Report on APPLICATION OF FINITE ELEMENT METHOD IN DYNAMIC EIGENVALUE ANALYSIS in TMHL62

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1. Introduction

It is crucial to note that resonant vibrations can occur in various types of rotating machines, which can result in significant issues. These vibrations arise when the eigenfrequency of a component coincides with a multiple of the machine's rotational speed.To prevent resonance, it is essential to ensure that the eigenfrequencies do not align with any of the vibrational frequencies that are caused by the lower most multiples of the rotational speed.In this task we are supposed to compute the lower-most eigenfrequencies using a very simplified geometry of the first blade in the compressor of the GT10C gas turbine which was manufactured by ALSTOM POWER in Finspång, Sweden.We are advised to make slight alterations to the blade's thickness, as long as they do not increase the root stress caused by centrifugal loads. However, any changes in the blade's geometry should not involve uniformly altering the thickness of the entire blade.

2. Subtask 1: Theoretical explanation

In dynamic eigenvalue analysis using model decomposition, we decompose the original dynamic system into smaller subsystems or modes, each of which can be analyzed separately. This is done by expressing the displacement field of the structure as a linear combination of a set of basis functions or mode shapes, each of which oscillates at a distinct frequency. This set of mode shapes and frequencies is obtained by solving the generalized eigenvalue problem:

$$\left(\vec{K} - \omega_i^2 \vec{M}\right) \vec{\Psi}_i = 0 \tag{1}$$

where $\vec{K}$ is the stiffness matrix, $\vec{M}$ is the mass matrix, $\vec{\Psi}$ is the eigen vector, and ω is the eigenvalue.

By solving this eigenvalue problem, we obtain a set of eigenvalues and eigenvectors that describe the dynamic behavior of the structure. The eigenvectors represent the mode shapes, or the way the structure deforms at each natural frequency, and the eigenvalues represent the natural frequencies. The stiffening effect of the centrifugal force can be explained by considering the pre-tension it creates in the body. To include this effect in a modal analysis, the geometric stiffness can be incorporated.

$$\left[\vec{K} + \vec{K_\sigma}\left(\vec{\sigma_o}\right) - \omega_i^2 \vec{M}\right] \vec{\Psi}_i = 0 \tag{2}$$

Where, $\vec{K_\sigma}$ is stress stiffening.

2.1 Analytical solution

In this task, I calculated a flat rectangular plate fixed along one edge and free along the others.

The equations are used here are KTH Handbook of formulas.
Data:
Young's modulus= 103 GPa
Density= 4540
Poisson's ratio= 0.22
Thickness= 10 mm
Width= 220 mm

$$D = \frac{Et^3}{12\left(1 - v^2\right)} \tag{3}$$

$$\omega = \frac{\lambda}{a^2}\left(\sqrt{\frac{D}{\rho t}}\right) \tag{4}$$

$$f = \frac{\omega}{2\pi} \tag{5}$$

Where, D is the plate stiffness,f is the frequency, E is the youngs modulus, v is the poisson's ratio, t is the thickness, a is the width of the plate and ρ is the density.
The following outcomes were derived from an analytical solution,

Table 1: Results of analytical solution

Mode	**Frequency**
1	161.9
2	396.2
3	993.7
4	1272.8

The FEM analysis for the same rectangular flat yielded the subsequent results.

Table 2: Results of FEM analysis

Mode	Frequency
1	162.02
2	404.61
3	996.09
4	1255.9

The results obtained from the FEM analysis were found to be in agreement with those obtained from analytical solution, indicating a accuracy and consistency between the two methods.

2.2 Blade Analysis

The figure below shows the simplified geometry of a blade Fig. 1

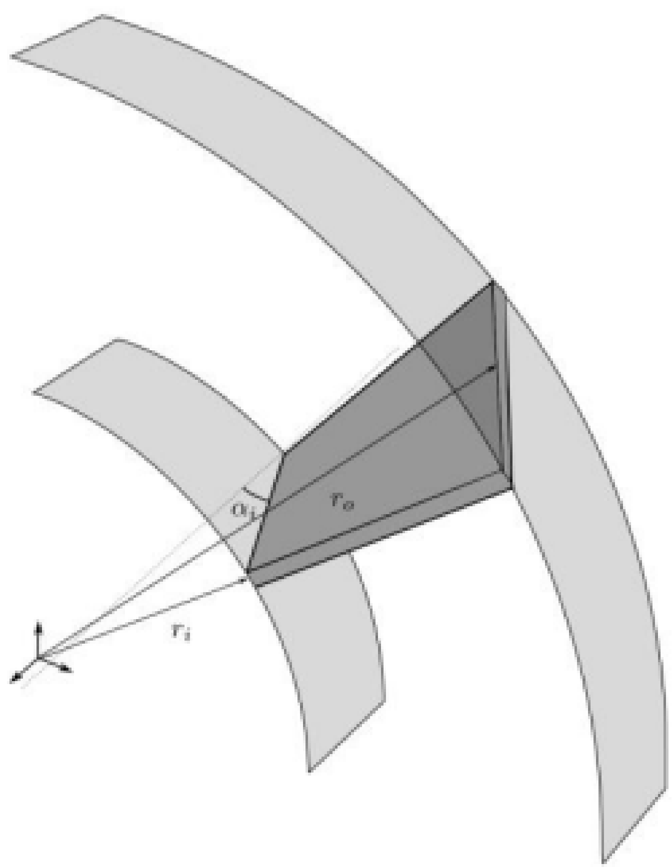

Figure 1: Simplified geometry of the blade

The blade is manufactured in the Titanium alloy and as inner radius of r_i=220 mm , an outer radius r_o= 440 mm , and a uniform width b= 100 mm . The blade is twisted at an angle α_i=20° with respect to the axial direction at the inner radius and α_o=36° at the outer radius, with all edges being straight lines. The initial thickness of the blade is assumed to be constant at 10 mm. An eigenvalue analysis was conducted on the blade across a range of five rotational speeds, spanning from 0 to 9600 rpm. For this particular task, I have selected five specific rotational speeds, namely 9500 rpm, 8600 rpm, 6600 rpm, 3500 rpm, and 2050 rpm.

2.3 Blade with constant thickness

In this task, firstly the blade with constant thickness of 10mm was subjected to analysis as shown in Fig. 2, and the following findings were obtained as shown in Table-3.

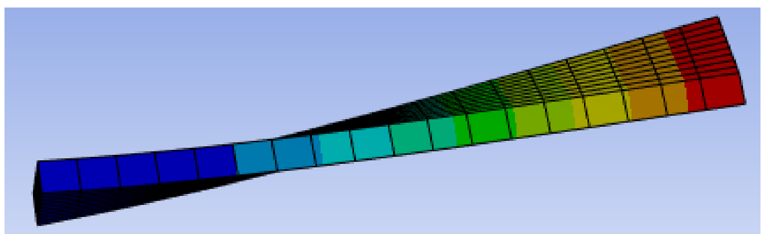

Figure 2: Blade with constant thickness

Table 3: Results of blade with constant thickness

RPM	rad/s	Mode 1	Mode 2	Mode 3	Mode 4
9500	994.85	269.59	804.09	1081.3	1506.6
8600	900.58	253.61	797.95	1057.6	1499
6600	691.15	220.38	786.33	1010.7	1485.7
3500	366.51	179.16	774.15	958.36	1472.7
2050	214.67	166.79	770.98	944.21	1469.5

A slight modification was made to the blade's thickness by 14 mm to the inner radius while adjusting the outer radius to 6 mm. The new analysis was conducted as shown in Fig. 3 and following results were obtained shown in Table-4

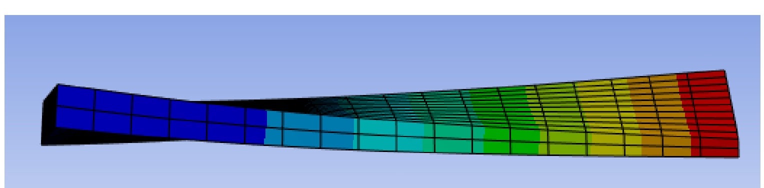

Figure 3: Blade with variable thickness

Table 4: Results of blade with variable thickness

RPM	rad/s	Mode 1	Mode 2	Mode 3	Mode 4
9500	994.85	338.55	973.9	1196.4	1824.6
8600	900.58	324.27	968.75	1175.6	1820.4
6600	691.15	295.51	959.06	1135.4	1812.5
3500	366.51	262.13	948.96	1092.2	1804.5
2050	214.67	252.74	946.35	1080.8	1802.5

2.4 Discussion

In the blade with constant thickness, the resonance occurred at the chosen RPM as shown below in Fig. 4. This suggest that the natural frequency of the blade at that RPM was close to the excitation frequency caused by the rotation of the blade.This is a problematic because it will lead to excessive vibrations and potentially cause damage or failure of the blade. By contrast, the blade with variable thickness did not exhibit resonance, which indicates that its natural frequency was higher than the excitation frequency as shown below in Fig. 5.This suggests that the variable thickness design was more effective at avoiding resonance.After analyzing the data presented in the table, it is evident that there is a significant difference in the frequencies of the blades, despite having identical RPM values for both constant and variable thickness blades. This difference in frequency can be attributed to the varying thickness across the length of the blade, which gives it an aerodynamic shape(Note: In current design i have not considered all the aspects of aerodynamics designing,however,I have changed the thickness of blade which gives better results than the constant thickness).A constant thickness blade maintains a uniform thickness throughout its length this can result in a reduced efficiency and increased drag forces, leading to a lower frequency of the blade despite the same RPM value.

The blade thickness has been increased near the root of the blade to provide additional stiffness and support . In contrast, the thickness has been reduced towards the tip of the blade, as shown in Fig. 3. The Tables-3 and Table-4 indicate that modifying the thickness of the blade resulted in an increase in stiffness. As stiffness and natural frequency are directly proportional to each other from equation-3, the natural frequency of the blade increased.In this case of a blade, the centrifugal force generated by its rotation causes tension stress to be developed in the blade, which leads to stress stiffening. This stress stiffening results in an increase in the stiffness of the blade, which leads to an increase in its natural frequencies.

The mass distribution can have a significant impact on the natural frequency of the blade. This is because the natural frequency is inversely proportional to the square root of the mass per unit length of the blade. The total mass of the blade remains the same for both the constant thickness and variable thickness designs. However, the distribution of mass along the centroid of x, y, and z differs between the two designs.

In the case of constant thickness design has a uniform thickness throughout the blade, which can result in a less optimized mass distribution. This can lead to a less balanced blade and potentially higher vibrational amplitudes at certain modes, as observed in the Campbell diagram. On other hand the variable thickness design, the mass distribution is optimized to achieve a desired natural frequency and mode shape by varying the thickness of the blade along its length.

2.5 Campbell diagrams

The below case shows the campbell diagram of the blade with constant thickness as shown in Fig. 4

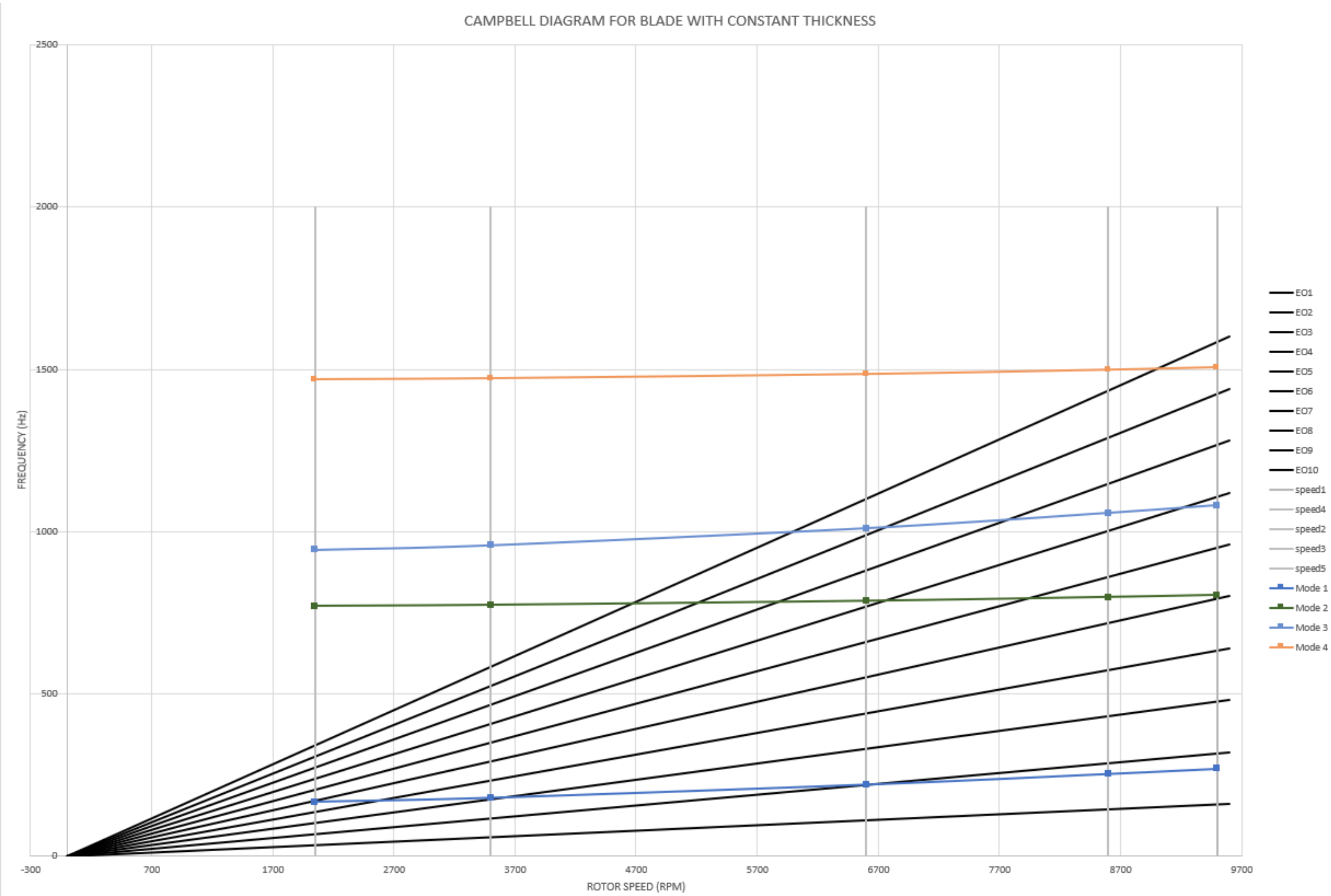

Figure 4: Campbell diagram for blade with constant thickness

The below case shows the campbell diagram of the blade with variable thickness as shown in Fig. 5

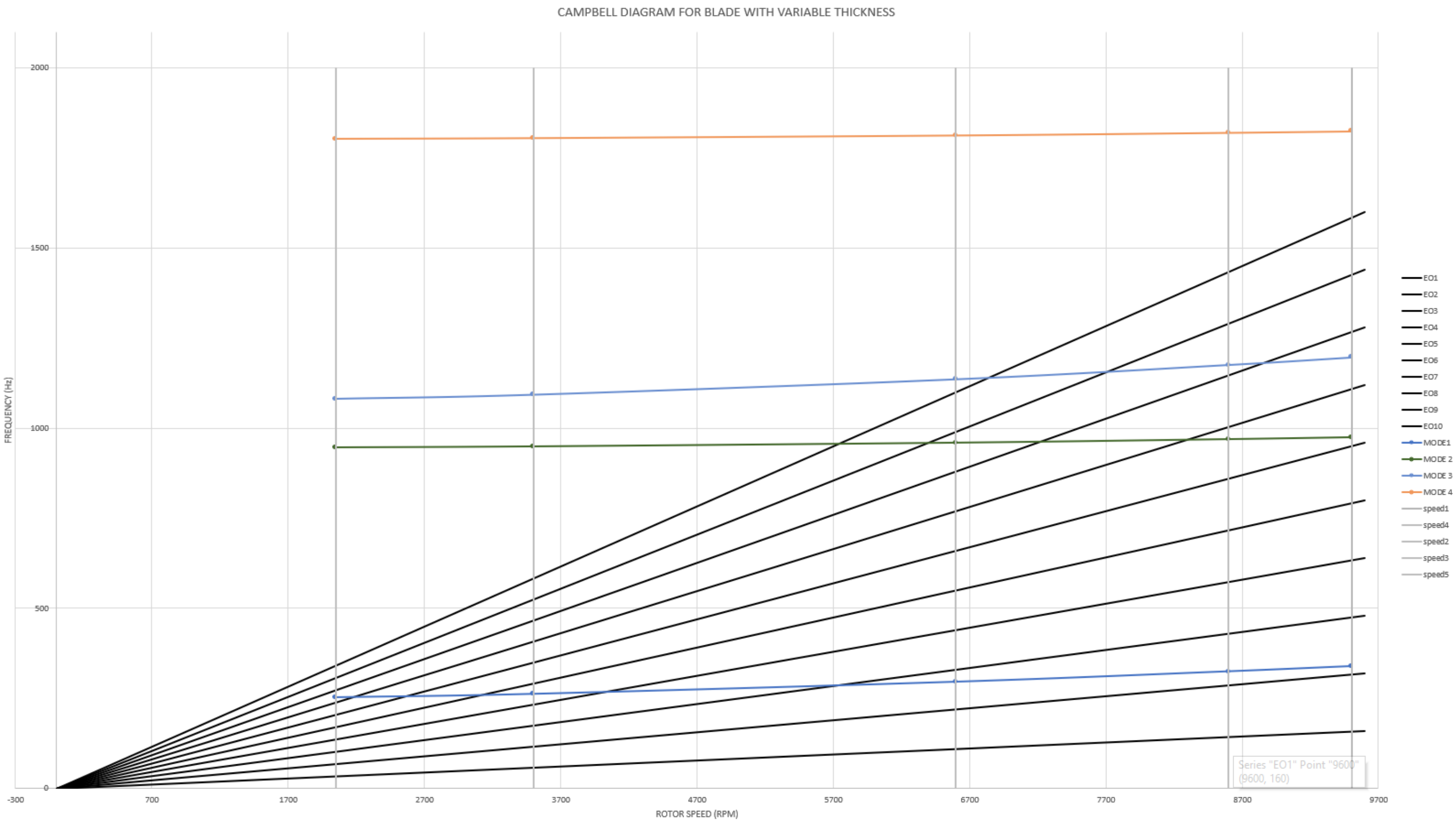

Figure 5: Campbell diagram for blade with variable thickness

References

(1) KTH Handbook formulas
(2) SIEMENS Basic Dynamic Analysis User's Guide
https://docs.plm.automation.siemens.com/data$_{s}ervices/resources/nxnastran/12/help/tdoc/en_{U}S/pdf/$