Problem Set 2

Digital Signal Processing (DSP) (ECC603)

April 5, 2025

Question 1.

The input to a causal LTI system is:

$$x[n] = u[-n-1] + \left(\frac{1}{2}\right)^n u[n]$$

The Z-transform of the output of this system is

$$Y(z) = \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})}$$

(a) Determine the impulse response $h[n]$

(b) Sketch the pole-zero plot of $\cos(\frac{n\pi}{2})h[n]$

Question 2.

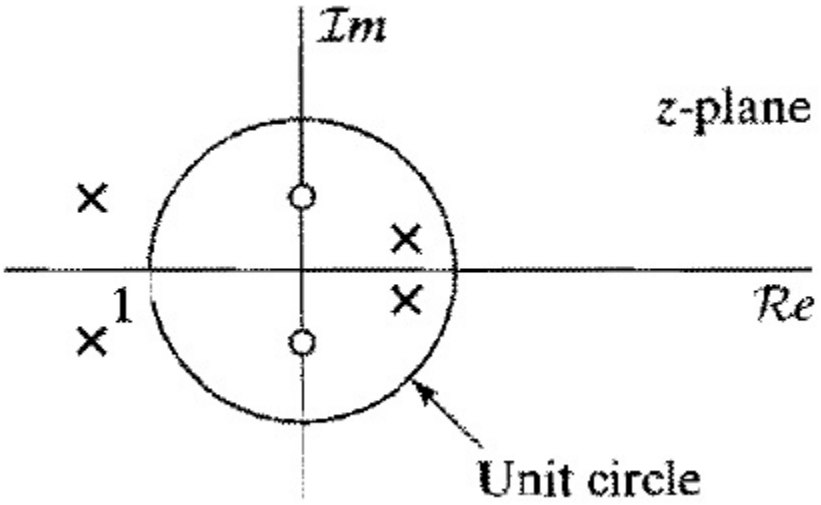
The system function of an LTI system has the pole-zero plot shown in the figure below. Specify whether each of the following statements is true/false/cannot be determined from the information given:

(a) The system is stable.

(b) The system is causal.

(c) If the system is causal, then it must be stable.

(d) If the system is stable, then it must have a two-sided impulse response.



Question 3.

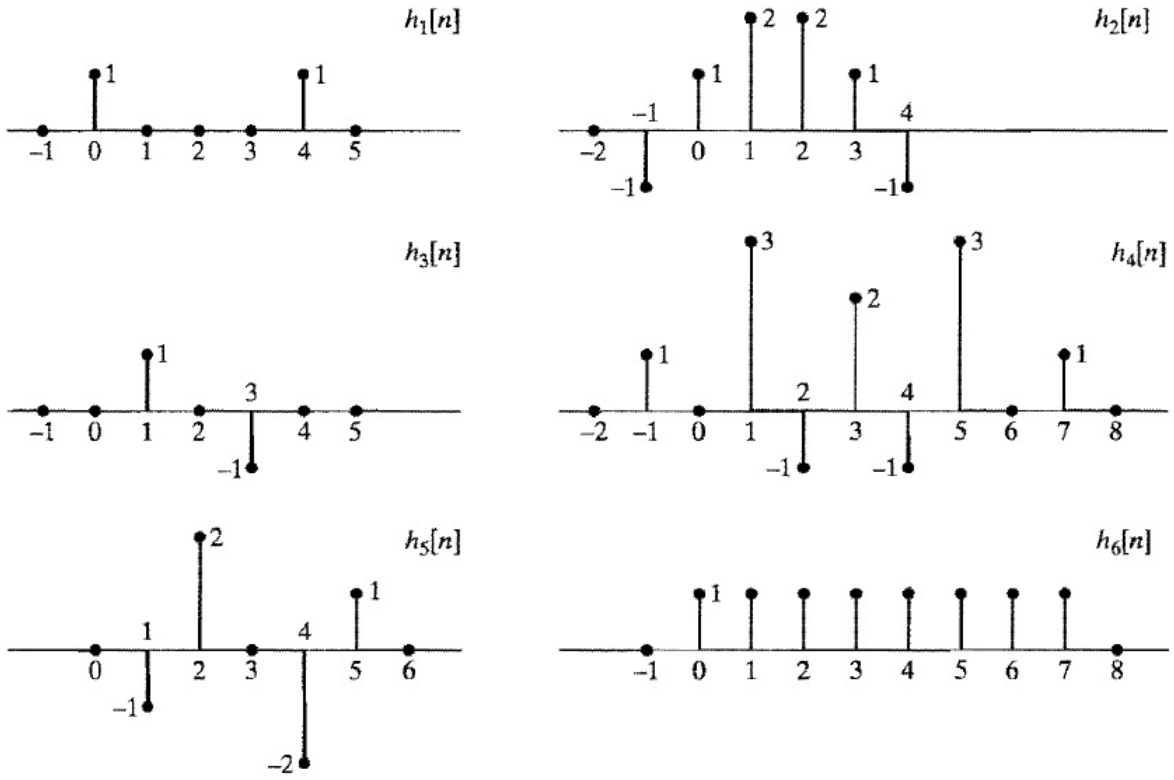
Consider a stable LTI system with input $x[n]$ and output $y[n]$. The input and output satisfy the difference equation:

$$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n]$$

(a) Plot the poles and zeros of the system function in the z-plane.
(b) Determine the impulse response $h[n]$.

Question 4.

Figure below shows the impulse responses for several different LTI systems. Determine the group-delay associated with each system.



Question 5.

Consider a linear time-invariant system with input $x[n]$ and output $y[n]$ for which

$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n].$$

The system may or may not be stable or causal.

By considering the pole–zero pattern associated with the preceding difference equation, determine three possible choices for the impulse response of the system. Show that each choice satisfies

the difference equation. Indicate which choice corresponds to a stable system and which choice corresponds to a causal system.

Question 6.

Determine the group delay for $0 < \omega < \pi$ for each of the following sequences:

(a)

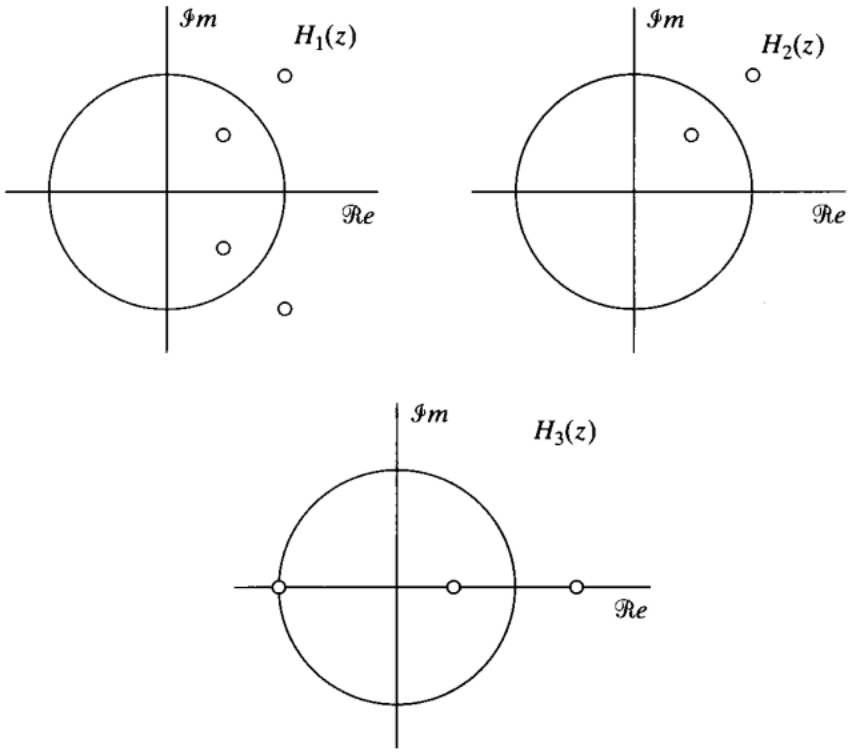
$$x_1[n] = \begin{cases} n-1, & 1 \le n \le 5, \\ 9-n, & 5 < n \le 9, \\ 0, & \text{otherwise.} \end{cases}$$

(b)

$$x_2[n] = \left(\frac{1}{2}\right)^{|n-1|} + \left(\frac{1}{2}\right)^{|n|}.$$

Question 7.

Following Figure shows just the zero locations for several different system functions. For each plot, state whether the system function could be a generalized linear-phase system implemented by a linear constant-coefficient difference equation with real coefficients.



Question 8.

A causal discrete-time LTI system is described by

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$$

where $x[n]$ and $y[n]$ are the input and output of the system, respectively.

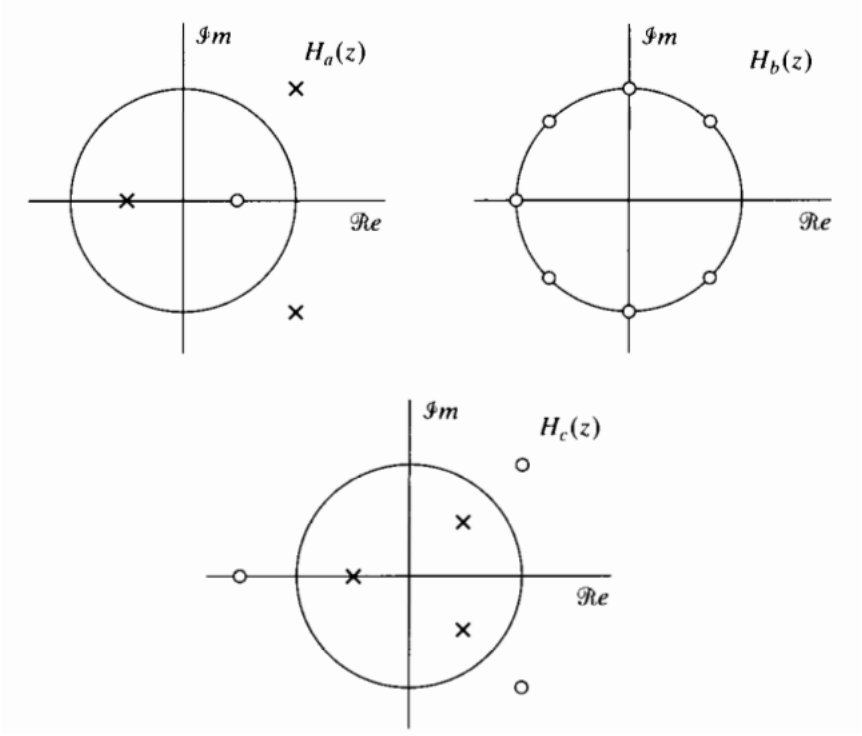
(a) Determine the system function $H(z)$.

(b) Find the impulse response $h[n]$ of the system.

(c) Find the step response $s[n]$ of the system.

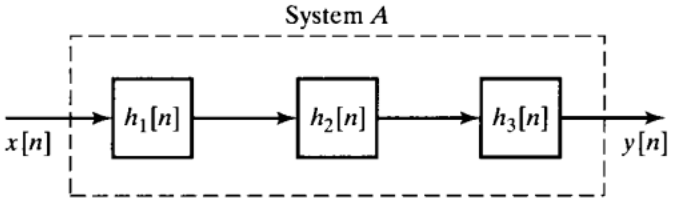
Question 9.

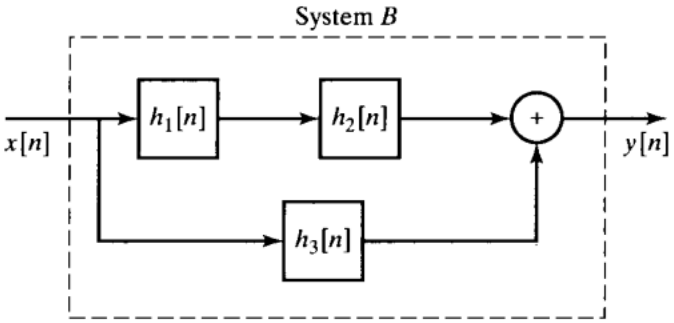
Figure below shows the pole–zero plots for three different causal LTI systems with real impulse responses. Indicate which of the following properties apply to each of the systems pictured: stable, IIR, FIR, minimum phase, all-pass, generalized linear phase.

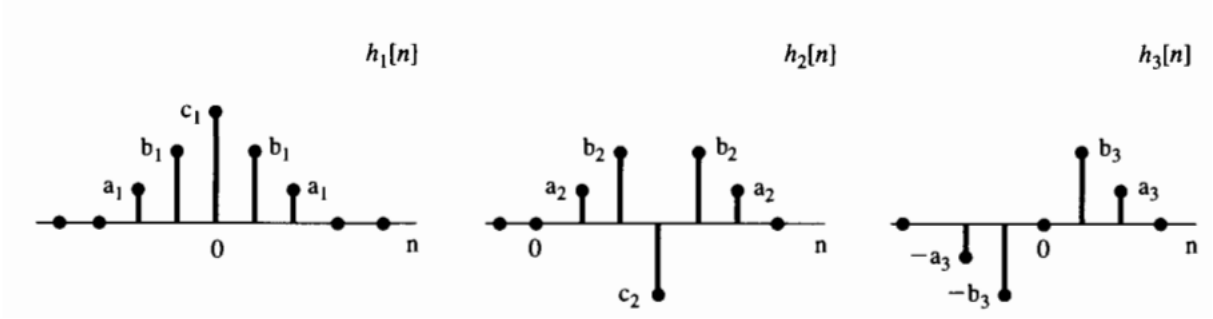


Question 10.

Figure below shows two different interconnections of three systems. The impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ are as shown in the next Figure. Determine whether system A and/or system B is a generalized linear-phase system.







Question 11.

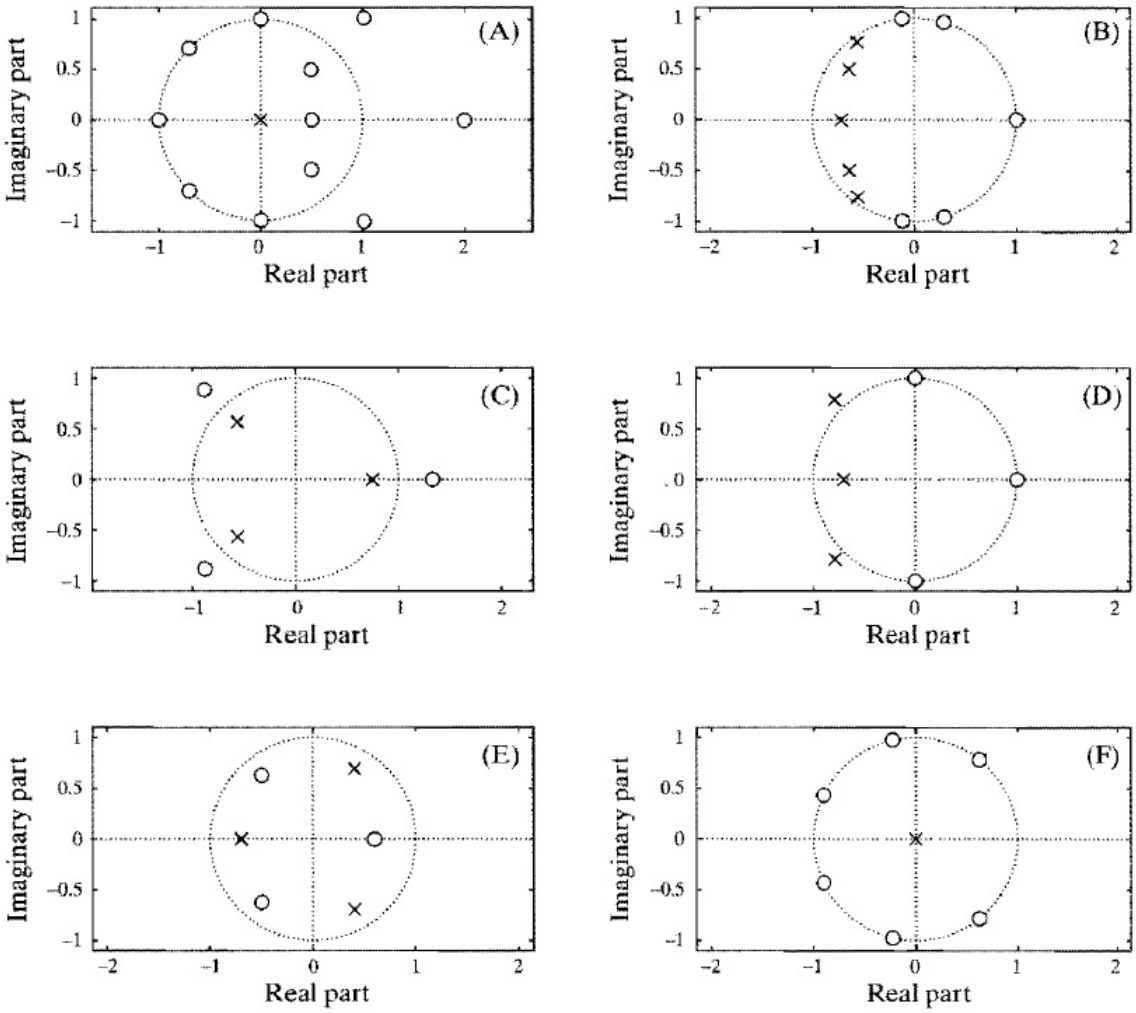
Find the N-point DFT of the following sequences $x[n]$:

(a) $x[n] = \delta[n]$

(b) $x[n] = u[n] - u[n-N]$

Question 12.

The pole-zero plots of six different causal LTI systems are shown in the figure below:



(a) Which systems are IIR systems?
(b) Which systems are FIR systems?
(c) Which systems are Stable systems?
(d) Which systems are Minimum-phase systems?
(e) Which systems are All-pass systems?
(f) Which systems have corresponding stable and causal inverse systems?