# Lecture-1

## Recap of Intro

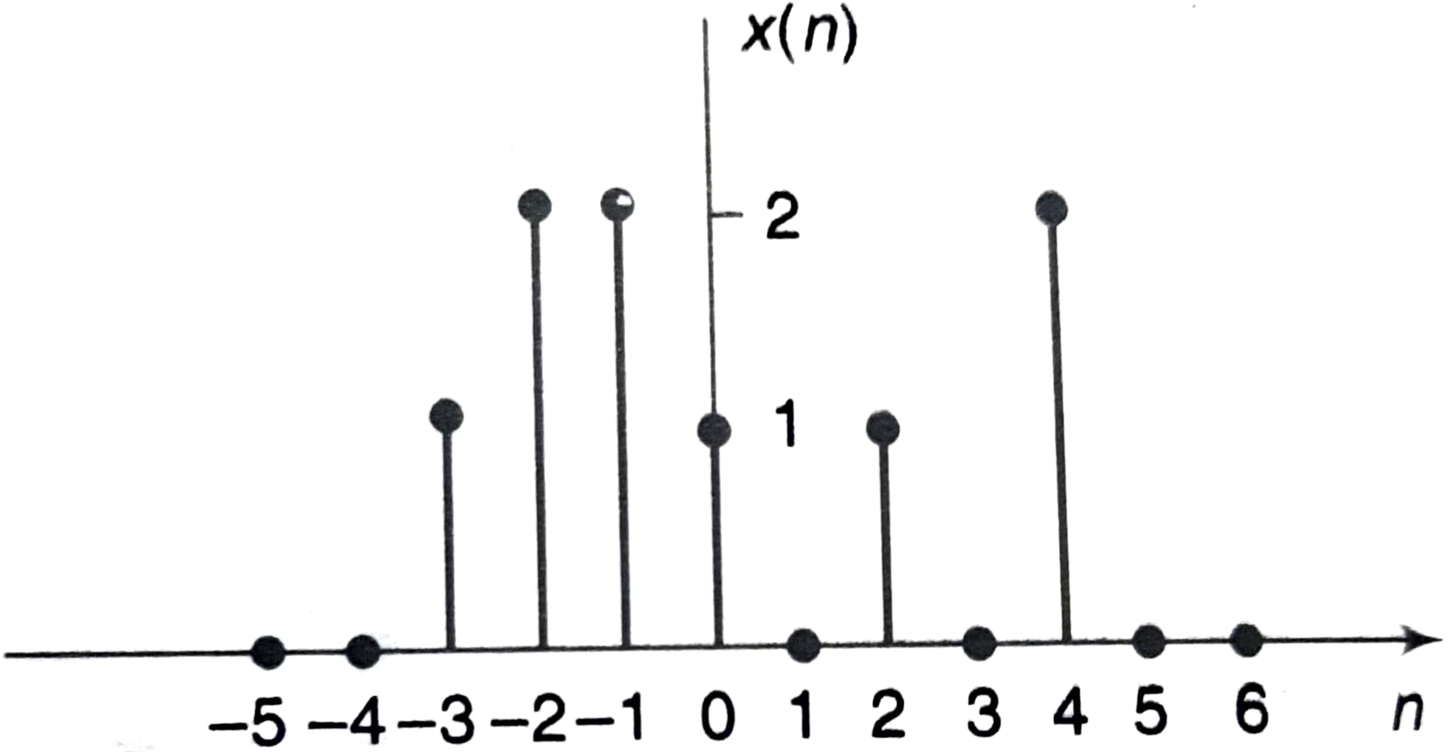
- Discrete-time Signal represention in time domain
- Introduction of DTS ~ LTI. System.
- Response of a discrete-time LTI System.

## Discrete-time Signal representation

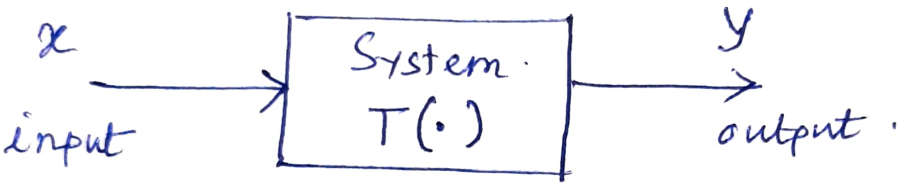
- Graphical representation
- Sequence form.

Suppose $x(n)$ is a discrete-time Signal.

$$x(n) = \{x_n\} = \{1, 2, 2, \underset{\uparrow}{1}, 01, 0, 2\}.$$


x(n)
2
1
−5 −4 −3 −2 −1 0 1 2 3 4 5 6
n

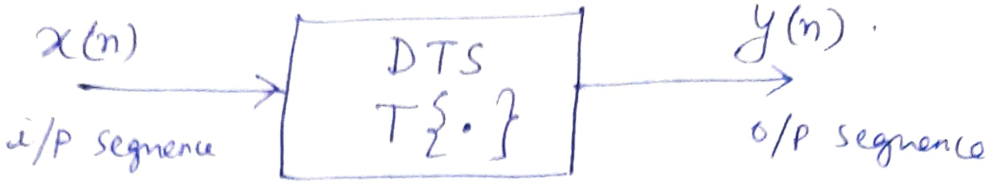
# System Representation




$$\boxed{y = T\{x\}}$$ → Mathematical notation

- System can be as a transformation (or mapping) of $x$ into $y$.
- $T$ is a operator representing some well-defined rule by which $x$ is transformed into $y$.
- We will restrict our attention to the single-input & single-output system.

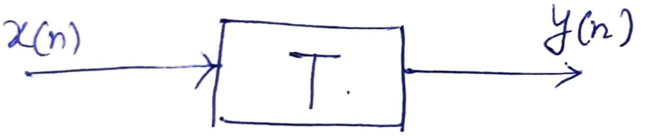
# Discrete-time System (DTS)




$$y(n) = T\{x(n)\}.$$ → General Mathematical notation of a DTS.

## Linear DTS.

↳ which follows the principle of Superposition theorem.




$$y(n) = T\,[x(n)]$$

i/P      O/P

$$x_1(n) \Rightarrow y_1(n) = T\,[x_1(n)]$$

$$x_2(n) \Rightarrow y_2(n) = T\,[x_2(n)].$$

$$x'(n) = C_1 x_1(n) + C_2 x_2(n).$$

$$y'(n) = T\left[C_1 x_1(n) + C_2 x_2(n)\right]$$

$$= C_1\, T\,[x_1(n)] + C_2\, T\,[x_2(n)]$$

$$= C_1 y_1(n) + C_2 y_2(n).$$

## Which one is linear?

1. $y[n] = m x[n]$

2. $y[n] = m x[n] + b$

## Time-invariant DTS. (shift-invariant)

$$x(n) \longrightarrow \boxed{T} \longrightarrow y(n).$$

$$y[n] = T[x(n)].$$

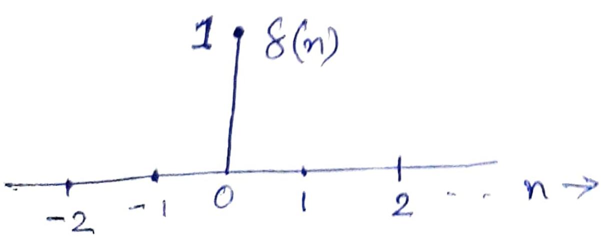
$$x'(n) = x(n-K).$$

what would be new o/p?

$$y'(n) = T[x(n-K)] = y(n-K).$$

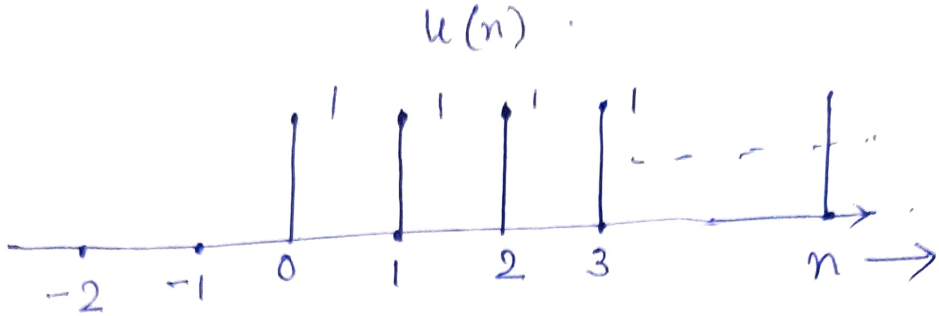
## Basic Discrete-time Sequence.

### 1 Unit-impulse Sequence (Sample sequence):




$$\delta(n) = 1, \quad n = 0$$
$$= 0, \quad \text{otherwise.}$$

## ② Unit Step sequence.




$$u(n) = 1, \quad n = 0, 1, 2, \dots, \infty \quad (n \geq 0).$$
$$= 0, \quad \text{otherwise}.$$

- How can we write $u(n)$ in terms of $\delta(n)$?
- How can we write $\delta(n)$ in terms of $u(n)$?
- How can we represent any arbitary sequence $x(n) = \{x_n\}$ in terms of $\delta(n)$?

$\Rightarrow$ $u(n)$ can be decomposed into several subsequences of ~~dett~~ unit-sample sequence.

$$\boxed{u(n) = \sum_{k=0}^{\infty} \delta(n-k)}$$

$$\Rightarrow \quad \boxed{\delta(n) = u(n) - u(n-1)}$$

$\Rightarrow$ Any arbitary sequence $x(n) = \{ \ldots, x(-1), x(0), x(1), x(2), \ldots \}$ — All are constants (values)

$= \{x_n\}$.

This sequence can be also expressed as a summation of several subsequences.

$x(n)$ can be represented in terms of delayed (shifted) ~~this~~ Scaled impulse sequence.

↳ linearly combining Scaled impulse sequence.

$$\boxed{x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)}$$

↳ General expression of $x(n)$ in terms of $\delta(n)$.

## Response of a LTI System.

$$x(n) \longrightarrow \boxed{T} \longrightarrow y(n).$$

Let's define impulse response,

$$\underset{i/p}{\delta(n)} \longrightarrow \underset{o/p}{h(n)}.$$

~~i.e. $h(n) = T[\delta(n)]$~~

$$\boxed{\therefore\ h(n) = T[\delta(n)]}$$

So, what would be expression of $y(n)$ interms of $x(n)$ and $h(n)$ ?

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, T[\delta(n-k)]$$

(as the given system is a ~~LTI~~ linear system)

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

(as the given system is a time-invariant system)

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

It indicates that a discrete-time LTI system is completely characterized by its impulse response.

This equation also defines the linear convolution of two sequences $x(n)$ & $h(n)$.

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$