Prime factors of a number N that is N = a.b.c.….k, where a,b,c .. k are prime numbers. There are three possible combinations.

1. All the prime factors are <=SQRT(N)
2. Only one prime factor >SQRT(N), all others must be <SQRT(N)
3. There is no prime number <= SQRT(N), then the only prime factor >SQRT(N) is N that is the number itself. Hence, the number N is a prime number.

Proof:

Let’s say ‘N’ is any composite number. It can be written as

N = a\*b, where a and b are any other numbers.

Let’s assume that a >SQRT(N) and b >SQRT(N), then it implies that a\*b >N which contradicts the statement N=a\*b. Hence, a<=SQRT(N) or/and b<=SQRT(N). That is both of them can’t be greater than SQRT(N) simultaneously. The possible combinations are described below.

1. The numbers a, b <=SQRT(N)
2. Only one of the numbers a and b can be >SQRT(N) but there must be a which is <SQRT(N)

Keeping the generality, since a and b can be any numbers (primes or composites), let’s say the number a is a prime number that we found by decomposing the number N into two numbers a and b. Hence, a and b can belong to one of the above-defined combinations.

1. The numbers a, b <=SQRT(N), in this case, the number a which is a prime number and the prime factors of b will all be <=SQRT(N). That is the number N has all its prime factors <=SQRT(N)
2. The number a which is a prime number >SQRT(N), then the number b, which is a composite number, has to be less than SQRT(N) which implies that all the prime factors of b are less than SQRT(N). Hence, it means if N has a prime factor which is >SQRT(N) then all other primes must be <=SQRT(N).

Code Snippets

* PrimeFactors: <https://drive.google.com/file/d/1_lYwQ0s1yRjlwtOHgHiprDxRkBzZxPMa/view?usp=sharing>
* PrimeNumbers: <https://drive.google.com/file/d/1lD_KdHgWEJSR89Bk4X1cZJtF92TtRiXc/view?usp=sharing>