Answers and Explanations

SECTION 3
Verbal Reasoning
25 Questions with Explanations

For each of Questions 1 to 4, select one answer choice unless otherwise instructed.

Questions 1 to 3 are based on the following reading passage.

Whether the languages of the ancient American peoples were used for expressing abstract universal concepts can be clearly answered in the case of Nahuatl. Nahuatl, like Greek and German, is a language that allows the formation of extensive compounds. By the combination of radicals or semantic elements, single compound words
line 5 can express complex conceptual relations, often of an abstract universal character.

The *tlamatinime* (those who know) were able to use this rich stock of abstract terms to express the nuances of their thought. They also availed themselves of other forms of expression with metaphorical meaning, some probably original, some derived from Toltec coinages. Of these forms, the most characteristic in Nahuatl is the juxtaposition
10 of two words that, because they are synonyms, associated terms, or even contraries, complement each other to evoke one single idea. Used metaphorically, the juxtaposed terms connote specific or essential traits of the being they refer to, introducing a mode of poetry as an almost habitual form of expression.

Description

This passage claims that Nahuatl was used to express abstract universal concepts, by combining semantic elements, and goes on to explain that the *tlamatinime* used these terms to express subtle distinctions.

For the following question, consider each of the choices separately and select all that apply.

1. Which of the following can be inferred from the passage regarding present-day research relating to Nahuatl?

   A Some record or evidence of the thought of the *tlamatinime* is available.

   B For at least some Nahuatl expressions, researchers are able to trace their derivation from another ancient American language.

   C Researchers believe that in Nahuatl, abstract universal concepts are always expressed metaphorically.

Explanation

Choices A and B are correct.

Choice A is correct: the *tlamatinime* are mentioned in the first sentence of the second paragraph, where it says they were able to use Nahuatl's stock of abstract terms "to express the nuances of their thought." This suggests that there is some evidence of what those thoughts were, and therefore Choice A can be inferred.

Choice B is correct: according to the next sentence, Nahuatl speakers used "forms of expression with metaphorical meaning," some of which were probably "original" and others "derived from Toltec coinages." That researchers know certain Nahuatl expressions are derived from Toltec suggests that they are able to trace the derivation of some Nahuatl expressions from another language besides Nahuatl, and therefore Choice B may be inferred.

Choice C is incorrect: the passage says that in Nahuatl there are single compound words that can express conceptual relations of an "abstract universal character" and mentions "other forms of expression with metaphorical meaning," but it does not indicate whether metaphorical words or phrases are the only way that abstract universal concepts are expressed in Nahuatl, or whether researchers believe this about Nahuatl. Therefore Choice C cannot be inferred.

2. Select the sentence in the passage in which the author introduces a specific Nahuatl mode of expression that is not identified as being shared with certain European languages.

Explanation

The passage introduces two specific Nahuatl modes of expression. One is the formation of single compound words that are capable of expressing complex conceptual relations (first paragraph); the other is the juxtaposition of two related words to evoke a single idea (second paragraph). In the formation of compounds Nahuatl is described as being "like Greek and German," but the second mode is not identified as being shared with other languages. Therefore the **sixth sentence** ("Of these forms … one single idea") is the best choice.

3. In the context in which it appears, "coinages" (line 9) most nearly means

   (A) adaptations
   (B) creations
   (C) idiosyncrasies
   (D) pronunciations
   (E) currencies

Explanation

"Coinage" has two senses that are represented among the answer choices: in one sense it denotes coins and currency, while in the other it denotes things — especially words —that are invented. The fifth sentence draws a contrast between linguistic expressions original to Nahuatl and those derived from Toltec. In this context of original versus derived language, "coinages" means "inventions," not "currencies." Of the answer choices given, "creations" is the nearest equivalent of "coinages" in the sense of "inventions," and therefore **Choice B** is the best answer.

Question 4 is based on the following reading passage.

At a certain period in Earth's history, its atmosphere contained almost no oxygen, although plants were producing vast quantities of oxygen. As a way of reconciling these two facts, scientists have hypothesized that nearly all of the oxygen being produced was taken up by iron on Earth's surface. Clearly, however, **this explanation is inadequate.** New studies show that **the amount of iron on Earth's surface was not sufficient to absorb anywhere near as much oxygen as was being produced.** Therefore, something in addition to the iron on Earth's surface must have absorbed much of the oxygen produced by plant life.

4. In the argument given, the two portions in **boldface** play which of the following roles?

(A) The first is a claim made by the argument in support of a certain position; the second is that position.

(B) The first is a judgment made by the argument about a certain explanation; the second is that explanation.

(C) The first expresses the argument's dismissal of an objection to the position it seeks to establish; the second is that position.

(D) The first sums up the argument's position with regard to a certain hypothesis; the second provides grounds for that position.

(E) The first is a concession by the argument that its initial formulation of the position it seeks to establish requires modification; the second presents that position in a modified form.

Explanation

The passage presents an argument and the question asks you to identify the role the portions highlighted in boldface play in that argument. The first step in responding is to read through the passage quickly to get an understanding of what is being said. Then it is possible to go back and assess how the parts of the passage fit together into an argument.

In this passage the first sentence presents two pieces of information that seem to be in conflict — the atmosphere contained almost no oxygen even though plants were producing so much of it. The second sentence presents a hypothetical explanation that has been proposed for reconciling the discrepancy — that oxygen was absorbed by iron. The next sentence calls this hypothetical explanation inadequate and the following sentence gives a reason for that judgment — that there was insufficient iron for the proposed explanation to work. Finally, the last sentence draws the conclusion that there must have been something in addition to iron to absorb the oxygen.

Since the highlighted portions in the passage represent the main content of the third and fourth sentences, the task in this question is to find the answer choice whose two parts fit those sentences' roles. It can be seen that answer Choice D fits the requirement: the third sentence does sum up the argument's position about a hypothesis, and the fourth sentence gives grounds for the third. Therefore **Choice D** is the correct answer.

For Questions 5 to 8, select one entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

5. In her later years, Bertha Pappenheim was an apostle of noble but already (i)_____

notions, always respected for her integrity, her energy, and her resolve but increasingly out of step and ultimately (ii)_____ even her own organization.

Blank (i)

(A) anachronistic
(B) accepted
(C) exotic

Blank (ii)

(D) emulated by
(E) appreciated by
(F) alienated from

Explanation

The sentence is clearly conveying a contrast since "but" is used twice to indicate something positive and something negative about Pappenheim. The clue to the negative aspect is in the later part of the sentence, where "out of step" leads both to "anachronistic" as the answer for the first blank and "alienated from" as the answer for the second.

Thus, the correct answer is **anachronistic** (Choice A) and **alienated from** (Choice F).

6. The reception given to Kimura's radical theory of molecular evolution shows that when _____ fights orthodoxy to a draw, then novelty has seized a good chunk of space from convention.

(A) imitation
(B) reaction
(C) dogmatism
(D) invention
(E) caution

Explanation

The sentence sets up two parallel, contrasting concepts. The word in the blank must contrast with "orthodoxy," and since "convention" in the second contrasting pair is synonymous with "orthodoxy," the correct answer should be roughly synonymous with "novelty." The word "invention" is the best choice.

Thus, the correct answer is **invention** (Choice D).

7. The (i)_____ of Vladimir Nabokov as one of North America's literary giants has thrown the spotlight on his peripheral activities and has thus served to (ii)_____ his efforts as an amateur entomologist.

Blank (i)

(A)	stigmatization
(B)	lionization
(C)	marginalization

Blank (ii)

(D)	foreground
(E)	transcend
(F)	obscure

Explanation

It is possible to analyze this sentence by starting with either blank. Broadly, it states that something that has happened to Nabokov has called attention to some of his peripheral activities. It would hardly make sense for what had happened to be either stigmatization or marginalization, since both of those activities represent a turning away from him, not a calling attention to him. So the correct answer for the first blank is "lionization," since to lionize means to treat as important. Then, since we are told that Nabokov is a literary giant, entomology must be one of his peripheral activities, so the correct answer for the second blank must be "foreground," which also means "call attention to." Spotlighting something would not result in transcending it or obscuring it, so neither of the other choices is correct.

Thus, the correct answer is **lionization** (Choice B) and **foreground** (Choice D).

8. Mathematicians have a distinctive sense of beauty: they strive to present their ideas and results in a clear and compelling fashion, dictated by _____ as well as by logic.

(A)	caprice
(B)	aesthetics
(C)	obligation
(D)	methodologies
(E)	intellect

Explanation

The opening statement attributes a "sense of beauty" to mathematicians, and the remainder of the sentence after the colon spells out that observation. Filling in the blank will supply some aspect that balances "logic" and reinforces the view that mathematicians have a sense of beauty; "aesthetics" is the best choice.

Thus, the correct answer is **aesthetics** (Choice B).

For each of Questions 9 to 13, select <u>one</u> answer choice unless otherwise instructed.

Questions 9 to 12 are based on the following reading passage.

Animal signals, such as the complex songs of birds, tend to be costly. A bird, by singing, may forfeit time that could otherwise be spent on other important behaviors such as foraging or resting. Singing may also advertise an individual's location to rivals or predators and impair the ability to detect their approach. Although these types of cost may be important, discussions of the cost of singing have generally focused on energy costs. Overall the evidence is equivocal: for instance, while Eberhardt found increases in energy consumption during singing for Carolina wrens, Chappell found no effect of crowing on energy consumption in roosters.

To obtain empirical data regarding the energy costs of singing, Thomas examined the relationship between song rate and overnight changes in body mass of male nightingales. Birds store energy as subcutaneous fat deposits or "body reserves"; changes in these reserves can be reliably estimated by measuring changes in body mass. If singing has important energy costs, nightingales should lose more body mass on nights when their song rate is high. Thomas found that nightingales reached a significantly higher body mass at dusk and lost more mass overnight on nights when their song rate was high.

These results suggest that there may be several costs of singing at night associated with body reserves. The increased metabolic cost of possessing higher body mass contributes to the increased overnight mass loss. The strategic regulation of evening body reserves is also likely to incur additional costs, as nightingales must spend more time foraging in order to build up larger body reserves. The metabolic cost of singing itself may also contribute to increased loss of reserves. This metabolic cost may arise from the muscular and neural activity involved in singing or from behaviors associated with singing. For example, birds may expend more of their reserves on thermoregulation if they spend the night exposed to the wind on a song post than if they are in a sheltered roost site. Thomas's data therefore show that whether or not singing per se has an important metabolic cost, metabolic costs associated with singing can have an important measurable effect on a bird's daily energy budget, at least in birds with high song rates such as nightingales.

Description

The passage mentions various ways in which singing is costly to a bird, but soon focuses on the main topic: the energy costs of singing. The second paragraph then discusses a particular experiment designed to assess the energy costs of singing for nightingales, and the third paragraph identifies a range of different associated costs.

9. The primary purpose of the passage is to

(A) compare the different types of cost involved for certain birds in singing
(B) question a hypothesis regarding the energy costs of singing for certain birds
(C) present evidence suggesting that singing has an important energy cost for certain birds
(D) discuss the benefits provided to an organism by a behavior that is costly in energy

Ⓔ describe an experiment that supports an alternative model of how birdsong functions

Explanation

As mentioned above, the first paragraph mentions various costs associated with birdsong, but from that point onward, the focus of the passage is on evidence concerning the energy costs of singing, for nightingales in particular. Thus, the correct answer is **Choice C**. Although the passage mentions other costs, it does not compare them with one another, so Choice A is incorrect. Because the passage does not question any hypotheses, discuss benefits, or advance an alternative model of birdsong, Choices B, D, and E are incorrect.

For the following question, consider each of the choices separately and select all that apply.

10. The passage implies that during the day before a night on which a male nightingale's song rate is high, that nightingale probably does which of the following?

    A Expends less of its reserves on thermoregulation than on other days

    B Stores more energy as body reserves than on other days

    C Hides to avoid predators

Explanation

Choice B is correct.

Choice A is incorrect: the only reference to thermoregulation comes in line 24 and discusses nighttime activity, not daytime activity.

Choice B is correct: the second paragraph explains that birds store energy as fat deposits that can be estimated by measuring body mass, and that body mass at dusk was significantly higher in nightingales on nights when their song rate was higher.

Choice C is incorrect: while the passage does say that singing exposes birds to predators (line 3), it says nothing to suggest that they make special efforts to hide before singing, and in fact it says that nightingales spend extra time foraging (line 20).

11. Select the sentence in the first or second paragraph that presents empirical results in support of a hypothesis about the energy costs of singing.

Explanation

Only two sentences in the relevant portion of the passage contain information that might be considered to be empirical results. The last sentence of the first paragraph contains information about increases in energy consumption but only the last sentence of the second paragraph provides results in support of the only hypothesis in the passage, that nightingales should lose more body mass on nights when their song rate is high. Thus, **sentence 9** ("Thomas found … high") is the correct choice.

For the following question, consider each of the choices separately and select all that apply.

12. It can be inferred from the passage that compared with other costs of singing, which of the following is true of the energy costs of singing?

A They are the single greatest cost to an individual bird.

B They have generally received more attention from scientists.

C They vary less from one bird species to another.

Explanation

Choice B is correct.

Choice A is incorrect: you might infer that energy costs of singing are significant but no information is given to suggest that they are greater than other costs.

Choice B is correct: lines 4—5 say that discussions of the costs of singing have generally focused on energy costs.

Choice C is incorrect: the only mention of differences across species occurs in the discussion of the findings of Eberhardt and Chappell. These findings relate to energy costs alone and, if anything, suggest that energy costs vary considerably.

Question 13 is based on the following reading passage.

In the past ten years, there have been several improvements in mountain-climbing equipment. These improvements have made the sport both safer and more enjoyable for experienced climbers. Despite these improvements, however, the rate of mountain-climbing injuries has doubled in the past ten years.

13. Which of the following, if true, best reconciles the apparent discrepancy presented in the passage?

(A) Many climbers, lulled into a false sense of security, use the new equipment to attempt climbing feats of which they are not capable.

(B) Some mountain-climbing injuries are caused by unforeseeable weather conditions.

(C) Mountain climbing, although a dangerous sport, does not normally result in injury to the experienced climber.

(D) In the past ten years there have been improvements in mountain-climbing techniques as well as in mountain-climbing equipment.

(E) Although the rate of mountain-climbing injuries has increased, the rate of mountain-climbing deaths has not changed.

Explanation

In this question you are asked to identify the fact that would best reconcile the apparent discrepancy that the passage presents. The discrepancy is that despite improvements in mountain climbing equipment that have made climbing safer, the incidence of mountain-climbing injuries has greatly increased. Choice A explains how this could have happened — the improvements in equipment have led climbers to attempt feats that are beyond their level of skill. Therefore, **Choice A** is the correct answer.

None of the other choices provides information that resolves the discrepancy. Neither Choice B nor Choice C relates to conditions that have changed over the relevant ten-year period. Choices D and E do relate to the relevant period. But if, as Choice D says, techniques as well as equipment have improved, that fact by itself only makes the increase in injuries more puzzling. Choice E provides more data about the consequences of climbing accidents, but doesn't suggest any explanation for the increase in injuries.

For Questions 14 to 17, select one entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

14. Unenlightened authoritarian managers rarely recognize a crucial reason for the low levels of serious conflict among members of democratically run work groups: a modicum of tolerance for dissent often prevents _____.

(A) demur
(B) schism
(C) cooperation
(D) compliance
(E) shortsightedness

Explanation

The blank must be filled with a word that describes a problem that a work group can suffer, a problem that can be a cause of (or associated with) serious conflict. Of the answer choices, only "schism" fits this description.

Thus, the correct answer is **schism** (Choice B).

15. The novelist devotes so much time to avid descriptions of his characters' clothes that the reader soon feels that such _____ concerns, although worthy of attention, have superseded any more directly literary aims.

(A) didactic
(B) syntactical
(C) irrelevant
(D) sartorial
(E) frivolous

Explanation

The "concerns" described by the adjective that fills the blank relate to clothing, so "sartorial" fits the blank. Although these concerns could also be described as "irrelevant" or "frivolous," neither of these choices is correct because the sentence identifies the concerns as "worthy of attention."

Thus, the correct answer is **sartorial** (Choice D).

16. Belanger dances with an (i)_____ that draws one's attention as if by seeking to (ii) _____ it; through finesse and understatement, he manages to seem at once intensely present and curiously detached.

Blank (i)

(A)	undemonstrative panache
(B)	unrestrained enthusiasm
(C)	unattractive gawkiness

Blank (ii)

(D)	focus
(E)	overwhelm
(F)	deflect

Explanation

The point of the sentence is to emphasize contradictory aspects of Belanger's dancing: we are told, for example, that he seems "at once intensely present and curiously detached." Looking at the second blank with this point in mind, we can see that the sentence is saying that Belanger draws attention in some way that would not normally be a means of doing so. The only choice that fits, therefore, is "deflect"; focusing or overwhelming attention would certainly be expected to draw it. And since employing "unrestrained enthusiasm" or "unattractive gawkiness" would not be ways of deflecting attention, the correct choice for the first blank is "undemonstrative panache," another paradoxical term, since "panache" means "dash or flamboyance in style."

Thus, the correct answer is **undemonstrative panache** (Choice A) and **deflect** (Choice F).

17. The most striking thing about the politician is how often his politics have been (i) _____ rather than ideological, as he adapts his political positions at any particular moment to the political realities that constrain him. He does not, however, piously (ii) _____ political principles only to betray them in practice. Rather, he attempts in subtle ways to balance his political self-interest with a (iii) _____, viewing himself as an instrument of some unchanging higher purpose.

Blank (i)

(A)	quixotic
(B)	self-righteous
(C)	strategic

Blank (ii)

(D)	brandish
(E)	flout
(F)	follow

Blank (iii)

G	profound cynicism
H	deeply felt moral code
I	thoroughgoing pragmatism

Explanation

Since the politician is portrayed as adapting political positions to political realities, blank (i) should be filled with "strategic," which is also the only choice that provides the required contrast with "ideological." The second blank, *brandishing* political principles is what a politician might do piously, while *flouting* is not pious and *following* principles does not make sense when combined with "betray[ing] them in practice." The third blank requires something that would have to be balanced against "political self-interest" and that would be embraced in service of an "unchanging higher purpose," making "deeply felt moral code" the only viable choice.

Thus, the correct answer is **strategic** (Choice C), **brandish** (Choice D), and **deeply felt moral code** (Choice H).

For each of Questions 18 to 20, select <u>one</u> answer choice unless otherwise instructed.

Questions 18 to 20 are based on the following reading passage.

The condition of scholarship devoted to the history of women in photography is confounding. Recent years have witnessed the posthumous inflation of the role of the hobbyist Alice Austen into that of a pioneering documentarian while dozens of notable senior figures — Marion Palfi, whose photographs of civil-rights activities in the South served as early evidence of the need for protective legislation, to name one — received scant attention from scholars. And, while Naomi Rosenblum's synoptic *History of Women Photographers* covers the subject through 1920 in a generally useful fashion, once she reaches the 1920s, when the venues, forms, applications, and movements of the medium expanded exponentially, she resorts to an increasingly terse listing of unfamiliar names, with approaches and careers summarized in a sentence or two.

line 5

10

Description

The passage expresses dismay at the current state of scholarship concerning the history of women in photography: some figures receive disproportionate attention, and past 1920 Rosenblum's book is too sketchy to be useful.

18. The author of the passage cites Rosenblum's book most likely in order to

A suggest that the works documented most thoroughly by historians of women in photography often do not warrant that attention

B offer an explanation for the observation that not all aspects of the history of women in photography have received the same level of attention

(C) provide an example of a way in which scholarship on the history of women in photography has been unsatisfactory

(D) suggest that employing a strictly chronological approach when studying the history of women in photography may be unproductive

(E) provide support for the notion that certain personalities in women's photography have attained undue prominence

Explanation

As mentioned above, the topic of the passage is the unsatisfactory condition of scholarship devoted to the history of women in photography. Since Rosenblum's book is clearly presented as an example of this unsatisfactory scholarship, **Choice C** is the correct answer. Choice D may seem appealing, because a strictly chronological approach might be inadequate to represent the explosive growth of the field in the 1920s. However, the sentence does not develop this idea, and this is not the reason for mentioning Rosenblum.

For the following question, consider each of the choices separately and select all that apply.

19. Which of the following statements about Marion Palfi is supported by the passage?

[A] Marion Palfi's photographs would have received greater recognition from historians had her work been done in an era when most aspects of photography were static rather than in a state of transition.

[B] Alice Austen has achieved greater notoriety than has Marion Palfi primarily because the subjects that Austen photographed were more familiar to her contemporaries.

[C] In addition to providing a record of certain historical events, Marion Palfi's photographs played a role in subsequent events.

Explanation

Choice C is correct.

Choice A is incorrect: the passage does not state whether the period in which Palfi was working was an era when photography was static or in transition.

Choice B is incorrect: the passage does not state the nature of the subjects Austen photographed, nor compare their relative familiarity to those photographed by Palfi.

Choice C is correct: Palfi's photographs played a role in subsequent events because they served as early evidence of the need for protective legislation.

20. In the context in which it appears, "inflation" (line 2) most nearly means

(A) exaggeration

(B) acquisition

(C) evaluation

(D) distortion

(E) attenuation

Explanation

The term "hobbyist" suggests Austen's relative lack of seriousness as a photographer when compared with "senior figures," yet her role has been elevated to that of a "pioneering documentarian" at the expense of these other figures. Choice D may be appealing in that this elevation could be considered a form of distortion, but Choice A is more specific as well as more in line with the dictionary definition of "inflated" as "expanded to an abnormal or unjustifiable volume or level." Thus, **Choice A**, "exaggeration," is the correct answer.

For Questions 21 to 25, select the <u>two</u> answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole <u>and</u> produce completed sentences that are alike in meaning.

21. The plan, which the engineers said would save the aquifer by reducing pumping to _____ levels, has passed a governmental environmental review but faces opposition from outdoor and environmental groups.

A innocuous

B feasible

C practicable

D minimal

E remedial

F benign

Explanation

If the engineers think that the reduced levels will save the aquifer, they may describe the reduced levels as innocuous, minimal, remedial, or benign. Of these words, only "innocuous" and "benign" produce sentences with the same meaning. The two words "feasible" and "practicable" are similar in meaning, but do not fit the context well, because they imply that the current levels are not feasible or practicable, conflicting with the implication that the current levels, though perhaps undesirable, are nevertheless entirely feasible.

Thus, the correct answer is **innocuous** (Choice A) and **benign** (Choice F).

22. Though feminist in its implications, Yvonne Rainer's 1974 film _____ the filmmaker's active involvement in feminist politics.

A antedated

B cloaked

C portrayed

D preceded

E renewed

F represented

Explanation

The words that fill the blank must fit with the idea that Rainer's film has some feminist implications, but that those are limited compared with her other activities. From the six words offered as answer choices, the pair "antedated" and "preceded" and the pair "portrayed" and "represented" each produce sentences that are similar in meaning. However, only "antedated" and "preceded" make sense in the context of the sentence: Rainer's 1974 film exhibits feminist themes in a limited way because it came before she became active in feminist politics.

Thus, the correct answer is **antedated** (Choice A) and **preceded** (Choice D).

23. Congress is having great difficulty developing a consensus on energy policy, primarily because the policy objectives of various members of Congress rest on such _____ assumptions.

A commonplace

B disparate

C divergent

D fundamental

E trite

F trivial

Explanation

The words that fill the blank must help explain the difficulty of developing a consensus. A lack of agreement on the assumptions that underlie Congress members' policy objectives would contribute to such a difficulty. Accordingly, "disparate" and "divergent" are the best choices because they both indicate disagreement among the members. Although the words "trite" and "trivial" are similar in meaning, triteness and triviality do not help to explain the difficulty in developing a consensus.

Thus, the correct answer is **disparate** (Choice B) and **divergent** (Choice C).

24. During the opera's most famous aria, the tempo chosen by the orchestra's conductor seemed _____, without necessary relation to what had gone before.

A arbitrary

B capricious

C cautious

D compelling

E exacting

F meticulous

Explanation

Any of the offered words could possibly describe a conductor's choice of tempo. However, the phrase "without necessary relation to what had gone before" is presented as an elaboration on the

word in the blank. Among the answer choices, only "arbitrary" and "capricious" could be elaborated that way; none of the other choices would be explained by the final phrase.

Thus, the correct answer is **arbitrary** (Choice A) and **capricious** (Choice B).

25. Because they had expected the spacecraft Voyager 2 to be able to gather data only about the planets Jupiter and Saturn, scientists were _____ the wealth of information it sent back from Neptune twelve years after leaving Earth.

A anxious for

B confident in

C thrilled about

D keen on

E elated by

F eager for

Explanation

In the sentence, the words "expected" and "only" imply that the data received from the spacecraft exceeded scientists' expectations. Therefore, the words that fill the blank should describe a reaction to results that are better than hoped for, and the choices "thrilled about" and "elated by" both express such a reaction. The scientists may well also have been eager for, or keen on, the information, but their eagerness is not well explained by the unexpectedness of the information.

Thus, the correct answer is **thrilled about** (Choice C) and **elated by** (Choice E).

SECTION 4
Verbal Reasoning
25 Questions with Explanations

For Questions 1 to 4, select the <u>two</u> answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole <u>and</u> produce completed sentences that are alike in meaning.

1. Only by ignoring decades of mismanagement and inefficiency could investors conclude that a fresh infusion of cash would provide anything other than a _____ solution to the company's financial woes.

A complete

B fleeting

C momentary

D premature

E trivial

F total

Explanation

The key phrases that indicate how the blank for this question should be completed are "Only by ignoring decades of mismanagement and inefficiency" and "provide anything other than." Taken together, these phrases indicate that the sentence will not envision a very beneficial or successful resolution of the "financial woes." Among the answer choices, "complete" and "total" are quite close in meaning and would clearly create two sentences very similar in meaning. But those two sentences would be internally contradictory, suggesting that doing something unwise would completely solve a problem. "Fleeting" and "momentary" suggest that the event referred to ("a fresh infusion of cash") might have some beneficial effect, but that it would ultimately not resolve the problem.

Thus, the correct answer is **fleeting** (Choice B) and **momentary** (Choice C).

2. Some scientists argue that carbon compounds play such a central role in life on Earth because of the possibility of _____ resulting from the carbon atom's ability to form an unending series of different molecules.

   A diversity

   B deviation

   C variety

   D reproduction

   E stability

   F invigoration

Explanation

The key phrase that indicates how the blank for this question should be completed is "the ability to form an unending series of different molecules." Among the answer choices, "diversity" and "variety" clearly fit logically with "unending" and "different" and create two very similar sentences. No other pair of choices here would produce two sentences as similar in meaning as those created by placing "diversity" and "variety" in the blank. Thus, the correct answer is **diversity** (Choice A) and **variety** (Choice C).

3. Given the flood of information presented by the mass media, the only way for someone to keep abreast of the news is to rely on _____ accounts.

   A synoptic

   B abridged

   C sensational

   D copious

   E lurid

   F understated

Explanation

The key phrase that indicates how the blank for this question should be completed is "the only way for someone to keep abreast of the news." Among the answer choices, "synoptic" and "abridged," while not synonymous in the strict sense, both fit the logic of this description, "synoptic" because of its emphasis on breadth and generality as opposed to detail, and "abridged" because of its obvious focus on brevity. "Sensational" and "lurid" would create two similar sentences but do not fit the logic for completing the blank, since we would not be relying on sensational or lurid accounts in order to keep abreast of the news.

Thus, the correct answer is **synoptic** (Choice A) and **abridged** (Choice B).

4. Always circumspect, she was reluctant to make judgments, but once arriving at a conclusion, she was _____ in its defense.

   A deferential

   B intransigent

   C lax

   C negligent

   E obsequious

   F resolute

Explanation

The key phrases that indicate how the blank for this question should be completed are: "circumspect," "reluctant," and "but once." Taken together they point to completing the blank with something that is opposite in some way to the two cited adjectives. Among the answer choices, "intransigent" and "resolute," although not strictly synonymous, both fit the logic of the description given here for completing the blank and create sentences that are similar in meaning. "Lax" and "negligent" are clearly similar in meaning and would create sentences similar in meaning, but they continue the sentiment voiced in the initial clause rather than contrasting with it. "Deferential" and "obsequious" are also similar in meaning, but their emphasis on "politeness," while not strictly synonymous with reluctance and circumspection, like "lax" and "negligent" fail to pick up on the expected contrast.

Thus, the correct answer is **intransigent** (Choice B) and **resolute** (Choice F).

For each of Questions 5 to 9, select <u>one</u> answer choice unless otherwise instructed.

Questions 5 and 6 are based on the following reading passage.

When marine organisms called phytoplankton photosynthesize, they absorb carbon dioxide dissolved in seawater, potentially causing a reduction in the concentration of atmospheric carbon dioxide, a gas that contributes to global warming. However, phytoplankton flourish only in surface waters where iron levels are sufficiently high.

line 5 Martin therefore hypothesized that adding iron to iron-poor regions of the ocean could help alleviate global warming. While experiments subsequently confirmed that such a procedure increases phytoplankton growth, field tests have shown that such growth does

not significantly lower atmospheric carbon dioxide. When phytoplankton utilize carbon dioxide for photosynthesis, the carbon becomes a building block for organic
10 matter, but the carbon leaks back into the atmosphere when predators consume the phytoplankton and respire carbon dioxide.

Description

The paragraph presents a hypotheses about reducing global warming by adding iron to iron-poor areas of the ocean and explains why adding the iron does not have the hoped-for benefit.

For the following question, consider each of the choices separately and select all that apply.

5. It can be inferred from the passage that Martin's hypothesis includes which of the following elements?

   A A correct understanding of how phytoplankton photosynthesis utilizes carbon dioxide

   B A correct prediction about how the addition of iron to iron-poor waters would affect phytoplankton growth

   C An incorrect prediction about how phytoplankton growth would affect the concentration of atmospheric carbon dioxide

Explanation

All three choices are correct. Martin's hypothesis was that adding iron to iron-poor regions of the ocean could help alleviate global warming.

Choice A is correct: the passage presents Martin as using the standard understanding of how phytoplankton photosynthesize as a basis for the hypothesis.

Choice B is correct: the passage states that experiments confirmed that adding iron to iron-poor regions increases phytoplankton growth in those regions. Therefore, Martin's prediction about this was correct.

Choice C is correct: it can be inferred that in Martin's hypothesis the means by which adding iron in certain regions could alleviate global warming is that phytoplankton increase in those regions and absorb atmospheric carbon dioxide. The passage states that predators who consume phytoplankton respire carbon dioxide, so that the carbon dioxide absorbed by phytoplankton reenters the atmosphere. Therefore, Martin's prediction about this was incorrect.

6. It can be inferred that the author of the passage mentions predators (line 10) primarily in order to

   (A) help explain why Martin's hypothesis is incorrect

   (B) identify one effect of adding iron to iron-poor waters

   (C) indicate how some carbon dioxide is converted to solid organic matter

   (D) help account for differences in the density of phytoplankton between different regions of the ocean

   (E) point out a factor that was not anticipated by the scientists who conducted the field tests

mentioned in the passage

Explanation

Lines 7—11 of the paragraph present the evidence against Martin's hypothesis. Lines 7—8 present field test results showing that Martin's hypothesis is incorrect, and the last sentence explains these results: the reason the increased phytoplankton resulting from the addition of iron do not reduce atmospheric carbon dioxide is that while the phytoplankton absorb carbon dioxide, the gas reenters the atmosphere when it is respired by phytoplankton predators. Therefore **Choice A** is correct: predators are mentioned to explain why Martin's hypothesis is incorrect. Choice B is not correct because while predators' consumption of phytoplankton and respiration of carbon dioxide might be considered one indirect consequence of adding iron to iron-poor waters, identifying a consequence is not the primary function of the mention of predators. Choice C is incorrect because the reference to predators is used to explain how carbon dioxide reappears as a gas, and Choice D is incorrect because no connection is suggested between predators and the distribution of phytoplankton. Choice E is not correct because it is Martin who did not anticipate this factor, rather than the scientists who conducted the field tests.

Question 7 is based on the following reading passage.

Sparva, unlike Treland's other provinces, requires automobile insurers to pay for any medical treatment sought by someone who has been involved in an accident; in the other provinces, insurers pay for nonemergency treatment only if they preapprove the treatment. Clearly, Sparva's less restrictive policy must be the explanation for the fact that altogether insurers there pay for far more treatments after accidents than insurers in other provinces, even though Sparva does not have the largest population.

Description

The passage tells us that in Sparva automobile insurers pay for far more medical treatments after accidents than they do in Treland's other provinces. The passage concludes that the explanation is to be found in the difference in legal requirements for insurers in Sparva as compared to other provinces.

7. Which of the following, if true, most strengthens the argument?

(A) Car insurance costs more in Sparva than in any other province.

(B) The cost of medical care in Sparva is higher than the national average.

(C) Different insurance companies have different standards for determining what constitutes emergency treatment.

(D) Fewer insurance companies operate in Sparva than in any other province.

(E) There are fewer traffic accidents annually in Sparva than in any of the provinces of comparable or greater population.

Explanation

The question asks you to identify among the answer choices a fact that would support the

passage's argument. The explanation offered in the passage can be supported by ruling out other explanations that might, given the information presented in the passage, appear likely. One obvious explanation for there being more medical treatments in Sparva is that there are more accidents there. Choice E rules out that explanation. So **Choice E** strengthens the argument in the passage and is the correct answer. Choices A and D each present consequences that are likely results of insurers in Sparva having to pay for more medical treatments. But neither bears on the cause of insurers having to pay for more treatments. Choice B does not strengthen the argument and may weaken it. A higher cost of medical care provides additional motivation for people to seek insurance payments to cover whatever post-accident care they receive. So Choice B might weaken the argument by providing an alternative explanation for insurers paying for more medical treatments in Sparva. According to the passage, whether treatment is emergency treatment is, in other provinces, an important criterion in determining insurers' responsibility. But since this criterion does not apply in Sparva, Choice C is not directly relevant to the point that the passage is trying to establish.

Questions 8 and 9 are based on the following reading passage.

Elements of the Philosophy of Newton, published by Voltaire in 1738, was an early attempt to popularize the scientific ideas of Isaac Newton. In the book's frontispiece, Voltaire is seen writing at his desk, and over him a shaft of light from heaven, the light of truth, passes through Newton to Voltaire's collaborator Madame du Châtelet; she reflects that light onto the inspired Voltaire. Voltaire's book commanded a wide audience, according to Feingold, because "he was neither a mathematician nor a physicist, but a literary giant aloof from the academic disputes over Newtonian ideas." In other words, Voltaire's amateurism in science "was a source of his contemporary appeal, demonstrating for the first time the accessibility of Newton's ideas to nonspecialists."

Description

The paragraph describes Voltaire's book and gives some reasons for its success.

For the following question, consider each of the choices separately and select all that apply.

8. Which of the following statements about Voltaire's *Elements of the Philosophy of Newton* can be inferred from the passage?

   A Voltaire's literary stature helped secure a large audience for this attempt to popularize Newton's ideas.

   B Voltaire's status as a nonscientist was an advantage in this effort to bring Newtonian science to the attention of the general public.

   C The frontispiece of the book implies that Voltaire's understanding of Newton's ideas was not achieved without assistance.

Explanation

All three choices are correct.

Choice A is correct: the paragraph states that one of the reasons Voltaire's book commanded a wide audience is that he was "a literary giant."

Choice B is correct: the paragraph states that Voltaire's amateurism in science demonstrated that nonspecialists could also understand Newton's ideas.

Choice C is correct: the paragraph refers to Voltaire's collaborator, Madame du Châtelet. In the image described, she serves as the intermediary between Newton and Voltaire, conveying Newton's ideas to Voltaire.

9. Select the sentence that describes an allegory for Voltaire's acquisition of knowledge concerning Newton's ideas.

Explanation

In the image described in the second sentence, Voltaire acquires his knowledge of Newton through Madame du Châtelet, who conveys Newton's ideas — his "light of truth"—to Voltaire. The only other sentence that contains figurative language is the next sentence, in which Voltaire is described as "a literary giant aloof … from disputes." However, this image refers not to Voltaire's acquisition of knowledge of Newtonian ideas, but rather to his attitude regarding Newtonian ideas. Therefore **sentence 2** ("In the book's … Voltaire") is the correct choice.

For Questions 10 to 13, select one entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

10. Ironically, the writer so wary of (i)_____ was (ii)_____ with ink and paper, his novel running to 2,500 shagreen-bound folio pages — a fortune in stationery at the time.

Blank (i)

(A) probity
(B) extravagance
(C) disapprobation

Blank (ii)

(D) acquisitive
(E) illiberal
(F) profligate

Explanation

The last part of the sentence provides most of the context needed to fill in the two blanks. The novel was extremely long and required vast amounts of paper. Among the choices for the second blank, only "profligate" matches this lack of restraint. The word "Ironically" indicates that what the writer was "wary of" was something similar to profligacy; of the choices for the first blank, "extravagance" is the closest.

Thus, the correct answer is **extravagance** (Choice B) and **profligate** (Choice F).

11. What readers most commonly remember about John Stuart Mill's classic exploration of the liberty of thought and discussion concerns the danger of (i)_____: in the absence of challenge, one's opinions, even when they are correct, grow weak and flabby. Yet Mill had another reason for encouraging the liberty of thought and discussion: the danger of partiality and incompleteness. Since one's opinions, even under the best circumstances, tend to (ii) _____, and because opinions opposed to one's own rarely turn out to be completely (iii)_____, it is crucial to supplement one's opinions with alternative points of view.

Blank (i)

(A)	tendentiousness
(B)	complacency
(C)	fractiousness

Blank (ii)

(D)	embrace only a portion of the truth
(E)	change over time
(F)	focus on matters close at hand

Blank (iii)

(G)	erroneous
(H)	antithetical
(I)	immutable

Explanation

An overview of the passage suggests that the first sentence is relatively self-contained and that the blank is answerable without the succeeding sentences, where the topic shifts slightly. The colon after the first blank signals that what follows will define the word in the blank and will explain what danger Mill was concerned about. It says that without challenge, one's opinions grow "weak and flabby" and therefore one becomes *complacent*, not tendentious or fractious. A quick reading of the next two sentences suggests that the topic will be another danger that Mill described, "the danger of partiality and incompleteness." Free and open discussion needs to take place because each person's opinion tends to "embrace only a portion of the truth" and others' views are partially right, or never completely "erroneous." The other choices for the second and third blanks deal with change, immediacy, or antithesis, none of which relate to the second danger of "partiality" or "incompleteness."

Thus, the correct answer is **complacency** (Choice B), **embrace only a portion of the truth** (Choice D), and **erroneous** (Choice G).

12. Just as the authors' book on eels is often a key text for courses in marine vertebrate zoology, their ideas on animal development and phylogeny _____ teaching in this area.

(A)	prevent

(B)	defy
(C)	replicate
(D)	inform
(E)	use

Explanation

The "just as" structure indicates that the second half of the sentence should somehow parallel the idea presented in the first half (i.e., the idea that the authors' book on eels is a "key text" in marine vertebrate zoology). Among the choices given, "inform" is clearly the best choice. "Prevent" and "defy" work in the opposite direction, while "use" and "replicate" would suggest that the authors' ideas are drawing upon the teaching in this area rather than the other way around. "Inform" leads to a meaning that nicely matches the first half of the sentence.

Thus, the correct answer is **inform** (Choice D).

13. Mechanisms develop whereby every successful species can _____ its innate capacity for population growth with the constraints that arise through its interactions with the natural environment.

(A)	enhance
(B)	replace
(C)	produce
(D)	surpass
(E)	reconcile

Explanation

A quick overview of the sentence indicates that the blank should be filled with a verb that indicates what a successful species does with its "innate capacity for population growth" in the face of certain constraints on that growth. This analysis suggests that the correct answer will have something to do with adjusting that capacity in the face of these constraints. Of the choices given, "reconcile" is closest to that meaning. None of the other options make for a meaningful, coherent sentence. "Enhance," for example, may fit nicely with "its innate capacity," but it does not make sense with "constraints."

Thus, the correct answer is **reconcile** (Choice E).

For each of Questions 14 to 16, select <u>one</u> answer choice unless otherwise instructed.

Questions 14 to 16 are based on the following reading passage.

It would be expected that a novel by a figure as prominent as W. E. B. DuBois would attract the attention of literary critics. Additionally, when the novel subtly engages the issue of race, as DuBois' *The Quest of the Silver Fleece* (1911) does, it would be a surprise not to encounter an

abundance of scholarly work about that text. But though valuable scholarship has examined DuBois' political and historical thought, his novels have received scant attention. Perhaps DuBois the novelist must wait his turn behind DuBois the philosopher, historian, and editor. But what if the truth lies elsewhere: what if his novels do not speak to current concerns?

Description

The paragraph first presents reasons for critical interest in DuBois' novels, but then goes on to explain that there has in fact been very little such interest and speculates as to why that might be.

14. Which of the following can be inferred from the passage regarding DuBois' *The Quest of the Silver Fleece*?

(A) The lack of attention devoted to *The Quest of the Silver Fleece* can be attributed to the fact that it was DuBois' first novel.

(B) Among DuBois' novels, *The Quest of the Silver Fleece* is unusual in that it has received scant attention from scholars.

(C) *The Quest of the Silver Fleece* has at least one feature that typically would attract the attention of literary scholars.

(D) *The Quest of the Silver Fleece,* given its subtle exploration of race, is probably the best novel written by DuBois.

(E) Much of the scholarly work that has focused on *The Quest of the Silver Fleece* has been surprisingly critical of it.

Explanation

Choice C is correct. The second sentence states that *The Quest of the Silver Fleece* subtly engages the issue of race and implies that such an issue would attract the attention of literary scholars. The passage provides no information about whether *The Quest of the Silver Fleece* is DuBois' first novel (Choice A), whether it received more or less scholarly attention than his other novels (Choice B), whether it is better than any of his other novels (Choice D), nor about what scholars have said about it (Choice E).

15. In the fourth sentence ("Perhaps DuBois … editor."), the author of the passage is most likely suggesting that

(A) scholars will find that DuBois' novels are more relevant to current concerns than is his work as philosopher, historian, and editor

(B) more scholarly attention will be paid to *The Quest of the Silver Fleece* than to DuBois' other novels

(C) DuBois' novels will come to overshadow his work as philosopher, historian, and editor

(D) DuBois' novels may eventually attract greater scholarly interest than they have to date

(E) it will be shown that DuBois' work as philosopher, historian, and editor had an important influence on his work as novelist

Explanation

The fourth sentence speculates that once DuBois scholars have exhausted potential avenues of research in the fields of philosophy, history, and editing, they will turn to his novels, so **Choice D** is the correct answer. None of the other choices fits the metaphor in "Perhaps DuBois the novelist must wait his turn."

16. Which of the following best describes the central issue with which the passage is concerned?

(A) The perfunctoriness of much of the critical work devoted to DuBois' novels
(B) The nature of DuBois' engagement with the issue of race in *The Quest of the Silver Fleece*
(C) Whether DuBois' novels are of high quality and relevant to current concerns
(D) The relationship between DuBois the novelist and DuBois the philosopher, historian, and editor
(E) The degree of consideration that has been given to DuBois' novels, including *The Quest of the Silver Fleece*

Explanation

The passage focuses on the scant attention given to DuBois' novels, *The Quest of the Silver Fleece* in particular. The first two sentences give reasons to expect greater attention, while the last two offer speculations about the explanation for the scant attention. Thus, **Choice E** is correct. The issues described in the other answer choices are all marginal to the passage, if they are mentioned at all.

For Questions 17 to 20, select one entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

17. In the midst of so many evasive comments, this forthright statement, whatever its intrinsic merit, plainly stands out as _____.

(A) a paradigm
(B) a misnomer
(C) a profundity
(D) an inaccuracy
(E) an anomaly

Explanation

The sentence offers a contrast between "many evasive statements" and a single "forthright statement." On that basis alone, one might expect an answer such as "an anomaly." Do any of the other options make for a meaningful, coherent sentence? "A paradigm" is appealing, as is "a profundity," since the forthright statement is clearly presented as something positive. However, we are not in a position to call it paradigmatic or profound, since the sentence withholds judgment on "its intrinsic merit." The same reasoning allows us to eliminate "inaccuracy" and "misnomer." The straightforwardly descriptive "anomaly" is clearly the best choice.

Thus, the correct answer is **an anomaly** (Choice E).

18. The activists' energetic work in the service of both woman suffrage and the temperance movement in the late nineteenth century (i)_____ the assertion that the two movements were (ii)_____.

Blank (i)

(A) undermines
(B) supports
(C) underscores

Blank (ii)

(D) diffuse
(E) inimical
(F) predominant

Explanation

The sentence is about the implications of the activists' energetic work for some assertion about the woman suffrage and temperance movements. The second blank, however, obscures the nature of that assertion. But it is clear that the "energetic work" could either support an assertion that the two movements were similar, or undermine an assertion that the two movements were opposed. "Supports" is offered as a choice for the first blank (as is the somewhat similar "underscores"), but there is no corresponding term in the second blank, nothing along the lines of "similar" or "compatible." "Undermines" and "inimical" make for the only meaningful statement.

Thus, the correct answer is **undermines** (Choice A) and **inimical** (Choice E).

19. There is nothing quite like this movie, and indeed I am not altogether sure there is much more to it than its lovely (i) _____. At a moment when so many films strive to be as (ii)_____ as possible, it is gratifying to find one that is so subtle and puzzling.

Blank (i)

(A) peculiarity
(B) pellucidity
(C) conventionality

Blank (ii)

(D) indirect
(E) assertive
(F) enigmatic

Explanation

The two sentences provide the reader with quite a bit of information about the movie. There is "nothing quite like it" and it is "subtle and puzzling." "Peculiarity" is clearly a solid fit for the first blank, while "conventionality" clearly does not work, given the fact that there is "nothing quite like it." That leaves "pellucidity," which, while it could fit logically in the first sentence in isolation, does not fit the later claim that the movie is "subtle and puzzling." The second blank needs simply to provide a contrast with "subtle and puzzling." Of the choices offered, only "assertive" clearly does that.

Thus, the correct answer is **peculiarity** (Choice A) and **assertive** (Choice E).

20. Wills argues that certain malarial parasites are especially (i)_____ because they have more recently entered humans than other species and therefore have had (ii)_____ time to evolve toward (iii)_____. Yet there is no reliable evidence that the most harmful *Plasmodium* species has been in humans for a shorter time than less harmful species.

Blank (i)

A	populous
B	malignant
C	threatened

Blank (ii)

D	ample
E	insufficient
F	adequate

Blank (iii)

G	virulence
H	benignity
I	variability

Explanation

The "Yet" that begins the second sentence indicates that Wills' position would be supported by evidence that the newer parasites are in humans, the more harmful they are. So Wills' position must be that more recent parasites are especially harmful, implying that "malignant" is the correct choice for the first blank. What follows "therefore" is a potential explanation for the trend that Wills expects, namely an evolution toward harmlessness, implying "benignity" for the third blank, with newer species having had "insufficient" time (second blank) to evolve toward harmlessness.

Thus, the correct answer is **malignant** (Choice B), **insufficient** (Choice E), and **benignity** (Choice H).

For each of questions 21 to 25, select one answer choice unless otherwise instructed.

Question 21 is based on the following reading passage.

Saturn's giant moon Titan is the only planetary satellite with a significant atmosphere and the only body in the solar system other than Earth that has a thick atmosphere dominated by molecular nitrogen. For a long time, the big question about Titan's atmosphere was how it could be so thick, given that Jupiter's moons Ganymede and Callisto, which are the same size as Titan, have none. The conditions for acquiring and retaining a thick nitrogen atmosphere are now readily understood. The low temperature of the protosaturnian nebula enabled Titan to acquire the moderately volatile compounds methane and ammonia (later converted to nitrogen) in addition to water. The higher temperatures of Jupiter's moons, which were closer to the Sun, prevented them from acquiring such an atmosphere.

Description

The paragraph discusses Titan's thick atmosphere and explains the conditions under which a body can have a thick atmosphere.

21. According to the passage, Titan differs atmospherically from Ganymede and Callisto because of a difference in

(A) rate of heat loss
(B) proximity to the Sun
(C) availability of methane and ammonia
(D) distance from its planet
(E) size

Explanation

According to the last two sentences of the paragraph, Titan was able to acquire an atmosphere because of a prevailing low temperature, but Ganymede and Callisto could not because they were at a higher temperature. Because the reason for this difference in temperature was their respective distances from the Sun, **Choice B** is correct. The passage says nothing about differences in rate of heat loss, availability of methane and ammonia, or distance from their planets, and it explicitly states that the three moons are the same size.

Question 22 is based on the following reading passage.

Observations of the Arctic reveal that the Arctic Ocean is covered by less ice each summer than the previous summer. If this warming trend continues, within 50 years the Arctic Ocean will be ice free during the summer months. This occurrence would in itself have little or no effect on global sea levels, since the melting of ice floating in water does not affect the water level. However, serious consequences to sea levels would eventually result, because _____.

22. Which of the following most logically completes the passage?

(A) large masses of floating sea ice would continue to form in the wintertime

(B) significant changes in Arctic sea temperatures would be accompanied by changes in sea temperatures in more temperate parts of the world

(C) such a warm Arctic Ocean would trigger the melting of massive land-based glaciers in the Arctic

(D) an ice-free Arctic Ocean would support a very different ecosystem than it does presently

(E) in the spring, melting sea ice would cause more icebergs to be created and to drift south into shipping routes

Explanation

To logically complete the passage's open-ended "because," something is needed that will explain why the continuation of the warming trend would have serious consequences for sea levels. The passage explains that the melting of the Arctic Ocean ice will not affect sea levels because the contribution that the water contained in that ice makes to sea levels is the same whether the water is frozen or liquid. But Choice C points to a way in which increasing temperatures in the Arctic could add water to the ocean, namely by melting ice on the land. So **Choice C** logically completes the passage and is the correct answer.

Given that the passage has already explained that melting sea ice does not affect sea levels, the formation of sea ice described in Choice A does not explain why there would be consequences for sea levels.

Choices B, D, and E all describe possible consequences of increased temperatures in the Arctic, but none of these consequences suggests a mechanism by which sea levels would change. So none of these options provides a logical completion for the passage.

Questions 23 to 25 are based on the following reading passage.

In a recent study, David Cressy examines two central questions concerning English immigration to New England in the 1630s: what kinds of people immigrated and why? Using contemporary literary evidence, shipping lists, and customs records, Cressy finds that most adult immigrants were skilled in farming or crafts, were literate, and were
line 5 organized in families. Each of these characteristics sharply distinguishes the 21,000 people who left for New England in the 1630s from most of the approximately 377,000 English people who had immigrated to America by 1700.

With respect to their reasons for immigrating, Cressy does not deny the frequently noted fact that some of the immigrants of the 1630s, most notably the organizers and
10 clergy, advanced religious explanations for departure, but he finds that such explanations usually assumed primacy only in retrospect. When he moves beyond the principal actors, he finds that religious explanations were less frequently offered, and he concludes that most people immigrated because they were recruited by promises of material improvement.

Description

The passage discusses Cressy's answers to the questions posed in the first sentence. The immigrants were skilled, literate, and in families, and they apparently immigrated to have a better

life materially, rather than religiously.

For the following question, consider each of the choices separately and select all that apply.

23. The passage indicates that Cressy would agree with which of the following statements about the organizers among the English immigrants to New England in the 1630s?

A Some of them offered a religious explanation for their immigration.

B They did not offer any reasons for their immigration until some time after they had immigrated.

C They were more likely than the average immigrant to be motivated by material considerations.

Explanation

Choice A is correct.

Choice A is correct: the organizers are mentioned in the second paragraph, where the passage says that Cressy "does not deny" that organizers "advanced religious explanations" for leaving England and immigrating to New England in the 1630s. This suggests that Cressy would agree with the statement in choice A about the organizers.

Choice B is incorrect: in lines 10—11, the passage says that Cressy finds that religious reasons for immigration "assumed primacy" only in retrospect, but this is not the same as Cressy's concluding that no reasons were given at the time of immigration. Therefore it cannot be inferred that Cressy would agree with the statement in Choice B.

Choice C is incorrect: the passage refers in lines 13—14 to "promises of material improvement" as a factor that in Cressy's view motivated most immigrants other than "the principal actors." This suggests that Cressy regards the principal actors, such as organizers, as having been less, not more, motivated by material considerations than average immigrants were. Therefore it cannot be inferred that Cressy would agree with the statement in Choice C.

24. Select the sentence that provides Cressy's opinion about what motivated English immigrants to go to New England in the 1630s.

Explanation

The last sentence says that Cressy "concludes that most people immigrated because they were recruited by promises of material improvement." Because this suggests that Cressy believes immigrants were motivated by these promises to go to New England, **sentence 5** ("When he … improvement) is the correct choice. The preceding sentence suggests that Cressy does not believe religion was a primary motive influencing immigrants' decision to immigrate in the 1630s. Thus, although this sentence provides an opinion of Cressy's concerning some immigrants' stated reasons for immigrating, it does not say what motive he believes was actually behind the immigration, and therefore does not answer the question.

25. In the passage, the author is primarily concerned with

(A) summarizing the findings of an investigation
(B) analyzing a method of argument
(C) evaluating a point of view
(D) hypothesizing about a set of circumstances
(E) establishing categories

Explanation

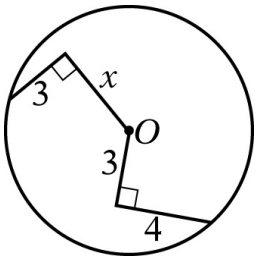
The passage is about Cressy's investigation of English immigration to New England in the 1630s, and it summarizes his findings concerning who immigrated and why. **Choice A**, "summarizing the findings of an investigation," is therefore the best description of the author's primary concern in the passage. The passage does not analyze a method of argument, so Choice B is incorrect. Choice C is incorrect because the passage is not primarily concerned with evaluating a point of view: it does not assess the merits or demerits of Cressy's viewpoint. The passage is concerned with reporting Cressy's findings, not with hypothesizing or with establishing categories, so Choices D and E are incorrect.

SECTION 5
Quantitative Reasoning
25 Questions with Explanations

For each of Questions 1 to 9, select one of the following answer choices.

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

1.



O is the center of the circle above.

Quantity A

x

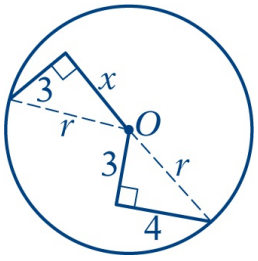
Quantity B

5

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare x with 5, where x is the length of a line segment from the center of the circle to a point inside the circle. In a circle the easiest line segments to deal with are the radius and the diameter. Looking at the figure in the question, you can see that you can draw two radii, each of which "completes" a right triangle, as shown in the figure below.



Since in one of the triangles the lengths of both legs are known, you can use that triangle to determine the length of the radius of the circle. The triangle has legs of length 3 and 4. If the length of the radius is r, then, using the Pythagorean theorem, you can see that

$$\begin{aligned} r^2 &= 3^2 + 4^2 \text{ or} \\ r^2 &= 9 + 16 \text{ or} \\ r^2 &= 25, \text{ and thus, } r = 5 \end{aligned}$$

Since the length of the radius of the circle is 5 and the line segment of length x is clearly shorter than the radius, you know that $x < 5$, and the correct answer is **Choice B**.

You could also see that the two triangles are congruent, and so $x = 4$, again yielding **Choice B**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

2.

Runner A ran $\frac{4}{5}$ kilometer and Runner B ran 800 meters.

Quantity A

The distance that A ran

Quantity B

The distance that B ran

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare two measurements, one given in kilometers and the other in meters. It would be easier to compare these measurements if they were both given in meters or both given in kilometers.

If you choose to convert the distance that Runner *B* ran from meters to kilometers, you need to use the conversion 1 meter is equal to $\frac{1}{1{,}000}$ kilometer. Since *B* ran 800 meters, it follows that *B* ran $(800)\left(\frac{1}{1{,}000}\right)$, or $\frac{4}{5}$ kilometer, which is the same distance that *A* ran.

If you choose to convert the distance that Runner *A* ran from kilometers to meters, you need to use the conversion 1 kilometer is equal to 1,000 meters. Since *A* ran $\frac{4}{5}$ kilometer, it follows that *A* ran $\left(\frac{4}{5}\right)(1{,}000)$, or 800 meters, which is the same distance that *B* ran. Either way, *A* and *B* ran the same distance, and the correct answer is **Choice C**.

This explanation uses the following strategy.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

3.

$$x < y < z$$

Quantity A

$\frac{x+y+z}{3}$

Quantity B

y

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are given that $x < y < z$, and you are asked to compare $\frac{x+y+z}{3}$ with y.

Two approaches that you could use to solve this problem are:

1: Search for a mathematical relationship between the two quantities.
2: Plug in numbers for the variables.

Approach 1: Note that $\frac{x+y+z}{3}$ is the average of the three numbers *x, y*, and *z* and that *y* is the median. Is the average of 3 numbers always equal to the median? The average could equal the median, but in general they do not have to be equal. Therefore, the correct answer is **Choice D**.

Approach 2: When you plug in numbers for the variables, it is a good idea to consider what kind of numbers are appropriate to plug in and to choose numbers that are easy to work with, if possible.

Since $\frac{x+y+z}{3}$ is the average of the three numbers *x, y*, and *z* and you are comparing it to

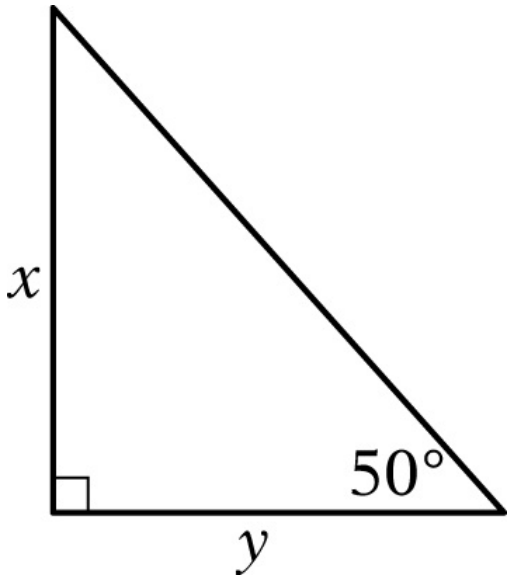
the median, you may want to try plugging in numbers that are evenly spaced and plugging in numbers that are not evenly spaced.

You can plug in numbers that are both evenly spaced and easy to work with. For example, you can plug in $x = 1$, $y = 2$, and $z = 3$. In this case,
$\frac{x+y+z}{3} = \frac{1+2+3}{3} = \frac{6}{3} = 2$, and so $\frac{x+y+z}{3} = y$.

You can also plug in numbers that not are not evenly spaced and are easy to work with. For example, you can plug in $x = 3$, $y = 6$, and $z = 12$. In this case,
$\frac{x+y+z}{3} = \frac{3+6+12}{3} = \frac{21}{3} = 7$, and $\frac{x+y+z}{3} > y$. Since in the first case, $\frac{x+y+z}{3}$ is equal to y and in the second case, it is greater than y, the relationship between the two quantities $\frac{x+y+z}{3}$ and y cannot be determined from the information given. The correct answer is **Choice D**.

This explanation uses the following strategies.
Strategy 10: Trial and Error
Strategy 11: Divide into Cases
Strategy 12: Adapt Solutions to Related Problems
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

4.



Quantity A

$\frac{x}{y}$

Quantity B

1

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

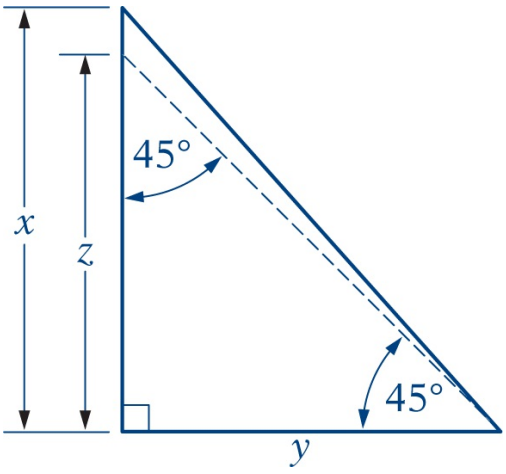
One way you can solve this problem is by using the following fact:

- If ABC is a triangle and the measure of angle A is greater than the measure of angle B, then the side opposite angle A is longer than the side opposite angle B.

Since the third angle of the triangle measures 40°, you can use the fact above to conclude that the side opposite the 50° angle is longer than the side opposite the 40° angle. So $x > y$, and $\frac{x}{y} > 1$, which yields **Choice A**.

You can also solve this problem without using the fact above. Instead, you can use the strategy of adapting solutions to related problems to determine the relationship between x and y.

Note that the angles in the triangle differ only a little from the angles in a 45°– 45°– 90° triangle. How do the lengths of the legs of a 45°– 45°– 90° triangle compare to the lengths of the legs of the triangle in the question? To make the comparison, add a line segment to the triangle so that the line segment cuts the 50° angle in two parts, making a 45° angle with the horizontal base, as shown in the following figure:



The 45°– 45°– 90° triangle has two 45° angles, so $z = y$, and $\frac{z}{y} = 1$. Since $\frac{z}{y} = 1$ and $x > z$, it follows that $\frac{x}{y} > 1$. The correct answer is **Choice A**.

This explanation uses the following strategies.

Strategy 6: Add to a Geometric Figure
Strategy 8: Search for a Mathematical Relationship
Strategy 12: Adapt Solutions to Related Problems

5.

$$0 < x < y < 1$$

Quantity A

$1 - y$

Quantity B

$y - x$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Two approaches that you could use to solve this problem are:

1: Translate from algebra to a number line.
2: Plug in values for the variables.

Approach 1: The following figure represents the information given in the problem on a number line.

On the number line, $1 - y$ is the distance between 1 and y, and $y - x$ is the distance between y and x. If y is exactly halfway between x and 1, then $1 - y$ is equal to $y - x$; and if y is not halfway between x and 1, then $1 - y$ is not equal to $y - x$. But y can be any number between x and 1, so the correct answer is **Choice D**.

Approach 2: Since this problem involves subtraction, it is a good idea to choose values for x and y that are close to each other as well as values that are far apart. For example, if $x = 0.4$ and $y = 0.5$, then $1 - y = 0.5$ and $y - x = 0.1$; and if $x = 0.1$ and $y = 0.9$, then $1 - y = 0.1$ and $y - x = 0.8$. This shows that the relationship cannot be determined, and the correct answer is **Choice D**.

This explanation uses the following strategies.

Strategy 3: Translate from an Algebraic to a Graphical Representation
Strategy 10: Trial and Error
Strategy 11: Divide into Cases
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

6.

p is the probability that event E will occur, and s is the probability that event E will not occur.

Quantity A

$p + s$

Quantity B

ps

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Since event E will either occur or not occur, it follows that $p + s = 1$, and the value of Quantity A is always 1. Since Quantity B is the product of the two probabilities p and s, you need to

look at its value for the cases $p = 1$, $p = 0$, and $0 < p < 1$.

If $p = 1$, then $s = 0$; similarly, if $p = 0$, then $s = 1$. In both cases, ps is equal to 0.

If $0 < p < 1$, both p and s are positive and less than 1, so ps is positive and less than 1.

Since Quantity A is equal to 1 and Quantity B is less than 1, the correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 11: Divide into Cases

7.

X is the set of all integers n that satisfy the inequality $2 \leq |n| \leq 5$.

Quantity A

The absolute value of the greatest integer in X

Quantity B

The absolute value of the least integer in X

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

When comparing these quantities, it is important to remember that a nonzero number and its negative have the same absolute value. For example, $|-2| = |2| = 2$. Keeping this in mind, you can see that the positive integers 2, 3, 4, and 5 and the negative integers −2, −3, −4, and −5 all satisfy the inequalities $2 \leq |n| \leq 5$, and that these are the only such integers. Thus, the set X consists of the integers −5, −4, −3, −2, 2, 3, 4, and 5. The greatest of these integers is 5, and its absolute value is 5. The least of these integers is −5, and its absolute value is also 5. Therefore, Quantity A is equal to Quantity B. The correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

8.

x and m are positive numbers, and m is a multiple of 3.

Quantity A

$$\frac{x^m}{x^3}$$

Quantity B

$x^{m/3}$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Since $\frac{x^m}{x^3} = x^{m-3}$, you need to compare x^{m-3} with $x^{m/3}$. Since the base in both expressions is the same, a good strategy to use to solve this problem is to plug in numbers for

m in both expressions and compare them.

You know that m is a multiple of 3, so the least positive integer you can plug in for m is 3.

If $m = 3$, then $x^{m-3} = 1$ and $x^{m/3} = x$. Since x can be any positive number, its relationship to 1 cannot be determined from the information given. This example is sufficient to show that the relationship between $\frac{x^m}{x^3}$ and $x^{m/3}$ cannot be determined from the information given. The correct answer is **Choice D**.

This explanation uses the following strategies.
Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 10: Trial and Error

9.

A random variable Y is normally distributed with a mean of 200 and a standard deviation of 10.

Quantity A

The probability of the event that the value of Y is greater than 220

Quantity B

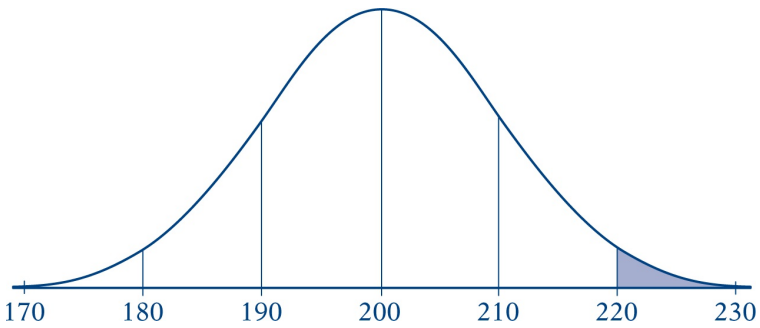
$\frac{1}{6}$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

This problem involves a normal distribution with mean 200 and standard deviation 10. Thus, the value of 210 is 1 standard deviation above the mean, and the value of 220 is 2 standard deviations above the mean. To compare Quantity A with Quantity B, it is not necessary to exactly determine the probability of the event that the value of Y is greater than 220. Remember that in any normal distribution, almost all of the data values, or about 95% of them, fall within 2 standard deviations on either side of the mean. This means that less than 5% of the values in this distribution will be greater than 220. Thus, the probability of the event that the value of Y is greater than 220 must be less than 5%, or $\frac{1}{20}$, and this is certainly less than $\frac{1}{6}$. The correct answer is **Choice B**.

Another approach to this problem is to draw a normal curve, or "bell-shaped curve," that represents the probability distribution of the random variable Y, as shown in the figure below.



The curve is symmetric about the mean 200. The values of 210, 220, and 230 are equally spaced to the right of 200 and represent 1, 2, and 3 standard deviations, respectively, above the mean. Similarly, the values of 190, 180, and 170 are 1, 2, and 3 standard deviations, respectively, below the mean. Quantity A, the probability of the event that the value of Y is greater than 220, is equal to the area of the shaded region as a fraction of the total area under the curve.

From the figure, you can see that the area under the normal curve has been divided into 6 regions and that these regions are not equal in area. The shaded region is one of the two smallest of the 6 regions, so its area must be less than $\frac{1}{6}$ of the total area under the curve. The correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 2: Translate from Words to a Figure or Diagram
Strategy 8: Search for a Mathematical Relationship

10. The ratio of $\frac{1}{3}$ to $\frac{3}{8}$ is equal to the ratio of

(A) 1 to 8
(B) 8 to 1
(C) 8 to 3
(D) 8 to 9
(E) 9 to 8

Explanation

Multiplying both parts of a ratio by the same positive number produces an equivalent ratio. While you could multiply both fractions in the ratio by any number, 24 is a good number to choose because it is the least common multiple of 3 and 8. Thus, multiplying both $\frac{1}{3}$ and $\frac{3}{8}$ by 24, you get that the ratio of $\frac{1}{3}$ to $\frac{3}{8}$ is equal to the ratio of 8 to 9. The correct answer is **Choice D**.

An alternate approach to this problem is to express the ratio of $\frac{1}{3}$ to $\frac{3}{8}$ as the fraction $\frac{\frac{1}{3}}{\frac{3}{8}}$. This fraction is equivalent to $\left(\frac{1}{3}\right)\left(\frac{8}{3}\right)$, or $\frac{8}{9}$. The correct answer is **Choice D**.

This explanation uses the following strategy.
Strategy 5: Simplify an Arithmetic or Algebraic Representation

11. A reading list for a humanities course consists of 10 books, of which 4 are biographies and the rest are novels. Each student is required to read a selection of 4 books from the list, including 2 or more biographies. How many selections of 4 books satisfy the

requirements?

(A) 90
(B) 115
(C) 130
(D) 144
(E) 195

Explanation

The requirement to select 4 books, including 2 or more biographies, means that you have to consider three cases. A student can choose 4 biographies and no novels, or 3 biographies and 1 novel, or 2 biographies and 2 novels.

Case 1: Choose 4 biographies. This case is easy, as there is only 1 way to choose all four biographies and no novels.

In the other two cases, you have to find the number of ways of choosing the biographies and the number of ways of choosing the novels and then multiply these two numbers.

Case 2: Choose 3 biographies and 1 novel. First, you need to find the number of ways of choosing 3 biographies out of 4. If you think of this as not choosing 1 out of the 4, you see that there are 4 choices. The number of ways of choosing 1 novel out of the 6 novels is 6. Therefore, the total number of choices is (4)(6) = 24.

Case 3: Choose 2 biographies and 2 novels. First, you need to find the number of ways of choosing 2 biographies out of 4. This number is sometimes called "4 choose 2" or the number of combinations of 4 objects taken 2 at a time. If you remember the combinations formula, you know that the number of combinations is $\frac{4!}{2!(4-2)!}$ (which is denoted symbolically as $\binom{4}{2}$ or ${}_4C_2$). The value of $\frac{4!}{2!(4-2)!}$ is $\frac{(4)(3)(2!)}{(2)(2!)} = \frac{(4)(3)}{2} = 6$. Thus, there are 6 ways to choose 2 biographies out of 4. Similarly, the number of ways to choose 2 novels out of 6 is $\frac{6!}{2!4!} = \frac{(6)(5)}{2} = 15$. Thus, the total number of ways to choose 2 biographies and 2 novels is (6)(15) = 90.

Adding the number of ways to choose the books for each of the three cases, you get a total of 1 + 24 + 90 = 115. The correct answer is **Choice B**.

This explanation uses the following strategies.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 11: Divide into Cases

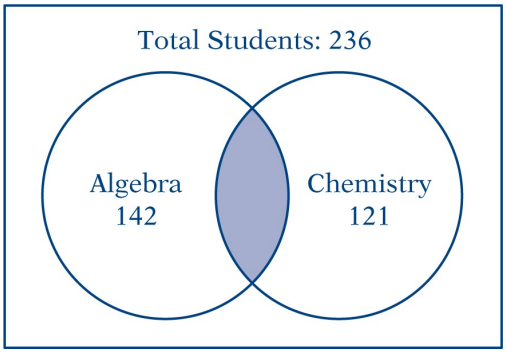
For the following question, enter your answer in the box.

12. In a graduating class of 236 students, 142 took algebra and 121 took chemistry. What is the greatest possible number of students that could have taken both algebra and chemistry?

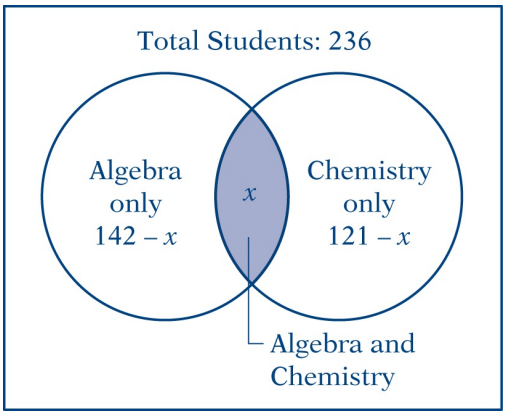
_____ students

Explanation

This is the type of problem for which drawing a Venn diagram is usually helpful. The Venn diagram below is one you could draw to represent the information given in the question.

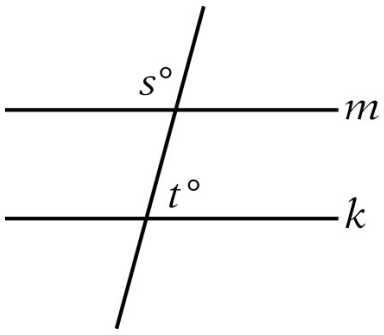


Note that the algebra and chemistry numbers given do not separate out the number of students who took both algebra and chemistry, and that this question asks for the greatest possible number of such students. It is a good idea, therefore, to redraw the Venn diagram with the number of students who took both algebra and chemistry separated out. The revised Venn diagram looks like the one below.



To solve this problem you want the greatest possible value of x. It is clear from the diagram that x cannot be greater than 142 nor greater than 121, otherwise $142 - x$ or $121 - x$ would be negative. Hence, x must be less than or equal to 121. Since there is no information to exclude $x = 121$, the correct answer is **121**.

This explanation uses the following strategy.
Strategy 2: Translate from Words to a Figure or Diagram

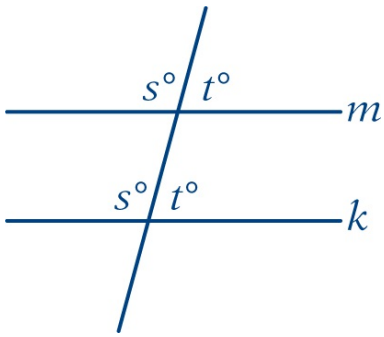


13. In the figure above, if $m|k$ and $s = t + 30$, then $t =$

(A) 30
(B) 60
(C) 75
(D) 80
(E) 105

Explanation

When trying to solve a geometric problem, it is often helpful to add any known information to the figure. Since corresponding angles have equal measures, you could place two more angle measures on the figure, as shown below.



Now, from the figure, you can see that $s + t = 180$. Therefore, since it is given that $s = t + 30$, you can substitute $t + 30$ for s into the equation $s + t = 180$ and get that $(t + 30) + t = 180$, which can be simplified as follows.

$$\begin{aligned}(t+30)+t &= 180\\ 2t &= 150\\ t &= 75\end{aligned}$$

The correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

14. If $2x = 3y = 4z = 20$, then $12xyz =$

(A) 16,000
(B) 8,000
(C) 4,000
(D) 800
(E) 10

Explanation

One approach you can use to solve this problem is to find the values of all three variables.

$$2x = 20, \quad \text{or} \quad x = 10$$
$$3y = 20, \quad \text{or} \quad y = \frac{20}{3}$$
$$4z = 20, \quad \text{or} \quad z = 5$$

So $12xyz = 12(10)\left(\frac{20}{3}\right)(5) = 4{,}000$, and the correct answer is **Choice C**.

Another approach you can use to solve this problem is to notice that $12xyz = \frac{(2x)(3y)(4z)}{2} = \frac{(20)(20)(20)}{2} = 4{,}000$. Therefore, the correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

For the following question, select all the answer choices that apply.

15. The total amount that Mary paid for a book was equal to the price of the book plus a sales tax that was 4 percent of the price of the book. Mary paid for the book with a $10 bill and received the correct change, which was less than $3.00. Which of the following statements must be true?

    Indicate all such statements.

    A The price of the book was less than $9.50.

    B The price of the book was greater than $6.90.

    C The sales tax was less than $0.45.

Explanation

For this problem you may find it helpful to translate the given information into an algebraic expression. Since the price of the book is unknown, you can call it x dollars, and then the total amount that Mary paid is x dollars plus 4% of x dollars, or $1.04x$ dollars. The problem states that Mary received some change from a $10 bill, so $1.04x$ dollars must be less than $10. Since the change was less than $3.00, the total amount Mary paid for the book must have been greater than $7.00. You can express this information algebraically by the inequality

$$7.00 < 1.04x < 10.00$$

Solving the inequality for x by dividing by 1.04, and rounding, you get

$$6.73 < x < 9.62$$

So you see that x, the price of the book, must be between \$6.73 and \$9.62, and each price in between is possible. With this information, you can quickly examine the first two statements. Choice A is not necessarily true because the price could be as high as \$9.61, and Choice B is not necessarily true because the price could be as low as \$6.74.

To examine Choice C, you could compute the tax for the greatest possible price, which would be 4% of 9.61, or $(0.04)(9.61) \approx 0.38$. Since this greatest possible tax is less than \$0.45, Choice C must be true.

You can also quickly see that Choice C must be true if you note that 4% of \$10.00 would only be \$0.40, and since the price must be less than \$10.00, the tax must be less than \$0.40. The correct answer consists of **Choice C**.

This explanation uses the following strategies.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

16. If $\frac{1}{(2^{11})(5^{17})}$ is expressed as a terminating decimal, how many nonzero digits will the decimal have?

(A) One
(B) Two
(C) Four
(D) Six
(E) Eleven

Explanation

To convert the fraction to a decimal, it is helpful to first write the fraction in powers of 10. Using the rules of exponents, you can write the following.

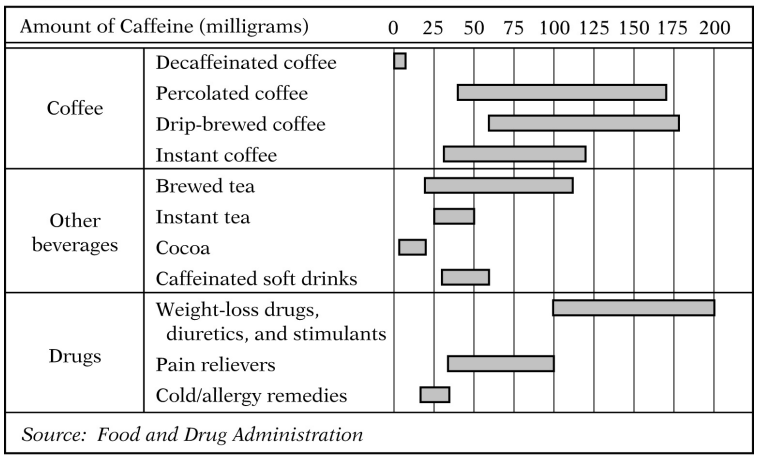
$$\begin{aligned}
\frac{1}{(2^{11})(5^{17})} &= \frac{1}{(2^{11})(5^{11+6})} \\
&= \frac{1}{(2^{11})(5^{11})(5^{6})} \\
&= \frac{1}{(10^{11})(5^{6})} \\
&= \left(\frac{1}{5}\right)^{6}(10^{-11}) \\
&= (0.2)^{6}(10^{-11}) \\
&= ((2)(10)^{-1})^{6}(10^{-11}) \\
&= (2^{6})(10^{-6})(10^{-11}) \\
&= (2^{6})(10^{-17}) \\
&= (64)(10^{-17})
\end{aligned}$$

So the decimal has two nonzero digits, 6 and 4. The correct answer is **Choice B**.

This explanation uses the following strategy.
Strategy 5: Simplify an Arithmetic or Algebraic Representation

Questions 17 to 20 are based on the following data.

VARIATION IN THE AMOUNT OF CAFFEINE IN COMMON BEVERAGES AND DRUGS*



*Based on 5-ounce cups of coffee, tea, and cocoa; 12-ounce cups of soft drinks; and single doses of drugs.

17. The least amount of caffeine in a 5-ounce cup of drip-brewed coffee exceeds the greatest amount of caffeine in a 5-ounce cup of cocoa by approximately how many milligrams?

(A) 160
(B) 80
(C) 60
(D) 40
(E) 20

Explanation

Each horizontal bar in the bar graph shows the possible number of milligrams of caffeine in each of the common beverages and drugs. The least possible amount of caffeine in a 5-ounce cup of drip-brewed coffee is about 60 milligrams, and the greatest possible amount of caffeine in a 5-ounce cup of cocoa is about 20 milligrams. So, the difference is approximately $60 - 20$, or 40 milligrams. The correct answer is **Choice D**.

To check your answer, it is useful to try to solve the problem using another method as well to see if you get the same answer. To solve this problem in another way, note that the distance between each pair of adjacent vertical grid lines represents 25 milligrams of caffeine, and the distance between the high end of the cocoa bar and the low end of the drip-brewed coffee bar is a little more than the distance between a pair of adjacent grid lines. Therefore, the answer is between 25 and 50. Among the choices, only Choice D is between 25 and 50, so the correct answer is **Choice D**.

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 9: Estimate

For the following question, enter your answer in the box.

18. For how many of the 11 categories of beverages and drugs listed in the graph can the amount of caffeine in the given serving size be less than 50 milligrams?

[ ] categories

Explanation

In the graph, the left edge of each bar tells you what is the least possible amount of caffeine in the corresponding beverage or drug. A beverage or drug can have less than 50 milligrams of caffeine if the left edge of its bar lies to the left of the vertical line corresponding to 50 milligrams of caffeine. From the graph, you see that there are 9 bars for which this is true. There are only 2 bars that lie entirely to the right of the 50-milligram line — the bar for drip-brewed coffee and the bar for weight-loss drugs, diuretics, and stimulants. So there are 9 categories of beverages and drugs that can have less than 50 milligrams of caffeine in the given serving size. The correct answer is **9**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

19. Approximately what is the minimum amount of caffeine, in milligrams, consumed per day by a person who daily drinks two 10-ounce mugs of percolated coffee and one 12-ounce cup of a caffeinated soft drink?

(A) 230
(B) 190
(C) 140
(D) 110
(E) 70

Explanation

According to the bar graph, the minimum amount of caffeine in a 5-ounce cup of percolated coffee is approximately 40 milligrams. Therefore, the minimum amount of caffeine in two 10-ounce cups of percolated coffee, which is the same as the minimum amount of caffeine in four 5-ounce cups, is approximately (40)(4), or 160 milligrams. The minimum amount of caffeine in a 12-ounce caffeinated soft drink is approximately 30 milligrams. So, the minimum amount of caffeine in two 10-ounce mugs of percolated coffee and one 12-ounce caffeinated soft drink is approximately 160 + 30, or 190 milligrams. The correct answer is **Choice B**.

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 9: Estimate

20. Which of the following shows the four types of coffee listed in order according to the

range of the amounts of caffeine in a 5-ounce cup, from the least range to the greatest range?

(A) Decaffeinated, instant, percolated, drip-brewed

(B) Decaffeinated, instant, drip-brewed, percolated

(C) Instant, decaffeinated, drip-brewed, percolated

(D) Instant, drip-brewed, decaffeinated, percolated

(E) Instant, percolated, drip-brewed, decaffeinated

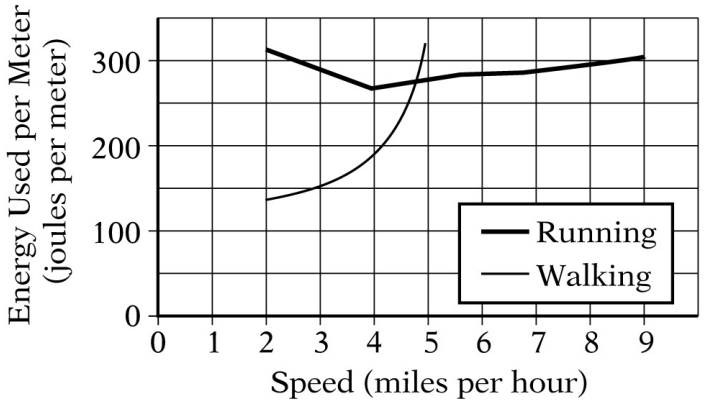
Explanation

For each of the four types of coffee, the range of the amounts of caffeine is the greatest possible amount minus the least possible amount. In the graph, this difference is represented by the length of the corresponding bar, so you can order the four types of coffee according to the lengths of their corresponding bars, from shortest to longest. From the graph, you can see that the order is decaffeinated coffee, instant coffee, drip-brewed coffee, percolated coffee. The correct answer is **Choice B**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

ENERGY USED PER METER VERSUS SPEED
DURING RUNNING AND WALKING



21. If s is a speed, in miles per hour, at which the energy used per meter during running is twice the energy used per meter during walking, then, according to the graph above, s is between

(A) 2.5 and 3.0

(B) 3.0 and 3.5

(C) 3.5 and 4.0

(D) 4.0 and 4.5

(E) 4.5 and 5.0

Explanation

This question is about the speed at which the energy used per meter during running is twice that used per meter during walking. Graphically, this is the speed for which the running energy is twice as high as the walking energy. Looking at the graph, you can see that for speeds greater than or equal to 3.0 miles per hour, the running energy is less than twice the walking energy, so the desired speed must be less than 3.0. In fact, the desired speed is between 2.0 (the lowest speed on the graph) and 3.0. Looking at the answer choices, you see that there is only one answer choice that is between 2.0 and 3.0; namely, Choice A, which says the desired speed is between 2.5 and 3.0. The correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

22. If $n = 2^3$, then $n^n =$

(A) 2^6

(B) 2^{11}

(C) 2^{18}

(D) 2^{24}

(E) 2^{27}

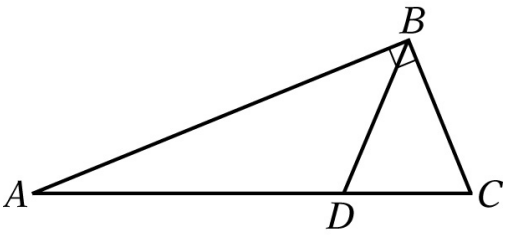
Explanation

When answering a question in which you are asked to calculate the value of an expression, it is often helpful to look at the answer choices first to see what form they are in. In this question the answer choices are all in the form 2 raised to a power, so you should try to achieve that form. It is given that $n = 2^3 = 8$. Therefore, $n^n = (2^3)^8 = 2^{24}$. The correct answer is **Choice D**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

For the following question, select all the answer choices that apply.



The length of AB is $10\sqrt{3}$.

23. Which of the following statements individually provide(s) sufficient additional information to determine the area of triangle ABC above?

Indicate all such statements.

A DBC is an equilateral triangle.

B ABD is an isosceles triangle.

C The length of BC is equal to the length of AD.

D The length of BC is 10.

E The length of AD is 10.

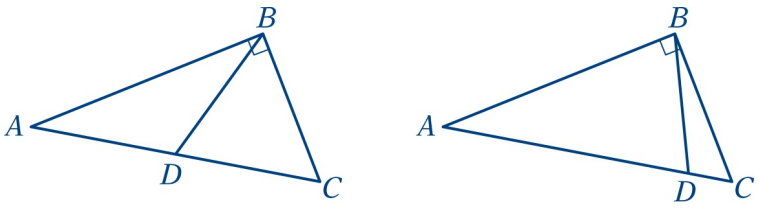
Explanation

From the figure you know that ABC is a right triangle with its right angle at vertex *B*. You also know that point *D* is on the hypotenuse AC. You are given that the length of AB is $10\sqrt{3}$. However, because the figure is not necessarily drawn to scale, you don't know the lengths of AD, DC, and BC. In particular, you don't know where *D* is on AC.

The area of a triangle is $\frac{1}{2}$(base)(height). Thus, the area of right triangle ABC is equal to $\frac{1}{2}$ of the length of AB times the length of BC. You already know that the length of AB is $10\sqrt{3}$. Any additional information that would allow you to calculate the length of BC would be sufficient to find the area of triangle ABC. You need to consider each of the five statements individually, as follows.

Statement A: DBC is an equilateral triangle. This statement implies that angle DCB is a 60° angle; and therefore, triangle ABC is a 30°– 60°– 90° triangle. Thus the length of BC can be determined, and this statement provides sufficient additional information to determine the area of triangle ABC.

Statement B: ABD is an isosceles triangle. There is more than one way in which triangle ABD can be isosceles. Below are two redrawn figures showing triangle ABD as isosceles. In the figure on the left, the length of AD is equal to the length of DB; and in the figure on the right, the length of AB is equal to the length of AD.



Either of the figures could have been drawn with the length of BC even longer. So, statement B does not provide sufficient additional information to determine the area of triangle ABC.

Statement C: The length of BC is equal to the length of AD. You have no way of finding the length of AD without making other assumptions, so statement C does not provide sufficient additional information to determine the area of triangle ABC.

Statement D: The length of BC is 10. The length of BC is known, so the area of triangle ABC can be found. Statement D provides sufficient additional information to determine the

area of triangle ABC.

Statement E: The length of AD is 10. The relationship between AD and BC is not known, so statement E does not provide sufficient additional information to determine the area of triangle ABC.

Statements A and D individually provide sufficient additional information to determine the area of triangle ABC. Therefore, the correct answer consists of **Choices A and D**.

This explanation uses the following strategies.

Strategy 8: Search for a Mathematical Relationship

Strategy 14: Determine What Additional Information Is Sufficient to Solve a Problem

For the following question, enter your answer in the box.

$$a_1, a_2, a_3, \ \ldots, a_n, \ \ldots$$

24. In the sequence above, each term after the first term is equal to the preceding term plus the constant c. If $a_1 + a_3 + a_5 = 27$, what is the value of $a_2 + a_4$?

$a_2 + a_4 =$ ☐

Explanation

Note that answering this question requires information only about the first five terms of the sequence. So it is a good idea to work with the relationships among these five terms to see what is happening.

Since you are given that in this sequence each term after a_1 is c greater than the previous term, you can rewrite the first five terms of the sequence in terms of a_1 and c as follows.

$$\begin{aligned} a_2 &= a_1 + c \\ a_3 &= a_2 + c = a_1 + 2c \\ a_4 &= a_1 + 3c \\ a_5 &= a_1 + 4c \end{aligned}$$

From the question, you know that $a_1 + a_3 + a_5 = 27$, and from the equations above, $a_1 + a_3 + a_5 = a_1 + (a_1 + 2c) + (a_1 + 4c) = 3a_1 + 6c$. So you can conclude that $3a_1 + 6c = 27$, or $a_1 + 2c = 9$.

To find $a_2 + a_4$, you can express a_2 and a_4 in terms of a_1 and c and simplify as follows.

$$\begin{aligned} a_2 + a_4 &= (a_1 + c) + (a_1 + 3c) \\ &= 2a_1 + 4c \\ &= 2(a_1 + 2c) \end{aligned}$$

But $a_1 + 2c = 9$, so $a_2 + a_4 = 2(9) = 18$. The correct answer is **18**.

This explanation uses the following strategies.

Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 7: Find a Pattern

25. A desert outpost has a water supply that is sufficient to last 21 days for 15 people. At the same average rate of water consumption per person, how many days would the water supply last for 9 people?

(A) 28.0

(B) 32.5

(C) 35.0

(D) 37.5

(E) 42.0

Explanation

The water supply is enough for 15 people to survive 21 days. Assuming the same average rate of water consumption per person, 1 person would have enough water to last for (15)(21) = 315 days. Therefore, 9 people would have enough water for $\frac{1}{9}$ of the 315 days, or 35 days. The correct answer is **Choice C**.

Another approach to solving this problem is to recognize that the water supply would last $\frac{15}{9}$ as many days for 9 people as it would for 15 people. Therefore, since the water supply would last 21 days for 15 people, it would last $\left(\frac{15}{9}\right)(21)$, or 35 days for 9 people. The correct answer is **Choice C**.

This explanation uses the following strategy.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

SECTION 6
Quantitative Reasoning
25 Questions with Explanations

For each of Questions 1 to 9, select one of the following answer choices.

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

1.

Country	Value of 1 United States Dollar

Argentina	0.93 peso
Kenya	32.08 shillings

Quantity A

The dollar value of 1 Argentine peso according to the table above

Quantity B

The dollar value of 1 Kenyan shilling according to the table above

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

When you are answering Quantitative Comparison questions, it is a good time-saving idea to see whether you can determine the relative sizes of the two quantities being compared without doing any calculations.

In the table accompanying this question, both the value of the Argentine peso and the value of the Kenyan shilling are compared to the United States dollar.

Without doing any calculations, you can see from the information given that 1 United States dollar is worth a little less than 1 Argentine peso, so 1 peso is worth more than 1 United States dollar. On the other hand, 1 United States dollar is worth 32.08 Kenyan shillings, so 1 Kenyan shilling is worth only a small fraction of 1 United States dollar. The correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 9: Estimate

2.

k is a digit in the decimal $1.3k5$, and $1.3k5$ is less than 1.33.

Quantity A

k

Quantity B

1

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

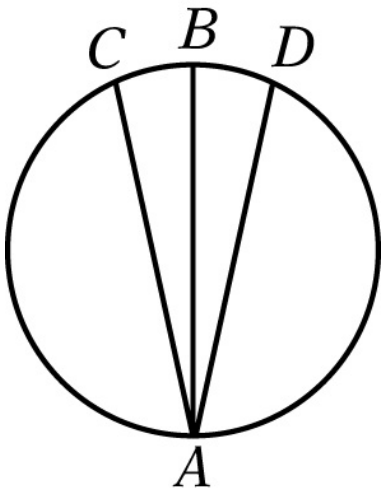
In this question, you are given that k is a digit in the decimal $1.3k5$ and that $1.3k5$ is less than 1.33. So you can see that $1.30 < 1.3k5 < 1.33$. Therefore, $1.3k5$ must equal 1.305 or 1.315 or 1.325, and the digit k must be 0, 1, or 2. The correct answer is **Choice D**.

This explanation uses the following strategies.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

Strategy 13: Determine Whether a Conclusion Follows from the Information Given

3.



AB is a diameter of the circle above.

Quantity A

The length of AB

Quantity B

The average (arithmetic mean) of the lengths of AC and AD

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Recall that in a circle, any diameter is longer than any other chord that is not a diameter. You are given that AB is a diameter of the circle. It follows that AC and AD are chords that are not diameters, since there is only one diameter with endpoint A. Hence, AB is longer than both AC and AD. Note that the average of two numbers is always less than or equal to the greater of the two numbers. Therefore, the average of the lengths of AC and AD, which is Quantity B, must be less than the length of AB, which is Quantity A. The correct answer is **Choice A**.

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

4.

$$st = \sqrt{10}$$

Quantity A

s^2

Quantity B

$\frac{10}{t^2}$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare s^2 with $\frac{10}{t^2}$. Since it is given that $st = \sqrt{10}$, it follows that $(st)^2 = \left(\sqrt{10}\right)^2$, and $s^2t^2 = 10$. Dividing both sides of the equation $s^2t^2 = 10$ by t^2, you get $s^2 = \frac{10}{t^2}$. The correct answer is **Choice C**.

You can look at this problem in another way. You can use the fact that $st = \sqrt{10}$ to express Quantity A in terms of *t*. Since $st = \sqrt{10}$, it follows that $s = \frac{\sqrt{10}}{t}$, and Quantity A is equal to $\left(\frac{\sqrt{10}}{t}\right)^2 = \frac{10}{t^2}$, which is the same as Quantity B. The correct answer is **Choice C**.

This explanation uses the following strategy.
Strategy 5: Simplify an Arithmetic or Algebraic Representation

5.

Three consecutive integers have a sum of −84.

Quantity A

The least of the three integers

Quantity B

−28

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

Two approaches you could use to solve this problem are:

1: Translate from words to algebra.
2: Determine a mathematical relationship between the two quantities.

Approach 1: You can represent the least of the three consecutive integers by *x*, and then the three integers would be represented by *x*, *x* + 1, and *x* + 2. It is given that the sum of the three integers is −84, so $x + (x + 1) + (x + 2) = -84$. You can solve this equation for *x* as follows.

$$x + (x + 1) + (x + 2) = -84$$
$$3x + 3 = -84$$
$$3x = -87$$
$$x = -29$$

Since the least of the three integers, −29, is less than −28, the correct answer is **Choice B**.

Approach 2: You could ask yourself what would happen if the least of the three consecutive integers was −28. The three consecutive integers would then be −28, −27, and −26, and their sum would be −81. But you were given that the sum of the three consecutive integers is −84, which is less than −81. Therefore, −28 is greater than the least of the three consecutive integers, and the correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

6.

In the xy-plane, the equation of line k is $3x - 2y = 0$.

Quantity A

The x-intercept of line k

Quantity B

The y-intercept of line k

Ⓐ Ⓑ Ⓒ Ⓓ

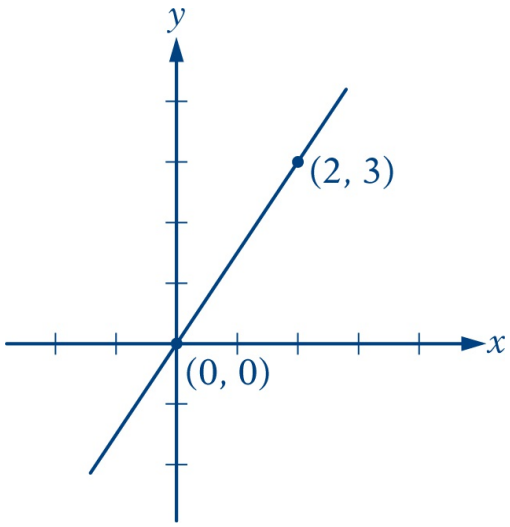
Explanation

Two approaches you could use to solve this problem are:

1: Reason algebraically.
2: Reason geometrically.

Approach 1: To solve this problem algebraically, note that given the equation of a line in the xy-plane, the x-intercept of the line is the value of x when y equals 0, and the y-intercept of the line is the value of y when x equals 0. The equation of line k is $3x - 2y = 0$. If $y = 0$, then $x = 0$; and if $x = 0$, then $y = 0$. Therefore, both the x-intercept and y-intercept of the line are equal to 0, which means that the line passes through the origin. The correct answer is **Choice C**.

Approach 2: To solve this problem geometrically, graph the line with equation $3x - 2y = 0$ in the xy-plane. Since two points determine a straight line, you can do this by plotting two points on the line and drawing the line they determine. The points (0,0) and (2,3) lie on the line, and the graph of the line in the xy-plane is shown in the following figure.



As you can see, the line passes through the origin, and so it crosses both the x-axis and the y-axis at (0,0). The correct answer is **Choice C**.

This explanation uses the following strategies.
Strategy 3: Translate from an Algebraic to a Graphical Representation
Strategy 8: Search for a Mathematical Relationship

7.

n is a positive integer that is divisible by 6.

Quantity A

The remainder when n is divided by 12

Quantity B

The remainder when n is divided by 18

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

One way to compare the two quantities is to plug in a few values of n. If you plug in $n = 36$, you find that both the remainder when n is divided by 12 and the remainder when n is divided by 18 are equal to 0, so Quantity A is equal to Quantity B. However, if you plug in $n = 18$, you find that the remainder when n is divided by 12 is 6 and the remainder when n is divided by 18 is 0, so Quantity B is greater than Quantity A. Therefore, the correct answer is **Choice D**.

Another way to compare the two quantities is to find all of the possible values of Quantity A and Quantity B. The positive integers that are divisible by 6 are 6, 12, 18, 24, 30, 36, etc. When dividing each of these integers by 12, you get a remainder of either 0 or 6, so Quantity A is either 0 or 6. When dividing each of these integers by 18, you get a remainder of either 0 or 6 or 12, so Quantity B is either 0 or 6 or 12. Note that when the value of Quantity B is 12, the value of Quantity A, 0 or 6, is less than the value of Quantity B; but when the value of Quantity B is 0, the value of Quantity A is greater than or equal to the value of Quantity B. Thus, the correct answer is **Choice D**.

This explanation uses the following strategies.
Strategy 10: Trial and Error
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

8.

$$\frac{1-x}{x-1}=\frac{1}{x}$$

Quantity A

x

Quantity B

$-\frac{1}{2}$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

One approach you could use to solve this problem is to solve the equation $\frac{1-x}{x-1}=\frac{1}{x}$ for x. Since fractions are defined only when the denominator is not equal to 0, the denominators of both of the fractions in the equation are nonzero. Therefore, $x \neq 0$ and $x \neq 1$.

To solve the equation for x, begin by multiplying both sides of the equation by the common denominator $x(x-1)$ to get $x(1-x)=(x-1)(1)$. Then proceed as follows.

$$\begin{aligned} x(1-x) &= (x-1)(1) \\ x-x^2 &= x-1 \\ x^2 &= 1 \end{aligned}$$

Since $x^2=1$ and $x \neq 1$, it follows that $x=-1$.

Quantity A is equal to −1 and Quantity B is equal to $-\frac{1}{2}$. Therefore, Quantity B is greater, and the correct answer is **Choice B**.

Another approach is to notice that for all values of $x \neq 1$, the value of $\frac{1-x}{x-1}$ is equal to −1. You can try plugging in a few numbers for x to see that this is true. For example, if you plug in $x=7$, you get $\frac{1-7}{7-1}=\frac{-6}{6}=-1$.

You can also show that for all values of $x \neq 1$, the value of $\frac{1-x}{x-1}$ is equal to −1 algebraically by rewriting $1-x$ as $-(x-1)$. Thus, $\frac{1-x}{x-1}=\frac{-(x-1)}{(x-1)}=-1$. Because the left side of the equation $\frac{1-x}{x-1}=\frac{1}{x}$ is equal to −1, it follows that $-1=\frac{1}{x}$, and so $x=-1$. Therefore, Quantity A is equal to −1, which is less than Quantity B, $-\frac{1}{2}$, and the correct answer is **Choice B**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

9. In a set of 24 positive integers, 12 of the integers are less than 50. The rest are greater than 50.

Quantity A

The median of the 24 integers

Quantity B

50

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In general, the median of a set of n positive integers, where n is even, is obtained by ordering the integers from least to greatest and then calculating the average (arithmetic mean) of the two middle integers. For this set of 24 integers, you do not know the values of the two middle integers; you know only that half of the integers are less than 50 and the other half are greater than 50. If the two middle integers in the list are 49 and 51, the median is 50; and if the two middle integers are 49 and 53, the median is 51. Thus the relationship cannot be determined from the information given, and the correct answer is **Choice D**.

This explanation uses the following strategies.

Strategy 11: Divide into Cases
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

10. The fabric needed to make 3 curtains sells for $8.00 per yard and can be purchased only by the full yard. If the length of fabric required for each curtain is 1.6 yards and all of the fabric is purchased as a single length, what is the total cost of the fabric that needs to be purchased for the 3 curtains?

Ⓐ $40.00
Ⓑ $38.40
Ⓒ $24.00
Ⓓ $16.00
Ⓔ $12.80

Explanation

Since 1.6 yards of fabric are required for each curtain, it follows that (3)(1.6), or 4.8, yards of fabric are required to make the 3 curtains. The fabric can be purchased only by the full yard, so 5 yards of fabric would need to be purchased. Since the fabric sells for $8.00 per yard, the total cost of the fabric is $40.00. The correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

For the following question, select all the answer choices that apply.

11. In the xy-plane, line k is a line that does not pass through the origin. Which of the following statements individually provide(s) sufficient additional information to conclude that the slope of line k is negative?

    Indicate all such statements.

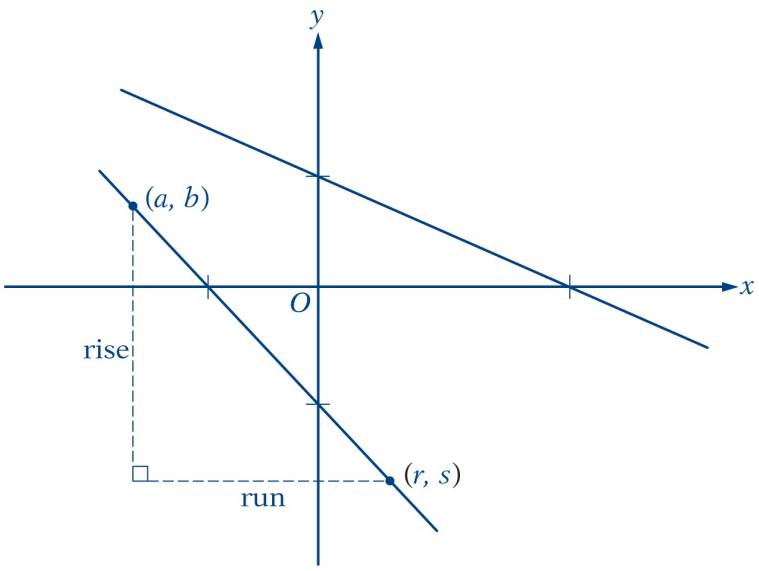
    A The x-intercept of line k is twice the y-intercept of line k.

    B The product of the x-intercept and the y-intercept of line k is positive.

    C Line k passes through the points (a, b) and (r, s), where $(a - r)(b - s) < 0$.

Explanation

For questions involving a coordinate system, it is often helpful to draw a figure to visualize the problem situation. If you draw some lines with negative slopes in the xy-plane, such as those in the figure below, you see that for each line that does not pass through the origin, the x- and y-intercepts are either both positive or both negative. Conversely, you can see that if the x- and y-intercepts of a line have the same sign then the slope of the line is negative.



You can use this fact to examine the information given in the first two statements. Remember that you need to evaluate each statement by itself.

Choice A states that the x-intercept is twice the y-intercept, so you can conclude that both intercepts have the same sign, and thus the slope of line k is negative. So the information in Choice A is sufficient to determine that the slope of line k is negative.

Choice B states that the product of the x-intercept and the y-intercept is positive. You know that the product of two numbers is positive if both factors have the same sign. So this information is also sufficient to determine that the slope of line k is negative.

To evaluate Choice C, it is helpful to recall the definition of the slope of a line passing through two given points. You may remember it as "rise over run." If the two points are (a, b)

and (r, s), then the slope is $\frac{b-s}{a-r}$.

Choice C states that the product of the quantities $(a - r)$ and $(b - s)$ is negative. Note that these are the denominator and the numerator, respectively, of $\frac{b-s}{a-r}$, the slope of line k. So you can conclude that $(a - r)$ and $(b - s)$ have opposite signs and the slope of line k is negative. The information in Choice C is sufficient to determine that the slope of line k is negative.

So each of the three statements individually provides sufficient information to conclude that the slope of line k is negative. The correct answer consists of **Choices A, B, and C**.

This explanation uses the following strategies.

Strategy 3: Translate from an Algebraic to a Graphical Representation

Strategy 14: Determine What Additional Information Is Sufficient to Solve a Problem

	Distance from Centerville (miles)
Freight train	$-10t + 115$
Passenger train	$-20t + 150$

12. The expressions in the table above give the distance from Centerville to each of two trains t hours after 12:00 noon. At what time after 12:00 noon will the trains be equidistant from Centerville?

(A) 1:30

(B) 3:30

(C) 5:10

(D) 8:50

(E) 11:30

Explanation

The distance between the freight train and Centerville at t hours past noon is $-10t + 115$. The distance between the passenger train and Centerville at t hours past noon is $-20t + 150$. To find out at what time the distances will be the same you need to equate the two expressions and solve for t as follows.

$$-10t + 115 = -20t + 150$$
$$10t + 115 = 150$$
$$10t = 35$$
$$t = 3.5$$

Therefore, the two trains will be the same distance from Centerville at 3.5 hours past noon, or at 3:30. The correct answer is **Choice B**.

This explanation uses the following strategies.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 5: Simplify an Arithmetic or Algebraic Representation

13. The company at which Mark is employed has 80 employees, each of whom has a different salary. Mark's salary of $43,700 is the second-highest salary in the first quartile of the 80 salaries. If the company were to hire 8 new employees at salaries that are less than the lowest of the 80 salaries, what would Mark's salary be with respect to the quartiles of the 88 salaries at the company, assuming no other changes in the salaries?

(A) The fourth-highest salary in the first quartile

(B) The highest salary in the first quartile

(C) The second-lowest salary in the second quartile

(D) The third-lowest salary in the second quartile

(E) The fifth-lowest salary in the second quartile

Explanation

In this question you are told that Mark's salary is the second-highest in the first quartile. From this you can conclude that the word *quartile* refers to one of the four groups that are created by listing the data in increasing order and then dividing the data into four groups of equal size. When the salaries of the 80 employees are listed in order, the 20 lowest salaries (that is, the salaries in the first quartile) are the first 20 salaries in the list. Since Mark's salary is the second-highest in the first quartile, 18 salaries in that quartile are lower than his, and one salary in that quartile is higher than his. After the salaries of the 8 new employees are added, there are 26 salaries that are lower than Mark's. The lowest 22 of those would be in the first quartile of the 88 salaries, and the remaining 4 (salaries 23 to 26) would be in the second quartile, followed by Mark's salary. This puts Mark at the fifth-lowest salary in the second quartile. The correct answer is **Choice E**.

Another way to approach this problem is to think of all 80 salaries numbered in order from least to greatest, the lowest salary at the number 1 position and the greatest salary at the number 80 position. There are 20 positions in each quartile, and Mark's salary is at position 19. The diagram below shows the salary positions and the quartile into which each position falls. Note that position 19, where Mark's salary appears, is second-highest in the first quartile.

First quartile	Second quartile	Third quartile	Fourth quartile
1	21	41	61
2	22	42	62
3	23	43	63
⋮	⋮	⋮	⋮
18	38	58	78
19 ← Mark's	39	59	79
20 salary	40	60	80

To see what Mark's position is with respect to the quartiles of the 88 salaries, you need add the 8 new salaries to the list, renumber the list from 1 to 88, and put 22 salaries in each

quartile. Because the 8 new salaries are less than the original 80 salaries, they must be listed in positions 1 through 8, and all salaries in the original list must move up by 8 positions in the renumbered list. In particular, Mark's salary moves from position 19 to position 27. The diagram below shows the renumbered list. Note that Mark's salary is in position 27, the fifth position in the second quartile.

First quartile	Second quartile	Third quartile	Fourth quartile
1	23	45	67
2 } New salaries	24	46	68
⋮	25	47	69
8	26	48	70
9 ← Salary at position 1 of original list	27 ← Mark's salary	49	71
⋮	⋮	⋮	⋮
20	42	64	86
21	43	65	87
22	44	66	88

Since Mark's salary is in the fifth position in the second quartile and the salaries are listed in order from least to greatest, Mark's salary would be the fifth-lowest in the second quartile. The correct answer is **Choice E**.

This explanation uses the following strategies.
Strategy 2: Translate from Words to a Figure or Diagram
Strategy 8: Search for a Mathematical Relationship

For the following question, enter your answer in the box.

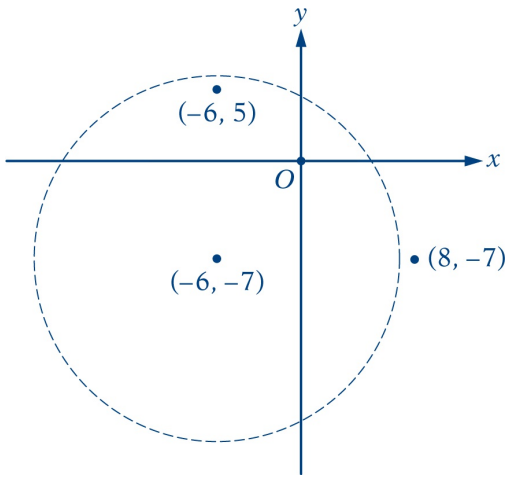
14. In the xy-plane, the point with coordinates $(-6, -7)$ is the center of circle C. The point with coordinates $(-6, 5)$ lies inside C, and the point with coordinates $(8, -7)$ lies outside C. If m is the radius of C and m is an integer, what is the value of m ?

$m =$ ☐

Explanation

A strategy that is often helpful in working with geometry problems is drawing a figure that represents the given information as accurately as possible.

In this question you are given that the point with coordinates $(-6, -7)$ is the center of circle C, the point with coordinates $(-6, 5)$ lies inside circle C, and the point with coordinates $(8, -7)$ lies outside circle C, so you could draw the following figure.



From the figure, you can see that the distance between (−6, −7) and (−6, 5) is 7 + 5, or 12, and the radius of C must be greater than 12. You can also see that the distance between (−6, −7) and (8, −7) is 6 + 8, or 14, and the radius of C must be less than 14. Therefore, since the radius is an integer greater than 12 and less than 14, it must be 13. The correct answer is **13**.

This explanation uses the following strategy.
Strategy 2: Translate from Words to a Figure or Diagram

15. If $-\frac{m}{19}$ is an even integer, which of the following must be true?

(A) m is a negative number.

(B) m is a positive number.

(C) m is a prime number.

(D) m is an odd integer.

(E) m is an even integer.

Explanation

An even integer is a multiple of 2. If $-\frac{m}{19}$ is an even integer, it must equal 2 times some integer k. This means that $-\frac{m}{19} = 2k$, or $m = -19(2k) = 2(-19k)$, which is a multiple of 2. Thus m must be an even integer, and the correct answer is **Choice E**. You can see that none of the other choices can be the correct answer by evaluating them as follows.

(A) m does not have to be a negative number for $-\frac{m}{19}$ to be even. For example, if $m = 38$, then $-\frac{m}{19} = -2$, which is an even number.

(B) m does not have to be a positive number for $-\frac{m}{19}$ to be even. For example, if $m = -38$, then $-\frac{m}{19} = 2$, which is an even number.

(C) The number used in (A), $m = 38$, shows that m does not have to be a prime number. In fact, because m is the product of at least two prime numbers (2 and 19), m cannot be

a prime number.

(D) Since m must be an even integer, m cannot be an odd integer.

This explanation uses the following strategies.
Strategy 8: Search for a Mathematical Relationship
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

For the following question, select all the answer choices that apply.

16. The integer v is greater than 1. If v is the square of an integer, which of the following numbers must also be the square of an integer?

Indicate all such numbers.

A $81v$

B $25v + 10\sqrt{v} + 1$

C $4v^2 + 4\sqrt{v} + 1$

Explanation

If v is the square of an integer, then $\sqrt{v}$ is an integer. You can use this fact, together with the fact that the product and the sum of integers are also integers, to examine the first two choices.

Choice A: The positive square root of $81v$ is $9\sqrt{v}$, which is an integer. So $81v$ is the square of an integer.

Choice B: $25v + 10\sqrt{v} + 1 = (5\sqrt{v} + 1)^2$ and $5\sqrt{v} + 1$ is an integer. So $25v + 10\sqrt{v} + 1$ is the square of an integer.

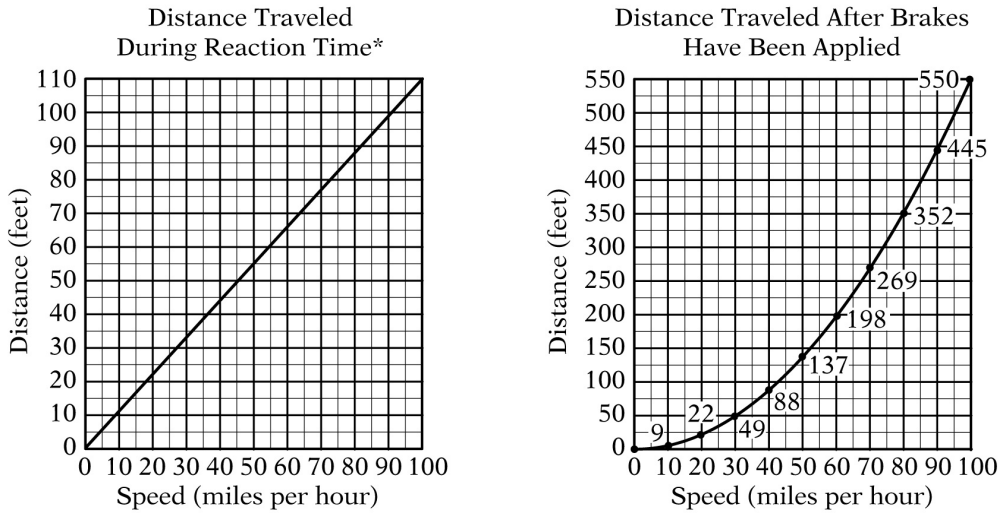
Choice C: Since there is no obvious way to factor the given expression, you may suspect that it is not the square of an integer. To show that a given statement is not true, it is sufficient to find one counterexample. In this case, you need to find one value of v such that v is the square of an integer but $4v^2 + 4\sqrt{v} + 1$ is not the square of an integer. If $v = 4$, then $4v^2 + 4\sqrt{v} + 1 = 64 + 8 + 1 = 73$, which is not the square of an integer. This proves that $4v^2 + 4\sqrt{v} + 1$ does not have to be the square of an integer.

The correct answer consists of **Choices A and B**.

This explanation uses the following strategies.
Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

Questions 17 to 20 are based on the following data.

DISTANCE TRAVELED BY A CAR ACCORDING TO THE CAR'S SPEED
WHEN THE DRIVER IS SIGNALED TO STOP



*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

Note: Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.

17. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

(A) 43
(B) 47
(C) 51
(D) 55
(E) 59

Explanation

The data accompanying questions 17 to 20 consists of two graphs. It is a good idea to look at the graphs before you try to answer the questions, so you can become familiar with the information contained in the graphs. Then, as you read each question, you should think about which of the graphs contains the information you need to solve the problem. It could be that all the information you need to solve the problem is contained in one of the graphs, or it could be that you need to get information from both of the graphs.

The graph on the left shows the relationship between the speed of the automobile and the distance it traveled during the reaction time. Therefore, the answer to this question is found using this graph by reading the speed, in miles per hour, corresponding to a distance of 52 feet. A distance of 52 feet is a little above the distance of 50 feet on the vertical axis of the graph. On the graph, the speed corresponding to a distance of 52 feet is a little less than 50 miles per hour. The correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 9: Estimate

18. Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

(A) 130
(B) 110
(C) 90
(D) 70
(E) 40

Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. At a speed of 40 miles per hour, the distance traveled during reaction time is a little less than 45 feet, and the distance traveled after the brakes have been applied is 88 feet. Since 45 + 88 = 133, the correct answer is **Choice A**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 9: Estimate

19. Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

(A) 50
(B) 55
(C) 60
(D) 65
(E) 70

Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. A good strategy for solving this problem is to calculate the total stopping distance for the speeds given in the options. For a speed of 50 miles per hour, the distance traveled during reaction time is about 55 feet, and the distance traveled after the brakes have been applied is 137 feet; therefore, the total stopping distance is about 55 + 137, or 192 feet. For a speed of 55 miles per hour, the distance traveled during reaction time is about 60 feet, and the distance traveled after the brakes have been applied is about 170 feet; therefore, the total stopping distance is about 60 + 170, or 230 feet. Since the speeds in the

remaining choices are greater than 55 miles per hour and both types of stopping distances increase as the speed increases, it follows that the total stopping distances for all the remaining choices are greater than 200 feet. The correct answer is **Choice A**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 9: Estimate

20. The total stopping distance for the car traveling at 60 miles per hour is approximately what percent greater than the total stopping distance for the car traveling at 50 miles per hour?

(A) 22%
(B) 30%
(C) 38%
(D) 45%
(E) 52%

Explanation

To solve this problem you need to find the total stopping distance at 50 miles per hour and at 60 miles per hour, find their difference, and then express the difference as a percent of the shorter total stopping distance. You need to use both graphs to find the total stopping distances. At 50 miles per hour, the total stopping distance is approximately 55 + 137 = 192 feet; and at 60 miles per hour it is approximately 66 + 198 = 264 feet. The difference of 72 feet as a percent of 192 feet is $\frac{72}{192} = 0.375$, or approximately 38%. The correct answer is **Choice C**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 9: Estimate

21. What is the least positive integer that is <u>not</u> a factor of <u>not</u> a factor of $25!$ and is <u>not</u> a prime number?

(A) 26
(B) 28
(C) 36
(D) 56
(E) 58

Explanation

Note that <u>not</u> a factor of $25!$ is equal to the product of all positive integers from 1 to 25, inclusive. Thus, every positive integer less than or equal to 25 is a factor of <u>not</u> a factor of

$25!$. Also, any integer greater than 25 that can be expressed as the product of different positive integers less than 25 is a factor of not a factor of $25!$. In view of this, it's reasonable to consider the next few integers greater than 25, including answer choices A and B.

Choice A, 26, is equal to (2)(13). Both 2 and 13 are factors of not a factor of $25!$, so 26 is also a factor of not a factor of $25!$. The same is true for 27, or (3)(9), and for Choice B, 28, or (4)(7). However, the next integer, 29, is a prime number greater than 25, and as such, it has no positive factors (other than 1) that are less than or equal to 25. Therefore, 29 is the least positive integer that is not a factor of not a factor of $25!$. However, the question asks for an integer that is not a prime number, so 29 is not the answer.

At this point, you could consider 30, 31, 32, etc., but it is quicker to look at the rest of the choices. Choice C, 36, is equal to (4)(9). Both 4 and 9 are factors of not a factor of $25!$, so 36 is also a factor of not a factor of $25!$. Choice D, 56, is equal to (4)(14). Both 4 and 14 are factors of not a factor of $25!$, so 56 is also a factor of not a factor of $25!$. Choice E, 58, is equal to (2)(29). Although 2 is a factor of not a factor of $25!$, the prime number 29, as noted earlier, is not a factor of not a factor of $25!$, and therefore 58 is not a factor of not a factor of $25!$. The correct answer must be **Choice E**.

The explanation above uses a process of elimination to arrive at Choice E, which is sometimes the most efficient way to find the correct answer. However, one can also show directly that the correct answer is 58. For if a positive integer n is not a factor of not a factor of $25!$, then one of the following must be true:

(i) n is a prime number greater than 25, like 29 or 31, or a multiple of such a prime number, like 58 or 62;
(ii) n is so great a multiple of some prime number less than 25, that it must be greater than 58.

To see that (i) or (ii) is true, recall that every integer greater than 1 has a unique prime factorization, and consider the prime factorization of not a factor of $25!$. The prime factors of not a factor of $25!$ are 2, 3, 5, 7, 11, 13, 17, 19, and 23, some of which occur more than once in the product not a factor of $25!$. For example, there are 8 positive multiples of 3 less than 25, namely 3, 6, 9, 12, 15, 18, 21, and 24. The prime number 3 occurs once in each of these multiples, except for 9 and 18, in which it occurs twice. Thus, the factor 3 occurs 10 times in the prime factorization of not a factor of $25!$. The same reasoning can be used to find the number of times that each of the prime factors occur, yielding the prime factorization $25! = (2^{22})(3^{10})(5^{6})(7^{3})(11^{2})(13)(17)(19)(23)$. Any integer whose prime factorization is a combination of one or more of the factors in the prime factorization of not a factor of $25!$, perhaps with lesser exponents, is a factor of not a factor of $25!$. Equivalently, if the positive integer n is not a factor of not a factor of $25!$, then, restating (i) and (ii) above, the prime factorization of n must

(i) include a prime number greater than 25; or
(ii) have a greater exponent for one of the prime numbers in the prime factorization of not a factor of $25!$.

For (ii), the least possibilities are 2^{23}, 3^{11}, 5^7, 7^4, 11^3, 13^2, 17^2, 19^2, and 23^2. Clearly, all of these are greater than 58. The least possibility for (i) that is not a prime number is 58, and the least possibility for (ii) is greater than 58, so **Choice E** is the correct answer.

This explanation uses the following strategies.
Strategy 8: Search for a Mathematical Relationship
Strategy 10: Trial and Error

22. If $0 < a < 1 < b$, which of the following is true about the reciprocals of a and b ?

(A) $1 < \frac{1}{a} < \frac{1}{b}$

(B) $\frac{1}{a} < 1 < \frac{1}{b}$

(C) $\frac{1}{a} < \frac{1}{b} < 1$

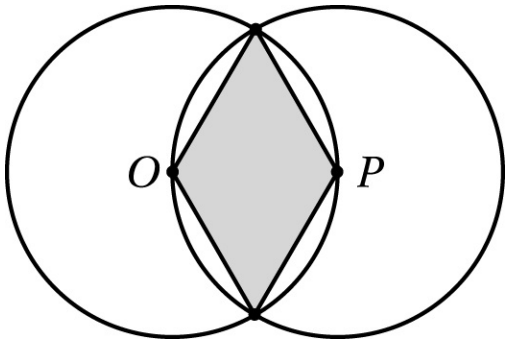
(D) $\frac{1}{b} < 1 < \frac{1}{a}$

(E) $\frac{1}{b} < \frac{1}{a} < 1$

Explanation

To answer this question, you must first look at the answer choices. Note that all of the choices are possible orderings of the quantities $\frac{1}{a}$, $\frac{1}{b}$, and 1 from least to greatest. So to answer the question, you must put the three quantities in order from least to greatest. The inequality $0 < a < 1 < b$ tells you that $0 < a < 1$ and that $b > 1$. Since a is a value between 0 and 1, the value of $\frac{1}{a}$ must be greater than 1. Since b is greater than 1, the value of $\frac{1}{b}$ must be less than 1. So you know that $\frac{1}{a} > 1$ and that $\frac{1}{b} < 1$, or combined in one expression, $\frac{1}{b} < 1 < \frac{1}{a}$, and the correct answer is **Choice D**.

This explanation uses the following strategy.
Strategy 5: Simplify an Arithmetic or Algebraic Representation



23. In the figure above, O and P are the centers of the two circles. If each circle has radius r,

what is the area of the shaded region?

(A) $\frac{\sqrt{2}}{2}r^2$

(B) $\frac{\sqrt{3}}{2}r^2$

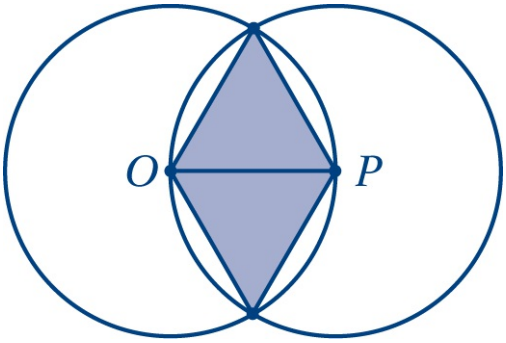
(C) $\sqrt{2}r^2$

(D) $\sqrt{3}r^2$

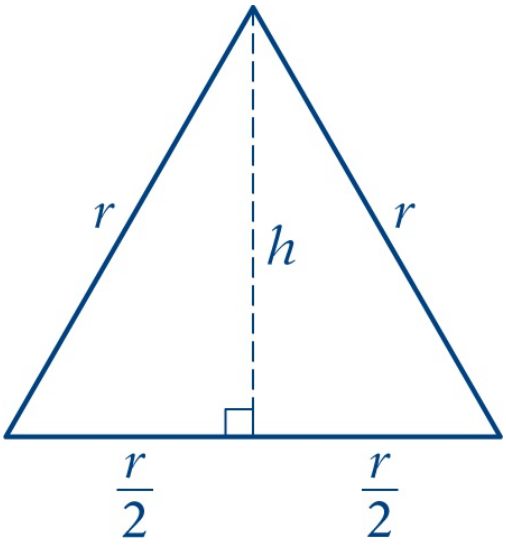
(E) $2\sqrt{3}r^2$

Explanation

If a geometric problem contains a figure, it can be helpful to draw additional lines and add information given in the text of the problem to the figure. For circles, the helpful additional lines are often radii or diameters. In this case, drawing radius OP will divide the shaded region into two triangles, as shown in the figure below.



The two circles have the same radius, *r*. Therefore, in each of the triangles, all three sides have length *r*, and each of the triangles is equilateral. If you remember from geometry that the height of an equilateral triangle with sides of length *r* is $\frac{\sqrt{3}}{2}r$, you could use that fact in solving the problem. However, if you do not remember what the height is, you can use the following figure to help you find the height.



Using the Pythagorean theorem, you get

$$\left(\frac{r}{2}\right)^2 + h^2 = r^2$$
$$\frac{r^2}{4} + h^2 = r^2$$
$$h^2 = \frac{3}{4}r^2$$
$$h = \frac{\sqrt{3}}{2}r$$

So the area of the equilateral triangle is $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(r)\left(\frac{\sqrt{3}}{2}r\right) = \frac{\sqrt{3}}{4}r^2$. Since the shaded region consists of 2 equilateral triangles with sides of length r, the area of the shaded region is $(2)\left(\frac{\sqrt{3}}{4}r^2\right) = \frac{\sqrt{3}}{2}r^2$, and the correct answer is **Choice B.**

This explanation uses the following strategies.
Strategy 6: Add to a Geometric Figure
Strategy 8: Search for a Mathematical Relationship

For the following question, enter your answer in the boxes.

24. Of the 20 lightbulbs in a box, 2 are defective. An inspector will select 2 lightbulbs simultaneously and at random from the box. What is the probability that neither of the lightbulbs selected will be defective?

    Give your answer as a fraction.

$$\frac{\square}{\square}$$

Explanation

The desired probability corresponds to the fraction

$$\frac{\text{the number of ways that 2 lightbulbs, both of which are not defective, can be chosen}}{\text{the number of ways that 2 lightbulbs can be chosen}}$$

In order to calculate the desired probability, you need to calculate the values of the numerator and the denominator of this fraction.

In the box there are 20 lightbulbs, 18 of which are not defective. The numerator of the fraction is the number of ways that 2 lightbulbs can be chosen from the 18 that are not defective, also known as the number of combinations of 18 objects taken 2 at a time.

If you remember the combinations formula, you know that the number of combinations is $\frac{18!}{2!(18-2)!}$ (which is denoted symbolically as $\binom{18}{2}$ or ${}_{18}C_2$). Simplifying, you get

$$\frac{18!}{2!16!} = \frac{(18)(17)(16!)}{(2)(16!)} = \frac{(18)(17)}{2} = 153$$

Similarly, the denominator of the fraction is the number of ways that 2 lightbulbs can be chosen from the 20 in the box, which is $\binom{20}{2} = \frac{20!}{2!18!} = \frac{(20)(19)(18!)}{(2)(18!)} = \frac{(20)(19)}{2} = 190$. Therefore, the probability that neither of the lightbulbs selected will be defective is $\frac{153}{190}$. The correct answer is $\mathbf{\frac{153}{190}}$ (or any equivalent fraction).

Another approach is to look at the selection of the two lightbulbs separately. The problem states that lightbulbs are selected simultaneously. However, the timing of the selection only ensures that the same lightbulb is not chosen twice. This is equivalent to choosing one lightbulb first and then choosing a second lightbulb without replacing the first. The probability that the first lightbulb selected will not be defective is $\frac{18}{20}$. If the first lightbulb selected is not defective, there will be 19 lightbulbs left to choose from, 17 of which are not defective. Thus, the probability that the second lightbulb selected will not be defective is $\frac{17}{19}$. The probability that both lightbulbs selected will not be defective is the product of these two probabilities. Thus, the desired probability is $\left(\frac{18}{20}\right)\left(\frac{17}{19}\right) = \frac{153}{190}$. The correct answer is $\mathbf{\frac{153}{190}}$ (or any equivalent fraction).

This explanation uses the following strategies.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 5: Simplify an Arithmetic or Algebraic Representation

25. What is the perimeter, in meters, of a rectangular playground 24 meters wide that has the same area as a rectangular playground 64 meters long and 48 meters wide?

(A) 112

(B) 152

(C) 224

(D) 256

(E) 304

Explanation

The area of the rectangular playground that is 64 meters long and 48 meters wide is (64)(48) = 3,072 square meters. The second playground, which has the same area, is 24 meters wide and $\frac{3,072}{24} = 128$ meters long. Therefore, the perimeter of the second playground is (2)(24) + (2)(128) = 304 meters. The correct answer is **Choice E**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation