Answers and Explanations

SECTION 3
Verbal Reasoning
25 Questions with Explanations

For questions 1 to 8, select <u>one</u> entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

1. Although plant and animal species that become established in ecosystems where they did not originate are sometimes referred to by the alarming term "invasive species," many such species are _____ in their new environments.

(A)	innocuous
(B)	conspicuous
(C)	robust
(D)	menacing
(E)	distinctive

Explanation

The sentence begins with "Although," indicating that the correct answer will contrast in tone with the "alarming term 'invasive species.'" The only answer choice that provides the necessary contrast is "innocuous." All the other choices are consistent with being alarming.

Thus, the correct answer is **innocuous** (Choice A).

2. Far from being _____ the corporate world because of cutbacks, serious researchers are playing a growing role in innovation at many firms.

(A)	lured to
(B)	enchanted with
(C)	banished from
(D)	protected by
(E)	immured in

Explanation

The words "Far from being" and the mention of "cutbacks" imply that the correct answer will create a contrast with the idea that "serious researchers are playing a growing role in innovation." Being "banished from" the corporate world and playing a growing role in it are strongly contrasted. Choices A, B, D, and E do not create any such contrast.

Thus, the correct answer is **banished from** (Choice C).

3. The brief survey, published under the title *The Work of Nature: How the Diversity of Life Sustains Us*, is surprisingly (i)_____. Indeed it makes several longer treatments of the effects of lost biodiversity seem (ii)_____.

Blank (i)

(A) distorted
(B) objective
(C) comprehensive

Blank (ii)

(D) redundant
(E) pithy
(F) premature

Explanation

In this question, it is hard to select an answer for either blank in isolation. The "brief survey," like any survey, could in fact be surprisingly "distorted," "objective," or "comprehensive." The contrast in the following sentence with "longer treatments" suggests that the brevity of the survey is important. Of the three choices, only "comprehensive" is particularly unexpected of a *brief* survey.

Provisionally accepting "comprehensive" makes it easier to analyze the second blank. If the short survey is surprisingly comprehensive, then longer treatments may not convey any useful additional information, making them "redundant." Longer works are unlikely to seem "pithy" in comparison to shorter ones, and "premature" makes little sense in this context.

Reading the sentence again with "comprehensive" and "redundant" filling the blanks confirms that these two choices result in a coherent whole.

Thus, the correct answer is **comprehensive** (Choice C) and **redundant** (Choice D).

4. The government has no choice but to (i)_____ the incessant demands for land reform, and yet any governmental action that initiated land reform without requisite attention to agrarian reform would (ii)_____ the overall goal of economic modernization.

Blank (i)

(A) anticipate
(B) heed
(C) silence

Blank (ii)

(D) delineate
(E) condone
(F) compromise

Explanation

The sentence informs us that the government has an "overall goal," and the use of "and yet" indicates that initiating land reform without attending to agrarian reform would have some negative consequence for that goal. Thus, the only answer choice that fits the second blank is "compromise." The use of "and yet" also implies that the government's response to the demands for land reform must be in line with initiating such reform, so Choice C, "silence," is incorrect. The description of the demands as "incessant" implies that the demands already exist, so "anticipate" is incorrect. Only "heed" describes an appropriate response.

Thus, the correct answer is **heed** (Choice B) and **compromise** (Choice F).

5. Certain music lovers yearn for (i)_____, but when it is achieved, there is something missing; perhaps they feel uncomfortable in a world where nothing discernible is (ii) _____.

Blank (i)

(A) novelty
(B) beauty
(C) flawlessness

Blank (ii)

(D) wrong
(E) visionary
(F) changed

Explanation

The structure of the sentence alerts us that it will describe a kind of contradiction or paradox: music lovers want something, but when they get what they want, they discover some cause for dissatisfaction. Among the answer choices, the only two words that produce such a paradox are "flawlessness" and "wrong"—the music lovers long for flawlessness but are unsatisfied with a world in which nothing is wrong.

Thus, the correct answer is **flawlessness** (Choice C) and **wrong** (Choice D).

6. Putting a cash value on the ecological services provided by nature—such as the water filtration "service" provided by a forested watershed—has, historically, been a (i) _____ process. Early attempts at such valuation resulted in impressive but (ii) _____ figures that were seized on by environmental advocates, and then, when these figures were later (iii)_____, they were used by opponents to tar the whole idea.

Blank (i)

(A) dispassionate

B	problematic
C	straightforward

Blank (ii)

D	redundant
E	unsound
F	understated

Blank (iii)

G	ignored
H	discredited
I	confirmed

Explanation

The correct response for the first blank cannot be determined without considering the second sentence. The correct choice for the second blank, however, can be determined more readily. Neither Choice D, "redundant," nor Choice F, "understated," makes sense when coupled with the preceding "impressive but." Since the figures were "used by opponents to tar the whole idea," Choice E, "unsound," is the word that makes the most sense in this context.

Once "unsound" is selected for the second blank, it follows that "confirmed" cannot be correct for the third blank. And if the figures were "used by opponents," then they cannot have been "ignored." Since the figures were unsound, it is natural that they would later be "discredited."

Now that the figures have been characterized as unsound and discredited, it is possible to identify the correct response for the first blank. From the second sentence it is clear that the process of putting a cash value on the ecological services provided by nature is neither "dispassionate" nor "straightforward." It is instead "problematic."

Thus, the correct answer is **problematic** (Choice B); **unsound** (Choice E); and **discredited** (Choice H).

7. Only with the discovery of an ozone hole over Antarctica in 1985 did chemical companies finally relinquish their opposition to a ban on chlorofluorocarbons (CFCs), which destroy ozone. The discovery suggested that strong political action to halt production of CFCs might be (i)_____, and fortunately, the chemical industry no longer felt compelled to oppose such action: although companies had recently (ii)_____ their research into CFC substitutes, studies they had initiated years earlier had produced (iii)_____ results.

Blank (i)

A	imminent
B	imprudent
C	premature

Blank (ii)

(D) corroborated
(E) publicized
(F) curtailed

Blank (iii)

(G) encouraging
(H) inconclusive
(I) unsurprising

Explanation

According to the first sentence, chemical companies opposed a ban on CFCs and then changed their stance in 1985 with the discovery of an ozone hole.

In the first blank, only "imminent" is compatible with the discovery of an ozone hole linked to CFCs: such a discovery would suggest that "strong political action" is required, not that it is "imprudent" or "premature."

What follows the colon in the second sentence explains why the chemical industry "no longer felt compelled to oppose" a ban on CFCs. In order for the industry to drop its opposition, the outcome of the studies into CFC substitutes must have been positive. Among the choices for the third blank, only "encouraging" has a sufficiently positive connotation.

Finally, the word "although" after the colon indicates that the second blank should contrast with the third blank in some way. Since the completed third blank now indicates that studies of CFC substitutes have been successful, "curtailed" makes the most sense in the second blank. "Corroborated" and "publicized" do not contrast appropriately with the success of the studies.

Thus, the correct answer is **imminent** (Choice A); **curtailed** (Choice F); and **encouraging** (Choice G).

8. The incipient (i)_____ regarding taxes could affect trade between the two countries much more than the (ii)_____ banana imports, which has been going on for years. Unfortunately, the trade regulators seem to be ignoring both disagreements.

Blank (i)

(A) row
(B) accord
(C) investigation

Blank (ii)

(D) profitable dealing in
(E) predicament regarding
(F) festering dispute over

Explanation

The words "both disagreements" at the end of the second sentence indicate that both blanks should be filled with words or phrases that are synonyms for "disagreement." "Row" and "festering dispute over" are the only choices that describe kinds of disagreement. (Note that the word "row" has many meanings, one of which is "a quarrel.")

Thus, the correct answer is **row** (Choice A) and **festering dispute over** (Choice F).

For each of questions 9 to 14, select <u>one</u> answer choice unless otherwise instructed.

Questions 9 and 10 are based on the following reading passage.

Fossil bones of the huge herbivorous dinosaurs known as sauropods were first discovered and studied between 1840 and 1880, providing evidence for the gargantuan dimensions of the adults. The shape of sauropod teeth suggested what they ate. But aside from trackways, or series of fossilized footprints—which established that sauropods

line 5 at least occasionally lived in herds—fossils incorporating direct evidence of other behavior, such as reproductive behavior, have been almost nonexistent. Because no modern land animals even approach sauropod size, scientists have also lacked a living analogue to use as a guide to possible sauropod behavior. Until the recent discovery of fossilized sauropod nesting grounds, scientists were thus uncertain whether sauropods

10 laid eggs or gave birth to live young.

Description

The passage outlines what was learned about sauropods after the discovery of their fossilized bones in the nineteenth century, including what has been inferred about their behavior from the fossil record.

For the following question, consider each of the choices separately and select all that apply.

9. Which of the following can be inferred from the passage regarding the evidence provided by sauropod teeth?

   A The teeth allow inferences to be made about sauropod social behavior.

   B The shape of the teeth indicates that sauropods were herbivorous.

   C The teeth have no resemblance to those of any modern land animal.

Explanation

Choice B is the only correct answer.

Choice A is incorrect: the passage mentions that fossilized footprints permit the inference that sauropods exhibited herd behavior, but there is no indication that this or any other social behavior can be inferred from sauropod teeth.

Choice B is correct: the passage states that sauropods were herbivorous (feeding on plants)

and that "the shape of sauropod teeth suggested what they ate."

Choice C is incorrect: the passage says that there are no modern land animals similar in size to sauropods, not that there are no such animals with similar teeth.

For the following question, consider each of the choices separately and select all that apply.

10. Which of the following can be inferred from the passage regarding the recently discovered fossilized sauropod nesting grounds?

A They are among the few fossils incorporating direct evidence of sauropod behavior.

B They confirm the evidence provided by trackways about sauropod behavior.

C They have forced a reevaluation of theories regarding the nature of sauropod herd behavior.

Explanation

Choice A is the only correct answer.

Choice A is correct: the last sentence of the passage implies that the discovery of fossilized sauropod nesting grounds resolved the question of whether sauropods laid eggs or gave birth to live young and therefore provided evidence of sauropods' reproductive behavior. Until this discovery, except for the trackways that showed herd behavior, "fossils incorporating direct evidence" of sauropod behavior were "almost nonexistent" (lines 5—6).

Choice B is incorrect: according to the passage, trackways provided evidence of herd behavior, whereas fossilized nesting grounds provided evidence concerning reproductive behavior. There is no information in the passage to suggest that the nesting grounds confirmed evidence provided by the trackways, or even that the trackway evidence needed confirming.

Choice C is incorrect: the passage presents fossilized nesting grounds as providing evidence about reproductive behavior, not herd behavior. Nothing in the passage suggests that there are theories of the nature of sauropod herd behavior that have been reevaluated.

Questions 11 to 14 are based on the following reading passage.

Some researchers contend that sleep plays no role in the consolidation of declarative memory (i.e., memory involving factual information). These researchers note that people with impairments in rapid eye movement (REM) sleep continue to lead normal lives, and they argue that if sleep were crucial for memory, then these individuals
line 5 would have apparent memory deficits. Yet the same researchers acknowledge that the cognitive capacities of these individuals have never been systematically examined, nor have they been the subject of studies of tasks on which performance reportedly depends on sleep. Even if such studies were done, they could only clarify our understanding of the role of REM sleep, not sleep in general.

10 These researchers also claim that improvements of memory overnight can be explained by the mere passage of time, rather than attributed to sleep. But recent studies of memory performance after sleep—including one demonstrating that sleep stabilizes declarative memories from future interference caused by mental activity during

wakefulness—make this claim unsustainable. Certainly there are memory
15 consolidation processes that occur across periods of wakefulness, some of which neither depend on nor are enhanced by sleep. But when sleep is compared with wakefulness, and performance is better after sleep, then some benefit of sleep for memory must be acknowledged.

Description

The passage presents and then rebuts two arguments made by researchers who question the contribution of sleep to the consolidation of declarative memory (memory involving factual information). The first argument is that people with impairments to REM sleep continue to lead normal lives. In response, the passage says that these researchers themselves acknowledge the absence of systematic study of such individuals' cognitive abilities, study that would be necessary in order to fully support the researchers' claim. The passage also points out that the researchers' claim applies only to REM sleep rather than to sleep in general. The second claim is that improvements of memory that occur overnight might be explained merely by the passage of time. In response, the passage cites research findings that demonstrate the role of sleep in stabilizing declarative memory.

11. The primary purpose of the passage is to

(A) present the evidence that supports a particular claim regarding REM sleep and memory
(B) describe how various factors contribute to the effect of sleep on memory
(C) argue against a particular position regarding sleep's role in memory
(D) summarize the most prevalent theory regarding sleep and memory
(E) defend the importance of the consolidation of declarative memory

Explanation

As described above, the purpose of the passage as a whole is to argue against the view held by some researchers that sleep plays no role in the consolidation of declarative memory. Therefore, **Choice C** is correct. Choice A is incorrect: the passage does mention REM sleep twice in the first paragraph, but its primary purpose is not to examine REM sleep in particular, and it does not present evidence related to REM sleep. Choice B is incorrect: the passage is concerned with the effect of sleep on memory, but not with any factors that contribute to that effect. Choice D is incorrect: the passage does not summarize a theory. Instead, it cites a claim and then assesses and rejects that claim. Choice E is incorrect: although the passage is about the consolidation of declarative memory, it does nothing to defend its importance.

12. According to the author of the passage, which of the following generalizations about memory and sleep is true?

(A) There are some memory-consolidation processes that have nothing to do with sleep.
(B) Sleep is more important to the consolidation of declarative memory than to the consolidation of other types of memory.
(C) REM sleep is more important to memory consolidation than is non-REM sleep.

(D) There are significant variations in the amount of sleep that people require for the successful consolidation of memory.

(E) It is likely that memory is more thoroughly consolidated during wakefulness than during sleep.

Explanation

The passage states that "there are memory-consolidation processes that occur across periods of wakefulness." Accordingly, **Choice A** is correct. Choices B, C, and D are incorrect: the passage does not discuss types of memory other than consolidative memory, the relative importance to consolidative memory of REM and non-REM sleep, or differences among individuals in the amount of sleep they require. Choice E is also incorrect: the passage suggests that the truth is the opposite of what this answer choice states. The last sentence of the passage indicates that performance on memory tasks has been found to be better after sleep than after periods of wakefulness.

13. Which of the following best describes the function of the sentence in lines 14—16 ("Certainly … sleep")?

(A) It provides the reasoning behind a claim about the role of sleep in memory consolidation.

(B) It explains why a previous claim about sleep and memory is unsustainable.

(C) It demonstrates why wakefulness is central to the process of declarative memory consolidation.

(D) It emphasizes the limited role sleep plays in the process of declarative memory consolidation.

(E) It concedes that the consolidation of declarative memory does not depend entirely on one factor.

Explanation

The cited sentence begins with the word "Certainly," a clue that the sentence will concede that the researchers are not entirely wrong: in this instance, they are not wrong about memory consolidation occurring during periods of wakefulness. Thus, **Choice E** is correct. Choice A is incorrect: the sentence deals with memory consolidation during wakefulness, not with the role of sleep in memory consolidation. Choice B is incorrect: the sentence does follow an assertion that the researchers' claim is unsustainable, but it does not explain why it is unsustainable. Choice C is incorrect: the sentence does not demonstrate anything. It acknowledges that memory consolidation occurs during wakeful periods but does not identify wakefulness as central to the process. Choice D is incorrect: while the sentence does acknowledge that some memory-consolidation processes are not dependent on sleep, it does not go so far as to claim that sleep plays a limited role in memory consolidation generally.

14. The importance of the study mentioned in lines 12—14 is that it

(A) reveals the mechanism by which declarative memory is stabilized during sleep

(B) identifies a specific function that sleep plays in the memory-consolidation process

(C) demonstrates that some kinds of mental activity can interfere with memory consolidation

(D) suggests that sleep and wakefulness are both important to memory consolidation

(E) explains how the passage of time contributes to memory consolidation

Explanation

The question asks what "the importance of the study mentioned in lines 12—14" is. The study is described as having shown that sleep stabilizes declarative memory from future interference caused by mental activity during wakefulness. This protection of memory from interference is the "specific function" played by sleep mentioned in Choice B. Therefore, **Choice B** is correct. Choice A is incorrect: there is no description of any mechanism, or specific process, by which declarative memory is stabilized. Although Choices C, D, and E each involve issues connected with the study, those connections are all tangential.

For questions 15 to 19, select the <u>two</u> answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole <u>and</u> produce completed sentences that are alike in meaning.

15. In American Indian art, the supposed distinction between modern and traditional was fabricated by critics, and when artists have control over interpretation of their own work, the distinction appears, happily, to have been _____.

[A] eliminated

[B] reinforced

[C] put to rest

[D] intensified

[E] recognized

[F] established

Explanation

By characterizing the distinction as "supposed" and "fabricated," the sentence indicates that the distinction has no basis in reality. Accordingly, when the sentence reports a happy outcome, this must mean that the distinction has been abandoned or rejected. Only "eliminated" and "put to rest" convey that sense; all the other answer choices suggest that the distinction is maintained, or even strengthened.

Thus, the correct answer is **eliminated** (Choice A) and **put to rest** (Choice C).

16. Notwithstanding their _____ regarding other issues, township residents have consistently passed the board of education's annual budget.

[A] accord

[B] indecision

C consensus

D disagreement

E divergence

F enthusiasm

Explanation

By using the word "Notwithstanding," the sentence sets up a contrast between the township residents' behavior regarding the "other issues" and their behavior regarding the board's annual budget, which they have "consistently passed." "Accord" and "consensus" are similar in meaning but do not provide the required contrast. Only "disagreement" and "divergence" provide the necessary contrast and lead to two sentences nearly alike in meaning. "Indecision" fits the context, but there is no other word among the possible choices that matches it closely.

Thus, the correct answer is **disagreement** (Choice D) and **divergence** (Choice E).

17. Some of the company's supporters charged that the negative report had been motivated by a broader political assault on the company that was designed to help market rivals who would like to see the company _____.

A reined in

B bolstered

C indemnified

D propped up

E manacled

F lionized

Explanation

The "market rivals" would clearly like to see the company experience some negative outcome. Only "reined in" and "manacled" describe such an outcome; the other choices all describe positive results for the company.

Thus, the correct answer is **reined in** (Choice A) and **manacled** (Choice E).

18. Skeptics contend that any scheme for charging visitors to Web sites that rewards the vendor adequately would require steep prices, _____ the kind of frequent, casual use of Web sites that surfers now take for granted.

A bridling

B exciting

C forbidding

D inhibiting

E provoking

F reversing

Explanation

The sentence concerns skeptics' reaction to a plan to generate revenue by charging visitors to Web sites. To justify the skeptics' reaction, the "steep prices" must be associated with a decrease in visitor volume. Only "bridling" and "inhibiting" are consistent with this logic and result in sentences nearly alike in meaning. "Forbidding" is too strong: steep prices might dissuade a casual visitor, but they would not forbid one. Although "exciting" and "provoking" can be similar in meaning, they do not fit the logic of the sentence.

Thus, the correct answer is **bridling** (Choice A) and **inhibiting** (Choice D).

19. It seems obvious that Miles Davis' _____ the Juilliard School, which resulted in his decision to drop out, was based on the school's training of musicians for a kind of music that he did not want to play.

A disaffection with

B dislocation of

C disentanglement from

D subversion of

E displacement of

F estrangement from

Explanation

The sentence asserts a logical relationship between Davis' attitude toward the school (as indicated in the blank) and his "decision to drop out." Only "disaffection with" and "estrangement from" are consistent with a decision to drop out and result in sentences nearly alike in meaning.

Thus, the correct answer is **disaffection with** (Choice A) and **estrangement from** (Choice F).

For each of questions 20 to 25, select <u>one</u> answer choice unless otherwise instructed.

Question 20 is based on the following reading passage.

Astronomers found a large body orbiting close to the star Upsilon Andromedae. The standard theory of planet formation holds that no planet that large could be formed so close to a star, leading to the suggestion that the body is a companion star. A subsequent discovery puts that suggestion in doubt: two other large bodies were found orbiting close to Upsilon Andromedae, and the standard theory of companion stars allows for at most one companion star.

20. Which of the following, if true, most helps to resolve the status of the orbiting body without casting doubt on the two standard theories mentioned?

(A) The smaller a planet orbiting a star is, and the farther away it is from the star, the less likely it is to be discovered.

(B) If a planet's orbit is disturbed, the planet can be drawn by gravity toward the star it is orbiting.

(C) The largest of the bodies orbiting Upsilon Andromedae is the farthest away from the star, and the smallest is the nearest.

(D) It is likely that there are many stars, in addition to Upsilon Andromedae and the Sun, that are orbited by more than one smaller body.

(E) In most cases of companion stars, the smaller companion is much fainter than the larger star.

Explanation

The passage outlines a conflict between two standard theories—one of planet formation, the other of companion stars—and observations of one large body, and later two others, orbiting close to a star. The question asks what would resolve this conflict without casting doubt on either one of the theories.

Choice B is correct: if, as it asserts, it is possible for a planet to be formed relatively far from a star and later move closer to it, then the observed large bodies found close to Upsilon Andromedae can be planets without casting doubt on the standard theory of planet formation. This explanation also leaves the standard theory of companion stars intact.

Choice A is incorrect because it describes difficulties with discovering a small planet far from a star, not anything pertaining to a large body near a star. Choice C is incorrect as well, since whatever the relative size and position of the three bodies may be, all three appear to be too close according to the standard theories. Choice D is incorrect because the pervasiveness of stars with multiple orbiting bodies has nothing to do with the status of the large bodies discussed in the passage. Choice E is similarly irrelevant and thus incorrect: information about the brightness of a star relative to its companion star does not help clarify the status of the large bodies discussed in the passage.

Question 21 is based on the following reading passage.

In Gilavia, the number of reported workplace injuries has declined 16 percent in the last five years. However, perhaps part of the decline results from injuries going unreported: many employers have introduced safety-incentive programs, such as prize drawings for which only employees who have a perfect work-safety record are eligible. Since a workplace injury would disqualify an employee from such programs, some employees might be concealing injury, when it is feasible to do so.

21. Which of the following, if true in Gilavia, most strongly supports the proposed explanation?

(A) In the last five years, there has been no decline in the number of workplace injuries leading to immediate admission to a hospital emergency room.

(B) Employers generally have to pay financial compensation to employees who suffer work-related injuries.

(C) Many injuries that happen on the job are injuries that would be impossible to conceal

and yet would not be severe enough to require any change to either the employee's work schedule or the employee's job responsibilities.

(D) A continuing shift in employment patterns has led to a decline in the percentage of the workforce that is employed in the dangerous occupations in which workplace injuries are likely.

(E) Employers who have instituted safety-incentive programs do not in general have a lower proportion of reported workplace injuries among their employees than do employers without such programs.

Explanation

The question asks what would support the claim that the decline in reported workplace injuries in Gilavia may be the result of incentives for workers to not report those injuries that they can conceal. If the number of injuries that cannot be concealed—such as injuries requiring immediate emergency care—has not declined in the same period, that could help bolster the claim that the decline in overall reported injuries may be a result of concealable injuries going unreported rather than an actual decline in workplace injuries in general, so **Choice A** is correct.

If employers have to provide financial compensation to employees injured on the job, employees would have an incentive to report injuries. More reported injuries would not support the author's argument, making Choice B incorrect. Choice C is incorrect because the fact that some injuries that cannot be concealed do not result in lost time or changed responsibilities has nothing to do with whether concealable injuries are going unreported. While a decline in dangerous occupations could well result in a decrease in workplace injuries, this fact would challenge the author's argument, not support it, so Choice D is incorrect. Similarly, if employers with safety-incentive programs do not see any drop in reported injuries compared to employers without such programs, the author's argument would be weakened, not supported, making Choice E incorrect.

Questions 22 and 23 are based on the following reading passage.

The attribution of early-nineteenth-century English fiction is notoriously problematic. Fewer than half of new novels published in Britain between 1800 and 1829 had the author's true name printed on the title page. Most of these titles have subsequently been attributed, either through the author's own acknowledgment of a previously anonymous or pseudonymous work or through bibliographical research. One important tool available to researchers is the list of earlier works "by the author" often found on title pages. But such lists are as likely to create new confusion as they are to solve old problems. Title pages were generally prepared last in the publication process, often without full authorial assent, and in the last-minute rush to press, mistakes were frequently made.

line 5

10

Description

The passage discusses the reasons why identifying the authors of early-nineteenth-century British fiction poses significant challenges. The passage explains that few authors during this period used their real names and goes on to describe how title pages can facilitate—but also hamper—efforts to attribute these works.

For the following question, consider each of the choices separately and select all that apply.

22. The passage suggests that which of the following factors contributes to the "notoriously problematic" (line 1) nature of authorial attribution in early-nineteenth-century English fiction?

A The unwillingness of any writers to acknowledge their authorship of works that were originally published anonymously or pseudonymously

B The possibility that the title page of a work may attribute works written by other authors to the author of that work

C The possibility that the author's name printed on a title page is fictitious

Explanation

Choices B and C are correct.

Choice A is incorrect: the passage mentions that the attribution of early-nineteenth-century fiction was sometimes achieved when the author came forward to acknowledge a previously anonymous work (lines 4—5), so Choice A can be eliminated.

Choice B is correct: in lines 5—7, the passage mentions that "one important tool available to researchers is the list of earlier works 'by the author' often found on title pages," but goes on to say that these title pages were prepared hastily and "frequently" contained mistakes (lines 8—10). Since the mistake most likely to "create new confusion" would be the inclusion of works not written by the author, Choice B may be inferred.

Choice C is correct: in lines 2—3, the passage states, "Fewer than half of the new novels published in Britain … had the author's true name printed on the title page." Line 5 suggests that pseudonyms—fictitious names—were commonly used. Hence, Choice C may be inferred.

For the following question, consider each of the choices separately and select all that apply.

23. The passage suggests that which of the following is frequently true of the title pages of early-nineteenth-century English novels?

A The title page was prepared for printing in a hurried manner.

B Material on the title page was included without the author's knowledge or approval.

C Information on the title page was deliberately falsified to make the novel more marketable.

Explanation

Choices A and B are correct.

Choice A is correct: the passage mentions that title pages were prepared last and that mistakes often occurred "in the last-minute rush to press" (line 9). This indicates that title pages were often prepared for printing in a hurried manner; hence, Choice A can be inferred.

Choice B is correct: the passage includes the detail that title pages were often prepared for printing "without full authorial assent" (line 9); hence, Choice B can be inferred.

Choice C is incorrect: nowhere does the passage speculate about commercial motives for falsifying information on title pages. Choice C, therefore, cannot be inferred.

Questions 24 and 25 are based on the following reading passage.

The more definitions a given noun has, the more valuable is each one. Multiple definitions, each subtly different from all the others, convey multiple shades of meaning. They expand the uses of the word; language is enriched, thought is widened, and interpretations increase or dilate to fill the potentialities of association. The very impossibility

line 5 of absoluteness in the definition of certain nouns adds to the levels of connotation they may reach. The inner life of a writer often says more than most readers can know; the mind of a reader can discover truths that go beyond the intent or perhaps even the comprehension of the writer. And all of it finds expression because a word can mean many things.

Description

The passage claims that the capacity of words to have multiple meanings can greatly enhance the resources of a language. The passage describes how this increases possibilities for interpretation and the expression of ideas, thus enriching the relationship between readers and writers.

24. In the context in which it appears, "shades" (line 2) most nearly means

(A) reminders
(B) nuances
(C) obscurities
(D) coverings
(E) degrees

Explanation

In the context in which it appears, "shades" is used to refer to the subtle distinctions in meaning that are made possible by "multiple definitions, each subtly different from all the others." Since the sentence deals with multiple definitions conveying meaning, Choice C ("obscurities") and the more literal Choice D ("coverings") may be eliminated. In deciding between Choice B and Choice E, one should bear in mind that the sentence focuses on subtle differences in meaning as opposed to different degrees of emphasis for the same meaning. **Choice B**, "nuances," best captures this sense of slight variations in meaning and is therefore the correct answer.

25. The passage suggests that a writer's use of nouns that have multiple definitions can have which of the following effects on the relationship between writer and reader?

(A) It can encourage the reader to consider how the writer's life might have influenced the work.
(B) It can cause the reader to become frustrated with the writer's failure to distinguish between subtle shades of meaning.

(C) It can allow the reader to discern in a work certain meanings that the writer did not foresee.

(D) It allows the writer to provide the reader with clues beyond the word itself in order to avoid ambiguity.

(E) It allows the writer to present unfamiliar ideas to the reader more efficiently.

Explanation

Lines 7—8 clearly indicate that multiple meanings of words enable readers to "discover truths that go beyond the intent or perhaps even the comprehension of the writer"; hence, **Choice C** is the correct answer. Choices A, B, D, and E all deal with topics that are not mentioned in the passage: the writer's life, the reader's frustration, the avoidance of ambiguity, and the question of how efficiently multiple definitions can aid in the presentation of unfamiliar ideas.

SECTION 4
Verbal Reasoning
25 Questions with Explanations

For questions 1 to 8, select <u>one</u> entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

1. The unexplained digressions into the finer points of quantum electrodynamics are so _____ that even readers with a physics degree would be wise to keep a textbook handy to make sense of them.

(A)	uninteresting
(B)	controversial
(C)	unsophisticated
(D)	frustrating
(E)	humorless

Explanation

An initial reading of this sentence might suggest that the blank should be filled with a word like "complex" that indicates how hard it is to "make sense of" the digressions. However, there is no such word among the answer choices. Focusing on the second half of the sentence suggests a different interpretation. According to the sentence, it would be "wise to" make sense of the digressions, and a textbook would help the reader to do so. If the digressions are "uninteresting," "unsophisticated," or "humorless," the sentence provides no reason to think it would be wise to make sense of them, and if they are "controversial," it provides no reason to think that a textbook would help. Only if the digressions are "frustrating" does the sentence make a coherent whole.

Thus, the correct answer is **frustrating** (Choice D).

2. The belief that politicians might become _____ after their election to office led to the appointment of ethics officers at various levels of government.

(A)	scrupulous
(B)	entrenched
(C)	venal
(D)	puzzled
(E)	artificial

Explanation

If a certain belief led to the appointment of ethics officers, that belief must concern some ethical issue. Of the choices provided, only "venal" fits that context. Although several of the other choices are not necessarily positive characteristics, none of them involves ethics.

Thus, the correct answer is **venal** (Choice C).

3. Even the charisma and technical prowess of two fine actors are not _____ the task of fully invigorating a gray domestic drama with a tired tale to tell.

(A)	required for
(B)	interested in
(C)	preferred for
(D)	adequate to
(E)	inferior to

Explanation

The "task" described in the second half of the sentence clearly presents the actors with a challenge. The "even the" followed by positive characteristics indicates that the actors did not meet that challenge; "adequate to" is the only answer choice that conveys this sense.

Thus, the correct answer is **adequate to** (Choice D).

4. There may be a threshold below which blood pressure reductions become _____ given that a long-running study showed no decreased heart risk for drops in blood pressure below a certain point.

(A)	worthwhile
(B)	indiscernible
(C)	arduous
(D)	significant
(E)	superfluous

Explanation

The portion of the sentence that begins with "given that" provides a reason for a conclusion reached in the first part of the sentence. Since the study "showed no decreased heart risk for

drops in blood pressure below a certain point," that point may be a threshold below which reductions in blood pressure provide no benefit; that is, they may be "superfluous."

Thus, the correct answer is **superfluous** (Choice E).

5. Unlike the problems in recent financial scandals, issues raised by the regulators in this case appear largely to pertain to unwieldy accounting rules that are open to widely divergent interpretations—not to (i)_____ transactions designed to (ii)_____ corporate malfeasance.

Blank (i)

(A) sham
(B) unpremeditated
(C) justifiable

Blank (ii)

(D) cloak
(E) ameliorate
(F) illuminate

Explanation

The "Unlike" at the beginning of the sentence and the "not to" that follows the dash set up a contrast between the relatively innocent problems in the current case and the issues involved in the "recent financial scandals." Clearly, these latter issues must have involved wrongdoing. Looking at the second blank, only transactions designed to "cloak" corporate malfeasance would qualify: both ameliorating and illuminating malfeasance are positive actions. For the first blank, only "sham" fits; "unpremeditated" or "justifiable" transactions could not be designed to cloak malfeasance.

Thus, the correct answer is **sham** (Choice A) and **cloak** (Choice D).

6. Everyone has routines that govern their work. The myth is that artists are somehow different, that they reject (i)_____, but of course that's not true: most artists work as the rest of us do, (ii)_____, day by day, according to their own customs.

Blank (i)

(A) latitude
(B) habit
(C) materialism

Blank (ii)

(D) impetuously
(E) ploddingly

F	sporadically

Explanation

The passage conveys the sense that artists are like everyone else in that they have "routines that govern their work." This view is contrasted with a myth that artists are "somehow different." In the first blank, only "habit" is something whose rejection presents a contrast with being governed by work routines. Rejecting "latitude" might well match being governed by work routines, and though "materialism" is sometimes rejected by artists, it is not relevant to having work routines. The second blank describes how artists "work as the rest of us do"; only "ploddingly" is consistent with the emphasis on routines and "day by day" work.

Thus, the correct answer is **habit** (Choice B) and **ploddingly** (Choice E).

7. Repression of painful memories is sometimes called "willed forgetting." Yet true forgetting is (i)_____ than the phenomenon of repressed memory. In spite of the effort that it (ii) _____, repressing unwanted memories is less (iii)_____ than truly forgetting them, for repressed memories are prone to come back.

Blank (i)

A	less controlled
B	different in its effect
C	far more common

Blank (ii)

D	eases
E	conveys
F	entails

Blank (iii)

G	permanent
H	arduous
I	immediate

Explanation

This question is best answered by first completing the third blank.

The third sentence sets up a comparison between repressing memories and forgetting them. The word "for" indicates that the last part of the sentence—"repressed memories are prone to come back"—presents the basis of that comparison. Choice G, "permanent," is the only choice that is related to the tendency to come back.

Working backward, the sentence begins with "In spite of," suggesting that the correct choice for the second blank is contrary to what one might expect. One would ordinarily expect that something entailing effort would be more rather than less permanent. Neither "eases" nor

"conveys" sets up such an expectation.

Filling the second and third blanks makes it possible to fill the first blank. Nothing in the completed text suggests that true forgetting is "more common" or "less controlled" than the repression of painful memories, but it does suggest that true forgetting is different in its effect—it is more permanent. Thus, Choice B, "different in its effect," is correct.

Thus, the correct answer is **different in its effect** (Choice B), **entails** (Choice F), and **permanent** (Choice G).

8. Rather than viewing the Massachusetts Bay Colony's antinomian controversy as the inevitable (i)_____ of the intransigent opposing forces of radical and (ii)_____ beliefs, male and female piety, (iii)_____ and secular power, and the like, as other critics have, Winship argues that the crisis was not "fixed and structural."

Blank (i)

(A) dissolution
(B) melding
(C) collision

Blank (ii)

(D) revolutionary
(E) orthodox
(F) questionable

Blank (iii)

(G) clerical
(H) civil
(I) cerebral

Explanation

The words "Rather than" indicate that the other critics, unlike Winship, think of the controversy as "fixed and structural." Since both "dissolution" and "melding" of "intransigent opposing forces" would tend to lessen the controversy, only "collision" (Choice C) fits the first blank. The second and third blanks appear in a series of examples of such opposing forces; only "orthodox" contrasts with "radical" in the second blank and only "clerical" contrasts with "secular" in the third blank.

Thus, the correct answer is **collision** (Choice C), **orthodox** (Choice E), and **clerical** (Choice G).

For each of questions 9 to 14, select <u>one</u> answer choice unless otherwise instructed.

Questions 9 to 12 are based on the following reading passage.

Until recently, many anthropologists assumed that the environment of what is now the southwestern United States shaped the social history and culture of the region's indigenous peoples. Building on this assumption, archaeologists asserted that adverse environmental conditions and droughts were responsible for the disappearances and migrations of southwestern populations from many sites they once inhabited.

However, such deterministic arguments fail to acknowledge that local environmental variability in the Southwest makes generalizing about that environment difficult. To examine the relationship between environmental variation and sociocultural change in the Western Pueblo region of central Arizona, which indigenous tribes have occupied continuously for at least 800 years, a research team recently reconstructed the climatic, vegetational, and erosional cycles of past centuries. The researchers found it impossible to provide a single, generally applicable characterization of environmental conditions for the region. Rather, they found that local areas experienced different patterns of rainfall, wind, and erosion, and that such conditions had prevailed in the Southwest for the last 1,400 years. Rainfall, for example, varied within and between local valley systems, so that even adjacent agricultural fields can produce significantly different yields.

The researchers characterized episodes of variation in southwestern environments by frequency: low-frequency environmental processes occur in cycles longer than one human generation, which generally is considered to last about 25 years, and high-frequency processes have shorter cycles. The researchers pointed out that low-frequency processes, such as fluctuations in stream flow and groundwater levels, would not usually be apparent to human populations. In contrast, high-frequency fluctuations such as seasonal temperature variations are observable and somewhat predictable, so that groups could have adapted their behaviors accordingly. When the researchers compared sequences of sociocultural change in the Western Pueblo region with episodes of low- and high-frequency environmental variation, however, they found no simple correlation between environmental process and sociocultural change or persistence.

Although early Pueblo peoples did protect themselves against environmental risk and uncertainty, they responded variously on different occasions to similar patterns of high-frequency climatic and environmental change. The researchers identified seven major adaptive responses, including increased mobility, relocation of permanent settlements, changes in subsistence foods, and reliance on trade with other groups. These findings suggest that groups' adaptive choices depended on cultural and social as well as environmental factors and were flexible strategies rather than uncomplicated reactions to environmental change. Environmental conditions mattered, but they were rarely, if ever, sufficient to account for sociocultural persistence and change. Group size and composition, culture, contact with other groups, and individual choices and actions were —barring catastrophes such as floods or earthquakes—more significant for a population's survival than were climate and environment.

Description

The passage describes research that bears on a presumed historical relationship between environmental variation and sociocultural change among indigenous people of the southwestern United States. The author mentions in the first paragraph that many anthropologists believed until recently that environmental variations explain changes in the human populations of the region. The passage then goes on to point out studies that show problems with this explanation, including the lack of generally applicable characterizations of the environment in the region and lack of correlation between environmental changes and sociocultural changes. In the final paragraph the author mentions an alternative explanation in researchers' findings suggesting that responses to environmental changes varied according to differing factors such as group size and composition, culture, contact with other groups, and individual choices.

9. The passage is primarily concerned with

(A) explaining why certain research findings have created controversy
(B) pointing out the flaws in a research methodology and suggesting a different approach
(C) presenting evidence to challenge an explanation and offering an alternative explanation
(D) elucidating the means by which certain groups have adapted to their environment
(E) defending a long-held interpretation by presenting new research findings

Explanation

As the description above indicates, **Choice C** is the best answer: the passage introduces an explanation, presents evidence that challenges it, and offers an alternative explanation. The passage does not mention the creation of controversy or discuss flaws in research methodology; therefore, Choices A and B are incorrect. Although the passage reports findings that different groups used different adaptive responses to environmental conditions, there is no focus on the adaptations used by particular groups, so Choice D is incorrect. The passage presents recent research findings but not in defense of a long-held interpretation; therefore, Choice E is incorrect.

10. Which of the following findings would most strongly support the assertion made by the archaeologists mentioned in line 3?

(A) A population remained in a certain region at least a century after erosion wore away much of the topsoil that sustained grass for their grazing animals.
(B) The range of a certain group's agricultural activity increased over a century of gradual decrease in annual rainfall.
(C) As winters grew increasingly mild in a certain region, the nomadic residents of the region continued to move between their summer and winter encampments.
(D) An agricultural population began to trade for supplies of a grain instead of producing the grain in its own fields as it had in the past.
(E) A half century of drought and falling groundwater levels caused a certain population to abandon their settlements along a riverbank.

Explanation

The archaeologists mentioned in line 3 asserted that adverse environmental conditions caused southwestern populations to move or disappear. The question asks which finding would support this assertion.

Choices A, B, and C all describe populations that did *not* move away or disappear in the face of environmental changes, and hence are all incorrect. Choice D is incorrect because it does not mention a change in environmental conditions and therefore cannot support an assertion about the effects of changing environmental conditions. **Choice E** is the best answer: it mentions an adverse environmental change (a long drought) that caused a population to leave the site it had inhabited, which would support the archaeologists' assertion that such environmental changes caused such population changes.

11. The fact that "adjacent agricultural fields can produce significantly different yields" (lines 16—17) is offered as evidence of the

(A) unpredictability of the climate and environment of the southwestern United States
(B) difficulty of producing a consistent food supply for a large population in the Western Pueblo region
(C) lack of water and land suitable for cultivation in central Arizona
(D) local climatic variation in the environment of the southwestern United States
(E) high-frequency environmental processes at work in the southwestern United States

Explanation

Choice D is the correct answer: the second paragraph says rainfall variations between local valleys cause different agricultural yields between adjacent fields and gives this as an example of how climate is not uniform within the Southwest but rather can vary significantly from place to place. Choice A is incorrect: while such variability might give rise to unpredictability, that is not how the difference in agricultural yields is being used as evidence in the passage. Choices B and C are incorrect: the passage does not make or report a claim about feeding large populations, nor does it assert that central Arizona lacks land suitable for cultivation. Choice E is incorrect: a discussion of high- and low-frequency processes occurs in the third paragraph, but the author does not present geographic differences in rainfall and agricultural yield as either a high- or a low-frequency environmental process.

12. It can be inferred from the passage that which of the following activities is NOT an example of a population responding to high-frequency environmental processes?

(A) Developing watertight jars in which to collect and store water during the rainy season
(B) Building multistory dwellings in low-lying areas to avoid the flash flooding that occurs each summer
(C) Moving a village because groundwater levels have changed over the last generation
(D) Trading with other groups for furs from which to make winter clothes.
(E) Moving one's herds of grazing animals each year between summer and winter pastures

Explanation

The phrasing of the question indicates that all but one of the answer choices are examples of a population responding to a high-frequency environmental process. You are asked to choose the one answer choice that does not provide such an example. Choices A, B, D, and E are incorrect because they all present responses to high-frequency environmental processes: developing water-storage jars to adapt to seasonal rainfall variations, adapting dwellings in response to seasonal flooding, trading to acquire clothing in adaptation to seasonal temperature variations, and moving grazing herds seasonally. **Choice C** is the best answer: the passage mentions fluctuations in ground water levels as a *low*-frequency process (lines 21—22); moving a village because of a change that takes place over the course of a generation is *not* a response to a high-frequency process.

Questions 13 and 14 are based on the following reading passage.

Arctic sea ice comes in two varieties. Seasonal ice forms in winter and then melts in summer, while perennial ice persists year-round. To the untrained eye, all sea ice looks similar, but by licking it, one can estimate how long a particular piece has been floating around. When ice begins to form in seawater, it forces out salt, which has no place
line 5 in the crystal structure. As the ice gets thicker, the rejected salt collects in tiny pockets of brine too highly concentrated to freeze. A piece of first-year ice will taste salty. Eventually, if the ice survives, these pockets of brine drain out through fine, veinlike channels, and the ice becomes fresher; multiyear ice can even be melted and drunk.

Description

The passage describes two varieties of Arctic sea ice and explains how the freezing process causes seasonal ice to taste much saltier than perennial ice.

For the following question, consider each of the choices separately and select all that apply.

13. The passage mentions which of the following as being a characteristic of seasonal ice?

A It is similar in appearance to perennial ice.

B It is typically filled with fine, veinlike channels.

C It tastes saltier than perennial ice.

Explanation

Choices A and C are correct.

Choice A is correct: the passage states that "to the untrained eye, all sea ice looks similar" (lines 2—3).

Choice B is incorrect: it is clear that perennial ice contains fine, veinlike channels, but the passage does not mention whether seasonal ice contains them.

Choice C is correct: in lines 6—8, the passage establishes that first-year ice tastes salty but eventually gets fresher if the ice survives.

14. In the context in which it appears, "fine" (line 7) most nearly means

(A) acceptable
(B) elegant
(C) precise
(D) pure
(E) small

Explanation

"Fine" appears in the context of an explanation of how the brine drains out; in such a context, it must be being used to describe a physical characteristic of the channels. In addition, the word "Eventually" implies that the draining is a slow process. Only **Choice E**, "small," helps to explain why the process is slow and is therefore the best choice. None of the other choices contributes to the explanation.

For questions 15 to 18, select the <u>two</u> answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole <u>and</u> produce completed sentences that are alike in meaning.

15. It would have been disingenuous of the candidate to appear _____ when her opponent won the election, but she congratulated the victor nonetheless.

[A] gracious
[B] ecstatic
[C] crestfallen
[D] indifferent
[E] euphoric
[F] disgruntled

Explanation

To answer the question, one must understand what sort of reaction on the part of a losing candidate would appear "disingenuous." Certainly "ecstatic" and "euphoric" reactions would be highly disingenuous or insincere. "Gracious" also fits the blank, but there is no other word offered that is nearly alike in meaning.

Thus, the correct answer is **ecstatic** (Choice B) and **euphoric** (Choice E).

16. As market forces penetrate firms and bid up the value of attributes of labor that are more measurable than is the knowledge born of experience, it can be expected that trends in wages will not _____ those whose main value lies in such experiential knowledge.

[A] favor
[B] aid
[C] affect
[D] forsake

E betray

F differentiate

Explanation

The sentence states that market forces are bidding up the value of certain attributes of labor that are "more measurable than is the knowledge born of experience." The blank has to do with trends in wages for those whose main value in the labor force lies in "experiential knowledge." Since experiential knowledge appears to be losing value in the bidding war for labor, the blank needs to be filled in a way that leads to something negative. Given the "not" that precedes the blank, "favor" and "aid" make for such an outcome and result in sentences alike in meaning.

Thus, the correct answer is **favor** (Choice A) and **aid** (Choice B).

17. The point we might still take from the First World War is the old one that wars are always, as one historian aptly put it, _____: they produce unforeseeable results.

A unsurprising

B astounding

C conventional

D ruinous

E stunning

F devastating

Explanation

The colon after the blank indicates a definitional relationship between the blanked word and the phrase that follows the colon. The two answer choices for which "they produce unforeseeable results" would most clearly serve as a definition are "astounding" and "stunning." While "ruinous" and "devastating" might be adjectives describing the effects of war, they clearly do not fit the logical structure of this sentence, since they are not by definition "unforeseeable."

Thus, the correct answer is **astounding** (Choice B) and **stunning** (Choice E).

18. This is the kind of movie—stuffed with intimations of faraway strife and people in suits talking frantically on cell phones and walkie-talkies—that is conventionally described as a political thriller, but the film is as apolitical as it is _____.

A intense

B unprecedented

C subtle

D humdrum

E refined

F dull

Explanation

The sentence suggests that the film is not well described by the conventional term "political thriller." The film is not political but rather apolitical, and the phrase "as apolitical as it is …" sets up a parallel between "apolitical" and the blanked word; therefore, the blanked word should go against the term "political thriller" in the same way that "apolitical" does. "Humdrum" and "dull" are the opposite of "thrilling" and are therefore the best choices.

Thus, the correct answer is **humdrum** (Choice D) and **dull** (Choice F).

For each of questions 19 to 25, select <u>one</u> answer choice unless otherwise instructed.

Questions 19 and 20 are based on the following reading passage.

Historians credit repeated locust invasions in the nineteenth century with reshaping United States agriculture west of the Mississippi River. Admonished by government entomologists, farmers began to diversify. Wheat had come to nearly monopolize the region, but it was particularly vulnerable to the locusts. In 1873, just before the locusts' most withering offensive, nearly two-thirds of Minnesota farmland was producing wheat; by the invasions' last year, that fraction had dropped to less than one-sixth. Farmers learned that peas and beans were far less vulnerable to the insects, and corn was a more robust grain than wheat. In addition to planting alternative crops, many farmers turned to dairy and beef production. Although pastures were often damaged by the locusts, these lands were almost always left in better shape than the crops were.

line 5

10

Description

The passage explains how the damage caused by repeated invasions of locusts in the nineteenth century caused farmers west of the Mississippi River to diversify. Since wheat, the dominant crop in the region, was especially susceptible to damage from locusts, it made sense for farmers to lower their wheat production and raise their production of other crops and animals less vulnerable to locust invasions.

For the following question, consider each of the choices separately and select all that apply.

19. According to the passage, before the recommendations by the government entomologists, which of the following was true about farming west of the Mississippi River?

A Farmers focused primarily on growing wheat.

B Peas and beans had not yet been planted in the region.

C A relatively small portion of farmland was devoted to crops other than wheat.

Explanation

Choices A and C are correct.

Choice A is correct: according to the passage, "wheat had come to nearly monopolize the region" prior to the recommendations of government entomologists.

Choice B is incorrect: although wheat was the dominant crop, there is no indication that peas and beans had not been planted in the region prior to the admonishments of government entomologists.

Choice C is correct: given that wheat was the dominant crop, only a relatively small portion of farmland could have been devoted to other crops.

20. In the context in which it appears, "robust" (line 8) most nearly means

(A) crude
(B) demanding
(C) productive
(D) vigorous
(E) rich

Explanation

In discussing the advantages of less vulnerable crops, the author describes corn as "robust." Of the choices presented, "vigorous" is most similar in meaning to "robust." Neither "crude" nor "demanding" is an advantage, and although being "productive" or "rich" might be desirable, neither matches the meaning of "robust" in this context. Therefore, **Choice D** is the correct answer.

Question 21 is based on the following reading passage.

In 1998 the United States Department of Transportation received nearly 10,000 consumer complaints about airlines; in 1999 it received over 20,000. Moreover, the number of complaints per 100,000 passengers also more than doubled. In both years the vast majority of complaints concerned flight delays, cancellations, mishandled baggage, and customer service. Clearly, therefore, despite the United States airline industry's serious efforts to improve performance in these areas, passenger dissatisfaction with airline service increased significantly in 1999.

21. Which of the following, if true, most seriously weakens the argument?

(A) Although the percentage of flights that arrived on time dropped slightly overall, from 77 percent in 1998 to 76 percent in 1999, some United States airlines' 1999 on-time rate was actually better than their 1998 on-time rate.
(B) The number of passengers flying on United States airlines was significantly higher in 1999 than in 1998.
(C) Fewer bags per 1,000 passengers flying on United States airlines were lost or delayed in 1999 than in 1998.
(D) The appearance in 1999 of many new Internet sites that relay complaints directly to the Department of Transportation has made filing a complaint about airlines much easier for consumers than ever before.
(E) Although the number of consumer complaints increased for every major United States airline in 1999, for some airlines the extent of the increase was substantial, whereas for

others it was extremely small.

Explanation

The passage describes two different year-over-year increases in airline passenger complaints: both the absolute number of complaints and the rate of complaints more than doubled from 1998 to 1999. From these facts, the author of the passage concludes that passenger dissatisfaction with airline service significantly increased in the same period.

Choice D is the correct answer: it weakens the argument because it presents a scenario in which the increase in complaints and in the rate of complaints could merely be the result of an easier means of filing complaints, not an actual increase in passenger dissatisfaction.

Choice A and Choice C are incorrect because neither presents a scenario that bears directly on the claim that passenger dissatisfaction increased from 1998 to 1999. Choice E is incorrect: the fact that some airlines experienced a smaller increase than others does not change the fact that all airlines experienced an increase and thus cannot weaken the argument. Choice B could be correct if the passage discussed only the change in the absolute number of complaints, since more passengers could account for more complaints without entailing an increase in dissatisfaction. But the passage also says that the rate of complaints increased, making Choice B incorrect.

Questions 22 to 24 are based on the following reading passage.

Nineteenth-century architect Eugène-Emmanuel Viollet-le-Duc contended that Paris's Notre-Dame cathedral, built primarily in the late twelfth century, was supported from the very beginning by a system of flying buttresses—a series of exterior arches (flyers) and their supports (buttresses)—which permitted the construction of taller vaulted buildings with slimmer walls and interior supports than had been possible previously. Other commentators insist, however, that Notre-Dame did not have flying buttresses until the thirteenth or fourteenth century, when they were added to update the building aesthetically and correct its structural flaws. Although post-twelfth-century modifications and renovations complicate efforts to resolve this controversy—all pre-fifteenth-century flyers have been replaced, and the buttresses have been rebuilt and/or resurfaced—it is nevertheless possible to tell that both the nave and the choir, the church's two major parts, have always had flying buttresses. It is clear, now that nineteenth-century paint and plaster have been removed, that the nave's lower buttresses date from the twelfth century. Moreover, the choir's lower flyers have chevron (zigzag) decoration. Chevron decoration, which was characteristic of the second half of the twelfth century and was out of favor by the fourteenth century, is entirely absent from modifications to the building that can be dated with confidence to the thirteenth century.

Description

The passage describes a disagreement about when Notre-Dame cathedral was supported by flying buttresses, with Viollet-le-Duc arguing that buttresses were present from the cathedral's construction in the late twelfth century and others claiming the buttresses were built later. The author of the passage goes on to present evidence that suggests that Viollet-le-Duc's argument is correct.

22. The passage is primarily concerned with

(A) tracing the development of a controversy
(B) discussing obstacles to resolving a controversy
(C) arguing in support of one side in a controversy
(D) analyzing the assumptions underlying the claims made in a controversy
(E) explaining why evidence relevant to a controversy has been overlooked

Explanation

As the description above indicates, **Choice C** is correct: the passage supports one side in a controversy. Choice A is incorrect because while the passage describes a controversy, it makes no mention of how that controversy developed. The passage also does not discuss any obstacles to resolving the controversy, any assumptions underlying the claims in the controversy, or any reasons why pertinent evidence may have been overlooked, so Choice B, Choice D, and Choice E are all incorrect.

23. The claim of the "other commentators" (line 6) suggests that they believe which of the following about Notre-Dame?

(A) It was the inspiration for many vaulted cathedrals built in the thirteenth and fourteenth centuries.
(B) Its design flaws were not apparent until flying buttresses were added in the thirteenth or fourteenth century.
(C) Its flying buttresses are embellished with decoration characteristic of the thirteenth and fourteenth centuries.
(D) It had been modified in some respects before flying buttresses were added in the thirteenth or fourteenth century.
(E) It was originally constructed in an architectural style that was considered outmoded by the thirteenth or fourteenth century.

Explanation

The passage states that the "other commentators" claim that Notre-Dame first received flying buttresses when it was updated for aesthetic and structural reasons in the thirteenth or fourteenth century. This claim thus suggests that the aesthetics of Notre-Dame were then seen as out of date, making **Choice E** correct. Choice A is incorrect because the passage does not include any information about other cathedrals, let alone attribute a view of them to the other commentators. While the other commentators do suggest that the design of Notre-Dame was seen as flawed in the thirteenth or fourteenth century, they say that flying buttresses were added to correct these flaws, not that the flaws became apparent after the addition of the flying buttresses, which makes Choice B incorrect. Choice C is incorrect because the passage does not attribute any views of the embellishments on the flying buttresses to the other commentators; similarly, Choice D is incorrect because the passage does not describe the other commentators as discussing any modifications prior to the thirteenth or fourteenth century.

24. The author's argument concerning Notre-Dame's flying buttresses depends on which of the following assumptions about the choir's lower flyers?

(A) They accurately reproduce the decoration on the choir's original lower flyers.
(B) They have a type of decoration used exclusively for exterior surfaces.
(C) They were the models for the choir's original upper flyers.
(D) They were the models for the nave's original lower flyers.
(E) They were constructed after the nave's flyers were constructed.

Explanation

The author supports the claim that flying buttresses were present on Notre-Dame from the twelfth century by noting that the choir's lower flyers feature a chevron decoration that was characteristic of the twelfth century. But since all flyers constructed prior to the fifteenth century have been replaced, the chevron decorations can indicate only that flyers were present in the twelfth century if those decorations accurately reproduce the decorations that existed on the original flyers. Thus, **Choice A** is the correct answer.

Choice B is incorrect: whether chevron decorations are used only on the exterior is not a point of dispute in the passage. Choices C, D, and E are all incorrect: no part of the argument turns on any claim about the choir's upper flyers, the nave's lower flyers, or the sequence in which the choir's and the nave's flyers were constructed.

Question 25 is based on the following reading passage.

The average temperature of the lobster-rich waters off the coast of Foerkland has been increasing for some years. In warmer water, lobsters grow faster. In particular, lobster larvae take less time to reach the size at which they are no longer vulnerable to predation by young cod, the chief threat to their survival. Consequently, the survival rate of lobster larvae must be going up, and the lobster population in Foerkland's coastal waters is bound to increase.

25. Which of the following, if true, most seriously weakens the argument?

(A) There are indications that in recent years the fishing fleet operating off the coast of Foerkland has been taking cod at an unsustainably high rate.
(B) The increase in water temperatures off Foerkland has not been as pronounced as the increase in average soil temperatures in Foerkland.
(C) Because of their speeded-up growth, lobsters now get large enough to be legal catch before they reach reproductive maturity.
(D) Even though lobsters grow faster in warmer waters, warmer waters have no effect on the maximum size to which a lobster can eventually grow.
(E) Cod are a cold-water species, and the increasing water temperatures have caused a northward shift in Foerkland's cod population.

Explanation

The argument in the passage concludes that the survival rate of lobster larvae is increasing and

that the lobster population will increase. The basis for the conclusion is that the change in the water temperature, by speeding the growth of lobster larvae, has made them less vulnerable to predation by cod. However, Choice C points to a way that the faster growth of individual lobsters could create a threat to the population: lobsters that have not yet reproduced might be large enough to be legally caught. Thus, **Choice C** weakens the argument and is the correct answer.

Among the other choices, Choice A tends, if anything, to support the passage's conclusion, by suggesting further reduction in the risks of predation by cod. The other choices have no clear bearing on the argument.

SECTION 5
Quantitative Reasoning
25 Questions with Explanations

For each of Questions 1 to 9, select one of the following answer choices.

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

1.

Quantity A

$\dfrac{3^{-1}}{4^{-1}}$

Quantity B

$\dfrac{4}{3}$

(A) (B) (C) (D)

Explanation

In this question, you are asked to compare $\dfrac{3^{-1}}{4^{-1}}$ with $\dfrac{4}{3}$. Recall that if a is a nonzero number, then $a^{-1} = \dfrac{1}{a}$ and $\dfrac{1}{a^{-1}} = a$. Using these rules of exponents, you can see that

$$\frac{3^{-1}}{4^{-1}} = (3^{-1})\left(\frac{1}{4^{-1}}\right) = \left(\frac{1}{3}\right)(4) = \frac{4}{3}$$

Thus, $\dfrac{3^{-1}}{4^{-1}} = \dfrac{4}{3}$, and the correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

2.

$$x < 1 \text{ and } x \neq 0$$

Quantity A

$x^2 + 1$

Quantity B

$x^3 + 1$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that $x < 1$ and $x \neq 0$, and you are asked to compare $x^2 + 1$ with $x^3 + 1$. One way to approach this problem is to set up a comparison between the two quantities using a placeholder symbol to represent the relationship between them as follows.

$$x^2 + 1 \boxed{?} x^3 + 1$$

Then simplify the comparison.

Step 1: Subtract 1 from both sides to get

$$x^2 \boxed{?} x^3$$

Step 2: Since $x \neq 0$, you can divide both sides by the positive quantity x^2 to get

$$1 \boxed{?} x$$

Since you are given that $x < 1$, or $1 > x$, you can conclude that the placeholder $\boxed{?}$ in the simplified comparison $1 \boxed{?} x$ represents *greater than* $(>)$. Note that the strategy of simplifying the comparison requires you to consider whether the steps in the simplification are reversible. This is because you must arrive at a conclusion about the initial comparison, not the simplified comparison. If you follow the simplification steps in reverse, you can see that the placeholder in each step remains unchanged: $1 > x$ implies $x^2 > x^3$ because multiplying by the positive number x^2 retains the inequality *greater than* $(>)$. Also, $x^2 > x^3$ implies $x^2 + 1 > x^3 + 1$. Therefore, Quantity A is greater than Quantity B, and the correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

3.

$$x > 0$$

Quantity A

0.5% of x

Quantity B

$\frac{1}{2}x$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that $x > 0$, and you are asked to compare 0.5% of x with $\frac{1}{2}x$. Recall that $\frac{1}{2}$ of a number is the same as 50% of the number. Therefore, $\frac{1}{2}x$ is equal to 50% of x. Since $x > 0$, it follows that 50% of x is greater than 0.5% of x. The correct answer is **Choice B**.

You could also make the comparison by rewriting 0.5% of x as a fraction of x and then comparing the result with $\frac{1}{2}x$. Rewrite 0.5% of x as a fraction of x as follows.

$$\frac{0.5}{100}x = \frac{5}{1{,}000}x$$

Since $x > 0$, it follows that $\frac{5}{1{,}000}x$ is less than $\frac{1}{2}x$, and Quantity B is greater than Quantity A. The correct answer is **Choice B**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

4.

The median income of a group of College *C* graduates six months after graduation was $3,000 higher than the median income of a group of College *D* graduates six months after graduation.

Quantity A

The 75th percentile of the incomes of the group of College *C* graduates six months after graduation

Quantity B

The 75th percentile of the incomes of the group of College *D* graduates six months after graduation

Ⓐ Ⓑ Ⓒ Ⓓ

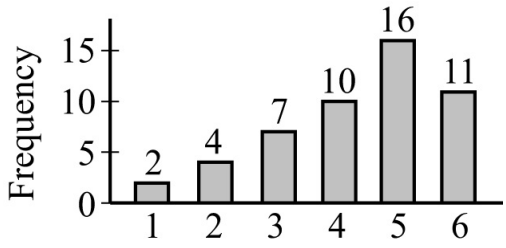
Explanation

In this question, you are asked to compare the 75th percentiles of the incomes of two groups of college graduates six months after graduation. The only information you are given is that the median income of the group in Quantity A is $3,000 greater than the median income of the group in Quantity B.

Recall that the median of a group of numbers is the middle number (or the average of the two middle numbers) when the numbers are listed from least to greatest. The median is also equal to the 50th percentile. The median does not indicate anything about the spread of the numbers in the group. In particular, for each group of incomes, you do not know how much greater than the median the 75th percentile of the group of incomes is, nor do you know the relationship between the 75th percentiles of the two groups. Since the relationship between Quantity A and Quantity B cannot be determined, the correct answer is **Choice D**.

This explanation uses the following strategies.
Strategy 8: Search for a Mathematical Relationship
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

5.



The graph above shows the frequency distribution of 50 integer values varying from 1 to 6.

Quantity A

The average (arithmetic mean) of the 50 values

Quantity B

The median of the 50 values

(A) (B) (C) (D)

Explanation

In this question, you are given a graph of the frequency distribution of 50 integer values and are asked to compare the average (arithmetic mean) with the median of the distribution.

In general, the median of a group of n values, where n is even, is obtained by ordering the values from least to greatest and then calculating the average (arithmetic mean) of the two middle values. So, for the 50 values shown in the graph, the median is the average of the 25th and 26th values, both of which are equal to 5. Therefore, the median of the 50 values is 5.

Once you know that the median of the 50 values is 5, the comparison simplifies to comparing the average of the 50 values with 5. You can make this comparison without actually calculating the average by noting from the graph that of the 50 values,

11 values are 1 unit above 5,

16 values are equal to 5,

10 values are 1 unit below 5, and

13 values are more than 1 unit below 5.

Since the part of the distribution that is below 5 contains 23 values—13 of which are more than 1 unit below 5—and the part of the distribution that is above 5 contains 11 values—none of which is more than 1 unit above 5—the average (arithmetic mean) of the 50 values must be less than 5. The correct answer is **Choice B**.

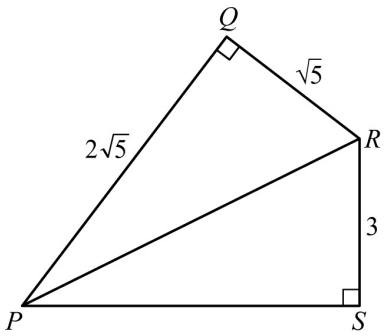
Alternatively, you can calculate the average of the 50 values as follows.

$$\frac{(1)(2)+(2)(4)+(3)(7)+(4)(10)+(5)(16)+(6)(11)}{50}=\frac{217}{50}$$

Thus the average of the 50 values, $\frac{217}{50}$, or 4.34, is less than the median of the 50 values, 5. The correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

6.



Quantity A

The area of triangle PQR

Quantity B

The area of triangle PSR

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the area of triangle PQR with the area of triangle PSR. Note that both triangles are right triangles and that line segment PR is the hypotenuse of both triangles. Recall that the area of a triangle is equal to one-half the product of a base and the height corresponding to the base. Also, for any right triangle, the lengths of the two legs of the triangle are a base and the corresponding height.

The area of triangle PQR: In the figure, it is given that the length of leg PQ is $2\sqrt{5}$ and the length of leg QR is $\sqrt{5}$. Therefore, you can conclude that the area of triangle PQR is $\frac{1}{2}(2\sqrt{5})(\sqrt{5})$, or 5.

The area of triangle PSR: To calculate the area of triangle PSR, you need to know the lengths of the legs PS and RS. From the figure, you know that the length of RS is 3, but you do not know the length of PS. How can you determine the length of PS? If, in addition to the length of RS, you knew the length of hypotenuse PR, you could use the Pythagorean theorem to determine the length of PS. So, to find the length of PS, you first need to find the length of hypotenuse PR.

Recall that PR is also the hypotenuse of triangle PQR. The lengths of legs PQ and QR of triangle PQR are $2\sqrt{5}$ and $\sqrt{5}$, respectively. By the Pythagorean theorem,

$$\begin{aligned}(PR)^2 &= (PQ)^2 + (QR)^2\\ &= \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2\\ &= 20 + 5\\ &= 25\end{aligned}$$

Thus, the length of PR is $\sqrt{25}$, or 5.

Returning to triangle PSR, you now know that the length of hypotenuse PR is 5 and the length of leg RS is 3. Therefore, by the Pythagorean theorem,

$$3^2 + (PS)^2 = 5^2$$
$$9 + (PS)^2 = 25$$
$$(PS)^2 = 25 - 9$$
$$(PS)^2 = 16$$

and the length of PS is 4.

Since legs PS and RS have lengths 4 and 3, respectively, the area of triangle PSR is $\frac{1}{2}(4)(3)$, or 6. Recall that you have already determined that the area of triangle PQR is 5. So Quantity B, the area of triangle PSR, is greater than Quantity A, the area of triangle PQR, and the correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

7.

Quantity A

The sum of the odd integers from 1 to 199

Quantity B

The sum of the even integers from 2 to 198

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

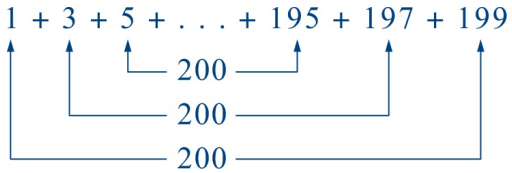
In this question, you are asked to compare the sum of the odd integers from 1 to 199 with the sum of the even integers from 2 to 198. Both of these sums involve many integers. How many integers are in each sum? Note that there are 200 integers from 1 to 200, where 100 of them are even and 100 of them are odd. The 100 odd integers are precisely the odd integers in Quantity A, whereas the 100 even integers include one more integer, 200, than the even integers in Quantity B. So Quantity A is the sum of 100 integers and Quantity B is the sum of 99 integers.

It would be very time-consuming to write out all the terms in each sum and add them together. Therefore, it is reasonable to find a more efficient way to calculate the sums or to find a way to compare the sums without actually calculating them. To find a more efficient way to calculate the two sums, it is often useful to look for ways to rearrange the terms in the sum so that they can be added more easily. You can begin by writing a few terms from the beginning and the end of the sum.

For the sum of the 100 odd integers from 1 to 199, you could write

$$1 + 3 + 5 + \ldots + 195 + 197 + 199$$

You can pair the odd integers in the sum and add the two integers in each pair as follows.



Note that the sum of the integers in each of the three pairs shown is 200. You can continue pairing terms in the sum in this way until all 100 terms have been rearranged in 50 pairs, where the sum of each pair is 200. It follows that

$$\begin{aligned} 1 + 3 + 5 + \ldots + 195 + 197 + 199 &= (1 + 199) + (3 + 197) + (5 + 195) + \ldots + (99 + 101) \\ &= 50(200) \\ &= 10{,}000 \end{aligned}$$

Now consider the sum of the 99 even integers from 2 to 198. For this sum, you could write

$$2 + 4 + 6 + \ldots + 194 + 196 + 198$$

In this sum, note that

the sum of the 1st and 99th terms is $2 + 198 = 200$

the sum of the 2nd and 98th terms is $4 + 196 = 200$

You can continue pairing terms in this way until 98 of the 99 terms in the sum have been rearranged into 49 pairs and the 50th term is unpaired. Note that the unpaired term is 100 (the 50th positive even integer). It follows that

$$\begin{aligned} 2 + 4 + \ldots + 98 + 100 + 102 + \ldots + 196 + 198 &= (2 + 198) + (4 + 196) + \ldots + (98 + 102) + 100 \\ &= 49(200) + 100 \end{aligned}$$

$= 9{,}900$

Therefore, Quantity A, 10,000, is greater than Quantity B, 9,900, and the correct answer is **Choice A**.

Alternatively, you can try to compare the two sums without actually calculating them. Recall that Quantity A is the sum of the 100 odd integers from 1 to 199, and Quantity B is the sum of the 99 even integers from 2 to 198. Write each sum with the terms in increasing order, as follows, pairing the *n*th term in Quantity B with the *n*th term in Quantity A and noting that there is no term in Quantity B that is paired with the 100th term, 199, in Quantity A.

Quantity A: $1 + 3 + 5 + \ldots + 193 + 195 + 197 + 199$

Quantity B: $2 + 4 + 6 + \ldots + 194 + 196 + 198$

Note that each of the 99 terms in Quantity B is 1 more than its paired term in Quantity A, so Quantity B is 99 more than the sum of all the terms in Quantity A excluding the last term, 199. Consequently, Quantity A is $199 - 99$ more than Quantity B—that is, 100 more than Quantity B—and the correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 7: Find a Pattern

8.

s and *t* are positive integers, and $32^s = 2^t$.

Quantity A

$\frac{s}{t}$

Quantity B

$\frac{1}{5}$

(A) (B) (C) (D)

Explanation

In this question, you are given that *s* and *t* are positive integers and that $32^s = 2^t$, and you are asked to compare $\frac{s}{t}$ with $\frac{1}{5}$. Since the expression $\frac{s}{t}$ involves the variables *s* and *t*, you need to look for a relationship between *s* and *t* using the equation $32^s = 2^t$.

If the two bases in this equation were equal, then the exponents would be equal. However, one of the bases is 32 and the other is 2. This suggests making the two bases equal by rewriting 32 as a power of 2 if it is possible to do so. In fact, $32 = 2^5$. Therefore, $32^s = (2^5)^s = 2^{5s}$, and the equation $32^s = 2^t$ can be rewritten as $2^{5s} = 2^t$. In the rewritten equation, the bases are equal, so you can conclude that $5s = t$.

Since $5s = t$, it follows that $\frac{s}{t} = \frac{1}{5}$. Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

9. In a quality-control test, 50 boxes—each containing 30 machine parts—were examined for defective parts. The number of defective parts was recorded for each box, and the average (arithmetic mean) of the 50 recorded numbers of defective parts per box was 1.12. Only one error was made in recording the 50 numbers: "1" defective part in a certain box was incorrectly recorded as "10".

Quantity A

The actual average number of defective parts per box

Quantity B

0.94

(A) (B) (C) (D)

Explanation

In this question, you are given that the number of defective parts in each of 50 boxes was recorded and that the average of the 50 recorded numbers was 1.12. You are also given that an error was made in recording one of the 50 numbers—the number 10 was recorded instead of the number 1—so the actual number of defective parts in this box is 9 less than the recorded number. Then you are asked to compare the actual average number of defective parts per box with 0.94.

To determine the actual average number of defective parts per box, first note that the sum of the 50 recorded numbers equals the average of the 50 recorded numbers times 50—that is, (1.12)(50), or 56.

Now you know that for 49 of the 50 boxes, the actual number of defective parts is equal to the recorded number; and for one box, the actual number is 9 less than the recorded number. From this you can conclude that the sum of the 50 actual numbers is equal to the sum of the 50 recorded numbers minus 9. So the sum of the actual numbers of defective parts is 56 − 9, or 47.

Therefore, the actual average number of defective parts per box is $\frac{47}{50}$, or 0.94. Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

10. In year *Y*, the population of Colorado was approximately half that of New Jersey, and the land area of Colorado was approximately 14 times that of New Jersey. The population density (number of persons per unit of land area) of Colorado in year *Y* was approximately how many times the population density of New Jersey?

(A) $\frac{1}{28}$

(B) $\frac{1}{14}$

Ⓒ $\frac{1}{7}$

Ⓓ $\frac{1}{4}$

Ⓔ $\frac{1}{2}$

Explanation

The information given in the question can be rewritten algebraically as follows.

$$\text{population of Colorado} \approx \left(\frac{1}{2}\right) \times (\text{population of New Jersey})$$
$$\text{land area of Colorado} \approx (14) \times (\text{land area of New Jersey})$$

Using the information given and the fact that population density is the number of persons per unit of land area, you can express the population density of Colorado in terms of the population density of New Jersey as follows.

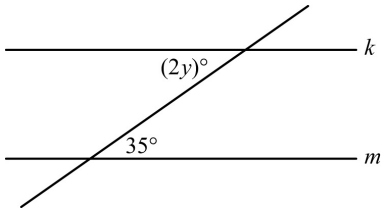
$$\begin{aligned}\text{population density of Colorado} &= \frac{\text{population of Colorado}}{\text{land area of Colorado}} \\ &\approx \frac{\left(\frac{1}{2}\right) \times (\text{population of New Jersey})}{(14) \times (\text{land area of New Jersey})} \\ &\approx \left(\frac{1}{2}\right) \times \frac{1}{14} \times \frac{\text{population of New Jersey}}{\text{land area of New Jersey}} \\ &\approx \left(\frac{1}{28}\right) \times (\text{population density of New Jersey})\end{aligned}$$

Thus, the population density of Colorado was approximately $\frac{1}{28}$ times the population density of New Jersey. The correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

For the following question, enter your answer in the box.



11. In the figure above, line k is parallel to line m. What is the value of y ?

$y =$ ☐

Explanation

In the figure, the angles with measures $(2y)°$ and 35° are between parallel lines k and m, and they are on opposite sides of the line that crosses k and m. Therefore, you can conclude that these two angles are congruent. So $2y = 35$, and $y = \frac{35}{2} = 17.5$. The correct answer is **17.5**.

This explanation uses the following strategy.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

12. The numbers in data set S have a standard deviation of 5. If a new data set is formed by adding 3 to each number in S, what is the standard deviation of the numbers in the new data set?

(A) 2
(B) 3
(C) 5
(D) 8
(E) 15

Explanation

Recall that the standard deviation of the numbers in a data set is a measure of the spread of the numbers about the mean of the numbers. The new data set is formed by adding the same number, 3, to each number in data set S. Thus, the mean of the numbers in the new data set is 3 more than the mean of the numbers in S, but the spread of the numbers in the new data set about the mean of the numbers in the new data set is the same as the spread of the numbers in S about the mean of the numbers in S. Because the standard deviation of the numbers in S is 5, the standard deviation of the numbers in the new data set is also 5. The correct answer is **Choice C**.

This explanation uses the following strategy.
Strategy 8: Search for a Mathematical Relationship

13. If $\frac{2y-3}{y} = \frac{3-y}{2}$, which of the following could be the value of y ?

(A) 4
(B) 1
(C) −1
(D) −3
(E) −5

Explanation

One approach to answer the question is to solve the equation for y as follows.

$$2(2y-3) = y(3-y)$$
$$4y-6 = 3y-y^2$$
$$y^2+y-6 = 0$$
$$(y+3)(y-2) = 0$$

Since a product equals 0 only if at least one of the factors equals 0,

$$y+3 = 0 \text{ or } y-2 = 0$$
$$y = -3 \text{ or } y = 2$$

Thus, there are two values of y that satisfy the equation, −3 and 2. The value −3 is Choice D, and the value 2 is not among the answer choices. The correct answer is **Choice D**.
Another approach is to determine, for each answer choice, whether the equation holds. To do this, you can substitute the answer choice for y in the equation $\frac{2y-3}{y} = \frac{3-y}{2}$, replace the equals sign in the equation by the place-holder symbol $\stackrel{?}{=}$, and then simplify to see whether the two expressions are in fact equal.

For Choice A, substituting $y = 4$ in the equation $\frac{2y-3}{y} = \frac{3-y}{2}$ and replacing the equals sign = with the placeholder symbol $\stackrel{?}{=}$ yields the relationship $\frac{2(4)-3}{4} \stackrel{?}{=} \frac{3-4}{2}$, which can be simplified as follows.

$$\frac{2(4)-3}{4} \stackrel{?}{=} \frac{3-4}{2}$$
$$\frac{8-3}{4} \stackrel{?}{=} \frac{-1}{2}$$
$$\frac{5}{4} \stackrel{?}{=} -\frac{1}{2}$$

Since $\frac{5}{4} \neq -\frac{1}{2}$, you can conclude that the placeholder symbol does not represent equality, and therefore the equation does not hold for $y = 4$.

If you continue evaluating the answer choices, you will find that the correct answer is **Choice D**, −3. To see that the equation $\frac{2y-3}{y} = \frac{3-y}{2}$ is true when $y = -3$, substitute $y = -3$ in the equation and replace the equals sign = with the placeholder symbol $\stackrel{?}{=}$. This yields the relationship $\frac{2(-3)-3}{-3} \stackrel{?}{=} \frac{3-(-3)}{2}$. This relationship can be simplified as follows.

$$\frac{2(-3)-3}{-3} \stackrel{?}{=} \frac{3-(-3)}{2}$$
$$\frac{-6-3}{-3} \stackrel{?}{=} \frac{3+3}{2}$$
$$\frac{-9}{-3} \stackrel{?}{=} \frac{6}{2}$$
$$3 \stackrel{?}{=} 3$$

Since 3 = 3, you can conclude that the placeholder symbol represents equality, and therefore the equation holds for $y = -3$. The correct answer is **Choice D**.

This explanation uses the following strategy.
Strategy 5: Simplify an Arithmetic or Algebraic Representation

For the following question, select all the answer choices that apply.

14. List K consists of the numbers -10, -5, 0, 5, and 10. Which of the following lists of numbers have the same range as the numbers in list K ?

    Indicate all such lists.

    A $-15, -1, 0, 1, 15$

    B $-7, -4, -2, 1, 13$

    C 0, 1, 2, 5, 8, 10

    D 2, 3, 5, 15, 19, 22

    E 4, 5, 6, 24

Explanation

Recall that the range of a list of numbers is defined as the difference between the greatest number and the least number in the list. The greatest number in list K is 10 and the least number is -10. Therefore, the range of the numbers in list K is $10 - (-10) = 10 + 10 = 20$. So, to answer the question, you need to consider each list of numbers given in the choices and determine whether that list of numbers has a range of 20.

Note that in each of the choices, the numbers are listed in order from least to greatest. Therefore, you need to look only at the first number and last number in each list to determine which lists have a range of 20. The ranges can be calculated quickly as follows.

Choice A: The greatest number is 15 and the least number is -15; therefore, the range is $15 - (-15) = 15 + 15 = 30$.

Choice B: The greatest number is 13 and the least number is -7; therefore, the range is $13 - (-7) = 13 + 7 = 20$.

Choice C: The greatest number is 10 and the least number is 0; therefore, the range is $10 - 0 = 10$.

Choice D: The greatest number is 22 and the least number is 2; therefore, the range is $22 - 2 = 20$.

Choice E: The greatest number is 24 and the least number is 4; therefore, the range is $24 - 4 = 20$.

In each of Choices B, D, and E, the range is 20. The correct answer consists of **Choices B, D, and E**.

This explanation uses the following strategy.

Strategy 8: Search for a Mathematical Relationship

15. Aisha's income in 2004 was 20 percent greater than her income in 2003. What is the ratio of Aisha's income in 2004 to her income in 2003 ?

    (A) 1 to 5

    (B) 5 to 6

    (C) 6 to 5

    (D) 5 to 1

(E) 20 to 1

Explanation

Because Aisha's income in 2004 was 20% greater than her income in 2003, her income in 2004 was equal to

(100% of her income in 2003) + (20% of her income in 2003)

which is 120% of her income in 2003. Therefore, the ratio of her income in 2004 to her income in 2003 is 120 to 100, which is equivalent to 6 to 5. The correct answer is **Choice C**.

Alternatively, to say that Aisha's income in 2004 was 20% greater than her income in 2003 is the same as saying that her income increased by $\frac{1}{5}$. Therefore, her income in 2004 was $\frac{6}{5}$ of her income in 2003, and the ratio of her income in 2004 to her income in 2003 is 6 to 5. The correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

16. Jacob's weekly take-home pay is n dollars. Each week he uses $\frac{4n}{5}$ dollars for expenses and saves the rest. At those rates, how many weeks will it take Jacob to save \$500, in terms of n ?

(A) $\frac{500}{n}$

(B) $\frac{2{,}500}{n}$

(C) $\frac{n}{625}$

(D) $\frac{n}{2{,}500}$

(E) $625n$

Explanation

It may be helpful to consider how you would determine the number of weeks it would take Jacob to save \$500 if you knew how much he saved each week. For example, suppose Jacob saved \$25 each week. At that rate, it is easy to see that it would take him 500 ÷ 25, or 20, weeks to save \$500. Using this example, you can see that the number of weeks it will take Jacob to save \$500 is equal to 500 divided by the amount he saves each week.

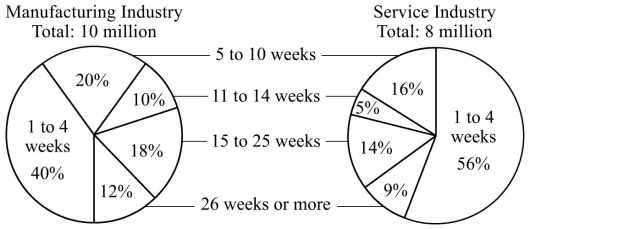
Now use the information given in the question to determine an algebraic expression representing the amount Jacob saved each week. In the question, you are given that Jacob's weekly expenses are $\frac{4n}{5}$ dollars. Therefore, the amount he saves each week is equal to his weekly take-home pay minus his weekly expenses, or $n - \frac{4n}{5} = \frac{5n}{5} - \frac{4n}{5} = \frac{5n-4n}{5} = \frac{n}{5}$ dollars.

Recall that you had already concluded that the number of weeks it will take Jacob to save \$500 is equal to 500 divided by the amount he saves each week. So the number of weeks it will take Jacob to save \$500 is $500 \div \frac{n}{5} = 500 \times \frac{5}{n} = \frac{2{,}500}{n}$. The correct answer is **Choice B**.

This explanation uses the following strategy.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

Questions 17 to 20 are based on the following data.

LENGTH OF UNEMPLOYMENT FOR WORKERS IN REGION *X* FOR TWO INDUSTRIES, 2003



Note: The circle graphs show the distributions of workers who were unemployed for at least 1 week in 2003, by length of unemployment, rounded to the nearest week.

17. In the circle graphs, the degree measure of the central angle of the sector representing the number of workers unemployed for 11 to 14 weeks is how much greater in the manufacturing industry graph than in the service industry graph?

(A) 5°
(B) 10°
(C) 15°
(D) 18°
(E) 20°

Explanation

Recall that in a circle graph, the degree measure of the central angle of a sector representing *n* percent of the data is equal to *n* percent of 360°.

The degree measure of the sector representing the number of workers unemployed for 11 to 14 weeks is 10% of 360°, or 36°, for the manufacturing industry graph and is 5% of 360°, or 18°, for the service industry graph. Since 36° − 18° = 18°, the measure of the central angle of that sector in the manufacturing industry graph is 18° greater than the measure of the central angle of the corresponding sector in the service industry graph. The correct answer is **Choice D**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

18. Which of the following could be the median length of unemployment, in weeks, for manufacturing industry workers who were unemployed for at least 1 week?

(A) 4
(B) 8
(C) 12
(D) 16
(E) 20

Explanation

Note that the sectors in the manufacturing industry circle graph separate the unemployed manufacturing industry workers into five groups by length of unemployment; also, the percent of workers within each of the five groups is given. Also note that there are 10 million lengths of unemployment, one length for each of the 10 million workers. Since the lengths are rounded to whole numbers of weeks, most of the 10 million lengths must be repetitions. The median length of unemployment is the average of the two middle lengths when the lengths are listed in order from least to greatest; that is, the median is the number at which 50% of the lengths have been listed.

To find the median length, first note that the group with the shortest unemployment lengths, 1 to 4 weeks, accounts for the first 40% of the lengths in the ordered list. Then, because the group with the next longer lengths, 5 to 10 weeks, accounts for the next 20% of the lengths in the list, the number at which 50% of the lengths have been listed is in this group. So the median length is in the 5-to-10 week interval. Among the answer choices, the only choice that is in the 5-to-10 week interval is Choice B, 8. The correct answer is **Choice B**.

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

19. If one of the workers in the manufacturing and service industries who were unemployed for at least 1 week will be randomly selected, what is the probability that the person selected will be a service industry worker who was unemployed for 26 weeks or more?

(A) 0.04
(B) 0.09
(C) 0.21
(D) 0.40
(E) 0.90

Explanation

The probability that the person selected will be a service industry worker who was unemployed for 26 weeks or more is equal to the following fraction.

$$\frac{\text{the number of service industry workers who were unemployed 26 weeks or more}}{\text{the number of workers in the two industries who were unemployed at least 1 week}}$$

The number of workers in the two industries who were unemployed for at least 1 week is the sum of the total numbers of workers represented by the two graphs, or 10 million + 8 million = 18 million.

According to the graph for the service industry, the number of workers who were unemployed for 26 weeks or more is 9% of 8 million, or 0.72 million. Therefore, the desired probability is $\frac{0.72 \text{ million}}{18 \text{ million}} = \frac{72}{1{,}800} = 0.04$. The correct answer is **Choice A**.

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

Strategy 8: Search for a Mathematical Relationship

20. The ratio of the number of manufacturing industry workers who were unemployed for 5 to 10 weeks to the number of service industry workers who were unemployed for 5 to 10 weeks is closest to which of the following?

(A) 5 to 4
(B) 6 to 5
(C) 3 to 2
(D) 5 to 2
(E) 7 to 6

Explanation

According to the graphs, the number of manufacturing industry workers who were unemployed for 5 to 10 weeks is 20% of 10 million, or 2 million; and the number of service industry workers who were unemployed for 5 to 10 weeks is 16% of 8 million, or 1.28 million. Thus, the ratio of the two numbers is 2 to 1.28.

To answer the question, you must now determine which of the answer choices is closest to the ratio 2 to 1.28. A good way to compare ratios is to express each ratio as a fraction and then as a decimal, and then to compare the decimals. The ratio 2 to 1.28 can be expressed as the fraction $\frac{2}{1.28}$, which is equal to the decimal 1.5625.

Now look at the answer choices. As you go through the answer choices, keep in mind that you are trying to determine which is closest to 1.5625.

Choice A: 5 to 4 can be expressed as $\frac{5}{4}$, which is equal to 1.25.

Choice B: 6 to 5 can be expressed as $\frac{6}{5}$, which is equal to 1.2.

Choice C: 3 to 2 can be expressed as $\frac{3}{2}$, which is equal to 1.5. Note that this ratio is close to 1.5625.

Choice D: 5 to 2 can be expressed as $\frac{5}{2}$, which is equal to 2.5.

Choice E: 7 to 6 can be expressed as $\frac{7}{6}$, which is approximately equal to 1.17.

Of the five choices, the ratio in Choice C is closest to 1.5625. The correct answer is **Choice C**.

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 5: Simplify an Arithmetic or Algebraic Representation

For the following question, select all the answer choices that apply.

21. If $|t + 3| > 5$, which of the following could be the value of t?

Indicate all such values.

A -9

B -6

C -2

D 0

E 2

F 3

Explanation

One way to approach this question is to substitute each of the answer choices into the inequality and determine which ones satisfy the inequality. If you do this, you will see that Choice A, -9, and Choice F, 3, satisfy the inequality, but the other answer choices do not. The correct answer consists of **Choices A and F**.

An algebraic approach to the question is to note that the inequality $|t + 3| > 5$ is satisfied whenever $t + 3 > 5$ or $t + 3 < -5$, that is, whenever $t > 2$ or $t < -8$. Therefore, all values of t greater than 2 or less than -8 satisfy the inequality $|t + 3| > 5$. The only answer choices that meet those conditions are -9, Choice A, and 3, Choice F. The correct answer consists of **Choices A and F**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

22. The operation $\otimes$ is defined for all integers x and y as $x \otimes y = xy - y$. If x and y are positive integers, which of the following CANNOT be zero?

A $x \otimes y$

B $y \otimes x$

C $(x - 1) \otimes y$

D $(x + 1) \otimes y$

E $x \otimes (y - 1)$

Explanation

In the formula $x \otimes y = xy - y$, the variables x and y are placeholders that can be replaced by integers or by expressions representing integers. Here are two examples.

If x is replaced by 3 and y is replaced by 4, then the formula gives

$$3 \otimes 4 = (3)(4) - 4 = 12 - 4 = 8$$

If x is replaced by $x - 1$ and y is replaced by 2, then the formula gives

$$(x - 1) \otimes 2 = ((x - 1)(2)) - 2 = 2x - 2 - 2 = 2x - 4$$

Scanning the answer choices, you can see that all of them are of the form

"first expression" $\otimes$ "second expression"

For each answer choice, you must determine whether the answer choice can be equal to 0 for some positive integers x and y. Are there positive integers x and y for which the answer choice is equal to 0? If not, then that answer choice is the correct answer.

Choice A: $x \otimes y$. Using the formula, try to find positive integers x and y for which $x \otimes y = 0$, that is, for which $xy - y = 0$. To solve this equation, note that factoring y out of the left-hand side of the equation $xy - y = 0$ gives the equation $(x - 1)y = 0$. So now you must find positive integers x and y such that the product of the two numbers $x - 1$ and y is 0. Since the product of two numbers is 0 only if at least one of the numbers is 0, it follows that the product of $x - 1$ and y will be 0 if $x = 1$, no matter what the value of y is. For example, if $x = 1$ and $y = 2$, then $x \otimes y = 1 \otimes 2 = (1)(2) - 2 = 0$, and both x and y are positive integers. Therefore, Choice A is not correct, since there are positive integers x and y for which $x \otimes y = 0$.

Choice B: $y \otimes x$. This is similar to Choice A, except the x and y are interchanged. Therefore, you might try the example in Choice A but with the values of x and y interchanged: $y = 1$ and $x = 2$. Using the formula, $y \otimes x = yx - x = (1)(2) - 2 = 0$. Therefore, Choice B is not correct, since there are positive integers x and y for which $y \otimes x = 0$.

Choice C: $(x - 1) \otimes y$. Using the formula, try to find positive integers x and y for which $(x - 1) \otimes y = 0$, that is, for which $(x - 1)y - y = 0$. Factoring y out of the left-hand side of the equation $(x - 1)y - y = 0$ yields $(x - 1 - 1)y = (x - 2)y = 0$. Here the product of the two numbers $x - 2$ and y is 0. So the product will be 0 if $x = 2$, no matter what the value of y is. For example, if $x = 2$ and $(x - 1) \otimes y = (2 - 1) \otimes 10 = 1 \otimes 10 = (1)(10) - 10 = 0$, and both x and y are positive integers. Therefore, Choice C is not correct, since there are positive integers x and y for which $(x - 1) \otimes y = 0$.

Choice D: $(x + 1) \otimes y$. Using the formula, try to find positive integers x and y for which $(x + 1) \otimes y = 0$, that is, for which $(x + 1)y - y = 0$. Factoring y out of the left-hand side of the equation $(x + 1)y - y = 0$ yields $(x + 1 - 1)y = xy = 0$. Here the product of x and y is 0, so $x = 0$ or $y = 0$. Since both x and y must be positive but 0 is not positive, it follows that there are no positive integers x and y for which $(x + 1) \otimes y = 0$. The correct answer is **Choice D**.

Choice E: $x \otimes (y - 1)$ cannot be correct, since Choice D is correct, but Choice E is considered here for completeness. Using the formula, try to find positive integers x and y for which $x \otimes (y - 1) = 0$, that is, for which $x(y - 1) - (y - 1) = 0$. Factoring $y - 1$ out of the left-hand side of the equation $x(y - 1) - (y - 1) = 0$ yields $(x - 1)(y - 1) = 0$. Here the product of the two numbers $x - 1$ and $y - 1$ is 0. So the product will be 0 if $x = 1$ or $y = 1$, no matter what the value of the other variable is. For example, if $x = 20$ and $y = 1$, then $x \otimes (y - 1) = 20 \otimes (1 - 1) = 20 \otimes 0 = (20)(0) - 0 = 0$, and both x and y are positive integers. Therefore, Choice E is not correct, since there are positive integers x and y for which

$x \otimes (y - 1) = 0$.

This explanation uses the following strategies.
Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 10: Trial and Error
Strategy 11: Divide into Cases
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

23. *P, Q*, and *R* are three points in a plane, and *R* does not lie on line PQ. Which of the following is true about the set of all points in the plane that are the same distance from all three points?

(A) It contains no points.
(B) It contains one point.
(C) It contains two points.
(D) It is a line.
(E) It is a circle.

Explanation

First consider just two of the three points, say *P* and *Q*, and the set of points in the plane that are the same distance from them. Clearly the midpoint of line segment PQ is such a point. Are there others? You may recall from geometry that the points on the line that bisects PQ and is perpendicular to PQ are all the points that are equidistant from *P* and *Q*. Similarly, the points in the plane that lie on the perpendicular bisector of line segment PR are all the points that are equidistant from points *P* and *R*.

Because *R* does not lie on line PQ, line segments PQ and PR do not lie on the same line, and so their respective perpendicular bisectors are not parallel. Therefore, you can conclude that the two perpendicular bisectors intersect at a point. The point of intersection is on both perpendicular bisectors, so it is equidistant from *P* and *Q* as well as from *P* and *R*. Therefore, the point of intersection is equidistant from all three points. Are there any other points that are equidistant from *P, Q*, and *R*? If there were, they would be on both perpendicular bisectors, but in fact only one point lies on both lines. The correct answer is **Choice B**.

This explanation uses the following strategy.
Strategy 8: Search for a Mathematical Relationship

24. If $x < y < 0$, which of the following inequalities must be true?

(A) $y + 1 < x$
(B) $y - 1 < x$
(C) $xy^2 < x$
(D) $xy < y^2$

(E) $xy < x^2$

Explanation

The conditions stated in the question, $x < y < 0$, tell you that x and y are negative numbers and that y is greater than x. Keep this in mind as you evaluate each of the inequalities in the answer choices, to see whether the inequality must be true.

Choice A: $y + 1 < x$. According to the conditions given in the question, y is greater than x. Since y is greater than x and $y + 1$ is greater than y, it follows that $y + 1$ is greater than x. So it cannot be true that $y + 1 < x$. Therefore, Choice A is not the correct answer.

Choice B: $y - 1 < x$. While it is true that both x and $y - 1$ are less than y, it may not be true that $y - 1 < x$ x. Consider what happens if $y = -2$ and $x = -7$. In this case, the inequality $y - 1 < x$ becomes $-3 < -7$, which is false. Therefore, Choice B is not the correct answer.

Choice C: $xy^2 < x$. Note that y^2 is positive and x is negative, so xy^2 is negative. Is the negative number xy^2 less than the negative number x? It depends on whether $y^2 > 1$ or $y^2 < 1$. Consider what happens if $x = -4$ and $y = -\frac{1}{2}$, where $y^2 < 1$. In this case, the inequality $xy^2 < x$ becomes $-1 < -4$, which is false. Therefore, Choice C is not the correct answer.

Choice D: $xy < y^2$. Since y is a negative number, multiplying both sides of the inequality $x < y$ by y reverses the inequality, resulting in the inequality $xy > y^2$. So it cannot be true that $xy < y^2$. Therefore, Choice D is not the correct answer.

Since Choices A through D have been eliminated, the correct answer is Choice E.

Choice E: You can show that the inequality in Choice E, $xy < x^2$, must be true as follows: Multiply both sides of the given inequality $x < y$ by x to obtain the inequality $x^2 > xy$, reversing the direction of the inequality because x is negative. Therefore, the inequality $xy < x^2$ must be true, and the correct answer is **Choice E**.

This explanation uses the following strategies.

Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 10: Trial and Error
Strategy 11: Divide into Cases
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

For the following question, enter your answer in the box.

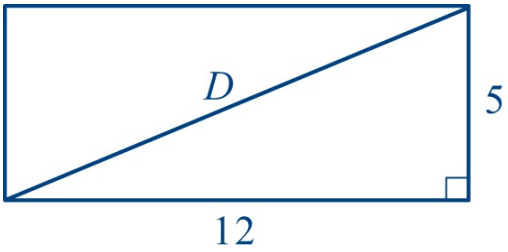
25. What is the length of a diagonal of a rectangle that has width 5 and perimeter 34 ?

Explanation

In this question, you are given that a rectangle has width 5 and perimeter 34, and you are asked to find the length of a diagonal of the rectangle. Let L and W represent the length and width of the rectangle, respectively, and let D represent the length of a diagonal. Note that you are not given L but you are given that $W = 5$ and that the perimeter is 34. Because the perimeter is equal to $L + L + W + W$, or $2(L + W)$, you can determine L as follows.

$$2(L + 5) = 34$$
$$L + 5 = 17$$
$$L = 12$$

The following figure shows a rectangle of length 12, width 5, and diagonal of length D.



From the figure, you can see that the diagonal is the hypotenuse of a right triangle with legs of length 5 and 12. Therefore, by the Pythagorean theorem,

$$5^2 + 12^2 = D^2$$
$$25 + 144 = D^2$$
$$169 = D^2$$
$$13 = D$$

The length of the diagonal is 13, so the correct answer is **13**.

This explanation uses the following strategies.

Strategy 2: Translate from Words to a Figure or Diagram
Strategy 8: Search for a Mathematical Relationship

SECTION 6
Quantitative Reasoning
25 Questions with Explanations

For each of Questions 1 to 9, select one of the following answer choices.

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

1.

A circle is inscribed in a square with sides of length 5.

Quantity A

The circumference of the circle

Quantity B

15

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that a circle is inscribed in a square with sides of length 5 and are asked to compare the circumference of the circle with 15. Since the circle is inscribed in the square, the diameter of the circle is equal to the length of a side of the square, or 5. Thus, the circumference of the circle is 5π. Because π is greater than 3, it follows that 5π is greater than 15. Therefore, Quantity A is greater than Quantity B, and the correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

2.

$$2u + v = 14$$
$$uv = 0$$

Quantity A

u

Quantity B

v

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare u with v, given that $2u + v = 14$ and $uv = 0$.

Consider the equation $uv = 0$. Since a product can equal 0 only if at least one of the factors in the product equals 0, you know that $u = 0$ or $v = 0$, or both. But since you are also given that $2u + v = 14$, it follows that u and v cannot both equal 0.

Knowing that either $u = 0$ or $v = 0$, you can substitute 0 into the equation $2u + v = 14$ for either u or v in order to determine the relationship between u and v if it is possible to do so from the information given. If $u = 0$, then $2u + v = 14$ simplifies to $v = 14$. In this case, u is less than v. However, if $v = 0$, then $2u + v = 14$ simplifies to $2u = 14$, or $u = 7$. And in this case, u is greater than v.

In the first case, $u < v$, and in the second case, $u > v$. Therefore, the relationship between the two quantities u and v cannot be determined from the information given, and the correct

answer is **Choice D**.

This explanation uses the following strategies.
Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

3.

Quantity A

$950^{2,000}$

Quantity B

$10^{6,000}$

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the quantity $950^{2,000}$ with the quantity $10^{6,000}$. Note that both quantities are written in the form "base to a power." If the bases were equal, you would be able to compare the quantities by comparing the powers. Because powers of 10 are easier to work with than powers of 950, it is reasonable to try to compare the quantities by rewriting the quantity $950^{2,000}$ as a power of 10. Unfortunately, there is no obvious way to do that. However, if you can approximate 950 by a power of 10, you may then be able to use the approximation to compare the quantity $950^{2,000}$ with the quantity $10^{6,000}$.

Note that 950 is close to, but a little less than, 1,000, or 10^3. Raising both sides of the inequality $950 < 10^3$ to the power 2,000 gives the inequality $950^{2,000} < (10^3)^{2,000}$. Since $(10^3)^{2,000} = 10^{6,000}$, you can conclude that $950^{2,000} < 10^{6,000}$. Thus, Quantity A is less than Quantity B, and the correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 9: Estimate

4.

Set A consists of 40 integers, and set B consists of 150 integers. The number of integers that are in both set A and set B is 20.

Quantity A

The total number of integers that are in set A or set B, or both

Quantity B

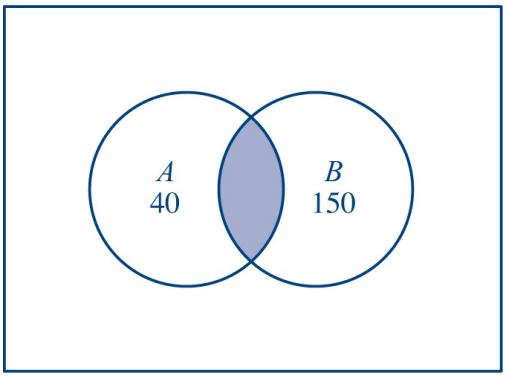
170

Ⓐ Ⓑ Ⓒ Ⓓ

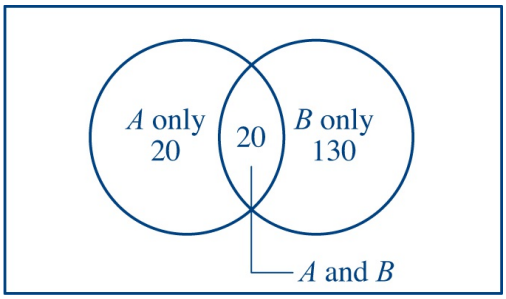
Explanation

In this question, you are given that the number of integers in set A is 40, the number of integers in set B is 150, and the number of integers that are in both A and B is 20. You are asked to compare the total number of integers that are in set A or set B, or both, with 170.

This is the type of question for which a Venn diagram is usually helpful to represent the information given. The following Venn diagram is a representation of the integers in sets A and B.



Note that there is no number in the shaded region of the diagram—the region representing the integers in both A and B. In fact, the number of integers in both A and B is included in both the number of integers in A and the number of integers in B. It is a good idea, therefore, to redraw the Venn diagram so that the numbers are separated into three categories: the integers in A only, the integers in B only, and the integers in both A and B. The revised Venn diagram follows.



Observe that summing the numbers of integers in set A only, set B only, and both A and B yields the total number of integers that are in set A or set B, or both. Therefore, Quantity A is 20 + 130 + 20, or 170, and the correct answer is **Choice C**.

Another approach is to realize that if you listed the integers in set A and the integers in set B, you would have listed the integers that are in both A and B twice and all of the other integers once. So the total number of integers in set A or set B, or both, is equal to

$$(\text{number in set } A) + (\text{number in set } B) - (\text{number in both sets})$$

Thus, the number of integers in set A or set B, or both, is $40 + 150 - 20$, or 170, and the correct answer is **Choice C**.

This explanation uses the following strategies.

Strategy 2: Translate from Words to a Figure or Diagram

Strategy 8: Search for a Mathematical Relationship

5.

x is a negative integer.

Quantity A

2^x

Quantity B

3^{x+1}

(A) (B) (C) (D)

Explanation

In this question, you are asked to compare 2^x with 3^{x+1}, given that x is a negative integer. One way to approach this problem is to plug a value of x in both expressions and compare the results.

You are given that x is a negative integer, so the greatest integer you can plug in for x is -1.

For $x = -1$, it follows that $2^x = 2^{-1} = \frac{1}{2}$ and $3^{x+1} = 3^{-1+1} = 3^0 = 1$.

In this case, 2^x is less than 3^{x+1}. However, to conclude that Quantity B is greater, it is not sufficient for 2^x to be less than 3^{x+1} for one particular value of x; the relationship would need to be true for all negative integer values of x. To analyze this relationship further, plug in another value of x, for example, -2.

For $x = -2$, it follows that $2^x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ and $3^{x+1} = 3^{-2+1} = 3^{-1} = \frac{1}{3}$.

Again, 2^x is less than 3^{x+1}, but note that these values are closer together than the previous values of 2^x and 3^{x+1}. It appears that the relationship between the quantities may differ for smaller values of x, so now try plugging in -3 for x.

For $x = -3$, it follows that $2^x = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $3^{x+1} = 3^{-3+1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

In this case, 2^x is greater than 3^{x+1}.

Since 2^x is less than 3^{x+1} for $x = -1$ and 2^x is greater than 3^{x+1} for $x = -3$, the relationship between these two quantities cannot be determined from the information given. The correct answer is **Choice D**.

Since both quantities are algebraic expressions, another way to approach the comparison is to set up a placeholder relationship, denoted by $\boxed{?}$, between the two quantities and then to simplify to see what conclusions you can draw. As you simplify and draw conclusions, keep in mind that x is a negative integer.

$$2^x \boxed{?} 3^{x+1}$$
$$2^x \boxed{?} 3(3^x)$$
$$\frac{2^x}{3^x} \boxed{?} 3$$
$$\left(\frac{2}{3}\right)^x \boxed{?} 3$$

For any value of x (including negative integer values of x), the value of 3^x is positive, so dividing by 3^x does not affect any inequality that might be represented by the placeholder. Since each step in this simplification is reversible, the simplification reduces the problem to comparing $\left(\frac{2}{3}\right)^x$ with 3, given that x is a negative integer. Note that $\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^n$, where $n = -x$; so the problem can be reduced further to comparing $\left(\frac{3}{2}\right)^n$ with 3, given that n is a positive integer.

Because $\frac{3}{2}$ is greater than 1, the value of $\left(\frac{3}{2}\right)^n$ becomes greater as n becomes larger. For small values of n, $\left(\frac{3}{2}\right)^n$ is less than 3, but for large values of n, $\left(\frac{3}{2}\right)^n$ is greater than 3. Therefore, the relationship between Quantity A and Quantity B cannot be determined from the given information, and the correct answer is **Choice D**.

This explanation uses the following strategies.
Strategy 10: Trial and Error
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

6.

$$(x + 3)(y - 4) = 0$$

Quantity A

xy

Quantity B

−12

Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are given that $(x + 3)(y - 4) = 0$, and you are asked to compare the product xy with −12. Since $(x + 3)(y - 4) = 0$ and the product of two numbers can equal 0 only if at least one of the numbers in the product equals 0, you can conclude that $x = -3$ or $y = 4$, or both.

Consider the case $x = -3$. When $x = -3$, you can choose any number as the value of y and the equation $(x + 3)(y - 4) = 0$ will be satisfied. Depending on the particular value of y you choose, the product xy may be greater than, less than, or equal to −12. For example, if $y = 1$, then $xy = -30$ is greater than −12; and if $y = 10$, then $xy = -30$ is less than −12.

Since different examples for the value of y yield different relationships between Quantities A and B, the relationship cannot be determined from the information given. The correct answer is **Choice D**.

This explanation uses the following strategies.
Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 10: Trial and Error
Strategy 13: Determine Whether a Conclusion Follows from the Information Given

7.

Geoff used $630 to buy a new guitar. This amount was 15 percent of his earnings last summer.

Quantity A

The amount of Geoff's earnings last summer not used to buy the new guitar

Quantity B

$3,570

(A) (B) (C) (D)

Explanation

In this question, you are asked to compare the amount of Geoff's earnings last summer not used to buy a new guitar with the amount $3,570. You are given that Geoff used 15% of his earnings last summer, or $630, to buy the new guitar. So the relationship between Geoff's earnings last summer and the amount he spent to buy the guitar can be expressed by the equation

$$(0.15) \times (\text{Geoff's earnings last summer}) = \$630$$

Therefore, you can conclude that Geoff's earnings last summer totaled $\frac{\$630}{0.15}$, or $4,200. Since Geoff earned $4,200 last summer and spent $630 of his earnings to buy the guitar, the amount he did not spend to buy the guitar was $4,200 − $630, or $3,570. Therefore, Quantity A is equal to Quantity B, and the correct answer is **Choice C**.

This explanation uses the following strategy.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

8.

Set S consists of 5 objects.

Quantity A

The number of subsets of set S that consist of 1 object

Quantity B

The number of subsets of set S that consist of 4 objects

(A) (B) (C) (D)

Explanation

In this question, you are given that set S consists of 5 objects and are asked to compare the number of subsets of set S that consist of 1 object with the number of subsets of set S that consist of 4 objects. Recall that a set R is a subset of set S if all of the members of R are also members of S.

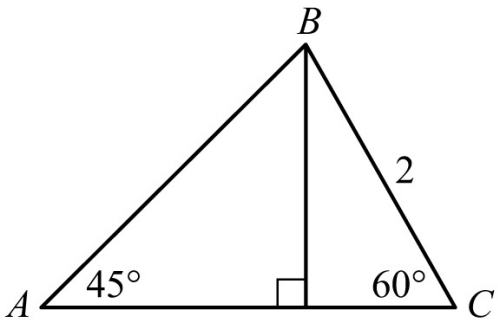
Note that if you select 1 object from set S, there are 4 objects in S that you have not selected; that is to say, each subset of S with 1 object corresponds to a subset of S with 4 objects. Therefore, the number of subsets of S with 1 object is equal to the number of subsets of S with 4 objects. Since Quantity A is equal to Quantity B, the correct answer is **Choice C**.

Another approach to solving this problem is to consider a particular set of 5 objects and determine all of the subsets consisting of 1 object and all of the subsets consisting of 4 objects. For example, let $S = \{a, b, c, d, e\}$. There are 5 subsets of S consisting of 1 object: $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, and $\{e\}$; there are 5 subsets of S consisting of 4 objects: $\{a, b, c, d\}$, $\{a, b, c, e\}$, $\{a, b, d, e\}$, $\{a, c, d, e\}$, and $\{b, c, d, e\}$. Clearly, the particular 5 objects in S do not change the fact that the number of subsets of S consisting of 1 object is equal to the number of subsets of S consisting of 4 objects. The correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 8: Search for a Mathematical Relationship

9.



Quantity A

The length of line segment AC

Quantity B

3

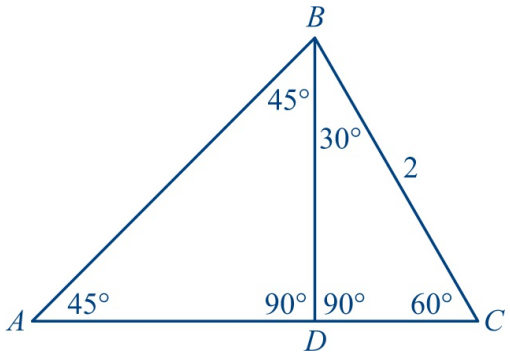
Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the length of line segment AC with 3.

Note that in the figure, the vertical line segment divides triangle ABC into two right triangles. Based on the fact that the sum of the measures of the angles in a triangle is 180°, you can conclude that the triangle to the left of the vertical line is a 45°– 45°– 90° right triangle and the triangle to the right of the vertical line is a 30°– 60°– 90° right triangle. The following

figure shows all of these angle measures, along with a new label *D* at the vertex of the right angles.



Note that the length of AC is equal to the length of AD plus the length of DC. Also note that AD is a leg of the 45°– 45°– 90° triangle and DC is a leg of the 30°– 60°– 90° triangle.

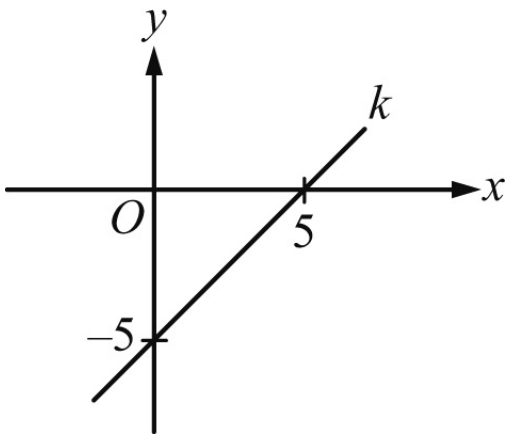
In the figure, you are given that the length of BC, the hypotenuse of the 30°– 60°– 90° triangle, is 2. No other lengths are given. Recall that if the length of the hypotenuse of a 30°– 60°– 90° triangle is 2, then the length of the side opposite the 30° angle is 1, and the length of the side opposite the 60° angle is $\sqrt{3}$. So, in the 30°– 60°– 90° triangle BDC, the length of DC (the side opposite the 30° angle) is 1, and the length of BD (the side opposite the 60° angle) is $\sqrt{3}$.

Now consider the 45°– 45°– 90° triangle ABD. Since this is an isosceles right triangle, its legs, AD and BD, have equal length. Since the length of BD is $\sqrt{3}$, the length of AD is also $\sqrt{3}$.

As noted above, the length of AC is equal to the length of AD plus the length of DC. Since the length of AD is $\sqrt{3}$ and the length of DC is 1, it follows that the length of AC is equal to $\sqrt{3} + 1$.

Recall that in the question you were asked to compare the length of AC with 3. Because $\sqrt{3}$ is less than 2, it follows that the length of AC, which is equal to $\sqrt{3} + 1$, is less than 2 + 1, or 3. Hence, Quantity B is greater than Quantity A, and the correct answer is **Choice B**.

This explanation uses the following strategies.
Strategy 6: Add to a Geometric Figure
Strategy 8: Search for a Mathematical Relationship



10. What is the slope of line k in the xy-plane above?

(A) −5
(B) −1
(C) 0
(D) 1
(E) 5

Explanation

Recall that if a line passes through the points with coordinates (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$, then the slope of the line is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

From the graph of line k in the xy-plane, you can conclude that the x-intercept of line k has coordinates (5, 0) and the y-intercept of line k has coordinates (0, −5). Thus, the slope of line k is

$$\frac{-5-0}{0-5} = \frac{-5}{-5} = 1$$

The correct answer is **Choice D**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

$$b - 3, b - 1, b + 2, b + 3, b + 4$$

11. The median of the five terms listed above is 5, where b is a constant. What is the average (arithmetic mean) of the five terms?

(A) 3

Ⓑ 4
Ⓒ 5
Ⓓ 6
Ⓔ 7

Explanation

To calculate the average of the five terms, you first need to use the information given in the question to determine the value of b.

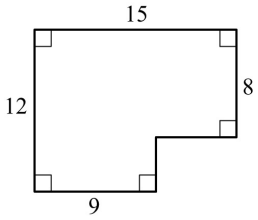
You are given that 5 is the median of the five terms $b - 3$, $b - 1$, $b + 2$, $b + 3$, and $b + 4$, where b is a constant. The median of five terms is the middle, or third, term when the terms are listed in increasing order. Observe that the five terms are already given in increasing order. Since $b + 2$ is the third term, you can conclude that $b + 2 = 5$ and thus $b = 3$.

Since $b = 3$, it follows that the values of the five terms are 0, 2, 5, 6, and 7, and the average of the five terms is $\frac{0+2+5+6+7}{5} = \frac{20}{5} = 4$. The correct answer is **Choice B**.

This explanation uses the following strategy.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

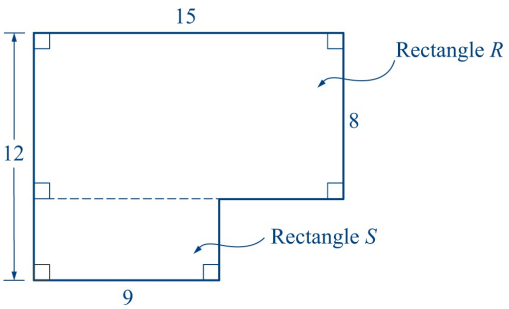
For the following question, enter your answer in the box.



12. What is the area of the region shown above?

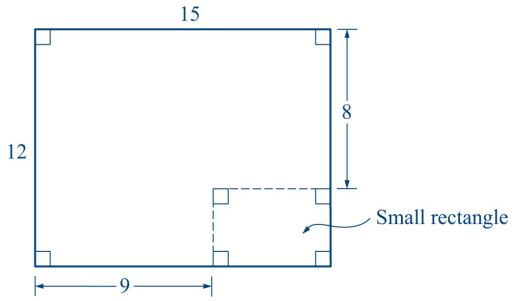
Explanation

In this question, you are asked to determine the area of the given region. One approach to solving this problem is to split the region into two rectangles, rectangle R and rectangle S, as follows.



The area of the region is the sum of the areas of the two rectangles. The area of rectangle R is (15)(8), or 120. Since the width of rectangle S is 12 − 8, or 4, the area of rectangle S is (9)(4), or 36. Thus, the area of the region is 120 + 36, or 156. The correct answer is **156**.

Another approach to solving this problem is to form the region by removing a small rectangle from a rectangle with length 15 and width 12, as follows.



The area of the region is the area of the rectangle with length 15 and width 12 minus the area of the small rectangle. The area of the rectangle with length 15 and width 12 is (15)(12), or 180. Since the length of the small rectangle is 15 − 9, or 6, and the width of the small rectangle is 12 − 8, or 4, the area of the small rectangle is (6)(4), or 24. Thus, the area of the region is 180 − 24, or 156. The correct answer is **156**.

This explanation uses the following strategies.
Strategy 6: Add to a Geometric Figure
Strategy 8: Search for a Mathematical Relationship

13. During a one-year study, biologists observed the number of fish in a certain pond as well as the percent of the fish that were catfish. At the beginning of the year, there were 300 fish in the pond, of which 15 percent were catfish; and at the end of the year, there were 400 fish in the pond, of which 10 percent were catfish. From the beginning of the year to the end of the year, the number of catfish in the pond

(A) decreased by more than 5%
(B) decreased by 5%
(C) did not change
(D) increased by 5%
(E) increased by more than 5%

Explanation

The answer choices indicate that the question is asking about the percent change in the number of catfish. The number of catfish in the pond at the beginning of the year was 15% of 300, or (0.15)(300), which is 45. The number of catfish in the pond at the end of the year was 10% of 400, or (0.10)(400), which is 40. Thus, the number of catfish decreased by 5.

The percent by which the number of catfish decreased from the beginning of the year to the end of the year is

$$\frac{\text{the decrease in the number of catfish over the year}}{\text{the number of catfish at the beginning of the year}} \times (100\%) = \left(\frac{5}{45}\right)(100\%) \approx 11\%$$

Thus, the number of catfish decreased by about 11%. This is a decrease of more than 5%, so the correct answer is **Choice A**.

This explanation uses the following strategy.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

For the following question, enter your answer in the box.

14. On a radio tower, a red light flashes every 6 seconds and a blue light flashes every 10 seconds. If both lights flash together at a certain time, how many seconds later will both lights flash together the next time?

[ ] seconds

Explanation

One way to approach this question is to look at the "flash times" for both lights to see what times they have in common. The following lists show the flash times for both lights as the numbers of seconds after the time at which both lights flashed together.

Red light: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, …
Blue light: 10, 20, 30, 40, 50, 60, 70, 80, 90, …

Note that 30 is the first number that is common to both lists. Therefore, if both lights flash together, they will flash together again 30 seconds later. The correct answer is **30**.

Alternatively, you may realize that if the lights flash together, the number of seconds that will elapse before they flash together the next time is the least common multiple of 6 and 10. To find the least common multiple of 6 and 10, begin by writing each integer as the product of its prime factors.

$$6 = (2)(3)$$
$$10 = (2)(5)$$

Since 2 is a factor in both products, but 3 and 5 are factors of only one of the products, the least common multiple of 6 and 10 is (2)(3)(5), or 30. Therefore, if both lights flash together, the next time they will flash together is 30 seconds later. The correct answer is **30**.

This explanation uses the following strategies.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 7: Find a Pattern

For the following question, select all the answer choices that apply.

15. If $a < b < 0$, which of the following numbers must be positive?

Indicate <u>all</u> such numbers.

A $a - b$

B $a^2 - b^2$

C ab

D $a^2 b$

E $a^2 b + ab^2$

Explanation

In this question, you are given that $a < b < 0$ and are asked to determine which of the answer choices must be positive. Note that the condition $a < b < 0$ means that *a* and *b* are negative and that $a < b$.

Choice A: $a - b$. In the question, it is given that $a < b$. Subtracting *b* from both sides of the inequality $a < b$ gives the inequality $a - b < 0$. Therefore, $a - b$ must be negative.

Choice B: $a^2 - b^2$. Since *a* and *b* are negative, you can square both sides of the inequality $a < b$ to get the inequality $a^2 > b^2$. Then you can subtract b^2 from both sides of the inequality $a^2 > b^2$ to conclude that $a^2 - b^2 > 0$. So $a^2 - b^2$ must be positive. Alternatively, note that $a^2 - b^2$ can be factored as $(a - b)(a + b)$. The factor $a - b$ is Choice A, which must be negative, and the factor $a + b$ is the sum of two negative numbers, which also must be negative. Thus, $a^2 - b^2$ is the product of two negative numbers, so it must be positive.

Choice C: ab. Because *a* and *b* are negative, you can conclude that their product ab must be positive.

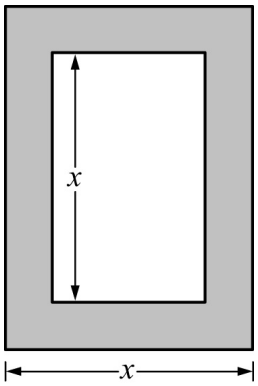
Choice D: $a^2 b$. Because $a^2 b$ can be written as $(a)(a)(b)$, which is the product of three negative numbers, you can conclude that $a^2 b$ must be negative.

Choice E: $a^2 b + ab^2$. By the reasoning in the explanation of Choice D, Choice E is the sum of two negative numbers. Therefore, you can conclude that $a^2 b + ab^2$ must be negative.

Choices B and C must be positive, and Choices A, D, and E must be negative. The correct answer consists of **Choices B and C**.

This explanation uses the following strategies.

Strategy 5: Simplify an Arithmetic or Algebraic Representation
Strategy 13: Determine Whether a Conclusion Follows from the Information Given



16. A flat rectangular picture, represented by the unshaded region in the figure above, is mounted in a flat rectangular frame, represented by the shaded region. The frame is 1 inch wide on all sides. For what value of x, in inches, is the area of the frame equal to the area of the picture?

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

Explanation

In this question, you are asked to determine the value of x for which the area of the frame is equal to the area of the picture. To do this, you need to express both the area of the frame and the area of the picture in terms of x and then find the value of x for which the two expressions are equal.

The area of the picture is the area of the inner rectangle, and the area of the frame is the area of the outer rectangle minus the area of the inner rectangle. Since the area of a rectangle is the length times the width, you need to know the length and width of the inner and outer rectangles.

In the figure, you are given that the length of the inner rectangle is x inches, but the width is not given. However, since you know that the width of the frame is 1 inch, it follows that the width of the inner rectangle is equal to the width of the outer rectangle minus 2 inches, or $x - 2$ inches. Thus, the area of the inner rectangle is $x(x - 2)$ square inches.

In the figure, you are given that the width of the outer rectangle is x inches, but the length is not given. However, since you know that the width of the frame is 1 inch, it follows that the length of the outer rectangle is equal to the length of the inner rectangle plus 2 inches, or $x + 2$ inches. Thus, the area of the outer rectangle is $x(x + 2)$ square inches.

Since the area of the frame is the area of the outer rectangle minus the area of the inner rectangle, the area of the frame is

$x(x+2) - x(x-2) = x^2 + 2x - x^2 + 2x = 4x$ square inches.

Now you are ready to set up the equation. Set the expression for the area of the picture equal to the expression for the area of the frame and solve the resulting equation for x, as follows.

$$x(x - 2) = 4x$$
$$x^2 - 2x = 4x$$
$$x^2 - 6x = 0$$
$$x(x - 6) = 0$$

There are two solutions to the equation, $x = 0$ and $x = 6$. Since x represents the length of a picture, in inches, the solution $x = 0$ does not make sense in this context. Therefore, when $x = 6$, the area of the picture equals the area of the frame. The correct answer is **Choice C**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 5: Simplify an Arithmetic or Algebraic Representation

Questions 17 to 20 are based on the following data.

PERCENT OF THE 300 PEOPLE IN GROUP 1 AND THE 400 PEOPLE IN GROUP 2 WHO HAVE SELECTED AILMENTS

Respiratory Ailment	Percent of People in Group 1 Who Have Ailment	Percent of People in Group 2 Who Have Ailment
Allergic sensitivity to endotoxins	14%	21%
Asthma (allergic)	3%	4%
Asthma (nonallergic)	2%	3%
Hay fever	4%	10%
Sneezing and itchy eyes	8%	11%
Wheezing (allergic)	5%	6%
Wheezing (nonallergic)	2%	5%

17. The number of people in group 2 who have hay fever is how much greater than the number of people in group 1 who have hay fever?

(A) 37
(B) 35
(C) 32
(D) 28
(E) 24

Explanation

In group 1, there are 300 people, 4% of whom have hay fever. Therefore, in group 1, there are (0.04)(300) people, or 12 people, who have hay fever. In group 2, there are 400 people, 10% of whom have hay fever. Therefore, in group 2, there are (0.10)(400) people, or 40 people, who have hay fever. Since 40 − 12 = 28, it follows that there are 28 more people in group 2

who have hay fever than there are in group 1. The correct answer is **Choice D**.

This explanation uses the following strategy.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

18. For the seven ailments, what is the median of the numbers of people in group 2 who have the ailments?

(A) 20
(B) 22
(C) 24
(D) 26
(E) 28

Explanation

The median of the seven numbers of people in group 2 is the middle number when the numbers are listed in increasing order. To find the middle number easily, first order from least to greatest the percents of people in group 2 who have the seven ailments, then find the median of the percents, and finally calculate the number of people corresponding to that percent.

The following table shows the percents of people in group 2 who have the seven ailments, ordered from least to greatest.

Respiratory Ailment	Percent of People in Group 2 Who Have Ailment
Asthma (nonallergic)	3%
Asthma (allergic)	4%
Wheezing (nonallergic)	5%
Wheezing (allergic)	6%
Hay fever	10%
Sneezing and itchy eyes	11%
Allergic sensitivity to endotoxins	21%

The median percent, which is the fourth number in the list, is 6%. Thus, the median number of people in group 2 who have the ailments is (0.06)(400), or 24, and the correct answer is **Choice C**.

This explanation uses the following strategies.
Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

19. The number of people in group 1 who have the ailment wheezing (allergic) is what percent greater than the number of people in group 1 who have the ailment wheezing

(nonallergic)?

(A) 50%
(B) 75%
(C) 150%
(D) 200%
(E) 300%

Explanation

In group 1, there are 300 people, 5% of whom have allergic wheezing and 2% of whom have nonallergic wheezing. That is, (0.05)(300) people, or 15 people, have allergic wheezing and (0.02)(300) people, or 6 people, have nonallergic wheezing. Therefore, in group 1, the number of people who have allergic wheezing exceeds the number who have nonallergic wheezing by 9, which is $\left(\frac{9}{6}\right)$(100%) greater than 6, or 150% greater than 6. The correct answer is **Choice C**.

This explanation uses the following strategy.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation

For the following question, enter your answer in the boxes.

20. What is the ratio of the number of people in group 2 with the ailment sneezing and itchy eyes to the total number of people in both groups with the ailment sneezing and itchy eyes?

    Give your answer as a fraction.

$$\frac{\square}{\square}$$

Explanation

In group 2, there are 400 people, 11% of whom have sneezing and itchy eyes. Therefore, in group 2, there are (0.11)(400) people, or 44 people, who have sneezing and itchy eyes. In group 1, there are 300 people, 8% of whom have sneezing and itchy eyes. Therefore, in group 1, there are (0.08)(300) people, or 24 people, who have sneezing and itchy eyes. So the total number of people in both groups who have sneezing and itchy eyes is 24 + 44, or 68. Thus, the ratio of the number of people in group 2 who have sneezing and itchy eyes to the total number of people in both groups who have sneezing and itchy eyes is $\frac{44}{68}$. The correct answer is $\mathbf{\frac{44}{68}}$ (or any equivalent fraction).

This explanation uses the following strategies.

Strategy 4: Translate from a Figure to an Arithmetic or Algebraic Representation
Strategy 5: Simplify an Arithmetic or Algebraic Representation

21. Of the people in a certain survey, 58 percent were at most 40 years old and 70 percent were at most 60 years old. If 252 of the people in the survey were more than 40 years old and at most 60 years old, what was the total number of people in the survey?

(A) 1,900
(B) 2,100
(C) 2,400
(D) 2,700
(E) 3,000

Explanation

In this question, it is given that of the people surveyed, 58% were at most 40 years old and 70% were at most 60 years old. Therefore, 70% − 58%, or 12%, of the people surveyed were more than 40 years old and at most 60 years old, and you are given that 252 people are in this group. Let x be the total number of people in the survey. Then 12% of x is 252, that is, $0.12x = 252$, and so $x = \frac{252}{0.12} = 2{,}100$. Therefore, the total number of people in the survey was 2,100, and the correct answer is **Choice B**.

This explanation uses the following strategies.

Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation
Strategy 8: Search for a Mathematical Relationship

22. If $x > 0$, which of the following is equal to 1.25 percent of x ?

(A) $\frac{x}{80}$
(B) $\frac{x}{8}$
(C) $\frac{x}{4}$
(D) $\frac{5x}{8}$
(E) $\frac{3x}{4}$

Explanation

Since the answer choices are fractional expressions, to answer the question you need to convert 1.25% of x to a fractional expression. Note that 1.25% is equivalent to the decimal 0.0125. Converting the decimal 0.0125 to a fraction and simplifying gives $0.0125 = \frac{125}{10{,}000} = \frac{1}{80}$. Thus, 1.25% of x is equal to $\frac{1}{80}$ of x, or $\frac{x}{80}$, and the correct answer is **Choice A**.

This explanation uses the following strategy.

Strategy 5: Simplify an Arithmetic or Algebraic Representation

23. Alice earns d dollars and has t percent of what she earns deducted for taxes. How much of

what she earns does Alice have left after taxes?

(A) $d(1 - 100t)$ dollars
(B) $d(1 - 10t)$ dollars
(C) $d(1 - t)$ dollars
(D) $d(1 - 0.1t)$ dollars
(E) $d(1 - 0.01t)$ dollars

Explanation

Recall that t percent can be expressed as $\frac{t}{100}$, or $0.01t$. Therefore, the amount that Alice has deducted for taxes, which is t percent of d dollars, can be expressed as $0.01td$ dollars. The amount that Alice has left after taxes is the amount that she earns minus the amount that she has deducted for taxes, or $d - 0.01td$ dollars. Note that $d - 0.01td$ is an algebraic expression with two terms, each containing d as a factor. Factoring out d from each term results in the algebraic expression $d(1 - 0.01t)$. The correct answer is **Choice E**.

This explanation uses the following strategy.
Strategy 1: Translate from Words to an Arithmetic or Algebraic Representation

For the following question, select all the answer choices that apply.

24. A student made a conjecture that for any integer n, the integer $4n + 3$ is a prime number. Which of the following values of n could be used to disprove the student's conjecture?

Indicate <u>all</u> such values.

[A] 1
[B] 3
[C] 4
[D] 6
[E] 7

Explanation

Recall that a prime number is an integer greater than 1 that has no positive divisors other than 1 and itself.

The answer choices for this question are integer values of n. Any of the answer choices for which the integer $4n + 3$ is <u>not</u> a prime number could be used to disprove the conjecture that for any integer n, the integer $4n + 3$ is a prime number.

To answer this question, you must determine for each of the answer choices whether the integer $4n + 3$ is a prime number. The evaluations are as follows:

Choice A: For $n = 1$, the integer $4n + 3$ is $4(1) + 3$, or 7, which is a prime number.
Choice B: For $n = 3$, the integer $4n + 3$ is $4(3) + 3$, or 15, which is not a prime number.
Choice C: For $n = 4$, the integer $4n + 3$ is $4(4) + 3$, or 19, which is a prime number.
Choice D: For $n = 6$, the integer $4n + 3$ is $4(6) + 3$, or 27, which is not a prime number.

Choice E: For $n = 7$, the integer $4n + 3$ is $4(7) + 3$, or 31, which is a prime number.

Therefore, of the answer choices, only Choices B and D, that is, $n = 3$ and $n = 6$, result in integers $4n + 3$ that are not prime numbers. Thus, the correct answer consists of **Choices B and D**.

This explanation uses the following strategies.

Strategy 10: Trial and Error

Strategy 13: Determine Whether a Conclusion Follows from the Information Given

25. Eight points are equally spaced on a circle. If 4 of the 8 points are to be chosen at random, what is the probability that a quadrilateral having the 4 points chosen as vertices will be a square?

(A) $\frac{1}{70}$

(B) $\frac{1}{35}$

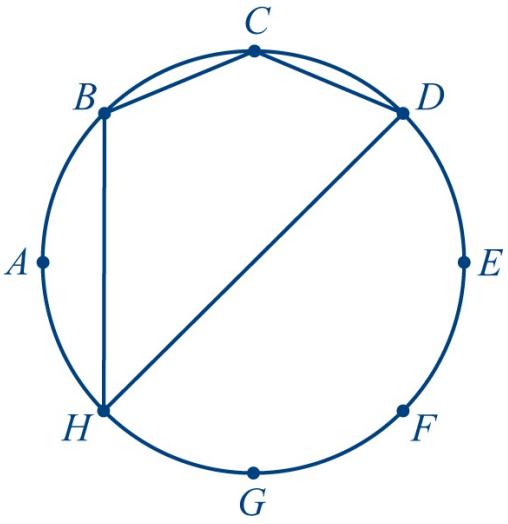
(C) $\frac{1}{7}$

(D) $\frac{1}{4}$

(E) $\frac{1}{2}$

Explanation

For questions involving geometry, it is often helpful to draw a figure representing the information in the question as accurately as possible. The figure below shows a circle with 8 equally spaced points, labeled A through H, and quadrilateral $BCDH$, which is one of the many quadrilaterals that have 4 of the 8 equally spaced points as vertices.



The probability that a quadrilateral having the 4 points chosen as vertices will be a square is equal to the following fraction.

$$\frac{\text{the number of squares that can be drawn using 4 of the 8 points as vertices}}{\text{the number of quadrilaterals that can be drawn using 4 of the 8 points as vertices}}$$

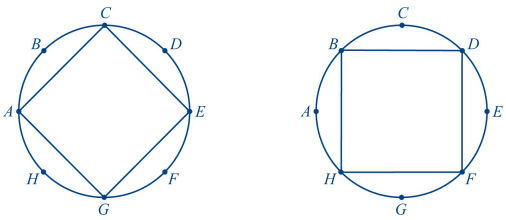
To calculate the desired probability, you need to determine the number of squares and the number of quadrilaterals that can be drawn using 4 of the 8 points as vertices.

To determine the number of quadrilaterals, first note that since the 8 points lie on a circle, every subset of 4 of the 8 points determines a unique quadrilateral. Therefore, the number of quadrilaterals that can be drawn using 4 of the 8 points as vertices is equal to the number of ways of choosing 4 points from the 8 points shown. The number of ways of choosing 4 points from the 8 points shown (also called the number of combinations of 8 objects taken 4 at a time) is equal to $\frac{8!}{4!(8-4)!}$. You can calculate the value of this expression as follows.

$$\begin{aligned}\frac{8!}{4!(8-4)!} &= \frac{(8)(7)(6)(5)(4!)}{(4)(3)(2)(1)(4!)} \\ &= \frac{(8)(7)(6)(5)}{(4)(3)(2)} \\ &= 70\end{aligned}$$

Thus, there are 70 quadrilaterals that can be drawn using 4 of the 8 points as vertices.

Because the points are equally spaced around the circle, there are only 2 squares that can be drawn using 4 of the 8 points as vertices, namely $ACEG$ and $BDFH$, as shown in the following figures.



Therefore, the probability that the quadrilateral will be a square is $\frac{2}{70}$, or $\frac{1}{35}$, and the correct answer is **Choice B**.

This explanation uses the following strategies.

Strategy 2: Translate from Words to a Figure or Diagram

Strategy 8: Search for a Mathematical Relationship