sequence}} = H(e^{j\omega})\, e^{j\omega n}$$

Comparing (1) & (2) →

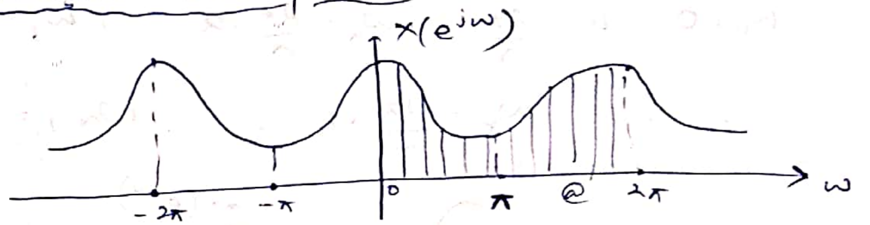
$$\boxed{Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})}$$

→ Frequency domain

$$\boxed{y(n) = x(n) * h(n)} \to \text{Time-domain.}$$

Discrete Fourier Transform.

$x[n] \longleftrightarrow X(e^{jw})$



Inverse DTFT :

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} \, dw.$$

→ entire envelope has to be taken either $(-\pi$ to $\pi)$ or $(0$ to $2\pi)$

↳ but within this envelope, to obtain $x(n)$, there are infinite pts. in domain. (since above is continuous.) Thus we take

For $x[n]$ → finite length sequence.
→ N-point sequence.

$x[n]$ → given for $n = 0, 1, 2, \ldots, N-1$.

$x[n] = 0$, outside $0 \leq n \leq N-1$.

$$DTFT\{x[n]\} = \sum_{n=0}^{N-1} x[n] e^{-jwn}.$$

* @ we just want N-points {actually infinite pts. exist b/w 0 to 2π}

i.e, N-samples.

enough to get back to $x[n]$.

6/9/19

frequency domain; $x(n) \xleftrightarrow{DTFT} X(e^{jw})$

Period : 2π

{A fn. is periodic if $f(x+T) = f(x)$, where T is period}.

To check for $X(e^{jw})$; replace ω with $(\omega + 2\pi m)$.

$$X(e^{j(\omega+2\pi m)}) = X(e^{jw} \cdot e^{j2\pi m}) = X(e^{jw}).$$

It's magnitude & phase spectrum :- periodic with ω

To get back $x[n]$: $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} \, dw$

- $x[n] \longrightarrow$ N-length sequence
  N-point sequence

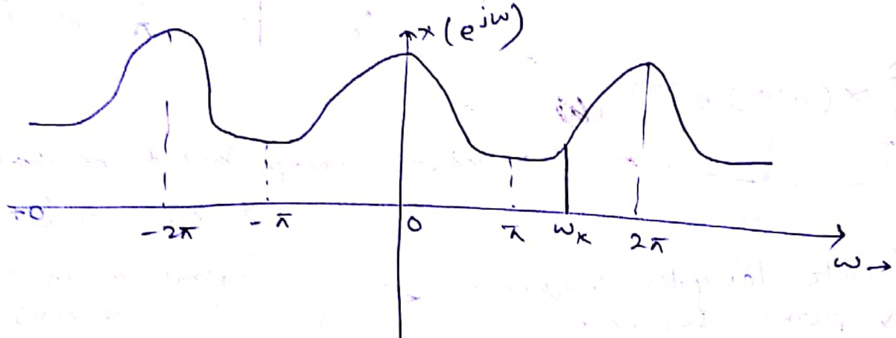
$x[n] = 0$, outside $0 \le n \le N-1$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n]\, e^{-j\omega n}$$

taken at pts:-

$\omega_0 = 0$; $\omega_1 = \frac{2\pi}{N}$; $\omega_2 = \frac{2\pi}{N} \times 2$; $\omega_3 = \frac{2\pi}{N} \times 3$ . . . . . .

. . . . . $\omega_k = \frac{2\pi}{N} \times k$ . . . . . $\omega_{N-1} = \frac{2\pi}{N}(N-1)$



DTFT @ $\omega_k = \frac{2\pi}{N} k$

$$X(k) = X\left(e^{j\frac{2\pi}{N}k}\right) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

k^{th} pt. DFT of $x[n]$. As such we will have n pt. DFTs of $x[n]$

$$X(k) = \sum_{r=0}^{N-1} x[r]\, e^{-j\frac{2\pi}{N}kr}$$