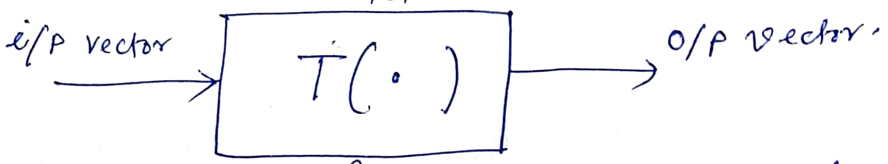
DSP

Recap · → Connection between LTI system's properties & impulse response.

→ Case-study.

"eigen"

Linear transformation system.



↳ defined by a matrix $A_{n\times n}$.

$$T[\vec{v}] = A\bar{v} = \lambda \bar{v}$$

Eigen vector. | Eigen value. Corresponding to this eigen vector $\bar{v}$.

e.g. $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$\bar{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\qquad \bar{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$A\bar{v}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A\bar{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The concept of Eigenvector can be extended to analyze the Eigen sequence of discrete-time LTI system.

non-Causal sequence

$$x(n) = a^n \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

Complex exponential function.

$$y(n) = h(n) * x(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, a^{n-k}$$

$$= \underbrace{\left[\sum_{k=-\infty}^{\infty} h(k)\, a^{-k}\right]}_{\lambda_a} a^n$$

a^n : Eigen sequence of LTI System.

$$a^n \longrightarrow \lambda_a \cdot a^n$$

$$a_1^n \longrightarrow \lambda_{a_1} \cdot a_1^n$$

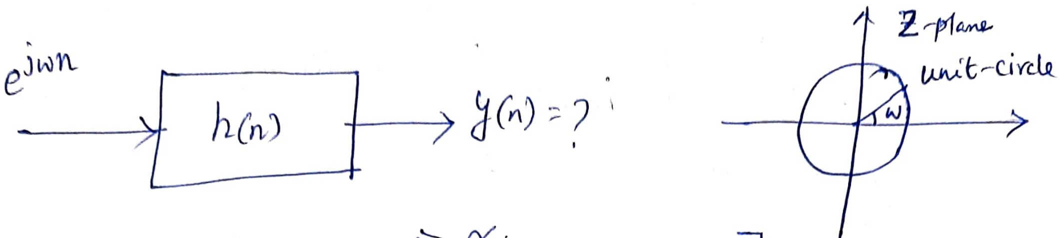
$$a_2^n \longrightarrow \lambda_{a_2} \cdot a_2^n$$

$$\left[c_1 a_1^n + c_2 a_2^n\right] \xrightarrow{T} c_1 T\left[a_1^n\right] + c_2 T\left[a_2^n\right]$$

$$= c_1 \lambda_{a_1} a_1^n + c_2 \lambda_{a_2} a_2^n .$$

Now, let's consider $x(n) = a^n$.

$\Rightarrow a = e^{jw}$. $\quad w \rightarrow \underset{\text{unit}}{\text{rad (angle)}}$.



$$y(n) = \Big[\underbrace{\sum_{K=-\infty}^{\infty} h(K) e^{-jwK}}\Big] e^{jwn}$$

eigen value of e^{jwn}

This is a polynomial in e^{jw} (power-series)

Let's ~~Consider~~, define,

$$H(e^{jw}) \triangleq \sum_{K=-\infty}^{\infty} h(K) e^{-jwK}.$$

By definition it would be called as Discrete-time Fourier Transform (DTFT).

of $h(n)$.

$$h(n) \overset{DTFT}{\longleftrightarrow} \sum_{n=-\infty}^{\infty} h(n) e^{-jwn} = H(e^{jw}).$$

$$h(t) \overset{CTFT}{\longleftrightarrow} \int_{-\infty}^{\infty} h(t)\, e^{-j\Omega t}\, dt = H(e^{j\Omega}).$$

Analog Signal Counterpart.

$\Omega \Rightarrow$ Analog freq (rad/sec)

$w \Rightarrow$ Digital freq. (rad)

$\boxed{w = \Omega T}$ $\rightarrow$ From Sampling Theory.