Strong Topological Rigidity of Non-Compact Orientable Surfaces

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Outline of the talk

(1) Topological rigidity.

(2) Non-homeomorphic homotopy equivalent n-manifolds.

(3) Pseudo proper homotopy equivalence.

(4) Strong topological rigidity of non-compact orientable surfaces.

(5) Some consequences and applications of (4).

(6) Sketch of proof of (4).

N.B. All manifolds with or without boundary will be assumed to be orientable. A surface is a connected, two-dimensional manifold with an empty boundary.

Sumanta Das. Strong Topological Rigidity of Non-Compact Orientable Surfaces. Algebr. Geom. Topol. (to appear), 2023. Available at arXiv:2111.11194v3.

Motivation

- Are two n-dimensional homotopy equivalent manifolds homeomorphic? More generally,
- Is every homotopy equivalence between two n-dimensional homotopy equivalent manifolds homotopic to a homeomorphism?

Topological Rigidity

Def. We call a closed manifold M *topologically rigid* if any homotopy equivalence $N \to M$ with a closed manifold N as the source is homotopic to a homeomorphism.

Borel Conj.[a] Every closed aspherical manifold is topologically rigid.

Rem. Mostow Rigidity Thm. $\equiv$ Borel Conj. $\Longrightarrow$ Generalized Top. Poincaré Conj.

[a] Andrew Ranicki. “The birth of the Borel conjecture”. Available at `https://www.maths.ed.ac.uk/~v1ranick/surgery/borel.pdf`.

Topological rigidity in dimension two

Dehn-Nielsen-Baer-Epstein Thm.[a] A homotopy equivalence between any two closed surfaces is homotopic to a homeomorphism.

[a] Max Dehn. Papers on group theory and topology. Translated from the German and with introductions and an appendix by John Stillwell, With an appendix by Otto Schreier. Springer-Verlag, New York, 1987. ISBN: 0-387-96416-9.

Topological rigidity in dimension three

• A homotopy equivalence between two closed prime Haken 3-manifolds is homotopic to a homeomorphism. [a]

• A homotopy equivalence from a closed irreducible 3-manifold to a closed hyperbolic 3-manifold is homotopic to a homeomorphism. [b]

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[a] Friedhelm Waldhausen. "On irreducible 3-manifolds which are sufficiently large". In: Ann. of Math. (2) 87 (1968), pp. 56–88. ISSN: 0003-486X.

[b] David Gabai, G. Robert Meyerhoff, and Nathaniel Thurston. "Homotopy hyperbolic 3-manifolds are hyperbolic". In: Ann. of Math. (2) 157.2 (2003), pp. 335–431. ISSN: 0003-486X.

Topological rigidity in high dimensions

If M is a closed connected Riemannian manifold with non-positive sectional curvatures and N is a closed aspherical topological manifold such that $\dim M = \dim N \geq 5$. Then, any isomorphism $\pi_1(N) \to \pi_1(M)$ is induced by a homeomorphism from $N \to M$. [a]

[a] F. T. Farrell and L. E. Jones. “Topological rigidity for compact non-positively curved manifolds”. In: Differential geometry: Riemannian geometry (Los Angeles, CA, 1990). Vol. 54. Proc. Sympos. Pure Math. Amer. Math. Soc., Providence, RI, 1993, pp. 229– 274.

Non-homeomorphic homotopy equivalent 4-manifolds

Two simply connected, closed topological 4-manifolds are

$$\begin{array}{ccc} \text{homotopy equivalent} & \iff^{a} & \text{same intersection form;} \\ \text{homeomorphic} & \iff^{b} & \text{same intersection form and same Kirby-Siebenmann invariant.} \end{array}$$

Any unimodular symmetric $\mathbb{Z}$-bilinear form q gives, up to homeomorphism, at most two simply connected, closed topological 4-manifolds:

$$\begin{array}{ccc} q \text{ is even} & \rightsquigarrow & \text{unique;} \\ q \text{ is odd} & \rightsquigarrow & \text{precisely two (same I.F. but different ks).} \end{array}$$

[a] John Milnor. "On simply connected 4-manifolds". In: Symposium internacional de topologia algebraica. Universidad Nacional Autónoma de México and UNESCO, Mexico City, 1958, pp. 122–128.

[b] Michael Hartley Freedman. "The topology of four-dimensional manifolds". In: J. Differential Geometry 17.3 (1982), pp. 357–453. ISSN: 0022-040X.

Non-homeomorphic homotopy equivalent 3-manifolds

• Lens spaces can be classified up to homotopy[a] and homeomorphism[b] types; for example, $L(7,1)$ and $L(7,2)$ are homotopy equivalent but not homeomorphic.

• Generalizing a construction given by J. H. C. Whitehead, uncountably many contractible open subsets of $\mathbb{R}^3$ can be constructed such that any two of them are not homeomorphic. [c]

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[a] Paul Olum. "Mappings of manifolds and the notion of degree". In: Ann. of Math. (2) 58 (1953), pp. 458–480. ISSN: 0003-486X.

[b] E. J. Brody. "The topological classification of the lens spaces". In: Ann. of Math. (2) 71 (1960), pp. 163–184. ISSN: 0003-486X.

[c] D. R. McMillan Jr. "Some contractible open 3-manifolds". In: Trans. Amer. Math. Soc. 102 (1962), pp. 373–382. ISSN: 0002-9947.

Non-homeomorphic homotopy equivalent surfaces

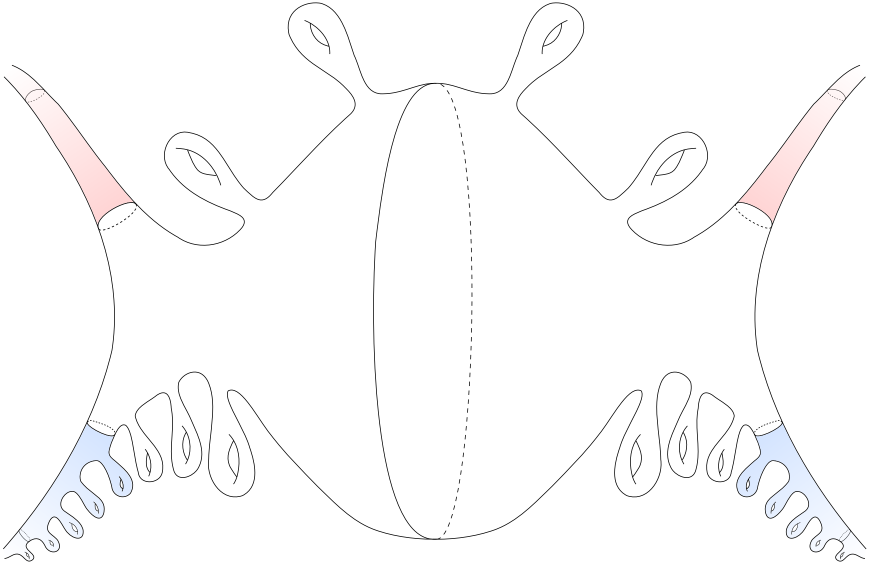
Any non-compact surface Σ is homotopy equivalent to $\bigvee_{\mathscr{A}} \mathbb{S}^1$ for an at most countable set $\mathscr{A}$. More explicitly, $|\mathscr{A}| = 2g + p - 1$ if $\Sigma = S_{g,0,p}$ and $|\mathscr{A}| = \aleph_0$ if Σ is of infinite-type.

• $S_{g,0,p}$ and $S_{g',0,p'}$ are homotopy equivalent if $2g + p = 2g' + p'$, and homeomorphic if $(g, p) = (g', p')$.

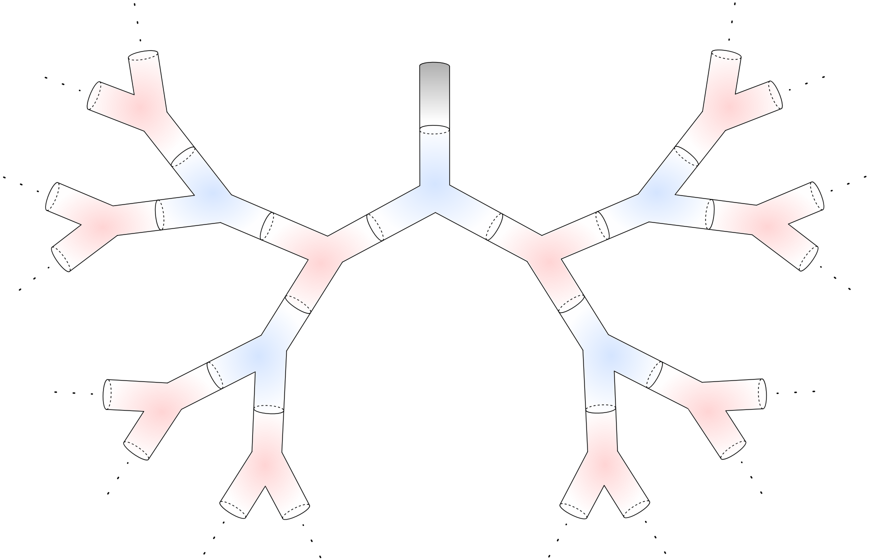
• Up to homotopy equivalence, there is exactly one infinite-type surface, but up to homeomorphism, there are $2^{\aleph_0}$ many infinite-type surfaces. [a]

[a] Sumanta Das. Strong Topological Rigidity of Non-Compact Orientable Surfaces. Algebr. Geom. Topol. (to appear), 2023. Available at arXiv:2111.11194v3.

The surface with two planar ends and two non-planar ends:



The surface obtained from removing Cantor set from the sphere:



Are non-compact surfaces topologically rigid in the proper category?

The proper category: spaces with proper maps

Def. A proper map $f\colon X \to Y$ is said to be *proper homotopy equivalence* (in short, PHE) if there exists a proper map $g\colon Y \to X$ such that fg is properly homotopic to Id_Y and gf is properly homotopic to Id_X.

$$\text{Clear}\Big\Downarrow \qquad \Big\Uparrow ??$$

Def. A homotopy equivalence is said to be *pseudo proper homotopy equivalence* (in short, PPHE) if it is a proper map.

PPHE is a weaker notion than PHE

• Consider $\varphi\colon \mathbb{C} \ni z \longmapsto z^2 \in \mathbb{C}$ and $\psi\colon \mathbb{S}^1 \times \mathbb{R} \ni (z,x) \longmapsto \big(z,|x|\big) \in \mathbb{S}^1 \times \mathbb{R}$; each of these is a PPHE, but none of them is a PHE as *the degree of a PHE is* ± 1, though $\deg(\varphi) = \pm 2$ and $\deg(\psi) = 0$.

• The binary symmetry of the cantor tree gives a two-fold branched covering, which is a PPHE but not PHE since *the induced map of a PHE between the spaces of ends is bijective*.

• If M is a non-compact, connected, contractible, boundaryless oriented manifold, then composing a proper map $M \to [0,\infty)$ with a non-surjective proper map $[0,\infty) \to M$ gives a PPHE which is not PHE *since a PHE is a surjective map*.

PPHE + ?? = PHE

• A PPHE between two connected, finite-dimensional, locally finite simplicial complexes is a PHE if and only if it induces a homeomorphism on the spaces of ends and isomorphisms on all proper homotopy groups. [a]

• If $f\colon M \to N$ is a PPHE between two simply-connected, non-compact, boundaryless n-dimensional smooth manifolds, where both M and N both are simply-connected at infinity, then f is a PHE if and only if $\deg(f) = \pm 1$. [b]

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[a] Edward M. Brown. "Proper homotopy theory in simplicial complexes". In: Topology Conference (Virginia Polytech. Inst. and State Univ., Blacksburg, Va., 1973). Lecture Notes in Math., Vol. 375. Springer, Berlin, 1974, pp. 41–46.

[b] F. T. Farrell, L. R. Taylor, and J. B. Wagoner. "The Whitehead theorem in the proper category". In: Compositio Math. 27 (1973), pp. 1–23. ISSN: 0010-437X.

Strong topological rigidity of non-compact surfaces

Thm. Let $f\colon \Sigma' \to \Sigma$ be a PPHE between two non-compact surfaces. Then Σ' is homeomorphic to Σ. If we further assume that Σ is homeomorphic to neither the plane nor the punctured plane, then f is a PHE, and there exists a homeomorphism $g_{\text{homeo}}\colon \Sigma \to \Sigma'$ as a proper homotopy inverse of f. [a]

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[a] Sumanta Das. Strong Topological Rigidity of Non-Compact Orientable Surfaces. Algebr. Geom. Topol. (to appear), 2023. Available at arXiv:2111.11194v3.

Some consequences and applications of the strong topological rigidity

Proper rigidity

Def. A non-compact topological manifold M without boundary is said to be *properly rigid* if, whenever N is another boundaryless topological manifold of the same dimension and $h\colon N \to M$ is a PHE, then h is properly homotopic to a homeomorphism. [a]

Thm. Let $n \geq 3$, and let $f\colon N \to \mathbb{R}^n$ be a PHE, where N is a boundaryless topological n-manifold. Then N is homeomorphic to $\mathbb{R}^n$. [b] [c] [d] Hence, f is properly homotopic to a homeomorphism. [e]

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[a] Stanley Chang and Shmuel Weinberger. A Course on Surgery Theory. Vol. 211. Annals of Mathematics Studies. Princeton University Press, Princeton, NJ, 2021, pp. xii+430. ISBN: 9780691200354.

[b] C. H. Edwards Jr. “Open 3-manifolds which are simply connected at infinity”. In: Proc. Amer. Math. Soc. 14 (1963), pp. 391–395. ISSN: 0002-9939.

[c] Michael Hartley Freedman. “The topology of four-dimensional manifolds”. In: J. Differential Geometry 17.3 (1982), pp. 357–453. ISSN: 0022-040X.

[d] L. C. Siebenmann. “On detecting Euclidean space homotopically among topological manifolds”. In: Invent. Math. 6 (1968), pp. 245–261. ISSN: 0020-9910.

[e] D. B. A. Epstein. “The degree of a map”. In: Proc. London Math. Soc. (3) 16 (1966), pp. 369–383. ISSN: 0024-6115.

Folklore Thm. Any finite-type non-compact surface is properly rigid.

Thm. Let Σ' and Σ be non-compact surfaces. Then Σ' is homeomorphic to Σ iff $\mathrm{g}(\Sigma') = \mathrm{g}(\Sigma)$ and there exists a ring isomorphism $\theta\colon \mathrm{H}^0_{\mathrm{e}}(\Sigma;\mathbb{Z}_2) \to \mathrm{H}^0_{\mathrm{e}}(\Sigma';\mathbb{Z}_2)$ with $\theta\big(\mathrm{I}_3(\Sigma)\big) = \mathrm{I}_3(\Sigma')$, where w.r.t. $\mathbb{Z}_2$-coefficients,

$$\begin{array}{rcl} \mathrm{I}_1 & := & \left\{x \in \mathrm{H}^1_{\mathrm{c}} : x \smile \mathrm{H}^1_{\mathrm{c}} = 0\right\}, \\ \mathrm{I}_2 & := & \left\{ y \in \mathrm{H}^1 : y \smile \mathrm{I}_1 = 0 \right\}, \\ \mathrm{I}_3 & := & \left\{ z \in \mathrm{H}^0_{\mathrm{e}} : z \smile \mathrm{I}_2 = 0 \right\}, \\ \mathrm{g} & := & \operatorname{rank} \mathrm{H}^1_{\mathrm{c}}\big/\mathrm{I}_1. \end{array}$$

Therefore, if there is a PHE from Σ' to Σ, then Σ' is homeomorphic to Σ. [a]

Thm. Any non-compact surface, possibly of infinite-type, is properly rigid. [b]

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[a] Martin Edward Goldman. Open surfaces and an algebraic study of ends. Thesis (Ph.D.)–Yale University. ProQuest LLC, Ann Arbor, MI, 1967, p. 88.

[b] Sumanta Das. Strong Topological Rigidity of Non-Compact Orientable Surfaces. Algebr. Geom. Topol. (to appear), 2023. Available at arXiv:2111.11194v3.

Classification of π_1-injective proper maps

Nielsen's Thm. Any π_1-injective map between two closed surfaces is homotopic to a covering map. [a]

Thm. Let $f : \Sigma' \to \Sigma$ be a π_1-injective proper map between two non-compact surfaces. If Σ is homeomorphic to neither the plane nor the punctured plane, then f is properly homotopic to a finite sheeted covering map. [b]

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[a] Allan L. Edmonds. "Deformation of maps to branched coverings in dimension two". In: Ann. of Math. (2) 110.1 (1979), pp. 113–125. ISSN: 0003-486X.

[b] I will write its proof somewhere, maybe in my thesis.

Characterizing the image of the Dehn-Nielsen-Baer-Epstein map

Let **S** be an aspherical surface. The natural map $\sigma\colon \mathrm{MCG}^{\pm}(\mathbf{S}) \to \mathrm{Out}\big(\pi_1(\mathbf{S})\big)$ is an injective group homomorphism. Each element of image(σ) is induced by a homotopy equivalence $\mathbf{S} \to \mathbf{S}$ homotopic to homeomorphism.

- If **S** is closed, σ is surjective.
- If $\mathbf{S} = S_{g,0,p}$ with $2 - 2g - p < 0$, then image(σ) is the subgroup of $\mathrm{Out}\big(\pi_1(\mathbf{S})\big)$ consisting of elements that preserve the set of conjugacy classes of *simple closed curves* surrounding individual punctures. [a]

Thm. If **S** is a non-compact other than the plane and the punctured plane, then each element of image(σ) is induced by a homotopy equivalence that preserves the geometric intersection number of the free homotopy classes of any two *closed curves*. [b]

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[a] Benson Farb and Dan Margalit. A primer on mapping class groups. Vol. 49. Princeton Mathematical Series. Princeton University Press, Princeton, NJ, 2012, pp. xiv+472. ISBN: 978-0-691-14794-9.

[b] Sumanta Das, Siddhartha Gadgil, and Ajay Kumar Nair. “The Goldman bracket characterizes homeomorphisms between non-compact surfaces”. In: arXiv preprint (2023). Available at arXiv:2307.02769v1.

Sketch of proof of strong topological rigidity

Infinite-type surfaces only!

Idea obtained from (compact) Haken 3-manifold theory

- Hierarchy: Cut the codomain along a suitable pairwise disjoint collection $\mathscr{C}$ of π_1-injective submanifolds of codim one to get a collection $\mathscr{S}$ smaller pieces of codim zero.
- Surgery on f along $\mathscr{C}$: Homotope f so that $\mathscr{C}' := f^{-1}(\mathscr{C})$ becomes a pairwise disjoint collection of π_1-injective submanifolds of codim one and $f|\mathscr{C}' \to \mathscr{C}$ becomes a homeomorphism.
- Rigidity on parts of split f: Cut the domain along $\mathscr{C}'$ to get a collection $\mathscr{S}'$ of codim zero submanifolds. Apply boundary relative rigidity on each of these smaller pieces of $\mathscr{S}'$ and $\mathscr{S}$ to get a collection of homeomorphisms.
- Required Homeomorphism: Paste all those homeomorphisms to get the final homeomorphism.

Friedhelm Waldhausen. “On irreducible 3-manifolds which are sufficiently large”. In: Ann. of Math. (2) 87 (1968), pp. 56–88. ISSN: 0003-486X.

John Hempel. 3-manifolds. Reprint of the 1976 original. AMS Chelsea Publishing, Providence, RI, 2004, pp. xii+195. ISBN: 0-8218-3695-1.

Benson Farb and Dan Margalit. A primer on mapping class groups. Vol. 49. Princeton Mathematical Series. Princeton University Press, Princeton, NJ, 2012, pp. xiv+472. ISBN: 978-0-691-14794-9.

Obstructions in modifying the aforesaid idea for infinite-type surfaces

Target: Properly homotope a PPHE $f\colon \Sigma' \to \Sigma$ between infinite-type surfaces to a homeomorphism.

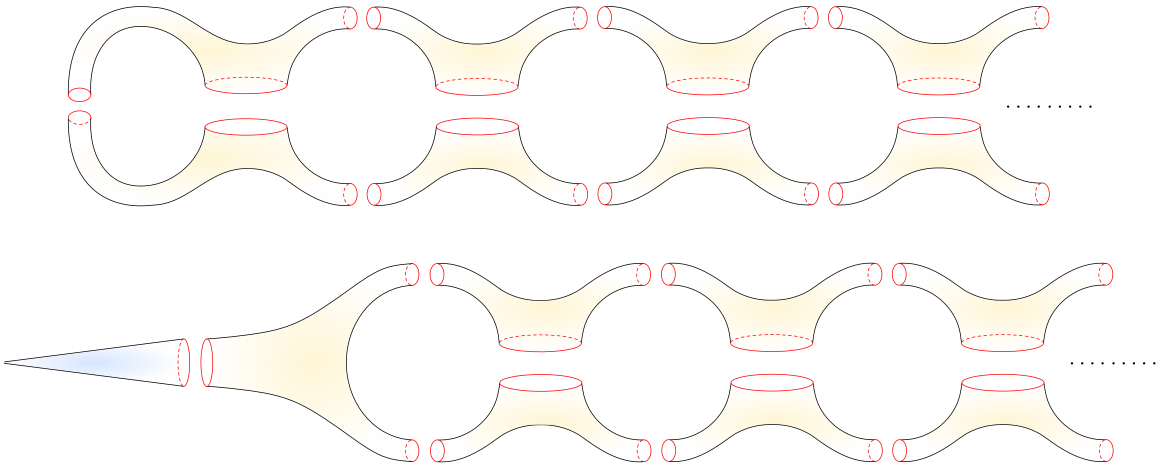
0. Induction on the genus can't be considered here, unlike the closed surface theory.
1. To decompose Σ into finite-type bordered surfaces, we need a pairwise disjoint *infinite collection* $\mathscr{C}$ of essential circles.
2. A *proper homotopy* of f is needed so that $\mathscr{C}' := f^{-1}(\mathscr{C})$ becomes a pairwise disjoint collection of circles on Σ'.
3. How to remove all inessential components from $\mathscr{C}'$. Unlike the compact surface theory, the number of inessential components of $\mathscr{C}'$ can be *infinite*.
4. Further, how to remove remaining (*infinitely many*) unnecessary components from $\mathscr{C}'$ so that $f|\mathscr{C}' \to \mathscr{C}$ becomes a homeomorphism.
5. What's the guarantee that $f^{-1}(\mathscr{C})$ remains *non-empty* even after a proper homotopy?

1. Decomposition into pair of pants and punctured disks

Since Σ is of infinite-type, there is a collection $\mathscr{C}$ of smoothly embedded circles on Σ such that

- $\mathscr{C}$ is countably infinite but locally finite and
- $\mathscr{C}$ decomposes Σ into bordered sub-surfaces such that a complementary component of this decomposition is homeomorphic to either the pair of pants or the punctured disk.

A decomposition of (punctured) Loch Ness Monster surface:



2. Whitney approximation and transversality in proper category

Properly homotope the PPHE $f\colon \Sigma' \to \Sigma$ to make it smooth as well as transverse to $\mathscr{C}$. Thus, after this proper homotopy,

- $f^{-1}(\mathscr{C})$ is either empty or an at most countable, locally-finite, pairwise disjoint collection of smoothly embedded circles on Σ' such that
- $f^{-1}(\mathcal{C})$ is either empty or has finitely many components for each component $\mathcal{C}$ of $\mathscr{C}$.

3. Arbitrarily large disk bounded by a component of $f^{-1}(\mathscr{C})$ doesn't exist

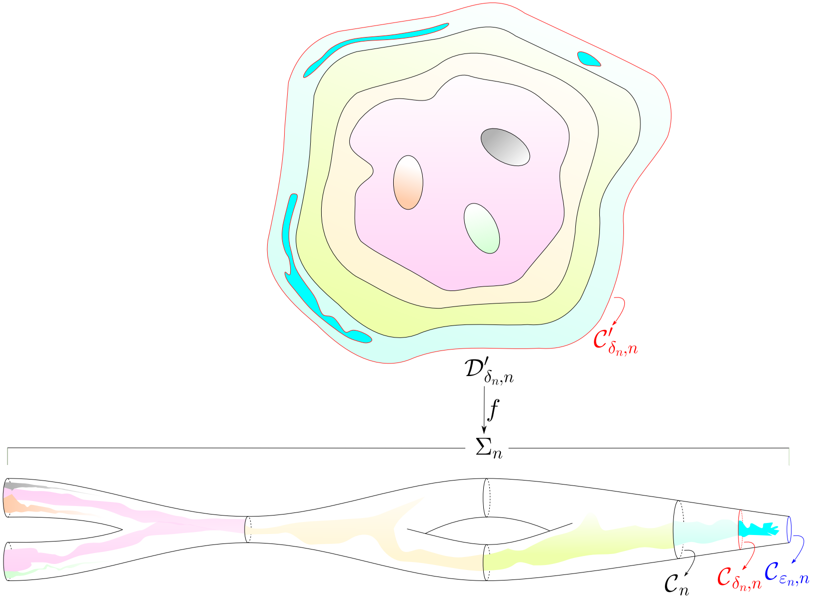
Since Σ' is not homeomorphic to $\mathbb{R}^2$ and $f^{-1}(\mathscr{C})$ is a locally finite, pairwise disjoint collection, there do not exist infinitely many components $\mathcal{C}'_1, \mathcal{C}'_2, \ldots$ of $f^{-1}(\mathscr{C})$ bounding the disks $\mathcal{D}'_1, \mathcal{D}'_2, \ldots$ in Σ', respectively such that $\mathcal{C}'_n \subseteq \mathrm{int}(\mathcal{D}'_{n+1})$ for each n.

4. Removing all outermost disks simultaneously

A disk $\mathcal{D}' \subset \Sigma'$ bounded by a component of $f^{-1}(\mathscr{C})$ is called an *outermost disk*, if given another disk $\mathcal{D}'' \subset \Sigma$ bounded by a component of $f^{-1}(\mathscr{C})$, then either $\mathcal{D}'' \subseteq \mathcal{D}'$ or $\mathcal{D}' \cap \mathcal{D}'' = \varnothing$.

Thus, considering all these outermost disks simultaneously, a proper homotopy of f exists to remove all trivial components from $f^{-1}(\mathscr{C})$ such that each non-trivial component of $f^{-1}(\mathscr{C})$ has an open neighborhood on which this proper homotopy is stationary.

Removing an outermost disk:



5. Mapping each component of $f^{-1}(\mathscr{C})$ onto a component of $\mathscr{C}$ homeomorphically

Being an isomorphism, $\pi_1(f)$ preserves primitiveness. Therefore, f can be properly homotoped to send each component $\mathcal{C}'$ of $f^{-1}(\mathscr{C})$ homeomorphically onto a component $\mathcal{C}$ of $\mathscr{C}$ so that the restriction of f to a small one-sided tubular neighborhood $\mathcal{C}' \times [1,2]$ of $\mathcal{C}'$ (on either side of $\mathcal{C}'$) can be described by the following homeomorphism:

$$\mathcal{C}' \times [1,2] \ni (z,t) \longmapsto \big(f(z),t\big) \in \mathcal{C} \times [1,2].$$

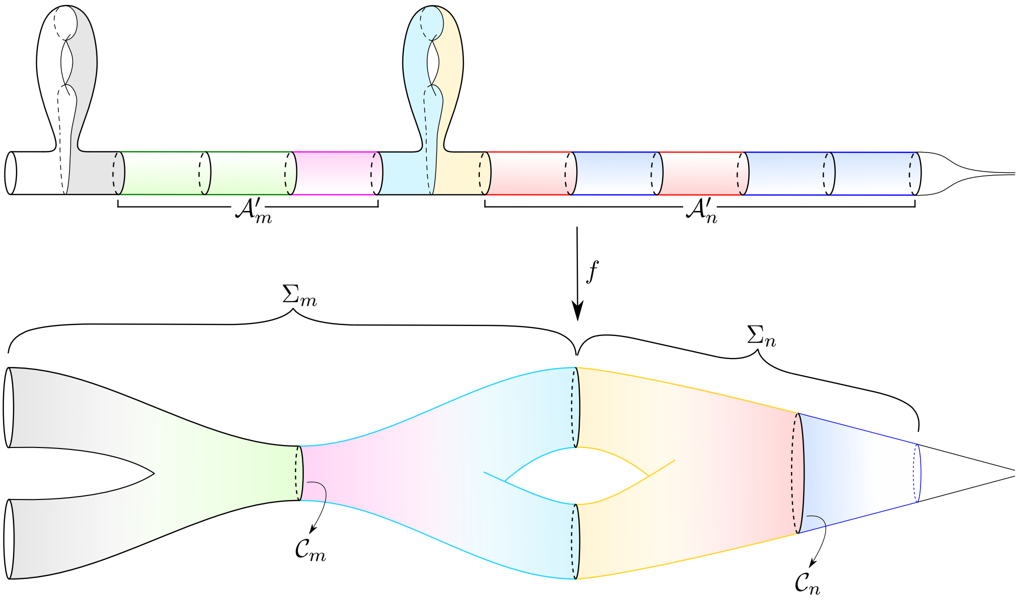
6. Compressing all outermost annuli simultaneously

Since f has (ordinary) homotopy left-inverse, for any component $\mathcal{C}$ of $\mathscr{C}$, any two components of $f^{-1}(\mathcal{C})$ co-bound an annulus in Σ'. The outermost annulus for each component $\mathcal{C}$ of $\mathscr{C}$ is the biggest annulus $\mathcal{A}'$ in Σ' co-bounded by two components of $f^{-1}(\mathcal{C})$.

Therefore, f can be properly homotoped so that

- the homotopy is relative to the boundary of each outermost annulus and
- after the homotopy, f sends each outermost annulus to a component of $\mathscr{C}$.

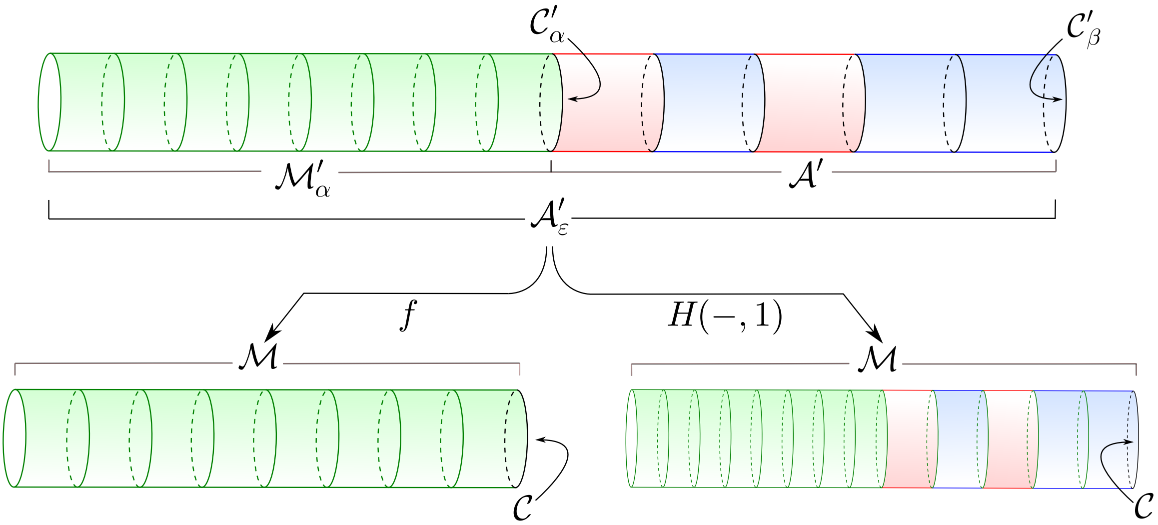
Compressing two outermost annuli:



7. Removing all outermost annuli simultaneously

Now, properly homotope f to push each outermost annulus $\mathcal{A}'$ to an one-sided small tubular neighborhood $\mathcal{M}'_\alpha$ of a component $\mathcal{C}'_\alpha$ of $\partial\mathcal{A}'$ such that for each component $\mathcal{C}$ of $\mathscr{C}$, either $f^{-1}(\mathcal{C})$ is empty or $f\big|f^{-1}(\mathcal{C}) \to \mathcal{C}$ is a homeomorphism.

Removing an outermost annulus:



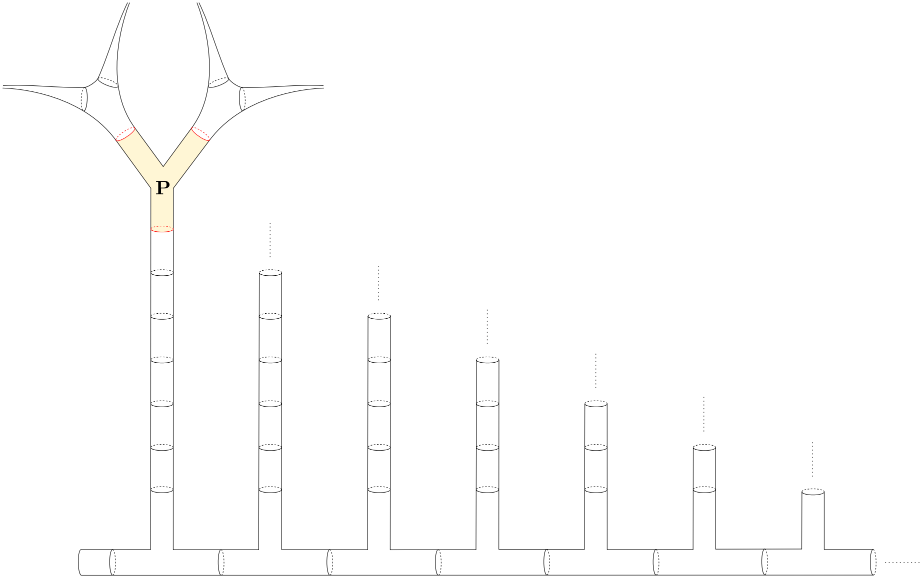
8. Essentail pair of pants in the codomain

If $\mathbf{P}$ is a pair of pants in Σ such that $\Sigma \setminus \mathbf{P}$ has at least two components and every component of $\Sigma \setminus \mathbf{P}$ has a non-abelian fundamental group; then

$$g^{-1}(\operatorname{int} \mathbf{P}) \neq \varnothing \neq g^{-1}(\mathbf{C}),$$

where $\mathbf{C}$ is any component of $\partial \mathbf{P}$ and g is properly homotopic to f.

Essential pair of pants in a planar surface with infinitely many isolated ends:



9. Showing $\deg(f) = \pm 1$ using bijectivity of $\pi_1(f)$

After a proper homotopy, we can assume that $f|f^{-1}(\partial \mathbf{P}) \to \partial \mathbf{P}$ is a homeomorphism.

Now, the topological rigidity of compact bordered surfaces applied on the π_1-bijective map $f|f^{-1}(\mathbf{P}) \to \mathbf{P}$ says that $f\colon \Sigma' \to \Sigma$ can be properly homotoped relative to $\Sigma' \setminus \operatorname{int} f^{-1}(\mathbf{P})$ to map $f^{-1}(\mathbf{P})$ homeomorphically onto $\mathbf{P}$. Therefore, $\deg(f) = \pm 1$.

10. Finishing the proof of strong topological rigidity

Since a map of non-zero degree is surjective, after a proper homotopy, we may assume that $f|f^{-1}(\mathcal{C}) \to \mathcal{C}$ is a homeomorphism for every component $\mathcal{C}$ of $\mathscr{C}$.

For a bordered sub-surface $\mathbf{S}$ of Σ obtained as a complementary component of the decomposition of Σ by $\mathscr{C}$, use the topological rigidity of compact bordered surfaces and the Alexander trick to properly homotope $f\big|f^{-1}(\mathbf{S}) \to \mathbf{S}$ relative to $\partial f^{-1}(\mathbf{S})$ to a homeomorphism $h_{\mathbf{S}} \colon f^{-1}(\mathbf{S}) \to \mathbf{S}$.

Paste all $h_{\mathbf{S}}$ to get a homeomorphism properly homotopic to f.

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