Problem Set 1

Digital Signal Processing (DSP) (ECC603)

April 2, 2025

Question 1.

The impulse response of a discrete-time LTI system is given by

$h[n] = (1/2)^n u[n]$

Let $y[n]$ be the output of the system with the input

$x[n] = 2\delta[n] + \delta[n-3]$

Find y[1] and y[4].

Question 2.

For each of the following impulse responses of the LTI system, indicate whether or not the system is causal:

(a) $h[n] = (-\frac{1}{2})^n u[n-1]$

(b) $h[n] = u[n+3] + u[n-2] - 2u[n-7]$

Please also comment on the stability of these systems with justification.

Question 3.

The impulse response of discrete-time system is given by:

$h[n] = \delta[n-1] - \delta[n-3]$

What is the value of the step response of the system at n=3?

Question 4.

The impulse response of a linear time-variant system is given as:

$$h[n] = \begin{cases} -2\sqrt{2}, & \text{if } n = 1, -1 \\ 4\sqrt{2}, & \text{if } n = 2, -2 \\ 0, & \text{otherwise} \end{cases}$$

If the input to the system is the sequence $e^{j\pi n/4}$, then what will be the output of the system?

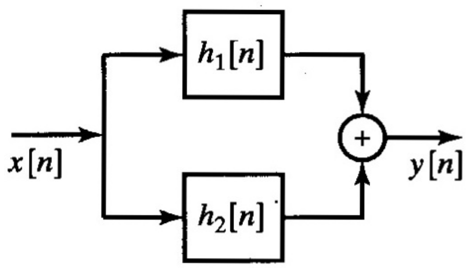
Question 5.

Consider the parallel combination of two LTI systems as shown in the figure. The impulse responses of the systems are:

$h1[n] = 2\delta[n+2] - 3\delta[n+1]$

$h2[n] = \delta[n-2]$

If the input $x[n]$ is a unit-step sequence, then what is the energy of $y[n]$?



Question 6.

If $x[n] = (\frac{1}{3})^{|n|} - (\frac{1}{2})^n u[n]$, then find the Z-transform of $x[n]$ and ROC in the z-plane.

Question 7.

The 8-point DFT of a real-valued sequence is given as $[5, A, B, C, 0, 3+4j, 0, 1+3j]$

Determine the energy of the sequence.

Question 8.

Consider a causal discrete-time system whose output $y[n]$ and input $x[n]$ are related by:

$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$

(a) Find its system function $H(z)$. (b) Find its impulse response $h[n]$.

Question 9.

Find the inverse z-transform of

$$X(z) = \frac{2 + z^{-2} + 3z^{-4}}{z^2 + 4z + 3}, \ |z| > 0$$

Question 10.

Let $X(k) = 2k - 1, (0 \le k \le 7)$ be 8-point DFT of a sequence $x[n]$.

Find out the value of $\sum_{n=0}^{3} x[2n]$.

Question 11.

When input to a causal LTI system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + 2^n u[-n-1]$$

The corresponding output is

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]$$

(a) Find the system function H(z). Plot "pole(s)" and "zero(s)" of H(z).

(b) Find the impulse response $h[n]$ of the system.

(c) Write a difference equation that is satisfied by the given input and output.

(d) Is the system stable? Provide a justification for your answer.

(e) Is the system FIR or IIR?

Question 12.

Assuming a continuous-time signal is given as:

$$x(t) = 10\cos(2\pi \cdot 5500t) + 5\sin(2\pi \cdot 7500t), \quad \text{for } t \ge 0,$$

sampled at a sampling rate of $8,000$ Hz.

(a) Sketch the spectrum of the sampled signal up to 20 kHz.

(b) Sketch the recovered analog signal spectrum if an ideal lowpass filter with a cutoff frequency of 4 kHz is used to filter the sampled signal in order to recover the original signal.

(c) Determine the frequency/frequencies of aliasing noise.

(d) A 1024-point DFT is applied to analyze the spectrum. Calculate the frequency resolution.

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Question 13.

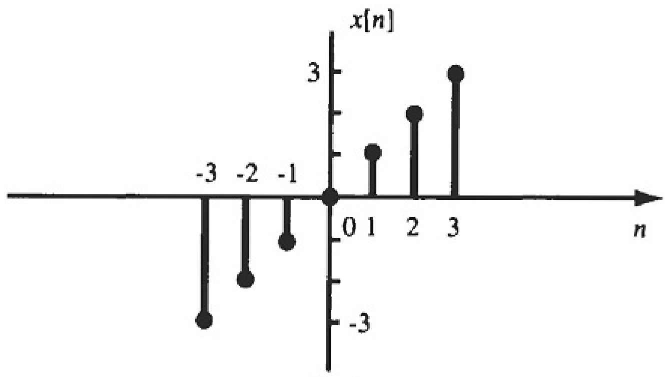

Fig.-1

The Fourier transform of the sequence $x[n]$ shown in Fig.1 can be given as:
$X(e^{j\omega}){=}Aj\sum_{k=B}^{C} k\sin(k\omega)$. Compute the values of A, B and C.

Question 14.

Consider a **length-4** sequence:

$$x[n] = \{-1, -2, 0, 1\}, \quad 0 \le n \le 3$$

Compute the **4-point DFT** $X[k]$ using the **DFT matrix approach**.

Question 15.

We use the DFT to compute the magnitude spectrum of a sampled data sequence with a sampling rate $f_s{=}$ 20 kHz. The desired frequency resolution is 0.5 Hz. Determine the number of data points would be used by the radix-2 FFT algorithm and actual frequency resolution in Hz, assuming that the sufficient number of data samples are available for processing.

Question 16.

Consider a IIR filter with transfer function as follows:

$$H(z) = \frac{1 + 2z^{-1}}{1 - 1.5z^{-1} + 0.9z^{-2}}$$

Write the difference equation of this system and also draw the Direct Form-II structure.

Question 17.

Given an FIR filter transfer function: $H(z) = 0.2 + 0.5z^{-1} - 0.3z^{-2} + 0.5z^{-3} + 0.2z^{-4}$
perform the linear-phase FIR filter realization with minimal resource utilization.

Question 18.

Consider a causal discrete-time LTI system whose output $y[n]$ and input $x[n]$ are related by a difference equation as follows: $y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n] - x[n-1]$

(a) Find its *system function*

(b) Find the impulse response $h[n]$ of the system. Is the system stable? Is $h[n]$ absolutely summable?

(c) Find its step response.

Question 19.

Two 4-point sequences are defined as $g[n] = \cos(\frac{\pi n}{2})$ and $h[n] = 2^n$ for n=0, 1, 2, 3.

(a) Calculate 4-point DFTs $G(k)$ and $H(k)$ using matrix method.

(b) Calculate 4-point circular convolution of $g[n]$ and $h[n]$ using graphical method.

(c) Calculate the inverse DFT of the product of $G(k)$ and $H(k)$ and compare it with the previous result obtained from step (b).

Question 20.

Let $x[n] = \{1, \underset{\uparrow}{-2}, 3\}$ be the input sequence to a discrete-time LTI system. If $h[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1\}$ is given as the impulse response of the system, then compute the output sequence of the system. Show the steps of your computation graphically.