FFT

- What
- Why
- How

Consider the DFT of a finite length of sequence

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}} = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

Similarly, the IDFT becomes,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_n^{-kn}, \quad n = 0, 1, \ldots\ldots\ldots\ldots N-1$$

(1) N complex multiplications for each value of k.

(2) $(N - 1)$ complex additions for each value of k.

(3) N^2 complex multiplications, for N values of k.

(4) $N(N - 1)$ complex additions, for N values of k.

DIT-FFT (Radix-2)

$$X(k) = \sum_{n \text{ even}} x(n)\, W_N^{nk} + \sum_{n \text{ odd}} x(n)\, W_N^{nk}$$

$$X(k) = \sum_{m=0}^{N/2-1} x(2m)\, W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m+1)\, W_N^{(2m+1)k}$$

$$X(k) = \sum_{m=0}^{N/2-1} f_1(m)\, W_N^{2mk} + W_N^{k} \sum_{m=0}^{N/2-1} f_2(m)\, W_N^{2mk}$$

where, $f_1(m)$ has elements $x(0), x(2), x(4), \ldots\ldots = x(2m)$.

and $f_2(m)$ has elements $x(1), x(3), x(5) \ldots\ldots = x(2m + 1)$.

$$X(k) = \sum_{m=0}^{N/2-1} f_1(m)\, W_{N/2}^{mk} + W_N^{k} \sum_{m=0}^{N/2-1} f_2(m)\, W_{N/2}^{mk}.$$

$$\boxed{X(k) = F_1(k) + W_N^{k}\, F_2(k)}$$

where, $F_1(k)$ and $F_2(k)$ are the $\frac{N}{2}$ point DFTs of N/2 length sequence $f_1(m)$ and $f_2(m)$

$$F_1(k) = \sum_{m=0}^{N/2-1} f_1(m)\, W_{N/2}^{mk} = \sum_{n=0}^{N/2-1} x(2n)\, W_{N/2}^{kn}$$

$$F_2(k) = \sum_{m=0}^{N/2-1} f_2(m)\, W_{N/2}^{mk} = \sum_{n=0}^{N/2-1} x(2n+1)\, W_{N/2}^{kn}$$

If N=8, then…………….

$$X(0) = F_1(0) + W_N^0 F_2(0)$$
$$X(1) = F_1(1) + W_N^1 F_2(1)$$
$$X(2) = F_1(2) + W_N^2 F_2(2)$$
$$X(3) = F_1(3) + W_N^3 F_2(3)$$
$$X(4) = F_1(4) + W_N^4 F_2(4)$$
$$X(5) = F_1(5) + W_N^5 F_2(5)$$
$$X(6) = F_1(6) + W_N^6 F_2(6)$$
$$X(7) = F_1(7) + W_N^7 F_2(7)$$

$$F_1(k + N/2) = F_1(k)$$
$$F_2(k + N/2) = F_2(k)$$

$$\begin{aligned} F_1(0) &= F_1(4) \\ F_1(1) &= F_1(5) \\ F_1(2) &= F_1(6) \\ F_1(3) &= F_1(7) \end{aligned} \qquad \text{and} \qquad \begin{aligned} F_2(0) &= F_2(4) \\ F_2(1) &= F_2(5) \\ F_2(2) &= F_2(6) \\ F_2(3) &= F_2(7) \end{aligned}$$

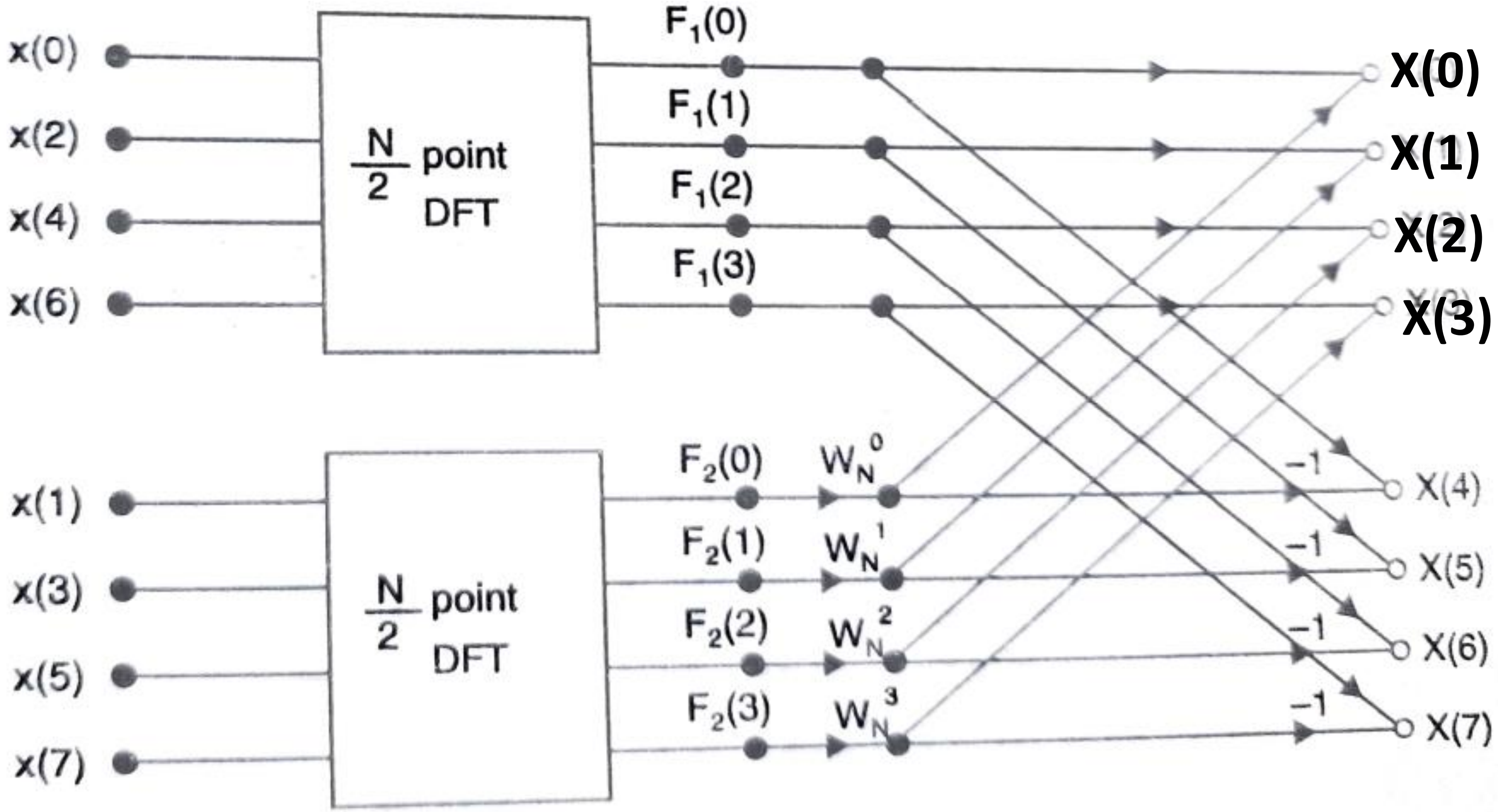
$$X(0) = F_1(0) + W_N^0 F_2(0)$$
$$X(1) = F_1(1) + W_N^1 F_2(1)$$
$$X(2) = F_1(2) + W_N^2 F_2(2)$$
$$X(3) = F_1(3) + W_N^3 F_2(3)$$
$$X(4) = F_1(0) + W_N^4 F_2(0)$$
$$X(5) = F_1(1) + W_N^5 F_2(1)$$
$$X(6) = F_1(2) + W_N^6 F_2(2)$$
$$X(7) = F_1(3) + W_N^7 F_2(3).$$

$$\boxed{W_N^{(k+N/2)} = -W_N^{k}}$$

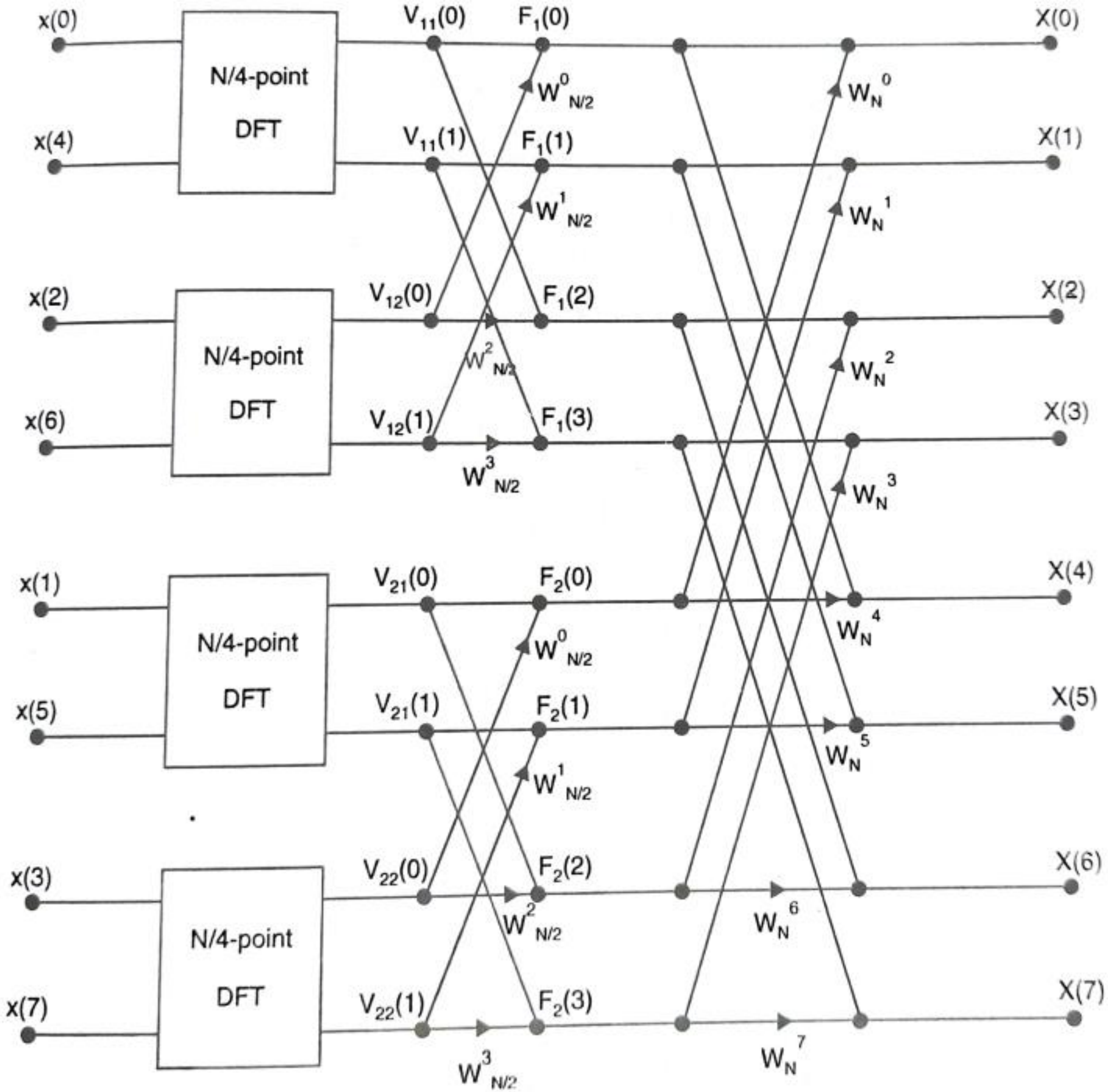
$$\boxed{\begin{aligned} W_N^{4} &= -W_N^{0} \\ W_N^{5} &= -W_N^{1} \\ W_N^{6} &= -W_N^{2} \\ W_N^{7} &= -W_N^{3} \end{aligned}}$$

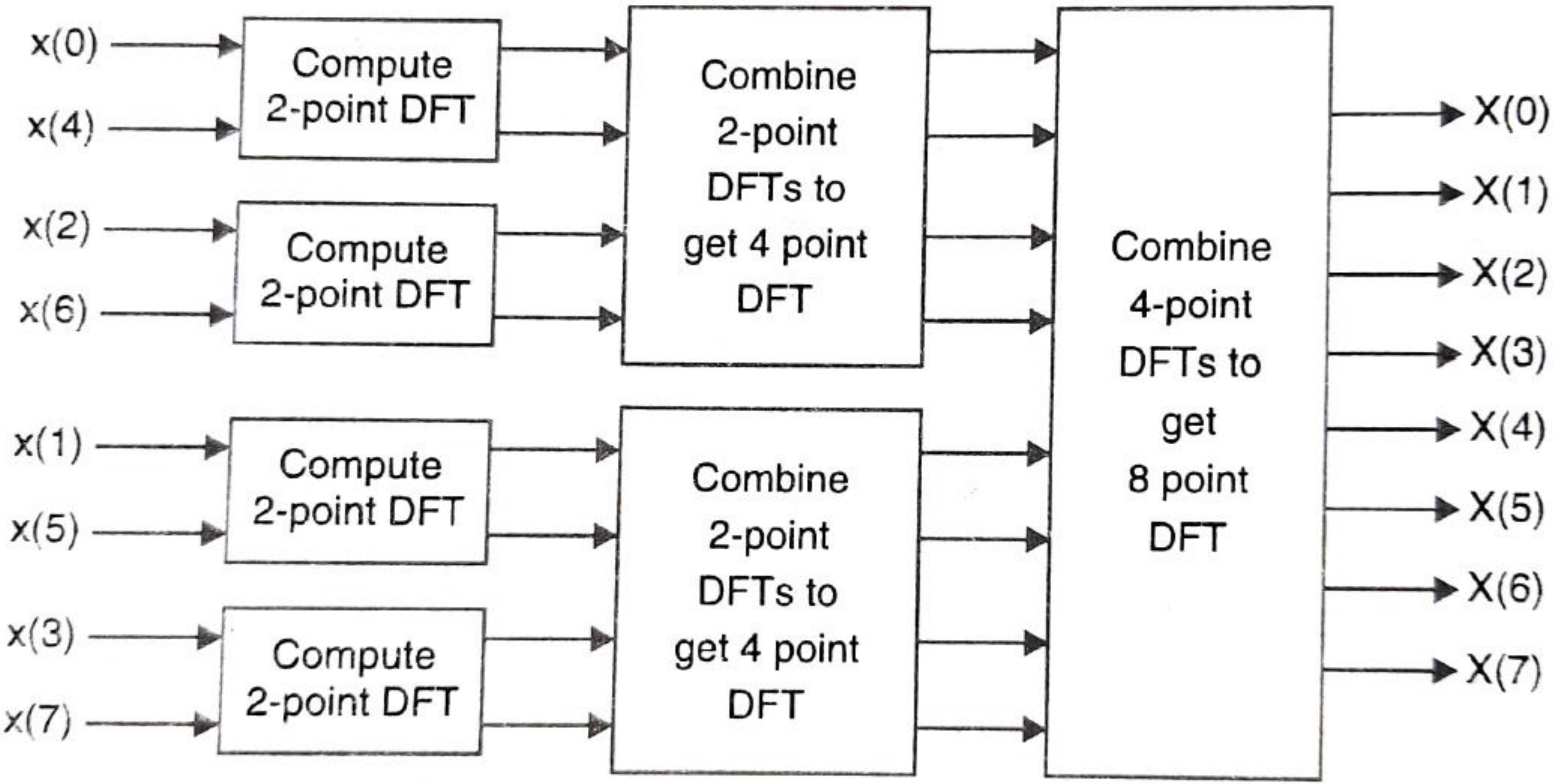
$$X(k) = F_1(k) + W_N^{k} F_2(k).$$

$$X(k + N/2) = F_1(k) - W_N^{k} F_2(k)$$



x(0)
x(2)
x(4)
x(6)
x(1)
x(3)
x(5)
x(7)
N/2 point DFT
N/2 point DFT
F1(0)
F1(1)
F1(2)
F1(3)
F2(0)
F2(1)
F2(2)
F2(3)
W_N^0
W_N^1
W_N^2
W_N^3
−1
−1
−1
−1
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)


x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
N/4-point DFT
N/4-point DFT
N/4-point DFT
N/4-point DFT
V11(0)
V11(1)
V12(0)
V12(1)
V21(0)
V21(1)
V22(0)
V22(1)
F1(0)
F1(1)
F1(2)
F1(3)
F2(0)
F2(1)
F2(2)
F2(3)
W0 N/2
W1 N/2
W2 N/2
W3 N/2
W0 N/2
W1 N/2
W2 N/2
W3 N/2
WN0
WN1
WN2
WN3
WN4
WN5
WN6
WN7
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)


x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
Compute 2-point DFT
Compute 2-point DFT
Compute 2-point DFT
Compute 2-point DFT
Combine 2-point DFTs to get 4 point DFT
Combine 2-point DFTs to get 4 point DFT
Combine 4-point DFTs to get 8 point DFT
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)

The combined butterfly diagram is shown below :