Appendix A

Math Facts for the GRE

In This Chapter:

Algebra

Absolute Value

The absolute value of a number is its distance from zero. Absolute values are always positive.

To solve any equation or inequality that has an absolute value, solve it once for the positive version of what's in the absolute value, and solve it once for the negative version of what's in the absolute value.

For example: $|x + 3| = 13$

$$\begin{array}{rcr} (x+3) = 13 & \text{and} & -(x+3) = 13 \\ x+3 = 13 & & -x-3 = 13 \\ & & -x = 16 \\ x = 10 & \text{or} & x = -16 \end{array}$$

Algebraic Translations

- "is," "are," "was," "were": =
- "of": multiply
- "more than," "total," "sum": add
- "percent": /100
- "what": the unknown variable, or *x*
- "the ratio of [this] to [that]": this/that

- "There are 3/5 as many A's as B's": A = 3/5 (B)

For example: "What is 8 more than 25% of 6% of $\frac{2}{3}$ of 12?"

translates to: "$x = 8 + \frac{25}{100} \times \frac{6}{100} \times \frac{2}{3} \times 12$"

$$x = 8 + .25 \times .06 \times \frac{2}{3} \times 12$$

$$x = 8 + .12$$

$$x = 8.12$$

Inequalities

When simplifying an inequality and multiplying or dividing by a negative, flip the greater than or less than sign.

For example: if $-3x > 6$, then $x < -2$.

Overlapping Sets

Formula for overlapping sets:

Total = Group 1 + Group 2 − Both + Neither

For example:

There are 526 students at Grover Cleveland High School. 156 take Mandarin. 432 take Spanish.

If 231 take both Spanish and Mandarin, how many take neither?

This translates to:

526 = 156 + 432 − 231 + Neither

526 = 357 + Neither

169 = Neither

Quadratics

Know the factored and foiled forms of these common quadratics:

$(x + y)(x + y) = x^2 + 2xy + y^2$

$(x - y)(x - y) = x^2 - 2xy + y^2$

$(x + y)(x - y) = x^2 - y^2$

Rates and Work

Distance = Rate × Time
Work = Rate × Time
Working together? Add the rates.
Working against each other (for example, the faster car trying to catch the slower car)? Subtract the rates.

$$\text{Average speed for the entire trip} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Sequences

In the notation a_n, the n indicates the "slot" or "place" in the sequence. So, a_1 refers to the first element in the sequence, a_2 refers to the second, and so on.

The expression a_n itself indicates the "*n*th" number of the sequence, or any number in the sequence.

The expression a_{n-1} means "the previous term." The expression a_{n-2} means "two terms before." The expression a_{n+1} means "the following term."

Translate a sequence rule into a set of instructions.

> For example: $a_n = 2(a_{n-1}) - 5$ is saying "For any given term in the sequence, take the previous term, multiply by 2, and then subtract 5."

Arithmetic

Divisibility/Primes

Prime numbers = numbers that are divisible only by 1 and themselves
Smallest prime number = 2
Only even prime number = 2
Primes from 1 to 50: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
Factors = numbers that divide evenly into another number

For example: the factors of 12 are 1, 12, 2, 6, 3, and 4.

Multiples = numbers that another number multiplies up to

For example: multiples of 4 include 4, 8, 12, 16, 20…

Prime factors = the prime building blocks that make up another number

For example: the prime factorization of 12 is $2 \times 2 \times 3$, or $2^2 3$.

The greatest common factor of two numbers is the product of the prime factors shared between those two numbers (multiply only the numbers they have in common).

For example, to find the greatest common factor of 16 and 28, start by finding the prime factors of each. 16 is $2 \times 2 \times 2 \times 2$, or 2^4. 28 is $2 \times 2 \times 7$, or $2^2 7$. 2^4 and $2^2 7$ have in common two 2s, or 2^2. The greatest common factor of 16 and 28 is thus 2×2, or 4.

Exponents/Roots

When multiplying two exponents with the same base, add the powers.

For example: $3^2 \times 3^5 = 3^{2+5} = 3^7$

When dividing two exponents with the same base, subtract the powers.

For example: $\frac{5^9}{5^6} = 5^{9-6} = 5^3$

When raising an exponent to another power, multiply the exponents.

For example: $(x^7)^3 = x^{7\times3} = x^{21}$

When the powers match, you can combine bases.

For example: $2^x \times 3^x = (2 \times 3)^x = 6^x$

If the bases don't match and the powers don't match, you can't do anything.

For example, $2^2 \times 3^5$ cannot be simplified further.

If you're adding or subtracting numbers that have the same base, factor:

For example: $2^5 + 2^8 = 2^5\,(1 + 2^3) = 2^5\,(9)$

Negative exponents … just reciprocate.

For example: $5^{-9} = \frac{1}{5^9}$

Fractions

Adding or subtracting fractions? Find a common denominator and add the numerators.

For example: $\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

Multiplying fractions? Multiply across the numerators and the denominators.

For example: $\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$

Dividing by a fraction? Multiply by the reciprocal.

For example: $\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$

Odds/Evens

Even +/− Odd = Odd
Even +/− Even = Even
Odd +/− Odd = Even
Even × (Even or Odd) = Even
Odd × Odd = Odd
$\text{Even}^{\text{any power}}$ = Even

$\text{Odd}^{\text{any power}} = \text{Odd}$
Code for even: multiple of 2; $x = 2y$
Code for odd: prime greater than 2; $x = 2y + 1$

Remainders

Remainders are the integer left over after division.

For example: the remainder when 7 is divided by 5 is 2.

"when n is divided by 12, the remainder is 4" = n could be any multiple of $12 + 4$

Order of Operations

Simplify expressions in the correct order:

Parentheses

Exponents

Multiplication

Division

Addition

Subtraction

Positives and Negatives

Pos × Pos = Pos

Pos × Neg = Neg

Neg × Neg = Pos

Subtracting a negative? Add a positive.

For example, $5 - -2 = 5 + 2 = 7$.

Inequalities as code for positive and negative:

$x > 0$ means x is positive.

$y < 0$ means y is negative.

$x + y > 0$ means "at least one is positive."

$x + y < 0$ means “at least one is negative.”

$x - y > 0$ means $x > y$, and $x - y < 0$ means $x < y$.

$xy > 0$ or $\frac{x}{y} > 0$ means that x and y are the same sign (either both are positive or both are negative).

$xy < 0$ or $\frac{x}{y} < 0$ means that x and y are the different signs (one is positive and the other is negative).

$x^2 < x$ means x is a fraction between 0 and 1.

Statistics

Combinatorics

When you have several options and want to find the total number of possibilities, multiply the options.

> For example: if you have 3 appetizer options, 5 main dish options, and 2 dessert options, you have 3 × 5 × 2 = 30 total meal options.

Factorial—the exclamation point—indicates the product of an integer and all the integers below it. For example: 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1.

When order matters (e.g., ordering 1st, 2nd, and 3rd place), use the formula $\frac{(\text{Pool!})}{(\text{Out!})}$.

> For example: if you are finding how many ways Gold, Silver, and Bronze medals could be awarded to 7 competitors, the formula would look like $\frac{7!}{4!}$.

When order does not matter (e.g., picking a team), use the formula $\frac{(\text{Pool!})}{(\text{In!})(\text{Out!})}$.

> For example: if you are picking a team of 3 members from 7 candidates, you would use the formula $\frac{7!}{3!4!}$.

Percents

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

Looking for a percent increase or decrease? Use the percent change formula:

$$\frac{\text{percent change}}{100} = \frac{\text{difference}}{\text{original}}$$

> For example: if the price of a shirt is cut from \$80 to \$60, find the percent decrease using the above formula. The difference between \$80 and \$60 is \$20, and the original, or the shirt you started with, is \$80. Thus, plug in

those numbers: $\frac{\text{percent change}}{100} = \frac{20}{80}$. Solve for the percent change to find 25%.

Trying to change from a fraction to a percent? Multiply by 100.

For example: $\frac{1}{4} \times 100 = 100/4 = 25\%,$

$\frac{2}{7} \times 100 = \frac{200}{7}\ \%$

Common fraction to percent translations:

$\frac{1}{2} = 50\%$

$\frac{1}{4} = 25\%$

$\frac{1}{5} = 20\%$

$\frac{1}{8} = 12.5\%$

Average, Median, Range, and Spread

$$\text{Average} = \frac{\text{sum of the elements}}{\text{number of elements}}$$

Range = biggest number in a set − smallest number in a set

Median = middle number when list is put in order. If there is no middle number, the median is the average of the two middle numbers.

In evenly spaced sets (such as consecutive integers), the average equals the median.

Number of terms in an evenly spaced set = range + 1.

“Normally distributed” data can be visualized as a bell curve. The data are symmetrical around the mean. The mean and median are the same in a normally distributed set.

For example, let's say GRE scores are normally distributed, with a mean of 300 and a standard deviation of 12. We can visualize like this:

The below percents are true within any normal distribution:

Percent of numbers within 1 standard deviation of the mean in both directions (normal distribution): 68%

Percent of numbers within 2 standard deviations of the mean in both directions (normal distribution): 95%

Percent of numbers within 3 standard deviations of the mean in both directions (normal distribution): 99.7%

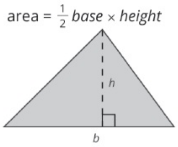
Probability

Probability: desired outcomes/total outcomes

Geometry

Area of a Triangle

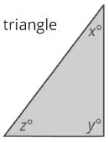
area = $\frac{1}{2}$ *base* × *height*



Angles in a Triangle

All angles in a triangle sum to 180°.

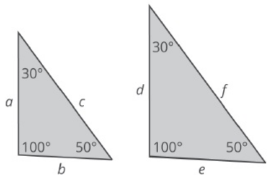
$x + y + z = 180$



Similar Triangles

Similar triangles have all the same angles and sides that are proportionate.

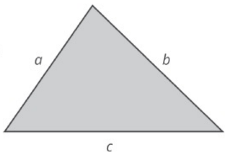
For example, a:c equals d:f.



Third Side Rule

The third side of a triangle is greater than the difference of the other two sides and less than their sum.

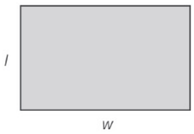
$(a + b) > c > (b - a)$



Area of a Rectangle

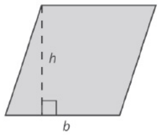
area of a rectangle = *length* × *width*

area of a square = *side*^{2}



Area of a Parallelogram

area of a parallelogram = *base* × *height*



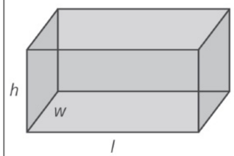
Sum of Angles in a Polygon

sum of angles in a polygon = $180(n - 2)$, where n = number of sides

Volume and Surface Area of a Box

volume = $l \times w \times h$

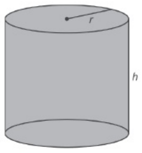
surface area = $2lw + 2wh + 2lh$



Volume and Surface Area of a Cylinder

volume = $\pi r^2 h$

surface area = $2\pi r^2 + 2\pi rh$

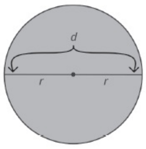


Circles

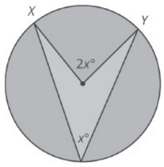
diameter = $2r$

circumference = πd or $2\pi r$

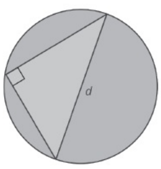
area = πr^2



A central angle's vertex is the center point of the circle. An inscribed angle's vertex is on the circle itself. When these share an arc, the inscribed angle is half the size of the central angle.

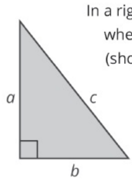


If one of the sides of an inscribed triangle is a diameter of the circle, then the triangle must be a right triangle.



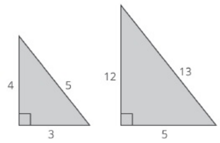
Pythagorean Theorem

In a right triangle, $a^2 + b^2 = c^2$, where a and b are the legs (shorter sides) and c is the hypotenuse.



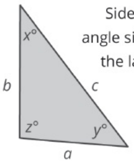
Pythagorean Triples

The most comment triples are 3–4–5, 5–12–13, and 6–8–10.

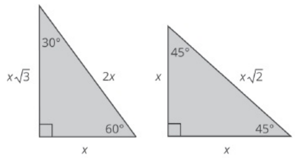


Angle/Side Relationship

Side length corresponds with angle size. Angle z and side c are the largest. Angle x and side a are the smallest.



Special Right Triangles

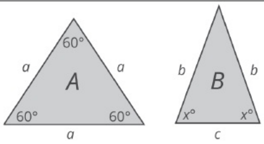


Isosceles and Equilateral

equilateral: all sides are the same length

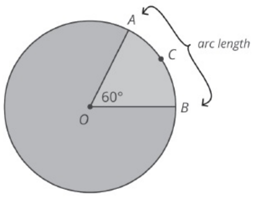
isosceles: two sides are the same length

In an equilateral triangle, all angles are 60°.



Sector

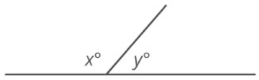
Sector area and arc length are proportional to angle size. Here, the angle is 60°. Since there are 360° in a circle, the sector (grey area) takes up $\frac{1}{6}$ of the circle. Thus, its area is $\frac{1}{6}$ of the total area, and its arc length is $\frac{1}{6}$ of the circumference.



Line and Angles

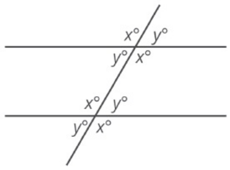
All angles in a line add up to 180°.

$x + y = 180$



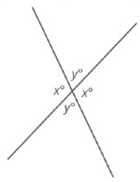
Parallel Lines

When a line intersects two parallel lines, the corresponding angles are equal.



Intersecting Lines

When two lines intersect, opposite angles are equal.

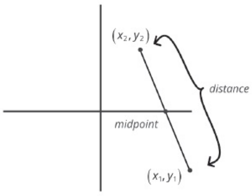


Distance and Midpoint on a Coordinate Plane

If you have endpoints (x_1, y_1) and (x_2, y_2) of a line on a coordinate plane:

midpoint = $\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$

distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Slope, x-, y-intercept

$y = mx + b$, where m = slope, b = y-intercept

slope = $\frac{\text{rise}}{\text{run}}$ or $\frac{(y_2 - y_1)}{(x_2 - x_1)}$

The y-intercept is the y-coordinate on a line at which $x = 0$.

The x-intercept is the x-coordinate on a line at which $y = 0$.