

Chapter 33

Math Practice Sections

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Answers to Math Mixed Mini Sets

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Answers to Math Practice Section 1

Math Practice Section 2: Medium Difficulty

Answers to Math Practice Section 2

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Math Mixed Mini Sets

Use these mini mixed sets to practice your timing and decision-making. Treat them like half sections, and complete each one in 17.5 minutes before reviewing your work. The sets are designed to contain a realistic mix of questions, spanning different topics, levels, and question types.

Quant Set 1

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

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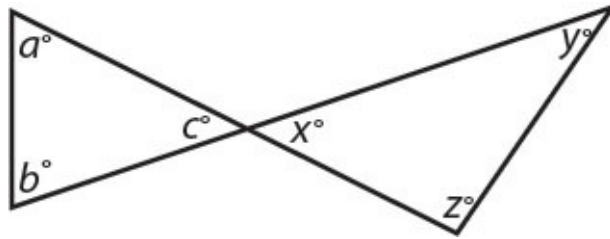
One worker strings 2 violins in 3 minutes. All workers string violins at the same constant rate.

Quantity A

- The number of minutes required for
1. 12 workers to string 720 violins

Quantity B

- The number of violins that 5
- workers can string in 24 minutes
-



- | | | | |
|----|--------------------------|--|--------------------------|
| | <u>Quantity A</u> | | <u>Quantity B</u> |
| 2. | $a + b + x$ | | $c + y + z$ |

The positive integer a is divisible by 2 and $0 < ab < 1$.

- | | | | |
|----|--------------------------|--|--------------------------|
| | <u>Quantity A</u> | | <u>Quantity B</u> |
| 3. | b | | $\frac{1}{2}$ |

In a normally distributed set of data, the mean is 12 and the standard deviation is less than 3.

- | | | | |
|----|---|--|--|
| | <u>Quantity A</u> | | <u>Quantity B</u> |
| 4. | The number of data points in the set between 9 and 15 | | 60% of the total number of data points |

5. In a certain box of cookies, $\frac{3}{4}$ of all the cookies have nuts and $\frac{1}{3}$ of all the cookies have both nuts and fruit. What fraction of all the cookies in the box have nuts but no fruit?

- (A) $\frac{1}{4}$
- (B) $\frac{5}{12}$
- (C) $\frac{1}{2}$

(D) $\frac{7}{12}$

(E) $\frac{5}{6}$

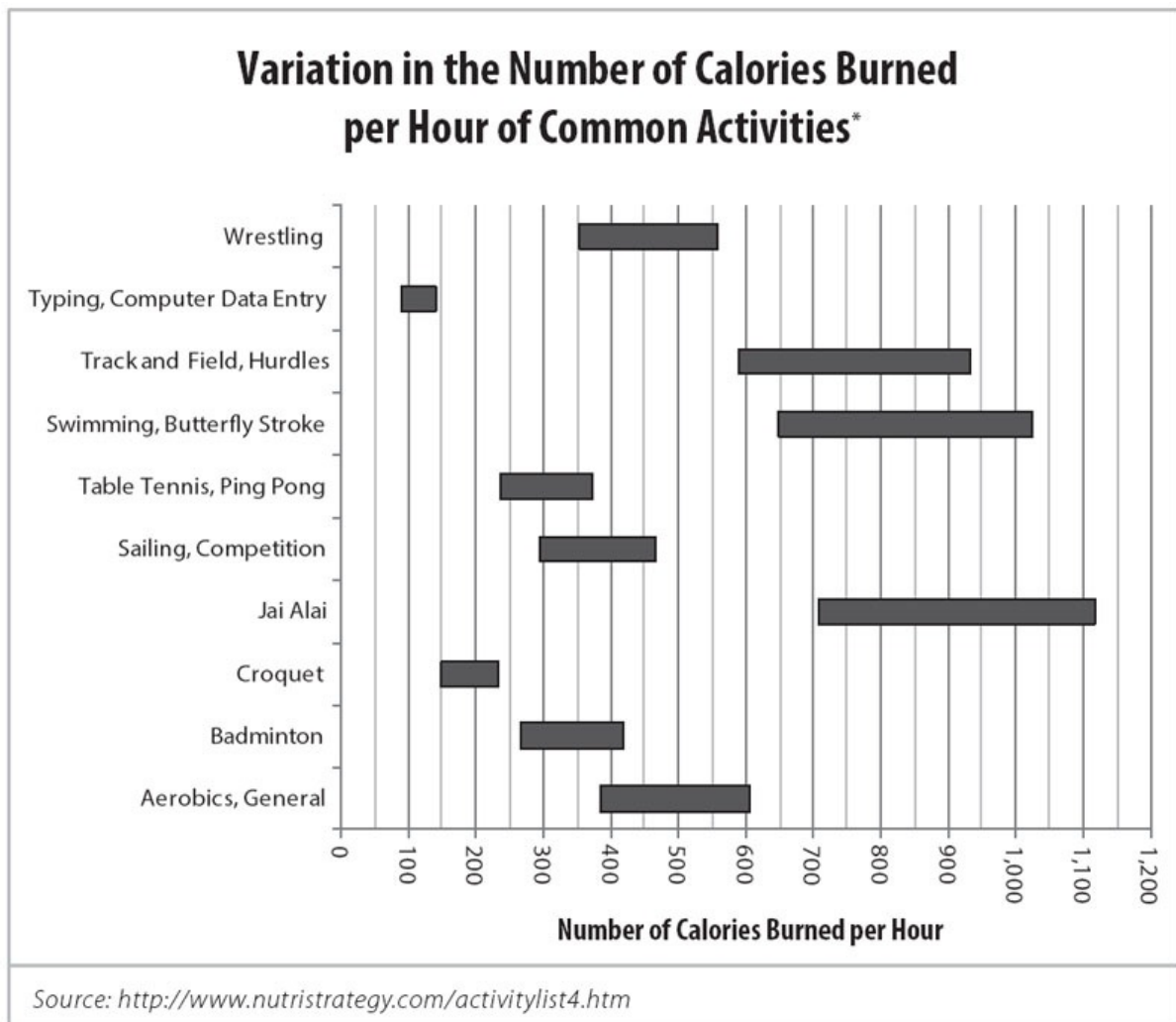
6. If $b \neq 0$ and $\frac{a}{b} > 0$, which of the following inequalities must be true?

Indicate all such inequalities.

☐ $a > b$

☐ $b > 0$

☐ $ab > 0$



* Based on body weight of exercise subject. The lower limit represents the calories burned by a person weighing 130 pounds, while the upper limit represents the calories burned by a person weighing 205 pounds.

7. Which of the following statements could be true?

Indicate all such statements.

- ☐ A person weighing between 130 and 205 pounds performs one of the above activities for 10 hours yet burns fewer calories than another person in the same weight range performing another activity for 1 hour.
- ☐ A 175-pound person playing jai alai for 1 hour burns fewer calories than a 180-pound person swimming the butterfly stroke for 1 hour.
- ☐ If all the people in question weigh between 130 and 205 pounds, the average calories burned by one person playing table tennis for 1 hour is more than the total calories burned by two people typing for 3 hours.

8. Which combination of activities burns the fewest calories total?

- (A) A 130-pound person playing badminton for 1 hour and a 205-pound person playing table tennis for 1 hour
- (B) A 130-pound person wrestling for 1 hour and a 205-pound person running track and field, hurdles for 1 hour
- (C) A 130-pound person typing for 1 hour and a 205-pound person swimming the butterfly stroke for 1 hour
- (D) A 130-pound person sailing in a competition for 1 hour and a 205-pound person doing aerobics for 1 hour
- (E) A 130-pound person typing for 1 hour and a 205-pound person playing croquet for 1 hour

9. A circle has a circumference of 16. What is its area?

- (A) $\frac{8}{\pi}$
- (B) $\frac{8}{\pi^2}$
- (C) $\frac{64}{\pi}$
- (D) $\frac{64}{\pi^2}$

(E) 64π

10. If $y = 4x + 10$ and $y = 7x - 5$, what is the value of y ?

Quant Set 2

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

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(B) Quantity B is greater.

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At a certain college, the ratio of students to professors is 8 : 1 and the ratio of students to administrators is 5 : 2. No person is in more than one category (for instance, there are no administrators who are also students).

<u>Quantity A</u>	<u>Quantity B</u>
1. The fractional ratio of professors to administrators	$\frac{5}{8}$

<u>Quantity A</u>	<u>Quantity B</u>
2. The number of ways 1st, 2nd, and 3rd place prizes could be awarded to 3 out of 6 contestants	The number of ways 1st, 2nd, 3rd, 4th, and 5th place prizes could be awarded to 5 contestants

3. If $x \neq -y$, what is the value of $\frac{x^2 + 2xy + y^2}{2(x + y)^2}$?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{x + y}$

(D) xy

(E) $2xy$

4. If $x^2 = y^2$, which of the following must be true?

☐ $x = y$

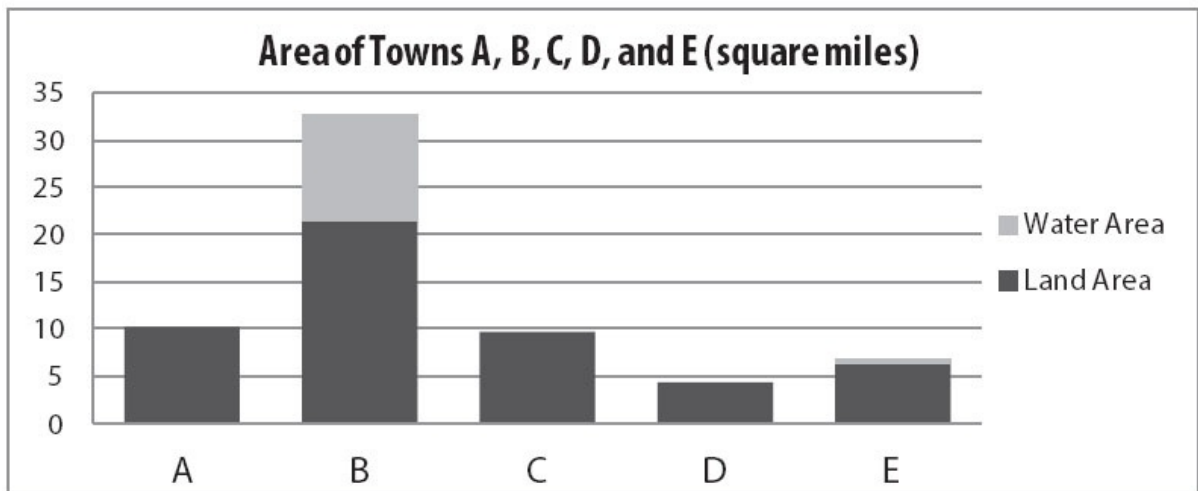
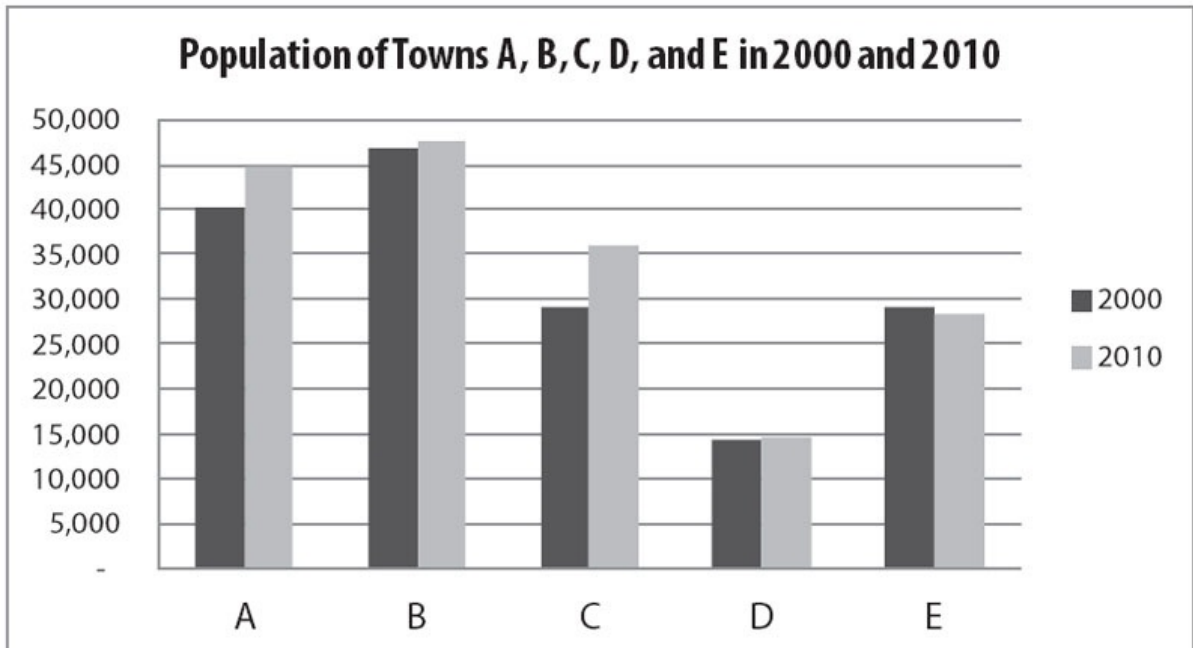
☐ $x^2 - y^2 = 0$

☐ $|x| - |y| = 0$

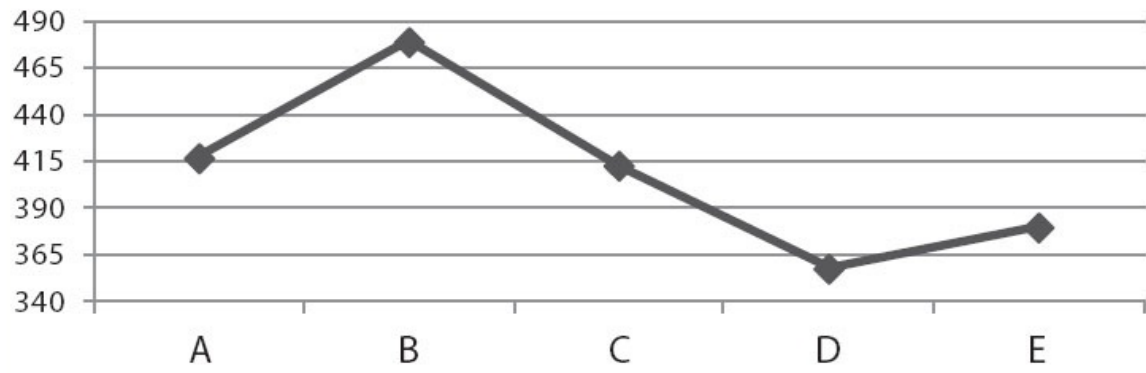
5. A 10-foot ladder leans against a vertical wall and forms a 60-degree angle with the floor. If the ground below the ladder is horizontal, how far above the ground is the top of the ladder?

(A) 5 feet

- (B) $5\sqrt{3}$ feet
- (C) 7.5 feet
- (D) 10 feet
- (E) $10\sqrt{3}$ feet



**Elevation (feet above sea level)
of Towns A, B, C, D, and E**



6. In what town did the population increase by the greatest percent between 2000 and 2010?

- (A) Town A
- (B) Town B
- (C) Town C
- (D) Town D
- (E) Town E

7. The water area of town B is most nearly equal to the sum of the land areas of which two towns?

Indicate two such towns.

- ☐ Town A
- ☐ Town B
- ☐ Town C
- ☐ Town D
- ☐ Town E

8. Which two towns have approximately the same elevation in feet above sea level?

Indicate two such towns.

- ☐ Town A
- ☐ Town B
- ☐ Town C
- ☐ Town D
- ☐ Town E

9. If x is odd, all EXCEPT which one of the following must be odd?

- (A) $x^2 + 4x + 6$
- (B) $x^3 + 5x + 3$
- (C) $x^4 + 6x + 7$
- (D) $x^5 + 7x + 1$
- (E) $x^6 + 8x + 4$

10. If a right circular cylinder's radius is halved and its height doubled, by

what percent will the volume increase or decrease?

- (A) 50% decrease
- (B) 0%
- (C) 25% increase
- (D) 50% increase
- (E) 100% increase

Quant Set 3

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Quantity A

$$\sqrt{3} + \sqrt{6}$$

Quantity B

$$\sqrt{9}$$

1.

The corner store sells yams and plantains by the pound. A pound of plantains cost \$0.30 less than twice the cost of a pound of yams.

	<u>Quantity A</u>	<u>Quantity B</u>
2.	The cost of two pounds of yams and two pounds of plantains	The cost of three pounds of plantains

In the coordinate plane, lines j and k are parallel and the product of their slopes is positive.

The x -intercept of line j is greater than the x -intercept of line k .

	<u>Quantity A</u>	<u>Quantity B</u>
3.	The y -intercept of line j	The y -intercept of line k

	<u>Quantity A</u>	<u>Quantity B</u>
4.	The standard deviation of the dataset 10, 20, 30	The standard deviation of the dataset 10, 20, 20, 20, 20, 20, 30

Number of Hours Worked per Week per Employee at Marshville Toy Company

# of Employees	Hours Worked Per Week
4	15
9	25
15	35
27	40
5	50

5. What is the median number of hours worked per week per employee at Marshville Toy Company?

- (A) 25
- (B) 30
- (C) 35
- (D) 37.5
- (E) 40

6. What is the average (arithmetic mean) number of hours worked per week per employee at Marshville Toy Company?

(A) 32

(B) 33

(C) 35

(D) $35\frac{2}{3}$

(E) $36\frac{1}{3}$

7. What is the positive difference between the mode and the range of the number of hours worked per week per employee at Marshville Toy Company?

(A) 0

(B) 4

(C) 5

(D) 8

(E) 26

8. If $3^2 9^2 = 3^x$, what is the value of x ?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

9. $\frac{1}{4}$ of all the juniors and $\frac{2}{3}$ of all the seniors of a certain school are going

on a trip. If there are $\frac{2}{3}$ as many juniors as seniors, what fraction of the junior and senior students are not going on the trip?

(A) $\frac{4}{9}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

(E) $\frac{5}{6}$

10. What is the value of $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$?

Give your answer as a fraction.

Quant Set 4

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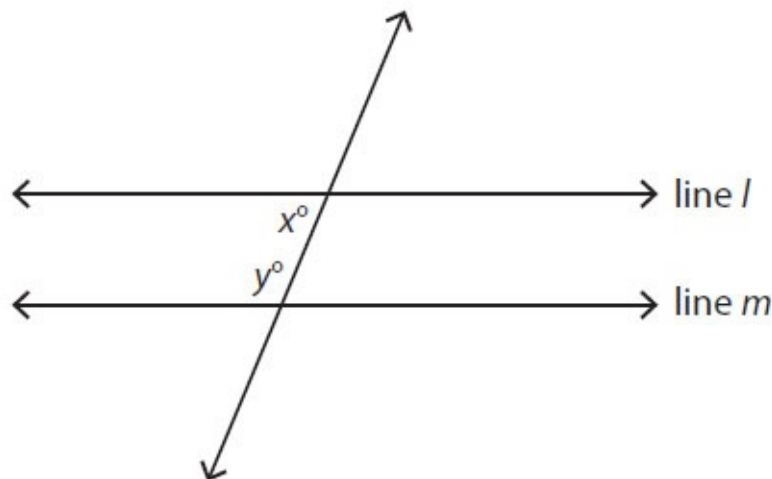
Quantity A

The number of possible 4-person teams that can be selected from 6 people

1.

Quantity B

The number of possible 2-person teams that can be selected from 6 people



Lines l and m are parallel.

Quantity A

2.

$$x + 2y$$

Quantity B

$$180$$

The average (arithmetic mean) weight of 4 people is 85 kilograms. Two of the people weigh 75 and 90 kilograms, respectively.

Quantity A

The average (arithmetic mean) weight of the other two people, in kilograms

3.

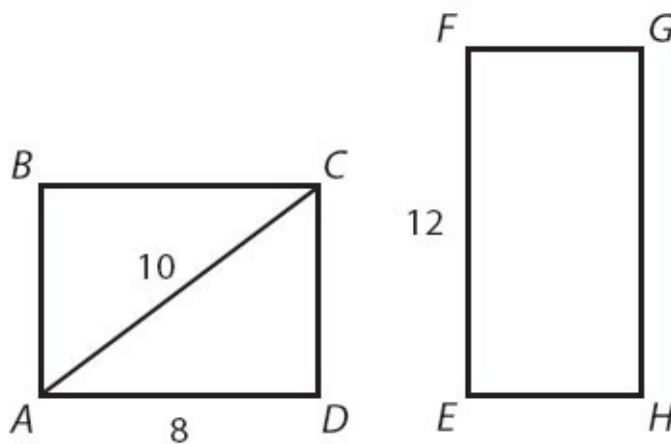
Quantity B

85 kilograms

4. If $x^2 - y^2 = 0$ and $xy \neq 0$, which of the following must be true?

Indicate all such statements.

- ☐ $x = y$
☐ $|x| = |y|$
☐ $\frac{x^2}{y^2} = 1$



5. Rectangles $ABCD$ and $EFGH$ have equal areas. What is the length of side FG ?

	Change in Total Revenue (2011–2012)	Percent Change in Number of Distinct Customers (2011–2012)	Percent Change in Total Costs (2011–2012)
Store W	–\$400,000	+2%	+15%
Store X	+\$520,000	+14%	+4%
Store Y	–\$365,000	+5%	+12%
Store Z	+\$125,000	–7%	–20%

6. For which store was the revenue per distinct customer greatest in 2012?

- (A) Store W
 (B) Store X
 (C) Store Y
 (D) Store Z

- (E) It cannot be determined from the information given.
7. Between 2011 and 2012, total costs per distinct customer increased by the greatest percent at which store?
- (A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(D) It cannot be determined from the information given.
8. At which of the following stores could the profit in 2012 have been less than that same store's profit in 2011?

Indicate all such stores.

- ☐ Store W
☐ Store X
☐ Store Y
☐ Store Z
☐ None of the above

9. Which of the following statements must be true?
- (A) Of the four stores, store X had the greatest percent increase in revenue from 2011 to 2012.
(B) Per customer revenue increased at store Z from 2011 to 2012.
(C) Of the four stores, store W had the greatest increase in total costs from 2011 to 2012.
(D) Of the four stores, store Y had the highest percentage of repeat customers.
(E) In 2012, store W and store Z combined had fewer distinct customers than did store X.

10. If \sqrt{x} is an integer and $xy^2 = 36$, how many values are possible for the integer y ?
- (A) Two
(B) Three
(C) Four

- (D) Six
(E) Eight

Answers to Mixed Mini Sets

Quant Set 1

1. **(A).** First, figure out the individual rate for 1 worker: $2 \text{ violins} \div 3 \text{ minutes}$
 $= \frac{2}{3}$ violin per minute. (Always divide work by time to get a rate.) Now
apply $W = RT$ separately to Quantity A and Quantity B.

Quantity A:

$$R = 12 \times \text{the individual rate} = 12 \times \frac{2}{3} = 8 \text{ violins per minute}$$

$$W = 720 \text{ violins}$$

Solve for T in $W = RT$:

$$720 = 8T$$

$$90 = T$$

Quantity B:

$$R = 5 \times \text{the individual rate} = 5 \times \frac{2}{3} = \frac{10}{3} \text{ violins per minute}$$

$$T = 24 \text{ minutes}$$

Solve for W in $W = RT$:

$$W = \left(\frac{10}{3} \right) (24)$$

$$W = 80$$

Since $90 > 80$, Quantity A is greater.

2. **(C).** Since c and x are vertical angles, they are equal. This means their positions in the quantities can be switched to put all the angles in the same triangle together:

Quantity A

$$a + b + c$$

Quantity B

$$x + y + z$$

The three angles inside a triangle sum to 180° , so the two quantities are equal.

3. **(B).** If the positive integer a is divisible by 2, it is a positive even integer. Thus, the minimum value for a is 2. Therefore, since $ab < 1$, b must be less than $\frac{1}{2}$.

4. **(A).** If the standard deviation is 3, then 1 standard deviation below the mean is 9 and 1 standard deviation above the mean is 15, so about 68% of the data is between 9 and 15 (in a normal distribution, it is always the case that about 68% of the data is within 1 standard deviation of the mean).

Since the *actual* standard deviation is *less than* 3, about 68% of the data is found within an *even smaller range* than 9 to 15. For instance, the standard deviation could be 1, and then about 68% of the data would be between 11 and 13. Or the standard deviation could be 2.5, and then about 68% of all the data would be found between 9.5 and 14.5.

Since 68% of the data is found within an *even smaller range* than 9 to 15, the range from 9 to 15 contains *more than* 68% of the data, so it definitely contains more than 60% of all the data points.

Don't be confused by the use of "number of data points." While the actual total number of data points is still unknown, it is definitely true that Quantity A is equal to a larger percent of that total than is Quantity B.

5. **(B).** Since $\frac{3}{4}$ of the cookies have nuts and $\frac{1}{3}$ of the cookies *also* have fruit, subtract $\frac{3}{4}$ from $\frac{1}{3}$ to get all the cookies with nuts but no fruit:

$$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

Alternatively, pick numbers. Since you will be dividing by 4 and 3, pick a number divisible by 4 and 3. If there are 12 cookies, then 9 have nuts and 4 of

them have nuts *and* fruit, so 5—and thus $\frac{5}{12}$ of the total—would have nuts but no fruit.

6. **$ab > 0$ only.** If $\frac{a}{b} > 0$, then both a and b must have the same sign. That is, a and b are either both positive or both negative. The 1st inequality could be true, but is not necessarily true. The relative values of a and b are not indicated by the inequality in the question stem. The 2nd inequality could be true, but is not necessarily true. If a were negative, b could be negative. The 3rd inequality must be true, as it indicates that a and b have the same sign.

7. **1st and 2nd statements only.** Consider each statement individually.

The first statement could be true. A 130-pound person typing for 10 hours would burn less than 1,000 calories, which is less than the number of calories burned by a 205-pound person doing one of several activities on the chart for 1 hour (certainly jai alai and swimming the butterfly stroke burn more than 1,000 calories).

The second statement could be true. In general, the range of calories burned per hour is greater for jai alai than for swimming the butterfly stroke. The people in question are about the same weight, but it cannot be assumed that the number of calories burned is a function of weight in this range or that the relationship is linear. All that matters is that the calorie burning ranges for the two activities overlap, and both people fall in the weight range, so it could be true that a 175-pound person playing jai alai for 1 hour burns fewer calories than a 180-pound person swimming the butterfly stroke for 1 hour.

The third statement must be false. The average calories burned by one person playing table tennis for 1 hour is a maximum of about 375. Two people typing for 3 hours burn as many calories total as one person typing for $2 \times 3 = 6$ hours, which is a minimum of about 550. "At most 375" cannot be greater than "at least 550."

8. **(E).** For each combination of activities, look at the minimum value on the chart for the 130-pound person and the maximum value on the chart for the 205-pound person:

$$(A) \text{ Badminton (minimum) + Table tennis (maximum) } = 275 + 375 = 650$$

$$(B) \text{ Wrestling (minimum) + Track and field, hurdles (maximum) } = 350 + 925 = 1,275$$

$$(C) \text{ Typing (minimum) + Swimming, butterfly stroke (maximum) } = \text{under } 100 + 1025 = \text{under } 1,125$$

$$(D) \text{ Sailing, competition (minimum) + Aerobics (maximum) } = 300 + 600 = 900$$

$$(E) \text{ Typing (minimum) + Croquet (maximum) } = \text{under } 100 + \text{over } 225 = \text{about } 325, \text{ CORRECT}$$

Alternatively, note that typing and croquet are the two activities that burn the fewest calories per hour overall. A 130 pound person and a 205-pound person each doing 1 hour of an activity shown on the chart would only burn fewer calories total if both people were typing.

9. **(C).** Since the circumference formula is $C = 2\pi r$:

$$16 = 2\pi r$$

Note that the circumference is just 16, not 16π , so the radius is going to look a bit unusual. Divide both sides by 2π to solve for r :

$$\frac{16}{2\pi} = r$$

$$\frac{8}{\pi} = r$$

Now, plug the radius $\frac{8}{\pi}$ into the area formula for a circle:

$$A = \pi \left(\frac{8}{\pi} \right)^2$$

$$A = \pi \times \frac{64}{\pi^2}$$

$$A = \frac{64}{\pi}$$

10. **30.** Since each equation is already solved for y , set the right side of each equation equal to the other.

$$4x + 10 = 7x - 5$$

$$10 = 3x - 5$$

$$15 = 3x$$

$$5 = x$$

Quant Set 2

1. **(B).** One way to approach this problem is to pick a smart number for the number of students, which shows up in both ratios. In the first ratio, students are represented by 8, so you want the smart number of students to be a multiple of 8. Likewise, in the second ratio, students are represented by 5, so you want the smart number of students to be a multiple of 5 as well. So pick 40 for the number of students.

From here, solve for the number of professors:

$$\frac{\text{Students}}{\text{Professors}} = \frac{40}{\text{Professors}} = \frac{8}{1}$$

$$40 = 8 \times \text{Professors}$$

$$5 = \text{Professors}$$

Likewise, solve for the number of administrators:

$$\frac{\text{Students}}{\text{Administrators}} = \frac{40}{\text{Administrators}} = \frac{5}{2}$$

$$40 \times 2 = 5 \times \text{Administrators}$$

$$16 = \text{Administrators}$$

Therefore, the ratio of professors to administrators is 5 : 16. In fractional ratio form, this is $\frac{5}{16}$. Comparing the two quantities, both have the same

numerator, but the denominator of Quantity A is greater, making it the smaller value. In fact, Quantity B is exactly twice as great as Quantity A.

2. **(C)**. In this problem, order matters; if Mari comes in 1st place and Rohit comes in 2nd, there is a different outcome than when Rohit places 1st and Mari places 2nd. Use the fundamental counting principle to solve. To determine Quantity A, make three slots (one for each prize). Six people are available to win 1st, and then five people could win 2nd, and four people could win 3rd:

$$\underline{6} \quad \underline{5} \quad \underline{4}$$

Multiply: $(6)(5)(4) = 120$.

For Quantity B, make 5 slots, one for each prize. Five people can win 1st prize, then 4 people can win 2nd prize, and so on:

$$\underline{5} \quad \underline{4} \quad \underline{3} \quad \underline{2} \quad \underline{1}$$

Multiply $(5)(4)(3)(2)(1) = 120$. The two quantities are equal.

3. **(B)**. First, recognize that $x^2 + 2xy + y^2 = (x + y)^2$. This is one of the “special products” you need to memorize for the exam. Factor the top, then cancel:

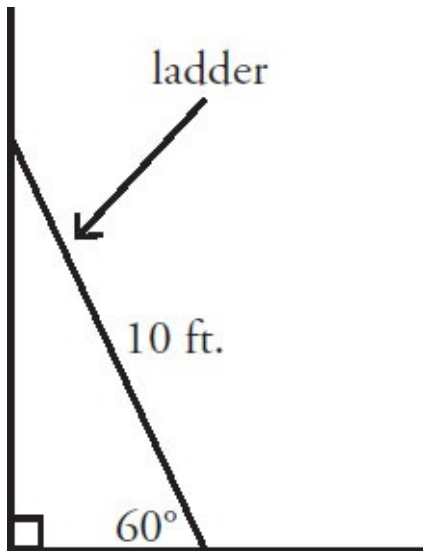
$$\frac{x^2 + 2xy + y^2}{2(x+y)^2} = \frac{\cancel{(x+y)}^2}{2\cancel{(x+y)}^2} = \frac{1}{2}$$

4. **$x^2 - y^2 = 0$ and $|x| - |y| = 0$ only.** When you take the square root of $x^2 = y^2$, the result is not $x = y$. Actually, it is $|x| = |y|$. After all, if $x^2 = y^2$, the variables could represent 2 and -2, 5 and 5, -1 and -1, etc. The information about the signs of x and y is lost when the numbers are squared; thus, taking the square root results in absolute values, which allow both sign possibilities for x and y . Thus, the first choice is not necessarily true.

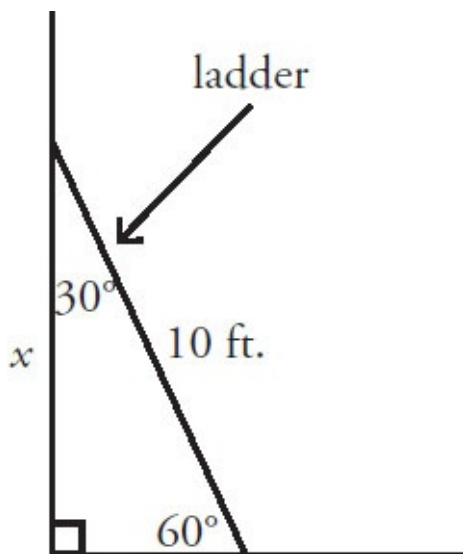
From $x^2 = y^2$, subtract y^2 from both sides to yield the second choice, providing algebraic proof that it must be true.

To prove the third choice, take the square root of both sides of $x^2 = y^2$ to get $|x| = |y|$, then subtract $|y|$ from both sides.

5. **(B).** First, draw a diagram and label all the givens:



Since the wall is vertical and the floor is horizontal, the angle where they meet is 90° . So the triangle is 30–60–90. The question asks for the vertical distance from the top of the ladder to the floor, so represent this length as x :



In any 30–60–90 triangle, the short leg (opposite the 30° angle) is the hypotenuse divided by 2, making the floor side equal to $10 \div 2 = 5$ feet. The longer leg (opposite the 60° angle) is $\sqrt{3}$ times the short leg. So $x = 5\sqrt{3}$ feet.

6. **(C).** The difference in population between 2000 and 2010 is the difference in the heights of the dark- and light-gray bars in the population bar chart. Population decreased in town E, and barely changed in towns B and D. Thus

towns A and C remain. The difference is about 5,000 for town A (45,000 – 40,000), but more than 5,000 for town C (more than 35,000—less than 30,000). The question asks for the town with greatest percent increase in population, so use the percent change formula: Percent Change =

$\left(\frac{\text{Difference}}{\text{Original}} \times 100 \right) \%$. Not only is the population increase greatest for town C, the population of town C is smaller than for town A. The percent increase in population for town A was $\left(\frac{5,000}{40,000} \times 100 \right) \% = 12.5\%$, but the percent increase in population for town C was about $\left(\frac{7,000}{29,000} \times 100 \right) \% \approx 24\%$.

7. Town D and Town E only. In the bar chart for area, dark gray represents the land area and light gray (stacked on top) represents the water area. Thus, to find water area, subtract the height of the dark-gray land area bar from the total height of the stacked bars.

Water area of town B is about $32 - 21 = 11$, although the vertical scale is admittedly not that precise. Water area for town B is a little more than 10, since the top of the dark-gray bar is slightly *closer* to the horizontal line for 20 than the top of the light-gray bar is to the horizontal line for 30. Similarly, water area for town B must be less than 12.5, as the top of the light-gray bar is halfway between 30 and 35, but the top of the dark-gray bar is clearly higher than 20.

Towns C, D, and E all have land area less than 10 square miles (i.e., all are below horizontal grid line for 10). Adding the land area of town C (a bit less than 10) to that of town D (about 4), the result is too high. The sum of land area in towns D and E is about $4 + \text{about } 6$ (certainly less than 7.5), for a result closest to 11.

8. Town A and Town C only. In the elevation chart, the towns are on the x-axis and the elevation (in feet above sea level) is on the y-axis. Two towns have the same elevation if marked at the same y value, that is, if their data points are on the same horizontal line. Towns A and C are both close to the horizontal line for 415.

9. (C). For even and odd questions, you can either think it out logically or plug in a number. Since one choice requires raising the number to the 6th power, pick something small! Plug in $x = 1$:

(A) $x^2 + 4x + 6 = 1 + 4 + 6 = 11$

(B) $x^3 + 5x + 3 = 1 + 5 + 3 = 9$

(C) $x^4 + 6x + 7 = 1 + 6 + 7 = 14$

(D) $x^5 + 7x + 1 = 1 + 7 + 1 = 9$

(E) $x^6 + 8x + 4 = 1 + 8 + 4 = 13$

For the logic approach, remember that an odd number raised to an integer power is always odd, an odd number multiplied by an odd number is always odd, and an odd number multiplied by an even number is always even:

$$(A) x^2 + 4x + 6 = \text{odd} + \text{even} + \text{even} = \text{odd}$$

$$(B) x^3 + 5x + 3 = \text{odd} + \text{odd} + \text{odd} = \text{odd}$$

$$(C) x^4 + 6x + 7 = \text{odd} + \text{even} + \text{odd} = \text{even}$$

$$(D) x^5 + 7x + 1 = \text{odd} + \text{odd} + \text{odd} = \text{odd}$$

$$(E) x^6 + 8x + 4 = \text{odd} + \text{even} + \text{even} = \text{odd}$$

10. **(A).** According to the formula for the volume of a right circular cylinder, the original volume is $V = \pi r^2 h$. To halve the radius, replace r with $\frac{r}{2}$. To double the height, replace h with $2h$. The only caveat: be sure to use parentheses!

$$V = \pi \left(\frac{r}{2} \right)^2 (2h) = \frac{2\pi r^2 h}{2^2} = \frac{\pi r^2 h}{2}$$

Thus, the volume, which was once $\pi r^2 h$, is now $\frac{\pi r^2 h}{2}$. In other words, it has been cut in half, or reduced by 50%.

Alternatively, plug in numbers. If the cylinder originally had radius 2 and height 1, the volume would be $V = \pi(2)^2(1) = 4\pi$. If the radius were halved to become 1 and the height were doubled to become 2, the volume would be $V = \pi(1)^2(2) = 2\pi$. Again, the volume is cut in half, or reduced by 50%.

Quant Set 3

1. **(A).** You may *not* add $\sqrt{3}$ and $\sqrt{6}$ to get $\sqrt{9}$, but you can put each value in the calculator. $\sqrt{3} = 1.732 \dots$ and $\sqrt{6} = 2.449 \dots$, and their sum is about 4.18. Since Quantity B is $\sqrt{9} = 3$, Quantity A is greater.

2. **(A).** Represent the cost per pound of the vegetables by their first letters. The given information translates to $p = 2y - 0.30$ because “twice” means multiply by two and “less” indicates subtraction. Then, translate the quantities

into algebraic expressions:

$$\text{Quantity A} = 2y + 2p$$

$$\text{Quantity B} = 3p$$

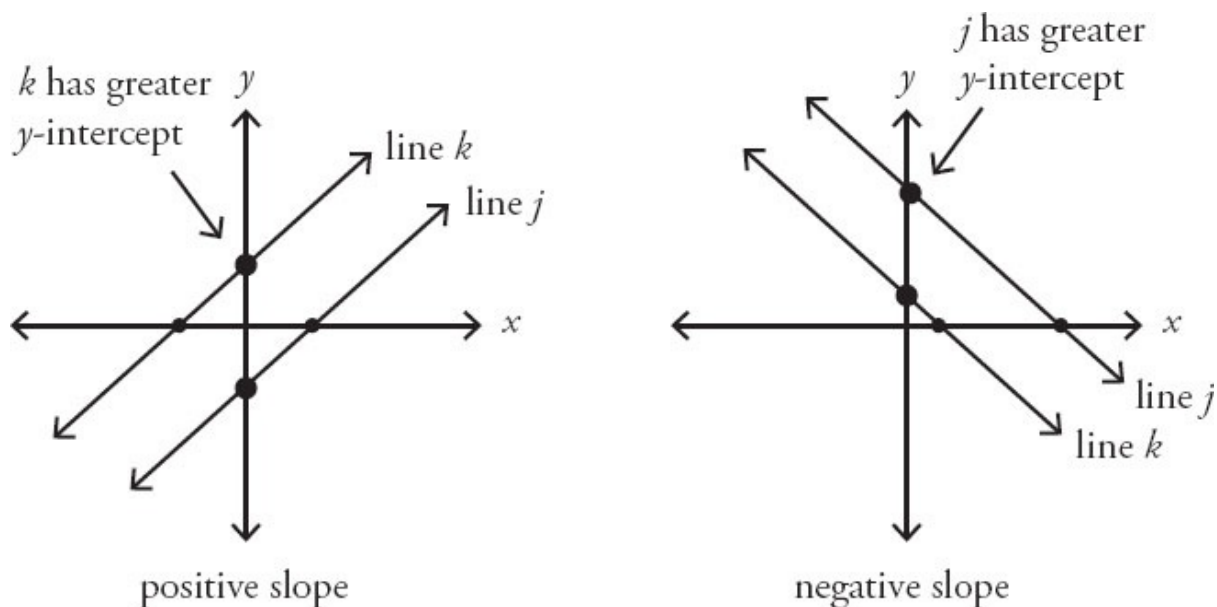
Since $2p$ is common to both quantities, it can be subtracted without changing their relative values:

$$\text{Quantity A} = 2y$$

$$\text{Quantity B} = p$$

Since $p = 2y - 0.30$, Quantity B is 0.30 less than Quantity A.

3. **(D)**. Parallel lines have the same slope. Since the product of the two slopes is positive, either both slopes are positive or both slopes are negative. Here are two examples in which line j has a greater x -intercept, as specified by the problem:



If the slopes are positive, k will have the greater y -intercept, but if the slopes are negative, j will have the greater y -intercept. The relationship cannot be determined from the information given.

4. **(A)**. Standard deviation is a measure of the data's spread from the mean. While the two sets have the same range ($30 - 10 = 20$), they do *not* have the same spread. The four extra terms in Quantity B are identical to the mean, meaning that, on average, the data in Quantity B is closer to the mean than the data in Quantity A. Thus, Quantity A is more spread out, on average, and has the larger standard deviation. Do not compute the actual standard deviations to find the answer here.

5. **(E)**. The median of any list is the middle number, if there is an odd number of terms, or the average of the two middle numbers, if there is an even number of terms. However, here the median number of hours worked per week is not the middle number in the list 15, 25, 35, 40, 50 because this does not account for the frequency with which each of those numbers occurs. The actual list includes the value 15 four times (once for each of the 4 employees who works 15 hours per week), the value 25 nine times (once for each of the 9 employees who works 25 hours per week), and so on.

There are 60 employees. Thus, the middle of this list is the average of the

30th and 31st values. Since $4 + 9 + 15 = 28$, the 30th and 31st values fall into the next highest group—the group of 27 people who work 40 hours per week. The median number of hours worked per week per employee is 40.

6. **(D).** The average number of hours worked per week is not the average of 15, 25, 35, 40, and 50. The calculation must account for how many people work each number of hours:

$$\frac{4(15) + 9(25) + 15(35) + 27(40) + 5(50)}{60} = 35.\overline{6} \text{ hours}$$

The answer is $35.\overline{6}$ or $35\frac{2}{3}$.

7. **(C).** The mode is the number that appears in the list with the greatest frequency. Since 27 people worked 40 hours a week and every other group has fewer than 27 people, the mode is 40. The range is the highest value in the list minus the lowest value in the list: $50 - 15 = 35$. The positive difference between 40 and 35 is $40 - 35 = 5$.

8. **(E).** In order to compare or combine exponents with different bases, convert to the same base if possible. Since $9 = 3^2$:

$$3^2(3^2)^2 = 3^x$$

Multiply exponents in accordance with the exponent formula, $(a^b)^c = a^{bc}$:

$$3^23^4 = 3^x$$

Add the exponents to multiply numbers that have the same base:

$$3^6 = 3^x$$

Therefore, $x = 6$.

9. **(B).** If a question refers to fractions of different numbers that are *also* related by a fraction, try plugging in numbers. Since there are $\frac{2}{3}$ as many juniors as seniors, some convenient numbers are:

$$\text{Juniors} = 20$$

$$\text{Seniors} = 30$$

$$\text{Juniors going on trip} = \frac{1}{4}(20) = 5$$

$$\text{Seniors going on trip} = \frac{2}{3} (30) = 20$$

Out of 50 total students, 25 are going on the trip, so 25 are NOT going on the trip. The answer is $\frac{25}{50} = \frac{1}{2}$.

10. $\frac{71}{20}$ **(or any equivalent fraction)**. Sum the fractions by finding a common denominator, which is a multiple of 2, 3, 4, 5, and 6. The smallest number that works is 60.

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = \frac{30}{60} + \frac{40}{60} + \frac{45}{60} + \frac{48}{60} + \frac{50}{60} = \frac{30 + 40 + 45 + 48 + 50}{60} = \frac{213}{60} = \frac{71}{20}$$

Quant Set 4

1. **(C)**. This is a classic combinatorics problem in which *order doesn't matter*—that is, picking Javier and Sonya is the same as picking Sonya and Javier. A person is either on the team or not. Use the standard “order doesn't matter” formula:

$$\frac{\text{total!}}{\text{in!out!}}$$

For Quantity A:

$$\frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{(2)(1)4!} = \frac{6 \times 5}{2} = 15$$

For Quantity B:

$$\frac{6!}{4!2!} = \frac{6 \times 5 \times 4!}{4!(2)(1)} = \frac{6 \times 5}{2} = 15$$

The two quantities are equal. Note that it is not actually necessary to reduce each quantity. The factorials are the same in each, so the resulting quantities must be equal.

This will always work—when order doesn't matter, the number of ways to pick 4 and leave out 2 is the same as the number of ways to pick 2 and leave

out 4. Either way, it's one group of 4 and one group of 2. What actually happens to those groups (getting picked, not getting picked, getting a prize, losing a contest, etc.) is irrelevant to the ultimate solution.

2. **(A).** When two parallel lines are cut by a transversal, same-side interior angles are supplementary. Thus, $x + y = 180$. Since y is not 0 (the transversal and line m do not “overlap”), $x + 2y$ is greater than $x + y$, which also means that $x + 2y$ is greater than 180.

3. **(A)**. The formula for averages is: $\text{Average} = \frac{\text{Sum}}{\text{Number of terms}}$.

The sum of the weights of the people in this group is:

$$\text{Sum} = \text{Average} \times \text{Number of terms} = 85 \times 4 = 340$$

If two of the people weigh 75 and 90 kilograms, subtract them from the total to see what the other two people weigh, combined:

$$340 - 75 - 90 = 175 \text{ kg}$$

If the total weight of the other two people is 175, their average weight is:

$$\frac{175}{2} = 87.5 \text{ kg.}$$

Quantity A is greater.

4. $|x| = |y|$ and $\frac{x^2}{y^2} = 1$. Since $x^2 - y^2 = 0$, add y^2 to both sides to get $x^2 = y^2$.

It might look as though $x = y$, but this is not necessarily the case. For example, x could be 2 and y could be -2 . Algebraically, taking the square root of both sides of $x^2 = y^2$ does *not* yield $x = y$, but rather $|x| = |y|$. Thus, the 1st statement is not necessarily true and the 2nd statement is true. The 3rd statement is also true and can be generated algebraically:

$$x^2 - y^2 = 0$$

$$x^2 = y^2$$

$$\frac{x^2}{y^2} = 1$$

5. **4**. Triangle ACD is a right triangle and two of the side lengths are labeled, so the length of CD can be determined. Either use the Pythagorean theorem or recognize that this is one of the Pythagorean triplets: a 6–8–10 triangle. The length of side CD is 6. The area of rectangle $ABCD$ is $8 \times 6 = 48$.

The area of rectangle $EFGH$ is also 48, so $12 \times FG = 48$. The length of side FG is 4.

6. **(E)**. It may be tempting to select store Z, as revenue increased from 2011 to

2012 while the number of distinct customers decreased, but be careful when mixing absolute numbers and percents. Without knowing the revenue in 2012 (only the change from the previous year is known) or the number of customers (only the percent change from the previous year is known) for any of the stores, it cannot be determined from the information given.

7. **(A).** Because the question concerns costs per customer, given in percent change terms in the chart, and the question asks about percent change for this ratio, a comparison can be made among the stores.

The percent change formula in general is $\left(\frac{\text{Difference}}{\text{Original}} \times 100 \right) \%$. Thus, the percent change in total costs per distinct customer at a particular store is:

$$\left(\frac{\frac{\text{cost}_{2012}}{\text{customer}_{2012}} - \frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}} \times 100 \right) \%$$

This looks like a mess, but remember that both cost_{2012} and customer_{2012} can be written in terms of cost_{2011} and customer_{2011} , respectively, based on the percent changes given in the table. Then cost_{2011} and customer_{2011} are in each term of the fraction and can be canceled. For example, for store W, the percent change in total costs per distinct customer is:

$$\left(\frac{\frac{1.15 \times \text{cost}_{2011}}{1.02 \times \text{customer}_{2011}} - \frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}} \times 100 \right) \% = \left(\frac{\frac{1.15}{1.02} - 1}{1} \times 100 \right) \% = \left(\left(\frac{1.15}{1.02} - 1 \right) \times 100 \right) \%$$

In other words, the magnitude of percent change in total costs per distinct customer depends only on the ratio of $(1 + \text{Percent change in total costs})$ to $(1 + \text{Percent change in number of distinct customers})$. Perform this comparison for all of the stores:

$$(A) \text{ Store W: } \frac{1.15}{1.02} = 1.12745 \text{ Greatest.}$$

$$(B) \text{ Store X: } \frac{1.04}{1.14} = 0.91228$$

(C) Store Y: $\frac{1.12}{1.05} = 1.06667$

(D) Store Z: $\frac{0.80}{0.93} = 0.86022$

8. Store W, Store X, and Store Y only. Profit = Revenue – Cost.

Store W: Revenue decreased by \$400,000, and costs increased by 15%. Both changes negatively affect profit. CORRECT.

Store X: Revenue increased by \$520,000, but costs also increased by 4%. Profit in 2012 could be greater than, less than, or equal to profit in 2011, depending on the store's cost structure. Try sample numbers to show that profit could have decreased. If in 2011, revenue was \$20,000,000 and costs were \$15,000,000, the profit was \$5,000,000. In 2012, revenue would be \$20,520,000 and costs \$15,600,000, making profit \$4,920,000, less than in the previous year. CORRECT.

Store Y: Revenue decreased by \$365,000, and costs increased by 12%. Both changes negatively affect profit. CORRECT.

Store Z: Revenue increased by \$125,000 and costs decreased by 20%. Both changes positively affect profit.

9. (B). Consider each statement individually:

(A) While store X had the greatest increase in revenue, in dollars, it is impossible to calculate percent change in revenue

$\left(\frac{\text{Difference}}{\text{Original}} \times 100 \right) \%$ for any of the stores without information about the actual dollar amount of their revenue in either year. Not necessarily true.

(B) Per customer revenue is $\frac{\text{Revenue}}{\text{Number of customers}}$. Store Z

experienced an increase in revenue and a decrease in number of distinct customers, both of which increase per customer revenue. TRUE.

(C) While store W had the greatest percent increase in total costs, it is impossible to say whether this was the greatest increase in dollars without knowing the actual dollar amount of total costs for each of the stores that experienced a cost increase. Not necessarily true.

(D) The chart says nothing about repeat customers, only “distinct” customers. Not necessarily true.

(E) The chart says nothing about absolute numbers of distinct customers at any of the stores, only percent change from 2011 to 2012. Not necessarily true.

10. (E). If \sqrt{x} is an integer, then x must be a perfect square. If x is a perfect square and $xy^2 = 36$, then x could actually equal *any* of the perfect square factors of 36, which are 1, 4, 9, or 36. (Only consider positive factors, because in order to have a valid square root, x must be positive.) Thus, y^2 could equal 36, 9, 4, or 1, respectively.

If y^2 is positive, y itself could be positive or negative. Thus, $y = \pm 6, \pm 3, \pm 2$, or ± 1 , for a total of 8 possible values.

Math Practice Section 1: Easy Difficulty

Math Practice Section: Easy

20 Questions

35 Minutes

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box , you are to enter your own answer in the

box. For questions followed by a fraction-style numeric entry box

, you are to enter

your answer in the form of a fraction. You are not required to reduce fractions. For example, if the

answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as xy -planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, are drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

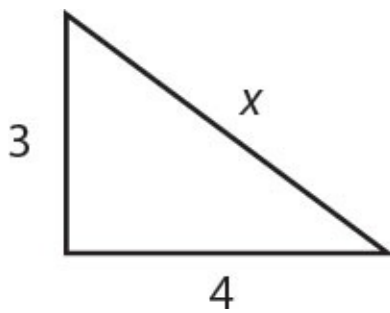
x , y , and z are consecutive integers such that $x < y < z$.

Quantity B

Quantity A

1.	y	$\frac{x+z}{2}$
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	<u>Quantity A</u>	<u>Quantity B</u>
2.	$(-3)^4$	$(-3)^{-3}$



	<u>Quantity A</u>	<u>Quantity B</u>
3.	x	5

	<u>Quantity A</u>	<u>Quantity B</u>
4.	$y^7 \times y^8 \times y^{-6}$	$3y^9$

$$xy > 0 \text{ and } yz < 0$$

	<u>Quantity A</u>	<u>Quantity B</u>
5.	xz	0

In 2011, it cost Tammy \$1.30 to manufacture each copy of her magazine, which she sold for \$2.30. In 2012, it cost Tammy \$1.50 to manufacture each copy of the same magazine, which she sold for \$3.00.

	<u>Quantity A</u>	<u>Quantity B</u>
6.	The percent by which Tammy's profit per copy of the magazine changed from 2011 to 2012	$33\frac{1}{3}\%$

List X consists of the numbers 4, 7, 9, 11, 24, 32.

List Y (not shown) consists of six unique numbers, each computed from the corresponding term in list X by dividing the number in list X by 2, then adding 5 to the result.

	<u>Quantity A</u>	<u>Quantity B</u>
7.	The range of list Y	6 less than the greatest number in list Y

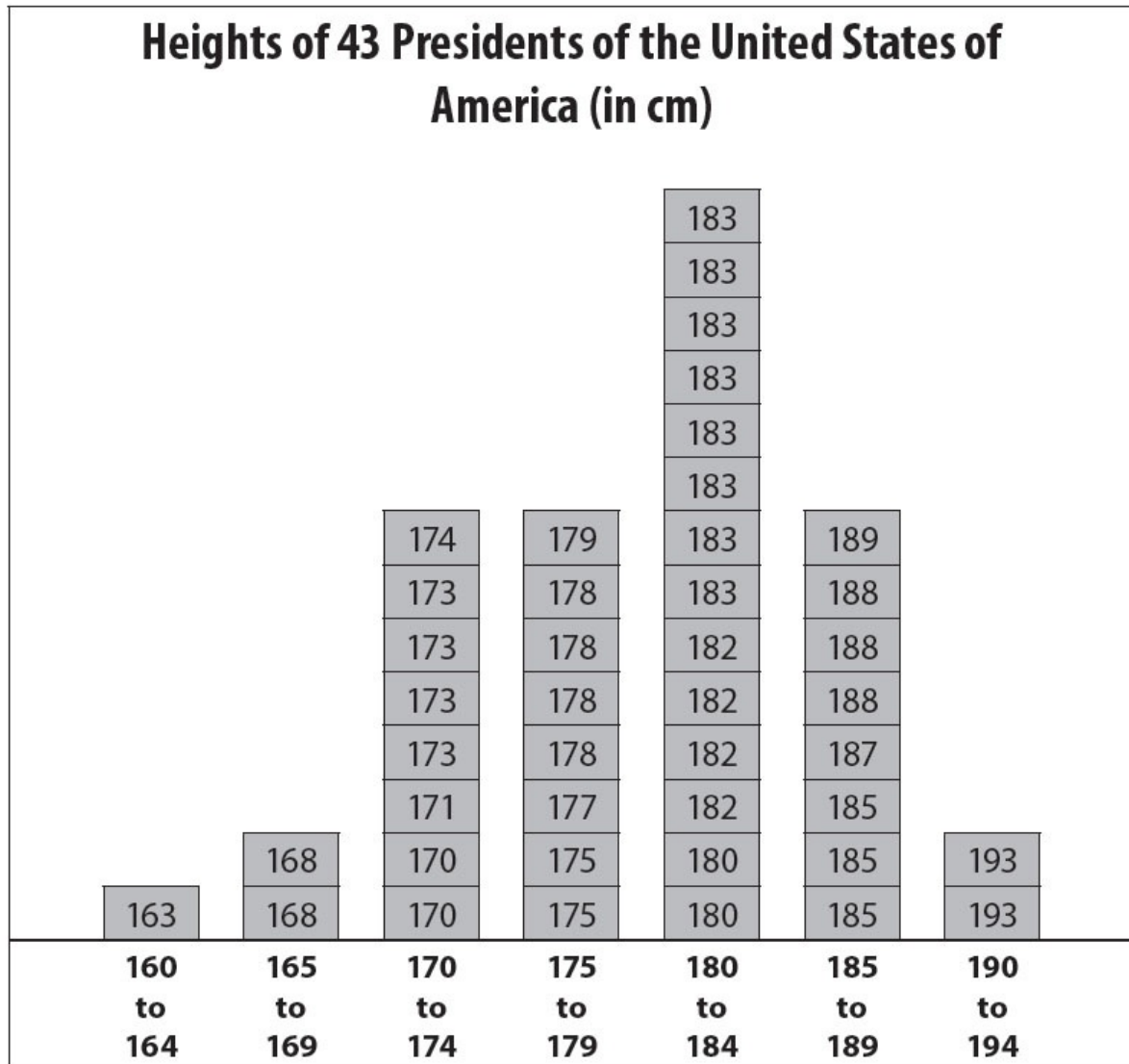
8. Which of the following represents the length of the diagonal d of a square with area a ?

- (A) $d = a^2$
- (B) $d = \sqrt{2a}$
- (C) $d = 2\sqrt{a}$
- (D) $d = a\sqrt{2}$
- (E) $d = a\sqrt{3}$

9. In an apartment complex, 60% of the apartments contain at least one television, and 20% of these apartments are equipped with cable. If every apartment that is equipped with cable contains at least one television, what percent of the apartments in the complex are not equipped with cable?

- (A) 8%
- (B) 12%
- (C) 16%
- (D) 88%
- (E) 92%

Questions 10–12 are based on the following chart.



10. What is the range of heights of the 43 U.S. presidents in the chart?
- (A) 30 cm
 (B) 34 cm
 (C) 35 cm
 (D) 163 cm
 (E) 178 cm
11. What is the median height of the 43 U.S. presidents in the chart, in centimeters?
- (A) 175
 (B) 177

(C) 178

(D) 180

(E) 182

12. Approximately what percent of U.S. presidents are 185 centimeters or taller?

- (A) 10%
- (B) 23%
- (C) 29%
- (D) 43%
- (E) 50%

13. If $m + 5 < \frac{3}{2}$, which of the following could be the value of m ?

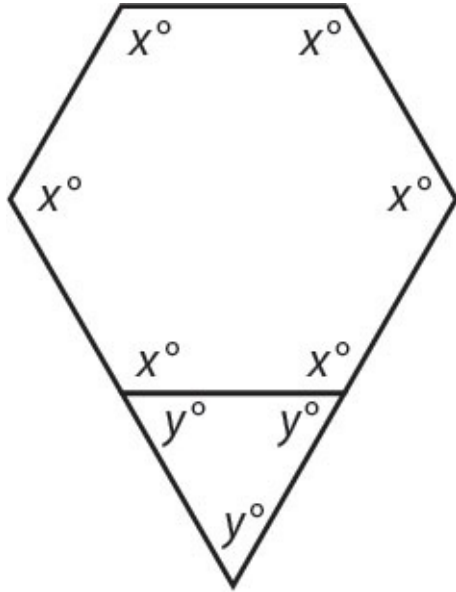
- (A) $-\frac{15}{4}$
- (B) $-\frac{7}{2}$
- (C) -2
- (D) $\frac{7}{2}$
- (E) 2

14. List M consists of the numbers 10, 20, 30, 40, 50.

Which of the following lists of numbers have an average (arithmetic mean) that is equal to the average of the numbers in list M?

Indicate all such lists.

- ☐ 0, 30, 60
- ☐ 10, 20, 30, 35, 50
- ☐ 10, 22, 30, 38, 50
- ☐ 0, 0, 0, 0, 150



15. What is the value of xy ?

Buying Habits of Customers Buying Toothpaste X at Chan's Grocery Store

Discount Type	Manufacturers' Coupon	Store Coupon	No Coupon
Percent of Customers	54%	43%	$x\%$

16. The table above summarizes all possible discount types for customers buying toothpaste X at a certain grocery store. No one used both types of coupon. If a person is selected randomly from among the customers buying toothpaste X at Chan's Grocery Store, what is the probability that this customer did not use a coupon?

- (A) 0.003
- (B) 0.03
- (C) 0.3
- (D) 0.33
- (E) 3.3

17. Company A can pave 500 feet of sidewalk in 6 hours, and company B can pave 1,000 feet of sidewalk in 8 hours. At these rates, how many more yards of sidewalk can company B pave in 9 hours than company A can pave in 9 hours? (3 feet = 1 yard)

- (A) 125
- (B) 166

(C) 333

(D) 375

(E) 500

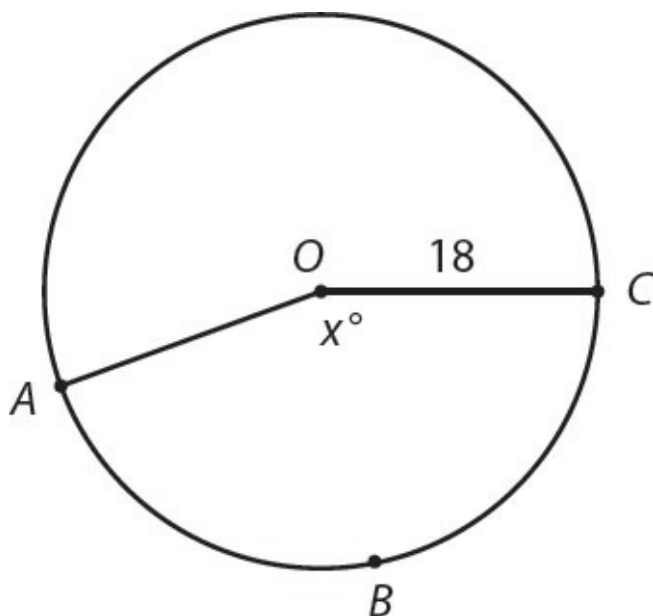
18. If the three sides of an equilateral triangle are equal to $4x$, $6y$, and 24 , respectively, what is the ratio of x to y ?

Give your answer as a fraction.

19. If the ratio of undergraduate students to graduate students is 7 to 4 and the ratio of graduate students to professors is 2 to 1, which could be the total number of undergraduate students, graduate students, and professors?

Indicate all such numbers.

- ☐ 520
☐ 640
☐ 2,600
☐ 10,000



20. If O is the center of the circle and $x = 160$, what is the perimeter of sector $ABCO$?

- (A) $18 + 8\pi$
(B) $18 + 16\pi$
(C) $36 + 8\pi$

(D) $36 + 16\pi$

(E) $36 + 24\pi$

Answers to Math Practice Section 1

1. **(C).** The average of three consecutive integers is always equal to the middle value and is always equal to the average of the smallest and largest terms. Since Quantity B represents the average of the smallest and largest terms, it is equal to the middle term y .

Alternatively, pick numbers. If x , y , and z are 1, 2, and 3, then Quantity A = 2 and Quantity B = $\frac{1+3}{2} = 2$. The two quantities are equal. Any other example of three consecutive numbers will also yield equal quantities.

2. **(A).** In Quantity A, $(-3)^4 = (-3)(-3)(-3)(-3) = 81$. In Quantity B, $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27}$. Quantity A is greater. Note that you can stop calculating as soon as you realize that one quantity is positive and one is negative. The negative base in both quantities suggests that you should check whether the exponents are odd or even. Even exponents “hide the sign” of the base, so a negative base to an even exponent is positive. On the other hand, a negative base to an odd exponent remains negative (even if the exponent is a *negative* odd).

3. **(D).** If this were a right triangle, the Pythagorean theorem would indicate the following:

$$3^2 + 4^2 = x^2$$

$$9 + 16 = x^2$$

$$25 = x^2$$

$$5 = x$$

However, the triangle is not known to be right (the Pythagorean theorem only applies to right triangles) as none of the angles are labeled. The Third Side Rule, which applies to all triangles regardless of angle measures, states that the third side of any triangle must be greater than the difference between the other two sides and less than the sum of the other two sides. So x must be greater than $4 - 3 = 1$ and less than $4 + 3 = 7$. Since x could be less than, greater than, or equal to 5, it cannot be determined which quantity is greater.

4. **(D).** Since the terms in Quantity A have the same base and are multiplied

together, simplify by adding the exponents:

$$y^7 \times y^8 \times y^{-6} = y^9$$

While y^9 may *seem* smaller than $3y^9$, this is only true if y is positive. If $y = 0$, the two quantities are equal. If y is negative, so is y^9 , and $3y^9$ is more negative than y^9 . Therefore, the relationship cannot be determined from the information given.

5. **(B)**. Since $xy > 0$, x and y have the same sign. Since $yz < 0$, y and z have opposite signs. Therefore, x and z have opposite signs. If x and z have opposite signs, their product is negative, which is less than 0. Quantity B is greater.

6. **(A)**. In order to calculate the percent change in profit from 2011 to 2012, first calculate the profits in each year based on the formula: Profit = Revenues – Costs.

Therefore,

Profit per each copy of the magazine in 2011 = $\$2.30 - \$1.30 = \$1.00$

Profit per each copy of the magazine in 2012 = $\$3.00 - \$1.50 = \$1.50$

To find the percent increase, use the percent change formula:

$$\text{Percent Change} = \left(\frac{\text{Difference}}{\text{Original}} \times 100 \right) \%$$
$$\text{Percent Change} = \left(\frac{0.50}{1.00} \times 100 \right) \% = 50\%$$

Be careful not to put the 2012 profit in the denominator. Mistakenly doing so would lead you to pick (C) erroneously. The “Original” profit is that for 2011.

Quantity A is greater.

7. **(B).** Since the terms in list Y are “each computed from the corresponding term in list X by dividing the number in list X by 2, then adding 5 to the result,” list Y consists of 7, 8.5, 9.5, 10.5, 17, 21. Therefore, you can solve:

Quantity A: The range is $21 - 7 = 14$.

Quantity B: 6 less than the greatest number in list Y = $21 - 6 = 15$.

Quantity B is greater.

8. **(B).** A square with area a has sides of \sqrt{a} . Use the Pythagorean theorem with \sqrt{a} for each leg and d for the hypotenuse:

$$(\sqrt{a})^2 + (\sqrt{a})^2 = d^2$$

$$a + a = d^2$$

$$2a = d^2$$

$$\sqrt{2a} = d$$

This is a match with choice (B). Alternatively, plug in numbers. If a square has side length 4, the area equals 16 and the diagonal would be:

$$4^2 + 4^2 = d^2$$

$$32 = d^2$$

$$\sqrt{32} = d$$

Plug $a = 16$ into each choice to see which yields $d = \sqrt{32}$. Only choice (B) works.

9. **(D)**. The easiest way to solve this problem is to choose a smart number for the total number of apartments in the apartment complex. As this is a percents problem, choose a total of 100 apartments. Since 60% of these apartments have at least one television, 60 apartments contain a television (or more than one television—it doesn't matter how many—only television at all vs. no television matters) and 40 apartments do not contain a television.

Since 20% of the apartments that contain a television are equipped with cable, 20% of 60 = $0.2 \times 60 = 12$ apartments have both television and cable.

“Every apartment that is equipped with cable contains at least one television” means that none of the 40 apartments without a television are equipped with cable. Thus, only 12 apartments are equipped with cable, meaning $100 - 12 = 88$ are not. Alternatively, $48 + 40 = 88$ apartments are not equipped with cable.

Since 88 out of 100 apartments are not equipped with cable, the answer is 88%.

Alternatively, you can assign the variable x to the total number of apartments in the apartment complex. Following the steps from above, $0.6x$ apartments contain a television and $(0.2)(0.6x) = 0.12x$ apartments are equipped with cable. From here, $x - 0.12x = 0.88x$ apartments, or 88% of the apartments in the complex, do not have cable.

10. **(A)**. The shortest U.S. president is 163 centimeters tall and the tallest is 193 centimeters tall. The range is the difference between the highest and lowest values, so $193 \text{ cm} - 163 \text{ cm} = 30 \text{ cm}$.

11. **(E)**. The median is the middle value if all the data points are arranged from least to greatest. With 43 data points, the median is the 22nd data point, because there are 21 data points that are less than or equal to that point, and 21 that are greater than or equal to that point. Counting up from the least value (or down from the greatest value), the 22nd data point is 182 cm.

12. **(B)**. From the chart, 10 U.S. presidents are 185 cm or taller, out of a total of 43. As a percent, this is $\left(\frac{10}{43} \times 100\right)\%$, or approximately 23%.

13. **(A)**. Solve the inequality:

$$m + 5 < \frac{3}{2}$$

$$m < \frac{3}{2} - 5$$

$$m < \frac{3}{2} - \frac{10}{2}$$

$$m < -\frac{7}{2}$$

Answer choice (A) is the only one that is less than $-\frac{7}{2}$. If needed, plug each answer choice into the calculator and compare decimal values to -3.5 .

14. **{0, 30, 60} and {10, 22, 30, 38, 50} and {0, 0, 0, 0, 150} only.** Certainly, you could average the list 10, 20, 30, 40, 50 (the average is 30) and then average the lists in all the answer choices to see which also averages to 30. However, you cannot afford to waste any time on the GRE.

Instead, note that the average of an evenly spaced set is equal to the median. Thus, the average of 10, 20, 30, 40, 50 is the median, or middle term, 30. In the first choice, the list 0, 30, 60 is also evenly spaced, so the average is 30.

In the second choice, the list 10, 20, 30, 35, 50 is the same as the original list (10, 20, 30, 40, 50) except for one number—the 40 has been changed to 35. Thus, the averages cannot be the same.

In the third choice, the list 10, 22, 30, 38, 50 is the same as the original list (10, 20, 30, 40, 50), but with 2 taken away from the fourth number and added to the second number. Since the sum didn't change, the average didn't either.

In the fourth choice, the average is the sum divided by the number of items, or $\frac{150}{5} = 30$.

15. **7,200.** Since every angle in the hexagon is labeled x° , the hexagon is equiangular. To find the sum of the degree measures in a polygon, use the formula $(n - 2)(180)$, where n is the number of sides. Since $n = 6$, then $(6 - 2)(180) = 720$, so the sum of the degrees in the hexagon is 720. Thus, $6x = 720$ and $x = 120$.

Since the triangle is equiangular, $3y = 180$ and $y = 60$.

Thus, the value of $xy = 120 \times 60 = 7,200$.

16. **(B).** Add $54\% + 43\% = 97\%$ to get the percent of customers who used a coupon. Only $100\% - 97\% = 3\%$ of customers did not use a coupon. Thus, for a person selected randomly from among the customers buying toothpaste X at Chan's Grocery Store, there is a 3%, or 0.03, probability that he or she did not use a coupon.

17. **(A).** Company A can pave 500 feet of sidewalk in 6 hours, and thus $\frac{500}{6}$ feet per hour. In 9 hours, company A can pave $\frac{500}{6} \times 9 = 750$ feet of sidewalk.

Company B can pave 1,000 feet of sidewalk in 8 hours, and thus $\frac{1000}{8} = 125$ feet per hour. In 9 hours, company B can pave $125 \times 9 = 1,125$ feet of sidewalk.

Thus, in 9 hours, company B can pave $1,125 - 750 = 375$ feet of sidewalk more than company A. Since 3 feet = 1 yard, divide by 3 to get the answer in the correct units: $375 \text{ feet} \div 3 \text{ feet per yard} = 125 \text{ yards}$.

18. $\frac{3}{2}$ (or any equivalent fraction). Since the sides of an equilateral triangle are all equal, $4x = 6y = 24$. With a three part equation, you can equate any two parts you wish.

For instance:

$$4x = 24$$

$$x = 6$$

$$6y = 24$$

$$y = 4$$

Thus, the ratio of x to y is 6 to 4, which reduces to 3 to 2. On the GRE, you do not need to reduce the answers to fraction numeric entry questions.

19. **520 and 2,600 only.** If the ratio of undergraduate students to graduate students is 7 to 4 and the ratio of graduate students to professors is 2 to 1:

Undergraduate	Graduate	Professors
7	4	
	2	1

Equate the ratios by making the two numbers under “Graduate” equal. To do this, double the second ratio. (If you change one number in the ratio 2 : 1, you must perform the same operation to the other number in that ratio.)

Undergraduate	Graduate	Professors
7	4	
	4	2

Now, collapse the ratios onto one line:

Undergraduate	Graduate	Professors
7	4	2

The ratio is 7 to 4 to 2. Since $7 + 4 + 2 = 13$ and numbers of people must be integers, the total number of people must be a multiple of 13. Only 520 and 2,600 qualify.

20. **(D).** If $x = 160$, then the sector is $\frac{160}{360} = \frac{4}{9}$ of the circle. Thus, arc ABC

is $\frac{4}{9}$ of the circumference. Since the circumference is equal to $2\pi r = 2\pi(18)$
 $= 36\pi$, take $\frac{4}{9}$ (36π) to yield 16π .

Thus, the perimeter of the sector is equal to two radii plus 16π , or $36 + 16\pi$.

Math Practice Section 2: Medium Difficulty

Math Practice Section: Medium

20 Questions

35 Minutes

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box , you are to enter your own answer in the

box. For questions followed by a fraction-style numeric entry box

, you are to enter

your answer in the form of a fraction. You are not required to reduce fractions. For example, if the

answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as xy -planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, are drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

Set M consists of all the integers between -2 and 12 , inclusive.

Set N consists of all the integers between 9 and 15 , inclusive.

Quantity A

1. The smallest integer in set M that is
also in set N

Quantity B

9

17% of p is equal to 18% of q , where p and q are positive.

Quantity A

2.

p

Quantity B

q

Circle A has area a . Semicircle B has area $\frac{a}{2}$.

Quantity A

Quantity B

3. The circumference of circle A Twice the perimeter of semicircle B
-

Quantity A

Quantity B

4. The standard deviation of the set 1, 5, 7, 19 The standard deviation of the set 0, 5, 7, 20

An isosceles triangle has a perimeter of 28. The shortest side has length 8.

Quantity A

Quantity B

5. The length of the longest side of the triangle 12

$$(3 - z)(z + 4) = 0$$

Quantity A

Quantity B

6. z 5

$$\begin{aligned} a &> b > c > d \\ ab &> 0 \\ ad &< 0 \end{aligned}$$

Quantity A

Quantity B

7. ac cd

-
8. If $12b = 2g$ and $4g - 3b = 63$, what is the value of $\frac{g}{b}$?

Give your answer as a fraction.

9. $81^3 + 27^4$ is equivalent to which of the following expressions?

Indicate all such expressions.

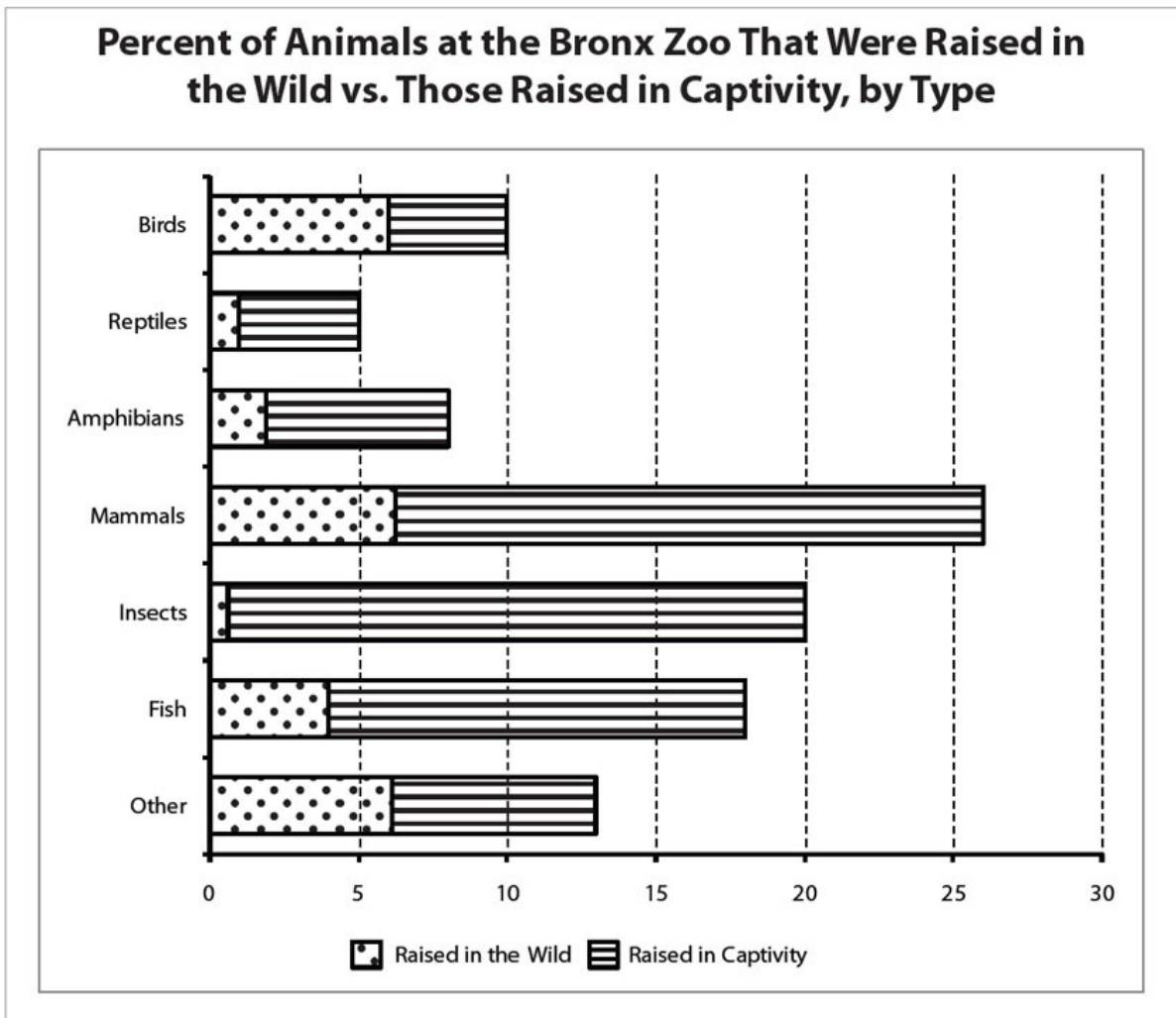
- ☐ $3^7(2)$
☐ $3^{12}(2)$

☐ $9^6(2)$

☐ 9^{12}

☐ 3^{24}

Questions 10–12 are based on the following chart.



10. Approximately what percent of all the zoo's animals are either mammals that were raised in the wild or amphibians raised in captivity?

- (A) 8
- (B) 12
- (C) 18
- (D) 34
- (E) 100

11. If the Bronx Zoo donated all of its insects and fish to other zoos, approximately what percent of the animals in the zoo would be birds raised in the wild?

- (A) 5
- (B) 9
- (C) 24
- (D) 32
- (E) 60

12. If the zoo currently has 80 total birds, what is the smallest number of birds that could be added such that at least 20% of the animals at the zoo would be birds?

- (A) 10
- (B) 80
- (C) 100
- (D) 125
- (E) 200

13. Trail mix is made by combining 3 pounds of nuts that cost x dollars per pound with 1 pound of chocolate that costs y dollars per pound and 2 pounds of dried fruit that costs z dollars per pound. What is the cost in dollars per pound for the trail mix?

- (A) $\frac{3x + y + 2z}{xyz}$
- (B) $3x + y + 2z$
- (C) $\frac{3x + y + 2z}{6}$
- (D) $6(3x + y + 2z)$
- (E) $\frac{x}{3} + y + \frac{2}{z}$

14. If $z = 3^4$, then $(3^z)^z =$

- (A) 3^{16}
- (B) 3^{81}

(C) 3^{324}

(D) 3^{405}

(E) $3^{6,561}$

15. Maurice entered a number into his calculator and erroneously divided the number by 0.03 instead of 0.0003, resulting in an incorrect result. Which of the following is a single operation that Maurice could perform on his calculator to correct the error?

Indicate all such operations.

- ☐ Multiply the incorrect product by 100
- ☐ Divide the incorrect product by 100
- ☐ Multiply the incorrect product by 0.01
- ☐ Divide the incorrect product by 0.01

16. A company's annual expenses are composed entirely of a fixed amount in costs, plus a variable amount that is directly proportional to the number of clients served. In 2009, the company served 450 clients and its total expense was \$830,000. In 2010, the company served 510 clients and its total expense was \$896,000. What is the company's fixed annual expense, in dollars?

- (A) 1,757
- (B) 1,844
- (C) 335,000
- (D) 485,000
- (E) 830,000

17. Which of the following lines is perpendicular to $4x + 5y = 9$ on the xy -plane?

(A) $y = \frac{5}{4}x + 2$

(B) $y = -\frac{5}{4}x + 9$

(C) $y = -4x + \frac{9}{5}$

(D) $y = \frac{4}{5}x - \frac{4}{5}$

(E) $y = -\frac{4}{5}x$

18. The tens digit is missing from the three-digit number 8 __ 9. If the tens digit is to be randomly selected from the ten different digits from 0 to 9, what is the probability that the resulting three-digit number will be a multiple of 9?

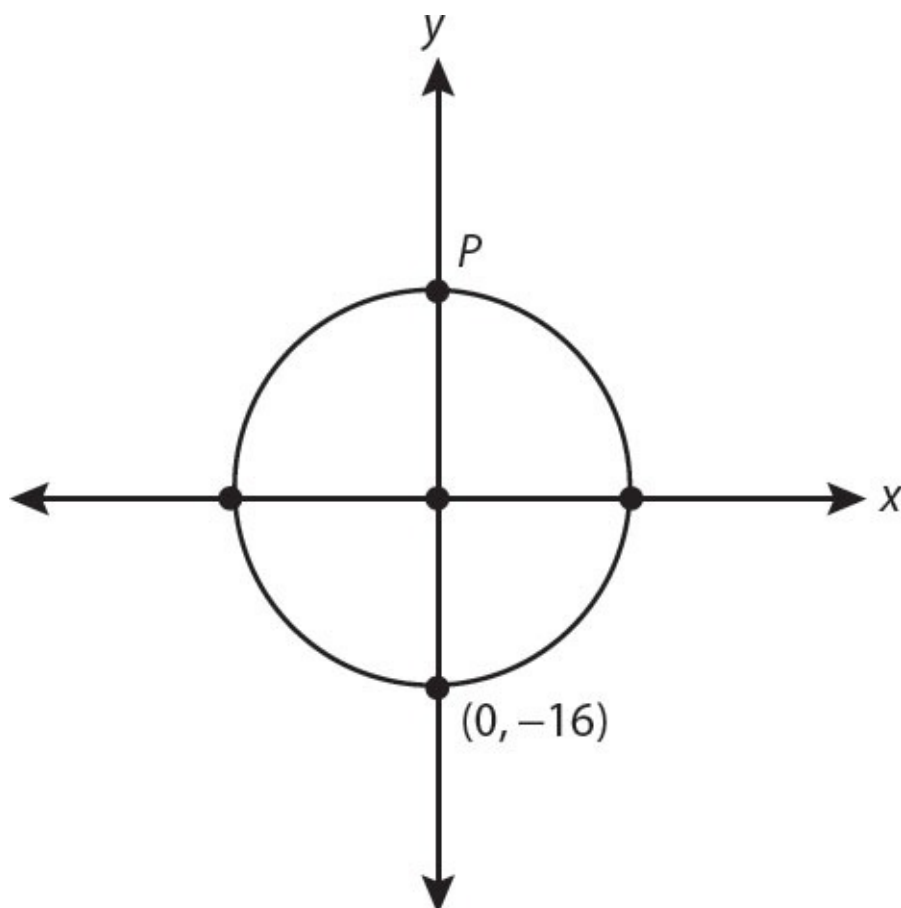
(A) 0.1

(B) 0.2

(C) 0.4

(D) 0.9

(E) 1



19. In the figure above, the circle is centered at $(0, 0)$. What is the distance between point P and the point $(-10, -8)$ (not shown on the graph)?
- (A) 18
(B) 20
(C) 22
(D) 24
(E) 26
20. If $f(-0.5) = 0$, which of the following could be $f(x)$?
- (A) $2x + 2$
(B) $4x - 2$
(C) $4x^2 - 1$
(D) $x^2 - 1$
(E) $(-x)^2 - 2.5$

Answers to Math Practice Section 2

1. **(C).** Set M consists of $-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$, and 12 . Quantity A is the *least* of these integers that is also in set N. The smallest integer in set N is 9, which is also in Set M, so Quantity A is 9. The two quantities are equal.

2. **(A).** Written out as algebra, “17% of p is equal to 18% of q ” is:

$$\frac{17}{100}p = \frac{18}{100}q$$

Solve for p . The easiest way to do this is to first multiply both sides of the equation by 100, then divide both sides by 17:

$$p = \frac{18}{17}q$$

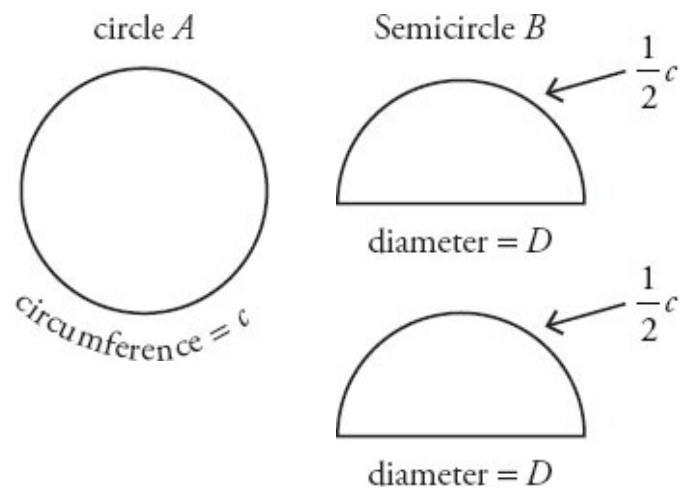
Since $\frac{18}{17}$ is greater than 1 and both variables are positive, p is greater than q .

(Note that it was necessary to know that both variables were positive! If they were negative, $p = \frac{18}{17}q$ would imply that p is more negative than q , so q would have been greater than p . Without information about sign, the answer would have been (D).)

3. **(B).** If the given semicircle has half the area of the circle, then the semicircle B is just one of the pieces you get after cutting circle A exactly in half through its center. However, that does *not* mean that the semicircle has half the perimeter. Observe:

Quantity A

Quantity B



The semicircle is drawn twice, as Quantity B refers to “twice the perimeter of semicircle B .” Note that Quantity A is equal to the circumference c , while Quantity B is equal to this same circumference, plus twice the length of the diameter. Quantity B is greater.

4. **(B)**. Standard deviation measures the variance from the mean; the more spread out a set is, the higher the deviation. The set in Quantity B is the same as the one in Quantity A, but with the smallest number *even smaller* and the largest number *even larger*, so the set in Quantity B is more spread out and thus has a greater standard deviation.

5. **(D)**. An isosceles triangle has two sides that are equal and a third side that is a different length. The isosceles triangle in this question has a perimeter of 28 and shortest side of length 8. Now, suppose that the shortest side is the one that is repeated, such that the triangle has two sides of length 8 and one other side of length x . This would mean:

$$8 + 8 + x = \text{Perimeter}$$

$$16 + x = 28$$

$$x = 12$$

This triangle would have lengths 8, 8, and 12 as the three legs. Test this triangle via the Third Side Rule: the length of any side of a triangle must be greater than the difference between the other two sides and less than the sum of the other two sides. The third side (x) must be greater than $8 - 8 = 0$ and less than $8 + 8 = 16$. Since 12 is between 0 and 16, this is a legal triangle.

On the other hand, consider the possibility that the other side, x , is repeated and the length 8 is used only once. In this case:

$$x + x + 8 = 28$$

$$2x = 20$$

$$x = 10$$

The sides of this triangle are 10, 10, and 8. Test this triangle via the Third Side Rule: the third side (8) must be greater than $10 - 10 = 0$ and less than $10 + 10 = 20$. Since 8 is between 0 and 20, this is a legal triangle.

For one triangle, the quantities in Quantity A and Quantity B would be equal, but for the other, Quantity B would be greater than Quantity A. Therefore, the relationship cannot be determined from the information given.

6. **(B)**. $(3 - z)(z + 4) = 0$, so either $(3 - z)$ or $(z + 4)$ must equal 0:

$$3 - z = 0$$

$$z = 3$$

OR

$$z + 4 = 0$$

$$z = -4$$

Thus, z is either 3 or -4 . Either way, Quantity B is greater.

7. **(D)**. If $ad < 0$, a and d have opposite signs. Because $a > d$, a must be positive and d must be negative. Similarly, if $ab > 0$, a and b have the same sign, so a and b are both positive. The remaining variable c can be positive, 0, or negative and still fall between b and d . If c is 0, the two quantities are equal. If c is positive, Quantity A is positive and

Quantity B is negative. If c is negative, Quantity B is positive and Quantity A is negative. The relationship cannot be determined from the information given.

Alternatively, pick numbers. If $a = 4$, $b = 3$, $c = 2$, and $d = -1$, then all the criteria of the problem are fulfilled and Quantity A is greater. But if $a = 4$, $b = 3$, $c = -5$, and $d = -10$, then all the criteria of the problem are still fulfilled, but Quantity B is greater.

8. $\frac{6}{1}$ **(or any equivalent fraction)**. Solve one equation for a single variable, and substitute into the other equation:

Eq. (1): $12b = 2g$	Eq. (2): $4g - 3b = 63$
$6b = g$	Isolate g in Eq. (1). Divide by 2.
$4(6b) - 3b = 63$	Substitute $(6b)$ for g in Eq. (2).
$24b - 3b = 63$	Solve for b . Simplify.
$21b = 63$	Combine like terms.
$b = 3$	Divide by 21.
$12(3) = 2g$	Substitute (3) for b in Eq. (1). Solve for g .
$36 = 2g$	Simplify.
$g = 18$	Divide by 2.

Since $b = 3$ and $g = 18$, $\frac{g}{b} = \frac{6}{1}$.

9. **$3^{12}(2)$ and $9^6(2)$ only.** To simplify $81^3 + 27^4$, note that both bases are powers of 3. Rewrite the bases and combine:

$$\begin{aligned}
 81^3 + 27^4 &= \\
 (3^4)^3 + (3^3)^4 &= \\
 3^{12} + 3^{12} &= \\
 3^{12}(1 + 1) &= \\
 3^{12}(2) &
 \end{aligned}$$

Since $3^{12}(2)$ appears in the choices, this is one answer. However, this is an “indicate all” question, so you should check whether any other choices are equivalent. One other choice, $9^6(2)$, also qualifies, since $9^6(2) = (3^2)^6(2) = 3^{12}(2)$.

10. **(B)**. Of the animals, 26% are mammals and about a quarter *of those* were raised in the wild: $\frac{1}{4}$ of 26% = about 6.5%.

Of the animals, 8% are amphibians and about three quarters *of those* were raised in captivity: $\frac{3}{4}$ of 8% = about 6 %.

In total, these two categories account for about 12% of all the zoo’s animals.

11. **(B)**. To solve this question, imagine that there were originally 100 animals in the zoo. If the zoo gives away all the insects and fish, then there are 38 fewer animals ($20 + 18$) in the zoo, or 62. But there are still 10 birds, which now make up about 16% of the zoo’s animals (use your calculator to find this if you don’t feel comfortable estimating). Of those, a little more than half were raised in the wild. Among the choices, only 9% is a little more than half of 16%.

12. **(C).** If the zoo has 80 birds, which make up 10% of the total number of animals at the zoo, then there are 800 animals total. To correctly calculate how many birds must be added, realize that any birds added increases not only the subtotal of 80 birds but also the total of 800 animals. If adding new animals (rather than trading reptiles for birds, for example), you cannot just double the number of birds to double the percent of the animals that are birds!

Thus, use the following inequality:

$$\frac{80 + x}{800 + x} \geq \frac{20}{100}$$

$$100(80 + x) \geq 20(800 + x)$$

$$8,000 + 100x \geq 16,000 + 20x$$

$$80x \geq 8,000$$

$$x \geq 100$$

At least 100 birds must be added such that at least 20% of the animals at the zoo would be birds (check: there would be 180 birds among 900 animals, or 20% of the total).

13. **(C).** This question is a tricky one, because even though it never uses the word *average* or the word *ratio*, it's more or less a combined ratio and averages question. The trail mix is nuts, chocolate, and dried fruit in a ratio of 3 : 1 : 2. For every 6 pounds of trail mix, there are 3 pounds of nuts, 1 pound of chocolate, and 2 pounds of dried fruit.

The cost of 6 pounds of trail mix is $3x + y + 2z$. However, to solve for the cost of one pound, divide by 6. You could also think of this as a kind of average:

$$\text{Average} = \frac{(\text{Sum})}{(\# \text{ of terms})} = \frac{(3x + y + 2z)}{6}, \text{ where each "term" is a pound.}$$

This is choice (C). Alternatively, pick numbers. For example:

$$x = 6$$

$$y = 5$$

$$z = 2$$

In this example, 3 pounds of nuts that cost $x = 6$ dollars per pound plus 1 pound of chocolate that costs $y = 5$ dollars per pound plus 2 pounds of dried

fruit that costs $z = 2$ dollars per pound would cost:

$$3(6) + 1(5) + 2(2) = 27$$

Thus, 6 pounds of trail mix (3 lb. nuts + 1 lb. chocolate + 2 lb. dried fruit) would cost \$27. So, 1 pound would cost one-sixth of that: $\frac{27}{6}$ or $\frac{9}{2}$ dollars, which is \$4.50.

Now, plug $x = 6$, $y = 5$, and $z = 2$ into the choices to see which answer yields \$4.50. Only (C) works.

14. **(E)**. Since $3^4 = 81$, $z = 81$. So, $(3^z)^z$ is equal to $(3^{81})^{81} = 3^{81 \times 81} = 3^{6,561}$.

15. **“Multiply the incorrect product by 100” and “Divide the incorrect product by 0.01” only.** Since 0.03 is 100 times greater than 0.0003, when Maurice accidentally divided by 0.03 instead of 0.0003, he divided by a number 100 times too big. Thus, multiplying by 100 will correct the error. Thus, the first choice is correct.

However, dividing by any quantity is the same as multiplying by its reciprocal. So, multiplying by 100 is the same as dividing by 0.01. Thus, the fourth choice is also correct.

Alternatively, pick a number. Divide by both 0.03 and 0.0003, and then check each answer to see which corrects the error. Choose 12 as the original number.

12 divided by 0.03 = 400	INCORRECT RESULT
12 divided by 0.0003 = 40,000	CORRECT RESULT

Now, perform the operation in each answer choice on the incorrect product, 400, to see which operations turn that product into 40,000. The first and the fourth operations listed as choices both work.

16. **(C).** Begin by constructing a function describing the situation in the problem. Using E for expenses, x for the number of clients, c for the expense per client, and f for fixed costs:

$$E(x) = xc + f$$

In words, expense as a function of the number of clients equals the number of clients multiplied by the variable cost per client, plus the fixed cost.

In 2009, the company served 450 clients and its total expense was \$830,000. Thus:

$$830,000 = 450c + f$$

In 2010, the company served 510 clients and its total expense was \$896,000. Thus:

$$896,000 = 510c + f$$

Since it is easier to isolate f than c in each equation, get f by itself for each equation and then set the opposite sides equal:

$$830,000 = 450c + f$$

$$f = 830,000 - 450c$$

$$896,000 = 510c + f$$

$$f = 896,000 - 510c$$

$$830,000 - 450c = 896,000 - 510c$$

$$830,000 + 60c = 896,000$$

$$60c = 66,000$$

$$c = 1,100$$

Plug $c = 1,100$ into either equation to find f :

$$f = 830,000 - 450(1,100)$$

$$f = 335,000$$

Alternatively, subtract $\$896,000 - \$830,000$ to get $\$66,000$, which must be the cost difference between serving 450 clients and serving 510 clients (a difference of 60 clients). Divide $\$66,000$ by 60 clients to get $\$1,100$, the variable cost per client. Then, multiply $\$1,100 \times 450 = \$495,000$ to get the variable cost of serving 450 clients, not counting the fixed cost. Finally, subtract this figure from the total cost of serving 450 clients to get the fixed cost: $\$830,000 - \$495,000 = \$335,000$. The numbers should look familiar; the point is that you can “reason through it” without strictly setting up equations.

17. **(A).** First, algebraically manipulate $4x + 5y = 9$ into $y = mx + b$ format, where m is the slope and b is the y -intercept:

$$4x + 5y = 9$$

$$5y = -4x + 9$$

$$y = -\frac{4}{5}x + \frac{9}{5}$$

Since $m = -\frac{4}{5}$, the slope is $-\frac{4}{5}$. Perpendicular lines have negative reciprocal

slopes. Thus, the correct answer has a slope of $\frac{5}{4}$. Only choice (A) qualifies.

18. **(A).** If the tens digit is to be randomly selected from the digits 0 to 9, there are ten possibilities for the completed number. Using your calculator, divide each by 9 to see which ones are multiples of 9:

809 ← not a multiple of 9

819 ← MULTIPLE OF 9

829 ← not a multiple of 9

839 ← not a multiple of 9

849 ← not a multiple of 9

859 ← not a multiple of 9

869 ← not a multiple of 9

879 ← not a multiple of 9

889 ← not a multiple of 9

899 ← not a multiple of 9

The answer is $\frac{1}{10}$, or 0.1.

Alternatively, a number is divisible by 9 if the sum of its digits is a multiple of 9. The existing digits sum to $8 + 9 = 17$, so the addition of 0 through 9 means that the sum of all three digits could be 17 through 26, inclusive. Only one multiple of 9 (i.e., 18) is found in this range.

19. **(E)**. Because the circle is centered at $(0, 0)$ and passes through $(0, -16)$, the radius of the circle is 16. Point P lies on the circle and the y -axis, so it lies exactly one radius above the origin. Point P 's coordinates are therefore $(0, 16)$. To find the distance between $(0, 16)$ and $(-10, -8)$, either use the distance formula, or draw a graph and make a right triangle on which you can use the Pythagorean theorem.

From the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

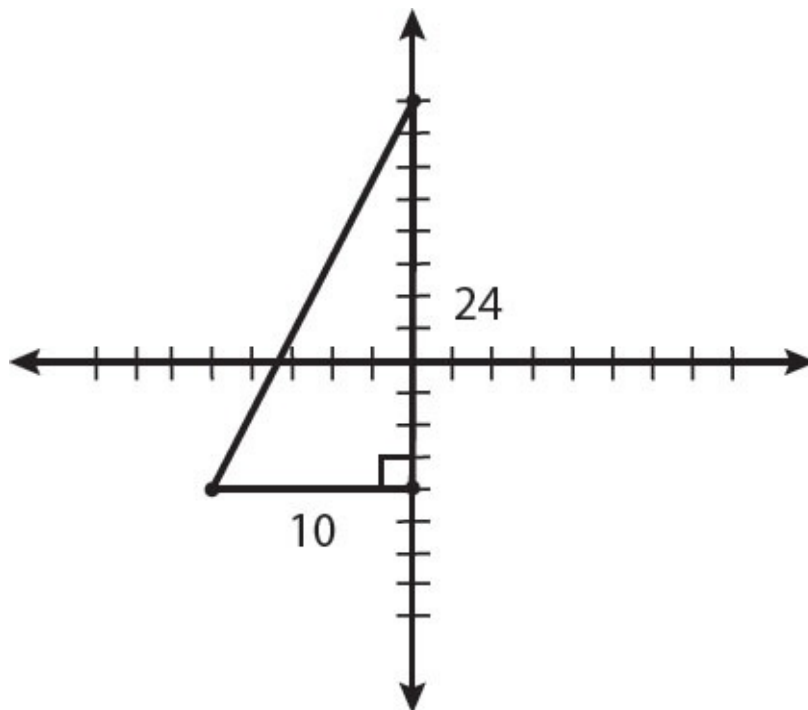
$$d = \sqrt{(-10 - 0)^2 + (-8 - 16)^2}$$

$$d = \sqrt{(-10)^2 + (-24)^2}$$

$$d = \sqrt{676}$$

$$d = 26$$

To use the triangle method, plot $(0, 16)$ and $(-10, -8)$, then drop a line down from $(0, 16)$ to make a right triangle. To do so, you will need to add the third point $(0, -8)$:



Use the coordinates to determine the lengths of the legs, then use the Pythagorean theorem (the hypotenuse is d):

$$24^2 + 10^2 = d^2$$

$$576 + 100 = d^2$$

$$676 = d^2$$

$$d = 26$$

20. **(C).** If $f(-0.5) = 0$, then the answer is 0 when $x = -0.5$. For each choice, plug in -0.5 for x . Only if the result is 0 could the choice be $f(x)$:

(A) $2x + 2 = 2(-0.5) + 2 = -1 + 2 = 1$

(B) $4x - 2 = 4(-0.5) - 2 = -2 - 2 = -4$

(C) $4x^2 - 1 = 4(-0.5)^2 - 1 = 4(0.25) - 1 = 1 - 1 = 0$, CORRECT

(D) $x^2 - 1 = (-0.5)^2 - 1 = 0.25 - 1 = -0.75$

(E) $(-x)^2 - 2.5 = -(-0.5))^2 - 2.5 = (0.5)^2 - 2.5 = 0.25 - 2.5 = -2.25$

Math Practice Section 3: Hard Difficulty

Math Practice Section: Hard

20 Questions

35 Minutes

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box , you are to enter your own answer in the

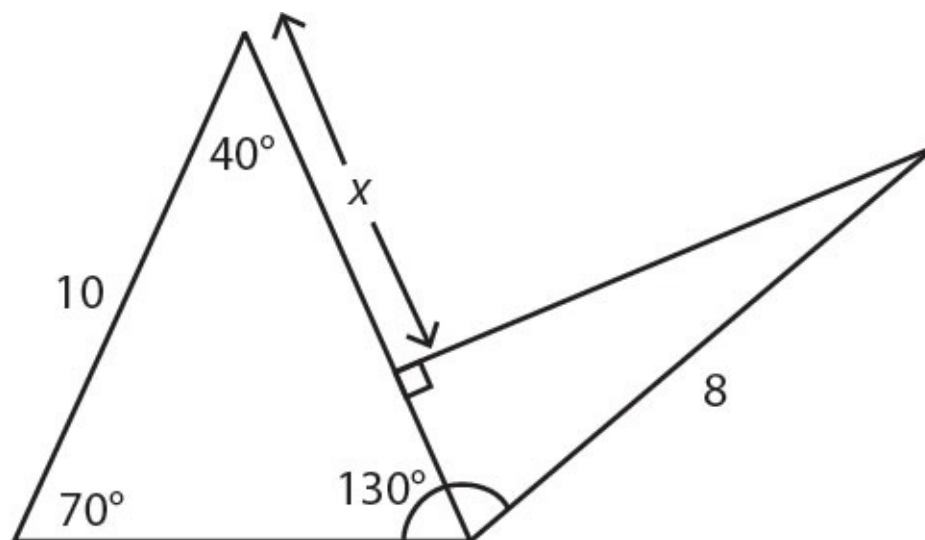
box. For questions followed by a fraction-style numeric entry box

, you are to enter

your answer in the form of a fraction. You are not required to reduce fractions. For example, if the

answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as xy -planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, are drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.



	<u>Quantity A</u>	<u>Quantity B</u>
1.	x	6
<hr/>		

	<u>Quantity A</u>	<u>Quantity B</u>
2.	$(z^6)^x \times z^{3x}$	z^{9x}
<hr/>		

For a group of test-takers, the scores on an aptitude test were normally distributed, had a mean of 154, and a standard deviation of 3.

	<u>Quantity A</u>	<u>Quantity B</u>
3.	The fraction of test-takers in the group who scored greater than 158	$\frac{1}{3}$

$$3x + 5y + 2z = 20$$

$$6x + 4z = 10$$

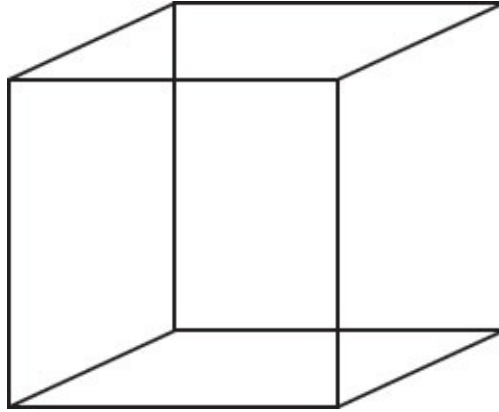
	<u>Quantity A</u>	<u>Quantity B</u>
4.	y by itself	2

Romero Automobiles sells cars only from manufacturer X and manufacturer Y. The range of the list prices of the cars from manufacturer X is \$22,000. The range of the list prices of the cars from manufacturer Y is \$15,000.

	<u>Quantity A</u>	<u>Quantity B</u>
5.	The range of the list prices of all automobiles sold by Romero Automobiles	\$22,000

The operation # is defined by $x\# = \frac{1}{x} + x$.

	<u>Quantity A</u>	<u>Quantity B</u>
6.	$(4\#)\#$	4.5



The cube above has side length of 4.

Quantity A

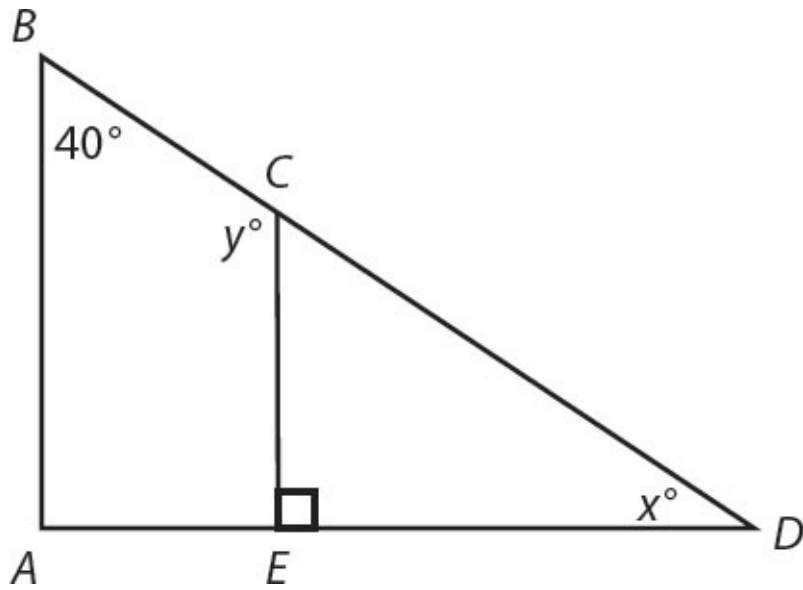
7. After selecting one vertex of the cube, the number of straight line segments longer than 4 that can be drawn from that vertex of the cube to another vertex of the cube.

Quantity B

When the cube is placed on a flat surface, the maximum number of edges of the cube that can be touching the flat surface at once.

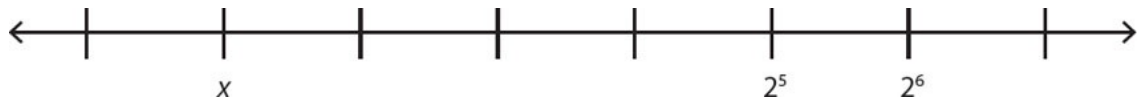
-
8. If $160^2 = 16x$, then x is equivalent to which of the following?

- (A) 10
- (B) $2^3 5$
- (C) $2^2 5^2$
- (D) $2^6 5^2$
- (E) $2^6 5^3$



9. In the triangle shown above, BA is parallel to CE . What is the value of $x + y$?

10. If the tick marks on the number line below are evenly spaced, what value is represented by x ?



- (A) 2^0
(B) 2
(C) $(-2)2^5$
(D) $(-3)2^5$
(E) $(-4)2^5$
11. If the volume of a cube is v , what is the surface area of the cube in terms of v ?

- (A) $6\sqrt{v}$
(B) $(\sqrt[2]{v})^3$
(C) $6(\sqrt[2]{v})^3$
(D) $(\sqrt[3]{v})^2$
(E) $6(\sqrt[3]{v})^2$

12. What is the area of an equilateral triangle with vertices at $(-1, -3)$, $(9, -3)$, and (m, n) where m and n are both positive numbers?

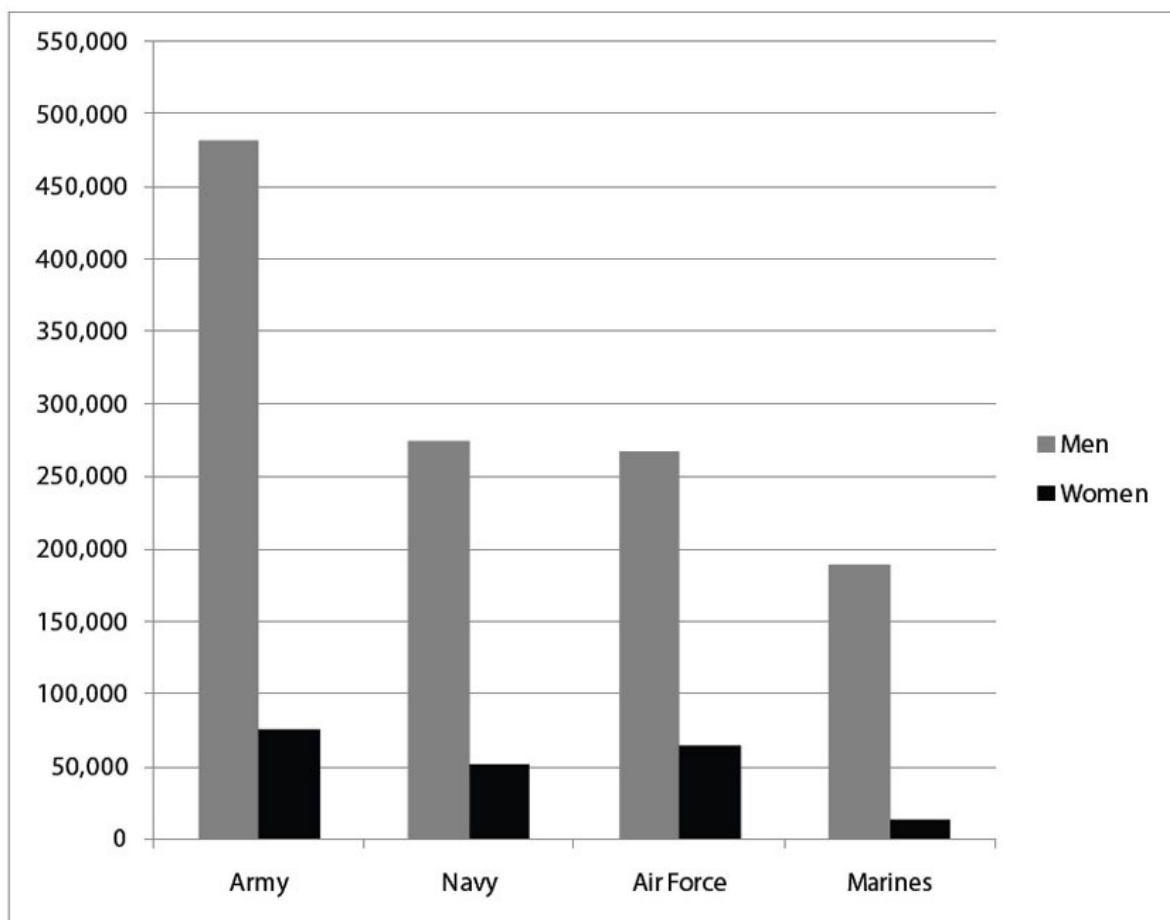
- (A) $25\sqrt{2}$
(B) $50\sqrt{2}$
(C) $10\sqrt{3}$
(D) $25\sqrt{3}$
(E) $50\sqrt{3}$

Questions 13–15 are based on the following charts.

Marital Status of Military Personnel by Gender and Branch

Marital Status		Army	Navy	Air Force	Marines
Single, no children less than 18 years old	Men	164,513	107,349	94,800	90,949
	Women	27,492	24,757	25,247	6,338
Single, with children less than 18 years old	Men	26,571	10,506	9,544	4,807
	Women	11,037	5,859	6,313	1,263
Married, spouse is also military personnel or retired military	Men	15,058	8,638	20,760	4,719
	Women	14,633	8,832	18,574	3,676
Married, spouse is a civilian	Men	275,953	147,255	142,573	88,233
	Women	21,687	11,175	13,982	1,858

Number of Military Personnel by Gender and Branch



13. Women who are single with children less than 18 years old are the greatest percent of all women within which military branch?
- (A) Army
 - (B) Navy
 - (C) Air Force
 - (D) Marines
 - (E) It cannot be determined from the information given.
14. If a man whose spouse is also military personnel or retired military were to be selected at random, what would the probability be that he was not in the Air Force?
- (A) 72%
 - (B) 58%
 - (C) 42%
 - (D) 24%
 - (E) 13%
15. Which of the following expressions approximates the number of women who would have to enlist in the Army to make the fraction of Army personnel who are women equal the fraction of Air Force personnel who are women?

(Assume that the number of men in the Army and the number of men and women in the Air Force remain unchanged from what is shown in the tables.)

- (A) $\frac{482,000 - 268,000}{75,000 - 64,000}$
- (B) $\frac{(482,000)(64,000) - (75,000)(268,000)}{482,000}$
- (C) $\frac{(482,000)(75,000) - (64,000)(268,000)}{482,000}$
- (D) $\frac{(482,000)(75,000)}{268,000} - 64,000$

$$(E) \quad \frac{(482,000)(64,000)}{268,000} - 75,000$$

16. A cable car travels from Seabreeze to Resortville, making two stops in between. Between Seabreeze and the first stop, the cable car travels $\frac{1}{3}$ of the total distance between Seabreeze and Resortville. Between the first stop and the second stop, the cable car travels $\frac{3}{5}$ of the remaining distance between the first stop and Resortville. What fraction of the entire distance from Seabreeze to Resortville remains between the second stop and Resortville?

- (A) $1 - \frac{1}{3} - \frac{3}{5}$
- (B) $1 - \frac{1}{3} - \frac{3}{5} \left(\frac{1}{3} \right)$
- (C) $1 - \frac{1}{3} - \frac{3}{5} \left(1 - \frac{1}{3} \right)$
- (C) $1 - \frac{1}{3} - \frac{1}{3} \left(1 - \frac{3}{5} \right)$
- (D) $1 - \frac{1}{3} - \frac{1}{5} \left(1 - \frac{1}{3} - \frac{1}{5} \right)$

17. If p and q are integers and $20p + 3q$ is odd, which of the following must be odd?

- (A) $p - q$
- (B) $p + 2q$
- (C) $3p + q$
- (D) $2p + q^2$
- (E) $3p + 3q$

18. In a study, 4,400 participants were surveyed regarding side effects of a new medication, and x percent reported experiencing drowsiness. If x is rounded to the nearest integer, the result is 8. Which of the following could be the number of survey participants who reported experiencing drowsiness?

Indicate all such values.

☐ 325

☐ 330

☐ 352

☐ 375

19. $\frac{5^3(4^{45} - 4^{43})27}{225^2}$ is equivalent to which of the following?

- (A) 4^{43}
- (B) 4^{45}
- (C) $4^{90}5^3$
- (D) $4^{86}5^33^3$
- (E) $4^{90}5^3 3^3$

20. The following table shows the price of a plane ticket for an April 1st flight, based on the date of purchase.

Price	When Purchased On
\$210	March 16
\$168	March 2
\$140	March 1

Harpreet purchased a ticket on March 1st. If he had purchased the ticket on March 2nd, he would have paid x percent more. If he had purchased the ticket on March 16th, he would have paid y percent more than he would have paid on March 2nd. What is the positive difference between x and y ?

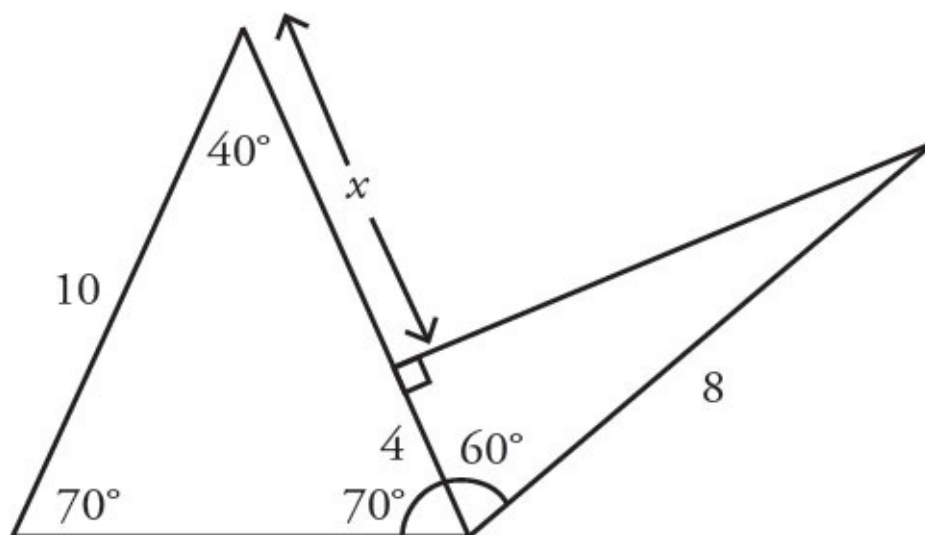
- (A) 5
- (B) 14
- (C) 20
- (D) 25
- (E) 28

Answers to Math Practice Section 3

1. **(C).** The leftmost triangle has two angles labeled 40° and 70° . Subtract these from 180° (the sum of the angles in any triangle) to determine that the third angle is 70° . Subtract $130 - 70 = 60^\circ$ to get the measure of the adjoining angle in the rightmost triangle.

Since the leftmost triangle is isosceles, the two long sides are each equal to 10.

Since the rightmost triangle is a 30–60–90 triangle, the sides are in the proportion $x : \sqrt{3}x : 2x$. Because the hypotenuse is 8 and also the $2x$ in the ratio, the shortest leg of this triangle is $x = 4$.



To calculate x , subtract: $10 - 4 = 6$. The two quantities are equal.

2. **(C).** The terms in Quantity A have the same base, so add the exponents: $(z^6)^x \times z^{3x} = z^{6x} \times z^{3x} = z^{9x}$. The two quantities are equal. Note that $(z^6)^x$ is interchangeable with $(z^x)^6$ and z^{6x} .

3. **(B).** For a normal distribution, approximately $\frac{2}{3}$ of the values are within 1 standard deviation of the mean. Thus, roughly $\frac{1}{6}$ of the population is more than a deviation above the mean, and $\frac{1}{6}$ is more than a deviation below.

Thus, about $\frac{1}{6}$ of the test-takers would score greater than 157 ($154 + 3 = 157$,

1 standard deviation above the mean), so an even smaller fraction of the test-takers would score greater than 158.

4. **(A).** In order to isolate y , eliminate both x and z . Because there are only two equations, both x and z must be eliminated at the same time if the value of y is to be determined.

Notice that the coefficients for x and z in the second equation (6 and 4, respectively) are exactly double their coefficients in the first equation (3 and 2, respectively). Divide the second equation by 2, making the coefficients the same.

$$3x + 5y + 2z = 20 \quad \rightarrow \quad 3x + 5y + 2z = 20$$

$$6x + 4z = 10 \quad \rightarrow \quad 3x + 2z = 5$$

Now subtract the second equation from the first:

$$\begin{array}{r}
 3x + 5y + 2z = 20 \\
 -(3x + 2z = 5) \\
 \hline
 5y = 15 \\
 y = 3
 \end{array}$$

Quantity A is greater.

5. **(D)**. The range of list prices of automobiles is found by subtracting the price of the least expensive automobile from the price of the most expensive automobile. Given just the range, there is not enough information to determine the maximum and minimum list price vehicles from either manufacturer. Before selecting (D), though, you should try to prove (D). Construct two examples in which the list prices of the cars from manufacturer X have a range of \$22,000 and the list prices of the cars from manufacturer Y have a range of \$15,000, but the overall ranges are drastically different:

EXAMPLE 1:

List prices of manufacturer X's cars range from \$10,000 to \$32,000.

List prices of manufacturer Y's cars range from \$10,000 to \$25,000.

Here, the overall range is the same as X's range, which is \$32,000 – \$10,000 = \$22,000.

EXAMPLE 2:

List prices of manufacturer X's cars range from \$10,000 to \$32,000.

List prices of manufacturer Y's cars range from \$100,000 to \$115,000.

Here, the overall range is \$115,000 – \$10,000 = \$105,000.

In Example 1, the range = \$22,000 and the quantities are equal. In Example 2, Quantity A is much greater than Quantity B. It is not possible to make the range any smaller than \$22,000 (the minimum range of all the prices cannot be *smaller* than the larger of the two ranges of each manufacturer's prices), but it can get much, much larger.

Note that the testing done above was very important! If Quantity B had read "\$21,999," the answer would be (A) rather than (D).

6. **(B)**. Start inside the parentheses (according to PEMDAS, always deal with parentheses first).

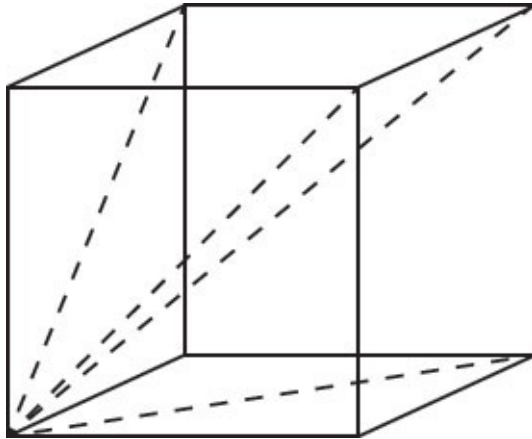
$$4\# = \frac{1}{4} + 4, \text{ or } \frac{17}{4}.$$

Since $4\# = \frac{17}{4}$, plug $\frac{17}{4}$ in for x to get $(4\#)\#$.

$$\text{Thus, } (4\#)\# = \frac{\frac{1}{17}}{\frac{17}{4}} + \frac{17}{4} = \frac{4}{17} + \frac{17}{4}.$$

While you could find a common denominator, it is more efficient to ballpark the value or use the calculator. Ballparking, $\frac{4}{17}$ is less than 0.25 and $\frac{17}{4}$ is exactly 4.25, so the sum is less than 4.5. Using the calculator, $\frac{4}{17} + \frac{17}{4}$ is about 4.485. Quantity B is greater.

7. **(C).** If a cube has side length of 4, all of the “straight line segments” connecting vertices of the cube *along an edge of the cube* will have length of 4. The only straight line segments between vertices that are longer than 4 are those that go diagonally through the cube or diagonally across a face. From a selected vertex of the cube, there are 3 diagonals across the adjacent faces of the cube, and 1 diagonal through the cube to the opposite vertex:



Thus, Quantity A is 4.

If a cube is placed on a flat surface, the maximum contact occurs when one cube face abuts the surface—and thus 4 cube edges touch the surface. There is no way to make more than 4 cube edges touch the flat surface at once. The two quantities are equal.

8. **(D).** The easiest first step is to divide both sides by 16. To do that, make sure you separate out 160^2 first. Notice that $160^2 = 160 \times 160 = 16 \times 10 \times 160$:

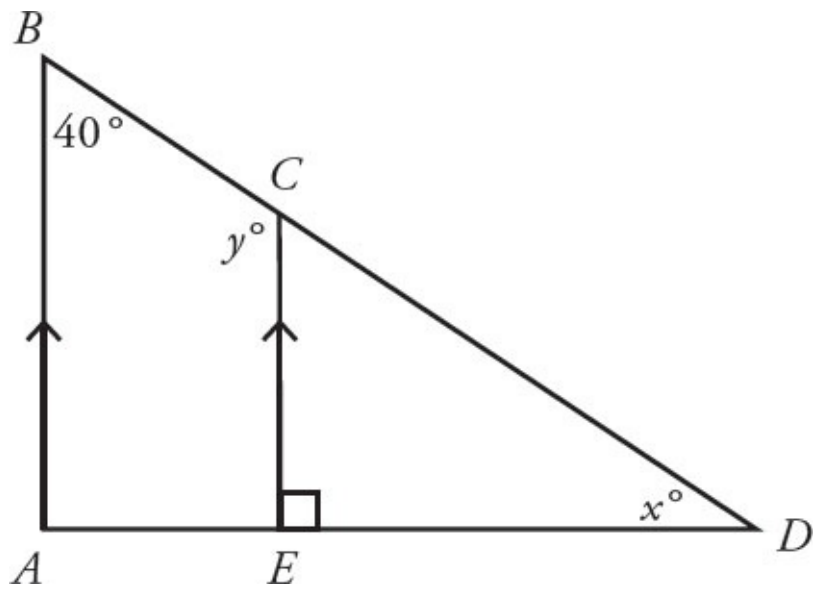
$$16 \times 10 \times 160 = 16x$$

$$10 \times 160 = x$$

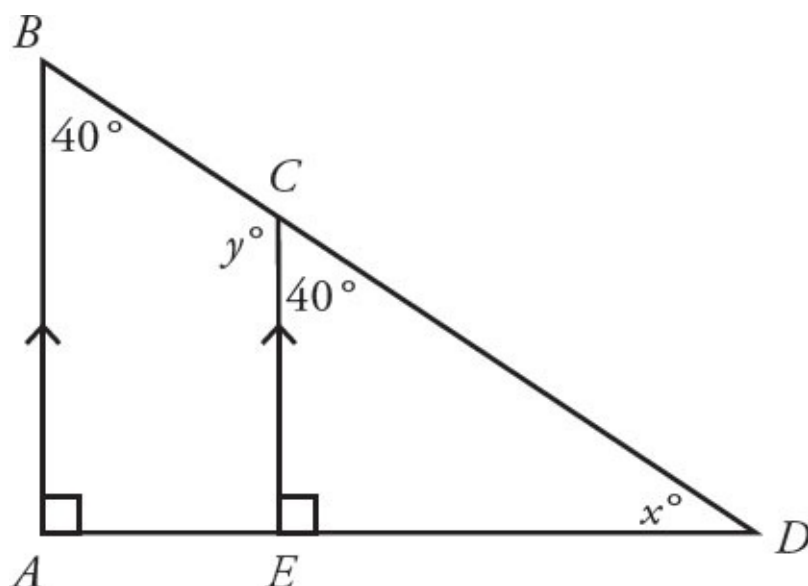
$$1,600 = x$$

None of the answer choices match this, so break 1,600 down into its primes ($1,600 = 100 \times 16 = 25 \times 4 \times 16 = 5^2 \times 2^2 \times 2^4 = 5^2 \times 2^6$) and see which choice is equivalent. Alternatively, multiply out the answer choices to see which equals 1,600. The correct choice is (D).

9. **190.** Redraw the figure, labeling all information given:



Since BA and CE are parallel, angle B and minor angle C are equivalent, as shown:



The two angles that meet at C make up a straight line, so they sum to 180° :

$$180 = y + 40$$

$$y = 140$$

The three angles of triangle CDE must sum to 180° , so:

$$180 = 40 + 90 + x$$

$$180 = 130 + x$$

$$x = 50$$

Therefore, $x + y = 140 + 50 = 190$.

10. **(D)**. At first glance, you might be tempted to think that each tick mark on this number line corresponds to a power of 2, but remember that powers grow exponentially (i.e., the distance between 2^5 and 2^6 is not the same as the distance between 2^1 and 2^2), whereas the tick marks in the diagram are evenly spaced. So, start by finding the distance between 2^5 and 2^6 .

Since $2^5 = 32$ and $2^6 = 64$, the difference between them is 32. That means the distance between each tick mark on the number line is 32. So to get from 2^5 to x , “walk back,” or subtract, four intervals of 32: $32 - 4(32) = -96$.

Multiply out the answers to see which one equals -96 . Only choice (D) works.

11. **(E)**. There are two ways to solve this question, with smart numbers or

algebra. Start with plugging-in. First, set a value for the volume. In this case, pick a perfect cube, so the side length and all other values will be integers. The smallest perfect cube (other than 1, which you should try never to use when doing plug-in questions) is 8.

A cube with a volume of 8 has a side length of 2, meaning each side has an area of 4. A cube has 6 faces, making the total surface area 24. (The equation for surface area is $\text{Surface Area} = 6s^2$.) The answer to this question is 24, based on these numbers.

Immediately eliminate any answer choices that have the square root of 8, as the result will not be an integer. The answer must be either (D) or (E). The cube root of 8 is 2. Answer choice (D) only squares it, yielding 4. In answer choice (E), that result is multiplied by 6, producing 24, which is the required answer. Thus, the answer is (E).

If you wanted to solve with algebra, you'd need to start by solving for a side of a cube with volume v :

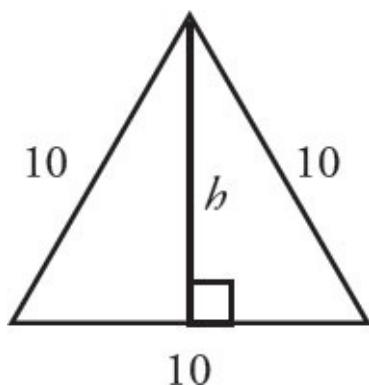
$$v = s^3 \quad \text{so} \quad s = \left(\sqrt[3]{v}\right)$$

The equation for the surface area of a cube is $6s^2$. In this case, substitution for s results in exactly the expression written in answer choice (E).

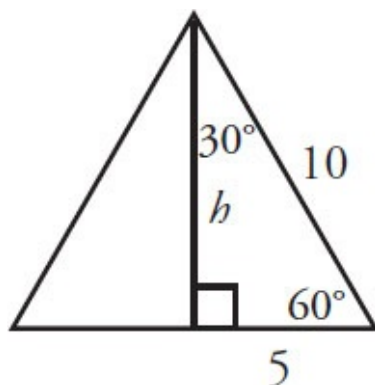
12. **(D)**. To find the area of an equilateral triangle with vertices at $(-1, -3)$, $(9, -3)$, and (m, n) , you do not need to find the values of m and n . To find the area of an equilateral triangle, you only need one side. First, find the distance between $(-1, -3)$ and $(9, -3)$.

Since these two points are on a horizontal line together (they share a y -coordinate), the distance is just the difference between their x -coordinates: $9 - (-1) = 10$.

An equilateral triangle with side 10 will have the same area regardless of where it is placed on an xy -coordinate plane, so the location of m and n is irrelevant. Instead, draw an equilateral triangle with sides equal 10. Drop a height down the middle:



Dividing a 60–60–60 triangle in this way creates two 30–60–90 triangles. The bottom side of the triangle is bisected by the height:



Using the properties of 30–60–90 triangles, h is equal to the shortest side

multiplied by the square root of 3. Thus, $h = 5\sqrt{3}$. (You may also wish to memorize that the height of an equilateral triangle is *always* equal to half the side multiplied by $\sqrt{3}$.)

Find the area of the triangle, using 10 as the base:

$$A = \frac{bh}{2} = \frac{10(5\sqrt{3})}{2} = 25\sqrt{3}$$

13. (A). Women who are single with children less than 18 years old as a fraction of all women is:

$$\frac{\text{\# single women with children}}{\text{\# women}}$$

Find the number of single mothers in each of the four services by looking at the first table, ***Marital Status of Military Personnel by Gender and Branch***. The number of women who are single with children is given as:

Army	11,037
Navy	5,859
Air Force	6,313
Marines	1,263

There are two ways to find the total number of women in each service, though. Either sum the exact number of women in each branch of the service across each of the marital statuses given in the first chart, or read an approximate number of women from the second bar chart, ***Number of Military Personnel by Gender and Branch***, then only bother to sum from the detailed chart if the two answers are very close to each other.

Since using the chart will be faster and GRE problems are designed to be solved quickly, try approximating from the bar chart first. The total number of women in each of the four services is approximately:

Army	75,000
Navy	50,000
Air Force	60,000
Marines	10,000

Now calculate the approximate percent of women who are single mothers in each branch of the service:

$$\text{Army} \quad \frac{11,000}{75,000} \approx 14.7\%$$

Navy

	$\frac{5,900}{50,000} \approx 11.8\%$
Air Force	$\frac{6,300}{60,000} \approx 10.5\%$
Marines	$\frac{1,300}{10,000} \approx 13\%$

The percent looks highest in the Army. At least, reason that the number of single mothers in the Army is about double the number of single women in either the Navy or Air Force, yet the total number of women in the Army is definitely less than double the total number of women in either the Navy or Air Force, making their percentage of women greater in the Army.

Just quickly check the actual totals for the Army and the Marines:

Army: single mothers = 11,037 and total women = 74,849. The percent is 14.7%.

Marines: single mothers = 1,263 and total women = 13,135. The percent is 9.6%.

Thus, the Army has the greatest percentage of women who are single and have dependents under the age of 18.

14. **(B).** The probability that a man whose spouse is also military personnel or retired military is NOT in the Air Force is given by the formula:

$$\frac{\# \text{ of men in the } \square \text{ married, military spouse } \square \text{ category who are NOT in the Air Force}}{\text{total } \# \text{ men in the } \square \text{ married, military spouse } \square \text{ category}}$$

All of the information needed to calculate both of these numbers is in the first table, ***Marital Status of Military Personnel by Gender and Branch.***

The total number of men married to a military spouse or retired military in each of the four services:

$$15,058 + 8,638 + 20,760 + 4,719 = 49,175$$

Then just subtract the number of Air Force men in this category to get the number of men in such marriages who are not in the Air Force:

$$49,175 - 20,760 = 28,415$$

And finally:

$$\frac{28,415}{49,175} = 0.5778 \approx 58\%$$

15. **(E).** In order to solve this problem, make the two ratios equal. The ratio in

question is $\frac{\text{Women}}{\text{Total}}$, but $\frac{\text{Women}}{\text{Men}}$ is simpler and works also because Total

depends only on Women and Men:

$$\frac{\text{resulting } \# \text{ of women in Army}}{\# \text{ of men in Army}} = \frac{\# \text{ of women in Air Force}}{\# \text{ of men in Air Force}}$$

There are two ways to find the number of women and men in the Army and Air Force. Either sum the exact number of women and men in each marital status for each branch of the service in question, or read an approximate number from the second bar chart, ***Number of Military Personnel by Gender***

and Branch.

Since the problem says to approximate and gives numbers in the answer choices that can serve as guidelines, approximation from the bar chart will be good enough.

The important thing is to focus on the *structure* of the math. Since adding women to the Army will change the number of women in the Army, use a variable to represent the additional women. Let x represent the number of women who would have to enlist in the Army in order to make the ratios equal.

$$\frac{\text{current \# of women in Army} + x}{\text{\# of men in Army}} = \frac{\text{\# of women in Air Force}}{\text{\# of men in Air Force}}$$

From the bar chart, look up the approximate numbers:

	<u>Army</u>	<u>AF</u>
# of women	75,000	60,000
# of men	475,000	270,000

The next step is to plug these approximate numbers into the equation and solve for x :

$$\frac{75,000 + x}{475,000} = \frac{60,000}{270,000}$$

$$\rightarrow 75,000 + x = 475,000 \times \frac{60,000}{270,000}$$

$$\rightarrow x = 475,000 \times \frac{60,000}{270,000} - 75,000$$

Looking at the answer choices, structurally, the answer must be (D) or (E), and the numbers in (E) are a better fit to the numbers approximated from the chart.

16. **(C).** Since the question concerns the “fraction of the entire distance from Seabreeze to Resortville,” think of the entire distance as equal to 1. Between Seabreeze and the first stop, the cable car travels $\frac{1}{3}$, leaving $\frac{2}{3}$ left to travel.

Between the first stop and the second stop, the cable car travels $\frac{3}{5}$ of the

remaining $\frac{2}{3}$, or $\frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$.

So far, the cable car has gone $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$. Thus, the remaining distance is $1 - \frac{11}{15} = \frac{4}{15}$. Only choice (C) is equal to $\frac{4}{15}$, although this takes some

manipulation of the choices to check.

Alternatively, construct a formula. The first leg of the journey leaves $1 - \frac{1}{3}$

left to travel. The second leg of the journey subtracts another $\frac{3}{5}$ of the

remaining $1 - \frac{1}{3}$, or $\frac{3}{5}\left(1 - \frac{1}{3}\right)$. Thus, the correct expression is

$$1 - \frac{1}{3} - \frac{3}{5}\left(1 - \frac{1}{3}\right).$$

17. **(D)**. If p and q are integers, then $20p$ is even regardless of whether p is even or odd. Since $20p + 3q$ is odd, $3q$ must be odd. If $3q$ is odd, then q is odd. Thus, q is odd, but p could be odd or even. The correct answer must be odd regardless of whether p is odd or even.

If p is odd, (A) is even, (B) is odd, (C) is even, (D) is odd, and (E) is even. Since the correct answer choice is the one that *must* be odd, only (B) and (D) are possibilities.

If p is even, (B) is even and (D) is odd. Thus, choice (D) is definitely odd and is the correct answer.

18. **330 and 352 only.** Using your calculator, convert each choice to a percent, and determine whether that percent would round up or down to 8%:

$$\frac{325}{4,400} \times 100 = 7.386\ldots\% \text{ This number would round down to } 7\%, \text{ not up to } 8\%.$$

$$\frac{330}{4,400} \times 100 = 7.5\%. \text{ This number rounds up to } 8\%, \text{ and thus this choice is correct.}$$

$$\frac{352}{4,400} \times 100 = 8\% \text{ exactly, and thus this choice is correct.}$$

$$\frac{375}{4,400} \times 100 = 8.522\ldots\% \text{ This number would round up to } 9\%, \text{ not down to } 8\%.$$

19. **(A).** When dealing with exponents, try to get (almost) everything in terms of common prime bases. Since all the answer choices have a base 4, leave those terms alone for now:

$$5^3 \text{ (already simplified)}$$

$$27 = 3^3$$

$$225^2 = 25^2 \times 9^2 = (5^2)^2 \times (3^2)^2 = 5^4 \times 3^4$$

Replacing all of these in the equation yields: $\frac{5^3(4^{45} - 4^{43})3^3}{5^4 3^4}$.

Cancel 5's and 3's in the top and bottom: $\frac{(4^{45} - 4^{43})}{5 \times 3} = \frac{(4^{45} - 4^{43})}{15}$.

Factor 4^{43} out of both terms in the numerator and simplify:

$$\frac{4^{43}(4^2 - 4^0)}{15} = \frac{4^{43}(16 - 1)}{15} = \frac{4^{43}(15)}{15} = 4^{43}.$$

20. **(A).** On March 1st, the ticket cost \$140. If he had purchased it on March 2nd, Harpreet would have paid \$168, which is \$28 more. To find x , use the

percent change formula:

$$\text{Percent Change} = \left(\frac{\text{Difference}}{\text{Original}} \times 100 \right) \% = \left(\frac{28}{140} \times 100 \right) \% = 20\%$$

Thus, $x = 20$. If he had purchased the ticket on March 16th, he would have paid \$210, which is \$42 more than the \$168 he would have paid on March 2nd. Again, use the Percent Change formula:

$$\left(\frac{42}{168} \times 100 \right) \% = 25\%$$

Thus, $y = 25$ and the positive difference between x and y is 5. ("Positive difference" just means to subtract the smaller number from the bigger one, or to subtract either number from the other and then take the absolute value.)