5.4 Answer Key

1. E
2. D
3. E
4. A
5. B
6. E
7. C
8. D
9. C
10. E
11. A
12. C
13. B
14. B
15. D
16. E
17. A
18. B
19. D
20. B
21. C
22. D
23. E
24. C
25. C
26. C
27. E
28. D
29. B
30. C
31. C
32. D
33. C
34. A
35. B
36. E
37. E
38. D
39. C
40. E
41. E
42. D
43. D
44. B
45. D
46. D
47. C
48. E
49. B
50. E
51. D
52. B
53. A
54. E
55. D
56. D
57. D
58. B
59. A
60. E
61. A
62. C
63. B
64. C
65. B
66. A
67. B
68. B
69. B
70. D
71. D
72. C
73. E
74. E
75. D
76. B
77. B
78. C
79. B
80. D
81. A
82. D
83. D
84. D
85. E
86. D
87. B
88. D
89. D
90. D
91. C
92. E
93. B
94. E
95. E
96. E
97. D
98. B
99. B
100. B
101. B
102. B
103. E
104. C
105. B
106. A
107. E
108. D
109. A
110. E
111. D
112. B
113. E
114. E
115. D
116. E
117. A
118. E
119. D
120. D

121.	B	151.	B	181.	B	211.	D
122.	D	152.	B	182.	B	212.	E
123.	D	153.	C	183.	A	213.	E
124.	E	154.	B	184.	B	214.	D
125.	D	155.	B	185.	D	215.	D
126.	C	156.	C	186.	C	216.	D
127.	D	157.	A	187.	C	217.	E
128.	B	158.	B	188.	B	218.	B
129.	A	159.	B	189.	E	219.	A
130.	D	160.	E	190.	A	220.	D
131.	A	161.	B	191.	A	221.	E
132.	E	162.	C	192.	D	222.	C
133.	D	163.	D	193.	E	223.	B
134.	B	164.	C	194.	D	224.	A
135.	A	165.	B	195.	E	225.	B
136.	A	166.	D	196.	A	226.	C
137.	A	167.	E	197.	C	227.	B
138.	E	168.	D	198.	C	228.	D
139.	D	169.	B	199.	E	229.	D
140.	D	170.	C	200.	A	230.	C
141.	D	171.	E	201.	D		
142.	C	172.	C	202.	A		
143.	C	173.	B	203.	B		
144.	A	174.	D	204.	B		
145.	D	175.	C	205.	D		
146.	E	176.	C	206.	D		
147.	A	177.	C	207.	D		
148.	B	178.	E	208.	D		
149.	E	179.	B	209.	D		
150.	E	180.	D	210.	D		

5.5 Answer Explanations

The following discussion is intended to familiarize you with the most efficient and effective approaches to the kinds of problems common to problem solving questions. The particular questions in this chapter are generally representative of the kinds of problem solving questions you will encounter on the GMAT exam. Remember that it is the problem solving strategy that is important, not the specific details of a particular question.

1. The price of a coat in a certain store is \$500. If the price of the coat is to be reduced by \$150, by what percent is the price to be reduced?

(A) 10%
(B) 15%
(C) 20%
(D) 25%
(E) 30%

Arithmetic Percents

A reduction of \$150 from \$500 represents a percent decrease of $\left(\frac{150}{500}\times 100\right)\% = 30\%$. Therefore, the price of the coat was reduced by 30%.

The correct answer is E.

2. On a vacation, Rose exchanged \$500.00 for euros at an exchange rate of 0.80 euro per dollar and spent $\frac{3}{4}$ of the euros she received. If she exchanged the remaining euros for dollars at an exchange rate of \$1.20 per euro, what was the dollar amount she received?

(A) \$60.00
(B) \$80.00
(C) \$100.00
(D) \$120.00
(E) \$140.00

Arithmetic Operations with rational numbers

At the exchange rate of 0.80 euro per dollar, Rose exchanged \$500.00 for $(0.80)(500) =$ 400 euros. She spent $\frac{3}{4}(400) = 300$ euros, had $400 - 300 = 100$ euros left, and exchanged them for dollars at the exchange rate of \$1.20 per euro. Therefore, the dollar amount she received was $(1.20)(100) = \$120.00$.

The correct answer is D.

x	x	x	y	y	v
v	x	x	y	w	w

3. Each of the 12 squares shown is labeled x, y, v, or w. What is the ratio of the number of these squares labeled x or y to the number of these squares labeled v or w?

(A) 1:2
(B) 2:3
(C) 4:3
(D) 3:2
(E) 2:1

Arithmetic Ratio and proportion

By a direct count, there are 8 squares labeled x or y (5 labeled x, 3 labeled y) and there are 4 squares labeled v or w (2 labeled v, 2 labeled w). Therefore, the ratio of the number of squares labeled x or y to the number of squares labeled v or w is 8:4, which reduces to 2:1.

The correct answer is E.

4. $\frac{1}{3}+\frac{1}{2}-\frac{5}{6}+\frac{1}{5}+\frac{1}{4}-\frac{9}{20}=$

(A) 0

(B) $\frac{2}{15}$

(C) $\frac{2}{5}$

(D) $\frac{9}{20}$

(E) $\frac{5}{6}$

Arithmetic Operations with rational numbers

A number that is divisible by each of the denominators is 60. Therefore, 60 can be used as a common denominator, which gives the following:

$$\frac{20}{60}+\frac{30}{60}-\frac{50}{60}+\frac{12}{60}+\frac{15}{60}-\frac{27}{60}=$$
$$\frac{20+30-50+12+15-27}{60}=\frac{0}{60}=0.$$

The correct answer is A.

5. Bouquets are to be made using white tulips and red tulips, and the ratio of the number of white tulips to the number of red tulips is to be the same in each bouquet. If there are 15 white tulips and 85 red tulips available for the bouquets, what is the greatest number of bouquets that can be made using all the tulips available?

(A) 3
(B) 5
(C) 8
(D) 10
(E) 13

Arithmetic Applied problems; Properties of numbers

Because all the tulips are to be used and the same number of white tulips will be in each bouquet, the number of white tulips in each bouquet times the number of bouquets must equal the total number of white tulips, or 15. Thus, the number of bouquets must be a factor of 15, and so the number must be 1, 3, 5, or 15. Also, the number of red tulips in each bouquet times the number of bouquets must equal the total number of red tulips, or 85. Thus, the number of bouquets must be a factor of 85, and so the number must be 1, 5, 17, or 85. Since the number of bouquets must be 1, 3, 5, or 15, and the number of bouquets must be 1, 5, 17, or 85, it follows that the number of bouquets must be 1 or 5, and thus the greatest number of bouquets that can be made is 5. Note that each of the 5 bouquets will have 3 white tulips, because (5)(3) = 15, and each of the 5 bouquets will have 17 red tulips, because (5)(17) = 85.

The correct answer is B.

6. 125% of 5 =

(A) 5.125
(B) 5.25
(C) 6
(D) 6.125
(E) 6.25

Arithmetic Percents

125% of 5 represents $\frac{125}{100}\times 5$, or $1.25\times 5=6.25$.

The correct answer is E.

7. Today Rebecca, who is 34 years old, and her daughter, who is 8 years old, celebrate their birthdays. How many years will pass before Rebecca's age is twice her daughter's age?

(A) 10
(B) 14
(C) 18
(D) 22
(E) 26

Algebra Applied problems

Let x be the desired number of years. In x years, Rebecca will be $34 + x$ years old and her daughter will be $8 + x$ years old. From the given information, it follows that $34 + x = 2(8 + x)$. The last equation is equivalent to $34 + x = 16 + 2x$, which has solution $x = 18$.

The correct answer is C.

8. When traveling at a constant speed of 32 miles per hour, a certain motorboat consumes 24 gallons of fuel per hour. What is the fuel consumption of this boat at this speed measured in miles traveled per gallon of fuel?

(A) $\frac{2}{3}$

(B) $\frac{3}{4}$

(C) $\frac{4}{5}$

(D) $\frac{4}{3}$

(E) $\frac{3}{2}$

Arithmetic Operations with rational numbers

If the motorboat consumes 24 gallons of fuel in 1 hour, then it consumes 1 gallon of fuel in $\frac{1}{24}$ hour. If the motorboat travels 32 miles in 1 hour, then it travels $\frac{32}{24} = \frac{4}{3}$ miles in $\frac{1}{24}$ hour, which is the length of time it takes to consume 1 gallon of fuel. Thus, the motorboat travels $\frac{4}{3}$ miles per gallon of fuel.

The correct answer is D.

9. A case contains c cartons. Each carton contains b boxes, and each box contains 100 paper clips. How many paper clips are contained in 2 cases?

(A) $100bc$

(B) $\frac{100b}{c}$

(C) $200bc$

(D) $\frac{200b}{c}$

(E) $\frac{200}{bc}$

Algebra Simplifying algebraic expressions

Each case has bc boxes, each of which has 100 paper clips. The total number of paper clips in 2 cases is thus $2(bc)(100) = 200bc$.

The correct answer is C.

10. A technician makes a round-trip to and from a certain service center by the same route. If the technician completes the drive to the center and then completes 10 percent of the drive from the center, what percent of the round-trip has the technician completed?

(A) 5%

(B) 10%

(C) 25%

(D) 40%

(E) 55%

Arithmetic Percents

In completing the drive to the service center, the technician has completed 50% of the round-trip. The drive from the center is the other 50% of the round-trip. In completing 10% of the drive from the center, the technician has completed an additional 10% of 50%, or 5% of the round-trip. Thus, the technician has completed 50% + 5% = 55% of the round-trip.

The correct answer is E.

11. Raffle tickets numbered consecutively from 101 through 350 are placed in a box. What is the probability that a ticket selected at random will have a number with a hundreds digit of 2 ?

(A) $\frac{2}{5}$

(B) $\frac{2}{7}$

(C) $\frac{33}{88}$

(D) $\frac{99}{250}$

(E) $\frac{100}{249}$

Arithmetic Probability

There are 250 integers from 101 to 350 inclusive, 100 of which (that is, 200 through 299) have a hundreds digit of 2. Therefore, the probability that a ticket selected from the box at random will have a hundreds digit of 2 can be expressed as $\frac{100}{250} = \frac{2}{5}$.

The correct answer is A.

12. When Leo imported a certain item, he paid a 7 percent import tax on the portion of the total value of the item in excess of \$1,000. If the amount of the import tax that Leo paid was \$87.50, what was the total value of the item?

(A) \$1,600
(B) \$1,850
(C) \$2,250
(D) \$2,400
(E) \$2,750

Algebra First-degree equations

Letting x represent the total value of the item, convert the words to symbols and solve the equation.

$$\begin{aligned} 7\% \text{ of value in excess of } \$1{,}000 &= 87.50 \\ 0.07(x-1{,}000) &= 87.50 \\ x-1{,}000 &= 1{,}250 \\ x &= 2{,}250 \end{aligned}$$

The correct answer is C.

13. A collection of 16 coins, each with a face value of either 10 cents or 25 cents, has a total face value of \$2.35. How many of the coins have a face value of 25 cents?

(A) 3
(B) 5
(C) 7
(D) 9
(E) 11

Algebra First-degree equations

Let x represent the number of coins each with a face value of 25 cents. Then, since there are 16 coins in all, $16 - x$ represents the number of coins each with a face value of 10 cents. The total face value of the coins is \$2.35 or 235 cents so,

$$\begin{aligned} 25x + 10(16 - x) &= 235 && \text{given} \\ 25x + 160 - 10x &= 235 && \text{distributive property} \\ 15x + 160 &= 235 && \text{combine like terms} \\ 15x &= 75 && \text{subtract 160 from both sides} \\ x &= 5 && \text{divide both sides by 15} \end{aligned}$$

Therefore, 5 of the coins have a face value of 25 cents.

The correct answer is B.

14. The numbers of cars sold at a certain dealership on six of the last seven business days were 4, 7, 2, 8, 3, and 6, respectively. If the number of cars sold on the seventh business day was either 2, 4, or 5, for which of the three values does the average (arithmetic mean) number of cars sold per business day for the seven business days equal the median number of cars sold per day for the seven days?

I. 2
II. 4
III. 5

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Arithmetic Statistics

Listed in numerical order, the given numbers are 2, 3, 4, 6, 7, and 8. If the seventh number were 2 or 4, then the numbers in numerical order would be 2, 2, 3, 4, 6, 7, and 8 or 2, 3, 4, 4, 6, 7, and 8. In either case the median would be 4 and the average would be $\frac{2+2+3+4+6+7+8}{7} = \frac{32}{7}$ or $\frac{2+3+4+4+6+7+8}{7} = \frac{34}{7}$, neither of which equals 4. So, for neither of the values in I or II does the average equal the median. If the seventh number were 5, then the numbers in numerical order would be 2, 3, 4, 5, 6, 7, and 8. The median would be 5 and the average would be $\frac{2+3+4+5+6+7+8}{7} = \frac{35}{7} = 5$. Thus, for the value in III, the average equals the median.

The correct answer is B.

15. If it is assumed that 60 percent of those who receive a questionnaire by mail will respond and 300 responses are needed, what is the minimum number of questionnaires that should be mailed?

(A) 400
(B) 420
(C) 480
(D) 500
(E) 600

Arithmetic Percents

From the given information, 60% of the minimum number of questionnaires is equal to 300. This has the form "60% of what number equals 300," and the number can be determined by dividing 300 by 60%. Performing the calculation gives $300 \div \frac{60}{100} = 300 \times \frac{100}{60} = 500$.

The correct answer is D.

16. If $1 < x < y < z$, which of the following has the greatest value?

(A) $z(x + 1)$
(B) $z(y + 1)$
(C) $x(y + z)$
(D) $y(x + z)$
(E) $z(x + y)$

Algebra Inequalities

This problem can be solved by calculating each of the options for a fixed and appropriate choice of values for each of the variables. For example, if $x = 2$, $y = 3$, and $z = 4$, then $1 < x < y < z$ and the values of the options are as follows:

A $z(x + 1) = 4(2 + 1) = 12$

B $z(y + 1) = 4(3 + 1) = 16$

C $x(y + z) = 2(3 + 4) = 14$

D $y(x + z) = 3(2 + 4) = 18$

E $z(x + y) = 4(2 + 3) = 20$

This problem can also be solved by the use of algebraic ordering principles in a way that does not assume the same answer option is true for each choice of values of x, y, and z such that $1 < x < y < z$. First, note that neither the value of the expression in A nor the value of the expression in B can ever have the greatest value, since the value of the expression in E is greater than the value of the expression in A (because $y > 1$) and the value of the expression in E is greater than the value of the expression in B (because $x > 1$). Also, the value of the expression in D is greater than the value of the expression in C, since $y > x$ implies $yz > xz$, which implies $xy + yz > xy + xz$, which implies $y(x + z) > x(y + z)$, and so the value of the expression in C cannot be the greatest. Finally, the value of the expression in E is greater than the value of the expression in D, since $z > y$ implies $xz > xy$, which implies $xz + yz > xy + yz$, which implies $z(x + y) > y(x + z)$, and so the value of the expression in D cannot be the greatest. Since none of the values of the expressions in A, B, C, or D can be the greatest, it follows that the value of the expression in E is the greatest.

The correct answer is E.

17. A rectangular garden is to be twice as long as it is wide. If 360 yards of fencing, including the gate, will completely enclose the garden, what will be the length of the garden, in yards?

(A) 120
(B) 140
(C) 160
(D) 180
(E) 200

Geometry Quadrilaterals; Perimeter

Let the width of the rectangle be x. Then the length is $2x$. Since the perimeter of a rectangle is twice the sum of the length and width, it follows that

$$360 = 2(x + 2x)$$
$$360 = 6x$$
$$120 = 2x$$

So, the length is 120.

The correct answer is A.

18. A rectangular floor that measures 8 meters by 10 meters is to be covered with carpet squares that each measure 2 meters by 2 meters. If the carpet squares cost $12 apiece, what is the total cost for the number of carpet squares needed to cover the floor?

(A) $200
(B) $240
(C) $480
(D) $960
(E) $1,920

Geometry Area (Rectangles)

The area of the floor is (10 m)(8 m) = 80 m^2 and the area of each carpet square is (2 m)(2 m) = 4 m^2. Therefore, the number of carpet squares needed to cover the floor is $\frac{80}{4} = 20$, and these 20 carpet squares have a total cost of (20)($12) = $240.

The correct answer is B.

19. If $893 \times 78 = p$, which of the following is equal to 893×79 ?

(A) $p + 1$
(B) $p + 78$
(C) $p + 79$
(D) $p + 893$
(E) $p + 894$

Arithmetic Properties of numbers

$$\begin{aligned} 893 \times 79 &= 893 \times (78 + 1) && \text{since } 79 = 78 + 1 \\ &= (893 \times 78) + 893 && \text{distributive property} \\ &= p + 893 && \text{since } p = 893 \times 78 \end{aligned}$$

The correct answer is D.

20. Thabo owns exactly 140 books, and each book is either paperback fiction, paperback nonfiction, or hardcover nonfiction. If he owns 20 more paperback nonfiction books than hardcover nonfiction books, and twice as many paperback fiction books as paperback nonfiction books, how many hardcover nonfiction books does Thabo own?

(A) 10
(B) 20
(C) 30
(D) 40
(E) 50

Algebra Simultaneous first-degree equations

Let F represent the number of paperback fiction books that Thabo owns; N_p, the number of paperback nonfiction books; and N_h, the number of hardcover nonfiction books. It is given that $F + N_p + N_h = 140$, $N_p = N_h + 20$, and $F = 2N_p = 2(N_h + 20)$. It follows that

$$\begin{aligned} F + N_p + N_h &= 140 && \text{given} \\ 2(N_h + 20) + (N_h + 20) + N_h &= 140 && \text{by substitution} \\ 4N_h + 60 &= 140 && \text{combine like terms} \\ 4N_h &= 80 && \text{subtract 60 from both sides} \\ N_h &= 20 && \text{divide both sides by 4} \end{aligned}$$

The correct answer is B.

21. If the average (arithmetic mean) of the four numbers 3, 15, 32, and $(N + 1)$ is 18, then $N =$

(A) 19
(B) 20
(C) 21
(D) 22
(E) 29

Arithmetic Statistics

From the given information and the definition of average, it follows that $\frac{3+15+32+(N+1)}{4} = 18$, or $\frac{51+N}{4} = 18$. Multiplying both sides of the last equation by 4 gives $51 + N = 72$. Therefore, $N = 72 - 51 = 21$.

The correct answer is C.

22. Abdul, Barb, and Carlos all live on the same straight road, on which their school is also located. The school is halfway between Abdul's house and Barb's house. Barb's house is halfway between the school and Carlos's house. If the school is 4 miles from Carlos's house, how many miles is Abdul's house from Carlos's house?

(A) $1\frac{1}{3}$
(B) 2
(C) 4
(D) 6
(E) 8

Geometry Applied problems

In the diagram, A represents the location of Abdul's house, S represents the location of the school, B represents the location of Barb's house, and C represents the location of Carlos's house. Because the school is halfway between Abdul's house and Barb's house, S is the midpoint of $\overline{AB}$, and because Barb's house is halfway between the school and Carlos's house, B is the midpoint of $\overline{SC}$. Therefore, $AS = SB = BC$. Finally, since $SC = 4$, it follows that $AS = SB = BC = 2$ and hence $AC = 2 + 2 + 2 = 6$.

The correct answer is D.

23. During a certain time period, Car X traveled north along a straight road at a constant rate of 1 mile per minute and used fuel at a constant rate of 5 gallons every 2 hours. During this time period, if Car X used exactly 3.75 gallons of fuel, how many miles did Car X travel?

(A) 36
(B) 37.5
(C) 40
(D) 80
(E) 90

Arithmetic Applied problems

The car traveled at a rate of $1\dfrac{\text{mi}}{\text{min}} = \left(1\dfrac{\text{mi}}{\cancel{\text{min}}}\right)\left(60\dfrac{\cancel{\text{min}}}{\text{hr}}\right) = 60\dfrac{\text{mi}}{\text{hr}}$ using fuel at a rate of 5 gallons every 2 hours, or $\dfrac{5}{2}\dfrac{\text{gal}}{\text{hr}}$. If the usage rate in miles per gallon was known, then that usage rate times 3.75 gallons would give the number of miles.

One approach to finding this usage rate is to consider whether multiplying or dividing the known rates $60\dfrac{\text{mi}}{\text{hr}}$ and $\dfrac{5}{2}\dfrac{\text{gal}}{\text{hr}}$ will lead to unit cancellations that result in miles per gallon. The following calculation shows that dividing these two known rates leads to the appropriate unit cancellations:

$$60\frac{\text{mi}}{\text{hr}} \div \frac{5}{2}\frac{\text{gal}}{\text{hr}} = 60\frac{\text{mi}}{\cancel{\text{hr}}} \times \frac{2}{5}\frac{\cancel{\text{hr}}}{\text{gal}} = 24\frac{\text{mi}}{\text{gal}}.$$

Therefore, after using 3.75 gal of fuel, the car has traveled $24\dfrac{\text{mi}}{\text{gal}} \times 3.75\text{ gal} = 90$ miles.

The correct answer is E.

24. Cheryl purchased 5 identical hollow pine doors and 6 identical solid oak doors for the house she is building. The regular price of each solid oak door was twice the regular price of each hollow pine door. However, Cheryl was given a discount of 25% off the regular price of each solid oak door. If the regular price of each hollow pine door was \$40, what was the total price of all 11 doors?

(A) \$320
(B) \$540
(C) \$560
(D) \$620
(E) \$680

Algebra Applied problems; Percents

The price of each pine door is \$40, so the price of 5 pine doors is 5(\$40) = \$200. The price of each oak door is twice that of a pine door, and thus \$80, which becomes (0.75)(\$80) = \$60 when the 25% discount is applied. Therefore, the price of 6 oak doors at the 25% discount is 6(\$60) = \$360, and hence the total price of all 11 doors is \$200 + \$360 = \$560.

The correct answer is C.

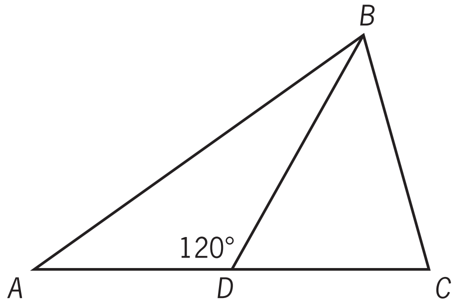
25. If $\left|y - \dfrac{1}{2}\right| < \dfrac{11}{2}$, which of the following could be a value of y?

(A) -11
(B) $-\dfrac{11}{2}$
(C) $\dfrac{11}{2}$
(D) 11
(E) 22

Algebra Inequalities; Absolute value

Since $\left|y-\frac{1}{2}\right|<\frac{11}{2}$ is equivalent to $-\frac{11}{2}<y-\frac{1}{2}<\frac{11}{2}$, or $-\frac{10}{2}<y<\frac{12}{2}$, select the value that lies between $-\frac{10}{2}=-5$ and $\frac{12}{2}=6$. That value is $\frac{11}{2}$.

The correct answer is C.



26. In the figure shown, $AC = 2$ and $BD = DC = 1$. What is the measure of angle ABD ?

(A) 15°
(B) 20°
(C) 30°
(D) 40°
(E) 45°

Geometry Triangles

Since $AC = AD + DC$, and it is given that $AC = 2$ and $DC = 1$, it follows that $AD = 1$. Therefore, $AD = BD = 1$ and ΔABD is an isosceles triangle where the measure of $\angle ABD$ is equal to the measure of $\angle BAD$. Letting $x°$ be the common degree measure of these two angles, it follows that $x° + x° + 120° = 180°$, or $2x° = 60°$, or $x° = 30°$.

The correct answer is C.

27. If $k^2 = m^2$, which of the following must be true?

(A) $k = m$
(B) $k = -m$
(C) $k = |m|$
(D) $k = -|m|$
(E) $|k| = |m|$

Algebra Simplifying algebraic expressions

One method of solving this is to first take the nonnegative square root of both sides of the equation $k^2 = m^2$ and then make use of the fact that $\sqrt{u^2} = |u|$. Doing this gives $|k| = |m|$. Alternatively, if (k, m) is equal to either of the pairs (1,1) or (–1,1), then $k^2 = m^2$ is true. However, each of the answer choices except $|k| = |m|$ is false for at least one of these two pairs.

The correct answer is E.

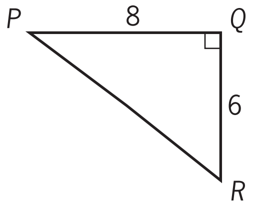
28. Makoto, Nishi, and Ozuro were paid a total of \$780 for waxing the floors at their school. Each was paid in proportion to the number of hours he or she worked. If Makoto worked 15 hours, Nishi worked 20 hours, and Ozuro worked 30 hours, how much was Makoto paid?

(A) \$52
(B) \$117
(C) \$130
(D) \$180
(E) \$234

Arithmetic Ratio and proportion

Makoto, Nishi, and Ozuro worked a total of $15 + 20 + 30 = 65$ hours and were paid a total of \$780. Each was paid in proportion to the number of hours he or she worked. Therefore, Makoto was paid $\frac{15}{65}(\$780) = \$180$.

The correct answer is D.



29. The figure above shows a path around a triangular piece of land. Mary walked the distance of 8 miles from P to Q and then walked the distance of 6 miles from Q to R. If Ted walked directly from P to R, by what percent did the distance that Mary walked exceed the distance that Ted walked?

(A) 30%
(B) 40%
(C) 50%
(D) 60%
(E) 80%

Geometry Pythagorean theorem

Mary walked a distance of $6 + 8 = 14$ miles. The distance that Ted walked, PR, can be found by using the Pythagorean theorem $6^2 + 8^2 = (PR)^2$, or $(PR)^2 = 100$. Taking square roots, it follows that Ted walked 10 miles. Therefore, the distance Mary walked exceeded the distance Ted walked by $14 - 10 = 4$ miles and 4 is 40% of 10.

The correct answer is B.

30. At a supermarket, John spent $\frac{1}{2}$ of his money on fresh fruits and vegetables, $\frac{1}{3}$ on meat products, and $\frac{1}{10}$ on bakery products. If he spent the remaining \$6 on candy, how much did John spend at the supermarket?

(A) \$60
(B) \$80
(C) \$90
(D) \$120
(E) \$180

Arithmetic Fractions

The amount spent was

$\frac{1}{2}+\frac{1}{3}+\frac{1}{10}=\frac{15}{30}+\frac{10}{30}+\frac{3}{30}=\frac{28}{30}=\frac{14}{15}$ of the total so the \$6 left was $\frac{1}{15}$ of the total. It follows that the total is $(15)(\$6) = \$90$.

The correct answer is C.

31. If $(1 - 1.25)N = 1$, then $N =$

(A) –400
(B) –140
(C) –4
(D) 4
(E) 400

Algebra Operations with rational numbers

Since $(1 - 1.25)N = -0.25N = -\frac{1}{4}N$, the equation becomes $-\frac{1}{4}N = 1$, which has solution $N = -4$.

The correct answer is C.

32. A carpenter constructed a rectangular sandbox with a capacity of 10 cubic feet. If the carpenter were to make a similar sandbox twice as long, twice as wide, and twice as high as the first sandbox, what would be the capacity, in cubic feet, of the second sandbox?

(A) 20
(B) 40
(C) 60
(D) 80
(E) 100

Geometry Volume

When all the dimensions of a three-dimensional object are changed by a factor of 2, the capacity, or volume, changes by a factor of $(2)(2)(2) = 2^3 = 8$. Thus the capacity of the second sandbox is $10(8) = 80$ cubic feet.

The correct answer is D.

33. The quotient when a certain number is divided by $\frac{2}{3}$ is $\frac{9}{2}$. What is the number?

(A) $\frac{4}{27}$

(B) $\frac{1}{3}$

(C) 3

(D) 6

(E) $\frac{27}{4}$

Arithmetic Operations with rational numbers

Recall the definition of division: $a \div b = c$ means $a = bc$. For example, if a certain number divided by 2 equals 3, then the number is $2 \times 3 = 6$. For this problem, a certain number divided by $\frac{2}{3}$ equals $\frac{9}{2}$, so the number is $\frac{2}{3} \times \frac{9}{2} = 3$.

This problem can also be solved by algebra. Let N be the number. Then $\frac{N}{\frac{2}{3}} = \frac{9}{2}$, or $N = \frac{2}{3} \times \frac{9}{2} = 3$.

The correct answer is C.

34. If a sphere with radius r is inscribed in a cube with edges of length e, which of the following expresses the relationship between r and e?

(A) $r = \frac{1}{2}e$

(B) $r = e$

(C) $r = 2e$

(D) $r = \sqrt{e}$

(E) $r = \frac{1}{4}e^2$

Geometry Volume

A sphere inscribed in a cube touches, but does not extend beyond, each of the 6 sides of the cube, and thus the diameter of the sphere is equal to the distance between a pair of opposite sides of the cube, which is equal to the edge length of the cube. Therefore, the radius of the sphere is equal to half the edge length of the cube, or $r = \frac{1}{2}e$.

The correct answer is A.

35. If $2x + y = 7$ and $x + 2y = 5$, then $\frac{x+y}{3} =$

(A) 1

(B) $\frac{4}{3}$

(C) $\frac{17}{5}$

(D) $\frac{18}{5}$

(E) 4

Algebra Simultaneous equations

Adding the equations $2x + y = 7$ and $x + 2y = 5$ gives $3x + 3y = 12$, or $x + y = 4$. Dividing both sides of the last equation by 3 gives $\frac{x+y}{3} = \frac{4}{3}$.

The correct answer is B.

36. City X has a population 4 times as great as the population of City Y, which has a population twice as great as the population of City Z. What is the ratio of the population of City X to the population of City Z?

(A) 1:8

(B) 1:4

(C) 2:1

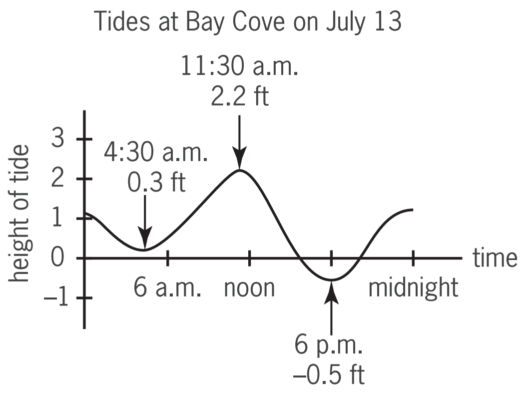
(D) 4:1

(E) 8:1

Arithmetic Ratio and Proportion

Let X, Y, and Z be the populations of Cities X, Y, and Z, respectively. It is given that $X = 4Y$, and $Y = 2Z$ or $Z = \frac{Y}{2}$. Then, $\frac{X}{Z} = \frac{4Y}{\frac{Y}{2}} = (4Y)\left(\frac{2}{Y}\right) = \frac{8}{1}$.

The correct answer is E.



37. The graph above shows the height of the tide, in feet, above or below a baseline. Which of the following is closest to the difference, in feet, between the heights of the highest and lowest tides on July 13 at Bay Cove?

(A) 1.7
(B) 1.9
(C) 2.2
(D) 2.5
(E) 2.7

Arithmetic Interpretation of graphs and tables

From the graph, the highest tide is 2.2 ft above the baseline and the lowest tide is 0.5 ft below the baseline. Therefore, the difference between the heights of the highest tide and the lowest tide is $[2.2 - (-0.5)]$ ft $= (2.2 + 0.5)$ ft $= 2.7$ ft.

The correct answer is E.

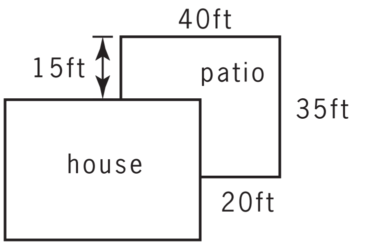
38. A manufacturer of a certain product can expect that between 0.3 percent and 0.5 percent of the units manufactured will be defective. If the retail price is $2,500 per unit and the manufacturer offers a full refund for defective units, how much money can the manufacturer expect to need to cover the refunds on 20,000 units?

(A) Between $15,000 and $25,000
(B) Between $30,000 and $50,000
(C) Between $60,000 and $100,000
(D) Between $150,000 and $250,000
(E) Between $300,000 and $500,000

Arithmetic Applied problems

The expected number of defective units is between 0.3% and 0.5% of 20,000, or between $(0.003)(20{,}000) = 60$ and $(0.005)(20{,}000) = 100$. Since each unit has a retail price of $2,500, the amount of money needed to cover the refunds for the expected number of defective units is between 60($2,500) and 100($2,500), or between $150,000 and $250,000.

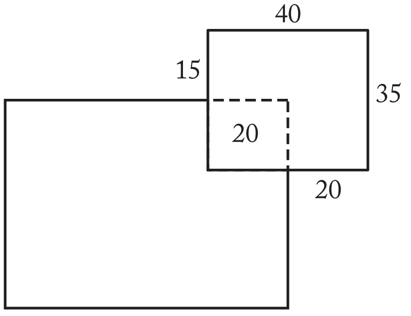
The correct answer is D.



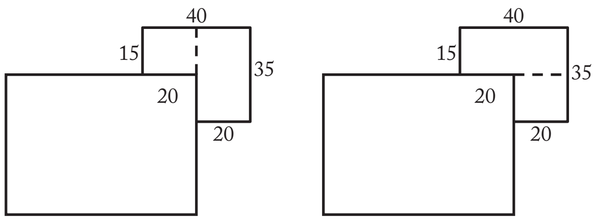
39. A flat patio was built alongside a house as shown in the figure above. If all angles shown are right angles, what is the area of the patio in square feet?

(A) 800
(B) 875
(C) 1,000
(D) 1,100
(E) 1,125

Geometry Area



The area of the patio can be calculated by imagining the patio to be a rectangle of dimensions 40 ft by 35 ft that has a lower-left square corner of dimensions 20 ft by 20 ft covered up, as shown in the figure above. The area of the patio will be the area of the uncovered part of the rectangle, and therefore the area of the patio, in square feet, is $(40)(35) - (20)(20) = 1{,}400 - 400 = 1{,}000$.



Alternatively, the area of the patio can be calculated by dividing the patio into two rectangles and adding the areas of the two rectangles. This can be done in two ways: by using rectangles of dimensions 20 ft by 35 ft and 20 ft by 15 ft, as shown in the figure above on the left (for a total area of 700 ft^2 + 300 ft^2 = 1,000 ft^2), or by using rectangles of dimensions 40 ft by 15 ft and 20 ft by 20 ft, as shown in the figure above on the right (for a total area of 600 ft^2 + 400 ft^2 = 1,000 ft^2).

The correct answer is C.

40. The sum of the weekly salaries of 5 employees is \$3,250. If each of the 5 salaries is to increase by 10 percent, then the average (arithmetic mean) weekly salary per employee will increase by

(A) \$52.50
(B) \$55.00
(C) \$57.50
(D) \$62.50
(E) \$65.00

Arithmetic Applied problems; Percents

Let S_1, S_2, S_3, S_4, and S_5 be the salaries, in dollars, of the 5 employees. Since the sum of the 5 salaries is 3,250, then $S_1 + S_2 + S_3 + S_4 + S_5 = 3{,}250$ and the average salary is $\frac{S_1 + S_2 + S_3 + S_4 + S_5}{5} = \frac{3{,}250}{5} = 650.$ After each salary is increased by 10%, the salaries will be $(1.1)S_1$, $(1.1)S_2$, $(1.1)S_3$, $(1.1)S_4$, and $(1.1)S_5$ and the average salary, in dollars, will be $\frac{(1.1)S_1 + (1.1)S_2 + (1.1)S_3 + (1.1)S_4 + (1.1)S_5}{5} =$ $1.1 \times \left(\frac{S_1 + S_2 + S_3 + S_4 + S_5}{5}\right) = 1.1 \times 650 = 715.$ Therefore, the increase in the average salary is \$715 – \$650 = \$65.

The correct answer is E.

41. A student's average (arithmetic mean) test score on 4 tests is 78. What must be the student's score on a 5th test for the student's average score on the 5 tests to be 80 ?

(A) 80
(B) 82
(C) 84
(D) 86
(E) 88

Arithmetic Statistics

The average of the student's first 4 test scores is 78, so the sum of the first 4 test scores is $4(78) = 312$. If x represents the fifth test score, then the sum of all 5 test scores is $312 + x$ and the average of all 5 test scores is $\frac{312 + x}{5}$. But the average of all 5 test scores is 80 so

$$\begin{aligned} \frac{312 + x}{5} &= 80 \\ 312 + x &= 400 \\ x &= 88 \end{aligned}$$

The correct answer is E.

42. Last week Chris earned x dollars per hour for the first 40 hours worked plus 22 dollars per hour for each hour worked beyond 40 hours. If last week Chris earned a total of 816 dollars by working 48 hours, what is the value of x ?

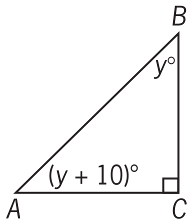
(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

Algebra Applied problems

Chris worked 40 hours at a rate of \$$x$ per hour, 48 – 40 = 8 hours at a rate of \$22 per hour, and earned a total of \$816.

$40x + 8(22) = 816$	given information
$40x + 176 = 816$	multiply 8 and 22
$40x = 640$	subtract 176 from both sides
$x = 16$	divide both sides by 40

The correct answer is D.



43. In the figure above, what is the ratio of the measure of angle B to the measure of angle A ?

(A) 2 to 3
(B) 3 to 4
(C) 3 to 5
(D) 4 to 5
(E) 5 to 6

Geometry Angles

Because the sum of the degree measures of the three interior angles of a triangle is 180, it follows that $y + (y + 10) + 90 = 180$. Therefore, $2y = 80$, and hence $y = 40$. The ratio of the measure of angle B to the measure of angle A can now be determined: $\frac{y}{y+10} = \frac{40}{50} = \frac{4}{5}$.

The correct answer is D.

44. If n is a prime number greater than 3, what is the remainder when n^2 is divided by 12 ?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 5

Arithmetic Properties of numbers

The simplest way to solve this problem is to choose a prime number greater than 3 and divide its square by 12 to see what the remainder is. For example, if $n = 5$, then $n^2 = 25$, and the remainder is 1 when 25 is divided by 12. A second prime number can be used to check the result. For example, if $n = 7$, then $n^2 = 49$, and the remainder is 1 when 49 is divided by 12. Because only one of the answer choices can be correct, the remainder must be 1.

For the more mathematically inclined, consider the remainder when each prime number n greater than 3 is divided by 6. The remainder cannot be 0 because that would imply that n is divisible by 6, which is impossible since n is a prime number. The remainder cannot be 2 or 4 because that would imply that n is even, which is impossible since n is a prime number greater than 3. The remainder cannot be 3 because that would imply that n is divisible by 3, which is impossible since n is a prime number greater than 3. Therefore, the only possible remainders when a prime number n greater than 3 is divided by 6 are 1 and 5. Thus, n has the form $6q + 1$ or $6q + 5$, where q is an integer, and, therefore, n^2 has the form $36q^2 + 12q + 1 = 12(3q^2 + q) + 1$ or $36q^2 + 60q + 25 = 12(3q^2 + 5q + 2) + 1$. In either case, n^2 has a remainder of 1 when divided by 12.

The correct answer is B.

45. $\dfrac{1}{1+\frac{1}{3}} - \dfrac{1}{1+\frac{1}{2}} =$

(A) $-\frac{1}{3}$
(B) $-\frac{1}{6}$
(C) $-\frac{1}{12}$
(D) $\frac{1}{12}$
(E) $\frac{1}{3}$

Arithmetic Operations with rational numbers

Perform the arithmetic calculations as follows:

$$\begin{aligned}
\frac{1}{1+\frac{1}{3}} - \frac{1}{1+\frac{1}{2}} &= \frac{1}{\frac{4}{3}} - \frac{1}{\frac{3}{2}} \\
&= \frac{3}{4} - \frac{2}{3} \\
&= \frac{9}{12} - \frac{8}{12} \\
&= \frac{9-8}{12} \\
&= \frac{1}{12}
\end{aligned}$$

The correct answer is D.

46. The positive two-digit integers x and y have the same digits, but in reverse order. Which of the following must be a factor of $x + y$?

(A) 6
(B) 9
(C) 10
(D) 11
(E) 14

Arithmetic Properties of numbers

Let m and n be digits. If $x = 10m + n$, then $y = 10n + m$. Adding x and y gives $x + y = (10m + n) + (10n + m) = 11m + 11n = 11(m + n)$, and therefore 11 is a factor of $x + y$.

The correct answer is D.

47. In a certain sequence of 8 numbers, each number after the first is 1 more than the previous number. If the first number is –5, how many of the numbers in the sequence are positive?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

Arithmetic Sequences

The sequence consists of eight consecutive integers beginning with –5:

$$-5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2$$

In this sequence exactly two of the numbers are positive.

The correct answer is C.

48. A total of s oranges are to be packaged in boxes that will hold r oranges each, with no oranges left over. When n of these boxes have been completely filled, what is the number of boxes that remain to be filled?

(A) $s - nr$
(B) $s - \frac{n}{r}$
(C) $rs - n$
(D) $\frac{s}{n} - r$
(E) $\frac{s}{r} - n$

Algebra Algebraic expressions

If s oranges are packed r oranges to a box with no oranges left over, then the number of boxes that will be filled is $\frac{s}{r}$. If n of these boxes are already filled, then $\frac{s}{r} - n$ boxes remain to be filled.

The correct answer is E.

49. If $0 < a < b < c$, which of the following statements must be true?

I. $2a > b + c$
II. $c - a > b - a$
III. $\frac{c}{a} < \frac{b}{a}$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

Algebra Inequalities

Given $0 < a < b < c$, Statement I is not necessarily true. If, for example, $a = 1$, $b = 2$, and $c = 3$, then $0 < a < b < c$, but $2a = 2(1) < 2 + 3 = b + c$.

Given $0 < a < b < c$, then $c > b$, and subtracting a from both sides gives $c - a > b - a$. Therefore, Statement II is true.

Given $0 < a < b < c$, Statement III is not necessarily true. If, for example, $a = 1$, $b = 2$, and $c = 3$, then $0 < a < b < c$, but $\frac{c}{a} = \frac{3}{1} > \frac{2}{1} = \frac{b}{a}$.

The correct answer is B.

50. In the xy-plane, the origin O is the midpoint of line segment PQ. If the coordinates of P are (r,s), what are the coordinates of Q?

(A) (r,s)
(B) $(s,-r)$
(C) $(-s,-r)$
(D) $(-r,s)$
(E) $(-r,-s)$

Algebra Coordinate geometry

Let (x,y) be the coordinates of Q. The midpoint of (r,s) and (x,y) is $\left(\frac{r+x}{2},\frac{s+y}{2}\right)$. Since the midpoint is $(0,0)$, it follows that $\frac{r+x}{2}=0$ and $\frac{s+y}{2}=0$. Therefore, $r+x=0$ and $s+y=0$, or $x=-r$ and $y=-s$, or $(x,y)=(-r,-s)$.

This problem can also be solved by observing that Q is the reflection of P about the origin, and so $Q=(-r,-s)$ (i.e., change the sign of each of the coordinates of P to obtain the coordinates of Q).

The correct answer is E.

51. Which of the following equations is NOT equivalent to $10y^2 = (x + 2)(x - 2)$?

(A) $30y^2 = 3x^2 - 12$
(B) $20y^2 = (2x - 4)(x + 2)$
(C) $10y^2 + 4 = x^2$
(D) $5y^2 = x^2 - 2$
(E) $y^2 = \frac{x^2-4}{10}$

Algebra Simplifying algebraic expressions

When $x = 2$ or $x = -2$, the equation becomes $10y^2 = 0$, or $y = 0$. Since, in the equation given in (D), y does not become 0 when $x = 2$, it follows that the equation given in (D) is not equivalent to the given equation. Alternatively, when each of the equations given in (A) through (E) is solved for $10y^2$ in terms of x, only the resulting equation in (D) fails to give an expression in terms of x that is equivalent to $(x + 2)(x - 2) = x^2 - 4$.

The correct answer is D.

	Monday	Tuesday	Wednesday	Thursday
Company *A*	45	55	50	50
Company *B*	10	30	30	10
Company *C*	34	28	28	30
Company *D*	39	42	41	38
Company *E*	50	60	60	70

52. The table shows the numbers of packages shipped daily by each of five companies during a 4-day period. The standard deviation of the numbers of packages shipped daily during the period was greatest for which of the five companies?

(A) *A*
(B) *B*
(C) *C*
(D) *D*
(E) *E*

Arithmetic Statistics

Since the standard deviation of a data set is a measure of how widely the data are scattered about their mean, find the mean number of packages shipped by each company and then determine the company for which the data is most widely scattered about its mean.

For Company A, the mean number of packages shipped is $\frac{45+55+2(50)}{4}=50$. Two of the data points are each 50 and the other two each differ from 50 by 5.

For Company B, the mean number of packages shipped is $\frac{2(10)+2(30)}{4}=20$. Each of the data points differs from 20 by 10. Thus, the data for Company B is more widely scattered about its mean of 20 than the data for Company A is about its mean of 50.

For Company C, the mean number of packages shipped is $\frac{34+2(28)+30}{4}=30$. One data point is 30, two others each differ from 30 by only 2, and the fourth data point differs from 30 by only 4. Therefore, the data for Company C is not as widely scattered about its mean of 30 as the data for Company B is about its mean of 20.

For Company D, the mean number of packages shipped is $\frac{39 + 42 + 41 + 38}{4} = 40$. Two of the data points each differ from 40 by only 1 and the other two each differ from 40 by only 2. Therefore, the data for Company D is not as widely scattered about its mean of 40 as the data for Company B is about its mean of 20.

For Company E, the mean number of packages shipped is $\frac{50 + 2(60) + 70}{4} = 60$. Two of the data points are each 60 and the other two each differ from 60 by 10. Therefore, the data for Company E is not as widely scattered about its mean of 60 as the data for Company B is about its mean of 20.

Thus, the data for Company B is most widely scattered about its mean and, therefore, the standard deviation of the number of packages shipped daily by the five companies is greatest for Company B.

For those interested, the standard deviations for the five companies can be calculated as follows:

For A: $\sqrt{\frac{(45-50)^2+(55-50)^2+2(50-50)^2}{4}}$

$=\sqrt{\frac{2(25)}{4}}=\sqrt{\frac{25}{2}}=\sqrt{12.5}$.

For B: $\sqrt{\frac{2(10-20)^2+2(30-20)^2}{4}}=\sqrt{\frac{4(100)}{4}}$

$=\sqrt{100}$.

For C: $\sqrt{\frac{(34-30)^2+2(28-30)^2+(30-30)^2}{4}}$

$=\sqrt{\frac{24}{4}}=\sqrt{6}$.

For D:

$\sqrt{\frac{(39-40)^2+(42-40)^2+(41-40)^2+(38-40)^2}{4}}$

$=\sqrt{\frac{10}{4}}=\sqrt{2.5}$.

For E: $\sqrt{\frac{(50-60)^2+2(60-60)^2+(70-60)^2}{4}}$

$=\sqrt{\frac{200}{4}}=\sqrt{50}$.

The correct answer is B.

53. Company Q plans to make a new product next year and sell each unit of this new product at a selling price of \$2. The variable costs per unit in each production run are estimated to be 40% of the selling price, and the fixed costs for each production run are estimated to be \$5,040. Based on these estimated costs, how many units of the new product will Company Q need to make and sell in order for their revenue to equal their total costs for each production run?

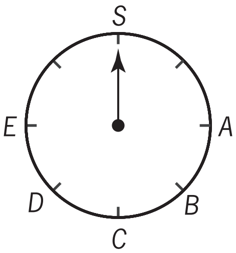
(A) 4,200
(B) 3,150
(C) 2,520
(D) 2,100
(E) 1,800

Algebra Applied problems

Let x be the desired number of units to be sold at a price of \$2 each. Then the revenue for selling these units is \$2x, and the total cost for selling these units is (40%)(\$2.00)x = \$0.80x plus a fixed cost of \$5,040.

revenue = total cost	given requirement
$2x = 0.8x + 5{,}040$	given information
$1.2x = 5{,}040$	subtract $0.8x$ from both sides
$x = 4{,}200$	divide both sides by 1.2

The correct answer is A.



54. The dial shown above is divided into equal-sized intervals. At which of the following letters will the pointer stop if it is rotated clockwise from *S* through 1,174 intervals?

(A) *A*
(B) *B*
(C) *C*
(D) *D*
(E) *E*

Arithmetic Properties of numbers

There are 8 intervals in each complete revolution. Dividing 8 into 1,174 gives 146 with remainder 6. Therefore, 1,174 intervals is equivalent to 146 complete revolutions followed by an additional 6 intervals measured clockwise from *S*, which places the pointer at *E*.

The correct answer is E.

Estimated Number of Home-Schooled Students by State, January 2001

State	Number (in thousands)
A	181
B	125
C	103
D	79
E	72

55. According to the table shown, the estimated number of home-schooled students in State A is approximately what percent greater than the number in State D ?

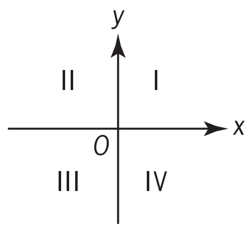
(A) 25%
(B) 55%
(C) 100%
(D) 125%
(E) 155%

Arithmetic Percents

The percent increase from the number in State D to the number in State A is

$= \left(\dfrac{181{,}000 - 79{,}000}{79{,}000} \times 100\right)\%$	expression for percent increase
$= \left(\dfrac{181 - 79}{79} \times 100\right)\%$	reduce fraction
$= \left(\dfrac{102}{79} \times 100\right)\%$	subtract
$\approx \left(\dfrac{100}{80} \times 100\right)\%$	approximate
$= (1.25 \times 100)\%$	divide
$= 125\%$	multiply

The correct answer is D.



56. The graph of the equation $xy = k$, where $k < 0$, lies in which two of the quadrants shown above?

(A) I and II
(B) I and III
(C) II and III
(D) II and IV
(E) III and IV

Algebra Coordinate geometry

If a point lies on the graph of $xy = k$, then the product of the point's x- and y-coordinates is k. Since k is negative, it follows that for any such point, the product of the point's x- and y-coordinates is negative. Therefore, for any such point, the point's x- and y-coordinates have opposite signs, and hence the point must be in quadrant II or in quadrant IV.

The correct answer is D.

57. When n liters of fuel were added to a tank that was already $\frac{1}{3}$ full, the tank was filled to $\frac{7}{9}$ of its capacity. In terms of n, what is the capacity of the tank, in liters?

(A) $\frac{10}{9}n$

(B) $\frac{4}{3}n$

(C) $\frac{3}{2}n$

(D) $\frac{9}{4}n$

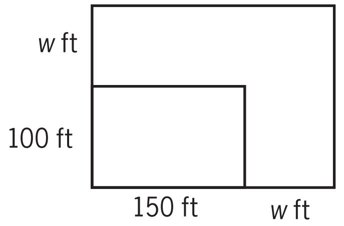
(E) $\frac{7}{3}n$

Algebra Applied problems

Let C represent the capacity of the tank, in liters. It follows that

$\frac{1}{3}C + n = \frac{7}{9}C$	given
$n = \frac{7}{9}C - \frac{1}{3}C$	subtract $\frac{1}{3}C$ from both sides
$n = \frac{4}{9}C$	combine like terms
$\frac{9}{4}n = C$	divide both sides by $\frac{4}{9}$

The correct answer is D.



Note: Not drawn to scale.

58. The smaller rectangle in the figure above represents the original size of a parking lot before its length and width were each extended by w feet to make the larger rectangular lot shown. If the area of the enlarged lot is twice the area of the original lot, what is the value of w?

(A) 25

(B) 50

(C) 75

(D) 100

(E) 200

Geometry Area

From the given information it follows that $(100 + w)(150 + w) = 2(100)(150)$, or $(100 + w)(150 + w) = (200)(150)$. This is a quadratic equation that can be solved by several methods. One method is by inspection. The left side is clearly equal to the right side when $w = 50$. Another method is by factoring. Expanding the left side gives $(100)(150) + 250w + w^2 = (200)(150)$, or $w^2 + 250w - (100)(150) = 0$. Factoring the left side gives $(w - 50)(w + 300) = 0$, which has $w = 50$ as its only positive solution.

The correct answer is B.

59. $\frac{1}{0.75-1} =$

(A) –4

(B) –0.25

(C) 0.25

(D) 0.75

(E) 4

Arithmetic Operations with rational numbers

Perform the arithmetic calculations as follows:

$$\frac{1}{0.75-1} = \frac{1}{\frac{3}{4}-1} = \frac{1}{-\frac{1}{4}} = -4$$

The correct answer is A.

60. Kevin invested \$8,000 for one year at a simple annual interest rate of 6 percent and invested \$10,000 for one year at an annual interest rate of 8 percent compounded semiannually. What is the total amount of interest that Kevin earned on the two investments?

(A) \$880

(B) \$1,088

(C) \$1,253

(D) \$1,280

(E) \$1,296

Arithmetic Applied problems

The amount of interest after one year is the total value of the investment after one year minus the total initial value of the investment. The total value of the investment after one year is $\$8{,}000(1.06) + \$10{,}000(1.04)^2 = \$8{,}480 + \$10{,}816$, so the amount of interest is $(\$8{,}480 + \$10{,}816) - (\$8{,}000 + \$10{,}000) = \$480 + \$816 = \$1{,}296$.

The correct answer is E.

61. The harvest yield from a certain apple orchard was 350 bushels of apples. If x of the trees in the orchard each yielded 10 bushels of apples, what fraction of the harvest yield was from these x trees?

(A) $\frac{x}{35}$

(B) $1-\frac{x}{35}$

(C) $10x$

(D) $35 - x$

(E) $350 - 10x$

Algebra Algebraic expressions

Since each of the x trees yielded 10 bushels, the total number of bushels yielded by these trees was $10x$. Since the yield of the entire orchard was 350 bushels, $\frac{10x}{350} = \frac{x}{35}$ was the fraction of the total yield from these x trees.

The correct answer is A.

62. If n is an integer, which of the following must be even?

(A) $n + 1$

(B) $n + 2$

(C) $2n$

(D) $2n + 1$

(E) n^2

Arithmetic Properties of integers

A quick look at the answer choices reveals the expression $2n$ in answer choice C. $2n$ is a multiple of 2 and hence must be even.

Since only one answer choice can be correct, the other answer choices need not be checked. However, for completeness:

A $n + 1$ is odd if n is even and even if n is odd. Therefore, it is not true that $n + 1$ must be even.

B $n + 2$ is even if n is even and odd if n is odd. Therefore, it is not true that $n + 2$ must be even.

D $2n + 1$ is odd whether n is even or odd. Therefore, it is not true that $2n + 1$ must be even.

E n^2 is even if n is even and odd if n is odd. Therefore, it is not true that n^2 must be even.

The correct answer is C.

63. The sum $\frac{7}{8}+\frac{1}{9}$ is between

(A) $\frac{1}{2}$ and $\frac{3}{4}$

(B) $\frac{3}{4}$ and 1

(C) 1 and $1\frac{1}{4}$

(D) $1\frac{1}{4}$ and $1\frac{1}{2}$

(E) $1\frac{1}{2}$ and 2

Arithmetic Operations with rational numbers

Since $\frac{1}{9}<\frac{1}{8}$, $\frac{7}{8}+\frac{1}{9}<\frac{7}{8}+\frac{1}{8}=1$, and answer choices C, D, and E can be eliminated. Since $\frac{7}{8}>\frac{6}{8}=\frac{3}{4}$, $\frac{7}{8}+\frac{1}{9}>\frac{3}{4}$, and answer choice A can be eliminated. Thus, $\frac{3}{4}<\frac{7}{8}+\frac{1}{9}<1$.

The correct answer is B.

64. Car X averages 25.0 miles per gallon of gasoline and Car Y averages 11.9 miles per gallon. If each car is driven 12,000 miles, approximately how many more gallons of gasoline will Car Y use than Car X ?

(A) 320
(B) 480
(C) 520
(D) 730
(E) 920

Arithmetic Applied problems

Car X uses 1 gallon of gasoline for every 25 miles it is driven, so Car X uses $\frac{1}{25}$ of a gallon for every 1 mile it is driven. Therefore Car X will use $(12{,}000)\left(\frac{1}{25}\right) = 480$ gallons of gasoline when it is driven 12,000 miles. Car Y uses 1 gallon of gasoline for every 11.9 or $\frac{119}{10}$ miles it is driven, so Car Y uses $\frac{10}{119}$ of a gallon for every 1 mile it is driven. Therefore Car Y will use $(12{,}000)\left(\frac{10}{119}\right) \approx (12{,}000)\left(\frac{10}{120}\right) = 1{,}000$ gallons of gasoline when it is driven 12,000 miles. Thus, Car Y will use approximately $1{,}000 - 480 = 520$ more gallons of gasoline than Car X.

The correct answer is C.

65. If y is an integer, then the least possible value of $|23 - 5y|$ is

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Arithmetic Absolute value; Operations with integers

Since y is an integer, $23 - 5y$ is also an integer. The task is to find the integer y for which $|23 - 5y|$ is the least. If $y \geq 0$, $-5y \leq 0$, and $23 - 5y \leq 23$. On the other hand, if $y \leq 0$, $-5y \geq 0$, and $23 - 5y \geq 23$. Therefore, the least possible value of $|23 - 5y|$ occurs at a nonnegative value of y. From the chart below, it is clear that the least possible integer value of $|23 - 5y|$ is 2, which occurs when $y = 5$.

y	$\lvert 23 - 5y\rvert$
0	23
1	18
2	13
3	8
4	3
5	2
6	7
7	12

Alternatively, since $|23 - 5y| \geq 0$, the minimum possible real value of $|23 - 5y|$ is 0. The integer value of y for which $|23 - 5y|$ is least is the integer closest to the solution of the equation $23 - 5y = 0$. The solution is $y = \frac{23}{5} = 4.6$ and the integer closest to 4.6 is 5.

The correct answer is B.

66. $\sqrt{80} + \sqrt{125} =$

(A) $9\sqrt{5}$
(B) $20\sqrt{5}$
(C) $41\sqrt{5}$
(D) $\sqrt{205}$
(E) 100

Arithmetic Operations with radical expressions

Rewrite each radical in the form $a\sqrt{b}$, where a and b are positive integers and b is as small as possible, and then add.

$$\begin{aligned}\sqrt{80} + \sqrt{125} &= \sqrt{16(5)} + \sqrt{25(5)} \\ &= \left(\sqrt{16}\right)\left(\sqrt{5}\right) + \left(\sqrt{25}\right)\left(\sqrt{5}\right) \\ &= 4\sqrt{5} + 5\sqrt{5} \\ &= 9\sqrt{5}\end{aligned}$$

The correct answer is A.

$$y = kx + 3$$

67. In the equation above, k is a constant. If $y = 17$ when $x = 2$, what is the value of y when $x = 4$?

(A) 34
(B) 31
(C) 14
(D) 11
(E) 7

Algebra First-degree equations

If $y = kx + 3$ and $y = 17$ when $x = 2$, then

$17 = 2k + 3$

$14 = 2k$

$7 = k$

Therefore, $y = 7x + 3$. When $x = 4$, $y = 7(4) + 3 = 31$.

The correct answer is B.

68. Which of the following is greatest?

(A) $10\sqrt{3}$
(B) $9\sqrt{4}$
(C) $8\sqrt{5}$
(D) $7\sqrt{6}$
(E) $6\sqrt{7}$

Arithmetic Operations on radical expressions

Since all the expressions represent positive numbers, the expression that has the greatest squared value will be the expression that has the greatest value.

$\left(10\sqrt{3}\right)^2 = 100 \times 3 = 300$ (not greatest)

$\left(9\sqrt{4}\right)^2 = 81 \times 4 = 324$ (greatest)

$\left(8\sqrt{5}\right)^2 = 64 \times 5 = 320$ (not greatest)

$\left(7\sqrt{6}\right)^2 = 49 \times 6 = 294$ (not greatest)

$\left(6\sqrt{7}\right)^2 = 36 \times 7 = 252$ (not greatest)

The correct answer is B.

69. Al and Ben are drivers for SD Trucking Company. One snowy day, Ben left SD at 8:00 a.m. heading east and Al left SD at 11:00 a.m. heading west. At a particular time later that day, the dispatcher retrieved data from SD's vehicle tracking system. The data showed that, up to that time, Al had averaged 40 miles per hour and Ben had averaged 20 miles per hour. It also showed that Al and Ben had driven a combined total of 240 miles. At what time did the dispatcher retrieve data from the vehicle tracking system?

(A) 1:00 p.m.
(B) 2:00 p.m.
(C) 3:00 p.m.
(D) 5:00 p.m.
(E) 6:00 p.m.

Algebra Applied problems

Let t be the number of hours after 8:00 a.m. that Ben drove. Then, in t hours, Ben drove $20t$ miles and Al, who began driving 3 hours after Ben began driving, drove $40(t - 3)$ miles. Therefore, their combined total distance at that time can be expressed as $(20t + 40t - 120)$ miles $= (60t - 120)$ miles. It follows that $60t - 120 = 240$, or $60t = 360$, or $t = 6$, which corresponds to 6 hours after 8:00 a.m. or 2:00 p.m.

The correct answer is B.

70. Of the land owned by a farmer, 90 percent was cleared for planting. Of the cleared land, 40 percent was planted with soybeans and 50 percent of the cleared land was planted with wheat. If the remaining 720 acres of cleared land was planted with corn, how many acres did the farmer own?

(A) 5,832
(B) 6,480
(C) 7,200
(D) 8,000
(E) 8,889

Arithmetic Applied problems; Percents

Corn was planted on 100% − (40% + 50%) = 10% of the cleared land, and the cleared land represents 90% of the farmer's land. Therefore, corn was planted on 10% of 90%, or (0.10)(0.90) = 0.09 = 9%, of the farmer's land. It is given that corn was planted on 720 acres, so if x is the number of acres the farmer owns, then $0.09x = 720$ and $x = \frac{720}{0.09} = 8{,}000$.

The correct answer is D.

71. At the start of an experiment, a certain population consisted of 3 animals. At the end of each month after the start of the experiment, the population size was double its size at the beginning of that month. Which of the following represents the population size at the end of 10 months?

(A) 2^3
(B) 3^2
(C) $2(3^{10})$
(D) $3(2^{10})$
(E) $3(10^2)$

Arithmetic Applied problems; Sequences

The population doubles each month, so multiply the previous month's population by 2 to get the next month's population. Thus, at the end of the 1st month the population will be (3)(2), at the end of the 2nd month the population will be (3)(2)(2), at the end of the 3rd month the population will be (3)(2)(2)(2), and so on. Therefore, at the end of the 10th month the population will be the product of 3 and ten factors of 2, which equals $3(2^{10})$.

The correct answer is D.

72. If $\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)=r\left(\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\frac{1}{18}\right)$, then $r =$

(A) $\frac{1}{3}$
(B) $\frac{4}{3}$
(C) 3
(D) 4
(E) 12

Arithmetic Operations with rational numbers

$$\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)=r\left(\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\frac{1}{18}\right)$$

$$\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)=r\left(\frac{1}{3}\right)\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)$$

$$1=r\left(\frac{1}{3}\right)$$

$$3=r$$

Alternatively,

$$\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)=r\left(\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\frac{1}{18}\right)$$

$$\left(\frac{20}{60}+\frac{15}{60}+\frac{12}{60}+\frac{10}{60}\right)=r\left(\frac{20}{180}+\frac{15}{180}+\frac{12}{180}+\frac{10}{180}\right)$$

$$\left(\frac{20+15+12+10}{60}\right)=r\left(\frac{20+15+12+10}{180}\right)$$

$$\left(\frac{20+15+12+10}{60}\right)\left(\frac{180}{20+15+12+10}\right)=r$$

$$\frac{180}{60}=r$$

$$3=r$$

The correct answer is C.

73. If x and y are positive integers such that y is a multiple of 5 and $3x + 4y = 200$, then x must be a multiple of which of the following?

(A) 3
(B) 6
(C) 7
(D) 8
(E) 10

Arithmetic Properties of numbers

Since it is given that y is a multiple of 5, let $y = 5q$, where q is a positive integer. It is given that $3x + 4y = 200$, so $3x + 4(5q) = 200$ or $3x + 20q = 200$. It follows that $3x = 200 - 20q = 20(10 - q)$. Since 3 is prime and is not a factor of 20, then 3 must be a factor of $10 - q$, which means the only possible values of q are 1, 4, and 7. This is summarized in the following table.

q	$10 - q$	$20(10 - q)$	$3x$	x
1	9	180	180	60
4	6	120	120	40
7	3	60	60	20

In each case, the value of x is a multiple of 10.

Alternatively, since $3x = 20(10 - q)$, the factors of the product of 3 and x must correspond to the factors of the product of 20 and $(10 - q)$. Since the factors of 20 include 10, then x must have a factor of 10, which means that x is a multiple of 10.

The correct answer is E.

74. Which of the following expressions can be written as an integer?

I. $\left(\sqrt{82} + \sqrt{82}\right)^2$

II. $(82)\left(\sqrt{82}\right)$

III. $\dfrac{\left(\sqrt{82}\right)\left(\sqrt{82}\right)}{82}$

(A) None
(B) I only
(C) III only
(D) I and II
(E) I and III

Arithmetic Operations with radical expressions

Expression I represents an integer because $\left(\sqrt{82} + \sqrt{82}\right)^2 = \left(2\sqrt{82}\right)^2 = (4)(82)$.
Expression II does not represent an integer because $(82)\sqrt{82} = \sqrt{82^3}$ and $82^3 = 2^3 \times 41^3$ is not a perfect square. Regarding this last assertion, note that the square of any integer has the property that each of its distinct prime factors is repeated an even number of times. For example, $24^2 = (2^3 \times 3)^2 = 2^6 \times 3^2$ has the prime factor 2 repeated 6 times and the prime factor 3 repeated twice. Expression III represents an integer, because $\dfrac{\left(\sqrt{82}\right)\left(\sqrt{82}\right)}{82} = \dfrac{82}{82} = 1$.

The correct answer is E.

75. Four staff members at a certain company worked on a project. The amounts of time that the four staff members worked on the project were in the ratio 2 to 3 to 5 to 6. If one of the four staff members worked on the project for 30 hours, which of the following CANNOT be the total number of hours that the four staff members worked on the project?

(A) 80
(B) 96
(C) 160
(D) 192
(E) 240

Arithmetic Ratio and proportion

For a certain value of x, the numbers of hours worked on the project by the four staff members are $2x$, $3x$, $5x$, and $6x$, for a total of $16x$. It is given that one of these four numbers is equal to 30. If $2x = 30$, then $x = 15$ and $16x = 16(15) = 240$, which is (E). If $3x = 30$, then $x = 10$ and $16x = 16(10) = 160$, which is (C). If $5x = 30$, then $x = 6$ and $16x = 16(6) = 96$, which is (B). If $6x = 30$, then $x = 5$ and $16x = 16(5) = 80$, which is (A).

The correct answer is D.

76. Pumping alone at their respective constant rates, one inlet pipe fills an empty tank to $\frac{1}{2}$ of capacity in 3 hours and a second inlet pipe fills the same empty tank to $\frac{2}{3}$ of capacity in 6 hours. How many hours will it take both pipes, pumping simultaneously at their respective constant rates, to fill the empty tank to capacity?

(A) 3.25
(B) 3.6
(C) 4.2
(D) 4.4
(E) 5.5

Arithmetic Applied problems

The first pipe can fill $\frac{1}{2}$ of the tank in 3 hours, which is equivalent to the rate of filling $\frac{1}{2} \div 3 = \frac{1}{6}$ of the tank per hour. The second pipe can fill $\frac{2}{3}$ of the tank in 6 hours, which is equivalent to the rate of filling $\frac{2}{3} \div 6 = \frac{1}{9}$ of the tank per hour. Together, they can fill the tank at a rate of $\frac{1}{6} + \frac{1}{9} = \frac{5}{18}$ of the tank per hour. Thus, when both pipes are used at the same time, they will fill the tank in $\frac{18}{5} = 3.6$ hours.

The correct answer is B.

77. In the *xy*-coordinate plane, which of the following points must lie on the line $kx + 3y = 6$ for every possible value of k?

(A) (1,1)

(B) (0,2)

(C) (2,0)

(D) (3,6)

(E) (6,3)

Algebra Coordinate geometry

Substituting the various answer choices for (x,y) into $kx + 3y = 6$ gives the following equations:

A $k + 3 = 6$

B $0 + 3(2) = 6$

C $2k + 3(0) = 6$

D $3k + 3(6) = 6$

E $6k + 3(3) = 6$

Each of these, except for the equation in B, holds for only one value of k. The equation in B does not include k and therefore holds for every value of k.

The correct answer is B.

78. If $x^2 - 2 < 0$, which of the following specifies all the possible values of x?

(A) $0 < x < 2$

(B) $0 < x < \sqrt{2}$

(C) $-\sqrt{2} < x < \sqrt{2}$

(D) $-2 < x < 0$

(E) $-2 < x < 2$

Algebra Inequalities

The corresponding equality $x^2 - 2 = 0$ has two solutions, $x = \sqrt{2}$ and $x = -\sqrt{2}$, and thus there are three intervals to test for inclusion in the solution of the inequality: $x < -\sqrt{2}$, $-\sqrt{2} < x < \sqrt{2}$, and $x > \sqrt{2}$. Choose $x = -2$, $x = 0$, and $x = 2$ from these intervals, respectively, to test whether the inequality holds. Then for these choices, the inequality becomes $(-2)^2 - 2 < 0$ (False), $(0)^2 - 2 < 0$ (True), and $(2)^2 - 2 < 0$ (False). Therefore, the solution consists of only the interval $-\sqrt{2} < x < \sqrt{2}$. Alternatively, the graph of $y = x^2 - 2$ is easily seen to be a parabola that opens upward with vertex at $(0,-2)$ and x-intercepts at $x = \sqrt{2}$ and $x = -\sqrt{2}$. The solution to the inequality is the set of the x-coordinates of the portion of this parabola that lies below the x-axis, which is $-\sqrt{2} < x < \sqrt{2}$.

The correct answer is C.

Book number	Pages in book	Total pages read
1	253	253
2	110	363
3	117	480
4	170	650
5	155	805
6	50	855
7	205	1,060
8	70	1,130
9	165	1,295
10	105	1,400
11	143	1,543
12	207	1,750

79. Shawana made a schedule for reading books during 4 weeks (28 days) of her summer vacation. She has checked out 12 books from the library. The number of pages in each book and the order in which she plans to read the books are shown in the table above. She will read exactly 50 pages each day. The only exception will be that she will never begin the next book on the same day that she finishes the previous one, and therefore on some days she may read fewer than 50 pages. At the end of the 28th day, how many books will Shawana have finished?

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11

Arithmetic Operations with integers

Book 1: 6 days—50 pages on each of Days 1–5, 3 pages on Day 6 [5(50) + 3 = 253]

Book 2: 3 days—50 pages on each of Days 7 and 8, 10 pages on Day 9 [2(50) + 10 = 110]

Book 3: 3 days—50 pages on each of Days 10 and 11, 17 pages on Day 12 [2(50) + 17 = 117]

Book 4: 4 days—50 pages on each of Days 13–15, 20 pages on Day 16 [3(50) + 20 = 170]

Book 5: 4 days—50 pages on each of Days 17–19, 5 pages on Day 20 [3(50) + 5 = 155]

Book 6: 1 day—50 pages on Day 21 [1(50) = 50]

Book 7: 5 days—50 pages on each of Days 22–25, 5 pages on Day 26 [4(50) + 5 = 205]

Book 8: 2 days—50 pages on Day 27, 20 pages on Day 28 [50 + 20 = 70]

At this point, Shawana has read on a total of 28 days and has finished 8 books.

The correct answer is B.

80. In Western Europe, x bicycles were sold in each of the years 1990 and 1993. The bicycle producers of Western Europe had a 42 percent share of this market in 1990 and a 33 percent share in 1993. Which of the following represents the decrease in the annual number of bicycles produced and sold in Western Europe from 1990 to 1993?

(A) 9% of $\frac{x}{100}$
(B) 14% of $\frac{x}{100}$
(C) 75% of $\frac{x}{100}$
(D) 9% of x
(E) 14% of x

Arithmetic Percents

Of the x bicycles sold in Western Europe in 1990, 42% of them were produced in Western Europe. It follows that the number of bicycles produced and sold in Western Europe in 1990 was $0.42x$. Similarly, of the x bicycles sold in Western Europe in 1993, 33% were produced in Western Europe. It follows that the number of bicycles produced and sold in Western Europe in 1993 was $0.33x$. Therefore, the decrease in the annual number of bicycles produced and sold in Western Europe from 1990 to 1993 was $0.42x - 0.33x = 0.09x$, which is 9% of x.

The correct answer is D.

81. If k is a positive integer, what is the remainder when $(k + 2)(k^3 - k)$ is divided by 6 ?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Algebra Properties of numbers

Since k can be any positive integer, the remainder must be the same regardless of the value of k. If $k = 2$, for example, then $(k + 2)(k^3 - k) = (2 + 2)(2^3 - 2) = (4)(6)$, which is a multiple of 6, and therefore, the remainder when divided by 6 is 0.

Alternatively, factor the given expression:

$$\begin{aligned}(k+2)(k^3-k) &= (k+2)(k)(k^2-1)\\ &= (k+2)(k)(k+1)(k-1)\end{aligned}$$

Now, rearrange the factors in ascending order $(k - 1)(k)(k + 1)(k + 2)$, and observe that for any positive integer k, the factors are 4 consecutive integers, two of which are even and one of which is divisible by 3. Therefore, $(k + 2)(k^3 - k)$ is divisible by both 2 and 3. Thus, $(k + 2)(k^3 - k)$ is divisible by 6 with 0 remainder.

The correct answer is A.

82. Which of the following fractions is closest to $\frac{1}{2}$?

(A) $\frac{4}{7}$
(B) $\frac{5}{9}$
(C) $\frac{6}{11}$
(D) $\frac{7}{13}$
(E) $\frac{9}{16}$

Arithmetic Fractions

Find the distance between each fraction and $\frac{1}{2}$:

$$\left|\frac{4}{7}-\frac{1}{2}\right| = \left|\frac{8}{14}-\frac{7}{14}\right| = \frac{1}{14}$$

$$\left|\frac{5}{9}-\frac{1}{2}\right| = \left|\frac{10}{18}-\frac{9}{18}\right| = \frac{1}{18}$$

$$\left|\frac{6}{11}-\frac{1}{2}\right| = \left|\frac{12}{22}-\frac{11}{22}\right| = \frac{1}{22}$$

$$\left|\frac{7}{13}-\frac{1}{2}\right| = \left|\frac{14}{26}-\frac{13}{26}\right| = \frac{1}{26}$$

$$\left|\frac{9}{16}-\frac{1}{2}\right| = \left|\frac{9}{16}-\frac{8}{16}\right| = \frac{1}{16}$$

Each distance is the reciprocal of a positive integer, the greatest of which is 26. Therefore, $\frac{1}{26}$ is the least of the distances and $\frac{7}{13}$ is the fraction closest to $\frac{1}{2}$.

Alternatively, asking which fraction is closest to $\frac{1}{2}$ is equivalent to asking which of the fractions, when doubled, is closest to 1. The fractions after doubling are $\frac{8}{7}, \frac{10}{9}, \frac{12}{11}, \frac{14}{13}$, and $\frac{18}{16} = \frac{9}{8}$. Each of these fractions can be expressed as $\frac{n+1}{n} = 1 + \frac{1}{n}$ for some integer n. Of these fractions, the one closest to 1 is the one for which $\frac{1}{n}$ is least, or in other words, the one for which n is greatest. That fraction is $\frac{14}{13}$, and therefore the closest of the original fractions to $\frac{1}{2}$ is $\left(\frac{1}{2}\right)\left(\frac{14}{13}\right) = \frac{7}{13}$.

The correct answer is D.

83. If $p \neq 0$ and $p - \frac{1-p^2}{p} = \frac{r}{p}$, then $r =$

(A) $p + 1$
(B) $2p - 1$
(C) $p^2 + 1$
(D) $2p^2 - 1$
(E) $p^2 + p - 1$

Algebra Simplifying algebraic expressions

$p - \dfrac{1-p^2}{p} = \dfrac{r}{p}$	given
$p^2 - (1-p^2) = r$	multiply both sides by p
$2p^2 - 1 = r$	combine like terms

The correct answer is D.

84. If the range of the six numbers 4, 3, 14, 7, 10, and x is 12, what is the difference between the greatest possible value of x and the least possible value of x ?

(A) 0
(B) 2
(C) 12
(D) 13
(E) 15

Arithmetic Statistics

The range of the six numbers 3, 4, 7, 10, 14, and x is 12. If x were neither the greatest nor the least of the six numbers, then the greatest and least of the six numbers would be 14 and 3. But, this cannot be possible because the range of the six numbers would be $14 - 3 = 11$ and not 12 as stated. Therefore, x must be either the greatest or the least of the six numbers. If x is the greatest of the six numbers, then 3 is the least, and $x - 3 = 12$. It follows that $x = 15$. On the other hand, if x is the least of the six numbers, then 14 is the greatest, and $14 - x = 12$. It follows that $x = 2$. Thus, there are only two possible values of x, namely 15 and 2, and so the difference between the greatest and least possible values of x is $15 - 2 = 13$.

The correct answer is D.

85. What number is 108 more than two-thirds of itself?

(A) 72
(B) 144
(C) 162
(D) 216
(E) 324

Algebra First-degree equations

Let x be the number that is 108 more than two-thirds of itself. Then, $108 + \frac{2}{3}x = x$. Solve for x as follows:

$$108 + \frac{2}{3}x = x$$
$$108 = \frac{1}{3}x$$
$$324 = x$$

The correct answer is E.

86. A doctor prescribed 18 cubic centimeters of a certain drug to a patient whose body weight was 120 pounds. If the typical dosage is 2 cubic centimeters per 15 pounds of body weight, by what percent was the prescribed dosage greater than the typical dosage?

(A) 8%
(B) 9%
(C) 11%
(D) 12.5%
(E) 14.8%

Arithmetic Percents

If the typical dosage is 2 cubic centimeters per 15 pounds of body weight, then the typical dosage for a person who weighs 120 pounds is $2\left(\frac{120}{15}\right) = 2(8) = 16$ cubic centimeters. The prescribed dosage of 18 cubic centimeters is, therefore, $\left(\left(\frac{18-16}{16}\right) \times 100\right)$% or 12.5% greater than the typical dosage.

The correct answer is D.

87. Company P had 15 percent more employees in December than it had in January. If Company P had 460 employees in December, how many employees did it have in January?

(A) 391
(B) 400
(C) 410
(D) 423
(E) 445

Arithmetic Percents

It is given that 460 is 115% of the number of employees in January. Therefore, the number of employees in January was

$$\frac{460}{1.15} = \frac{460}{1.15}\left(\frac{100}{100}\right) = \left(\frac{460}{115}\right)(100) = (4)(100) = 400.$$

The correct answer is B.

88. The function f is defined by $f(x) = \sqrt{x} - 10$ for all positive numbers x. If $u = f(t)$ for some positive numbers t and u, what is t in terms of u ?

(A) $\sqrt{\sqrt{u}+10}$

(B) $\left(\sqrt{u}+10\right)^2$

(C) $\sqrt{u^2+10}$

(D) $(u + 10)^2$

(E) $(u^2 + 10)^2$

Algebra Functions

The question can be answered by solving $u = \sqrt{t} - 10$ for t in terms of u. Adding 10 to both sides of this equation gives $u + 10 = \sqrt{t}$. Squaring both sides of the last equation gives $(u + 10)^2 = t$, which gives t in terms of u.

The correct answer is D.

89. A glass was filled with 10 ounces of water, and 0.01 ounce of the water evaporated each day during a 20-day period. What percent of the original amount of water evaporated during this period?

(A) 0.002%

(B) 0.02%

(C) 0.2%

(D) 2%

(E) 20%

Arithmetic Percents

Since 0.01 ounce of water evaporated each day for 20 days, a total of $20(0.01) = 0.2$ ounce evaporated. Then, to find the percent of the original amount of water that evaporated, divide the amount that evaporated by the original amount and multiply by 100 to convert the decimal to a percent. Thus, $\frac{0.2}{10} \times 100 = 0.02 \times 100$ or 2%.

The correct answer is D.

90. If m and p are positive integers and $m^2 + p^2 < 100$, what is the greatest possible value of mp ?

(A) 36

(B) 42

(C) 48

(D) 49

(E) 51

Arithmetic Operations with integers

Trying various integer values for m and corresponding values of p that satisfy $m^2 + p^2 < 100$ might be the quickest way to solve this problem. First, $m < 10$ and $p < 10$; otherwise, $m^2 + p^2 < 100$ is not true.

If $m = 9$, then for $m^2 + p^2 < 100$ to be true, $p < \sqrt{100 - 81} = \sqrt{19}$, so $p \le 4$, and the greatest possible value for mp is $(9)(4) = 36$.

Similarly, if $m = 8$, then $p < \sqrt{100 - 64} = \sqrt{36}$, so $p \le 5$, and the greatest possible value for mp is $(8)(5) = 40$.

If $m = 7$, then $p < \sqrt{100 - 49} = \sqrt{51}$, so $p \le 7$, and the greatest possible value for mp is $(7)(7) = 49$.

If $m = 6$, then $p < \sqrt{100 - 36} = \sqrt{64}$, so $p \le 7$, and the greatest possible value for mp is $(6)(7) = 42$.

If $m \le 5$ and $p \le 9$, it follows that $mp \le 45$.

Thus, the greatest possible value for mp is 49.

The correct answer is D.

91. If $\frac{x}{y} = \frac{c}{d}$ and $\frac{d}{c} = \frac{b}{a}$, which of the following must be true?

I. $\frac{y}{x} = \frac{b}{a}$

II. $\frac{x}{a} = \frac{y}{b}$

III. $\frac{y}{a} = \frac{x}{b}$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

Algebra Ratio and proportion

Equation I is true:

$\frac{x}{y} = \frac{c}{d}$ given

$\frac{y}{x} = \frac{d}{c}$ take reciprocals

$\frac{d}{c} = \frac{b}{a}$ given

$\frac{y}{x} = \frac{b}{a}$ use last two equations

Equation II is true:

$\frac{y}{x} = \frac{b}{a}$ Equation I (shown true)

$y = \frac{bx}{a}$ multiply both sides by x

$\frac{y}{b} = \frac{x}{a}$ divide both sides by b

Equation III is false, since otherwise it would follow that:

$y = \frac{bx}{a}$ from above

$\frac{y}{a} = \frac{bx}{a^2}$ divide both sides by a

$\frac{x}{b} = \frac{bx}{a^2}$ use Equation III (assumed true)

$x = \frac{b^2x}{a^2}$ multiply both sides by b

From this it follows that Equation III will hold only if $\frac{b^2}{a^2} = 1$, which can be false. For example, if $x = a = c = 1$ and $y = b = d = 2$ (a choice of values for which $\frac{x}{y} = \frac{c}{d}$ and $\frac{d}{c} = \frac{b}{a}$ are true), then $\frac{b^2}{a^2} \neq 1$ and Equation III is $\frac{2}{1} = \frac{1}{2}$, which is false.

The correct answer is C.

92. If k is an integer and $(0.0025)(0.025)(0.00025) \times 10^k$ is an integer, what is the least possible value of k?

(A) –12
(B) –6
(C) 0
(D) 6
(E) 12

Arithmetic Properties of numbers

Let $N = (0.0025)(0.025)(0.00025) \times 10^k$. Rewriting each of the decimals as an integer times a power of 10 gives $N = (25 \times 10^{-4})(25 \times 10^{-3})(25 \times 10^{-5}) \times 10^k = (25)^3 \times 10^{k-12}$. Since the units digit of $(25)^3$ is 5, it follows that if $k = 11$, then the tenths digit of N would be 5, and thus N would not be an integer; and if $k = 12$, then N would be $(25)^3 \times 10^0 = (25)^3$, which is an integer. Therefore, the least value of k such that N is an integer is 12.

The correct answer is E.

93. If $a(a + 2) = 24$ and $b(b + 2) = 24$, where $a \neq b$, then $a + b =$

(A) –48
(B) –2
(C) 2
(D) 46
(E) 48

Algebra Second-degree equations

$a(a + 2) = 24$	given
$a^2 + 2a = 24$	use distributive property
$a^2 + 2a - 24 = 0$	subtract 24 from both sides
$(a + 6)(a - 4) = 0$	factor

So, $a + 6 = 0$, which means that $a = -6$, or $a - 4 = 0$, which means $a = 4$. The equation with the variable b has the same solutions, and so $b = -6$ or $b = 4$.

Since $a \neq b$, then $a = -6$ and $b = 4$, which means $a + b = -6 + 4 = -2$, or $a = 4$ and $b = -6$, which means that $a + b = 4 + (-6) = -2$

The correct answer is B.

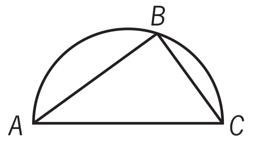
94. In a recent election, Ms. Robbins received 8,000 votes cast by independent voters, that is, voters not registered with a specific political party. She also received 10 percent of the votes cast by those voters registered with a political party. If N is the total number of votes cast in the election and 40 percent of the votes cast were cast by independent voters, which of the following represents the number of votes that Ms. Robbins received?

(A) $0.06N + 3{,}200$
(B) $0.1N + 7{,}200$
(C) $0.4N + 7{,}200$
(D) $0.1N + 8{,}000$
(E) $0.06N + 8{,}000$

Algebra Percents

If N represents the total number of votes cast and 40% of the votes cast were cast by independent voters, then 60% of the votes cast, or $0.6N$ votes, were cast by voters registered with a political party. Ms. Robbins received 10% of these, and so Ms. Robbins received $(0.10)(0.6N) = 0.06N$ votes cast by voters registered with a political party. Thus, Ms. Robbins received $0.06N$ votes cast by voters registered with a political party and 8,000 votes cast by independent voters, so she received $0.06N + 8{,}000$ votes in all.

The correct answer is E.



95. In the figure shown, the triangle is inscribed in the semicircle. If the length of line segment AB is 8 and the length of line segment BC is 6, what is the length of arc ABC ?

(A) 15π
(B) 12π
(C) 10π
(D) 7π
(E) 5π

Geometry Circles, triangles

Because ΔABC is inscribed in a semicircle, $\angle ABC$ is a right angle. Applying the Pythagorean theorem gives $(AB)^2 + (BC)^2 = (AC)^2$. Then substituting the given lengths, $8^2 + 6^2 = (AC)^2$, and so $(AC)^2 = 100$ and $AC = 10$. Thus, the diameter of the circle is 10, the circumference of the entire circle is 10π, and the length of arc ABC is half the circumference of the circle, or 5π.

The correct answer is E.

96. A manufacturer makes and sells 2 products, P and Q. The revenue from the sale of each unit of P is \$20.00 and the revenue from the sale of each unit of Q is \$17.00. Last year the manufacturer sold twice as many units of Q as P. What was the manufacturer's average (arithmetic mean) revenue per unit sold of these 2 products last year?

(A) \$28.50
(B) \$27.00
(C) \$19.00
(D) \$18.50
(E) \$18.00

Arithmetic Statistics

Let x represent the number of units of Product P the manufacturer sold last year. Then $2x$ represents the number of units of Product Q the manufacturer sold last year, and $x + 2x = 3x$ represents the total number of units of Products P and Q the manufacturer sold last year. The total revenue from the sale of Products P and Q was $\$(20x) + \$(17(2x)) = \$(54x)$, so the average revenue per unit sold was $\frac{\$(54x)}{3x} = \$18$.

The correct answer is E.

97. On a certain day, orangeade was made by mixing a certain amount of orange juice with an equal amount of water. On the next day, orangeade was made by mixing the same amount of orange juice with twice the amount of water. On both days, all the orangeade that was made was sold. If the revenue from selling the orangeade was the same for both days and if the orangeade was sold at \$0.60 per glass on the first day, what was the price per glass on the second day?

(A) \$0.15
(B) \$0.20
(C) \$0.30
(D) \$0.40
(E) \$0.45

Arithmetic Applied problems

The ratio of the amount of orangeade made and sold on the first day to amount of orangeade made and sold on the second day is 2:3, because the orangeade on the first day was 1 part orange juice and 1 part water, while on the second day it was 1 part orange juice and 2 parts water. Thus, the ratio of the number of glasses of orangeade made and sold on the first day to the number of glasses of orangeade made and sold on the second day is 2:3. Since the revenues for each day were equal and 2 glasses were sold on the first day for every 3 glasses that were sold on the second day, $2(\$0.60) = 3p$, where p represents the price per glass at which the orangeade was sold on the second day. Therefore, $p = \left(\frac{2}{3}\right)(\$0.60) = \$0.40$.

The correct answer is D.

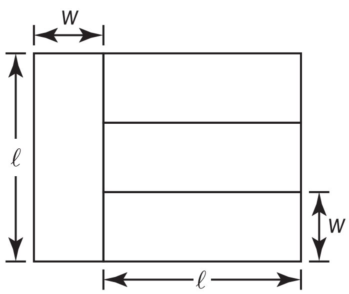
98. A worker carries jugs of liquid soap from a production line to a packing area, carrying 4 jugs per trip. If the jugs are packed into cartons that hold 7 jugs each, how many jugs are needed to fill the last partially filled carton after the worker has made 17 trips?

(A) 1
(B) 2
(C) 4
(D) 5
(E) 6

Arithmetic Remainders

Carrying 4 jugs per trip, the worker carries a total of $4(17) = 68$ jugs in 17 trips. At 7 jugs per carton, these jugs will completely fill 9 cartons with 5 jugs left over since $(9)(7) + 5 = 68$. To fill the 10th carton, $7 - 5 = 2$ jugs are needed.

The correct answer is B.



99. The figure shown above represents a modern painting that consists of four differently colored rectangles, each of which has length ℓ and width w. If the area of the painting is 4,800 square inches, what is the width, in inches, of each of the four rectangles?

(A) 15
(B) 20
(C) 25
(D) 30
(E) 40

Geometry Area

From the figure, $\ell = 3w$, and the area of the painting is $\ell(w + \ell)$. Substituting $3w$ for ℓ gives $3w(w + 3w) = 3w(4w) = 12w^2$. It is given that the area is 4,800 square inches, so $12w^2 = 4{,}800$, $w^2 = 400$, and $w = 20$.

The correct answer is B.

100. A certain fruit stand sold apples for \$0.70 each and bananas for \$0.50 each. If a customer purchased both apples and bananas from the stand for a total of \$6.30, what total number of apples and bananas did the customer purchase?

(A) 10
(B) 11
(C) 12
(D) 13
(E) 14

Algebra First-degree equations; Operations with integers

If each apple sold for \$0.70, each banana sold for \$0.50, and the total purchase price was \$6.30, then $0.70x + 0.50y = 6.30$, where x and y are positive integers representing the number of apples and bananas, respectively, the customer purchased.

$$0.70x + 0.50y = 6.30$$
$$0.50y = 6.30 - 0.70x$$
$$0.50y = 0.70(9 - x)$$
$$y = \frac{7}{5}(9 - x)$$

Since y must be an integer, $9 - x$ must be divisible by 5. Furthermore, both x and y must be positive integers. For $x = 1, 2, 3, 4, 5, 6, 7, 8$, the corresponding values of $9 - x$ are 8, 7, 6, 5, 4, 3, 2, and 1. Only one of these, 5, is divisible by 5.

Therefore, $x = 4$ and $y = \frac{7}{5}(9 - 4) = 7$ and the total number of apples and bananas the customer purchased is $4 + 7 = 11$.

The correct answer is B.

101. In the *xy*-plane, what is the slope of the line with equation $3x + 7y = 9$?

(A) $-\frac{7}{3}$
(B) $-\frac{3}{7}$
(C) $\frac{3}{7}$
(D) 3
(E) 7

Algebra Coordinate geometry

Since the given equation of the line is equivalent to $7y = -3x + 9$, or $y = -\frac{3}{7}x + \frac{9}{7}$, the slope of the line is $-\frac{3}{7}$. Alternatively, choose 2 points lying on the line and then use the slope formula for these 2 points. For example, substitute $x = 0$ in $7y = -3x + 9$ and solve for y to get $\left(0, \frac{9}{7}\right)$, substitute $y = 0$ in $7y = -3x + 9$ and solve for x to get (3,0), then use the slope formula to get $\frac{\frac{9}{7} - 0}{0 - 3} = \frac{\frac{9}{7}}{-3} = -\frac{3}{7}$.

The correct answer is B.

102. Working simultaneously and independently at an identical constant rate, 4 machines of a certain type can produce a total of x units of product P in 6 days. How many of these machines, working simultaneously and independently at this constant rate, can produce a total of $3x$ units of product P in 4 days?

(A) 24
(B) 18
(C) 16
(D) 12
(E) 8

Algebra Applied problems

Define a *machine day* as 1 machine working for 1 day. Then, 4 machines each working 6 days is equivalent to $(4)(6) = 24$ machine days. Thus, x units of product P were produced in 24 machine days, and $3x$ units of product P will require $(3)(24) = 72$ machine days, which is equivalent to $\frac{72}{4} = 18$ machines working independently and simultaneously for 4 days.

The correct answer is B.

103. At a certain school, the ratio of the number of second graders to the number of fourth graders is 8 to 5, and the ratio of the number of first graders to the number of second graders is 3 to 4. If the ratio of the number of third graders to the number of fourth graders is 3 to 2, what is the ratio of the number of first graders to the number of third graders?

(A) 16 to 15
(B) 9 to 5
(C) 5 to 16
(D) 5 to 4
(E) 4 to 5

Arithmetic Ratio and proportion

If F, S, T, and R represent the number of first, second, third, and fourth graders, respectively, then the given ratios are: (i) $\frac{S}{R} = \frac{8}{5}$, (ii) $\frac{F}{S} = \frac{3}{4}$, and (iii) $\frac{T}{R} = \frac{3}{2}$. The desired ratio is $\frac{F}{T}$. From (i), $S = \frac{8}{5}R$, and from (ii), $F = \frac{3}{4}S$. Combining these results, $F = \frac{3}{4}S = \frac{3}{4}\left(\frac{8}{5}R\right) = \frac{6}{5}R$. From (iii), $T = \frac{3}{2}R$. Then $\frac{F}{T} = \frac{\frac{6}{5}R}{\frac{3}{2}R} = \frac{6}{5}\cdot\frac{2}{3} = \frac{4}{5}$. So, the ratio of the number of first graders to the number of third graders is 4 to 5.

The correct answer is E.

104. The symbol Δ denotes one of the four arithmetic operations: addition, subtraction, multiplication, or division. If $6 \Delta 3 \le 3$, which of the following must be true?

I. $2 \Delta 2 = 0$
II. $2 \Delta 2 = 1$
III. $4 \Delta 2 = 2$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

Arithmetic Operations with integers

If Δ represents addition, subtraction, multiplication, or division, then $6 \Delta 3$ is equal to either $6 + 3 = 9$, or $6 - 3 = 3$, or $6 \times 3 = 18$, or $6 \div 3 = 2$. Since it is given that $6 \Delta 3 \le 3$, Δ represents either subtraction or division.

Statement I is true for subtraction since $2 - 2 = 0$ but not true for division since $2 \div 2 = 1$.

Statement II is not true for subtraction since $2 - 2 = 0$ but is true for division since $2 \div 2 = 1$.

Statement III is true for subtraction since $4 - 2 = 2$ and is true for division since $4 \div 2 = 2$. Therefore, only Statement III must be true.

The correct answer is C.

105. The average distance between the Sun and a certain planet is approximately 2.3×10^{14} inches. Which of the following is closest to the average distance between the Sun and the planet, in kilometers? (1 kilometer is approximately 3.9×10^4 inches.)

(A) 7.1×10^8
(B) 5.9×10^9
(C) 1.6×10^{10}
(D) 1.6×10^{11}
(E) 5.9×10^{11}

Arithmetic Measurement conversion

Convert to kilometers and then estimate.

$$\begin{aligned}\left(2.3\times10^{14}\text{ in}\right)\left(\frac{1\text{ km}}{3.9\times10^{4}\text{ in}}\right) &= \frac{2.3\times10^{14}}{3.9\times10^{4}}\text{km}\\ &= \frac{2.3}{3.9}\times10^{14-4}\text{km}\\ &\approx \frac{2}{4}\times10^{10}\\ &= 0.5\times10^{10}\\ &= 5\times10^{9}\end{aligned}$$

The correct answer is B.

106. If $mn \neq 0$ and 25 percent of n equals $37\frac{1}{2}$ percent of m, what is the value of $\frac{12n}{m}$?

(A) 18
(B) $\frac{32}{3}$
(C) 8
(D) 3
(E) $\frac{9}{8}$

Algebra Percents; First-degree equations

It is given that $(25\%)n = (37.5\%)m$, or $0.25n = 0.375m$. The value of $\frac{12n}{m}$ can be found by first finding the value of $\frac{n}{m}$ and then multiplying the result by 12. Doing this gives $\frac{12n}{m} = \left(\frac{0.375}{0.25}\right)(12) = 18$. Alternatively, the numbers involved allow for a series of simple equation transformations to be carried out, such as the following:

$0.25n = 0.375m$	given
$25n = 37.5m$	multiply both sides by 100
$50n = 75m$	multiply both sides by 2
$2n = 3m$	divide both sides by 25
$12n = 18m$	multiply both sides by 6
$\frac{12n}{m} = 18$	divide both sides by m

The correct answer is A.

107. In the coordinate plane, a circle has center $(2,-3)$ and passes through the point $(5,0)$. What is the area of the circle?

(A) 3π
(B) $3\sqrt{2}\pi$
(C) $3\sqrt{3}\pi$
(D) 9π
(E) 18π

Geometry Coordinate geometry; Circles; Area

The area of a circle is given by πr^2, where r is the radius of the circle. The value of r^2 is the square of the distance from the center to a point of the circle. Using the distance formula, $r^2 = (2-5)^2 + (-3-0)^2 = 9 + 9 = 18$. Therefore, the area of the circle is 18π.

The correct answer is E.

108. Last year Joe grew 1 inch and Sally grew 200 percent more than Joe grew. How many inches did Sally grow last year?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Arithmetic Percents

Joe grew 1 inch last year and Sally grew 200 percent more than Joe grew, so Sally grew 1 inch plus 200 percent of 1 inch or $1 + 2(1) = 3$ inches.

The correct answer is D.

109. The cost C, in dollars, to remove p percent of a certain pollutant from a pond is estimated by using the formula $C = \frac{100{,}000p}{100-p}$. According to this estimate, how much more would it cost to remove 90 percent of the pollutant from the pond than it would cost to remove 80 percent of the pollutant?

(A) \$500,000
(B) \$100,000
(C) \$50,000
(D) \$10,000
(E) \$5,000

Algebra; Arithmetic Simplifying algebraic expressions; Operations on rational numbers

Removing 90% of the pollutant from the pond would cost $\frac{(100{,}000)(90)}{100-90} = \frac{9{,}000{,}000}{10} =$ 900,000 dollars, and removing 80% of the pollutant would cost $\frac{(100{,}000)(80)}{100-80} = \frac{8{,}000{,}000}{20} = 400{,}000$ dollars. The difference is, then, \$900,000 – \$400,000 = \$500,000.

The correct answer is A.

110. If $xy \neq 0$ and $x^2y^2 - xy = 6$, which of the following could be y in terms of x?

I. $\frac{1}{2x}$

II. $-\frac{2}{x}$

III. $\frac{3}{x}$

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

Algebra Second-degree equations

$x^2y^2 - xy = 6$ given
$x^2y^2 - xy - 6 = 0$ subtract 6 from both sides
$(xy + 2)(xy - 3) = 0$ factor

So, $xy + 2 = 0$, which means $xy = -2$ and $y = -\frac{2}{x}$, or $xy - 3 = 0$, which means that $xy = 3$ and $y = \frac{3}{x}$. Thus, y in terms of x could be given by the expressions in II or III.

The correct answer is E.

111. At a certain instant in time, the number of cars, N, traveling on a portion of a certain highway can be estimated by the formula

$$N = \frac{20Ld}{600 + s^2}$$

where L is the number of lanes in the same direction, d is the length of the portion of the highway, in feet, and s is the average speed of the cars, in miles per hour. Based on the formula, what is the estimated number of cars traveling on a $\frac{1}{2}$ - mile portion of the highway if the highway has 2 lanes in the same direction and the average speed of the cars is 40 miles per hour? (5,280 feet = 1 mile)

(A) 155
(B) 96
(C) 80
(D) 48
(E) 24

Algebra Simplifying algebraic expressions

Substitute $L = 2$, $d = \frac{1}{2}(5,280)$, $s = 40$ into the given formula and calculate the value for N.

$$\begin{aligned} N &= \frac{20(2)\left(\frac{1}{2}\right)(5,280)}{600 + 40^2} \\ &= \frac{20(5,280)}{600 + 1,600} \\ &= \frac{20(5,280)}{2,200} \\ &= \frac{2(528)}{22} \\ &= \frac{528}{11} \\ &= 48 \end{aligned}$$

The correct answer is D.

112. $\sqrt{4.8 \times 10^9}$ is closest in value to

(A) 2,200
(B) 70,000
(C) 220,000
(D) 7,000,000
(E) 22,000,000

Arithmetic Operations on radical expressions

$\sqrt{4.8} \times \sqrt{10^9} = \sqrt{48 \times 10^8}$ substitute 48×10^8 for 4.8×10^9

$= \sqrt{48} \times \sqrt{10^8}$ $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$\approx \sqrt{49} \times \sqrt{10^8}$ $49 \approx 48$

and then

$\sqrt{49} \times \sqrt{10^8} = 7 \times 10^4$ $\sqrt{49} = 7$, $\sqrt{10^8} = \sqrt{(10^4)^2} = 10^4$

$= 70,000$

The correct answer is B.

113. Three printing presses, R, S, and T, working together at their respective constant rates, can do a certain printing job in 4 hours. S and T, working together at their respective constant rates, can do the same job in 5 hours. How many hours would it take R, working alone at its constant rate, to do the same job?

(A) 8
(B) 10
(C) 12
(D) 15
(E) 20

Algebra Applied problems

Let r be the portion of the job that printing press R, working alone, completes in 1 hour; and let s and t be the corresponding portions, respectively, for printing press S and printing press T. From the given information, it follows that $r + s + t = \frac{1}{4}$ and $s + t = \frac{1}{5}$. Subtracting these two equations gives $r = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$. It follows that printing press R, working alone, will complete $\frac{1}{20}$ of the job in 1 hour, and therefore printing press R, working alone, will complete the job in 20 hours.

The correct answer is E.

114. For a party, three solid cheese balls with diameters of 2 inches, 4 inches, and 6 inches, respectively, were combined to form a single cheese ball. What was the approximate diameter, in inches, of the new cheese ball? (The volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius.)

(A) 12
(B) 16
(C) $\sqrt[3]{16}$
(D) $3\sqrt[3]{8}$
(E) $2\sqrt[3]{36}$

Geometry Volume

Since the diameters of the cheese balls are given as 2 inches, 4 inches, and 6 inches, the radii of the cheese balls are 1 inch, 2 inches, and 3 inches, respectively. Using $V = \frac{4}{3}\pi r^3$, the combined volume of the 3 cheese balls is $\frac{4}{3}\pi\left(1^3 + 2^3 + 3^3\right)$ or $\frac{4}{3}\pi(36)$ cubic inches.

Thus, if R represents the radius of the new cheese ball, then the volume of the new cheese ball is $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(36)$ and $R^3 = 36$, from which it follows that $R = \sqrt[3]{36}$ inches. Therefore, the diameter of the new cheese ball is $2R = 2\sqrt[3]{36}$ inches.

The correct answer is E.

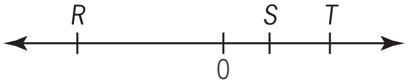
115. The sum of all the integers k such that $-26 < k < 24$ is

(A) 0
(B) −2
(C) −25
(D) −49
(E) −51

Arithmetic Operations on integers

In the sum of all integers k such that $-26 < k < 24$, the positive integers from 1 through 23 can be paired with the negative integers from −1 through −23. The sum of these pairs is 0 because $a + (-a) = 0$ for all integers a. Therefore, the sum of all integers k such that $-26 < k < 24$ is $-25 + (-24) + (23)(0) = -49$.

The correct answer is D.



116. The number line shown contains three points R, S, and T, whose coordinates have absolute values r, s, and t, respectively. Which of the following equals the average (arithmetic mean) of the coordinates of the points R, S, and T?

(A) s
(B) $s + t - r$
(C) $\frac{r - s - t}{3}$
(D) $\frac{r + s + t}{3}$
(E) $\frac{s + t - r}{3}$

Arithmetic Absolute value; Number line

Because point R is to the left of 0 on the number line, the coordinate of R is negative. It is given that r is the absolute value of the coordinate of R and so the coordinate of R is $-r$. Because points S and T are to the right of 0 on the number line, their coordinates are positive. It is given that s and t are the absolute values of the coordinates of S and T, and so the coordinates of S and T are s and t. The arithmetic mean of the coordinates of R, S, and T is $\frac{s+t-r}{3}$.

The correct answer is E.

117. Mark and Ann together were allocated n boxes of cookies to sell for a club project. Mark sold 10 boxes less than n and Ann sold 2 boxes less than n. If Mark and Ann have each sold at least one box of cookies, but together they have sold less than n boxes, what is the value of n?

(A) 11
(B) 12
(C) 13
(D) 14
(E) 15

Algebra Inequalities

Mark sold $n - 10$ boxes and Ann sold $n - 2$ boxes. Because each person sold at least one box, it follows that $n - 10 \geq 1$ and $n - 2 \geq 1$, which implies that $n \geq 11$. On the other hand, together they sold less than n boxes, so $(n - 10) + (n - 2) < n$, which implies that $n < 12$. Therefore, n is an integer such that $n \geq 11$ and $n < 12$, which implies that $n = 11$.

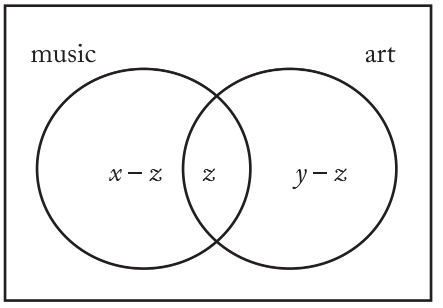
The correct answer is A.

118. A certain high school has 5,000 students. Of these students, x are taking music, y are taking art, and z are taking both music and art. How many students are taking neither music nor art?

(A) $5{,}000 - z$
(B) $5{,}000 - x - y$
(C) $5{,}000 - x + z$
(D) $5{,}000 - x - y - z$
(E) $5{,}000 - x - y + z$

Algebra Sets

Since x students are taking music, y students are taking art, and z students are taking both music and art, the number of students taking only music is $x - z$, and the number of students taking only art is $y - z$, as illustrated by the following Venn diagram.



Therefore, the number of students taking neither music nor art is
$5{,}000 - [(x - z) + z + (y - z)] = 5{,}000 - x - y + z$.

The correct answer is E.

119. Yesterday's closing prices of 2,420 different stocks listed on a certain stock exchange were all different from today's closing prices. The number of stocks that closed at a higher price today than yesterday was 20 percent greater than the number that closed at a lower price. How many of the stocks closed at a higher price today than yesterday?

(A) 484
(B) 726
(C) 1,100
(D) 1,320
(E) 1,694

Arithmetic Percents

Let n be the number of stocks that closed at a lower price today than yesterday. Then $1.2n$ is the number of stocks that closed at a higher price today than yesterday, and $1.2n$ is the value asked for. Because the total number of stocks is 2,420, it follows that $n + 1.2n = 2{,}420$, or $2.2n = 2{,}420$.

Therefore, $n = \frac{2{,}420}{2.2} = 1{,}100$, and hence
$1.2n = (1.2)(1{,}100) = 1{,}320$.

The correct answer is D.

120. Each person who attended a company meeting was either a stockholder in the company, an employee of the company, or both. If 62 percent of those who attended the meeting were stockholders and 47 percent were employees, what percent were stockholders who were not employees?

(A) 34%
(B) 38%
(C) 45%
(D) 53%
(E) 62%

Arithmetic Sets

Let M represent the number of meeting attendees. Then, since 62% of M or $0.62M$ were stockholders and 47% of M or $0.47M$ were employees, it follows that $0.62M + 0.47M = 1.09M$ were either stockholders, employees, or both. Since $1.09M$ exceeds M, the excess $1.09M - M = 0.09M$ must be the number of attendees who were both stockholders and employees, leaving the rest $0.62M - 0.09M = 0.53M$, or 53%, of the meeting attendees to be stockholders but not employees.

The correct answer is D.

121. A gym class can be divided into 8 teams with an equal number of players on each team or into 12 teams with an equal number of players on each team. What is the lowest possible number of students in the class?

(A) 20
(B) 24
(C) 36
(D) 48
(E) 96

Arithmetic Properties of numbers

The lowest value that can be divided evenly by 8 and 12 is their least common multiple (LCM). Since $8 = 2^3$ and $12 = 2^2(3)$, the LCM is $2^3(3) = 24$.

The correct answer is B.

Accounts	Amount Budgeted	Amount Spent
Payroll	$110,000	$117,000
Taxes	40,000	42,000
Insurance	2,500	2,340

122. The table shows the amount budgeted and the amount spent for each of three accounts in a certain company. For which of these accounts did the amount spent differ from the amount budgeted by more than 6 percent of the amount budgeted?

(A) Payroll only
(B) Taxes only
(C) Insurance only
(D) Payroll and Insurance
(E) Taxes and Insurance

Arithmetic Percents

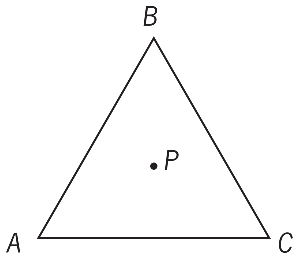
For Payroll, 6% of the budgeted amount is $(0.06)(\$110{,}000) = \$6{,}600$. Since $\$117{,}000 - \$110{,}000 = \$7{,}000 > \$6{,}600$, the amount spent differed from the amount budgeted by more than 6%.

For Taxes, 6% of the budgeted amount is $(0.06)(\$40{,}000) = \$2{,}400$. Since $\$42{,}000 - \$40{,}000 = \$2{,}000 < \$2{,}400$, the amount spent did not differ from the amount budgeted by more than 6%.

For Insurance, 6% of the budgeted amount is $(0.06)(\$2{,}500) = \$150$. Since $\$2{,}500 - \$2{,}340 = \$160 > \$150$, the amount spent differed from the amount budgeted by more than 6%.

Thus, the amount spent differed from the amount budgeted by more than 6% for Payroll and Insurance.

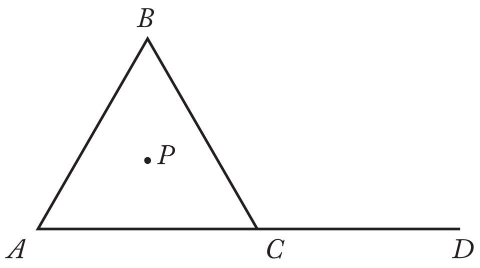
The correct answer is D.



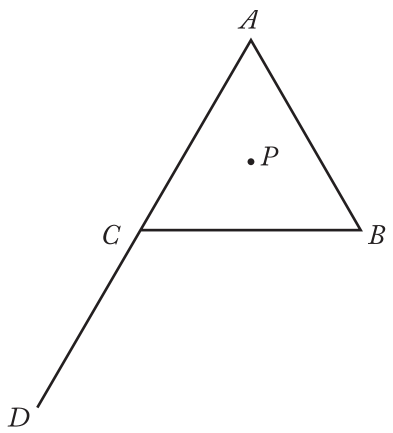
123. In the figure above, triangle *ABC* is equilateral, and point *P* is equidistant from vertices *A*, *B*, and *C*. If triangle *ABC* is rotated clockwise about point *P*, what is the minimum number of degrees the triangle must be rotated so that point *B* will be in the position where point *A* is now?

(A) 60
(B) 120
(C) 180
(D) 240
(E) 270

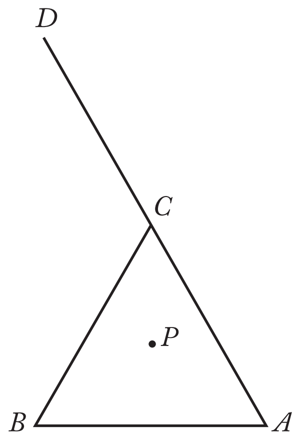
Geometry Angles



Since ΔABC is equilateral, the measure of $\angle ACB$ is $60°$. Therefore, the measure of $\angle BCD$ is $180° - 60° = 120°$. Rotating the figure clockwise about point P through an angle of $120°$ will produce the figure shown below.



Then rotating this figure clockwise about point P through an angle of $120°$ will produce the figure shown below.



In this figure, point B is in the position where point A was in the original figure. The triangle was rotated clockwise about point P through $120° + 120° = 240°$.

The correct answer is D.

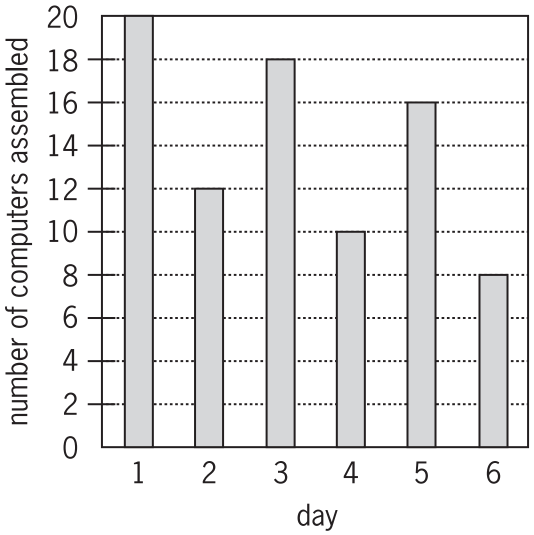
124. At least $\frac{2}{3}$ of the 40 members of a committee must vote in favor of a resolution for it to pass. What is the greatest number of members who could vote against the resolution and still have it pass?

(A) 19
(B) 17
(C) 16
(D) 14
(E) 13

Arithmetic Operations on rational numbers

If at least $\frac{2}{3}$ of the members must vote in favor of a resolution, then no more than $\frac{1}{3}$ of the members can be voting against it. On this 40-member committee, $\frac{1}{3}(40) = 13\frac{1}{3}$, which means that no more than 13 members can vote against the resolution and still have it pass.

The correct answer is E.



125. The graph shows the number of computers assembled during each of 6 consecutive days. From what day to the next day was the percent change in the number of computers assembled the greatest in magnitude?

(A) From Day 1 to Day 2
(B) From Day 2 to Day 3
(C) From Day 3 to Day 4
(D) From Day 4 to Day 5
(E) From Day 5 to Day 6

Arithmetic Percents

The following table shows the percent change from each day to the next and the magnitude of the percent change.

Time period	Percent change	Magnitude of percent change
From Day 1 to Day 2	$\left(\frac{12-20}{20}\times 100\right)\% = \left(-\frac{8}{20}\times 100\right)\% = -40\%$	40
From Day 2 to Day 3	$\left(\frac{18-12}{12}\times 100\right)\% = \left(\frac{6}{12}\times 100\right)\% = 50\%$	50
From Day 3 to Day 4	$\left(\frac{10-18}{18}\times 100\right)\% = \left(-\frac{8}{18}\times 100\right)\% \approx -44\%$	44
From Day 4 to Day 5	$\left(\frac{16-10}{10}\times 100\right)\% = \left(\frac{6}{10}\times 100\right)\% = 60\%$	60
From Day 5 to Day 6	$\left(\frac{8-16}{16}\times 100\right)\% = \left(-\frac{8}{16}\times 100\right)\% = -50\%$	50

The correct answer is D.

126. If $n = 20! + 17$, then n is divisible by which of the following?

I. 15
II. 17
III. 19

(A) None
(B) I only
(C) II only
(D) I and II
(E) II and III

Arithmetic Properties of numbers

Because 20! is the product of all integers from 1 through 20, it follows that 20! is divisible by each integer from 1 through 20. In particular, 20! is divisible by each of the integers 15, 17, and 19. Since 20! and 17 are both divisible by 17, their sum is divisible by 17, and hence the correct answer will include II. If n were divisible by 15, then $n - 20!$ would be divisible by 15. But, $n - 20! = 17$ and 17 is not divisible by 15. Therefore, the correct answer does not include I. If n were divisible by 19, then $n - 20!$ would be divisible by 19. But, $n - 20! = 17$ and 17 is not divisible by 19. Therefore, the correct answer does not include III.

The correct answer is C.

127. The product of two negative numbers is 160. If the lesser of the two numbers is 4 less than twice the greater, what is the greater number?

(A) –20
(B) –16
(C) –10
(D) –8
(E) –4

Algebra Second-degree equations

Let x and y be the two numbers, where x is the lesser of the two numbers and y is the number desired. From the given information it follows that $xy = 160$ and $x = 2y - 4$, from which it follows that $(2y - 4)y = 160$. Dividing both sides of the last equation by 2 gives $(y - 2)y = 80$. Thus, 80 is to be written as a product of two negative numbers, one that is 2 less than the other. Trying simple factorizations of 80 quickly leads to the value of y: $(-40)(-2) = 80$, $(-20)(-4) = 80$, $(-10)(-8) = 80$. Therefore, $y = -8$. Note that because -8 is one of the answer choices, it is not necessary to ensure there are no other negative solutions to the equation $(y - 2)y = 80$.

Alternatively, $(y - 2)y = 80$ can be written as $y^2 - 2y - 80 = 0$. Factoring the left side gives $(y + 8)(y - 10) = 0$, and $y = -8$ is the only negative solution.

The correct answer is D.

128. According to a certain estimate, the depth $N(t)$, in centimeters, of the water in a certain tank at t hours past 2:00 in the morning is given by $N(t) = -20(t - 5)^2 + 500$ for $0 \le t \le 10$. According to this estimate, at what time in the morning does the depth of the water in the tank reach its maximum?

(A) 5:30
(B) 7:00
(C) 7:30
(D) 8:00
(E) 9:00

Algebra Functions

When $t = 5$, the value of $-20(t - 5)^2 + 500$ is 500. For all values of t between 0 and 10, inclusive, except $t = 5$, the value of $-20(t - 5)^2$ is negative and $-20(t - 5)^2 + 500 < 500$. Therefore, the tank reaches its maximum depth 5 hours after 2:00 in the morning, which is 7:00 in the morning.

The correct answer is B.

129. After driving to a riverfront parking lot, Bob plans to run south along the river, turn around, and return to the parking lot, running north along the same path. After running 3.25 miles south, he decides to run for only 50 minutes more. If Bob runs at a constant rate of 8 minutes per mile, how many miles farther south can he run and still be able to return to the parking lot in 50 minutes?

(A) 1.5
(B) 2.25
(C) 3.0
(D) 3.25
(E) 4.75

Algebra Applied problems

After running 3.25 miles south, Bob has been running for $(3.25 \text{ miles})\left(8\dfrac{\text{minutes}}{\text{mile}}\right) = 26$ minutes. Thus, if t is the number of additional minutes that Bob can run south before turning around, then the number of minutes that Bob will run north, after turning around, will be $t + 26$. Since Bob will be running a total of 50 minutes after the initial 26 minutes of running, it follows that $t + (t + 26) = 50$, or $t = 12$. Therefore, Bob can run south an additional $\dfrac{12 \text{ minutes}}{8\dfrac{\text{minutes}}{\text{mile}}} = 1.5$ miles before turning around.

The correct answer is A.

130. Alex deposited x dollars into a new account that earned 8 percent annual interest, compounded annually. One year later Alex deposited an additional x dollars into the account. If there were no other transactions and if the account contained w dollars at the end of two years, which of the following expresses x in terms of w?

(A) $\frac{w}{1+1.08}$

(B) $\frac{w}{1.08+1.16}$

(C) $\frac{w}{1.16+1.24}$

(D) $\frac{w}{1.08+(1.08)^2}$

(E) $\frac{w}{(1.08)^2+(1.08)^3}$

Algebra Applied problems

At the end of the first year, the value of Alex's initial investment was $x(1.08)$ dollars, and after he deposited an additional x dollars into the account, its value was $[x(1.08) + x]$ dollars. At the end of the second year, the value was w dollars, where $w = [x(1.08) + x](1.08) = x(1.08)^2 + x(1.08) = x[(1.08)^2 + 1.08]$. Thus, $x = \frac{w}{1.08+(1.08)^2}$.

The correct answer is D.

131. M is the sum of the reciprocals of the consecutive integers from 201 to 300, inclusive. Which of the following is true?

(A) $\frac{1}{3} < M < \frac{1}{2}$

(B) $\frac{1}{5} < M < \frac{1}{3}$

(C) $\frac{1}{7} < M < \frac{1}{5}$

(D) $\frac{1}{9} < M < \frac{1}{7}$

(E) $\frac{1}{12} < M < \frac{1}{9}$

Arithmetic Estimation

Because $\frac{1}{300}$ is less than each of the 99 numbers $\frac{1}{201}, \frac{1}{202}, \ldots, \frac{1}{299}$, it follows that $\frac{1}{300}+\frac{1}{300}+\cdots+\frac{1}{300}$ (the sum of 99 identical values) is less than $\frac{1}{201}+\frac{1}{202}+\cdots+\frac{1}{299}$. Therefore, adding $\frac{1}{300}$ to both sides of this last inequality, it follows that $\frac{1}{300}+\frac{1}{300}+\cdots+\frac{1}{300}$ (the sum of 100 identical values) is less than

$\frac{1}{201}+\frac{1}{202}+\cdots+\frac{1}{299}+\frac{1}{300} = M$. Hence, $(100)\left(\frac{1}{300}\right) < M$ or $\frac{1}{3} < M$. Also, because $\frac{1}{200}$ is greater than each of the 100 numbers $\frac{1}{201}, \frac{1}{202}, \ldots, \frac{1}{300}$, it follows that $\frac{1}{200}+\frac{1}{200}+\cdots+\frac{1}{200}$ (the sum of 100 identical values) is greater than $\frac{1}{201}+\frac{1}{202}+\cdots+\frac{1}{300}$. Hence, $(100)\left(\frac{1}{200}\right) > M$ or $\frac{1}{2} > M$. From $\frac{1}{3} < M$ and $\frac{1}{2} > M$, it follows that $\frac{1}{3} < M < \frac{1}{2}$.

The correct answer is A.

132. Working simultaneously at their respective constant rates, Machines A and B produce 800 nails in x hours. Working alone at its constant rate, Machine A produces 800 nails in y hours. In terms of x and y, how many hours does it take Machine B, working alone at its constant rate, to produce 800 nails?

(A) $\frac{x}{x+y}$

(B) $\frac{y}{x+y}$

(C) $\frac{xy}{x+y}$

(D) $\frac{xy}{x-y}$

(E) $\frac{xy}{y-x}$

Algebra Applied problems

Let R_A and R_B be the constant rates, in nails per hour, at which Machines A and B work, respectively. Then it follows from the given information that $R_A + R_B = \frac{800}{x}$ and $R_A = \frac{800}{y}$. Hence, $\frac{800}{y} + R_B = \frac{800}{x}$, or $R_B = \frac{800}{x} - \frac{800}{y} = 800\left(\frac{1}{x} - \frac{1}{y}\right) = 800\left(\frac{y-x}{xy}\right)$. Therefore, the time, in hours, it would take Machine B to produce 800 nails is given by $\frac{800}{800\left(\frac{y-x}{xy}\right)} = \frac{xy}{y-x}$.

The correct answer is E.

133. In the Johnsons' monthly budget, the dollar amounts allocated to household expenses, food, and miscellaneous items are in the ratio 5:2:1, respectively. If the total amount allocated to these three categories is \$1,800, what is the amount allocated to food?

(A) \$900
(B) \$720
(C) \$675
(D) \$450
(E) \$225

Algebra Applied problems

Since the ratio is 5:2:1, let $5x$ be the money allocated to household expenses, $2x$ be the money allocated to food, and $1x$ be the money allocated to miscellaneous items. The given information can then be expressed in the following equation and solved for x.

$$5x + 2x + 1x = \$1,800$$

$8x = \$1,800$	combine like terms
$x = \$225$	divide both sides by 8

The money allocated to food is $2x = 2(\$225) = \$450$.

The correct answer is D.

	Number of Marbles in Each of Three Bags	Percent of Marbles in Each Bag That Are Blue (to the nearest tenth)
Bag P	37	10.8%
Bag Q	$\times$	66.7%
Bag R	32	50.0%

134. If $\frac{1}{3}$ of the total number of marbles in the three bags listed in the table above are blue, how many marbles are there in bag Q?

(A) 5
(B) 9
(C) 12
(D) 23
(E) 46

Algebra Percents

What is the value of x, the number of marbles in bag Q? From the given information and rounded to the nearest integer, bag P has $(37)(0.108) = 4$ blue marbles, bag Q has $(x)(0.667) = \frac{2}{3}x$ blue marbles, and bag R has $(32)(0.5) = 16$ blue marbles. Therefore, the total number of blue marbles is equal to $4 + \frac{2}{3}x + 16 = 20 + \frac{2}{3}x$. It is given that $\frac{1}{3}$ of the total number of marbles are blue, so the total number of blue marbles is also equal to $\frac{1}{3}(37 + x + 32) = \frac{1}{3}x + 23$. It follows that $20 + \frac{2}{3}x = \frac{1}{3}x + 23$, or $\frac{1}{3}x = 3$, or $x = 9$.

The correct answer is B.

135. $\frac{(0.0036)(2.8)}{(0.04)(0.1)(0.003)} =$

(A) 840.0
(B) 84.0
(C) 8.4
(D) 0.84
(E) 0.084

Arithmetic Operations with rational numbers

To make the calculations less tedious, convert the decimals to whole numbers times powers of 10 as follows:

$$\frac{(0.0036)(2.8)}{(0.04)(0.1)(0.003)}$$

$$=\frac{\left(36\times10^{-4}\right)\left(28\times10^{-1}\right)}{\left(4\times10^{-2}\right)\left(1\times10^{-1}\right)\left(3\times10^{-3}\right)}$$

$$=\frac{(36)(28)}{(4)(1)(3)}\times10^{(-4-1)-(-2-1-3)}$$

$$=\frac{(36)(28)}{(4)(1)(3)}\times10^{-5-(-6)}$$

$$=\frac{(36)(28)}{(4)(1)(3)}\times10^{-5+6}$$

$$=\left(\frac{36}{3}\right)\left(\frac{28}{4}\right)\times10^{-5+6}$$

$$=(12)(7)\times10^{1}$$

$$=84\times10$$

$$=840$$

The correct answer is A.

136. If n is an integer greater than 6, which of the following must be divisible by 3 ?

(A) $n(n+1)(n-4)$
(B) $n(n+2)(n-1)$
(C) $n(n+3)(n-5)$
(D) $n(n+4)(n-2)$
(E) $n(n+5)(n-6)$

Arithmetic Properties of numbers

The easiest and quickest way to do this problem is to choose an integer greater than 6, such as 7, and eliminate answer choices in which the value of the expression is not divisible by 3:

A $7(7+1)(7-4)=(7)(8)(3)$, which is divisible by 3, so A cannot be eliminated.

B $7(7+2)(7-1)=(7)(9)(6)$, which is divisible by 3, so B cannot be eliminated.

C $7(7+3)(7-5)=(7)(10)(2)$, which is not divisible by 3, so C can be eliminated.

D $7(7+4)(7-2)=(7)(11)(5)$, which is not divisible by 3, so D can be eliminated.

E $7(7+5)(7-6)=(7)(12)(1)$, which is divisible by 3, so E cannot be eliminated.

Choose another integer greater than 6, such as 8, and test the remaining answer choices:

A $8(8+1)(8-4)=(8)(9)(4)$, which is divisible by 3, so A cannot be eliminated.

B $8(8+2)(8-1)=(8)(10)(7)$, which is not divisible by 3, so B can be eliminated.

E $8(8+5)(8-6)=(8)(13)(2)$, which is not divisible by 3, so E can be eliminated.

Thus, A is the only answer choice that has not been eliminated.

For the more mathematically inclined, if n is divisible by 3, then the expression in each answer choice is divisible by 3. Assume, then, that n is not divisible by 3. If the remainder when n is divided by 3 is 1, then $n = 3q + 1$ for some integer q. All of the expressions $n - 4$, $n - 1$, $n + 2$, and $n + 5$ are divisible by 3 [i.e., $n - 4 = 3q - 3 = 3(q - 1)$, $n - 1 = 3q$, $n + 2 = 3q + 3 = 3(q + 1)$, $n + 5 = 3q + 6 = 3(q + 2)$], and none of the expressions $n - 6$, $n - 5$, $n - 2$, $n + 1$, $n + 3$, and $n + 4$ is divisible by 3. Therefore, if the remainder when n is divided by 3 is 1, only the expressions in answer choices A, B, and E are divisible by 3. On the other hand, if the remainder when n is divided by 3 is 2, then $n = 3q + 2$ for some integer q. All of the expressions $n - 5$, $n - 2$, $n + 1$, and $n + 4$ are divisible by 3 [i.e., $n - 5 = 3q - 3 = 3(q - 1)$, $n - 2 = 3q$, $n + 1 = 3q + 3 = 3(q + 1)$, $n + 4 = 3q + 6 = 3(q + 2)$], and none of the expressions $n - 6$, $n - 4$, $n - 1$, $n + 2$, $n + 3$, and $n + 5$ is divisible by 3. Therefore, if the remainder when n is divided by 3 is 2, only the expressions in answer choices A, C, and D are divisible by 3. Only the expression in answer choice A is divisible by 3 regardless of whether n is divisible by 3, has a remainder of 1 when divided by 3, or has a remainder of 2 when divided by 3.

The correct answer is A.

Age Category (in years)	Number of Employees
Less than 20	29
20–29	58
30–39	36
40–49	21
50–59	10
60–69	5
70 and over	2

137. The table above gives the age categories of the 161 employees at Company X and the number of employees in each category. According to the table, if m is the median age, in years, of the employees at Company X, then m must satisfy which of the following?

(A) $20 \le m \le 29$
(B) $25 \le m \le 34$
(C) $30 \le m \le 39$
(D) $35 \le m \le 44$
(E) $40 \le m \le 49$

Arithmetic Statistics

The median of 161 ages is the 81st age when the ages are listed in order. Since 29 of the ages are less than 20, the median age must be greater than or equal to 20. Since 58 of the ages are between 20 and 29, a total of $29 + 58 = 87$ of the ages are less than or equal to 29, and thus the median age is less than or equal to 29. Therefore, the median age is greater than or equal to 20 and less than or equal to 29.

The correct answer is A.

138. If x and y are positive numbers such that $x + y = 1$, which of the following could be the value of $100x + 200y$?

I. 80
II. 140
III. 199

(A) II only
(B) III only
(C) I and II
(D) I and III
(E) II and III

Algebra Simultaneous equations; Inequalities

Since $x + y = 1$, then $y = 1 - x$ and $100x + 200y$ can be expressed as $100x + 200(1 - x) = 200 - 100x$. Test each value.

I. If $200 - 100x = 80$, then $x = \frac{200-80}{100} = 1.2$ and $y = 1 - 1.2 = -0.2$.
Since y must be positive, 80 cannot be a value of $100x + 200y$.

II. If $200 - 100x = 140$, then $x = \frac{200-140}{100} = 0.6$ and $y = 1 - 0.6 = 0.4$, so 140 can be a value of $100x + 200y$.

III. If $200 - 100x = 199$, then $x = \frac{200-199}{100} = 0.01$ and $y = 1 - 0.01 = 0.99$, so 199 can be a value of $100x + 200y$.

The correct answer is E.

139. If X is the hundredths digit in the decimal $0.1X$ and if Y is the thousandths digit in the decimal $0.02Y$, where X and Y are nonzero digits, which of the following is closest to the greatest possible value of $\frac{0.1X}{0.02Y}$?

(A) 4
(B) 5
(C) 6
(D) 9
(E) 10

Arithmetic Operations with decimals; Place value

The greatest possible value of $\frac{0.1X}{0.02Y}$ will occur when $0.1X$ has the greatest possible value and $0.02Y$ has the least possible value. Since X and Y are nonzero digits, this means than X must be 9 and Y must be 1. The greatest possible value of $\frac{0.1X}{0.02Y}$ is then $\frac{0.19}{0.021} \approx 9.05$, which is closest to 9.

The correct answer is D.

140. Clarissa will create her summer reading list by randomly choosing 4 books from the 10 books approved for summer reading. She will list the books in the order in which they are chosen. How many different lists are possible?

(A) 6
(B) 40
(C) 210
(D) 5,040
(E) 151,200

Arithmetic Elementary combinatorics

Any of the 10 books can be listed first. Any of the 9 books remaining after the first book is listed can be listed second. Any of the 8 books remaining after the first and second books are listed can be listed third. Any of the 7 books remaining after the first, second, and third books are listed can be listed fourth. By the multiplication principle, there are $(10)(9)(8)(7) = 5{,}040$ different lists possible.

The correct answer is D.

141. If n is a positive integer and the product of all the integers from 1 to n, inclusive, is divisible by 990, what is the least possible value of n?

(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

Arithmetic Properties of numbers

For convenience, let N represent the product of all integers from 1 through n. Then, since N is divisible by 990, every prime factor of 990 must also be a factor of N. The prime factorization of 990 is $2 \times 3^2 \times 5 \times 11$, and therefore, 11 must be a factor of N. Then, the least possible value of N with factors of $2, 5, 3^2$, and 11 is $1 \times 2 \times 3 \times \cdots \times 11$, and the least possible value of n is 11.

The correct answer is D.

142. The probability that event M will not occur is 0.8 and the probability that event R will not occur is 0.6. If events M and R cannot both occur, which of the following is the probability that either event M or event R will occur?

(A) $\frac{1}{5}$
(B) $\frac{2}{5}$
(C) $\frac{3}{5}$
(D) $\frac{4}{5}$
(E) $\frac{12}{25}$

Arithmetic Probability

Let $P(M)$ be the probability that event M will occur, let $P(R)$ be the probability that event R will occur, and let $P(M \text{ and } R)$ be the probability that events M and R both occur. Then the probability that either event M or event R will occur is $P(M) + P(R) - P(M \text{ and } R)$. From the given information, it follows that $P(M) = 1.0 - 0.8 = 0.2$, $P(R) = 1.0 - 0.6 = 0.4$, and $P(M \text{ and } R) = 0$. Therefore, the probability that either event M or event R will occur is $0.2 + 0.4 - 0 = 0.6 = \frac{3}{5}$.

The correct answer is C.

143. The total cost for Company X to produce a batch of tools is \$10,000 plus \$3 per tool. Each tool sells for \$8. The gross profit earned from producing and selling these tools is the total income from sales minus the total production cost. If a batch of 20,000 tools is produced and sold, then Company X's gross profit per tool is

(A) \$3.00
(B) \$3.75
(C) \$4.50
(D) \$5.00
(E) \$5.50

Arithmetic Applied problems

The total cost to produce 20,000 tools is $\$10{,}000 + \$3(20{,}000) = \$70{,}000$. The revenue resulting from the sale of 20,000 tools is $\$8(20{,}000) = \$160{,}000$. The gross profit is

$\$160{,}000 - \$70{,}000 = \$90{,}000$, and the gross profit per tool is $\frac{\$90{,}000}{20{,}000} = \$4.50$.

The correct answer is C.

144. If Q is an odd number and the median of Q consecutive integers is 120, what is the largest of these integers?

(A) $\frac{Q-1}{2} + 120$

(B) $\frac{Q}{2} + 119$

(C) $\frac{Q}{2} + 120$

(D) $\frac{Q+119}{2}$

(E) $\frac{Q+120}{2}$

Arithmetic Statistics

For an odd number of data values, the median is the middle number. Thus, 120 is the middle number, and so half of the $Q-1$ remaining values are at most 120 and the other half of the $Q-1$ remaining values are at least 120. In particular, $\frac{Q-1}{2}$ data values lie to the right of 120 when the data values are listed in increasing order from left to right, and so the largest data value is $120 + \frac{Q-1}{2}$. Alternatively, it is evident that (B), (C), or (E) cannot be correct since these expressions do not have an integer value when Q is odd. For the list consisting of the single number 120 (i.e., if $Q = 1$), (D) fails because $\frac{Q+119}{2} = \frac{1+119}{2} = 60 \neq 120$ and (A) does not fail because $\frac{Q-1}{2} + 120 = \frac{1-1}{2} + 120 = 120$.

The correct answer is A.

145. A ladder of a fire truck is elevated to an angle of 60° and extended to a length of 70 feet. If the base of the ladder is 7 feet above the ground, how many feet above the ground does the ladder reach?

(A) 35

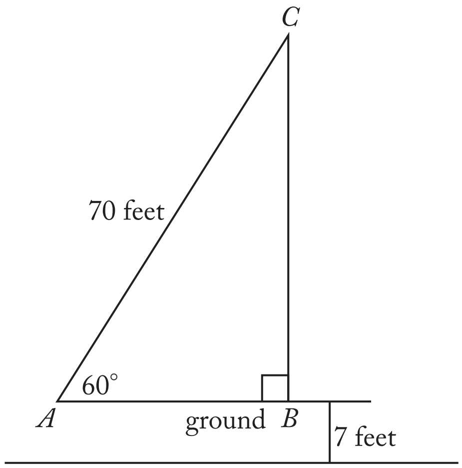
(B) 42

(C) $35\sqrt{3}$

(D) $7 + 35\sqrt{3}$

(E) $7 + 42\sqrt{3}$

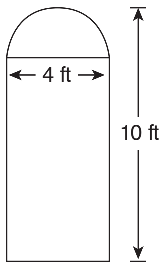
Geometry Triangles



Given the figure above, determine BC. Then add 7 to determine how far above the ground the ladder reaches.

Triangle ΔABC is a 30°–60°–90° triangle with hypotenuse $\overline{AC}$ of length 70 feet. Since the lengths of the sides of a 30°–60°–90° triangle are in the ratio $1{:}2{:}\sqrt{3}$, $\frac{AB}{AC} = \frac{1}{2}$ and so $AB = \frac{1}{2}AC = 35$, and $\frac{AB}{BC} = \frac{1}{\sqrt{3}}$ and so $BC = AB\sqrt{3} = 35\sqrt{3}$. Therefore, the ladder reaches $7 + 35\sqrt{3}$ feet above the ground.

The correct answer is D.



146. The window in the figure above consists of a rectangle and a semicircle with dimensions as shown. What is the area, in square feet, of the window?

(A) $40 + 8\pi$

(B) $40 + 2\pi$

(C) $32 + 8\pi$

(D) $32 + 4\pi$

(E) $32 + 2\pi$

Geometry Area

The semicircle has a radius of 2 ft, and thus its area is $\frac{1}{2}\pi(2^2) = 2\pi\ \text{ft}^2$. The rectangle has dimensions 4 ft by 8 ft, where $8 = 10 - 2$ is the full height of the window minus the radius of the semicircle, and thus has area $(4)(8) = 32\ \text{ft}^2$. Therefore, in square feet, the area of the window is $32 + 2\pi$.

The correct answer is E.

147. If there are fewer than 8 zeros between the decimal point and the first nonzero digit in the decimal expansion of $\left(\frac{t}{1{,}000}\right)^4$, which of the following numbers could be the value of t?

I. 3
II. 5
III. 9

(A) None
(B) I only
(C) II only
(D) III only
(E) II and III

Arithmetic Properties of numbers; Decimals

The decimal expansion of $\left(\frac{1}{1{,}000}\right)^4 = 10^{-12}$ has 11 zeros to the right of the decimal point followed by a single digit that is 1. Therefore, in the decimal expansion of $\left(\frac{t}{1{,}000}\right)^4 = t^4 \times 10^{-12}$, the number of zeros between the decimal point and the first nonzero digit is equal to 12 minus the number of digits in the integer t^4. Since this number of zeros is fewer than 8, it follows that the number of digits in the integer t^4 is greater than 4. The integer t cannot be 3, because the number of digits in $3^4 = 81$ is not greater than 4. The integer t cannot be 5, because the number of digits in $5^4 = 625$ is not greater than 4. The integer t cannot be 9, because the number of digits in $9^4 = 6{,}561$ is not greater than 4. Therefore, none of 3, 5, or 9 could be the value of t.

The correct answer is A.

148. A three-digit code for certain locks uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 according to the following constraints. The first digit cannot be 0 or 1, the second digit must be 0 or 1, and the second and third digits cannot both be 0 in the same code. How many different codes are possible?

(A) 144
(B) 152
(C) 160
(D) 168
(E) 176

Arithmetic Elementary combinatorics

Since the first digit cannot be 0 or 1, there are 8 digits possible for the first digit. Since the second digit must be 0 or 1, there are 2 digits possible for the second digit. If there were no other restrictions, all 10 digits would be possible for the third digit, making the total number of possible codes $8 \times 2 \times 10 = 160$. But, the additional restriction that the second and third digits cannot both be 0 in the same code eliminates the 8 codes 2-0-0, 3-0-0, 4-0-0, 5-0-0, 6-0-0, 7-0-0, 8-0-0, and 9-0-0. Therefore, there are $160 - 8 = 152$ possible codes.

The correct answer is B.

149. Jackie has two solutions that are 2 percent sulfuric acid and 12 percent sulfuric acid by volume, respectively. If these solutions are mixed in appropriate quantities to produce 60 liters of a solution that is 5 percent sulfuric acid, approximately how many liters of the 2 percent solution will be required?

(A) 18
(B) 20
(C) 24
(D) 36
(E) 42

Algebra Simultaneous equations

Let x represent the quantity of the 2% sulfuric acid solution in the mixture, from which it follows that the 2% sulfuric acid solution contributes $0.02x$ liters of sulfuric acid to the mixture. Let y represent the quantity of the 12% sulfuric acid solution in the mixture, from which it follows that the 12% sulfuric acid solution contributes $0.12y$ liters of sulfuric acid to the mixture. Since there are 60 liters of the mixture, $x + y = 60$. The quantity of sulfuric acid in the mixture, which is 5% sulfuric acid, is then $(0.05)(60) = 3$ liters. Therefore, $0.02x + 0.12y = 3$. Substituting $60 - x$ for y gives $0.02x + 0.12(60 - x) = 3$. Then,

$0.02x+0.12(60-x)=3$	given
$0.02x+7.2-0.12x=3$	use distributive property
$7.2-0.1x=3$	combine like terms
$-0.1x=-4.2$	subtract 7.2 from both sides
$x=42$	divide both sides by -0.1

The correct answer is E.

150. If Jake loses 8 pounds, he will weigh twice as much as his sister. Together they now weigh 278 pounds. What is Jake's present weight, in pounds?

(A) 131
(B) 135
(C) 139
(D) 147
(E) 188

Algebra Systems of equations

Let J represent Jake's weight and S represent his sister's weight. Then $J - 8 = 2S$ and $J + S = 278$. Solve the second equation for S and get $S = 278 - J$. Substituting the expression for S into the first equation gives

$$J-8=2(278-J)$$
$$J-8=556-2J$$
$$J+2J=556+8$$
$$3J=564$$
$$J=188$$

The correct answer is E.

151. For each student in a certain class, a teacher adjusted the student's test score using the formula $y = 0.8x + 20$, where x is the student's original test score and y is the student's adjusted test score. If the standard deviation of the original test scores of the students in the class was 20, what was the standard deviation of the adjusted test scores of the students in the class?

(A) 12
(B) 16
(C) 28
(D) 36
(E) 40

Arithmetic Statistics

Let n be the number of students in the class, let $\mu = \frac{\sum x}{n}$ be the mean of the students' unadjusted scores. It follows that the standard deviation of the unadjusted scores is $20 = \sqrt{\frac{\sum(x-\mu)^2}{n}}$.

To find the standard deviation of the adjusted scores, first find their mean: $\frac{\sum(0.8x+20)}{n} = \frac{0.8\sum x}{n} + \frac{20n}{n} = 0.8\mu + 20$. Then, subtract the adjusted mean from each adjusted score: $(0.8x + 20) - (0.8\mu + 20) = 0.8(x - \mu)$. Next, square each difference: $(0.8(x - \mu))^2 = 0.64(x - \mu)^2$. Next, find the average of the squared differences: $\frac{\sum 0.64(x-\mu)^2}{n} = \frac{0.64\sum(x-\mu)^2}{n}$. Finally, take the nonnegative square root:
$$\sqrt{\frac{0.64\sum(x-\mu)^2}{n}} = 0.8\sqrt{\frac{\sum(x-\mu)^2}{n}} = 0.8(20) = 16.$$

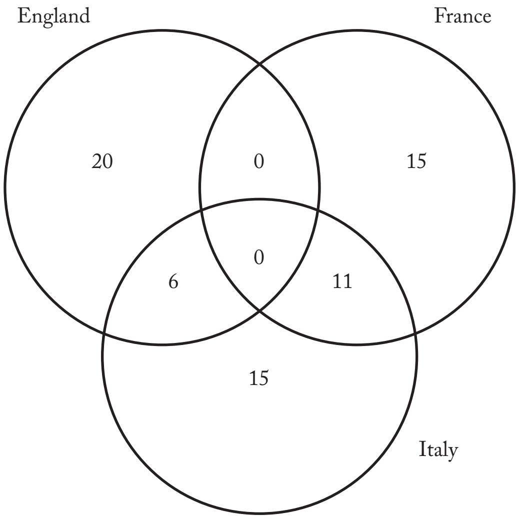
The correct answer is B.

152. Last year 26 members of a certain club traveled to England, 26 members traveled to France, and 32 members traveled to Italy. Last year no members of the club traveled to both England and France, 6 members traveled to both England and Italy, and 11 members traveled to both France and Italy. How many members of the club traveled to at least one of these three countries last year?

(A) 52
(B) 67
(C) 71
(D) 73
(E) 79

Arithmetic Applied problems

The numbers in the following diagram represent the numbers of members of the club who traveled to the indicated countries, and these numbers can be determined as follows. Since no members traveled to both England and France, both regions that form the overlap of England and France are labeled with 0. It follows that none of the 6 members who traveled to both England and Italy traveled to France, and so the region corresponding to England and Italy only is labeled with 6. It also follows that none of the 11 members who traveled to both France and Italy traveled to England, and so the region corresponding to France and Italy only is labeled with 11. At this point it can be determined that $26 - 6 = 20$ members traveled to England only, $26 - 11 = 15$ members traveled to France only, and $32 - (6 + 11) = 15$ members traveled to Italy only.



From the diagram it follows that $20 + 15 + 6 + 11 + 15 = 67$ members traveled to at least one of these three countries.

The correct answer is B.

153. A store reported total sales of \$385 million for February of this year. If the total sales for the same month last year was \$320 million, approximately what was the percent increase in sales?

(A) 2%
(B) 17%
(C) 20%
(D) 65%
(E) 83%

Arithmetic Percents

The percent increase in sales from last year to this year is 100 times the quotient of the difference in sales for the two years divided by the sales last year. Thus, the percent increase is

$$\begin{aligned}\frac{385-320}{320}\times 100 &= \frac{65}{320}\times 100\\ &= \frac{13}{64}\times 100\\ &\approx \frac{13}{65}\times 100\\ &= \frac{1}{5}\times 100\\ &= 20\%\end{aligned}$$

The correct answer is C.

154. When positive integer x is divided by positive integer y, the remainder is 9. If $\frac{x}{y} = 96.12$, what is the value of y?

(A) 96
(B) 75
(C) 48
(D) 25
(E) 12

Arithmetic Properties of numbers

The remainder is 9 when x is divided by y, so $x = yq + 9$ for some positive integer q. Dividing both sides by y gives $\frac{x}{y} = q + \frac{9}{y}$. But, $\frac{x}{y} = 96.12 = 96 + 0.12$. Equating the two expressions for $\frac{x}{y}$ gives $q + \frac{9}{y} = 96 + 0.12$. Thus, $q = 96$ and $\frac{9}{y} = 0.12$.

$$9 = 0.12y$$
$$y = \frac{9}{0.12}$$
$$y = 75$$

The correct answer is B.

155. If $x(2x + 1) = 0$ and $\left(x + \frac{1}{2}\right)(2x - 3) = 0$, then $x =$

(A) -3

(B) $-\frac{1}{2}$

(C) 0

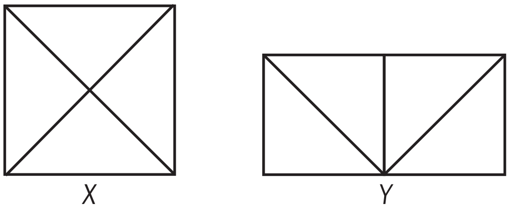
(D) $\frac{1}{2}$

(E) $\frac{3}{2}$

Algebra Second-degree equations; Simultaneous equations

Setting each factor equal to 0, it can be seen that the solution set to the first equation is $\left\{0, -\frac{1}{2}\right\}$ and the solution set to the second equation is $\left\{-\frac{1}{2}, \frac{3}{2}\right\}$. Therefore, $-\frac{1}{2}$ is the solution to both equations.

The correct answer is B.



156. Figures X and Y above show how eight identical triangular pieces of cardboard were used to form a square and a rectangle, respectively. What is the ratio of the perimeter of X to the perimeter of Y?

(A) 2:3

(B) $\sqrt{2}$:2

(C) $2\sqrt{2}$:3

(D) 1:1

(E) $\sqrt{2}$:1

Geometry Perimeter

Because Figure X is a square and the diagonals of a square are the same length, are perpendicular, and bisect each other, it follows that each triangular piece is a 45°–45°–90° triangle. Thus, the length of each side of the square is $a\sqrt{2}$, and the perimeter is $4a\sqrt{2}$. The perimeter of the rectangle is $2(a + 2a) = 6a$. It follows that the ratio of the perimeter of the square to the perimeter of the rectangle is $\frac{4a\sqrt{2}}{6a} = \frac{2\sqrt{2}}{3}$, or $2\sqrt{2} : 3$.

The correct answer is C.

157. A certain experimental mathematics program was tried out in 2 classes in each of 32 elementary schools and involved 37 teachers. Each of the classes had 1 teacher and each of the teachers taught at least 1, but not more than 3, of the classes. If the number of teachers who taught 3 classes is n, then the least and greatest possible values of n, respectively, are

(A) 0 and 13

(B) 0 and 14

(C) 1 and 10

(D) 1 and 9

(E) 2 and 8

Algebra Simultaneous equations; Inequalities

It is given that $2(32) = 64$ classes are taught by 37 teachers. Let k, m, and n be the number of teachers who taught, respectively, 1, 2, and 3 of the classes. Then $k + m + n = 37$ and $k + 2m + 3n = 64$. Subtracting these two equations gives $m + 2n = 64 - 37 = 27$, or $2n = 27 - m$, and therefore $2n \leq 27$. Because n is an integer, it follows that $n \leq 13$ and B cannot be the answer. Since $n = 0$ is possible, which can be seen by using $m = 27$ and $k = 10$ (obtained by solving $2n = 27 - m$ with $n = 0$, then by solving $k + m + n = 37$ with $n = 0$ and $m = 27$), the answer must be A.

It is not necessary to ensure that $n = 13$ is possible to answer the question. However, it is not difficult to see that $k = 23$, $m = 1$, and $n = 13$ satisfy the given conditions.

The correct answer is A.

158. For the positive numbers, n, $n + 1$, $n + 2$, $n + 4$, and $n + 8$, the mean is how much greater than the median?

(A) 0
(B) 1
(C) $n + 1$
(D) $n + 2$
(E) $n + 3$

Algebra Statistics

Since the five positive numbers n, $n + 1$, $n + 2$, $n + 4$, and $n + 8$ are in ascending order, the median is the third number, which is $n + 2$. The mean of the five numbers is

$$\frac{n+(n+1)+(n+2)+(n+4)+(n+8)}{5}$$
$$=\frac{5n+15}{5}$$
$$=n+3$$

Since $(n + 3) - (n + 2) = 1$, the mean is 1 greater than the median.

The correct answer is B.

159. The interior of a rectangular carton is designed by a certain manufacturer to have a volume of x cubic feet and a ratio of length to width to height of 3:2:2. In terms of x, which of the following equals the height of the carton, in feet?

(A) $\sqrt[3]{x}$
(B) $\sqrt[3]{\frac{2x}{3}}$
(C) $\sqrt[3]{\frac{3x}{2}}$
(D) $\frac{2}{3}\sqrt[3]{x}$
(E) $\frac{3}{2}\sqrt[3]{x}$

Geometry; Arithmetic Volume; Ratio and proportion

Letting c represent the constant of proportionality, the length, width, and height, in feet, of the carton can be expressed as $3c$, $2c$, and $2c$, respectively. The volume of the carton is then $(3c)(2c)(2c) = 12c^3$ cubic feet. But, the volume is x cubic feet, and so $12c^3 = x$. Then, $c^3 = \frac{x}{12}$ and $c = \sqrt[3]{\frac{x}{12}}$. The height of the carton is $2c$ and in terms of x, $2c = 2\sqrt[3]{\frac{x}{12}}$. Since $2 = \sqrt[3]{8}$ and $\left(\sqrt[3]{a}\right)\left(\sqrt[3]{b}\right) = \sqrt[3]{ab}$, the height of the carton can be expressed as $2\sqrt[3]{\frac{x}{12}} = \sqrt[3]{8\left(\frac{x}{12}\right)} = \sqrt[3]{\frac{2x}{3}}$.

The correct answer is B.

160. The present ratio of students to teachers at a certain school is 30 to 1. If the student enrollment were to increase by 50 students and the number of teachers were to increase by 5, the ratio of students to teachers would then be 25 to 1. What is the present number of teachers?

(A) 5
(B) 8
(C) 10
(D) 12
(E) 15

Algebra Applied problems

Let s be the present number of students, and let t be the present number of teachers. According to the problem, the following two equations apply:

$$\frac{30}{1} = \frac{s}{t} \quad \text{Current student to teacher ratio}$$

$$\frac{s+50}{t+5} = \frac{25}{1} \quad \text{Future student to teacher ratio}$$

Solving the first equation for s gives $s = 30t$. Substitute this value of s into the second equation, and solve for t.

$$\frac{30t+50}{t+5} = \frac{25}{1}$$

$$30t + 50 = 25t + 125 \quad \text{multiply both sides by } t + 5$$

$$5t = 75 \quad \text{simplify by subtraction}$$

$$t = 15$$

The correct answer is E.

161. What is the smallest integer n for which $25^n > 5^{12}$?

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

Arithmetic Operations with rational numbers

Because $5^2 = 25$, a common base is 5. Rewrite the left side with 5 as a base: $25^n = (5^2)^n = 5^{2n}$. It follows that the desired integer is the least integer n for which $5^{2n} > 5^{12}$. This will be the least integer n for which $2n > 12$, or the least integer n for which $n > 6$, which is 7.

The correct answer is B.

162. Sixty percent of the members of a study group are women, and 45 percent of those women are lawyers. If one member of the study group is to be selected at random, what is the probability that the member selected is a woman lawyer?

(A) 0.10
(B) 0.15
(C) 0.27
(D) 0.33
(E) 0.45

Arithmetic Probability

For simplicity, suppose there are 100 members in the study group. Since 60 percent of the members are women, there are 60 women in the group. Also, 45 percent of the women are lawyers so there are $0.45(60) = 27$ women lawyers in the study group. Therefore the probability of selecting a woman lawyer is $\frac{27}{100} = 0.27$.

The correct answer is C.

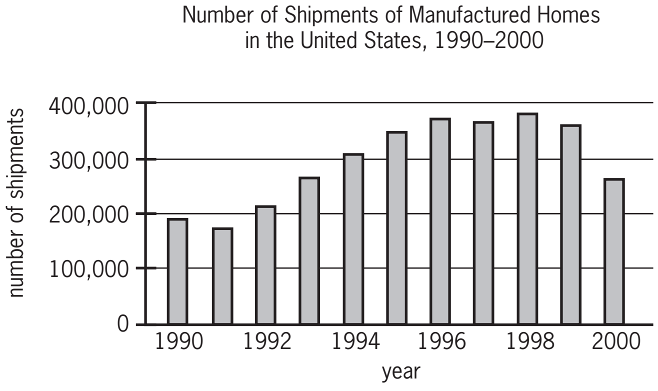
163. Each year for 4 years, a farmer increased the number of trees in a certain orchard by $\frac{1}{4}$ of the number of trees in the orchard the preceding year. If all of the trees thrived and there were 6,250 trees in the orchard at the end of the 4-year period, how many trees were in the orchard at the beginning of the 4-year period?

(A) 1,250
(B) 1,563
(C) 2,250
(D) 2,560
(E) 2,752

Arithmetic Operations on rational numbers

Increasing the number of trees each year by $\frac{1}{4}$ of the number of trees in the orchard the preceding year is equivalent to making the number of trees increase 25% per year, compounded yearly. If there were n trees at the beginning of the 4-year period, then there will be $1.25n$ trees at the end of the first year, $1.25(1.25n) = (1.25)^2n$ trees at the end of the second year, $1.25[(1.25)^2n] = (1.25)^3n$ trees at the end of the third year, and $1.25[(1.25)^3n] = (1.25)^4n$ trees at the end of the fourth year. Hence, $6{,}250 = (1.25)^4n$ and $n = \frac{6{,}250}{(1.25)^4}$. The arithmetic can be greatly simplified by rewriting $(1.25)^4$ as $\left(\frac{5}{4}\right)^4 = \frac{5^4}{4^4}$ and 6,250 as $(625)(10) = (5^4)(10)$. Then $\frac{6{,}250}{(1.25)^4} = (5^4)(10)\left(\frac{4^4}{5^4}\right) = (10)(4^4) = 2{,}560$.

The correct answer is D.



164. According to the chart shown, which of the following is closest to the median annual number of shipments of manufactured homes in the United States for the years from 1990 to 2000, inclusive?

(A) 250,000
(B) 280,000
(C) 310,000
(D) 325,000
(E) 340,000

Arithmetic Interpretation of graphs and tables; Statistics

From the chart, the approximate numbers of shipments are as follows:

Year	Number of shipments
1990	190,000
1991	180,000
1992	210,000
1993	270,000
1994	310,000
1995	350,000
1996	380,000
1997	370,000
1998	390,000
1999	360,000
2000	270,000

Since there are 11 entries in the table and 11 is an odd number, the median of the numbers of shipments is the 6th entry when the numbers of shipments are arranged in order from least to greatest. In order, from least to greatest, the first 6 entries are:

Number of shipments
180,000
190,000
210,000
270,000
270,000
310,000

The 6th entry is 310,000.

The correct answer is C.

165. For the positive integers a, b, and k, $a^k \| b$ means that a^k is a divisor of b, but a^{k+1} is not a divisor of b. If k is a positive integer and $2^k \| 72$, then k is equal to

(A) 2
(B) 3
(C) 4
(D) 8
(E) 18

Arithmetic Property of numbers

Since $72 = (2^3)(3^2)$, it follows that 2^3 is a divisor of 72 and 2^4 is not a divisor of 72. Therefore, $2^3 \| 72$, and hence $k = 3$.

The correct answer is B.

166. A certain characteristic in a large population has a distribution that is symmetric about the mean m. If 68 percent of the distribution lies within one standard deviation d of the mean, what percent of the distribution is less than $m + d$?

(A) 16%
(B) 32%
(C) 48%
(D) 84%
(E) 92%

Arithmetic Statistics

Since 68% lies between $m - d$ and $m + d$, a total of $(100 - 68)\% = 32\%$ lies to the left of $m - d$ and to the right of $m + d$. Because the distribution is symmetric about m, half of the 32% lies to the right of $m + d$. Therefore, 16% lies to the right of $m + d$, and hence $(100 - 16)\% = 84\%$ lies to the left of $m + d$.

The correct answer is D.

167. Four extra-large sandwiches of exactly the same size were ordered for m students, where $m > 4$. Three of the sandwiches were evenly divided among the students. Since 4 students did not want any of the fourth sandwich, it was evenly divided among the remaining students. If Carol ate one piece from each of the four sandwiches, the amount of sandwich that she ate would be what fraction of a whole extra-large sandwich?

(A) $\frac{m+4}{m(m-4)}$

(B) $\frac{2m-4}{m(m-4)}$

(C) $\frac{4m-4}{m(m-4)}$

(D) $\frac{4m-8}{m(m-4)}$

(E) $\frac{4m-12}{m(m-4)}$

Algebra Applied problems

Since each of 3 of the sandwiches was evenly divided among m students, each piece was $\frac{1}{m}$ of a sandwich. Since the fourth sandwich was evenly divided among $m - 4$ students, each piece was $\frac{1}{m-4}$ of the fourth sandwich. Carol ate 1 piece from each of the four sandwiches, so she ate a total of

$$(3)\frac{1}{m}+\frac{1}{m-4}=\frac{3(m-4)+m}{m(m-4)}=\frac{4m-12}{m(m-4)}$$

The correct answer is E.

168. Which of the following equations has $1 + \sqrt{2}$ as one of its roots?

(A) $x^2 + 2x - 1 = 0$

(B) $x^2 - 2x + 1 = 0$

(C) $x^2 + 2x + 1 = 0$

(D) $x^2 - 2x - 1 = 0$

(E) $x^2 - x - 1 = 0$

Algebra Second-degree equations

This problem can be solved by working backwards to construct a quadratic equation with $1 + \sqrt{2}$ as a root that does not involve radicals.

$x = 1 + \sqrt{2}$	set x to the desired value
$x - 1 = \sqrt{2}$	subtract 1 from both sides
$(x - 1)^2 = 2$	square both sides
$x^2 - 2x + 1 = 2$	expand the left side
$x^2 - 2x - 1 = 0$	subtract 2 from both sides

The correct answer is D.

169. In Country C, the unemployment rate among construction workers dropped from 16 percent on September 1, 1992, to 9 percent on September 1, 1996. If the number of construction workers was 20 percent greater on September 1, 1996, than on September 1, 1992, what was the approximate percent change in the number of unemployed construction workers over this period?

(A) 50% decrease

(B) 30% decrease

(C) 15% decrease

(D) 30% increase

(E) 55% increase

Arithmetic Percents

Let U_1 and U_2 be the numbers of unemployed construction workers on September 1, 1992, and September 1, 1996, respectively, and let N be the number of construction workers on September 1, 1992. Then, from the given information, $1.2N$ is the number of construction workers on September 1, 1996, $U_1 = 0.16N$, and $U_2 = 0.09(1.2N)$. Therefore, the percent change from September 1, 1992, to September 1, 1996, of unemployed construction workers is given by

$$\left(\frac{U_2 - U_1}{U_1}\times 100\right)\%$$
$$= \left(\frac{0.09(1.2N) - 0.16N}{0.16N}\times 100\right)\%$$
$$= \left(\frac{0.108 - 0.16}{0.16}\times 100\right)\%$$
$$= \left(\frac{108 - 160}{160}\times 100\right)\%$$
$$= \left(-\frac{13}{40}\times 100\right)\%$$
$$\approx \left(-\frac{13}{39}\times 100\right)\%$$
$$\approx \left(-\frac{1}{3}\times 100\right)\%$$
$$\approx -30\%$$

The correct answer is B.

170. In a box of 12 pens, a total of 3 are defective. If a customer buys 2 pens selected at random from the box, what is the probability that neither pen will be defective?

(A) $\frac{1}{6}$

(B) $\frac{2}{9}$

(C) $\frac{6}{11}$

(D) $\frac{9}{16}$

(E) $\frac{3}{4}$

Arithmetic Probability

Let A represent the event that the first pen purchased is not defective and let B represent the event that the second pen purchased is not defective, where A and B are dependent events. Since there are 3 defective pens in the box of 12 pens, there are 9 pens in the box that are not defective. Using standard probability notation, $P(A) = \frac{9}{12}$ and $P(B$, given that A has occurred$) = \frac{8}{11}$. (Event A has occurred so there are 11 pens left in the box and 8 of them are not defective.) Then, using the multiplication rule for dependent events, P(neither pen is defective) $= P(A$ and $B) = P(A) \times P(B$, given that A has occurred$) = \frac{9}{12}\times\frac{8}{11} = \frac{6}{11}$.

Alternately, the probability of selecting 2 pens, neither of which is defective, from a box containing 12 pens, 3 of which are defective, and therefore, 9 of which are non-defective is the number of ways to select 2 non-defective pens from 9 non-defective pens over the number of ways to select 2 pens from 12 pens

$$= \frac{\binom{9}{2}}{\binom{12}{2}} = \frac{\frac{9\times 8}{2}}{\frac{12\times 11}{2}} = \frac{6}{11}.$$

The correct answer is C.

171. At a certain fruit stand, the price of each apple is 40 cents and the price of each orange is 60 cents. Mary selects a total of 10 apples and oranges from the fruit stand, and the average (arithmetic mean) price of the 10 pieces of fruit is 56 cents. How many oranges must Mary put back so that the average price of the pieces of fruit that she keeps is 52 cents?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Algebra Statistics

If Mary selected x apples, then she selected $(10 - x)$ oranges. The average price of the 10 pieces of fruit is $\frac{40x + 60(10 - x)}{10} = 56$. From this,

$$\begin{aligned}
\frac{40x+60(10-x)}{10} &= 56 && \text{given}\\
40x+60(10-x) &= 560 && \text{multiply both sides by 10}\\
40x+600-60x &= 560 && \text{distribution property}\\
600-20x &= 560 && \text{combine like terms}\\
-20x &= -40 && \text{subtract 600 from both sides}\\
x &= 2 && \text{divide both sides by } -20
\end{aligned}$$

Thus, Mary selected 2 apples and 8 oranges. Next, let y be the number of oranges Mary needs to put back, so that the average price of the $[2 + (8 - y)]$ pieces of fruit Mary keeps is 52 cents. Then,

$$\begin{aligned}
\frac{(40)(2)+(60)(8-y)}{2+(8-y)} &= 52 && \text{given}\\
\frac{80+480-60y}{10-y} &= 52 && \text{distributive property}\\
80+480-60y &= 52(10-y) && \text{multiply both sides by } 10-y\\
560-60y &= 520-52y && \text{distributive property}\\
8y &= 40 && \text{subtract } 520-60y \text{ from both sides}\\
y &= 5 && \text{divide both sides by 8}
\end{aligned}$$

Therefore, Mary must put back 5 oranges, so that the average price of the fruit she keeps (that is, the average price of 2 apples and 3 oranges) is 52 cents.

The correct answer is E.

172. A pharmaceutical company received \$3 million in royalties on the first \$20 million in sales of the generic equivalent of one of its products and then \$9 million in royalties on the next \$108 million in sales. By approximately what percent did the ratio of royalties to sales decrease from the first \$20 million in sales to the next \$108 million in sales?

(A) 8%
(B) 15%
(C) 45%
(D) 52%
(E) 56%

Arithmetic Percents

The ratio of royalties to sales for the first \$20 million in sales is $\frac{3}{20}$, and the ratio of royalties to sales for the next \$108 million in sales is $\frac{9}{108} = \frac{1}{12}$. The percent decrease in the royalties to sales ratios is 100 times the quotient of the difference in the ratios divided by the ratio of royalties to sales for the first \$20 million in sales or

$$\begin{aligned}
\frac{\frac{1}{12}-\frac{3}{20}}{\frac{3}{20}} \times 100 &= \left(\frac{1}{12}-\frac{3}{20}\right) \times \frac{20}{3} \times 100\\
&= \left(\frac{1}{12}\times\frac{20}{3} - 1\right) \times 100\\
&= \left(\frac{5}{9} - 1\right) \times 100\\
&= -\frac{4}{9} \times 100\\
&\approx -0.44 \times 100\\
&\approx 45\% \text{ decrease}
\end{aligned}$$

The correct answer is C.

Times at Which the Door Opened from 8:00 to 10:00

8:00	8:06	8:30	9:05
8:03	8:10	8:31	9:11
8:04	8:18	8:54	9:29
8:04	8:19	8:57	9:31

173. The light in a restroom operates with a 15-minute timer that is reset every time the door opens as a person goes in or out of the room. Thus, after someone enters or exits the room, the light remains on for only 15 minutes unless the door opens again and resets the timer for another 15 minutes. If the times listed above are the times at which the door opened from 8:00 to 10:00, approximately how many minutes during this two-hour period was the light off?

(A) 10
(B) 25
(C) 35
(D) 40
(E) 70

Arithmetic Operations with integers

Look for two consecutive times that are more than 15 minutes apart

8:03 – 8:00 = 3 minutes

8:04 – 8:03 = 1 minute

8:04 – 8:04 = 0 minutes

8:06 – 8:04 = 2 minutes

8:10 – 8:06 = 4 minutes

8:18 – 8:10 = 8 minutes

8:19 – 8:18 = 1 minute

8:30 – 8:19 = 11 minutes

8:31 – 8:30 = 1 minute

8:54 – 8:31 = 23 minutes, so the light is off for 23 – 15 = 8 minutes

8:57 – 8:54 = 3 minutes

9:05 – 8:57 = 8 minutes

9:11 – 9:05 = 6 minutes

9:29 – 9:11 = 18 minutes, so the light is off for 18 – 15 = 3 minutes

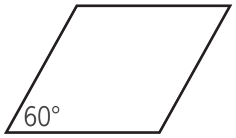
9:31 – 9:29 = 2 minutes

10:00 – 9:31 = 29 minutes, so the light is off for 29 – 15 = 14 minutes

Thus, the light is off for a total of 8 + 3 + 14 = 25 minutes during the two-hour period.

Alternatively, the light comes on at 8:00 when the door is opened and is scheduled to go off at 8:15. So, when the door is opened at 8:03, twice at 8:04, at 8:06, and 8:10, the light is still on, but by the door being opened at these times, the timer has been reset to turn the light off at 8:18, 8:19, 8:21, and 8:25, respectively. Therefore, the light is still on when the door is opened at 8:18, 8:19, 8:30, and 8:31, but the timer has been reset to turn the light off at 8:33, 8:34, 8:45, and 8:46, respectively. The light is therefore off from 8:46 until the door is opened again at 8:54, which is an interval of 8 minutes. The light is still on when the door is opened at 8:57, 9:05, and 9:11, but the timer has been reset to turn the light off at 9:12, 9:20, and 9:26, respectively. The light is off from 9:26 until the door is opened again at 9:29, which is an interval of 3 minutes. The light is still on when the door is opened again at 9:31, and the timer has been reset to turn the light off at 9:46. Since, according to the chart, the door is not opened again before 10:00, the light remains off from 9:46 until 10:00, which is an interval of 14 minutes. Thus, the light is off a total of 8 + 3 + 14 = 25 minutes during the two-hour period.

The correct answer is B.



174. The parallelogram shown has four sides of equal length. What is the ratio of the length of the shorter diagonal to the length of the longer diagonal?

(A) $\frac{1}{2}$

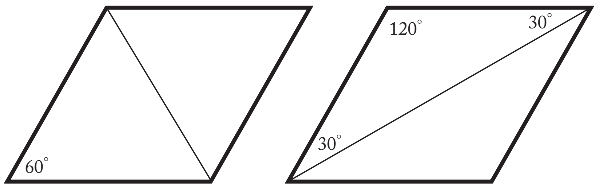
(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2\sqrt{2}}$

(D) $\frac{1}{\sqrt{3}}$

(E) $\frac{1}{2\sqrt{3}}$

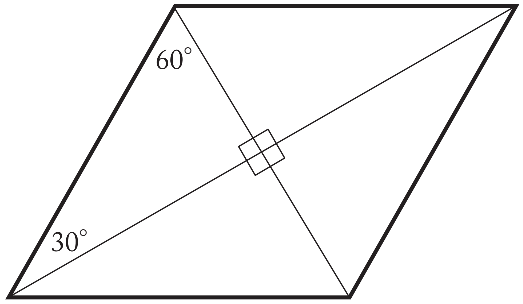
Geometry Quadrilaterals; Triangles



First, opposite angles of a parallelogram have equal measure, and the sum of the measures of adjacent angles is 180°. This means that each of the angles adjacent to the angle labeled 60° has measure 120°.

Since all four sides of the parallelogram have equal length, say x units, the shorter diagonal divides the parallelogram into two isosceles triangles. An isosceles triangle with one angle measuring 60° is equilateral, and so the shorter diagonal has length x units.

The longer diagonal divides the parallelogram into two isosceles triangles with one angle measuring 120° and each of the other angles measuring $\frac{180-120}{2} = 30°$, as shown in the figure above on the right.



Then, because the diagonals of a parallelogram are perpendicular and bisect each other, the two diagonals divide the parallelogram into four 30°-60°-90° triangles, each with hypotenuse x units long. The sides of a 30°-60°-90° triangle are in the ratio of $1:\sqrt{3}:2$ and so, if y represents the length of the side opposite the 60° angle, then $\frac{x}{2} = \frac{y}{\sqrt{3}}$ and $y = \frac{x\sqrt{3}}{2}$. But, y is half the length of the longer diagonal, so the longer diagonal has length $x\sqrt{3}$ units. Therefore, the ratio of the length of the shorter diagonal to the length of the longer diagonal is $\frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$.

The correct answer is D.

175. If p is the product of the integers from 1 to 30, inclusive, what is the greatest integer k for which 3^k is a factor of p ?

(A) 10

(B) 12

(C) 14

(D) 16

(E) 18

Arithmetic Properties of numbers

The table below shows the numbers from 1 to 30, inclusive, that have at least one factor of 3 and how many factors of 3 each has.

Multiples of 3 between 1 and 30	Number of factors of 3
3	1
$6 = 2 \times 3$	1
$9 = 3 \times 3$	2
$12 = 2 \times 2 \times 3$	1
$15 = 3 \times 5$	1
$18 = 2 \times 3 \times 3$	2
$21 = 3 \times 7$	1
$24 = 2 \times 2 \times 2 \times 3$	1
$27 = 3 \times 3 \times 3$	3
$30 = 2 \times 3 \times 5$	1

The sum of the numbers in the right column is 14. Therefore, 3^{14} is the greatest power of 3 that is a factor of the product of the first 30 positive integers.

The correct answer is C.

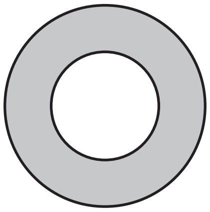
176. If $n = 3^8 - 2^8$, which of the following is NOT a factor of n?

(A) 97
(B) 65
(C) 35
(D) 13
(E) 5

Arithmetic Properties of numbers

Since $3^8 - 2^8$ is the difference of the perfect squares $(3^4)^2$ and $(2^4)^2$, then $3^8 - 2^8 = (3^4 + 2^4)(3^4 - 2^4)$. But $3^4 - 2^4$ is also the difference of the perfect squares $(3^2)^2$ and $(2^2)^2$ so $3^4 - 2^4 = (3^2 + 2^2)(3^2 - 2^2)$ and therefore $3^8 - 2^8 = (3^4 + 2^4)(3^2 + 2^2)(3^2 - 2^2)$. It follows that $3^8 - 2^8$ can be factored as $(81 + 16)(9 + 4)(9 - 4) = (97)(13)(5)$. Therefore, 7 is not a factor of $3^8 - 2^8$, and hence $35 = 5 \times 7$ is not a factor of $3^8 - 2^8$. It is easy to see that each of 97, 13, and 5 is a factor of $3^8 - 2^8$, and so is 65, since $65 = 5 \times 13$, although this additional analysis is not needed to arrive at the correct answer.

The correct answer is C.



177. In the figure shown, if the area of the shaded region is 3 times the area of the smaller circular region, then the circumference of the larger circle is how many times the circumference of the smaller circle?

(A) 4
(B) 3
(C) 2
(D) $\sqrt{3}$
(E) $\sqrt{2}$

Geometry Circles

Let R represent the radius of the larger circle and r represent the radius of the smaller circle. Then the area of the shaded region is the area of the larger circular region minus the area of the smaller circular region, or $\pi R^2 - \pi r^2$. It is given that the area of the shaded region is three times the area of the smaller circular region, and so $\pi R^2 - \pi r^2 = 3\pi r^2$. Then $R^2 - r^2 = 3r^2$, and so $R^2 = 4r^2$ and $R = 2r$. The circumference of the larger circle is $2\pi R = 2\pi(2r) = 2(2\pi r)$, which is 2 times the circumference of the smaller circle.

The correct answer is C.

178. Club X has more than 10 but fewer than 40 members. Sometimes the members sit at tables with 3 members at one table and 4 members at each of the other tables, and sometimes they sit at tables with 3 members at one table and 5 members at each of the other tables. If they sit at tables with 6 members at each table except one and fewer than 6 members at that one table, how many members will be at the table that has fewer than 6 members?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Arithmetic Properties of numbers

Let n be the number of members that Club X has. Since the members can be equally divided into groups of 4 each with 3 left over, and the members can be equally divided into groups of 5 each with 3 left over, it follows that $n - 3$ is divisible by both 4 and 5. Therefore, $n - 3$ must be a multiple of $(4)(5) = 20$. Also, because the only multiple of 20 that is greater than 10 and less than 40 is 20, it follows that $n - 3 = 20$, or $n = 23$. Finally, when these 23 members are divided into the greatest number of groups of 6 each, there will be 5 members left over, since $23 = (3)(6) + 5$.

The correct answer is E.

179. In order to complete a reading assignment on time, Terry planned to read 90 pages per day. However, she read only 75 pages per day at first, leaving 690 pages to be read during the last 6 days before the assignment was to be completed. How many days in all did Terry have to complete the assignment on time?

(A) 15
(B) 16
(C) 25
(D) 40
(E) 46

Algebra Applied problems

Let n be the number of days that Terry read at the slower rate of 75 pages per day. Then $75n$ is the number of pages Terry read at this slower rate, and $75n + 690$ is the total number of pages Terry needs to read. Also, $n + 6$ is the total number of days that Terry will spend on the reading assignment. The requirement that Terry average 90 pages per day is equivalent to $\frac{75n+690}{n+6} = 90$. Then

$$\frac{75n+690}{n+6} = 90$$
$$75n+690 = 90n+540$$
$$150 = 15n$$
$$10 = n$$

Therefore, the total number of days that Terry has to complete the assignment on time is $n + 6 = 10 + 6 = 16$.

The correct answer is B.

180. If $s > 0$ and $\sqrt{\frac{r}{s}} = s$, what is r in terms of s ?

(A) $\frac{1}{s}$
(B) $\sqrt{s}$
(C) $s\sqrt{s}$
(D) s^3
(E) $s^3 - s$

Algebra Equations

Solve the equation for r as follows:

$$\sqrt{\frac{r}{s}} = s$$
$$\frac{r}{s} = s^2 \quad \text{square both sides of the equation}$$
$$r = s^3 \quad \text{multiply both sides by } s$$

The correct answer is D.

181. If $3 < x < 100$, for how many values of x is $\frac{x}{3}$ the square of a prime number?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Nine

Arithmetic Properties of numbers

If $\frac{x}{3}$ is the square of a prime number, then possible values of $\frac{x}{3}$ are $2^2, 3^2, 5^2, 7^2, \ldots$. Therefore, possible values of x are $3 \times 2^2 = 12$, $3 \times 3^2 = 27$, $3 \times 5^2 = 75$, $3 \times 7^2 = 147$, …. Since only three of these values, namely 12, 27, and 75, are between 3 and 100, there are three values of x such that $\frac{x}{3}$ is the square of a prime number.

The correct answer is B.

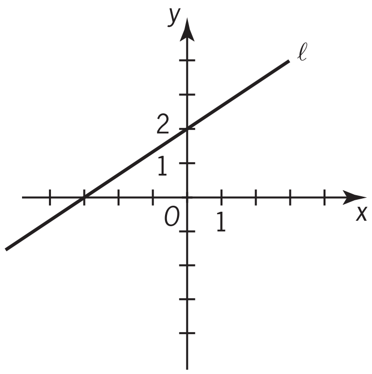
182. A researcher plans to identify each participant in a certain medical experiment with a code consisting of either a single letter or a pair of distinct letters written in alphabetical order. What is the least number of letters that can be used if there are 12 participants, and each participant is to receive a different code?

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

Arithmetic Elementary combinatorics

None of the essential aspects of the problem is affected if the letters are restricted to be the first n letters of the alphabet, for various positive integers n. With the 3 letters a, b, and c, there are 6 codes: a, b, c, ab, ac, and bc. With the 4 letters a, b, c, and d, there are 10 codes: a, b, c, d, ab, ac, ad, bc, bd, and cd. Clearly, more than 12 codes are possible with 5 or more letters, so the least number of letters that can be used is 5.

The correct answer is B.



183. The graph of which of the following equations is a straight line that is parallel to line ℓ in the figure above?

(A) $3y - 2x = 0$
(B) $3y + 2x = 0$
(C) $3y + 2x = 6$
(D) $2y - 3x = 6$
(E) $2y + 3x = -6$

Algebra Coordinate geometry

From the graph, line ℓ contains points (–3,0) and (0,2), so the slope of line ℓ is $\frac{0-2}{-3-0} = \frac{2}{3}$. Any line parallel to line ℓ has slope $\frac{2}{3}$. Rewrite each of the equations given in the answer choices in slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept, to find the equation whose graph is a line with slope $\frac{2}{3}$. For answer choice A, $3y - 2x = 0$ so $3y = 2x$ and $y = \frac{2}{3}x$. The graph of this equation is a line with slope $\frac{2}{3}$.

The correct answer is A.

184. An object thrown directly upward is at a height of h feet after t seconds, where $h = -16(t-3)^2 + 150$. At what height, in feet, is the object 2 seconds after it reaches its maximum height?

(A) 6
(B) 86
(C) 134
(D) 150
(E) 166

Algebra Applied problems

Since $(t-3)^2$ is positive when $t \neq 3$ and zero when $t = 3$, it follows that the *minimum* value of $(t-3)^2$ occurs when $t = 3$. Therefore, the *maximum* value of $-16(t-3)^2$, and also the maximum value of $-16(t-3)^2 + 150$, occurs when $t = 3$. Hence, the height 2 seconds after the maximum height is the value of h when $t = 5$, or $-16(5-3)^2 + 150 = 86$.

The correct answer is B.

185. Which of the following is equivalent to the pair of inequalities $x + 6 > 10$ and $x - 3 \leq 5$?

(A) $2 \leq x < 16$
(B) $2 \leq x < 4$
(C) $2 < x \leq 8$
(D) $4 < x \leq 8$
(E) $4 \leq x < 16$

Algebra Inequalities

Solve the inequalities separately and combine the results.

$$x + 6 > 10$$
$$x > 4$$
$$x - 3 \le 5$$
$$x \le 8$$

Since $x > 4$, then $4 < x$. Combining $4 < x$ and $x \le 8$ gives $4 < x \le 8$.

The correct answer is D.

186. David has d books, which is 3 times as many as Jeff and $\frac{1}{2}$ as many as Paula. How many books do the three of them have altogether, in terms of d?

(A) $\frac{5}{6}d$
(B) $\frac{7}{3}d$
(C) $\frac{10}{3}d$
(D) $\frac{7}{2}d$
(E) $\frac{9}{2}d$

Algebra Applied problems; Simultaneous equations

Let J be the number of books that Jeff has, and let P be the number of books Paula has. Then, the given information about David's books can be expressed as $d = 3J$ and $d = \frac{1}{2}P$. Solving these two equations for J and P gives $\frac{d}{3} = J$ and $2d = P$. Thus, $d + J + P = d + \frac{d}{3} + 2d = 3\frac{1}{3}d = \frac{10}{3}d$.

The correct answer is C.

187. There are 8 teams in a certain league and each team plays each of the other teams exactly once. If each game is played by 2 teams, what is the total number of games played?

(A) 15
(B) 16
(C) 28
(D) 56
(E) 64

Arithmetic Operations on rational numbers

Since no team needs to play itself, each team needs to play 7 other teams. In addition, each game needs to be counted only once, rather than once for each team that plays that game. Since two teams play each game, $\frac{8 \times 7}{2} = 28$ games are needed.

The correct answer is C.

188. At his regular hourly rate, Don had estimated the labor cost of a repair job as \$336 and he was paid that amount. However, the job took 4 hours longer than he had estimated and, consequently, he earned \$2 per hour less than his regular hourly rate. What was the time Don had estimated for the job, in hours?

(A) 28
(B) 24
(C) 16
(D) 14
(E) 12

Algebra Second-degree equations

Let r be Don's regular hourly rate and t be the number of hours he estimated the repair job to take. Then $rt = 336$ is Don's estimated labor cost. Since Don was paid \$336 for doing $t + 4$ hours of work at an hourly rate of $r - 2$, it also follows that $(r - 2)(t + 4) = 336$. Then

$$\begin{aligned}
(r-2)(t+4) &= 336 & & \\
rt - 2t + 4r - 8 &= 336 & & \\
-2t + 4r - 8 &= 0 & & \text{since } rt = 336 \text{ from above} \\
-2t^2 + 4rt - 8t &= 0 & & \text{multiply both sides by } t \\
-2t^2 + 4(336) - 8t &= 0 & & \text{since } rt = 336 \\
t^2 + 4t - 672 &= 0 & & \text{divide both sides by } -2 \\
(t-24)(t+28) &= 0 & & \text{factor}
\end{aligned}$$

Alternatively, from the third line above,

$$-2t+4r-8=0$$

$$-2t+4\left(\frac{336}{t}\right)-8=0 \quad \text{since } rt=336 \text{ from above gives } r=\frac{336}{t}$$

$$-2t^2+4(336)-8t=0 \quad \text{multiply both sides by } t$$

$$t^2+4t-672=0 \quad \text{divide both sides by } -2$$

$$(t-24)(t+28)=0 \quad \text{factor}$$

So, $t-24=0$, which means $t=24$, or $t+28=0$, which means $t=-28$. Since an estimated time cannot be negative, $t=24$.

The correct answer is B.

189. If $\frac{p}{q}<1$, and p and q are positive integers, which of the following must be greater than 1 ?

(A) $\sqrt{\frac{p}{q}}$

(B) $\frac{p}{q^2}$

(C) $\frac{p}{2q}$

(D) $\frac{q}{p^2}$

(E) $\frac{q}{p}$

Arithmetic Properties of numbers

Since p and q are positive integers, $0<\frac{p}{q}<1$.

A Since $\frac{p}{q}<1$, then $q>p$. Taking the square root of both sides of the inequality gives $\sqrt{q}>\sqrt{p}$. Then, $\sqrt{\frac{p}{q}}=\frac{\sqrt{p}}{\sqrt{q}}$, so here the denominator will still be larger than the numerator. CANNOT be greater than 1.

B Squaring the denominator increases the denominator, which decreases the value of the fraction. CANNOT be greater than 1.

C Multiplying the denominator by 2 increases the denominator, which decreases the value of the fraction. CANNOT be greater than 1.

D Since $\frac{p}{q}<1$, then $q>p$. When $p^2<q$, this expression will be greater than 1, but p^2 need not be less than q. For example, if $p=2$ and $q=100$, $\frac{p}{q}=\frac{2}{100}$ and $\frac{q}{p^2}=\frac{100}{2^2}=\frac{100}{4}=25>1$.
However, if $p=3$ and $q=4$, then $\frac{p}{q}=\frac{3}{4}$ and $\frac{q}{p^2}=\frac{4}{3^2}=\frac{4}{9}<1$. NEED NOT be greater than 1.

E Again, since $\frac{p}{q}<1$, then $q>p$. Thus, the reciprocal, $\frac{q}{p}$, always has a value greater than 1 because the numerator will always be a larger positive integer than the denominator. MUST be greater than 1.

The correct answer is E.

190. To mail a package, the rate is x cents for the first pound and y cents for each additional pound, where $x>y$. Two packages weighing 3 pounds and 5 pounds, respectively, can be mailed separately or combined as one package. Which method is cheaper, and how much money is saved?

(A) Combined, with a savings of $x-y$ cents

(B) Combined, with a savings of $y-x$ cents

(C) Combined, with a savings of x cents

(D) Separately, with a savings of $x-y$ cents

(E) Separately, with a savings of y cents

Algebra Applied problems

Shipping the two packages separately would cost $1x+2y$ for the 3-pound package and $1x+4y$ for the 5-pound package. Shipping them together (as a single 8-pound package) would cost $1x+7y$. By calculating the sum of the costs for shipping the two packages separately minus the cost for shipping the one combined package, it is possible to determine the difference in cost, as shown.

$((1x+2y)+(1x+4y))-(1x+7y)$	(cost for 3 lb. + cost for 5 lb.) − cost for 8 lb.
$=(2x+6y)-(1x+7y)$	combine like terms
$=2x+6y-1x-7y$	distribute the negative
$=x-y$	combine like terms

Since $x > y$, this value is positive, which means it costs more to ship two packages separately. Thus it is cheaper to mail one combined package at a cost savings of $x-y$ cents.

The correct answer is A.

191. If money is invested at r percent interest, compounded annually, the amount of the investment will double in approximately $\frac{70}{r}$ years. If Pat's parents invested \$5,000 in a long-term bond that pays 8 percent interest, compounded annually, what will be the approximate total amount of the investment 18 years later, when Pat is ready for college?

(A) \$20,000
(B) \$15,000
(C) \$12,000
(D) \$10,000
(E) \$9,000

Algebra Applied problems

Since the investment will double in $\frac{70}{r}=\frac{70}{8}=8.75\approx 9$ years, the value of the investment over 18 years can be approximated by doubling its initial value twice. Therefore, the approximate value will be $(\$5{,}000)(2)(2)=\$20{,}000$.

The correct answer is A.

192. On a recent trip, Cindy drove her car 290 miles, rounded to the nearest 10 miles, and used 12 gallons of gasoline, rounded to the nearest gallon. The actual number of miles per gallon that Cindy's car got on this trip must have been between

(A) $\frac{290}{12.5}$ and $\frac{290}{11.5}$
(B) $\frac{295}{12}$ and $\frac{285}{11.5}$
(C) $\frac{285}{12}$ and $\frac{295}{12}$
(D) $\frac{285}{12.5}$ and $\frac{295}{11.5}$
(E) $\frac{295}{12.5}$ and $\frac{285}{11.5}$

Arithmetic Estimation

The lowest number of miles per gallon can be calculated using the lowest possible miles and the highest amount of gasoline. Also, the highest number of miles per gallon can be calculated using the highest possible miles and the lowest amount of gasoline.

Since the miles are rounded to the nearest 10 miles, the number of miles is between 285 and 295. Since the gallons are rounded to the nearest gallon, the number of gallons is between 11.5 and 12.5. Therefore, the lowest number of miles per gallon is $\frac{\text{lowest miles}}{\text{highest gallons}}=\frac{285}{12.5}$ and the highest number of miles per gallon is $\frac{\text{highest miles}}{\text{lowest gallons}}=\frac{295}{11.5}$.

The correct answer is D.

193. Which of the following inequalities is an algebraic expression for the shaded part of the number line above?

(A) $|x| \le 3$
(B) $|x| \le 5$
(C) $|x-2| \le 3$
(D) $|x-1| \le 4$
(E) $|x+1| \le 4$

Algebra Inequalities

The number line above shows $-5 \le x \le 3$. To turn this into absolute value notation, as all the choices are written, the numbers need to be opposite signs of the same value.

Since the distance between −5 and 3 is 8 $(3-(-5)=8)$, that distance needs to be split in half with −4 to one side and 4 to the other. Each of these two values is 1 more than the values in the inequality above, so adding 1 to all terms in the inequality gives $-4 \le x+1 \le 4$, which is the same as $|x+1| \le 4$.

The correct answer is E.

194. In a small snack shop, the average (arithmetic mean) revenue was \$400 per day over a 10-day period. During this period, if the average daily revenue was \$360 for the first 6 days, what was the average daily revenue for the last 4 days?

(A) \$420
(B) \$440
(C) \$450
(D) \$460
(E) \$480

Arithmetic; Algebra Statistics; Applied problems

Let x be the average daily revenue for the last 4 days. Using the formula $\text{average} = \dfrac{\text{sum of values}}{\text{number of values}}$, the information regarding the average revenues for the 10-day and 6-day periods can be expressed as follows and solved for x:

$$\$400 = \frac{6(\$360)+4x}{10}$$

$\$4,000 = \$2,160 + 4x$	multiply both sides by 10
$\$1,840 = 4x$	subtract \$2,160 from both sides
$\$460 = x$	divide both sides by 4

The correct answer is D.

195. If y is the smallest positive integer such that 3,150 multiplied by y is the square of an integer, then y must be

(A) 2
(B) 5
(C) 6
(D) 7
(E) 14

Arithmetic Properties of numbers

To find the smallest positive integer y such that $3{,}150y$ is the square of an integer, first find the prime factorization of 3,150 by a method similar to the following:

$$\begin{aligned} 3{,}150 &= 10 \times 315 \\ &= (2\times 5)\times(3\times 105) \\ &= 2\times 5\times 3\times(5\times 21) \\ &= 2\times 5\times 3\times 5\times(3\times 7) \\ &= 2\times 3^2\times 5^2\times 7 \end{aligned}$$

To be a perfect square, $3{,}150y$ must have an even number of each of its prime factors. At a minimum, y must have one factor of 2 and one factor of 7 so that $3{,}150y$ has two factors of each of the primes 2, 3, 5, and 7. The smallest positive integer value of y is then $(2)(7) = 14$.

The correct answer is E.

196. If $[x]$ is the greatest integer less than or equal to x, what is the value of $[-1.6]+[3.4]+[2.7]$?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

Arithmetic Computation with integers

The greatest integer that is less than or equal to −1.6 is −2. It cannot be −1 because −1 is greater than −1.6. The greatest integer that is less than or equal to 3.4 is 3. It cannot be 4 because 4 is greater than 3.4. The greatest integer that is less than or equal to 2.7 is 2. It cannot be 3 because 3 is greater than 2.7. Therefore, $[-1.6]+[3.4]+[2.7] = -2+3+2 = 3$.

The correct answer is A.

197. In the first week of the year, Nancy saved \$1. In each of the next 51 weeks, she saved \$1 more than she had saved in the previous week. What was the total amount that Nancy saved during the 52 weeks?

(A) \$1,326
(B) \$1,352
(C) \$1,378
(D) \$2,652
(E) \$2,756

Arithmetic Operations on rational numbers

In dollars, the total amount saved is the sum of 1, (1 + 1), (1 + 1 + 1), and so on, up to and including the amount saved in the 52nd week, which was \$52. Therefore, the total amount saved in dollars was $1 + 2 + 3 + \ldots + 50 + 51 + 52$. This sum can be easily evaluated by grouping the terms as $(1 + 52) + (2 + 51) + (3 + 50) + \ldots + (26 + 27)$, which results in the number 53 added to itself 26 times. Therefore, the sum is $(26)(53) = 1{,}378$.

Alternatively, the formula for the sum of the first n positive integers is $\frac{n(n+1)}{2}$. Therefore, the sum of the first 52 positive integers is $\frac{52(53)}{2} = 26(53) = 1{,}378$.

The correct answer is C.

198. In a certain sequence, the term x_n is given by the formula $x_n = 2x_{n-1} - \frac{1}{2}(x_{n-2})$ for all $n \geq 2$. If $x_0 = 3$ and $x_1 = 2$, what is the value of x_3 ?

(A) 2.5
(B) 3.125
(C) 4
(D) 5
(E) 6.75

Algebra Simplifying algebraic expressions

Given the formula $x_n = 2x_{n-1} - \frac{1}{2}(x_{n-2})$ with $x_0 = 3$ and $x_1 = 2$, then

$$\begin{aligned} x_2 &= 2x_1 - \frac{1}{2}x_0 \\ &= 2(2) - \frac{1}{2}(3) \\ &= \frac{5}{2} \\ x_3 &= 2x_2 - \frac{1}{2}x_1 \\ &= 2\left(\frac{5}{2}\right) - \frac{1}{2}(2) \\ &= 5 - 1 \\ &= 4 \end{aligned}$$

The correct answer is C.

199. During a trip, Francine traveled x percent of the total distance at an average speed of 40 miles per hour and the rest of the distance at an average speed of 60 miles per hour. In terms of x, what was Francine's average speed for the entire trip?

(A) $\frac{180 - x}{2}$
(B) $\frac{x + 60}{4}$
(C) $\frac{300 - x}{5}$
(D) $\frac{600}{115 - x}$
(E) $\frac{12{,}000}{x + 200}$

Algebra Applied problems

Assume for simplicity that the total distance of Francine's trip is 100 miles. Then the table below gives all of the pertinent information.

Distance	Rate	Time = $\frac{\text{Distance}}{\text{Rate}}$
x	40	$\frac{x}{40}$
$100 - x$	60	$\frac{100 - x}{60}$

The total time for Francine's trip is

$$\frac{x}{40}+\frac{100-x}{60}=\frac{3x}{120}+\frac{2(100-x)}{120}$$
$$=\frac{3x+2(100-x)}{120}$$
$$=\frac{3x+200-2x}{120}$$
$$=\frac{x+200}{120}$$

Francine's average speed over the entire trip is

$$\frac{\text{total distance}}{\text{total time}}=\frac{100}{\frac{x+200}{120}}=\frac{12{,}000}{x+200}.$$

The correct answer is E.

200. If $n = (33)^{43} + (43)^{33}$, what is the units digit of n?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

Arithmetic Properties of numbers

If the units digit of an integer n is 3, then the units digits of n^1, n^2, n^3, n^4, n^5, n^6, n^7, and n^8 are, respectively, 3, 9, 7, 1, 3, 9, 7, and 1. Thus, the units digit of the powers of n form the sequence in which the digits 3, 9, 7, and 1 repeat indefinitely in that order. Since $43 = (10)(4) + 3$, the 43rd number in the sequence is 7, and therefore the units digit of $(33)^{43}$ is 7. Since $33 = (8)(4) + 1$, the 33rd number of this sequence is 3, and therefore, the units digit of $(43)^{33}$ is 3. Thus, the units digit of $(33)^{43} + (43)^{33}$ is the units digit of $7 + 3$, which is 0.

The correct answer is A.

201. Team A and Team B are competing against each other in a game of tug-of-war. Team A, consisting of 3 males and 3 females, decides to line up male, female, male, female, male, female. The lineup that Team A chooses will be one of how many different possible lineups?

(A) 9
(B) 12
(C) 15
(D) 36
(E) 720

Arithmetic Elementary combinatorics

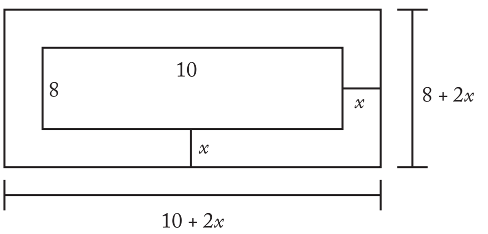
Any of the 3 males can be first in the line, and any of the 3 females can be second. Either of the 2 remaining males can be next, followed by either of the 2 remaining females. The last 2 places in the line are filled with the only male left followed by the only female left. By the multiplication principle, there are $3 \times 3 \times 2 \times 2 \times 1 \times 1 = 36$ different lineups possible.

The correct answer is D.

202. A border of uniform width is placed around a rectangular photograph that measures 8 inches by 10 inches. If the area of the border is 144 square inches, what is the width of the border, in inches?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

Algebra Second-degree equations



Note: Figure not drawn to scale.

Let x be the width, in inches, of the border. The photograph with the border has dimensions $(10 + 2x)$ inches and $(8 + 2x)$ inches with an area of $(10 + 2x)(8 + 2x) = (80 + 36x + 4x^2)$ square inches. The photograph without the border has dimensions 10 inches and 8 inches with an area of $(10)(8) = 80$ square inches. The area of the border is then the difference between the areas of the photograph with and without the border or $(80 + 36x + 4x^2) - 80 = 36x + 4x^2$ square inches. It is given that the area of the border is 144 square inches so,

$$36x + 4x^2 = 144$$
$$4x^2 + 36x - 144 = 0$$
$$x^2 + 9x - 36 = 0$$
$$(x - 3)(x + 12) = 0$$

So, $x - 3 = 0$, which means $x = 3$, or $x + 12 = 0$, which means $x = -12$.

Thus, after discarding $x = -12$ since the width of the border must be positive, $x = 3$.

The correct answer is A.

203. If $d = \dfrac{1}{2^3 \times 5^7}$ is expressed as a terminating decimal, how many nonzero digits will d have?

(A) One
(B) Two
(C) Three
(D) Seven
(E) Ten

Arithmetic Operations on rational numbers

It will be helpful to use the fact that a factor that is an integer power of 10 has no effect on the number of nonzero digits a terminating decimal has.

$$\begin{aligned}\frac{1}{2^3 \times 5^7} &= \frac{1}{2^3 \times 5^3} \times \frac{1}{5^4}\\ &= \left(\frac{1}{2 \times 5}\right)^3 \times \left(\frac{1}{5}\right)^4\\ &= \left(\frac{1}{10}\right)^3 \times \left(\frac{1}{5}\right)^4\\ &= 10^{-3} \times (0.2)^4\\ &= 10^{-3} \times (0.0016)\\ &= 0.0000016\end{aligned}$$

The correct answer is B.

204. For any positive integer n, the sum of the first n positive integers equals $\dfrac{n(n+1)}{2}$. What is the sum of all the even integers between 99 and 301 ?

(A) 10,100
(B) 20,200
(C) 22,650
(D) 40,200
(E) 45,150

Algebra Simplifying expressions; Arithmetic Computation with integers

The given formula translates into $1 + 2 + \ldots + n = \sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$. The sum of the even integers between 99 and 301 is the sum of the even integers from 100 through 300, or the sum of the 50th even integer through the 150th even integer. To get this sum, find the sum of the first 150 even integers and subtract the sum of the first 49 even integers. In symbols,

$$\begin{aligned}\sum_{k=1}^{150} 2k - \sum_{k=1}^{49} 2k = 2\sum_{k=1}^{150} k - 2\sum_{k=1}^{49} k\\ &= 2\left(\frac{150(150+1)}{2}\right) - 2\left(\frac{49(49+1)}{2}\right)\\ &= 150(151) - 49(50)\\ &= 50\left[3(151) - 49\right]\\ &= 50(453 - 49)\\ &= 50(404)\\ &= 20{,}200\end{aligned}$$

The correct answer is B.

205. How many prime numbers between 1 and 100 are factors of 7,150 ?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

Arithmetic Properties of numbers

To find the number of prime numbers between 1 and 100 that are factors of 7,150, find the prime factorization of 7,150 using a method similar to the following:

$$\begin{aligned}7{,}150 &= 10 \times 715\\ &= (2 \times 5) \times (5 \times 143)\\ &= 2 \times 5 \times 5 \times (11 \times 13)\end{aligned}$$

Thus, 7,150 has four prime factors: 2, 5, 11, and 13.

The correct answer is D.

206. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined as follows: $a_1 = 3$, $a_2 = 5$, and every term in the sequence after a_2 is the product of all terms in the sequence preceding it, e.g., $a_3 = (a_1)(a_2)$ and $a_4 = (a_1)(a_2)(a_3)$. If $a_n = t$ and $n > 2$, what is the value of a_{n+2} in terms of t?

(A) $4t$

(B) t^2

(C) t^3

(D) t^4

(E) t^8

Algebra Sequences

It is given that $a_n = (a_1)(a_2) \ldots (a_{n-1})$ and $a_n = t$. Therefore, $a_{n+1} = (a_1)(a_2) \ldots (a_{n-1})(a_n) = (a_n)(a_n) = t^2$ and $a_{n+2} = (a_1)(a_2) \ldots (a_n)(a_{n+1}) = (a_{n+1})(a_{n+1}) = (t^2)(t^2) = t^4$.

The correct answer is D.

207. Last year the price per share of Stock X increased by k percent and the earnings per share of Stock X increased by m percent, where k is greater than m. By what percent did the ratio of price per share to earnings per share increase, in terms of k and m?

(A) $\frac{k}{m}\%$

(B) $(k - m)\%$

(C) $\frac{100(k-m)}{100+k}\%$

(D) $\frac{100(k-m)}{100+m}\%$

(E) $\frac{100(k-m)}{100+k+m}\%$

Algebra Percents

If P and E are the price and earnings per share before the increase, then $\left(1+\frac{k}{100}\right)P$ and $\left(1+\frac{m}{100}\right)E$ are the price and earnings per share after the increase. Therefore, the percent increase in the ratio of price per share to earnings per share can be expressed as follows:

$$\left(\frac{(\text{ratio after increases})-(\text{ratio before increases})}{(\text{ratio before increases})}\times 100\right)\%$$

$$=\left[\left(\frac{(\text{ratio after increases})}{(\text{ratio before increases})}-1\right)\times 100\right]\%$$

$$=\left[\left(\frac{\dfrac{\left(1+\frac{k}{100}\right)P}{\left(1+\frac{m}{100}\right)E}}{\dfrac{P}{E}}-1\right)\times 100\right]\%$$

$$=\left[\left(\frac{\dfrac{\left(1+\frac{k}{100}\right)}{\left(1+\frac{m}{100}\right)}\cdot\dfrac{P}{E}}{\dfrac{P}{E}}-1\right)\times 100\right]\%$$

$$=\left[\left(\frac{\dfrac{\left(1+\frac{k}{100}\right)}{\left(1+\frac{m}{100}\right)}}{1}-1\right)\times 100\right]\%$$

$$=\left[\left(\frac{1+\frac{k}{100}}{1+\frac{m}{100}}-1\right)\times 100\right]\%$$

$$= \left[\left(\frac{1+\frac{k}{100}}{1+\frac{m}{100}} \cdot \frac{100}{100} - 1\right) \times 100\right]\%$$

$$= \left[\left(\frac{100+k}{100+m} - 1\right) \times 100\right]\%$$

$$= \left(\frac{(100+k)-(100+m)}{100+m} \times 100\right)\%$$

$$= \left(\frac{k-m}{100+m} \times 100\right)\%$$

$$= \frac{100(k-m)}{100+m}\%$$

The correct answer is D.

208. Of the 300 subjects who participated in an experiment using virtual-reality therapy to reduce their fear of heights, 40 percent experienced sweaty palms, 30 percent experienced vomiting, and 75 percent experienced dizziness. If all of the subjects experienced at least one of these effects and 35 percent of the subjects experienced exactly two of these effects, how many of the subjects experienced only one of these effects?

(A) 105
(B) 125
(C) 130
(D) 180
(E) 195

Arithmetic Applied problems

Let a be the number who experienced only one of the effects, b be the number who experienced exactly two of the effects, and c be the number who experienced all three of the effects. Then $a + b + c = 300$, since each of the 300 participants experienced at least one of the effects. From the given information, $b = 105$ (35% of 300), which gives $a + 105 + c = 300$, or $a + c = 195$ (Eq. 1). Also, if the number who experienced sweaty palms (40% of 300, or 120) is added to the number who experienced vomiting (30% of 300, or 90), and this sum is added to the number who experienced dizziness (75% of 300, or 225), then each participant who experienced only one of the effects is counted exactly once, each participant who experienced exactly two of the effects is counted exactly twice, and each participant who experienced all three of the effects is counted exactly 3 times. Therefore, $a + 2b + 3c = 120 + 90 + 225 = 435$. Using $b = 105$, it follows that $a + 2(105) + 3c = 435$, or $a + 3c = 225$ (Eq. 2). Then solving the system defined by Eq. 1 and Eq. 2,

$$\begin{cases} a + c = 195 \\ a + 3c = 225 \end{cases} \qquad \text{multiply 1st equation by } -3$$

$$\begin{cases} -3a - 3c = -585 \\ \quad a + 3c = 225 \end{cases} \qquad \text{add equations}$$

$-2a = -360$, or $a = 180$

The correct answer is D.

209. If $m^{-1} = \frac{1}{3}$, then m^{-2} is equal to

(A) -9
(B) -3
(C) $-\frac{1}{9}$
(D) $\frac{1}{9}$
(E) 9

Arithmetic Negative exponents

Using rules of exponents, $m^{-2} = m^{-1 \cdot 2} = \left(m^{-1}\right)^2$, and since $m^{-1} = -\frac{1}{3}$, $m^{-2} = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$.

The correct answer is D.

210. A photography dealer ordered 60 Model X cameras to be sold for $250 each, which represents a 20 percent markup over the dealer's initial cost for each camera. Of the cameras ordered, 6 were never sold and were returned to the manufacturer for a refund of 50 percent of the dealer's initial cost. What was the dealer's approximate profit or loss as a percent of the dealer's initial cost for the 60 cameras?

(A) 7% loss
(B) 13% loss
(C) 7% profit
(D) 13% profit
(E) 15% profit

Arithmetic Percents

Given that \$250 is 20% greater than a camera's initial cost, it follows that the initial cost for each camera was $\left(\$\frac{250}{1.2}\right)$. Therefore, the initial cost for the 60 cameras was $60\left(\$\frac{250}{1.2}\right)$. The total revenue is the sum of the amount obtained from selling 60 − 6 = 54 cameras for \$250 each and the $\left(\frac{1}{2}\right)\left(\$\frac{250}{1.2}\right)$ refund for each of 6 cameras, or $(54)(\$250)+(6)\left(\frac{1}{2}\right)\left(\$\frac{250}{1.2}\right)$. The total profit, as a percent of the total initial cost, is $\left(\frac{(\text{total revenue})-(\text{total initial cost})}{(\text{total initial cost})}\times 100\right)\% =$ $\left(\left(\frac{(\text{total revenue})}{(\text{total initial cost})}-1\right)\times 100\right)\%$. Using the numerical expressions obtained above, $\frac{(\text{total revenue})}{(\text{total initial cost})}-1$

$$=\frac{(54)(250)+6\left(\frac{1}{2}\right)\left(\frac{250}{1.2}\right)}{(60)\left(\frac{250}{1.2}\right)}-1 \quad \text{by substitution}$$

$$=\frac{54+3\left(\frac{1}{1.2}\right)}{(60)\left(\frac{1}{1.2}\right)}-1 \quad \text{by canceling 250s}$$

$$=\frac{54(1.2)+3}{60}-1 \quad \text{by multiplying top and bottom by 1.2 and then canceling 1.2}$$

$$=\frac{67.8}{60}-1$$

$$=1.13-1$$

$$=0.13$$

Finally, $(0.13 \times 100)\% = 13\%$, which represents a profit since it is positive.

The correct answer is D.

211. Seven pieces of rope have an average (arithmetic mean) length of 68 centimeters and a median length of 84 centimeters. If the length of the longest piece of rope is 14 centimeters more than 4 times the length of the shortest piece of rope, what is the maximum possible length, in centimeters, of the longest piece of rope?

(A) 82
(B) 118
(C) 120
(D) 134
(E) 152

Algebra Statistics

Let a, b, c, d, e, f, and g be the lengths, in centimeters, of the pieces of rope, listed from least to greatest. From the given information it follows that $d = 84$ and $g = 4a + 14$. Therefore, listed from least to greatest, the lengths are a, b, c, 84, e, f, and $4a + 14$. The maximum value of $4a + 14$ will occur when the maximum value of a is used, and this will be the case only if the shortest 3 pieces all have the same length. Therefore, listed from least to greatest, the lengths are a, a, a, 84, e, f, and $4a + 14$. The maximum value for $4a + 14$ will occur when e and f are as small as possible. Since e and f are to the right of the median, they must be at least 84 and so 84 is the least possible value for each of e and f. Therefore, listed from least to greatest, the lengths are a, a, a, 84, 84, 84, and $4a + 14$. Since the average length is 68, it follows that $\frac{a+a+a+84+84+84+(4a+14)}{7} = 68$, or $a = 30$.

Hence, the maximum length of the longest piece is $(4a + 14) = [4(30) + 14] = 134$ centimeters.

The correct answer is D.

212. What is the difference between the sixth and the fifth terms of the sequence 2, 4, 7, … whose nth term is $n+2^{n-1}$?

(A) 2
(B) 3
(C) 6
(D) 16
(E) 17

Algebra Simplifying algebraic expressions

According to the given formula, the sixth term of the sequence is $6 + 2^{6-1} = 6 + 2^5$ and the fifth term is $5 + 2^{5-1} = 5 + 2^4$. Then,

$$\begin{aligned}(6+2^5)-(5+2^4) &= (6-5)+(2^5-2^4)\\ &= 1+2^4(2-1)\\ &= 1+2^4\\ &= 1+16\\ &= 17\end{aligned}$$

The correct answer is E.

213. From the consecutive integers –10 to 10, inclusive, 20 integers are randomly chosen with repetitions allowed. What is the least possible value of the product of the 20 integers?

(A) $(-10)^{20}$
(B) $(-10)^{10}$
(C) 0
(D) $-(10)^{19}$
(E) $-(10)^{20}$

Arithmetic Properties of numbers

If –10 is chosen an odd number of times and 10 is chosen the remaining number of times (for example, choose –10 once and choose 10 nineteen times, or choose –10 three times and choose 10 seventeen times), then the product of the 20 chosen numbers will be $-(10)^{20}$. Note that $-(10)^{20}$ is less than $-(10)^{19}$, the only other negative value among the answer choices.

The correct answer is E.

214. The letters D, G, I, I, and T can be used to form 5-letter strings such as DIGIT or DGIIT. Using these letters, how many 5-letter strings can be formed in which the two occurrences of the letter I are separated by at least one other letter?

(A) 12
(B) 18
(C) 24
(D) 36
(E) 48

Arithmetic Elementary combinatorics

There are 6 ways to select the locations of the 2 occurrences of the letter I, and this number can be determined by listing all such ways as shown below, where the symbol * is used in place of the letters D, G, and T:

I*I**, I**I*, I***I, *I*I*, *I**I, **I*I

Alternatively, the number of ways to select the locations of the 2 occurrences of the letter I can be determined by using $\binom{5}{2} - 4 = \frac{5!}{(2!)(3!)} - 4 = 10 - 4 = 6$, which is the number of ways to select 2 of the 5 locations minus the 4 ways in which the 2 selected locations are adjacent.

For each of these 6 ways to select the locations of the 2 occurrences of the letter I, there are 6 ways to select the locations of the letters D, G, and T, which can be determined by using $3! = 6$ or by listing all such ways:

DGT, DTG, GDT, GTD, TDG, TGD

It follows that the number of ways to select the locations of the 5 letters to form 5-letter strings is $(6)(6) = 36$.

The correct answer is D.

215. Last Sunday a certain store sold copies of Newspaper A for \$1.00 each and copies of Newspaper B for \$1.25 each, and the store sold no other newspapers that day. If *r* percent of the store's revenue from newspaper sales was from Newspaper A and if *p* percent of the newspapers that the store sold were copies of Newspaper A, which of the following expresses *r* in terms of *p* ?

(A) $\frac{100p}{125-p}$
(B) $\frac{150p}{250-p}$
(C) $\frac{300p}{375-p}$
(D) $\frac{400p}{500-p}$
(E) $\frac{500p}{625-p}$

Algebra Simultaneous equations

Let N be the total number of newspapers that the store sold. Then, the number of copies of Newspaper A the store sold was $p\%$ of $N = \left(\frac{p}{100}\right)N$ and the revenue from those copies of Newspaper A, in dollars, was $(1.00)\left(\frac{p}{100}\right)N = \left(\frac{p}{100}\right)N$. The number of copies of Newspaper B the store sold was $(100 - p)\%$ of $N = \left(\frac{100-p}{100}\right)N$ and the revenue from those copies of Newspaper B, in dollars, was $(1.25)\left(\frac{100-p}{100}\right)N = \left(\frac{5}{4}\right)\left(\frac{100-p}{100}\right)N$. The store's total revenue from newspaper sales, in dollars, was $\left(\frac{p}{100}\right)N + \left(\frac{5}{4}\right)\left(\frac{100-p}{100}\right)N$, and the fraction of that revenue from the sale of Newspaper A was

$$\frac{\frac{p}{100}N}{\frac{p}{100}N + \left(\frac{5}{4}\right)\left(\frac{100-p}{100}\right)N} = \frac{\frac{p}{100}}{\frac{4p}{400} + \left(\frac{500-5p}{400}\right)}$$

$$= \frac{\frac{p}{100}}{\frac{4p+500-5p}{400}}$$

$$= \frac{\frac{p}{100}}{\frac{500-p}{400}}$$

$$= \left(\frac{p}{100}\right)\left(\frac{400}{500-p}\right)$$

$$= \frac{4p}{500-p}$$

Since r percent of the store's newspaper sales revenue was from Newspaper A, $\frac{r}{100} = \frac{4p}{500-p}$, and so $r = \frac{400p}{500-p}$.

The correct answer is D.

216. $\frac{0.99999999}{1.0001} - \frac{0.99999991}{1.0003} =$

(A) 10^{-8}
(B) $3(10^{-8})$
(C) $3(10^{-4})$
(D) $2(10^{-4})$
(E) 10^{-4}

Arithmetic Operations on rational numbers

Calculations with lengthy decimals can be avoided by writing 0.99999999 as $1 - 10^{-8}$, 0.99999991 as $1 - 9(10^{-8})$, 1.0001 as $1 + 10^{-4}$, and 1.0003 as $1 + 3(10^{-4})$. Doing this gives

$$\frac{1-10^{-8}}{1+10^{-4}} - \frac{1-9(10^{-8})}{1+3(10^{-4})}$$

$$= \frac{[1+10^{-4}][1-10^{-4}]}{1+10^{-4}} - \frac{1-9(10^{-8})}{1+3(10^{-4})}$$

$$= \frac{1-10^{-4}}{1} - \frac{1-9(10^{-8})}{1+3(10^{-4})}$$

$$= \frac{[1-10^{-4}][1+3(10^{-4})]-[1-9(10^{-8})]}{1+3(10^{-4})}$$

$$= \frac{1+3(10^{-4})-10^{-4}-3(10^{-8})-1+9(10^{-8})}{1+3(10^{-4})}$$

$$= \frac{2(10^{-4})+6(10^{-8})}{1+3(10^{-4})}$$

$$= \frac{[2(10^{-4})][1+3(10^{-4})]}{1+3(10^{-4})}$$

$$= 2(10^{-4})$$

The correct answer is D.

217. For the past n days, the average (arithmetic mean) daily production at a company was 50 units. If today's production of 90 units raises the average to 55 units per day, what is the value of n ?

(A) 30
(B) 18
(C) 10
(D) 9
(E) 7

Arithmetic; Algebra Statistics; Applied problems; Simultaneous equations

Let x be the total production of the past n days. Using the formula average $= \frac{\text{sum of values}}{\text{number of values}}$, the information in the problem can be expressed in the following two equations

$50 = \frac{x}{n}$ daily average of 50 units over the past n days

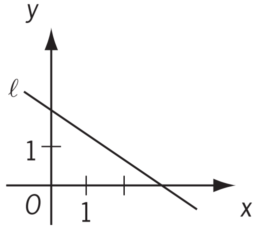
$55 = \frac{x+90}{n+1}$ increased daily average when including today's 90 units

Solving the first equation for x gives $x = 50n$. Then substituting $50n$ for x in the second equation gives the following that can be solved for n:

$$55 = \frac{50n+90}{n+1}$$

$55(n+1) = 50n+90$	multiply both sides by $(n+1)$
$55n+55 = 50n+90$	distribute the 55
$5n = 35$	subtract $50n$ and 55 from both sides
$n = 7$	divide both sides by 5

The correct answer is E.



218. In the coordinate system above, which of the following is the equation of line ℓ ?

(A) $2x - 3y = 6$
(B) $2x + 3y = 6$
(C) $3x + 2y = 6$
(D) $2x - 3y = -6$
(E) $3x - 2y = -6$

Geometry Simple coordinate geometry

The line is shown going through the points (0,2) and (3,0). The slope of the line can be found with the formula slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$ for two points (x_1,y_1) and (x_2,y_2). Thus, the slope of this line equals $\frac{0-2}{3-0} = -\frac{2}{3}$. Using the formula for a line of $y = mx + b$, where m is the slope and b is the y-intercept (in this case, 2), an equation for this line is $y = -\frac{2}{3}x + 2$. Since this equation must be compared to the available answer choices, the following further steps should be taken:

$$y = -\frac{2}{3}x + 2$$

$3y = -2x + 6$	multiply both sides by 3
$2x + 3y = 6$	add $2x$ to both sides

This problem can also be solved as follows. From the graph, when $x = 0$, y is positive; when $y = 0$, x is positive. This eliminates all but B and C. Of these, B is the only line containing (0,2). Still another way is to use (0,2) to eliminate A, C, and E, and then use (3,0) to eliminate D.

The correct answer is B.

219. If a two-digit positive integer has its digits reversed, the resulting integer differs from the original by 27. By how much do the two digits differ?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

Algebra Applied problems

Let the one two-digit integer be represented by $10t + s$, where s and t are digits, and let the other integer with the reversed digits be represented by $10s + t$. The information that the difference between the integers is 27 can be expressed in the following equation, which can be solved for the answer.

$$(10s + t) - (10t + s) = 27$$

$10s + t - 10t - s = 27$	distribute the negative
$9s - 9t = 27$	combine like terms
$s - t = 3$	divide both sides by 9

Thus, it is seen that the two digits s and t differ by 3.

The correct answer is A.

220. In an electric circuit, two resistors with resistances x and y are connected in parallel. In this case, if r is the combined resistance of these two resistors, then the reciprocal of r is equal to the sum of the reciprocals of x and y. What is r in terms of x and y?

(A) xy

(B) $x + y$

(C) $\frac{1}{x+y}$

(D) $\frac{xy}{x+y}$

(E) $\frac{x+y}{xy}$

Algebra Applied problems

Note that two numbers are reciprocals of each other if and only if their product is 1. Thus the reciprocals of r, x, and y are $\frac{1}{r}$, $\frac{1}{x}$, and $\frac{1}{y}$, respectively. So, according to the problem, $\frac{1}{r} = \frac{1}{x} + \frac{1}{y}$. To solve this equation for r, begin by creating a common denominator on the right side by multiplying the first fraction by $\frac{y}{y}$ and the second fraction by $\frac{x}{x}$:

$$\frac{1}{r} = \frac{1}{x} + \frac{1}{y}$$

$$\frac{1}{r} = \frac{y}{xy} + \frac{x}{xy}$$

$\frac{1}{r} = \frac{x+y}{xy}$	combine the fractions on the right side
$r = \frac{xy}{x+y}$	invert the fractions on both sides

The correct answer is D.

221. Xavier, Yvonne, and Zelda each try independently to solve a problem. If their individual probabilities for success are $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{5}{8}$, respectively, what is the probability that Xavier and Yvonne, but not Zelda, will solve the problem?

(A) $\frac{11}{8}$

(B) $\frac{7}{8}$

(C) $\frac{9}{64}$

(D) $\frac{5}{64}$

(E) $\frac{3}{64}$

Arithmetic Probability

Since the individuals' probabilities are independent, they can be multiplied to figure out the combined probability. The probability of Xavier's success is given as $\frac{1}{4}$, and the probability of Yvonne's success is given as $\frac{1}{2}$. Since the probability of Zelda's success is given as $\frac{5}{8}$, then the probability of her NOT solving the problem is $1 - \frac{5}{8} = \frac{3}{8}$.

Thus, the combined probability is $\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) = \frac{3}{64}$.

The correct answer is E.

222. If $\frac{1}{x}-\frac{1}{x+1}=\frac{1}{x+4}$, then x could be

(A) 0
(B) –1
(C) –2
(D) –3
(E) –4

Algebra Second-degree equations

Solve the equation for x. Begin by multiplying all the terms by $x(x+1)(x+4)$ to eliminate the denominators.

$$\frac{1}{x}-\frac{1}{x+1}=\frac{1}{x+4}$$

$$(x+1)(x+4)-x(x+4)=x(x+1)$$

$(x+4)(x+1-x)=x(x+1)$	factor the $(x+4)$ out front on the left side
$(x+4)(1)=x(x+1)$	simplify
$(x+4)=x^2+x$	distribute the x on the right side
$4=x^2$	subtract x from both sides
$\pm 2=x$	take the square root of both sides

Both – 2 and 2 are square roots of 4 since $\left(-2^2\right)=4$ and $\left(2^2\right)=4$. Thus, x could be – 2.

This problem can also be solved as follows. Rewrite the left side as, $\frac{(x+1)-x}{x(x+1)}=\frac{1}{x(x+1)}$, then set equal to the right side to get $\frac{1}{x(x+1)}=\frac{1}{x+4}$. Next, cross multiply: $(1)(x+4)=x(x+1)(1)$. Therefore, $x+4=x^2+x$, or $x^2=4$, so $x=\pm 2$.

The correct answer is C.

223. $\left(\frac{1}{2}\right)^{-3}\left(\frac{1}{4}\right)^{-2}\left(\frac{1}{16}\right)^{-1}=$

(A) $\left(\frac{1}{2}\right)^{-48}$
(B) $\left(\frac{1}{2}\right)^{-11}$
(C) $\left(\frac{1}{2}\right)^{-6}$
(D) $\left(\frac{1}{8}\right)^{-11}$
(E) $\left(\frac{1}{8}\right)^{-6}$

Arithmetic Operations on rational numbers

It is clear from the answer choices that all three factors need to be written with a common denominator, and they thus become

$$\left(\frac{1}{2}\right)^{-3}=\left(\frac{1}{2}\right)^{-3}$$

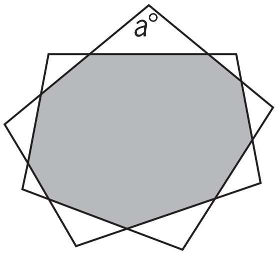
$$\left(\frac{1}{4}\right)^{-2}=\left(\left(\frac{1}{2}\right)^{2}\right)^{-2}=\left(\frac{1}{2}\right)^{-4}$$

$$\left(\frac{1}{16}\right)^{-1}=\left(\left(\frac{1}{2}\right)^{4}\right)^{-1}=\left(\frac{1}{2}\right)^{-4}$$

So, $\left(\frac{1}{2}\right)^{-3}\left(\frac{1}{4}\right)^{-2}\left(\frac{1}{16}\right)^{-1}=$

$$\left(\frac{1}{2}\right)^{-3}\left(\frac{1}{2}\right)^{-4}\left(\frac{1}{2}\right)^{-4}=\left(\frac{1}{2}\right)^{-3-4-4}=\left(\frac{1}{2}\right)^{-11}.$$

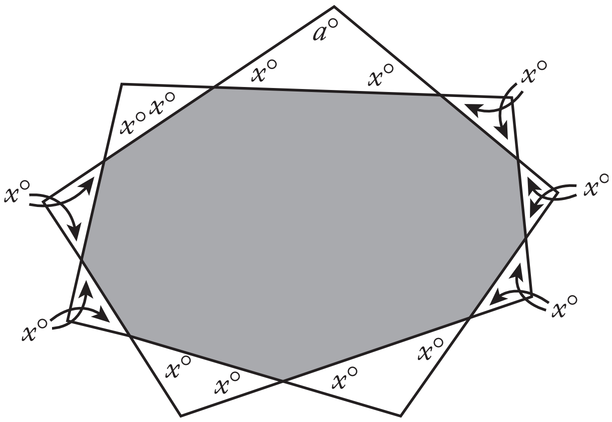
The correct answer is B.



224. The figure shown above consists of a shaded 9-sided polygon and 9 unshaded isosceles triangles. For each isosceles triangle, the longest side is a side of the shaded polygon and the two sides of equal length are extensions of the two adjacent sides of the shaded polygon. What is the value of *a* ?

(A) 100
(B) 105
(C) 110
(D) 115
(E) 120

Geometry Polygons



Let $x°$ represent the measure of each base angle of the triangle with vertex angle labeled $a°$. Each base angle of this triangle and one base angle of a triangle with which it shares a vertex are vertical angles and have the same measure. Thus, the base angles of these triangles also have measure $x°$. This pattern continues for the base angles of each pair of triangles that share a vertex, so each base angle of each of the 9 triangles has measure $x°$, as shown above. Also, the vertex angle of each of the 9 triangles has measure $a° = 180° - 2x°$.

Each interior angle of the shaded polygon has measure $(180 - x)°$ since each forms a straight angle with an angle that has measure $x°$, and the sum of the measures is $(9)(180 - x)°$. But the sum of the interior angles of a polygon with n sides is $(n - 2)(180°)$, so the sum of the interior angles of a 9-sided polygon is $(7)(180°) = 1{,}260°$. Therefore, $(9)(180 - x)° = 1{,}260°$ and $x = 40$. Finally, $a = 180 - 2x = 180 - 2(40) = 100$.

The correct answer is A.

225. List *T* consists of 30 positive decimals, none of which is an integer, and the sum of the 30 decimals is *S*. The <u>estimated</u> sum of the 30 decimals, *E*, is defined as follows. Each decimal in *T* whose tenths digit is even is rounded up to the nearest integer, and each decimal in *T* whose tenths digit is odd is rounded down to the nearest integer; *E* is the sum of the resulting integers. If $\frac{1}{3}$ of the decimals in *T* have a tenths digit that is even, which of the following is a possible value of $E - S$?

I. -16
II. 6
III. 10

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Arithmetic Operations on rational numbers

Since $\frac{1}{3}$ of the 30 decimals in T have an even tenths digit, it follows that $\frac{1}{3}(30) = 10$ decimals in T have an even tenths digit. Let T_E represent the list of these 10 decimals, let S_E represent the sum of all 10 decimals in T_E, and let E_E represent the estimated sum of all 10 decimals in T_E after rounding. The remaining 20 decimals in T have an odd tenths digit. Let T_O represent the list of these 20 remaining decimals, let S_O represent the sum of all 20 decimals in T_O, and let E_O represent the estimated sum of all 20 decimals in T_O after rounding. Note that $E = E_E + E_O$ and $S = S_E + S_O$ and hence $E - S = (E_E + E_O) - (S_E + S_O) = (E_E - S_E) + (E_O - S_O)$.

The least values of $E_E - S_E$ occur at the extreme where each decimal in T_E has tenths digit 8. Here, the difference between the rounded integer and the original decimal is greater than 0.1. (For

example, the difference between the integer 15 and 14.899 that has been rounded to 15 is 0.101.) Hence, $E_E - S_E > 10(0.1) = 1$. The greatest values of $E_E - S_E$ occur at the other extreme, where each decimal in T_E has tenths digit 0. Here, the difference between the rounded integer and the original decimal is less than 1. (For example, the difference between the integer 15 and 14.001 that has been rounded to 15 is 0.999.)
Hence, $E_E - S_E < 10(1) = 10$. Thus, $1 < E_E - S_E < 10$.

Similarly, the least values of $E_O - S_O$ occur at the extreme where each decimal in T_O has tenths digit 9. Here, the difference between the rounded integer and the original decimal is greater than −1. (For example, the difference between the integer 14 and 14.999 that has been rounded to 14 is −0.999.) Hence $E_O - S_O > 20(-1) = -20$.
The greatest values of $E_O - S_O$ occur at the other extreme where each decimal in T_O has tenths digit 1. Here, the difference between the rounded integer and the original decimal is less than or equal to −0.1. (For example, the difference between the integer 14 and 14.1 that has been rounded to 14 is −0.1.)
Hence, $E_O - S_O \le 20(-0.1) = -2$.
Thus, $-20 < E_O - S_O \le -2$.

Adding the inequalities $1 < E_E - S_E < 10$ and $-20 < E_O - S_O \le -2$ gives $-19 < (E_E - S_E) + (E_O - S_O) < 8$. Therefore, $-19 < (E_E + E_O) - (S_E + S_O) < 8$ and $-19 < E - S < 8$. Thus, of the values −16, 6, and 10 for $E - S$, only −16 and 6 are possible.

Note that if T contains 10 repetitions of the decimal 1.8 and 20 repetitions of the decimal 1.9, $S = 10(1.8) + 20(1.9) = 18 + 38 = 56$, $E = 10(2) + 20(1) = 40$, and $E - S = 40 - 56 = -16$. Also, if T contains 10 repetitions of the decimal 1.2 and 20 repetitions of the decimal 1.1, $S = 10(1.2) + 20(1.1) = 12 + 22 = 34$, $E = 10(2) + 20(1) = 40$, and $E - S = 40 - 34 = 6$.

The correct answer is B.

226. If $5 - \frac{6}{x} = x$, then x has how many possible values?

(A) None
(B) One
(C) Two
(D) A finite number greater than two
(E) An infinite number

Algebra Second-degree equations

Solve the equation to determine how many values are possible for x.

$$5 - \frac{6}{x} = x$$
$$5x - 6 = x^2$$
$$0 = x^2 - 5x + 6$$
$$0 = (x-3)(x-2)$$
$$x = 3 \text{ or } 2$$

The correct answer is C.

227. Seed mixture X is 40 percent ryegrass and 60 percent bluegrass by weight; seed mixture Y is 25 percent ryegrass and 75 percent fescue. If a mixture of X and Y contains 30 percent ryegrass, what percent of the weight of the mixture is X ?

(A) 10%
(B) $33\frac{1}{3}\%$
(C) 40%
(D) 50%
(E) $66\frac{2}{3}\%$

Algebra Applied problems

Let X be the amount of seed mixture X in the final mixture, and let Y be the amount of seed mixture Y in the final mixture. The final mixture of X and Y needs to contain 30 percent ryegrass seed, so any other kinds of grass seed are irrelevant to the solution to this problem. The information about the ryegrass percentages for X, Y, and the final mixture can be expressed in the following equation and solved for X.

$0.40X + 0.25Y = 0.30(X+Y)$	
$0.40X + 0.25Y = 0.30X + 0.30Y$	distribute the 0.30 on the right side
$0.10X = 0.05Y$	subtract $0.30X$ and $0.25Y$ from both sides
$X = 0.5Y$	divide both sides by 0.10

Using this, the percent of the weight of the combined mixture $(X + Y)$ that is X is

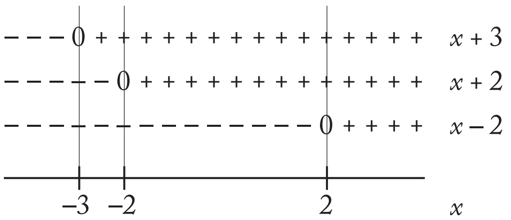
$$\frac{X}{X+Y} = \frac{0.5Y}{0.5Y+Y} = \frac{0.5Y}{1.5Y} = \frac{0.5}{1.5} = 0.33\overline{3} = 33\frac{1}{3}\%$$

The correct answer is B.

228. How many of the integers that satisfy the inequality $\frac{(x+2)(x+3)}{x-2} \geq 0$ are less than 5 ?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Algebra Inequalities



Pictorially, the number line above shows the algebraic signs of the expressions $(x + 3)$, $(x + 2)$, and $(x - 2)$. For example, $x + 3$ is 0 when $x = -3$, $x + 3$ is negative when $x < -3$, and $x + 3$ is positive when $x > -3$. The expression $\frac{(x+2)(x+3)}{x-2}$ will be positive in the intervals of the number line where the number of minus signs is even. Therefore $\frac{(x+2)(x+3)}{x-2}$ is positive for values of x such that $-3 < x < -2$ and for values of x such that $x > 2$. The only integer values of x in these intervals that are also less than 5 are 3 and 4. Also, $\frac{(x+2)(x+3)}{x-2}$ will be zero if and only if $(x + 2)(x + 3) = 0$, which has two integer solutions less than 5, namely, $x = -2$ and $x = -3$.

Therefore, there are four integers less than 5 that satisfy $\frac{(x+2)(x+3)}{x-2} \geq 0$ and those integers are −3, −2, 3, and 4.

Alternatively, $\frac{(x+2)(x+3)}{x-2}$ will be zero if and only if $(x + 2)(x + 3) = 0$, which has two integer solutions less than 5, namely, $x = -2$ and $x = -3$.

Also, $\frac{(x+2)(x+3)}{x-2}$ will be positive if $(x + 2)(x + 3)$ and $x - 2$ are both positive or both negative, and for no other values of x. On the one hand, $(x + 2)(x + 3)$ will be positive when $x + 2$ and $x + 3$ are both positive, which will be the case when $x > -2$ and $x > -3$ and thus when $x > -2$. On the other hand, $(x + 2)(x + 3)$ will be positive when $x + 2$ and $x + 3$ are both negative, which will be the case when $x < -2$ and $x < -3$ and thus when $x < -3$. So, $(x + 2)(x + 3)$ will be positive when $x < -3$ or $x > -2$. (This result can also be deduced from the fact that the graph of $y = (x + 2)(x + 3)$ is a parabola with x-intercepts $(-2,0)$ and $(-3,0)$ that opens upward.) Since $x - 2$ will be positive when $x > 2$, it follows that $(x + 2)(x + 3)$ and $x - 2$ are both positive when $x > 2$, which includes exactly two integer values less than 5, namely, $x = 3$ and $x = 4$. There are no integer values of x such that $(x + 2)(x + 3)$ and $x - 2$ are both negative, since $(x + 2)(x + 3)$ is negative if and only if x lies between −3 and −2 and there are no integers between −3 and −2. Therefore, there are exactly 4 integer values of x less than 5 such that $\frac{(x+2)(x+3)}{x-2} \geq 0$.

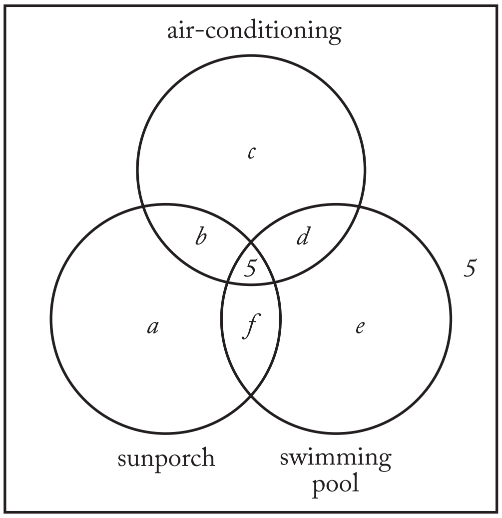
Two of the values, $x = -2$ and $x = -3$, arise from solutions to $\frac{(x+2)(x+3)}{x-2} = 0$, and two of the values, $x = 3$ and $x = 4$, arise from solutions to $\frac{(x+2)(x+3)}{x-2} > 0$.

The correct answer is D.

229. Of the 150 houses in a certain development, 60 percent have air-conditioning, 50 percent have a sunporch, and 30 percent have a swimming pool. If 5 of the houses have all three of these amenities and 5 have none of them, how many of the houses have exactly two of these amenities?

(A) 10
(B) 45
(C) 50
(D) 55
(E) 65

Arithmetic Sets



Since 60% of the 150 houses have air-conditioning, $b + c + d + 5 = 0.6(150) = 90$, so $b + c + d = 85$ (i). Similarly, since 50% have a sunporch, $a + b + f + 5 = 0.5(150) = 75$, so $a + b + f = 70$ (ii). Likewise, since 30% have a swimming pool, $d + e + f + 5 = 0.3(150) = 45$, so $d + e + f = 40$ (iii). Adding equations (i), (ii), and (iii) gives $(b + c + d) + (a + b + f) + (d + e + f) = 195$, or $a + 2b + c + 2d + e + 2f = 195$ (iv). But $a + b + c + d + e + f + 5 + 5 = 150$, or $a + b + c + d + e + f = 140$ (v). Subtracting equation (v) from equation (iv) gives $b + d + f = 55$, so 55 houses have exactly two of the amenities.

The correct answer is D.

230. The value of $\dfrac{2^{-14}+2^{-15}+2^{-16}+2^{-17}}{5}$ is how many times the value of $2^{(-17)}$?

(A) $\dfrac{3}{2}$
(B) $\dfrac{5}{2}$
(C) 3
(D) 4
(E) 5

Arithmetic Negative exponents

If the value of $\dfrac{2^{-14} + 2^{-15} + 2^{-16} + 2^{-17}}{5}$ is x times the value of 2^{-17}, then

$$x\left(2^{-17}\right)=\frac{2^{-14}+2^{-15}+2^{-16}+2^{-17}}{5}$$

$$x=\frac{\dfrac{2^{-14}+2^{-15}+2^{-16}+2^{-17}}{5}}{2^{-17}}$$

$$=\frac{2^{-14}+2^{-15}+2^{-16}+2^{-17}}{5}\times 2^{17}$$

$$=\frac{\left(2^{-14}+2^{-15}+2^{-16}+2^{-17}\right)\times 2^{17}}{5}$$

$$=\frac{2^{-14+17}+2^{-15+17}+2^{-16+17}+2^{-17+17}}{5}$$

$$=\frac{2^{3}+2^{2}+2^{1}+2^{0}}{5}$$

$$=\frac{8+4+2+1}{5}$$

$$=3$$

The correct answer is C.

7.0 Reading Comprehension

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Reading comprehension questions appear in the Verbal section of the GMAT® exam. The Verbal section uses multiple-choice questions to measure your ability to read and comprehend written material, to reason and evaluate arguments, and to correct written material to conform to standard written English. Because the Verbal section includes content from a variety of topics, you may be generally familiar with some of the material; however, neither the passages nor the questions assume knowledge of the topics discussed. Reading comprehension questions are intermingled with critical reasoning and sentence correction questions throughout the Verbal section of the test.

You will have 75 minutes to complete the Verbal section, or an average of about 1¾ minutes to answer each question. Keep in mind you will need time to read the written passages—and that time is not factored into the 1¾ minute average. Therefore, you should plan to proceed more quickly through the reading comprehension questions in order to give yourself enough time to read the passages thoroughly.

Reading comprehension questions begin with written passages up to 350 words long. The passages discuss topics from the social sciences, humanities, the physical or biological sciences, and such business-related fields as marketing, economics, and human resource management. The passages are accompanied by questions that will ask you to interpret the passage, apply the information you gather from the reading, and make inferences (or informed assumptions) based on the reading. For these questions, you will see a split computer screen. The written passage will remain visible on the left side as each question associated with that passage appears, in turn, on the right side. You will see only one question at a time. However, the number of questions associated with each passage may vary.

As you move through the reading comprehension sample questions, try to determine a process that works best for you. You might begin by reading a passage carefully and thoroughly. Some test-takers prefer to skim the passages the first time through, or even to read the first question before reading the passage. You may want to reread any sentences that present complicated ideas or introduce terms that are new to you. Read each question and series of answers carefully. Make sure you understand exactly what the question is asking and what the answer choices are.

If you need to, you may go back to the passage and read any parts that are relevant to answering the question. Specific portions of the passages may be indicated in the related questions.

The following pages describe what reading comprehension questions are designed to measure, present the directions that will precede questions of this type, and describe the various question types. This chapter also provides test-taking strategies, sample questions, and detailed explanations of all the questions. The explanations further illustrate the ways in which reading comprehension questions evaluate basic reading skills.

7.1 What Is Measured

Reading comprehension questions measure your ability to understand, analyze, and apply information and concepts presented in written form. All questions are to be answered on the basis of what is stated or implied in the reading material, and no specific prior knowledge of the material is required.

The GMAT reading comprehension questions evaluate your ability to do the following:

- **Understand words and statements.**
  Although the questions do not test your vocabulary (they will not ask you to define terms), they do test your ability to interpret special meanings of terms as they are used in the reading passages. The questions will also test your understanding of the English language. These questions may ask about the overall meaning of a passage.
- **Understand logical relationships between points and concepts.**
  This type of question may ask you to determine the strong and weak points of an argument or evaluate the relative importance of arguments and ideas in a passage.
- **Draw inferences from facts and statements.**
  The inference questions will ask you to consider factual statements or information presented in a reading passage and reach conclusions on the basis of that information.
- **Understand and follow the development of quantitative concepts as they are presented in written material.**
  This may involve the interpretation of numerical data or the use of simple arithmetic to reach conclusions about material in a passage.

There are six kinds of reading comprehension questions, each of which tests a different skill. The reading comprehension questions ask about the following areas:

Main idea

Each passage is a unified whole—that is, the individual sentences and paragraphs support and develop one main idea or central point. Sometimes you will be told the central point in the passage itself, and sometimes it will be necessary for you to determine the central point from the overall organization or development of the passage. You may be asked in this kind of question to

- recognize a correct restatement, or paraphrasing, of the main idea of a passage
- identify the author's primary purpose or objective in writing the passage
- assign a title that summarizes, briefly and pointedly, the main idea developed in the passage

Supporting ideas

These questions measure your ability to comprehend the supporting ideas in a passage and differentiate them from the main idea. The questions also measure your ability to differentiate ideas that are *explicitly stated* in a passage from ideas that are *implied* by the author but are not explicitly stated. You may be asked about

- facts cited in a passage
- the specific content of arguments presented by the author in support of his or her views
- descriptive details used to support or elaborate on the main idea

Whereas questions about the main idea ask you to determine the meaning of a passage *as a whole*, questions about supporting ideas ask you to determine the meanings of individual sentences and paragraphs that *contribute* to the meaning of the passage as a whole. In other words, these questions ask for the main point of *one small part* of the passage.

Inferences

These questions ask about ideas that are not explicitly stated in a passage but are *implied* by the author. Unlike questions about supporting details, which ask about information that is directly stated in a passage, inference questions ask about ideas or meanings that must be inferred from information that is directly stated. Authors can make their points in indirect ways, suggesting ideas without actually stating them. Inference questions measure your ability to understand an author's intended meaning in parts of a passage where the meaning is only suggested. These questions do not ask about meanings or implications that are remote from the passage; rather, they ask about meanings that are developed indirectly or implications that are specifically suggested by the author.

To answer these questions, you may have to

- logically take statements made by the author one step beyond their literal meanings
- recognize an alternative interpretation of a statement made by the author
- identify the intended meaning of a word used figuratively in a passage

If a passage explicitly states an effect, for example, you may be asked to infer its cause. If the author compares two phenomena, you may be asked to infer the basis for the comparison. You may be asked to infer the characteristics of an old policy from an explicit description of a new one. When you read a passage, you should concentrate not only on the explicit meaning of the author's words, but also on the more subtle meaning implied by those words.

Applying information to a context outside the passage itself

These questions measure your ability to discern the relationships between situations or ideas presented by the author and other situations or ideas that might parallel those in the passage. In this kind of question, you may be asked to

- identify a hypothetical situation that is comparable to a situation presented in the passage
- select an example that is similar to an example provided in the passage
- apply ideas given in the passage to a situation not mentioned by the author
- recognize ideas that the author would probably agree or disagree with on the basis of statements made in the passage

Unlike inference questions, application questions use ideas or situations *not* taken from the passage. Ideas and situations given in a question are *like* those given in the passage, and they parallel ideas and situations in the passage; therefore, to answer the question, you must do more than recall what you read. You must recognize the essential attributes of ideas and situations presented in the passage when they appear in different words and in an entirely new context.

Logical structure

These questions require you to analyze and evaluate the organization and logic of a passage. They may ask you

- how a passage is constructed—for instance, does it define, compare or contrast, present a new idea, or refute an idea?
- how the author persuades readers to accept his or her assertions
- the reason behind the author's use of any particular supporting detail
- to identify assumptions that the author is making
- to assess the strengths and weaknesses of the author's arguments
- to recognize appropriate counterarguments

These questions measure your ability not only to comprehend a passage but also to evaluate it critically. However, it is important for you to realize that logical structure questions do not rely on any kind of formal logic, nor do they require you to be familiar with specific terms of logic or argumentation. You can answer these questions using only the information in the passage and careful reasoning.

About the style and tone

Style and tone questions ask about the expression of a passage and about the ideas in a passage that may be expressed through its diction—the author's choice of words. You may be asked to deduce the author's attitude to an idea, a fact, or a situation from the words that he or she uses to describe it. You may also be asked to select a word that accurately describes the tone of a passage—for instance, "critical," "questioning," "objective," or "enthusiastic."

To answer this type of question, you will have to consider the language of the passage as a whole. It takes more than one pointed, critical word to make the tone of an entire passage "critical." Sometimes, style and tone questions ask what audience the passage was probably intended for or what type of publication it probably appeared in. Style and tone questions may apply to one small part of the passage or to the passage as a whole. To answer them, you must ask yourself what meanings are contained in the words of a passage beyond the literal meanings. Did the author use certain words because of their emotional content, or because a particular audience would expect to hear them? Remember, these questions measure your ability to discern meaning expressed by the author through his or her choice of words.

7.2 Test-Taking Strategies

1. **Do not expect to be completely familiar with any of the material presented in reading comprehension passages.**
   You may find some passages easier to understand than others, but all passages are designed to present a challenge. If you have some familiarity with the material presented in a passage, do not let this knowledge influence your choice of answers to the questions. Answer all questions on the basis of what is *stated or implied* in the passage itself.