Chapter 7

Arithmetic

In This Chapter…

Arithmetic

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

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(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

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	Quantity A	**Quantity B**
1.	39 – (25 – 17)	39 – 25 – 17

	Quantity A	**Quantity B**
2.	14 – 3(4 – 6)	(4)(–3)(2)(–1)

	Quantity A	**Quantity B**
3.	–5 × 1 ÷ 5	–6 × 1 ÷ 6

4. What is the value of 5 – (4 – (3 – (2 – 1))) ?

$\boxed{}$

	Quantity A	Quantity B
5.	$-\frac{2^3}{2}$	$(-2)^2$
6.	$5^3 - 5^2$	5
7.	$-10 - (-3)^2$	$-[10 + (-3)^2]$

8.	**Quantity A**	**Quantity B**
	(30,000,000)(2,000,000)	(15,000,000)(4,000,000)

9. What is the sum of the numbers in the grid below?

–2	–1	1	2	3	4
–4	–2	2	4	6	8
–6	–3	3	6	9	12
–8	–4	4	8	12	16
–10	–5	5	10	15	20
–12	–6	6	12	18	24

10. Molly worked at an amusement park over the summer. Every two weeks, she was paid according to the following schedule: at the end of the first 2 weeks, she received \$160. At the end of each subsequent 2-week period, she received \$1, plus an additional amount equal to the sum of all payments she had received in previous weeks. How much money was Molly paid during the full 10 weeks of summer?

$

A book with 80,000 words costs \$24 and a short story with 1,000 words costs \$1.

	Quantity A	**Quantity B**
11.	Cost per word of the book	Cost per word of the short story

Ticket Prices at the Natural History Museum

	Weekdays	Weekends & Holidays
Child (ages 5–18)	\$7	\$9

Adult (ages 19–64)	$14	$16
Senior (ages 65+)	$8	$10
*Children under age 5 attend free		

12.

Quantity A	**Quantity B**
The price for tickets at the Natural History Museum on a weekday for one 12-year-old and one 39-year-old	The price for tickets at the Natural History Museum on a weekend for one 4-yearold, two 8-year-olds, and one senior over 65 years old, after applying a coupon for $10 off the total cost

On a certain train, tickets cost \$6 each for children and \$9 each for adults. The total train ticket cost for a certain group of six passengers was between \$44 and \$50.

	Quantity A	**Quantity B**
13.	The number of children in the group	The number of adults in the group

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14. If 617 is divided by 49, the sum of the <u>tens</u> digit and the <u>tenths</u> digit of the resulting number is what value?

(A) 1

(B) 5

(C) 6

(D) 7

(E) 9

	Quantity A	**Quantity B**
15.	The number of days between May 30, 1917, and May 15, 1996, inclusive	The number of days between May 15, 1912, and May 30, 1991, inclusive

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Alfred's Coffee Shop offers a "buy six cups of coffee, get one free" discount, and Boris's Coffee Shop offers 15% off all orders of six or more cups of coffee. At both shops, the regular price of a single cup of coffee is \$2.60.

	Quantity A	**Quantity B**
16.	The total cost for one order of seven single cups of coffee from Alfred's	The total cost for one order of seven single cups of coffee from Boris's

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17. In a certain ancient kingdom, the standard unit of measure was the "crown," equal to 10 standard modern inches. An alternative unit of measure was the "scepter," equal to 14 standard modern inches. If a tower measured 70 crowns tall, how many scepters tall was it?

(A) 35

(B) 49

(C) 50

(D) 75

(E) 98

18. A total of \$450 was donated to charity by 25 employees. If 15 employees donated at least \$12 but less than \$19 and 9 employees donated at least \$19, what is the maximum amount, in dollars, that the last employee could have donated?

\$ $\boxed{}$

19. A tank has a capacity of 200 pints. How many gallons of water would it take to fill the tank to $\frac{3}{10}$ of its capacity? (1 gallon = 8 pints)

$\boxed{}$ gallons

---

1 kilogram = 2.2 pounds

	<u>Quantity A</u>	**<u>Quantity B</u>**
20.	The number of kilograms in 44 pounds	The number of pounds in 44 kilograms

---

21. If the formula for converting degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$, what is the value of *F* when *C* is 30?

(A) $-\frac{10}{9}$

(B) $\frac{338}{9}$

(C) 86

(D) $\frac{558}{5}$

(E) 112

22. On a trip, Joe's car traveled an average of 36 miles per gallon of fuel. Approximately how many kilometers did the car travel on 10 liters of fuel?

(5 miles = approximately 8 kilometers; 1 gallon = approximately 4 liters)

$\boxed{}$ kilometers

23. How many 1-inch square tiles would it take to cover the floor of a closet that has dimensions 5 feet by 4 feet? (1 foot = 12 inches)

(A) 20

(B) 240

(C) 1,440

(D) 2,160

(E) 2,880

Child A ate $\frac{3}{5}$ of a kilogram of chocolate and Child B ate 300 grams of chocolate. (1 kilogram = 1,000 grams)

	Quantity A	**Quantity B**
24.	The weight, in grams, of the chocolate that Child A ate	Twice the weight, in grams, of the chocolate that Child B ate

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25. Out of 5.5 billion bacteria grown for an experiment, 1 in 75 million has a particular mutation. Approximately how many of the bacteria have the mutation?

(A) 7

(B) 73

(C) 733

(D) 7,333

(E) 73,333

26. A particular nation's GDP (Gross Domestic Product) is $4.5 billion. If the population of the nation is 1.75 million, what is the per capita (per person) GDP, rounded to the nearest dollar?

(A) $3

(B) $25

(C) $257

(D) $2,571

(E) $25,714

27. Global GDP (Gross Domestic Product) was $69.97 trillion in 2011. If the world population for 2011 was best estimated at 6,973,738,433, approximately what was the global GDP per person?

(A) $10

(B) $100

(C) $1,000

(D) $10,000

(E) $100,000

28. The runners on a cross country team ran a 5-mile race at average (arithmetic mean) speeds ranging from 4 miles per hour to 7 miles per hour, inclusive. Which of the following are possible race completion times for individual members of the team?

Indicate <u>all</u> such times.

- ☐ 36 minutes
- ☐ 48 minutes
- ☐ 60 minutes
- ☐ 75 minutes
- ☐ 90 minutes
- ☐ 120 minutes

Arithmetic Answers

1. **(A).** First simplify inside the parentheses:

$$39 - (25 - 17) =$$
$$39 - 8 =$$
$$31$$

You could also distribute the minus sign to get 39 – 25 + 17 if you prefer. Quantity B is equal to –3, so Quantity A is greater. If you noticed right away that the minus sign would distribute in Quantity A but not Quantity B, you could have picked (A) without doing any arithmetic.

2. **(B).** This question is testing PEMDAS (Parentheses/Exponents, then Multiplication/Division, then Addition/Subtraction), at least in Quantity A. Make sure that you simplify inside the parentheses, and then multiply, before subtracting:

$$14 - 3(4 - 6) =$$
$$14 - 3(-2) =$$
$$14 + 6 =$$
$$20$$

Quantity B is $(4)(-3)(2)(-1) = 24$.

3. **(C).** The two quantities are equal. Note that in Quantity A:

$$-5 \times 1 \div 5 =$$
$$-5 \div 5 =$$
$$-1$$

In Quantity B:

$$-6 \times 1 \div 6 =$$
$$-6 \div 6 =$$
$$-1$$

4. **3.** Make sure to begin with the innermost parentheses:

$$5 - (4 - (3 - (2 - 1))) =$$
$$5 - (4 - (3 - 1)) =$$
$$5 - (4 - 2) =$$
$$5 - (2) =$$
$$3$$

5. **(B).** In Quantity A, the exponent should be computed before taking the negative of the value—in accordance with PEMDAS. Thus, you get –8/2 = –4.

In Quantity B:

$(-2)^2 =$
$(-2)(-2) =$
4

6. **(A).** Do not make the mistake of thinking that $5^3 - 5^2 = 5^1$. You cannot just subtract the exponents when you are subtracting two terms with the same base! Instead, compute the exponents and subtract:

$5^3 - 5^2 =$
$125 - 25 =$
100

Quantity A is greater. Alternatively, you could factor out 5^2 (this is an important technique for large numbers and exponents where pure arithmetic would be impractical):

$5^3 - 5^2 =$
$5^2(5^1 - 1) =$
$5^2(4) =$
100

7. **(C).** In Quantity A:

$-10 - (-3)^2 =$
$-10 - (9) =$
-19

In Quantity B:

$-[10 + (-3)^2] =$
$-[10 + (9)] =$
-19

8. **(C).** The GRE calculator will not be able to handle that many zeros. Start this calculation on paper. To make things easier, you could cancel as many zeros as you want, as long as you do the same operation to both quantities. For instance, you could divide both sides by 1,000,000,000,000 (just think of this as "1 with 12 zeros"), to get:

Quantity A **Quantity B**

(30)(2) (15)(4)

Or, just use a bit of logic: 30 million times 2 million is 60 million *million*, and 15 million times 4 million is also 60 million *million*. (A "million million" is a trillion, but this doesn't matter as long as you're sure that each Quantity will have the same number of zeros.)

9. **147.** There are several patterns in the grid, depending on whether you look by row or by column. Within each row, there are positive and negative terms at the beginning that cancel each other. For example, in the first row, you have –2 + 2 = 0 and –1 + 1 = 0. The only terms in the first row that contribute to the sum are 3 and 4, in the far-right columns. The same is true for the other rows.

Thus, the sum of the grid is equal to the sum of only the two far-right columns. The sum in the first row in those columns is 3 + 4 = 7; the sum in the next row is 6 + 8 = 14, etc. The sum in the final row is 18 + 24 = 42. Add 7 + 14 + 21 + 28 + 35 + 42 in your calculator to get 147.

10. **$2,575.** At the end of the first two weeks, Molly received $160. At the end of the fourth week, she received $1, plus $160 for the total she had been paid up to that point, for a total of $161. At the end of the sixth week, she received $1, plus ($160 + $161), or $321, for the total she had been paid up to that point, making the sixth week total $322. To keep track, put these values in a table:

Week #	Paid This Week($)	Cumulative Pay Including This Week ($)
2	160	160
4	160 + 1 = 161	160 + 161 = 321
6	321 + 1 = 322	321 + 322 = 643
8	643 + 1 = 644	643 + 644 = 1,287
10	1,287 + 1 = 1,288	1,287 + 1,288 = 2,575

11. **(B).** In Quantity A, $\frac{24}{80{,}000}$ = 0.0003, or 0.03 cents per word. In Quantity B, $\frac{1}{1{,}000}$ = 0.001, or 0.1 cents per word. Quantity B is greater.

Note that the calculation was not strictly necessary—it would have been more efficient to notice that the book costs 24 times the story but has 80 times the words. (Then remember to choose the greater number!)

12. **(A).** The ticket for the 4-year-old in Quantity B costs \$0 (children under age 5 attend free).

Quantity A: The price for tickets at the Natural History Museum on a weekday for one 12-year-old and one 39-year-old = \$7 + \$14 = \$21.

Quantity B: The price for tickets at the Natural History Museum on a weekend for one 4-year-old, two 8-year-olds, and one senior over 65 years old, after applying a coupon for \$10 off the total cost is equal to (\$0 + \$9 + \$9 + \$10) – \$10 = \$18.

Quantity A is greater.

13. **(D).** Even though the range of costs ($44 to $50) is fairly small, there is still more than one possibility. A good way to work this out is to start with the simplest scenario: 3 adults and 3 children. Their tickets would cost 3(9) + 3(6) = $45. That's in the range, so it's one possibility.

Since children's tickets are cheaper, you don't want to add more children to the mix (4 children, 2 adults will give you too small a total), but try switching out 1 child for 1 adult.

For 4 adults and 2 children, tickets would cost 4(9) + 2(6) = $48. Thus, Quantity A and Quantity B could be equal, or Quantity B could be greater, so the relationship cannot be determined from the information given.

14. **(C).** Divide 617 by 49 with the calculator to get 12.5918…. The **tens** digit is 1. The **tenths** digit is 5. The answer is 1 + 5 = 6.

15. **(B).** Calculating the number of days in each quantity would be time-consuming; each date range includes a lot of days! Instead, a faster approach is to compare the starting and ending dates for the two quantities.

Quantity A: The number of days between May 30, 1917 and May 15, 1996, inclusive.

Quantity B: The number of days between May 15, 1912 and May 30, 1991, inclusive.

All of the start and end dates are in May, but both the starting and ending years in Quantity B are 5 years earlier than those in Quantity A. Thus, the approximate whole number of years in both ranges is the same (about 79 years). However, the range in Quantity A starts later in the month and ends earlier in the month than the range in Quantity B. Both differences mean that Quantity B includes a greater number of days.

Alternatively, consider the following: the date range in Quantity A is about half a month less than 79 years, while the date range in Quantity B is about half a month greater than 79 years. Quantity B is greater.

16. **(A).** At Alfred's, an order of 7 single cups of coffee would cost 6($2.60) = $15.60, because the 7th cup is free.

At Boris's, an order of 7 single cups of coffee would receive the 15% discount: 7($2.60)(0.85) = $15.47.

Alternatively, because the non-discounted price of a single coffee ($2.60) and the number of single cups of coffee ordered is common to both quantities, an actual cost calculation is optional. Instead, you could compare the discounts in percent terms. At Alfred’s, “buy six drinks get one free” means that, for every seven drinks you purchase, the last one is free. That’s one in seven drinks free, or $\frac{1}{7}$ off, which is about $\left(\frac{1}{7}\times 100\right)\%$ = 14.29% off. This is smaller than the 15% discount at Boris’s, so the total cost at Alfred’s is greater. By the way, remember to pick the greater quantity (Quantity A), not the “better deal”!

17. **(C).** A tower that was 70 crowns tall was 70 crowns × 10 inches/crown = 700 inches tall. This same 700-inch tower, measured in scepters, would be $\frac{700\text{ inches}}{14\text{ inches/scepter}}$ = 50 scepters tall. Also, note that since the scepter is longer than the crown in absolute terms, fewer scepters will “fit” in the height of the tower, so any choices 70 or greater could be eliminated right away.

18. **$99.** To maximize the last employee's contribution, minimize everyone else's. If 15 employees could have donated a minimum of $12 and 9 employees could have donated a minimum of $19:

$$15(12) + 9(19) = 180 + 171 = 351$$

So, the minimum that all 24 of these employees could have given is $351. Therefore, the maximum that the 25th employee could have given is 450 – 351 = 99, or $99.

19. **7.5 gallons.**

First find out how many pints $\frac{3}{10}$ of the capacity is:

$$200 \times \frac{3}{10} = \frac{600}{10} = 60$$

Now convert pints to gallons:

$$60 \text{ pints} \times \frac{1 \text{ gallon}}{8 \text{ pints}} = \frac{60}{8} = 7.5 \text{ gallons}$$

20. **(B).** To compare the values, convert the quantity on the left from pounds to kilograms and the quantity on the right from kilograms to pounds:

<u>Quantity A</u>	**<u>Quantity B</u>**
$44 \text{ pounds} \times \frac{1 \text{ kilogram}}{2.2 \text{ pounds}}$	$44 \text{ kilograms} \times \frac{2.2 \text{ pounds}}{1 \text{ kilogram}}$

Before actually multiplying, notice that the Quantity A is divided by 2.2, while the Quantity B is multiplied by 2.2. Quantity B will be greater.

You could also solve this by noticing that the two quantities involve reverse calculations, with the same number of units (44). Since a kilogram is heavier than a pound, it takes more of the lighter pounds to equal 44 heavier kilograms than it takes of the heavier kilograms to equal 44 of the lighter pounds.

21. **(C).** Start by plugging 30 in for *C* in the equation:

$$30 = \frac{5}{9}(F - 32)$$

Now isolate *F*. Begin by multiplying both sides by $\frac{9}{5}$:

$$\frac{9}{5} \times 30 = F - 32$$

To multiply 30 by $\frac{9}{5}$ quickly, reduce before multiplying:

$$\frac{9}{1} \times 6 = F - 32$$

$$54 = F - 32$$

$$86 = F$$

22. **144 kilometers.** Convert miles per gallon to kilometers per liter by multiplying by the conversion ratios such that both the miles and gallons units are canceled out:

$$\frac{36 \text{ miles}}{1 \text{ gallon}} \times \frac{8 \text{ kilometers}}{5 \text{ miles}} = \frac{288 \text{ kilometers}}{5 \text{ gallons}}$$

$$\frac{288 \text{ kilometers}}{5 \text{ gallons}} \times \frac{1 \text{ gallon}}{4 \text{ liters}} = \frac{288 \text{ kilometers}}{20 \text{ liters}} = \frac{14.4 \text{ kilometers}}{1 \text{ liter}}$$

The car has 10 liters of fuel in the tank:

10 liters × 14.4 kilometers/liter = 144 kilometers

23. **(E).** There is a hidden trap in this question. Remember that the dimensions of this room are square feet, not feet (because 5 feet × 4 feet = 20 square feet). To avoid this trap, you should convert the dimensions to inches first, then multiply.

5 feet × 4 feet = 60 inches × 48 inches

The dimensions of the closet in inches are 60 inches by 48 inches, or 60 × 48 = 2,880 square inches. Each tile is 1 square inch, so it will take 2,880 tiles to cover the floor.

24. **(C).** $\frac{3}{5}$ of a kilogram is 600 grams. Twice 300 grams is also 600 grams.

The two quantities are equal.

25. **(B).** One good way to keep track of large numbers (especially those that won't fit in the GRE calculator!) is to use scientific notation (or a loose version thereof—for instance, 5.5 billion in scientific notation is 5.5×10^9, but it would be equally correct for your purposes to write it as 55×10^8).

5.5 billion = 5,500,000,000 = 5.5×10^9
75 million = 75,000,000 = 75×10^6

Since 1 in 75 million of the bacteria have the mutation, divide 5.5 billion by 75 million:

$\frac{5.5 \times 10^9}{75 \times 10^6}$, which can also be written as $\frac{5.5}{75} \times \frac{10^9}{10^6}$. Only $\frac{5.5}{75}$ needs to go in the calculator, to yield 0.0733333 … Since $\frac{10^9}{10^6}$ is 10^3, move the decimal three places to the right to get 73.333 …, or answer choice (B).

Or, write one number over the other and *cancel out the same number of zeros from the top and bottom* before trying to use the calculator:

$$\frac{5{,}500{,}000{,}000}{75{,}000{,}000} = \frac{5{,}500,\cancel{000{,}000}}{75,\cancel{000{,}000}} = \frac{5{,}500}{75} = 73.333\ldots$$

26. **(D).** This problem is asking you to divide \$4.5 billion by 1.75 million. When dealing with numbers that have many zeros, you can avoid mistakes by using scientific notation or by writing out the numbers and canceling zeros before using the calculator:

4.5 billion = 4,500,000,000 = 4.5×10^9
1.75 million = 1,750,000 = 1.75×10^6

$$\frac{4.5\times10^9}{1.75\times10^6} = 2.57142\ldots\times10^3 = 2{,}571.42\ldots$$

The answer is (D). Alternatively, write one number on top of the other in fully expanded form, and cancel zeros before using the calculator:

$$\frac{4{,}500{,}000{,}000}{1{,}750{,}000} = \frac{4{,}500{,}00\cancel{0{,}000}}{1{,}75\cancel{0{,}000}} = \frac{450{,}000}{175} = 2{,}571.42\ldots$$

27. **(D).** This problem is asking you to divide \$69.97 trillion by 6,973,738,433. When dealing with numbers that have many zeros, you can avoid mistakes by using scientific notation or by writing out the numbers and canceling zeros before using the calculator.

Before doing that, however, look at the answers—they are very far apart from one another, which gives you license to estimate. GDP is about 70 trillion. Population is about 7 billion. Thus:

$$\frac{70{,}000{,}000{,}000{,}000}{7{,}000{,}000{,}000} = \frac{70{,}000{,}\cancel{000{,}000{,}000}}{7{,}\cancel{000{,}000{,}000}} = 10{,}000$$

28. **48 minutes, 60 minutes, and 75 minutes.** Rate × Time = Distance, thus $\frac{\text{Distance}}{\text{Rate}}$ = Time.

The race times range from a maximum time of $\frac{5 \text{ miles}}{4 \text{ miles/hour}}$ = 1.25 hours = 75 minutes for the slowest runner to a minimum time of $\frac{5 \text{ miles}}{7 \text{ miles/hour}}$ = about 0.71429 hours = about 42.86 minutes for the fastest runner. All answers between (and including) 42.86 minutes and 75 minutes are correct.

Chapter 8

Algebra

In This Chapter…

Algebra

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

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1. If $4(–3x – 8) = 8(–x + 9)$, what is the value of x^2?

$\boxed{}$

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2. If $2x(4 – 6) = –2x + 12$, what is the value of x?

$\boxed{}$

---

3. If $x \neq 0$ and $\frac{3(6-x)}{2x} = –6$, what is the value of x?

$\boxed{}$

---

4. If $x \neq 2$ and $\frac{8-2(-4+10x)}{2-x} = 17$, what is the value of x?

$\boxed{}$

–5 is 7 more than –z.

	Quantity A	**Quantity B**
5.	z	–12

---

6. If $(x + 3)^2 = 225$, which of the following could be the value of $x - 1$?

(A) 13

(B) 12

(C) –12

(D) –16

(E) –19

$$x = 2$$

	Quantity A	**Quantity B**
7.	$x^2 - 4x + 3$	1

$$p = 300c^2 - c$$
$$c = 100$$

	Quantity A	**Quantity B**
8.	p	$29{,}000c$

$$-(x)^3 = 64$$

	Quantity A	**Quantity B**
9.	x^4	x^5

---

10. If $3t^3 - 7 = 74$, what is the value of $t^2 - t$?

(A) –3

(B) 3

(C) 6

(D) 9

(E) 18

11. If $x - y = 4$ and $2x + y = 5$, what is the value of x?

$\boxed{}$

12. $4x + y + 3z = 34$

$4x + 3z = 21$

What is the value of y?

$\boxed{}$

	Quantity A	Quantity B
13.	$(x + 2)(x - 3)$	$x^2 - x - 6$

$xy > 0$

	Quantity A	Quantity B
14.	$(2x - y)(x + 4y)$	$2x^2 + 8xy - 4y^2$

$x^2 - 2x = 0$

	Quantity A	Quantity B
15.	x	2

	Quantity A	Quantity B
16.	$d(d^2 - 2d + 1)$	$d(d^2 - 2d) + 1$

	Quantity A	Quantity B
17.	$xy^2z(x^2z + yz^2 - xy^2)$	$x^3y^2z^2 + xy^3z^3 - x^2y^4z$

$a = 2b = 4c$ and a, b, and c are integers.

	Quantity A	Quantity B
18.	$a + b$	$a + c$

$k = 2m = 4n$ and k, m, and n are non-negative integers.

	Quantity A	Quantity B
19.	km	kn

For the positive integers a, b, c, and d, a is half of b, which is one-third of c. The value of d is three times the value of c.

	Quantity A	Quantity B
20.	$\frac{a+b}{c}$	$\frac{a+b+c}{d}$

$$3x + 6y = 27$$
$$x + 2y + z = 11$$

	Quantity A	Quantity B
21.	$z + 5$	$x + 2y - 2$

---

22. If $(x - y) = \sqrt{12}$ and $(x + y) = \sqrt{3}$, what is the value of $x^2 - y^2$?

(A) 3

(B) 6

(C) 9

(D) 36

(E) It cannot be determined from the information given.

$$a \neq b$$

	Quantity A	Quantity B
23.	$\frac{a-b}{b-a}$	1

$$a = \frac{b}{2}$$
$$c = 3b$$

	Quantity A	**Quantity B**
24.	a	c

---

25. If $xy \neq 0$ and $x \neq -y$, $\dfrac{x^{36} - y^{36}}{(x^{18} + y^{18})(x^{9} + y^{9})}$

(A) 1

(B) $x^2 - y^2$

(C) $x^9 - y^9$

(D) $x^{18} - y^{18}$

(E) $\dfrac{1}{x^9 - y^9}$

$$x > y$$
$$xy \neq 0$$

	Quantity A	**Quantity B**
26.	$\dfrac{x^2}{y + \frac{1}{y}}$	$\dfrac{y^2}{x + \frac{1}{x}}$

---

27. If $x + y = -3$ and $x^2 + y^2 = 12$, what is the value of $2xy$?

$\boxed{}$

28. If $x - y = \frac{1}{2}$ and $x^2 - y^2 = 3$, what is the value of $x + y$?

29. If $x^2 - 2xy = 84$ and $x - y = -10$, what is the value of $|y|$?

30. Which of the following is equal to $(x - 2)^2 + (x - 1)^2 + x^2 + (x + 1)^2 + (x + 2)^2$?

(A) $5x^2$

(B) $5x^2 + 10$

(C) $x^2 + 10$

(D) $5x^2 + 6x + 10$

(E) $5x^2 - 6x + 10$

31. If $a = (x + y)^2$ and $b = x^2 + y^2$ and $xy > 0$, which of the following must be true?

Indicate <u>all</u> such statements.

☐ $a = b$

☐ $a > b$

☐ a is positive

32. a is directly proportional to b. If $a = 8$ when $b = 2$, what is a when $b = 4$?

(A) 10

(B) 16

(C) 32

(D) 64

(E) 128

Algebra Answers

1. **676.** Distribute, group like terms, and solve for x:

$$\begin{aligned} 4(-3x - 8) &= 8(-x + 9) \\ -12x - 32 &= -8x + 72 \\ -32 &= 4x + 72 \\ -104 &= 4x \\ -26 &= x \end{aligned}$$

Then, multiply 26 by 26 in the calculator (or –26 by –26, although the negatives will cancel each other out) to get x^2, which is 676.

2. **–6.**

$$\begin{aligned} 2x(4 - 6) &= -2x + 12 \\ 2x(-2) &= -2x + 12 \\ -4x &= -2x + 12 \\ -2x &= 12 \\ x &= -6 \end{aligned}$$

3. **–2.** $\dfrac{3(6-x)}{2x} = -6$

Multiply both sides by $2x$, distribute the left side, combine like terms, and solve:

$$\begin{aligned} 3(6 - x) &= -6(2x) \\ 18 - 3x &= -12x \\ 18 &= -9x \\ -2 &= x \end{aligned}$$

4. –**6.** $\frac{8-2(-4+10x)}{2-x}$ = 17

Multiply both sides by the expression 2 – *x*, distribute both sides, combine like terms, and solve:

$$\begin{aligned} 8-2(-4+10x) &= 17(2-x) \\ 8+8-20x &= 34-17x \\ 16-20x &= 34-17x \\ 16 &= 34+3x \\ -18 &= 3x \\ -6 &= x \end{aligned}$$

5. **(A).** Translate the question stem into an equation and solve for *z*:

$$\begin{aligned} -5 &= -z+7 \\ -12 &= -z \\ 12 &= z \end{aligned}$$

Because $z = 12 > -12$, Quantity A is greater.

6. **(E).** Begin by square-rooting both sides of the equation, but remember that 225 could be the square of either 15 or –15. (The calculator will not remind you of this! It's your job to keep this in mind). So:

$$\begin{aligned} x + 3 &= 15 \\ x &= 12 \\ \text{so, } x - 1 &= 11 \end{aligned}$$

OR

$$\begin{aligned} x + 3 &= -15 \\ x &= -18 \\ \text{so, } x - 1 &= -19 \end{aligned}$$

Only –19 appears in the choices.

7. **(B).** To evaluate the expression in Quantity A, replace x with 2.

$$\begin{aligned} x^2 - 4x + 3 &= \\ (2)^2 - 4(2) + 3 &= \\ 4 - 8 + 3 &= -1 < 1 \end{aligned}$$

Therefore, Quantity B is greater.

8. **(A).** To find the value of p, first replace c with 100 to find the value for Quantity A:

$$\begin{aligned} p &= 300c^2 - c \\ p &= 300(100)^2 - 100 \\ p &= 300(10{,}000) - 100 \\ p &= 3{,}000{,}000 - 100 = 2{,}999{,}900 \end{aligned}$$

Since $c = 100$, the value for Quantity B is 29,000(100) = 2,900,000. Quantity A is greater.

9. **(A).** First, solve for *x:*

$$-(x)^3 = 64$$

$$(x)^3 = -64$$

The GRE calculator will not do a cube root. As a result, cube roots on the GRE tend to be quite small and easy to puzzle out. What number times itself three times equals –64? The answer is $x = -4$.

Since x is negative, Quantity A is positive (a negative number times itself four times is positive) and Quantity B is negative (a negative number times itself five times is negative). No further calculations are needed to conclude that Quantity A is greater. Notice that solving for the value of x here was not strictly necessary. Knowing that the cube root of a negative number is negative gives you all the information you need to solve.

10. **(C).** First, solve for t:

$$\begin{aligned} 3t^3 - 7 &= 74 \\ 3t^3 &= 81 \\ t^3 &= 27 \\ t &= 3 \end{aligned}$$

Now, plug $t = 3$ into $t^2 - t$:

$$(3)^2 - 3 = 9 - 3 = 6$$

11. **3.** Notice that the first equation has the term $-y$ while the second equation has the term $+y$. While it is possible to use the substitution method, summing the equations together will make $-y$ and y cancel, so this is the easiest way to solve for x.

$$\begin{array}{rcl} x - \cancel{y} & = & 4 \\ 2x + \cancel{y} & = & 5 \\ \hline 3x & = & 9 \\ x & = & 3 \end{array}$$

12. **13.** This question contains only two equations, but three variables. To isolate y, both x and z must be eliminated. Notice that the coefficients of x and z are the same in both equations. Subtract the second equation from the first to eliminate x and z.

$$\begin{array}{rcl} 4x + y + 3z & = & 34 \\ -(4x \quad\; + 3z) & = & 21) \\ \hline y & = & 13 \end{array}$$

13. **(C).** FOIL the terms in Quantity A:

$$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

The two quantities are equal.

14. **(B).** FOIL the terms in Quantity A:

$$(2x - y)(x + 4y) = 2x^2 + 8xy - xy - 4y^2 = 2x^2 + 7xy - 4y^2$$

Since $2x^2$ and $-4y^2$ appear in both quantities, eliminate them. Quantity A is now equal to $7xy$ and Quantity B is now equal to $8xy$. Because $xy > 0$, Quantity B is greater. (Don't assume! If xy were zero, the two quantities would have been equal. If xy were negative, Quantity A would have been greater.)

15. **(D).** Factor $x^2 - 2x = 0$:

$$x^2 - 2x = 0$$
$$x(x - 2) = 0$$
$$x = 0 \text{ OR } (x - 2) = 0$$

$x = 0$ or 2.

Thus, Quantity A could be less than or equal to Quantity B. The relationship cannot be determined from the information given.

(Note that you *cannot* simply divide both sides of the original equation by x. It is illegal to divide by a variable unless it is certain that the variable does not equal zero.)

16. **(D).** In Quantity A, multiply d by every term in the parentheses:

$$d(d^2 - 2d + 1) =$$
$$(d \times d^2) - (d \times 2d) + (d \times 1) =$$
$$d^3 - 2d^2 + d$$

In Quantity B, multiply d by the two terms in the parentheses:

$$d(d^2 - 2d) + 1 =$$
$$(d \times d^2) - (d \times 2d) + 1 =$$
$$d^3 - 2d^2 + 1$$

Because $d^3 - 2d^2$ is common to both quantities, it can be ignored. The comparison is really between d and 1. Without more information about d, the relationship cannot be determined from the information given.

17. **(C).** In Quantity A, the term xy^2z on the outside of the parentheses must be multiplied by each of the three terms inside the parentheses. Then simplify the expression as much as possible.

Taking one term at a time, the first is $xy^2z \times x^2z = x^3y^2z^2$, because there are three factors of x, two factors of y, and two factors of z. Similarly, the second term is $xy^2z \times yz^2 = xy^3z^3$ and the third is $xy^2z \times (-xy^2) = -x^2y^4z$. Adding these three terms together gives the distributed form of Quantity A: $x^3y^2z^2 + xy^3z^3 - x^2y^4z$.

This is identical to Quantity B, so the two quantities are equal.

18. **(D).** Since a is common to both quantities, it can be ignored. The comparison is really between b and c. Because $2b = 4c$, it is true that $b = 2c$, so the comparison is really between $2c$ and c. Watch out for negatives. If the variables are positive, Quantity A is greater, but if the variables are negative, Quantity B is greater.

19. **(D).** If the variables are positive, Quantity A is greater. However, all three variables could equal zero, in which case the two quantities are equal. Watch out for the word "non-negative," which means "positive or zero."

20. **(C).** The following relationships are given: $a = \frac{b}{2}$, $b = \frac{c}{3}$, and $d = 3c$. Pick one variable and put everything in terms of that variable. For instance, variable a:

$$b = 2a$$
$$c = 3b = 3(2a) = 6a$$
$$d = 3c = 3(6a) = 18a$$

Substitute into the quantities and simplify.

Quantity A: $\frac{a+b}{c} = \frac{a+2a}{6a} = \frac{3a}{6a} = \frac{1}{2}$

Quantity B: $\frac{a+b+c}{d} = \frac{a+2a+6a}{18a} = \frac{9a}{18a} = \frac{1}{2}$

The two quantities are equal.

21. **(C).** This question may at first look difficult, as there are three variables and only two equations. However, notice that the top equation can be divided by 3, yielding $x + 2y = 9$. This can be plugged into the second equation:

$$\begin{aligned}(x + 2y) + z &= 11\\ (9) + z &= 11\\ z &= 2\end{aligned}$$

Quantity A is thus $2 + 5 = 7$. For Quantity B, remember that $x + 2y = 9$. Thus, Quantity B is $9 – 2 = 7$.

The two quantities are equal.

22. **(B).** The factored form of the Difference of Squares (one of the "special products" you need to memorize for the exam) is comprised of the terms given in this problem:

$$x^2 - y^2 = (x + y)(x - y)$$

Substitute the values $\sqrt{12}$ and $\sqrt{3}$ in place of $(x - y)$ and $(x + y)$, respectively:

$$x^2 - y^2 = \sqrt{12} \times \sqrt{3}$$

Combine 12 and 3 under the same root sign and solve:

$$x^2 - y^2 = \sqrt{12} \times \sqrt{3}$$

$$x^2 - y^2 = \sqrt{36}$$

$$x^2 - y^2 = 6$$

23. **(B).** Plug in any two unequal values for a and b, and Quantity A will always be equal to –1. This is because a negative sign can be factored out of the top or bottom of the fraction to show that the top and bottom are the same, except for their signs:

$$\frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = -1$$

24. **(D).** To compare a and c, put c in terms of a. Multiply the first equation by 2 to find that $b = 2a$. Substitute into the second equation: $c = 3b = 3(2a) = 6a$. If all three variables are positive, then $6a > a$. If all three variables are negative, then $a > 6a$. Finally, all three variables could equal zero, making the two quantities equal.

25. **(C).** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. This pattern works for any perfect square minus another perfect square. Thus, $x^{36} - y^{36}$ will factor according to this pattern. Note that $\sqrt{x^{36}} = \left(x^{36}\right)^{1/2} = x^{36/2} = x^{18}$, or $x^{36} = (x^{18})^2$. First, factor $x^{36} - y^{36}$ in the numerator, then cancel $x^{18} + y^{18}$ with the $x^{18} + y^{18}$ on the bottom:

$$\frac{x^{36}-y^{36}}{\left(x^{18}+y^{18}\right)\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{18}-y^{18}\right)}{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{9}+y^{9}\right)}=\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}$$

The $x^{18} - y^{18}$ in the numerator will also factor according to this pattern. Then cancel $x^9 + y^9$ with the $x^9 + y^9$ on the bottom:

$$\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{9}+y^{9}\right)}\left(x^{9}-y^{9}\right)}{\cancel{\left(x^{9}+y^{9}\right)}}=x^{9}-y^{9}$$

26. **(D).** It is possible to simplify first and then plug in examples, or to just plug in examples without simplifying. For instance, if $x = 2$ and $y = 1$:

Quantity A: $\dfrac{x^2}{y+\dfrac{1}{y}}=\dfrac{2^2}{1+\dfrac{1}{1}}=\dfrac{4}{2}=2$

Quantity B: $\dfrac{y^2}{x+\dfrac{1}{x}}=\dfrac{1^2}{2+\dfrac{1}{2}}=\dfrac{1}{\dfrac{5}{2}}=\dfrac{2}{5}$

In this case, Quantity A is greater. Next, try negatives. If $x = –1$ and $y = –2$ (remember, x must be greater than y):

Quantity A: $\dfrac{x^2}{y+\dfrac{1}{y}}=\dfrac{(-1)^2}{-2+\dfrac{1}{-2}}=\dfrac{1}{\dfrac{5}{-2}}=\dfrac{-2}{5}$

Quantity B: $\dfrac{y^2}{x+\dfrac{1}{x}}=\dfrac{(-2)^2}{(-1)+\dfrac{1}{-1}}=\dfrac{4}{-2}=-2$

Quantity A is still greater. However, before assuming that Quantity A is *always* greater, make sure you have tried every category of possibilities for x and y. What if x is positive and y is negative? For instance, $x = 2$ and $y = –2$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{-2+\frac{1}{-2}} = \frac{4}{-\frac{5}{2}} = 4 \times -\frac{2}{5} = -\frac{8}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(2)+\frac{1}{2}} = \frac{4}{\frac{5}{2}} = 4 \times \frac{2}{5} = \frac{8}{5}$$

In this case, Quantity B is greater. The relationship cannot be determined from the information given.

27. **–3.** One of the "special products" you need to memorize for the GRE is $x^2 + 2xy + y^2 = (x + y)^2$. Write this pattern on your paper, plug in the given values, and simplify, solving for $2xy$:

$$\begin{aligned} x^2 + 2xy + y^2 &= (x + y)^2 \\ (x^2 + y^2) + 2xy &= (x + y)^2 \\ (12) + 2xy &= (-3)^2 \\ 12 + 2xy &= 9 \\ 2xy &= -3 \end{aligned}$$

28. **6.** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. Write this pattern on your paper and plug in the given values, solving for $x + y$:

$$\begin{aligned} x^2 - y^2 &= (x + y)(x - y) \\ 3 &= (x + y)(1/2) \\ 6 &= x + y \end{aligned}$$

29. **4.** One of the "special products" you need to memorize for the exam is $x^2 - 2xy + y^2 = (x - y)^2$. Write this pattern on your paper and plug in the given values:

$$\begin{aligned} x^2 - 2xy + y^2 &= (x - y)^2 \\ 84 + y^2 &= (-10)^2 \\ 84 + y^2 &= 100 \\ y^2 &= 16 \\ y &= 4 \text{ or } -4, \text{ so } |y| = 4. \end{aligned}$$

30. **(B)**. First, multiply out (remember FOIL = First, Outer, Inner, Last) each of the terms in parentheses:

$(x^2 - 2x - 2x + 4) + (x^2 - 1x - 1x + 1) + (x^2) + (x^2 + 1x + 1x + 1) + (x^2 + 2x + 2x + 4)$

Note that some of the terms will cancel each other out (e.g., $-x$ and x, $-2x$ and $2x$):

$(x^2 + 4) + (x^2 + 1) + (x^2) + (x^2 + 1) + (x^2 + 4)$

Finally, combine:

$5x^2 + 10$

31. ***a* > *b* and *a* is positive.** Distribute for a: $a = (x + y)^2 = x^2 + 2xy + y^2$. Since $b = x^2 + y^2$, a and b are the same except for the "extra" $2xy$ in a. Since xy is positive, a is greater than b. The 1st statement is false and the 2nd statement is true.

Each term in the sum for a is positive: xy is given as positive, and x^2 and y^2 are definitely positive, as they are squared and not equal to zero. Therefore, a

$= x^2 + 2xy + y^2$ is positive. The 3rd statement is true.

32. **(B).** To answer this question, it is important to understand what is meant by the phrase “directly proportional.” It means that $a = kb$, where k is a constant. In alternative form: $\frac{a}{b} = k$, where k is a constant.

So, because they both equal the constant, $\frac{a_{\text{old}}}{b_{\text{old}}} = \frac{a_{\text{new}}}{b_{\text{new}}}$. Plugging in values:

$\frac{8}{2} = \frac{a_{\text{new}}}{4}$. Cross-multiply and solve:

$$32 = 2a_{\text{new}}$$
$$a_{\text{new}} = 16$$

Chapter 9

Inequalities and Absolute Values

In This Chapter…

Inequalities and Absolute Values

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{\quad\quad}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{\quad\quad}}{\boxed{\quad\quad}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

$$7y - 3 \leq 4y + 9$$

	Quantity A	**Quantity B**
1.	y	4

$$3|x - 4| = 16$$

	Quantity A	**Quantity B**
2.	x	$\frac{28}{3}$

---

3. If $6 < 2x - 4 < 12$, which of the following could be the value of x?

(A) 4

(B) 5

(C) 7

(D) 8

(E) 9

4. If $y < 0$ and $4x > y$, which of the following could be equal to $\frac{x}{y}$?

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) 4

$$3(x - 7) \geq 9$$
$$0.25y - 3 \leq 1$$

	Quantity A	Quantity B
5.	x	y

---

6. If $|1 - x| = 6$ and $|2y - 6| = 10$, which of the following could be the value of xy?

Indicate <u>all</u> such values.

☐ –40
☐ –14
☐ –10
☐ 56

7. If $2(x - 1)^3 + 3 \leq 19$, which of the following must be true?

(A) $x \geq 3$

(B) $x \leq 3$

(C) $x \geq -3$

(D) $x \leq -3$

(E) $x < -3$ or $x > 3$

8. If $3p < 51$ and $5p > 75$, what is the value of the integer p?

(A) 15

(B) 16

(C) 24

(D) 25

(E) 26

9. A bicycle wheel has spokes that go from a center point in the hub to equally spaced points on the rim of the wheel. If there are fewer than six spokes, what is the smallest possible angle between any two spokes?

(A) 18°

(B) 30°

(C) 40°

(D) 60°

(E) 72°

$$|-x| \geq 6$$
$$xy^2 < 0 \text{ and } y \text{ is an integer.}$$

	Quantity A	**Quantity B**
10.	x	-4

---

11. If $\frac{|x+4|}{2} > 5$ and $x < 0$, which of the following could be the value of x?

Indicate <u>all</u> such values.

☐ –6
☐ –14
☐ –18

$$|x^3| < 64$$

	Quantity A	**Quantity B**
12.	$-x$	$-\|x\|$

13. If $|3x + 7| \geq 2x + 12$, then which of the following is true?

(A) $x \leq \frac{-19}{5}$

(B) $x \geq \frac{-19}{5}$

(C) $x \geq 5$

(D) $x \leq \frac{-19}{5}$ or $x \geq 5$

(E) $\frac{-19}{5} \leq x \leq 5$

$|3 + 3x| < -2x$

	Quantity A	**Quantity B**
14.	$\|x\|$	4

---

15. If $|y| \leq -4x$ and $|3x - 4| = 2x + 6$, what is the value of x?

(A) -3

(B) $-\frac{1}{3}$

(C) $-\frac{2}{5}$

(D) $\frac{1}{3}$

(E) 10

---

x is an integer such that $-x|x| \geq 4$.

	Quantity A	**Quantity B**
16.	x	2

$|x| < 1$ and $y > 0$

	Quantity A	**Quantity B**
17.	$\lvert x\rvert + y$	xy

$|x| > |y|$ and $x + y > 0$

	Quantity A	**Quantity B**
18.	y	x

x and y are integers such that $|x|(y) + 9 < 0$ and $|y| \leq 1$.

	Quantity A	**Quantity B**
19.	x	-9

$p + |k| > |p| + k$

	Quantity A	**Quantity B**
20.	p	k

$|x| + |y| > |x + z|$

	Quantity A	**Quantity B**
21.	y	z

$b \neq 0$

$\frac{|a|}{b} > 1$

$a + b < 0$

	Quantity A	**Quantity B**
22.	a	0

---

23. If $f^2g < 0$, which of the following must be true?

(A) $f < 0$

(B) $g < 0$

(C) $fg < 0$

(D) $fg > 0$

(E) $f^2 < 0$

24. $\sqrt{96} < x\sqrt{6}$ and $\frac{x}{\sqrt{6}} < \sqrt{6}$. If x is an integer, which of the following is the value of x?

(A) 2

(B) 3

(C) 4

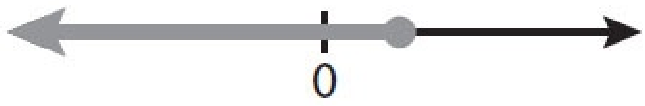
(D) 5

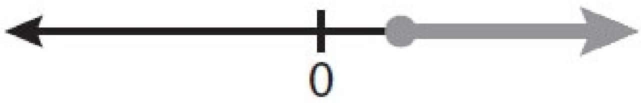
(E) 6

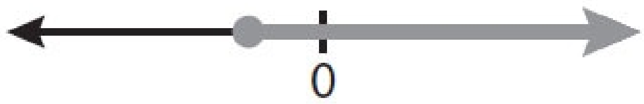
$$|x|y > x|y|$$

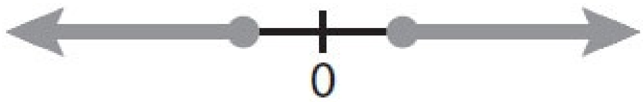
	Quantity A	Quantity B
25.	$(x + y)^2$	$(x - y)^2$

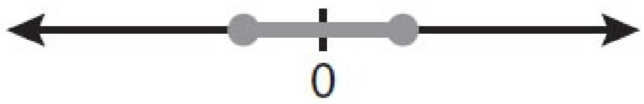
26. Which of the following could be the graph of all values of x that satisfy the inequality $4 - 11x \geq \dfrac{-2x+3}{2}$?

(A) 

(B) 

(C) 

(D) 

(E) 

$$-1 < a < 0 < |a| < b < 1$$

	Quantity A	Quantity B
27.	$\left(\dfrac{a^2\sqrt{b}}{\sqrt{a}}\right)^2$	$\dfrac{ab^5}{\left(\sqrt{b}\right)^4}$

$$x > |y| > z$$

	Quantity A	Quantity B
28.	$x + y$	$|y| + z$

29. The integers k, l, and m are consecutive even integers between 23 and 33.

Which of the following could be the average (arithmetic mean) of k, l, and m?

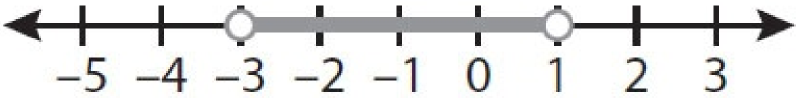
(A) 24

(B) 25

(C) 25.5

(D) 28

(E) 32

30. 

The number line above represents which of the following inequalities?

(A) $x < 1$

(B) $-6 < 2x < 2$

(C) $-9 < 3x < 6$

(D) $1 < 2x < 3$

(E) $x > -3$

31. For a jambalaya cook-off, there will be x judges sitting in a single row of x chairs. If x is greater than 3 but no more than 6, which of the following could be the number of possible seating arrangements for the judges?

Indicate <u>two</u> such numbers.

☐ 6
☐ 25
☐ 120
☐ 500
☐ 720

32. Which of the following inequalities is equivalent to $-\frac{a}{3b} < c$ for all non-zero values of a, b, and c?

Indicate <u>all</u> such inequalities.

☐ $\frac{a}{b} > -3c$
☐ $-\frac{a}{3} < bc$
☐ $a > -3bc$

$$|x + y| = 10$$
$$x \geq 0$$
$$z < y - x$$

<u>**Quantity A**</u>	<u>**Quantity B**</u>

33. z 10

$$0 < a < \frac{b}{2} < 9$$

	Quantity A	Quantity B
34.	$9 - a$	$\frac{b}{2} - a$

For all values of the integer p such that $1.9 < |p| < 5.3$, the function $f(p) = p^2$.

	Quantity A	Quantity B
35.	$f(p)$ for the greatest value of p	$f(p)$ for the least value of p

---

36. If $\left|\frac{a}{b}\right|$ and $\left|\frac{x}{y}\right|$ are reciprocals and $\frac{a}{b}\left(\frac{x}{y}\right) < 0$, which of the following must be true?

(A) $ab < 0$

(B) $\frac{a}{b}\left(\frac{x}{y}\right) < -1$

(C) $\frac{a}{b} < 1$

(D) $\frac{a}{b} = -\frac{y}{x}$

(E) $\frac{y}{x} > \frac{a}{b}$

37. If $mn < 0$ and $\frac{k}{m} + \frac{l}{n} < mn$, which of the following must be true?

(A) $km + ln < (mn)^2$

(B) $kn + lm < 1$

(C) $kn + lm > (mn)^2$

(D) $k + l > mn$

(E) $km > -ln$

38. If the reciprocal of the negative integer x is greater than the sum of y and z, then which of the following must be true?

(A) $x > y + z$

(B) y and z are positive.

(C) $1 > x(y + z)$

(D) $1 < xy + xz$

(E) $\frac{1}{x} > z - y$

39. If u and $-3v$ are greater than 0, and $\sqrt{u} < \sqrt{-3v}$, which of the following cannot be true?

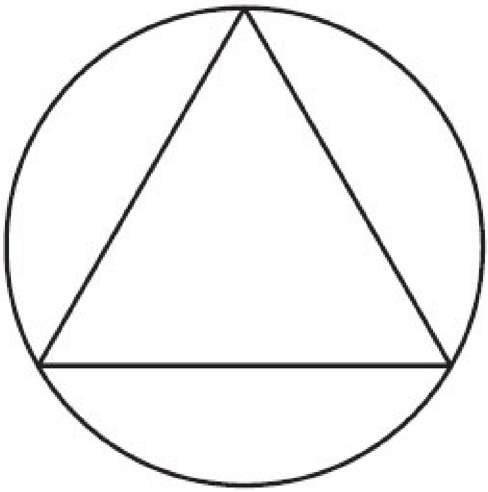
(A) $\frac{u}{3} < -v$

(B) $\frac{u}{v} > -3$

(C) $\sqrt{\frac{u}{-v}} < \sqrt{3}$

(D) $u + 3v > 0$

(E) $u < -3v$



40. In the figure above, an equilateral triangle is inscribed in a circle. If the arc bounded by adjacent corners of the triangle is between 4π and 6π long, which of the following could be the diameter of the circle?

(A) 6.5

(B) 9

(C) 11.9

(D) 15

(E) 23.5

Inequalities and Absolute Values Answers

1. **(D).** Solve the inequality algebraically:

$$\begin{aligned} 7y - 3 &\le 4y + 9 \\ 3y - 3 &\le 9 \\ 3y &\le 12 \\ y &\le 4 \end{aligned}$$

Because y could be less than or equal to 4, the relationship cannot be determined from the information given.

2. **(D).** Solve the inequality by first dividing both sides by 3 to isolate the absolute value. Then solve for the positive and negative possibilities of (x – 4), using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$3|x-4| = 16$$

$$|x-4| = \frac{16}{3}$$

$$+(x-4) = \frac{16}{3} \quad \text{or} \quad -(x-4) = -\frac{16}{3}$$

$$(x-4) = \frac{16}{3} \qquad (x-4) = -\frac{16}{3}$$

$$x - 4 = \frac{16}{3} \qquad x = -\frac{16}{3} + 4$$

$$x = \frac{16}{3} + 4 \qquad x = -\frac{16}{3} + \frac{12}{3}$$

$$x = \frac{16}{3} + \frac{12}{3} \qquad x = -\frac{4}{3}$$

$$x = \frac{28}{3}$$

Thus, x could be $\frac{28}{3}$ or $-\frac{4}{3}$, making the two quantities equal or Quantity B greater, respectively. The relationship cannot be determined from the information given.

3. **(C).** When manipulating a “three-sided” inequality, perform the same operations on all “sides.” Therefore, the first step to simplify this inequality would be to add 4 to all three sides to get: $10 < 2x < 16$. Next, divide all three sides by 2. The result is $5 < x < 8$. The only answer choice that fits within the parameters of this inequality is 7.

4. **(A).** If y is negative, then dividing both sides of the second inequality by y yields $\frac{4x}{y} < 1$. Remember to switch the direction of the inequality sign when multiplying or dividing by a negative (whether that negative is in number or variable form). Next, dividing both sides by 4 changes the inequality to $\frac{x}{y} < \frac{1}{4}$. The only answer choice less than $\frac{1}{4}$ is 0.

5. **(D).** Solve each inequality algebraically:

$$\begin{aligned} 3(x-7) &\geq 9 \\ x-7 &\geq 3 \\ x &\geq 10 \end{aligned}$$

$$\begin{aligned} 0.25y-3 &\leq 1 \\ 0.25y &\leq 4 \\ y &\leq 16 \end{aligned}$$

Since the ranges for x and y overlap, either quantity could be greater. For instance, x could be 11 and y could be 15 (y is greater), or x could be 1,000 and y could be –5 (x is greater). The relationship cannot be determined from the information given.

6. **–40, –14, and 56 only.** Solve each absolute value, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$|1-x| = 6$$

$$\begin{array}{lcl} +(1-x) = 6 & \text{or} & -(1-x) = 6 \\ (1-x) = 6 & & (1-x) = -6 \\ -x = 5 & & -x = -7 \\ x = -5 & & x = 7 \end{array}$$

$$x = -5 \text{ or } 7$$

$$|2y-6| = 10$$

$$+ (2y - 6) = 10 \quad \text{or} \quad -(2y - 6) = 10$$

$$(2y - 6) = 10 \qquad (2y - 6) = -10$$

$$2y = 16 \qquad 2y = -4$$

$$y = 8 \qquad y = -2$$

$$y = 8 \text{ or } -2$$

Since $x = –5$ or 7 and $y = 8$ or –2, calculate all four possible combinations for xy:

$$\begin{aligned} (-5)(8) &= -40 \\ (-5)(-2) &= 10 \\ (7)(8) &= 56 \\ (7)(-2) &= -14 \end{aligned}$$

Select –40, –14, and 56. (Do *not* pick –10, as xy could be 10, but not –10.)

7. **(B).**

$$\begin{aligned} 2(x - 1)^3 + 3 &\leq 19 \\ 2(x - 1)^3 &\leq 16 \\ (x - 1)^3 &\leq 8 \end{aligned}$$

Taking the cube root of an inequality is permissible here, because cubing a number, unlike squaring it, does not change its sign.

$$\begin{aligned} x - 1 &\leq 2 \\ x &\leq 3 \end{aligned}$$

8. **(B).** Dividing the first inequality by 3 results in $p < 17$. Dividing the second inequality by 5 results in $p > 15$. Therefore, $15 < p < 17$. Because p is an integer, it must be 16.

9. **(E).** In this scenario, if there are n spokes, there are n angles between them. Thus, the measure of the angle between spokes is $\frac{360°}{n}$. Since $n < 6$, rewrite this expression as $\frac{360°}{\text{(less than 6)}}$. Dividing by a "less than" produces a "greater than" result. Therefore, $\frac{360°}{\text{(less than 6)}}$ = greater than 60°. The only answer that is greater than 60° is (E). To verify, note that n can be at most 5, because n must be an integer. Because there are 360° in a circle, a wheel with

5 spokes would have $\frac{360°}{5}$ = 72° between adjacent spokes.

10. **(B).** First, solve the inequality for x, remembering the two cases that must be considered when dealing with absolute value: $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$|-x| \geq 6$$

$$+(-x) \geq 6 \quad \text{or} \quad -(-x) \geq 6$$

$$-x \geq 6 \qquad\qquad x \geq 6$$

$$x \leq -6$$

$$x \leq -6 \text{ or } x \geq 6$$

Because $xy^2 < 0$, neither x nor y equals zero. A squared term cannot be negative, so y^2 must be positive. For xy^2 to be negative, x must be negative. This rules out the $x \geq 6$ range of solutions for x. Thus, $x \leq -6$ is the only range of valid solutions. Since all values less than or equal to –6 are less than –4, Quantity B is greater.

11. –**18 only.** Solve the absolute value inequality by first isolating the absolute value:

$$\frac{|x+4|}{2} > 5$$

$$|x + 4| > 10$$

To solve the absolute value, use the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative. Here if $(x + 4)$ is positive or zero, that leaves:

$$x + 4 > 10$$

$$x > 6$$

This, however, is not a valid solution range, as the other inequality indicates that x is negative.

Solve for the negative case, that is, assuming that $(x + 4)$ is negative:

$$-(x + 4) > 10$$

$$(x + 4) < -10$$

$$x < -14$$

Note that this fits the other inequality, which states that $x < 0$.

If $x < -14$, only –18 is a valid answer.

12. **(D).** First, solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$|x^3| < 64$$

$+(x^3) < 64$	or	$-(x^3) < 64$
$x < 4$		$x^3 > -64$ (Flip the inequality sign when multiplying by –1.)
		$x > -4$

$$-4 < x < 4$$

x could be positive, negative, or zero. If *x* is positive or zero, the two quantities are equal. If *x* is negative, Quantity A is greater. The relationship cannot be determined from the information given.

13. **(D).** Solve $|3x + 7| \geq 2x + 12$, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$+(3x + 7) \geq 2x + 12$	or	$-(3x + 7) \geq 2x + 12$
$x + 7 \geq 12$		$-3x - 7 \geq 2x + 12$
$x \geq 5$		$-7 \geq 5x + 12$
		$-19 \geq 5x$
		$\frac{-19}{5} \geq x$

$x \le \frac{-19}{5}$ or $x \ge 5$

14. **(B).** Solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$|3 + 3x| < -2x$$

$$+(3 + 3x) < -2x \quad \text{or} \quad -(3 + 3x) < -2x$$

$$3 + 5x < 0 \qquad\qquad -3 - 3x < -2x$$

$$5x < -3 \qquad\qquad -3 < x$$

$$x < -\frac{3}{5}$$

$$-3 < x < \frac{3}{5}$$

Since x is between -3 and $-\frac{3}{5}$, its absolute value is between $\frac{3}{5}$ and 3.

Quantity B is greater.

15. **(C).** The inequality is not strictly solvable, as it has two unknowns. However, any absolute value cannot be negative. Putting $0 \le |y|$ and $|y| \le -4x$ together, $0 \le -4x$. Dividing both sides by -4 and flipping the inequality sign, this implies that $0 \ge x$.

Now solve the absolute value equation, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$|3x - 4| = 2x + 6$$

$$+(3x - 4) = 2x + 6 \quad \text{or} \quad -(3x - 4) = 2x + 6$$

$$3x - 4 = 2x + 6 \qquad\qquad -3x + 4 = 2x + 6$$

$$x - 4 = 6 \qquad\qquad 4 = 5x + 6$$

$$x = 10 \qquad\qquad -2 = 5x$$

$$x = 10 \text{ or } -\frac{2}{5}$$

If $x = 10$ or $-\frac{2}{5}$, but $0 \geq x$, then x can only be $-\frac{2}{5}$.

16. **(B).** If $-x|x| \geq 4$, $-x|x|$ is positive. Because $|x|$ is positive by definition, $-x|x|$ is positive only when $-x$ is also positive. This occurs when x is negative. For example, $x = -2$ is one solution allowed by the inequality: $-x|x| = -(-2) \times |-2| = 2 \times 2 = 4$.

So, Quantity A can be any integer less than or equal to –2, all of which are less than 2. Quantity B is greater.

17. **(A).** The inequality $|x| < 1$ allows x to be either a positive or negative fraction (or zero). Interpreting the absolute value sign, it is equivalent to $-1 < x < 1$. As indicated, y is positive.

When x is a negative fraction:

> Quantity A: $|x| + y$ = positive fraction + positive = positive
> Quantity B: xy = negative fraction × positive = negative
> Quantity A is greater in these cases.

When x is zero:

> Quantity A: $|x| + y = 0$ + positive = positive
> Quantity B: $xy = 0$ × positive = 0
> Quantity A is greater in this case.

When x is a positive fraction:

> Quantity A: $|x| + y$ = positive fraction + y = greater than y
> Quantity B: xy = positive fraction × y = less than y
> Quantity A is greater in these cases.

In all cases, Quantity A is greater.

18. **(B).** In general, there are four cases for the signs of x and y, some of which can be ruled out by the constraints of this question:

x	y	$x + y > 0$
pos	pos	True
pos	neg	True when $\|x\| > \|y\|$
neg	pos	False when $\|x\| > \|y\|$
neg	neg	False

Only the first two cases need to be considered for this question, since $x + y$ is not greater than zero for the third and fourth cases.

If x and y are both positive, $|x| > |y|$ just means that $x > y$.

If x is positive and y is negative, $x > y$ simply because positive > negative.

In both cases, $x > y$. Quantity B is greater.

19. **(D).** If y is an integer and $|y| \leq 1$, then $y = -1$, 0, or 1. The other inequality can be simplified from $|x|(y) + 9 < 0$ to $|x|(y) < -9$. In other words, $|x|(y)$ is negative. Because $|x|$ cannot be negative by definition, y must be negative, so only $y = -1$ is possible.

If $y = -1$, then $|x|(y) = |x|(-1) = -|x| < -9$. So, $-|x| = -10, -11, -12, -13$, etc.

Thus, $x = \pm 10, \pm 11, \pm 12, \pm 13$, etc. Some of these x values are greater than -9 and some are less than -9. Therefore, the relationship cannot be determined.

20. **(A).** In general, there are four cases for the signs of p and k, some of which can be ruled out by the constraints of this question:

p	k	$p + \|k\| > \|p\| + k$
pos	pos	Not true in this case: For positive numbers, absolute value "does nothing," so both sides are equal to $p + k$.
pos	neg	True for this case: p + (a positive absolute value) is greater than p + (a negative value).
neg	pos	Not true in this case: k + (a negative value) is less than k + (a positive absolute value).
neg	neg	Possible in this case: It depends on relative values. Both sides are a positive plus a negative.

Additionally, check whether p or k could be zero.

If $p = 0$, $p + |k| > |p| + k$ is equivalent to $|k| > k$. This is true when k is negative.

If $k = 0$, $p + |k| > |p| + k$ is equivalent to $p > |p|$. This is not true for any p value.

So, there are three possible cases for p and k values. For the second one, use the identity that $|a| = -a$ when a is negative:

p	k	Interpret:
pos	neg	$p = \text{pos} > \text{neg} = k$ $p > k$
neg	neg	$p + \|k\| > \|p\| + k$ $p + -(k) > -(p) + k$ $p - k > -p + k$ $2p - k > k$ $2p > 2k$

		$p > k$
0	neg	$p = 0 > \text{neg} = k$ $p > k$

In all the cases that are valid according to the constraint inequality, p is greater than k. Quantity A is greater.

21. **(D).** Given only one inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D).

For example, $x = 2$, $y = 5$, and $z = 3$.
Check that $|x| + |y| > |x + z|$: $|2| + |5| > |2 + 3|$ is $7 > 5$, which is true.
In this case, $y > z$ and Quantity A is greater.

Try to find another example such that $y < z$. Always consider negatives in inequalities and absolute value questions. Consider another example: $x = 2$, $y = -5$ and $z = 3$.
Check that $|x| + |y| > |x + z|$: $|2| + |-5| > |2 + 3|$ is $7 > 5$, which is true.
In this case, $z > y$ and Quantity B is greater.

Either statement could be greater. The relationship cannot be determined from the information given.

22. **(B).** If $\frac{|a|}{b}$ is greater than 1, then it is positive. Because $|a|$ is non-negative by definition, b would have to be positive. Thus, when multiplying both sides of the inequality by b, you do not have to flip the sign of the inequality:

$$\frac{|a|}{b} > 1$$

$$|a| > b$$

To summarize, $b > 0$ and $|a| > b$. Putting this together, $|a| > b > 0$.

In order for $a + b$ to be negative, a must be more negative than b is positive. For example, $a = -4$ and $b = 2$ agree with all the constraints so far. Note that a cannot be zero (because $\frac{|a|}{b} = 0$ in this case, not > 1) and a cannot be positive (because $a + b > 0$ in this case, not < 0).

Therefore, $a < 0$. Quantity B is greater.

23. **(B).** Neither f nor g can be zero, or f^2g would be zero. The square of either a positive or negative base is always positive, so f^2 is positive. In order for $f^2g < 0$ to be true, g must be negative. Therefore, the correct answer is (B). Answer choices (A), (C), and (D) are not correct because f could be either positive or negative. Answer choice (E) directly contradicts the truth that f^2 is positive.

24. **(D).** Solve the first inequality:

$$
\begin{aligned}
\sqrt{96} &< x\sqrt{6} \\
\frac{\sqrt{96}}{\sqrt{6}} &< x \\
\sqrt{16} &< x \\
4 &< x
\end{aligned}
$$

Solve the second inequality:

$$\frac{x}{\sqrt{6}} < \sqrt{6}$$

$$x < \sqrt{6}\sqrt{6}$$

$$x < \sqrt{36}$$

$$x < 6$$

Combining the inequalities gives $4 < x < 6$, and since x is an integer, x must be 5.

25. **(B).** In general, there are four cases for the signs of x and y, some of which can be ruled out by the constraint in the question stem. Use the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

x	y	$\lvert x\rvert y > x\lvert y\rvert$ is equivalent to:	True or False?
pos	pos	$xy > xy$	False: $xy = xy$
pos	neg	$xy > x(-y)$	False: xy is negative and $-xy$ is positive.
neg	pos	$(-x)y > xy$	True: xy is negative and $-xy$ is positive.
neg	neg	$(-x)y > x(-y)$	False: $-xy = -xy$

Note that if either x or y equals 0, that case would also fail the constraint.

The only valid case is when x is negative and y is positive:

Quantity A: $(x + y)^2 = x^2 + 2xy + y^2$
Quantity B: $(x - y)^2 = x^2 - 2xy + y^2$

Ignore (or subtract) $x^2 + y^2$ as it is common to both quantities. Thus:

Quantity A: $2xy$ = 2(negative)(positive) = negative
Quantity B: $-2xy$ = –2(negative)(positive) = positive

Quantity B is greater.

26. **(A).** First, solve $4 - 11x \geq \frac{-2x+3}{2}$ for x:

$$\begin{aligned} 4 - 11x &\geq \frac{-2x+3}{2} \\ 8 - 22x &\geq -2x + 3 \\ 5 - 22x &\geq -2x \\ 5 &\geq 20x \\ \frac{5}{20} &\geq x \\ \frac{1}{4} &\geq x \end{aligned}$$

Thus, the correct choice should show the gray line beginning to the right of zero (in the positive zone), and continuing indefinitely into the negative zone. Even without actual values (other than zero) marked on the graphs, only (A) meets these criteria.

27. **(A).** From $-1 < a < 0 < |a| < b < 1$, the following can be determined:

a is a negative fraction,
b is a positive fraction, and
b is more positive than *a* is negative (i.e., $|b| > |a|$, or *b* is farther from 0 on the number line than *a* is).

Using exponent rules, simplify the quantities:

Quantity A: $\left(\frac{a^2\sqrt{b}}{\sqrt{a}}\right)^2 = \frac{\left(a^2\right)^2\left(\sqrt{b}\right)^2}{\left(\sqrt{a}^2\right)} = \frac{a^4 b}{a} = a^3 b$

Quantity B: $\frac{ab^5}{\left(\sqrt{b}\right)^4} = \frac{ab^5}{\left(b^{\frac{1}{2}}\right)^4} = \frac{ab^5}{b^{\frac{1}{2}\times 4}} = \frac{ab^5}{b^2} = ab^3$

Dividing both quantities by *b* would be acceptable, as *b* is positive and doing so won’t flip the relative sizes of the quantities. It would be nice to cancel *a*’s, too, but it is problematic that *a* is negative. Dividing both quantities by a^2

would be okay, though, as a^2 is positive.

Divide both quantities by a^2b:

Quantity A: $\dfrac{a^3b}{a^2b} = a$

Quantity B: $\dfrac{ab^3}{a^2b} = \dfrac{b^2}{a}$

Just to make the quantities more similar in form, divide again by b, which is positive:

Quantity A: $\frac{a}{b}$

Quantity B: $\frac{b}{a}$

Both quantities are negative, as a and b have opposite signs. Remember that b is more positive than a is negative. (i.e., $|b| > |a|$, or b is farther from 0 on the number line than a is.) Thus, each fraction can be compared to –1:

Quantity A: $\frac{a}{b}$ is less negative than –1. That is, $-1 < \frac{a}{b}$.

Quantity B: $\frac{b}{a}$ is more negative than –1. That is, $\frac{b}{a} < -1$.

Therefore, Quantity A is greater.

28. **(D).** Given only a compound inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D). Always consider negatives in inequalities and absolute value questions.

For example, $x = 10$, $y = -9$, and $z = 8$.
Check that $x > |y| > z$: $10 > |-9| > 8$, which is true.
In this case, $x + y = 10 + (-9) = 1$ and $|y| + z = 9 + 8 = 17$. Quantity B is greater.

Try to find another example such that Quantity A is greater.
For example, $x = 2$, $y = 1$, and $z = -3$.
Check that $x > |y| > z$: $2 > |1| > -3$, which is true.
In this case, $x + y = 2 + 1 = 3$ and $|y| + z = 1 + (-3) = -2$. Quantity A is greater.

The relationship cannot be determined from the information given.

29. **(D).** The values for k, l, and m, respectively, could be any of the following three sets:

Set 1: 24, 26, and 28
Set 2: 26, 28, and 30
Set 3: 28, 30, and 32

For evenly spaced sets with an odd number of terms, the average is the middle value. Therefore, the average of k, l, and m could be 26, 28, or 30. Only answer choice (D) matches one of these possibilities.

30. **(B).** The number line indicates a range between, but not including, –3 and

1. However, $-3 < x < 1$ is not a given option. However, answer choice (B) gives the inequality $-6 < 2x < 2$. Dividing all three sides of this inequality by 2 yields $-3 < x < 1$.

31. **120 and 720 only.** If *x* is "greater than 3 but no more than 6," then *x* is 4, 5, or 6. If there are 4 judges sitting in 4 seats, they can be arranged $4! = 4 \times 3 \times 2 \times 1 = 24$ ways. If there are 5 judges sitting in 5 seats, they can be arranged

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. If there are 6 judges sitting in 6 seats, they can be arranged $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways. Thus, 24, 120, and 720 are all possible answers. Only 120 and 720 appear in the choices.

32. $\frac{a}{b} > -3c$ **only.** For this problem, use the rule that multiplying or dividing an inequality by a negative flips the inequality sign. Thus, multiplying or dividing an inequality by a variable should *not* be done unless you know *whether* to flip the inequality sign (i.e., whether the variable represents a positive or a negative number).

1st inequality: TRUE. Multiply both sides of the original inequality by –3 and flip the inequality sign.

2nd inequality: Maybe. Multiply both sides of the original inequality by b to get the 2nd inequality, but only if b is positive. If b is negative, the direction of the inequality sign would have to be changed.

3rd inequality: Maybe. Multiplying both sides of the original inequality by $-3b$ could lead to the 3rd inequality, but because the inequality sign flipped, this is only true if $-3b$ is negative (i.e., if b is positive).

33. **(B).** From $z < y - x$, the value of z depends on x and y. So, solve for x and y as much as possible. There are two cases for the absolute value equation: $|x + y| = 10$ means that $\pm(x + y) = 10$ or that $(x + y) = \pm 10$. Consider these two cases separately.

The positive case:

> $x + y = 10$, so $y = 10 - x$.
> Substitute into $z < y - x$, getting $z < (10 - x) - x$, or $z < 10 - 2x$.
> Because x is at least zero, $10 - 2x \leq 10$.
> Putting the inequalities together, $z < 10 - 2x \leq 10$.
> Thus, $z < 10$.

The negative case:

> $x + y = -10$, so $y = -10 - x$.
> Substitute into $z < y - x$, getting $z < (-10 - x) - x$, or $z < -10 - 2x$.
> Because x is at least zero, $-10 - 2x \leq -10$.
> Putting the inequalities together, $z < -10 - 2x \leq -10$.
> Thus, $z < -10$.

In both cases, 10 is greater than z. Quantity B is greater.

34. **(A).** The variable a is common to both quantities, and adding it to both

quantities to cancel will not change the relative values of the quantities:

Quantity A: $(9 - a) + a = 9$

Quantity B: $\left(\frac{b}{2} - a\right) + a = \frac{b}{2}$

According to the given constraint, $\frac{b}{2} < 9$, so Quantity A is greater.

35. **(C).** If p is an integer such that $1.9 < |p| < 5.3$, p could be 2, 3, 4, or 5, as well as –2, –3, –4, or –5. The greatest value of p is 5, for which the value of $f(p)$ is equal to $5^2 = 25$. The least value of p is –5, for which the value of $f(p)$ is equal to $(-5)^2 = 25$. Therefore, the two quantities are equal.

36. **(D).** If $\frac{a}{b}\left(\frac{x}{y}\right) < 0$, then the two fractions have opposite signs. Therefore, by the definition of reciprocals, $\frac{a}{b}$ must be the negative inverse of $\frac{x}{y}$, no matter which one of the fractions is positive. In equation form, this means $\frac{a}{b} = -\frac{y}{x}$, which is choice (D). The other choices are possible but not necessarily true.

37. **(C).** In order to get m and n out of the denominators of the fractions on the left side of the inequality, multiply both sides of the inequality by mn. The result is $kn + lm > (mn)^2$. The direction of the inequality sign changes because mn is negative. This is an exact match with (C), which must be the correct answer.

38. **(D).** The inequality described in the question is $0 > \frac{1}{x} > y + z$.

Multiplying both sides of this inequality by x, the result is $0 < 1 < xy + xz$. Notice that the direction of the inequality sign must change because x is negative. Therefore:

(A) Maybe true: true only if x equals –1.

(B) Maybe true: either y or z or both can be negative.

(C) False: the direction of the inequality sign is opposite the correct direction determined above.

(D) TRUE: it is a proper rephrasing of the original inequality.

(E) Maybe true: it is not a correct rephrasing of the original inequality.

39. **(D).** When the GRE writes a root sign, the question writers are indicating a non-negative root only. Therefore, both sides of this inequality are positive. Thus, you can square both sides without changing the direction of the inequality sign. So $u < -3v$. Now evaluate each answer choice:

(A) Must be true: divide both sides of $u < -3v$ by 3.

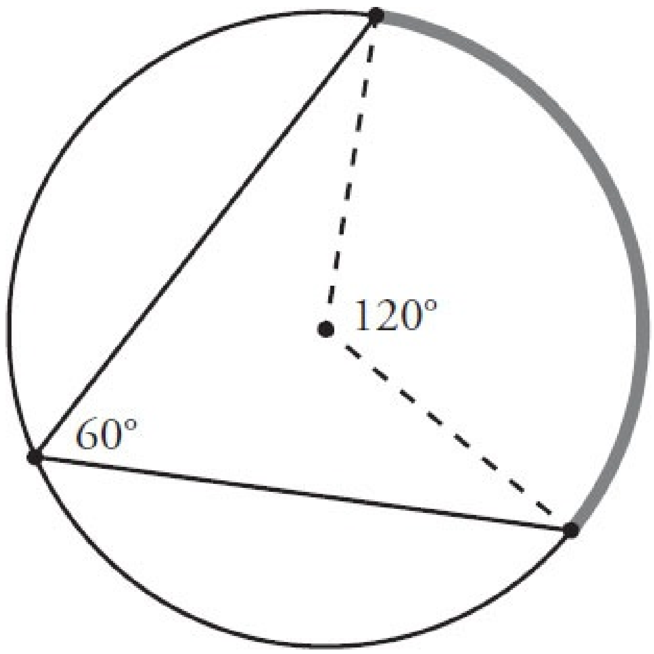
(B) Must be true: it is given that $-3v > 0$ and therefore, $v < 0$. Then, when dividing both sides of $u < -3v$ by v, you must flip the inequality sign and get $\frac{u}{v} > -3$.

(C) Must be true: this is the result after dividing both sides of the original inequality by $\sqrt{-v}$.

(D) CANNOT be true: adding $3v$ to both sides of $u < -3v$ results in $u + 3v < 0$, not $u + 3v > 0$.

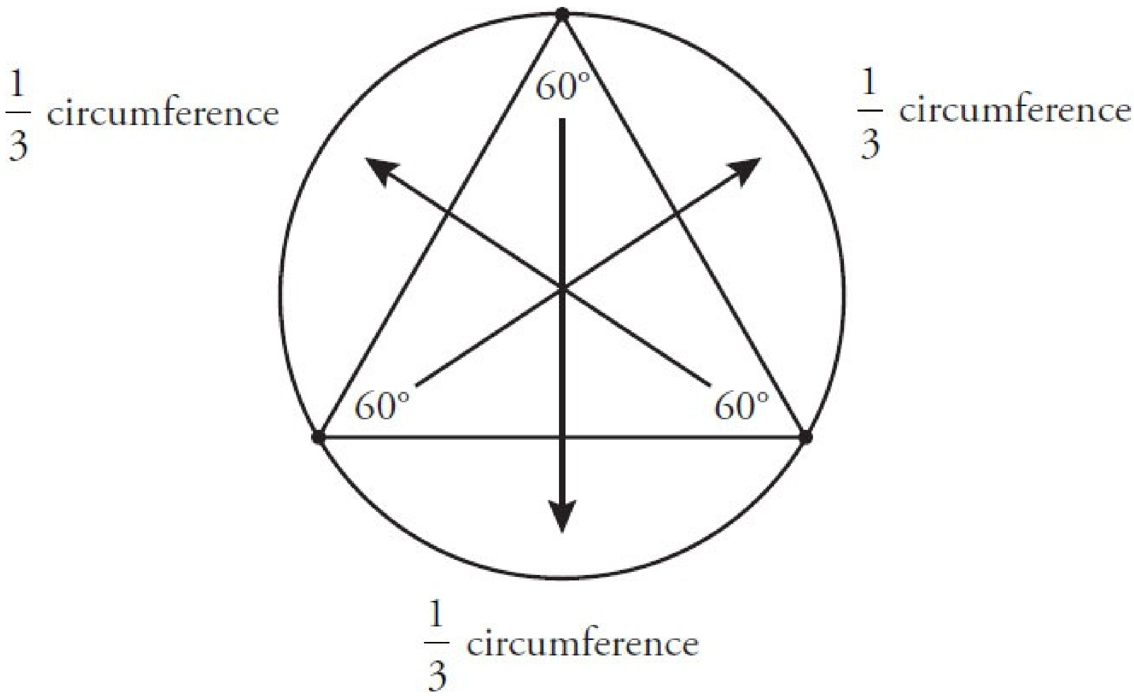
(E) Must be true: this is the result of squaring both sides of the original inequality.

40. **(D).** Since each of the three arcs corresponds to one of the 60° angles of the equilateral triangle, each arc represents $\frac{1}{3}$ of the circumference of the circle. The diagram below illustrates this for just one of the three angles in the triangle:



$$\frac{120°}{360°} = \frac{1}{3}$$

The same is true for each of the three angles:



Since each of the three arcs is between 4π and 6π, triple these values to determine that the circumference of the circle is between 12π and 18π.

Because circumference equals π times the diameter, the diameter of this circle must be between 12 and 18. Only choice (D) is in this range.

Chapter 10

Functions, Formulas, and Sequences

In This Chapter…

Functions, Formulas, and Sequences

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $f(x) = x^2 + 1$, what is the value of $f(2) + f(-2)$?

   (A) 0

   (B) 1

   (C) 4

   (D) 5

   (E) 10

2. If $h(x) = 2x^3 - 3$ and $h(m) = -19$, what is the value of m?

   (A) –3

   (B) –2

   (C) 2

   (D) 6,856

(E) 6,862

3. If $f(x) = x + 5$ and $f(2g) = 19$, what is the value of $f(3 - g)$?

4. If $f(a, b) = a^2b^4$, and $f(m, n) = 5$, what is the value of $f(3m, 2n)$?

5. If $f(x) = x^2 - 1$, what is the value of $f(y) + f(-1)$?

(A) $y^2 - 1$

(B) y^2

(C) $y^2 + 1$

(D) $y^2 - 2y$

(E) $y^2 - 2y - 1$

6. If $f(x) = \frac{x}{2} - 1$, what is the value of $f(f(10))$?

7. If $h(x) = 5x^2 + x$, then which of the following is equal to $h(a + b)$?

(A) $5a^2 + 5b^2$

(B) $5a^3 + 5b^3$

(C) $5a^2 + 5b^2 + a + b$

(D) $5a^3 + 10ab + 5b^3$

(E) $5a^2 + 10ab + 5b^2 + a + b$

8. If $\ulcorner x = 2x^2 + 2$, which of the following is equal to $\ulcorner 4$?

(A) $\ulcorner\ulcorner -1$

(B) $\ulcorner -2$

(C) $\ulcorner\ulcorner 2$

(D) $\ulcorner 17$

(E) $\ulcorner 34$

9. $\boxed{x}$ is defined as the least integer greater than x for all odd values of x, and the greatest integer less than x for all even values of x. What is the value of $\boxed{-2} - \boxed{5}$?

(A) -12

(B) -9

(C) –8

(D) –7

(E) 3

10. $g(x) = x^2 - 4$ and $g(c) = 12$. If $c < 0$, what is the value of $g(c - 2)$?

11. If $h(x) = 2x - 1$ and $g(x) = x^2 - 3$, what is the value of $h(g(5))$?

$\boxed{}$

12. If $h(x) = |3x| + 2$ and $g(x) = x^2 - 7$ and $g(m) = 29$, what is the value of $h(m)$?

$\boxed{}$

13. If $*x$ is defined as the square of one-half of x, what is the value of $\frac{*5}{*3}$?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

14. If $\sim x = |14x|$, which of the following must be true?

Indicate all such answers.

- ☐ $\sim 2 = \sim(-2)$
- ☐ $\sim 3 + \sim 4 = \sim 7$
- ☐ The minimum possible value of $\sim x$ is zero.

15. If $\#x$ is defined for all $x > -2$ as the square root of the number that is 2 more than x, what is the value of $\#7 - \#(-1)$?

$\boxed{}$

16. If $g(x) = \dfrac{x^2(4x+9)}{(3x-3)(x+2)}$, for which of the following x values is $g(x)$ undefined?

Indicate all such values of x.

- ☐ $-\frac{9}{4}$
- ☐ -2

- ☐ 0
- ☐ 1
- ☐ 2
- ☐ $\frac{9}{4}$

$$f(x) = 2x - 3$$
$$f(m) = -11$$

	Quantity A	Quantity B
17.	m	One-half of $f(m)$

18. The price of a phone call consists of a standard connection fee, which is constant, plus a per-minute charge. A 10-minute call costs $2.90 and a 16-minute call costs $4.40. How much does a 13-minute call cost?

(A) $3.55

(B) $3.57

(C) $3.58

(D) $3.65

(E) $3.77

19. The first three terms in an arithmetic sequence are 30, 33, and 36. What is the 80th term?

$\boxed{}$

20. The sequence S is defined by $S_n = 2(S_{n-1}) - 4$ for each integer $n \geq 2$. If $S_1 = 6$, what is the value of S_5?

(A) –20

(B) 16

(C) 20

(D) 24

(E) 36

21. The sequence S is defined by $S_n = S_{n-1} + S_{n-2} - 3$ for each integer $n \geq 3$. If $S_1 = 5$ and $S_2 = 0$, what is the value of S_6?

(A) –6

(B) –5

(C) –3

(D) –1

(E) 1

22. The sequence S is defined by $S_n = S_{n-1} + S_{n-2} - 1$ for each integer $n \geq 3$. If $S_1 = 11$ and $S_3 = 10$, what is the value of S_5?

(A) 0

(B) 9

(C) 10

(D) 18

(E) 19

23. The sequence S is defined by $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$ for each integer $n \geq 4$. If $S_1 = 4$, $S_2 = 0$, and $S_4 = -4$, what is the value of S_6?

(A) –2

(B) –12

(C) –16

(D) –20

(E) –24

24. The sequence P is defined by $P_n = 10(P_{n-1}) - 2$ for each integer $n \geq 2$. If $P_1 = 2$, what is the value of P_4?

25. The sequence S is defined by $S_{n-1} = \frac{1}{4}(S_n)$ for each integer $n \geq 2$. If $S_1 = -4$, what is the value of S_4?

(A) –256

(B) –64

(C) $-\frac{1}{16}$

(D) $\frac{1}{16}$

(E) 256

26. The sequence A is defined by $A_n = A_{n-1} + 2$ for each integer $n \geq 2$, and $A_1 = 45$. What is the sum of the first 100 terms in sequence A?

(A) 243

(B) 14,400

(C) 14,500

(D) 24,300

(E) 24,545

27. In a certain sequence, the term a_n is defined by the formula $a_n = a_{n-1} + 10$ for each integer $n \geq 2$. What is the positive difference between a_{10} and a_{15}?

(A) 5

(B) 10

(C) 25

(D) 50

(E) 100

28. For a physical fitness test, scores are determined by the expression $2ps - 45m$, where p and s are the numbers of push-ups and sit-ups an athlete can do in one minute for each activity and m is the number of minutes the athlete takes to run a mile. During the test, Abraham did 21 push-ups and 30 sit-ups and ran the mile in 10 minutes. Javed got the same score, but did 4 more push-ups and ran the mile in 12 minutes. How many sit-ups did Javed do in one minute?

(A) 16

(B) 19

(C) 25

(D) 27

(E) 35

29. If $a\#b = a^2\sqrt{b} - a$, where $b \geq 0$, what is the value of $(-4)\#4$?

(A) –36

(B) –28

(C) 12

(D) 28

(E) 36

30. The expression $x\$y$ is defined as $\frac{x^2}{y}$, where $y \neq 0$. What is the value of $9\$(6\$2)$?

(A) $\frac{1}{2}$

(B) $\frac{9}{4}$

(C) $\frac{9}{2}$

(D) 18

(E) 108

31. Amy deposited \$1,000 into an account that earns 8% annual interest compounded every 6 months. Bob deposited \$1,000 into an account that earns 8% annual interest compounded quarterly. If neither Amy nor Bob makes any additional deposits or withdrawals, in 6 months how much more money will Bob have in his account than will Amy have in hers?

(A) \$40

(B) \$8

(C) \$4

(D) \$0.40

(E) \$0.04

32. The half-life of an isotope is the amount of time required for 50% of a sample of the isotope to undergo radioactive decay. The half-life of the carbon-14 isotope is 5,730 years. How many years must pass until a sample that starts out with 16,000 carbon-14 isotopes decays into a sample with only 500 carbon-14 isotopes?

(A) 180 years

(B) 1,146 years

(C) 5,730 years

(D) 28,650 years

(E) 183,360 years

33. $f(x) = \dfrac{2-x}{5}$ and $g(x) = 3x - 2$. If $f(g(x)) = 1$, what is the value of x?

(A) $-\dfrac{5}{3}$

(B) $-\dfrac{1}{3}$

(C) $\dfrac{2}{3}$

(D) 1

(E) $\dfrac{5}{3}$

34. A certain investment doubled in value every 9 years. If Saidah had $25,125 in the investment when she was 27 years old, what was the value of the investment when she retired at 63 years old?

(A) $50,250

(B) $150,750

(C) $201,000

(D) $251,250

(E) $402,000

35. An archer's score is calculated by the formula $\frac{50b-10a}{10+s}$, where *b* is the number of bull's-eyes hit, *a* is the total number of arrows shot, and *s* is the time in seconds it took the archer to shoot. By how many points would an archer who took 10 seconds to shoot 10 arrows and hit all bull's-eyes beat an archer who shot twice as many arrows and hit half as many bull's-eyes in 15 seconds?

(A) 2

(B) 7

(C) 10

(D) 18

(E) 20

36. Each term of a certain sequence is calculated by adding a particular constant to the previous term. The 2nd term of this sequence is 27 and the 5th term is 84. What is the 1st term of this sequence?

(A) 20

(B) 15

(C) 13

(D) 12

(E) 8

37. If $a\#b = \frac{1}{2a-3b}$ and $a@b = 3a - 2b$, what is the value of $1@2 - 3\#4$?

(A) $-\frac{7}{6}$

(B) -1

(C) $-\frac{5}{6}$

(D) $\frac{2}{3}$

(E) $\frac{7}{6}$

38. In a certain sequence, the term a_n is defined by the formula $a_n = a_{n-1} + 5$ for each integer $n \geq 2$. If $a_1 = 1$, what is the sum of the first 75 terms of this sequence?

(A) 10,150

(B) 11,375

(C) 12,500

(D) 13,950

(E) 15,375

39. In a certain sequence, the term a_n is defined by the formula $a_n = 2 \times a_{n-1}$ for each integer $n \geq 2$. If $a_1 = 1$, what is the positive difference between the sum of the first 10 terms of the sequence and the sum of the 11th and 12th terms of the same sequence?

(A) 1

(B) 1,024

(C) 1,025

(D) 2,048

(E) 2,049

The operation @ is defined by the equation $a@b = (a - 1)(b - 2)$.

$x@5 = 3@x$

	Quantity A	**Quantity B**
40.	x	1

---

41. The wait time in minutes, w, for a table at a certain restaurant can be estimated by the formula $w = d^2 + kn$, where d is the number of diners in the party, k is a constant, and n is the number of parties ahead in line at the beginning of the wait. If a party of 4 has an estimated wait time of 40 minutes when 6 other parties are ahead of it, how many minutes would the estimated wait time be for a party of 6 if there are 3 parties ahead of it?

(A) 28

(B) 33

(C) 39

(D) 42

(E) 48

A certain sequence is defined by the formula $a_n = a_{n-1} - 7$.

$a_7 = 7$

	Quantity A	**Quantity B**
42.	a_1	–35

---

43. Monthly rent for units in a certain apartment building is determined by the formula $k\left(\dfrac{5r^2+10t}{f+5}\right)$ where *k* is a constant, *r* and *t* are the number of bedrooms and bathrooms in the unit, respectively, and *f* is the floor number of the unit. A 2-bedroom, 2-bathroom unit on the first floor is going for \$800/month. How much is the monthly rent on a 3-bedroom unit with 1 bathroom on the 3rd floor?

(A) \$825

(B) \$875

(C) \$900

(D) \$925

(E) \$1,000

	Quantity A	**Quantity B**
44.	The sum of all the multiples of 3 between 250 and 350	9,990

Town A has a population of 160,000 and is growing at a rate of 20% annually. Town B has a population of 80,000 and is growing at a rate of 50% annually.

	Quantity A	**Quantity B**
45.	The number of years until town B's population is greater than that of town A	3

---

46. If $f(x) = x^2$, which of the following is equal to $f(m + n) + f(m - n)$?

(A) $m^2 + n^2$

(B) $m^2 - n^2$

(C) $2m^2 + 2n^2$

(D) $2m^2 - 2n^2$

(E) m^2n^2

47. S is a sequence such that $S_n = (-1)^n$ for each integer $n \geq 1$. What is the sum of the first 20 terms in S?

$\boxed{}$

48. If $f(x, y) = x^2y$ and $f(a, b) = 6$, what is $f(2a, 4b)$?

$\boxed{}$

$f(x) = m$ where m is the number of distinct prime factors of x.

	Quantity A	**Quantity B**
49.	$f(30)$	$f(64)$

50. The sequence $a_1, a_2, a_3, \ldots, a_n$ is defined by $a_n = 9 + a_{n-1}$ for each integer $n \geq 2$. If $a_1 = 11$, what is the value of a_{35}?

51. In sequence Q, the first number is 3, and each subsequent number in the sequence is determined by doubling the previous number and then adding 2. In the first 10 terms of the sequence, how many times does the digit 8 appear in the units digit?

52. For which of the following functions $f(x)$ is $f(a + b) = f(a) + f(b)$?

(A) $f(x) = x^2$

(B) $f(x) = 5x$

(C) $f(x) = 2x + 1$

(D) $f(x) = \sqrt{x}$

(E) $f(x) = x - 2$

Sam invests a principal of $10,000, which earns interest over a period of years.

	Quantity A	**Quantity B**
53.	The final value of the investment after 2 years at 8% interest, compounded annually	The final value of the investment after 4 years at 4% interest, compounded annually

54. The number of years it would take for the value of an investment to double, at 26% interest compounded annually, is approximately which of the following?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

55. An investment is made at 12.5% annual simple interest. The number of years it will take for the cumulative value of the interest to equal the original investment is equal to which of the following?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

56. If $f(2a) = 2f(a)$ and $f(6) = 11$, what is the value of $f(24)$?

(A) 22

(B) 24

(C) 44

(D) 66

(E) 88

57. If $\frac{1}{2} f(x) = f\left(\frac{1}{2}x\right)$, which of the following is true for all values of $f(x)$?

(A) $f(x) = 2x + 2$

(B) $f(x) = 13x$

(C) $f(x) = x^2$

(D) $f(x) = x - 10$

(E) $f(x) = \sqrt{x - 4}$

Functions, Formulas, and Sequences Answers

1. **(E).** Use the notation "$f(x)$" and "$f(2)$" as an indication to substitute 2 for x in the given equation:

$$f(2) = (2)^2 + 1$$
$$f(2) = 5$$

Likewise, plug –2 in for x:

$$f(-2) = (-2)^2 + 1$$
$$f(-2) = 5$$

Now add: 5 + 5 = 10.

2. **(B).** Be careful with the notation here. The problem indicates that $h(m) = -19$, *not* that $h(-19)$ = something else. Do not plug –19 in for x; rather, plug m in for x and set the answer equal to –19:

$$\begin{aligned} 2m^3 - 3 &= -19 \\ 2m^3 &= -16 \\ m^3 &= -8 \\ m &= -2 \end{aligned}$$

3. **1.** The main function is $f(x) = x + 5$. The notation $f(2g)$ indicates that you should plug $2g$ in for all instances of x: $f(2g) = 2g + 5$, which is also given as 19. If $2g + 5 = 19$, then $2g = 14$, and $g = 7$.

The question asks for the value of $f(3 - \text{g})$, which is $f(3 - 7) = f(-4) = -4 + 5 = 1$.

4. **720.** Plug m and n into the function in place of a and b. If $f(m, n) = 5$, then:

$$m^2n^4 = 5$$

This cannot be further simplified, so continue to the second part of the problem: plug $3m$ and $2n$ into the function for a and b:

$$f(3m, 2n) = (3m)^2(2n)^4 = 9m^2\ 16n^4 = 144m^2n^4$$

Since $m^2n^4 = 5$, $144m^2n^4 = 144(5) = 720$.

5. **(A).** The question requires plugging y into the function, then plugging –1

into the function, then summing the two results:

$$\begin{aligned} f(y) &= y^2 - 1 \\ f(-1) &= (-1)^2 - 1 \\ f(-1) &= 0 \end{aligned}$$

Thus, $f(y) + f(-1) = y^2 - 1 + 0 = y^2 - 1$.

6. **1.** When dealing with "nested" functions, solve the innermost function first:

$$f(10) = \frac{10}{2} - 1 = 4$$

$$f(4) = \frac{4}{2} - 1 = 1$$

Thus, $f(f(10)) = 1$.

7. **(E).** Replace each x with the expression $(a + b)$ to solve for $h(a + b)$.

$$h(a + b) = 5(a + b)^2 + (a + b)$$

$$h(a + b) = 5(a^2 + 2ab + b^2) + a + b$$

$$h(a + b) = 5a^2 + 10ab + 5b^2 + a + b$$

8. **(A).** The question uses a made-up symbol in place of the traditional notation $f(x)$. To answer the question, "If $\lceil x = 2x^2 + 2$, which of the following is equal to $\lceil 4$?" plug 4 into the given function.

$$\lceil 4 = 2(4)^2 + 2$$

$$\lceil 4 = 34$$

Do not fall for trap answer choice (E). The correct answer is 34, which does not appear in the choices in that form. Trap choice (E) is $\lceil 34$, which equals $2(34)^2 + 2$; this is much greater than 34.

Instead, solve each answer choice until one equals 34. Choice (A), $\lceil\lceil -1$, uses the function symbol twice, so plug –1 into the function, then plug the resulting answer back into the function again:

$$\lceil -1 = 2(-1)^2 + 2 = 4$$

$$\lceil 4 = 2(4)^2 + 2 = 34$$

(Note: you do not need to complete this math if you notice that $\lceil 4$ must have the same value as the original $\lceil 4$ in the question stem.)

Thus, $\lceil\lceil -1$ = 34, choice (A), is correct. It is not necessary to try the other answer choices.

9. **(B).** This problem uses a made-up symbol that is then defined verbally, rather than with a formula. $\boxed{x}$ has two different definitions:

> If *x* is odd, $\boxed{x}$ equals the least integer greater than *x* (e.g., if *x* = 3, then the “least integer greater than 3” is equal to 4).
>
> If *x* is even, $\boxed{x}$ equals the greatest integer less than *x* (e.g., if *x* = 6, the “greatest integer less than *x*” is equal to 5).

Since –2 is even, $\boxed{-2}$ = the greatest integer less than –2, or –3.

Since 5 is odd, $\boxed{5}$ = the least integer greater than 5, or 6.

Thus, $\boxed{-2} - \boxed{5} = -3 - 6 = -9$.

10. **32.** For the function $g(x) = x^2 - 4$, plugging c in for x gives the answer 12. Thus:

$$\begin{aligned} c^2 - 4 &= 12 \\ c^2 &= 16 \\ c &= 4 \text{ or } -4 \end{aligned}$$

The problem indicates that $c < 0$, so c must be -4.

The problem then asks for $g(c - 2)$. Since $c = -4$, $c - 2 = -6$. Plug -6 into the function:

$$\begin{aligned} g(-6) &= (-6)^2 - 4 \\ g(-6) &= 36 - 4 = 32 \end{aligned}$$

11. **43.** The problem introduces two functions and asks for $h(g(5))$. When dealing with "nested" functions, begin with the innermost function:

$$\begin{aligned} g(5) &= 5^2 - 3 = 22 \\ h(22) &= 2(22) - 1 = 43 \end{aligned}$$

Thus, $h(g(5)) = 43$.

12. **20.** The problem introduces two functions as well as the fact that $g(m) = 29$. First, solve for m:

$$\begin{aligned} g(m) &= m^2 - 7 = 29 \\ m^2 &= 36 \\ m &= 6 \text{ or } -6 \end{aligned}$$

The question asks for $h(m)$:

$$h(6) = |3 \times 6| + 2 = |18| + 2 = 18 + 2 = 20$$

$h(–6)$ = $|3 \times –6| + 2 = |–18| + 2 = 18 + 2 = 20$

The answer is 20 for either value of *m*.

13. $\frac{25}{9}$ **(or any equivalent fraction).** This function defines a made-up symbol rather than using traditional function notation such as $f(x)$. Since $*x$ is defined as “the square of one-half of x”:

$$*x = \left(\frac{1}{2}x\right)^2$$

The question asks for *5 divided by *3:

$$*5 = (2.5)^2 = 6.25$$

$$*3 = (1.5)^2 = 2.25$$

Therefore, $\frac{*5}{*3} = \frac{6.25}{2.25} = \frac{625}{225} = \frac{25}{9}$.

Alternatively, you can reduce before squaring:

$$\frac{*5}{*3} = \frac{(2.5)^2}{(1.5)^2} = \left(\frac{2.5}{1.5}\right)^2 = \left(\frac{25}{15}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

14. **1st, 2nd, and 3rd.** This function defines a made-up symbol: $\sim x$ is equivalent to $|14x|$. The question asks which statements must be true, so test each one:

$$\begin{aligned} \sim 2 &= \sim(-2) \\ |14(2)| &= |14(-2)| \\ |28| &= |-28| \\ 28 &= 28 \end{aligned}$$

This first statement must be TRUE.

Similarly, test the second statement:

$$\begin{aligned} \sim 3 + \sim 4 &= \sim 7 \\ |14(3)| + |14(4)| &= |14(7)| \\ 42 + 56 &= 98 \\ 98 &= 98 \end{aligned}$$

This second statement must be TRUE.

Finally, the third statement is also TRUE. Since $\sim x$ is equal to a statement inside an absolute value, this value can never be negative. If $x = 0$, then the value of $|14x|$ is also 0. The minimum possible value for $\sim x$ is 0.

15. **2.** This function defines a made-up symbol, rather than using traditional

notation such as *f*(*x*). First, translate the function:

$\#x = \sqrt{x+2}$

The square root of any value greater than or equal to 0 is the *non-negative* square root of the value. That is, the square root of 4 is just +2, not –2. Thus:

$\#7 = \sqrt{7+2} = \sqrt{9} = 3$

$\#(-1) = \sqrt{-1+2} = \sqrt{1} = 1$

Finally, #7 – #(–1) = 3 – 1 = 2.

16. –**2 and 1.** The term “undefined” refers to the circumstance when the solution is not a real number—for example, division by 0 is considered “undefined.” There aren’t many circumstances that result in an undefined answer. Essentially, the GRE considers two main cases: the square root of a negative integer and division by 0. There are no square roots in this problem, but it’s possible that 0 could end up on the denominator of the fraction. Set each of the terms in the denominator equal to 0:

$$\begin{aligned} 3x - 3 &= 0 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} x + 2 &= 0 \\ x &= -2 \end{aligned}$$

Thus, if $x = 1$ or $x = -2$, then the denominator would be 0, making $g(x)$ undefined. All other values are acceptable.

17. **(A).** The problem gives a function, $f(x) = 2x - 3$, and then indicates that, when m is plugged in to the function, the answer is –11. Therefore:

$$\begin{aligned} 2m - 3 &= -11 \\ 2m &= -8 \\ m &= -4 \end{aligned}$$

Quantity A is equal to –4. The problem indicates that $f(m) = -11$, so Quantity B is equal to $\frac{11}{2} = -5.5$. Quantity A is greater.

18. **(D).** Since “the price of a phone call consists of a standard connection fee, which is a constant, plus a per-minute charge,” write a formula, using variables for the unknown information. Let c equal the connection fee and r equal the per-minute rate:

$$\begin{aligned} 2.90 &= c + r(10) \\ 4.40 &= c + r(16) \end{aligned}$$

Now, either substitute and solve or stack and combine the equation. Note that there is one *c* in each equation, so subtracting is likely to be fastest:

$$
\begin{array}{rrcl}
 & 4.40 & = & c + 16r \\
- & (2.90 & = & c + 10\mathrm{r}) \\
\hline
 & 1.50 & = & 6r \\
 & r & = & 0.25
\end{array}
$$

The calls cost 25 cents per minute. Note that most people will next plug *r* back into either equation to find *c*, but *c* isn't necessary to solve!

A 10-minute call costs \$2.90. That \$2.90 already includes the basic connection fee (which is a constant) as well as the per-minute fee for 10 minutes. The problem asks how much a 13-minute call costs. Add the cost for another 3 minutes (\$0.75) to the cost for a 10-minute call (\$2.90): 2.90 + 0.75 = \$3.65.

In fact, both the 10-minute and 16-minute calls include the same connection fee (which is a constant), so a shortcut can be used to solve. The extra 6 minutes for the 16-minute call cost a total of \$4.40 – \$2.90 = \$1.50. From there, calculate the cost per minute (1.5 ÷ 6 = 0.25) or notice that 13 minutes is halfway between 10 minutes and 16 minutes, so the cost for a 13-minute call must also be halfway between the cost for a 10-minute call and the cost for a 16-minute call. Add half of \$1.50, or \$0.75, to \$2.90 to get \$3.65.

19. **267.** While the sequence is clear (30, 33, 36, 39, 42, etc.), don't spend time counting to the 80th term. Instead, find a pattern. Each new term in the list adds 3 to the previous term, so determine how many times 3 needs to be added. (By the way, the term "arithmetic sequence" means a sequence in which the same number is added or subtracted for each new term.)

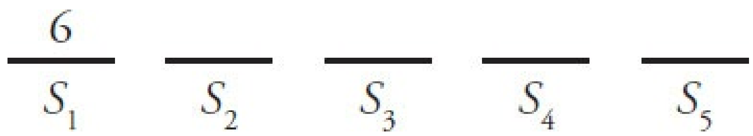
Start with the first term, 30. To get from the first term to the second term, start with 30 and add 3 *once*. To get from the first term to the third term, start with 30 and add 3 *twice*. In other words, for the third term, add one fewer instance of 3: twice rather than three times. To write this mathematically, say: 30 + 3(*n*-1), where *n* is the number of the term. (Note: it's not necessary to write this out, as long as you understand the pattern.)

To get to the 80th term, then, start with 30 and add 3 exactly 79 times:

$$30 + (79 \times 3) = 267$$

20. **(E).** The sequence $S_n = 2(S_{n-1}) - 4$ can be read as "to get any term in sequence *S*, double the previous term and subtract 4."

The problem gives S_1 (the first term) and asks for S_5 (the fifth term):



To get any term, double the previous term and subtract 4. To get S_2, double S_1 (which is 6) and subtract 4: $S_2 = 2(6) - 4 = 8$. Continue doubling each term and subtracting 4 to get the subsequent term:

6	8	12	20	36
S_1	S_2	S_3	S_4	S_5

21. **(A).** The sequence $S_n = S_{n-1} + S_{n-2} - 3$ can be read as “to get any term in sequence *S*, sum the two previous terms and subtract 3.”

The problem gives the first two terms and asks for the sixth term:

5	0				
S_1	S_2	S_3	S_4	S_5	S_6

To get any term, sum the two previous terms and subtract 3. So the third term will equal 5 + 0 – 3 = 2. The fourth term will equal 0 + 2 – 3 = –1. The fifth term will equal 2 + (–1) – 3 = –2. The sixth term will equal –1 + (–2) – 3 = –6:

5	0	2	–1	–2	–6
S_1	S_2	S_3	S_4	S_5	S_6

22. **(D).** The sequence $S_n = S_{n-1} + S_{n-2} - 1$ can be read as “to get any term in sequence *S*, sum the two previous terms and subtract 1.”

The problem gives the first term and the third term and asks for the fifth term:

11		10		
S_1	S_2	S_3	S_4	S_5

Within the sequence S_1 to S_3, the problem gives two values but not the middle one (S_2). What version of the formula would include those three terms?

$$\begin{aligned} S_3 &= S_2 + S_1 - 1 \\ 10 &= S_2 + (11) - 1 \\ 10 &= S_2 + 10 \\ 0 &= S_2 \end{aligned}$$

11	0	10		
S_1	S_2	S_3	S_4	S_5

To get each subsequent term, sum the two previous terms and subtract 1. Thus, S_4 = 10 + 0 – 1 = 9 and S_5 = 9 + 10 – 1 = 18:

11	0	10	9	18
S_1	S_2	S_3	S_4	S_5

23. **(E).** The sequence $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$ can be read as “to get any term in sequence *S*, sum the three previous terms and subtract 5.”

The problem gives the first, second, and *fourth* terms and asks for the sixth term:

4	0		−4		
S_1	S_2	S_3	S_4	S_5	S_6

Within the sequence S_1 to S_4, the problem gives three values but not the fourth (S_3). What version of the formula would include those four terms?

$$S_4 = S_3 + S_2 + S_1 - 5$$

$$-4 = S_3 + 4 + 0 - 5$$

$$-4 = S_3 - 1$$

$$-3 = S_3$$

Fill in the newly calculated value. To find each subsequent value, continue to add the three previous terms and subtract 5. Therefore, $S_5 = –4 + (–3) + 0 – 5 = –12$. $S_6 = –12 + (–4) + (–3) – 5 = –24$:

4	0	–3	–4	–12	–24
S_1	S_2	S_3	S_4	S_5	S_6

24. **1,778.** The sequence $P_n = 10(P_{n-1}) – 2$ can be read as "to get any term in sequence P, multiply the previous term by 10 and subtract 2."

The problem gives the first term and asks for the fourth:

2			
P_1	P_2	P_3	P_4

To get P_2 , multiply 2 × 10, then subtract 2 to get 18. Continue this procedure to find each subsequent term ("to get any term in sequence P, multiply the previous term by 10 and subtract 2"). Therefore, $P_3 = 10(18) – 2 = 178$. $P_4 = 10(178) – 2 = 1{,}778$.

2	18	178	1,778
P_1	P_2	P_3	P_4

25. **(A).** The sequence $S_{n-1} = \frac{1}{4}(S_n)$ can be read as "to get any term in sequence S, multiply the term *after* that term by $\frac{1}{4}$." Since this formula is "backwards" (usually, later terms are defined with regard to previous terms), solve the formula for S_n:

$$\begin{aligned} S_{n-1} &= \frac{1}{4}(S_n) \\ 4S_{n-1} &= S_n \\ S_n &= 4S_{n-1} \end{aligned}$$

This can be read as "to get any term in sequence S, multiply the previous term by 4."

The problem gives the first term and asks for the fourth:

–4			
S_1	S_2	S_3	S_4

To get S_2, multiply the previous term by 4: (4)(–4) = –16. Continue this procedure to find each subsequent term. Therefore, S_3 = (4)(–16) = –64. S_4 = (4)(–64) = –256:

–4	–16	–64	–256
S_1	S_2	S_3	S_4

26. **(B).** The first term of the sequence is 45, and each subsequent term is determined by adding 2. The problem asks for the sum of the first 100 terms, which cannot be calculated directly in the given time frame; instead, find the pattern. The first few terms of the sequence are 45, 47, 49, 51…

What's the pattern? To get to the 2nd term, start with 45 and add 2 once. To get to the 3rd term, start with 45 and add 2 twice. To get to the 100th term, then, start with 45 and add 2 ninety-nine times: 45 + (2)(99) = 243.

Next, find the sum of all odd integers from 45 to 243, inclusive. To sum up any evenly spaced set, multiply the average (arithmetic mean) by the number of elements in the set. To get the average, average the first and last terms.

Since $\frac{45+243}{2}$ = 144, the average is 144.

To find the total number of elements in the set, subtract 243 – 45 = 198, then divide by 2 (count only the odd numbers, not the even ones): $\frac{198}{2}$ = 99 terms. Now, add 1 (to count both endpoints in a consecutive set, first subtract and then "add 1 before you're done"). The list has 100 terms.

Multiply the average and the number of terms:

144 × 100 = 14,400

27. **(D).** This is an arithmetic sequence where the difference between successive terms is always +10. The difference between, for example, a_{10} and a_{11}, is exactly 10, regardless of the actual values of the two terms. The difference between a_{10} and a_{12} is 10 + 10 = 20, or 10 × 2 = 20, because there are two "steps," or terms, to get from a_{10} to a_{12}. Starting from a_{10}, there is a sequence of 5 terms to get to a_{15}. Therefore, the difference between a_{10} and a_{15} is 10 × 5 = 50.

28. **(D).** First, calculate Abraham's score: $2ps - 45m$, = 2(21)(30) – 45(10) = 1,260 – 450 = 810. Javed got a score of 810 also, but did 4 more push-ups than Abraham, or 21 + 4 = 25 push-ups. So, the formula for Javed's score is:

$$810 = 2ps - 45m$$

$$810 = 2(25)s - 45(12)$$

$$810 = 50s - 540$$

$$1{,}350 = 50s$$

$$s = 27$$

29. **(E).** This problem defines a function for the made-up symbol #. In this problem a = (–4) and b = 4. Plug the values into the function: $(-4)^2\sqrt{4} - (-4)$ = 16 × 2 + 4 = 36. Do not forget to keep the parentheses around the –4! Also note that only the positive root of 4 applies, because the problem has been presented in the form of a real number underneath the square root sign.

30. **(C).** This problem defines a function for the made-up symbol $. The order of operation rules (PEMDAS) stay the same even when the problem uses made-up symbols. First, calculate the value of the expression in parentheses, 6$2. Plug x = 6 and y = 2 into the function: $\frac{6^2}{2} = \frac{36}{2} = 18$. Replace 6$2 with 18 in the original expression to give 9$18. Again, plug x = 9 and y = 18 into the function: $9\$18 = \frac{9^2}{18} = \frac{81}{18} = \frac{9}{2}$.

31. **(D).** Both Amy and Bob start with \$1,000 and earn 8% interest annually; the difference is in how often this interest is compounded. Amy's interest is compounded twice a year at 4% each time (8% annual interest compounded 2 times a year means that she gets half the interest, or 4%, every 6 months). Bob's interest is compounded four times a year at 2% (8% divided by 4 times per year) each time. After 6 months, Amy has \$1,000 × 1.04 = \$1,040.00 (one interest payment at 4%) and Bob has \$1,000 × $(1.02)^2$ = \$1,040.40 (two interest payments at 2%). The difference is \$1,040.40 – \$1,040.00 = \$0.40.

Alternatively, Bob's interest could be calculated as two separate payments. After three months, Bob will have \$1,000 × 1.02 = \$1,020.00. After 6 months, Bob will have \$1,020 × 1.02 = \$1,040.40.

32. **(D).** After each half-life, the sample is left with half of the isotopes it started with in the previous period. After one half-life, the sample goes from 16,000 isotopes to 8,000. After two half-lives, it goes from 8,000 to 4,000. Continue this pattern to determine the total number of half-lives that have passed: 4,000 becomes 2,000 after 3 half-lives, 2,000 becomes 1,000 after 4 half-lives, 1,000 becomes 500 after 5 half-lives. The sample will have 500 isotopes after 5 half-lives. Thus, multiply 5 times the half-life, or 5 × 5,730 = 28,650 years.

Note that the answer choices are very spread apart. After determining that 5 half-lives have passed, estimate: 5 × 5,000 = 25,000 years; answer (D) is the only possible answer.

33. **(B).** Substitute the expression for $g(x)$ into the function for $f(x)$, and set the answer equal to 1. Since $g(x) = 3x - 2$, substitute the expression $3x - 2$ in for x in the expression for $f(x)$:

$$f(g(x)) = \frac{2-g(x)}{5} = \frac{2-(3x-2)}{5} = \frac{4-3x}{5}$$

Since $f(g(x)) = 1$, solve the equation $\frac{4-3x}{5} = 1$:

$$\begin{aligned} 4-3x &= 5 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

34. **(E).** The value of the investment doubled every 9 years. Calculate the amount of money at the end of each 9-year period:

Age	**Value**
27	$25,125
27 + 9 = 36	$50,250
36 + 9 = 45	$100,500
45 + 9 = 54	$201,000
54 + 9 = 63	$402,000

At age 63, the investor had $402,000.

35. **(D).** Calculate each of the archer's scores by plugging in the appropriate values for *b, a,* and *s*. For the first archer, *b* = *a* = *s* = 10 and the score is $\frac{(50\times 10)-(10\times 10)}{10+10}=\frac{400}{20}=20$. For the second archer, *b* = half of 10 = 5, *a* = twice as many as 10 = 20, and *s* = 15. The score for the second archer is $\frac{(50\times 5)-(10\times 20)}{10+15}=\frac{50}{25}=2$. The difference in scores is 20 – 2 = 18.

36. **(E).** Let *k* equal the constant added to a term to get the next term. If the 2nd term = 27, then the 3rd term = 27 + *k*, the 4th term = 27 + 2*k*, and the 5th term = 27 + 3*k*. The 5th term equals 84, so create an equation:

$$\begin{aligned} 27 + 3k &= 84 \\ 3k &= 57 \\ k &= 19 \end{aligned}$$

To find the 1st term, subtract *k* from the 2nd term. The 1st term = 27 – 19 = 8.

37. **(C).** This problem defines functions for the made-up symbols # and @. Substitute *a* = 1 and *b* = 2 into the function for *a*@*b*: 3(1) – 2(2) = –1. Substitute *a* = 3 and *b* = 4 into the function for *a*#*b*: $\frac{1}{2(3)-3(4)}=\frac{1}{-6}=-\frac{1}{6}$. Now, subtract: (–1) – $\left(-\frac{1}{6}\right)$ = –1 + $\frac{1}{6}$ = $-\frac{5}{6}$.

38. **(D).** This is an arithmetic sequence: each new number is created by adding 5 to the previous number in the sequence. Calculate the first few terms of the sequence: 1, 6, 11, 16, 21, and so on. Arithmetic sequences can be written in this form: $a_n = a_1 + k(n - 1)$, where *k* is the added constant and *n* is the number of the desired term. In this case, the function is: $a_n = 1 + 5(n - 1)$. The 75th term of this sequence is $a_{75} = 1 + 5(74) = 371$.

To find the sum of an arithmetic sequence, multiply the average value of the terms by the number of terms. The average of any evenly spaced set is equal to the midpoint between the first and last terms. The average of the 1st and

75th terms is $\frac{1+371}{2}$ = 186. There are 75 terms. Therefore, the sum of the first 75 terms = 186 × 75 = 13,950.

39. **(E).** This is a geometric sequence: each new number is created by multiplying the previous number by 2. Calculate the first few terms of the series to find the pattern: 1, 2, 4, 8, 16, and so on. Geometric sequences can be written in this form: $a_n = r^{n-1}$, where r is the multiplied constant and n is the number of the desired term. In this case, the function is $a_n = 2^{n-1}$.

The question asks for the difference between the sum of the first 10 terms and the sum of the 11th and 12th terms. While there is a clever pattern at play, it is hard to spot. If you don’t see the pattern, one way to solve is to use the calculator to add the first ten terms: 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1,023.

The 11th term plus the 12th term is equal to 1,024 + 2,048 = 3,072. Subtract 1,023 to get 2,049.

Alternatively, look for the pattern in the first few terms (1, 2, 4, 8, 16…): every term is equal to 1 more than the sum of the ones before it. For example, 1 + 2 = 3 and the next term is 4: 1 + 2 + 4 = 7 and the next term is 8. Thus, the sum of the first 10 terms of the sequence is 1 less than the 11th term. The 11th term = 2^{10} = 1,024, so the sum of the first 10 terms = 1,023. and the difference between the 10th and 11th terms equals 1. Add the value of the 12th term or 1 + 2,048 = 2,049.

40. **(B).** This problem defines a function for the made-up symbol @. Use the definition of the new symbol to rewrite the equation $x@5 = 3@x$ without the @ operator:

For $x@5$, $a = x$ and $b = 5$: $x@5 = (x - 1)(5 - 2) = 3x - 3$.
For $3@x$, $a = 3$ and $b = x$: $3@x = (3 - 1)(x - 2) = 2x - 4$.

Equating these two expressions gives you:

$$\begin{aligned} 3x - 3 &= 2x - 4 \\ x &= -1 \end{aligned}$$

Quantity B is greater.

41. **(E).** Start by solving for the constant, k. A party of 4 ($d = 4$) has an estimated wait time of 40 minutes ($w = 40$) when 6 other parties are ahead of it ($n = 6$). Plug these values into the formula:

$$\begin{aligned} w &= d^2 + kn \\ 40 &= 4^2 + k(6) \\ 40 &= 16 + 6k \\ 24 &= 6k \\ k &= 4 \end{aligned}$$

Then solve for the wait time for a party of 6 ($d = 6$) if there are 3 parties ahead of it ($n = 3$), using the constant $k = 4$ determined above:

$$\begin{aligned} w &= d^2 + kn \\ w &= 6^2 + 4(3) \\ w &= 36 + 12 \\ w &= 48 \text{ minutes} \end{aligned}$$

42. **(A).** The sequence $a_n = a_{n-1} - 7$ can be read as "to get any term in sequence a, subtract 7 from the previous term." The problem provides the 7th term; plug the term into the function in order to determine the pattern. Note that Quantity A asks for the value of a_1, so try to find the 6th term:

$$7 = a_6 - 7$$

$$a_6 = 14$$

In other words, each previous term will be 7 greater than the subsequent term. Therefore, $a_7 = 7$, $a_6 = 14$, $a_5 = 21$, and so on. The term a_1, then, is greater than the starting point, 7, and must also be greater than the negative value in Quantity B. Quantity A is greater. Note that the value in Quantity B is the result of incorrectly *subtracting* 7 six times, rather than adding it.

43. **(A).** First, solve for the constant k using the price information of the 2-bedroom, 2-bath unit ($m = 800$, $r = t = 2$, and $f = 1$):

$$
\begin{aligned}
800 &= k\left(\frac{5(2)^2 + 10(2)}{1+5}\right) \\
800 &= k\left(\frac{20+20}{6}\right) \\
800 &= k\left(\frac{20}{3}\right) \\
800\left(\frac{3}{20}\right) &= k \\
40(3) &= k \\
120 &= k
\end{aligned}
$$

Next, solve for the rent on the 3-bedroom, 1-bath unit on the 3rd floor ($r = 3$, $t = 1$, and $f = 3$):

$$
\begin{aligned}
m &= 120\left(\frac{5(3)^2 + 10(1)}{3+5}\right) \\
m &= 120\left(\frac{45+10}{8}\right) \\
m &= 120\left(\frac{55}{8}\right) \\
m &= 15(55)
\end{aligned}
$$

$m = 825$

44. **(B).** First, find the smallest multiple of 3 in this range: 250 is not a multiple of 3 ($2 + 5 + 0 = 7$, which is not a multiple of 3). The smallest multiple of 3 in this range is 252 ($2 + 5 + 2 = 9$, which is a multiple of 3). Next, find the largest multiple of 3 in this range. Since 350 is not a multiple of 3 ($3 + 5 + 0 = 8$), the largest multiple of 3 in this range is 348.

The sum of an evenly spaced set of numbers equals the average value multiplied by the number of terms. The average value is the midpoint between 252 and 348: $(252 + 348) \div 2 = 300$. To find the number of terms, first subtract $348 - 252 = 96$. This figure represents all numbers between 348 and 252, inclusive. To count only the multiples of 3, divide 96 by the 3: $96 \div 3 = 32$. Finally, "add 1 before you're done" to count both end points of the range: $32 + 1 = 33$.

The sum is $300 \times 33 = 9{,}900$. Since 9,900 is smaller than 9,990, Quantity B is greater.

45. **(A).** Set up a table and calculate the population of each town after every year; use the calculator to calculate town A's population. If you feel comfortable multiplying by 1.5 yourself, you do not need to use the calculator for town B. Instead, add 50% each time (e.g., from 80,000 add 50%, or 40,000, to get 120,000).

	Town A	**Town B**
Now	160,000	80,000
Year 1	160,000(1.2) = 192,000	80,000 + 40,000 = 120,000
Year 2	192,000(1.2) = 230,400	120,000 + 60,000 = 180,000
Year 3	230,400(1.2) = 276,480	180,000 + 90,000 = 270,000

Note that, after three years, town A still has more people than town B. It will take longer than 3 years, then, for town B to surpass town A, so Quantity A is greater.

46. **(C).** The problem provides the function $f(x) = x^2$ and asks for the quantity $f(m + n) + f(m - n)$. Plug into this function twice—first, to insert $m + n$ in place of x, and then to insert $m - n$ in place of x:

$$f(m + n) = (m + n)^2 = m^2 + 2mn + n^2$$
$$f(m - n) = (m - n)^2 = m^2 - 2mn + n^2$$

Now add the two:

$$(m^2 + 2mn + n^2) + (m^2 - 2mn + n^2) = 2m^2 + 2n^2$$

47. **0.** Adding 20 individual terms would take quite a long time. Look for a pattern. The first several terms in $S_n = (-1)^n$, where $n \geq 1$:

$$S_1 = (-1)^1 = -1$$
$$S_2 = (-1)^2 = 1$$
$$S_3 = (-1)^3 = -1$$
$$S_4 = (-1)^4 = 1$$

The terms alternate –1, 1, –1, 1, and so on. If the terms are added, every pair of –1 and 1 will add to zero; in other words, for an even number of terms, the sum will be zero. Since 20 is an even number, so the first 20 terms sum to zero.

48. **96.** The problem provides the function $f(x, y) = x^2y$ and also the fact that when a and b are plugged in for x and y, the answer is 6. In other words:

$$f(x, y) = x^2y$$

$$f(a, b) = a^2b = 6$$

The problem asks for the value of $f(2a,4b)$. First, plug $2a$ in for x and $4b$ in for y:

$$f(2a, 4b) = (2a)^2(4b)$$

$$f(2a, 4b) = 4a^2(4b)$$

$$f(2a, 4b) = 16a^2b$$

The problem already provides the value for the variables: $a^2b = 6$. Therefore, $16a^2b = 16(6) = 96$.

49. **(A).** The problem indicates that $f(x) = m$ where m is the number of distinct (or different) prime factors of x. For example, if $x = 6$, 6 has two distinct prime factors: 2 and 3. Therefore, the corresponding answer (m value) would be 2.

For Quantity A, $f(30)$: 30 has 3 distinct prime factors (2, 3, and 5), so $f(30) = 3$.

For Quantity B, $f(64)$: 64 is made of the prime factors 2, 2, 2, 2, 2, and 2). This is only one distinct prime factor, so $f(64) = 1$.

Quantity A is greater.

50. **317.** Each term in the sequence is 9 greater than the previous term. To make this clear, write a few terms of the sequence: 11, 20, 29, 38, etc.

a_{35} comes 34 terms after a_1 in the sequence. In other words, a_{35} is $34 \times 9 = 306$ greater than a_1.

Thus, $a_{35} = 11 + 306 = 317$.

51. **9.** After the first term in the sequence, every term has a units digit of 8:

$Q_1 = 3$
$Q_2 = 2(3) + 2 = 8$
$Q_3 = 2(8) + 2 = 18$
$Q_4 = 2(18) + 2 = 38$
$Q_5 = 2(38) + 2 = 78$
…

So 8 is the units digit nine out of the first ten times.

52. **(B).** The question asks which of the functions in the answer choices is such that performing the function on $a + b$ yields the same answer as performing the function to a and b individually and then adding those answers together.

The correct answer should be such that $f(a + b) = f(a) + f(b)$ is true for any values of a and b. Test some numbers, for example $a = 2$ and $b = 3$:

	$f(a + b) = f(5)$	$f(a) = f(2)$	$f(b) = f(3)$	Does $f(a + b) =$ $f(a) + f(b)$?
(A)	$f(5) = 5^2 = 25$	$f(2) = 2^2 = 4$	$f(3) = 3^2 = 9$	No
(B)	$f(5) = 5(5) = 25$	$f(2) = 5(2) = 10$	$f(3) = 5(3) = 15$	Yes
(C)	$f(5) = 2(5) + 1 = 11$	$f(2) = 2(2) + 1 = 5$	$f(3) = 2(3) + 1 = 7$	No
(D)	$f(5) = \sqrt{5}$	$f(2) = \sqrt{2}$	$f(3) = \sqrt{3}$	No
(E)	$f(5) = 5 - 2 = 3$	$f(2) = 2 - 2 = 0$	$f(3) = 3 - 2 = 1$	No

Alternatively, use logic—for what kinds of operations are performing the operation on two numbers and then summing results the same as summing the original numbers and then performing the operation? Multiplication or division would work, but squaring, square-rooting, adding, or subtracting would not. The correct function can contain *only* multiplication and/or division.

53. **(B).** Solve this problem by applying the compound interest formula:

$$V = P\left(1+\frac{r}{100}\right)^t$$

Since the principal *P* is the same in both cases, leave it out and just compare the rest:

Quantity A: $\left(1+\frac{r}{100}\right)^t = \left(1+\frac{8}{100}\right)^2 = (1.08)^2 = 1.08 \times 1.08 =$ 1.1664

Quantity B: $\left(1+\frac{r}{100}\right)^t = \left(1+\frac{4}{100}\right)^4 = (1.04)^4 =$ $1.04 \times 1.04 \times 1.04 \times 1.04 \approx 1.1699$

Quantity B is greater.

Alternatively, use logic. Notice that the *simple* interest in each case would be the same: 2 years of 8% *simple* interest (of an unchanging principal) is equal to 4 years of 4% *simple* interest of the same principal. Now go back to the compounded world. If the simple interest scenarios are the same, then it will always be true that the compounded scenario with *more frequent* compounding will result in greater principal in the end, because "interest on the interest" is earned more often:

2 periods of 8% compounded interest	<	4 periods of 4% compounded interest	<	8 periods of 2% compounded interest	<	16 periods of 1% compounded interest

The differences are small but real.

54. **(B).** Start with $1, and multiply by $\left(1+\frac{26}{100}\right) = 1.26$ for each year that

passes. In order for the amount to double, it would have to reach \$2:

End of Year 1: $\$1 \times 1.26 = \$1.26$
End of Year 2: $\$1.26 \times 1.26 = \$1.5876$
End of Year 3: $\$1.5876 \times 1.26 = \$2.000376 \approx \$2.00$

It takes 3 years for the investment to double in value. In terms of *simple* interest, it would take about 4 years (since 26% is just a tiny bit more than 25% = 1/4). The compounded case earns “interest on the interest,” though, so the investment grows more quickly.

55. **(E).** “Simple” interest means that the interest is calculated based on the initial amount every time; the interest earned is not included in future calculations. Each year, the investment pays 12.5%, or $\frac{1}{8}$, of the original investment as simple interest. As a result, it will take exactly 8 years for the cumulative interest to add up to the original investment.

Be careful not to apply the compound interest formula here. If the 12.5% interest is in fact compounded annually, it will take only about 6 years for the investment to double in value.

56. **(C).** This question concerns some function for which the full formula is not provided. The problem indicates that $f(2a) = 2f(a)$. In other words, this function is such that plugging in $2a$ is the same as plugging in a and then multiplying by 2. Plug $f(6) = 11$ into the equation $f(2a) = 2f(a)$:

$$\begin{aligned} f(2(6)) &= 2(11) \\ f(12) &= 22 \end{aligned}$$

Use the same process a second time. If $a = 12$ and $f(12) = 22$:

$$\begin{aligned} f(2(12)) &= 2(22) \\ f(24) &= 44 \end{aligned}$$

Alternatively, use logic. Plugging in $2a$, yields the same answer as plugging in a and then multiplying by 2. Plugging in 24 is the same as plugging in 6 a total of 4 times, and yields an answer 4 times as big as plugging in 6. Since plugging in 6 yields 11, plugging in 24 yields 44.

57. **(B).** The question is asking, “For which function is performing the function on x and THEN multiplying by $\frac{1}{2}$ the equivalent of performing the function on $\frac{1}{2}$ of x?”

The fastest method is to use logic: since the order of operations says that order does not matter with multiplication and division but *does* matter between multiplication and addition/subtraction, or multiplication and exponents, choose a function that has only multiplication and/or division.

Only answer choice (B) qualifies.

Alternatively, try each choice:

	$\frac{1}{2}f(x)$	$f\left(\frac{1}{2}x\right)$	equal?
(A)	$\frac{1}{2}(2x+2)=x+1$	$2\left(\frac{1}{2}x\right)+2=x+2$	No
(B)	$\frac{1}{2}(13x)=\frac{13x}{2}$	$13\left(\frac{1}{2}x\right)=\frac{13x}{2}$	Yes
(C)	$\frac{1}{2}x^2$	$\left(\frac{1}{2}x\right)^2=\frac{1}{4}x^2$	No
(D)	$\frac{1}{2}(x-10)=\frac{x}{2}-5$	$\frac{1}{2}x-10=\frac{x}{2}-10$	No
(E)	$\frac{1}{2}\sqrt{x-4}$	$\sqrt{\frac{1}{2}x-4}$	No

To confirm that the terms in choice (E) are equal, try plugging in a real number for *x*. If *x* = 8, then the left-hand value becomes 1 and the right-hand value becomes the square root of 0. The two values are *not* the same.

Chapter 11

Fractions and Decimals

In This Chapter…

Fractions and Decimals

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	**Quantity B**
1.	$-\frac{3}{4}+\frac{2}{3}$	$-\frac{3}{4}\times\frac{2}{3}$

2. At a convention of monsters, $\frac{2}{5}$ have no horns, $\frac{1}{7}$ have one horn, $\frac{1}{3}$ have two horns, and the remaining 26 have three or more horns. How many monsters are attending the convention?

   (A) 100

   (B) 130

   (C) 180

   (D) 210

(E) 260

3. Devora spends $\frac{1}{4}$ of her money on a textbook, and then buys a notebook that costs $\frac{1}{6}$ the price of the textbook. If there is no sales tax on the items and she makes no other purchases, what fraction of her original money does Devora have remaining?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

4. Which of the following are equal to 0.003482?

Indicate <u>all</u> such values.

☐ -0.003482×10^{-1}
☐ 0.3482×10^{-2}
☐ 34.82×10^{4}
☐ 34.82×10^{-4}
☐ $3{,}482 \times 10^{-6}$

5. Which of the following are equal to 12.12×10^{-3}?

Indicate <u>all</u> such values.

☐ -1.21×10^{3}
☐ 0.012
☐ 0.00001212×10^{3}
☐ 0.01212×10^{3}

6. 5 is how many fifths of 10?

(A) 2.5

(B) 5

(C) 10

(D) 20

(E) 50

	Quantity A	Quantity B
7.	$\frac{75}{4^2} \times \frac{3^2}{45} \times \frac{2^4}{45}$	$\frac{3^2}{4^2} \times \frac{2^2}{5^2} \times \frac{10}{3}$

8. In a certain class, $\frac{5}{12}$ of all the students are girls and $\frac{1}{4}$ of all the students are girls who take Spanish. What fraction of the girls take Spanish?

(A) $\frac{5}{48}$

(B) $\frac{5}{12}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

(E) $\frac{7}{12}$

9. $\frac{1}{5}$ of all the cars on a certain auto lot are red, and $\frac{2}{3}$ of all the red cars are convertibles. What fraction of all the cars are NOT red convertibles?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

10. Two identical pies were cut into a total of 16 equal pieces. If 1 of the resulting pieces was then split equally among 3 people, what fraction of a pie did each person receive?

(A) $\frac{1}{48}$

(B) $\frac{1}{24}$

(C) $\frac{1}{16}$

(D) $\frac{3}{16}$

(E) $\frac{3}{8}$

$xy \neq 0$

	Quantity A	**Quantity B**
11.	$2+\frac{1}{xy}$	$\frac{2xy+1}{xy}$

	Quantity A	Quantity B
12.	$\dfrac{\frac{1}{4}}{\frac{2}{3}-\frac{1-2}{\frac{1}{3}}}$	$\dfrac{\frac{1}{3}}{\frac{1}{4}-\frac{3-4}{\frac{2}{3}}}$

At store A, $\frac{3}{4}$ of the apples are red. At store B, which has twice as many apples, 0.375 of them are red.

	Quantity A	Quantity B
13.	The number of red apples at store A	The number of red apples at store B

14. A pot of soup was divided equally into 2 bowls. If Manuel ate $\frac{1}{4}$ of 1 of the bowls of soup and $\frac{2}{5}$ of the other bowl of soup, what fraction of the entire pot of soup did Manuel eat?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

15. Which of the following is equal to $\dfrac{\frac{ab}{c}}{\frac{cd}{a}}$ for all non-zero values of *a*, *b*, *c*, and *d*?

(A) *ac*

(B) *bd*

(C) $\dfrac{1}{bd}$

(D) $\dfrac{a^2b}{c^2d}$

(E) $\dfrac{ab^2}{cd^2}$

16. Which of the following is equal to $\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right)$?

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

17. Which of the following is equal to $\frac{-1}{2x}-\frac{1}{4y}+\frac{1}{xy}+\frac{1}{8}$?

(A) $\frac{(x-4)(2-y)}{8xy}$

(B) $\frac{(x-2)(y-4)}{8xy}$

(C) $\frac{(x-4)(y-2)}{8xy}$

(D) $\frac{(x+2)(4-y)}{8xy}$

(E) $$\frac{(x-2)(4-y)}{8xy}$$

---

x is a digit in the decimal 12.15*x*9, which, if rounded to the nearest hundredth, would equal 12.16.

	Quantity A	Quantity B
18.	x	4

---

19. What is the value of $\dfrac{(17^2)(22)(38)(41)(91)}{(19)(34)(123)(11)(119)(26)}$?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

20. In a decimal number, a bar over one or more consecutive digits means that the pattern of digits under the bar repeats without end. What fraction is equal to $7.58\overline{3}$?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

	Quantity A	**Quantity B**
21.	$\left(\dfrac{\sqrt{25}}{\sqrt{10}}\right)\left(\dfrac{\sqrt{8}}{\sqrt{15}}\right)$	$\left(\dfrac{\sqrt{51}}{\sqrt{46}}\right)\left(\dfrac{\sqrt{23}}{\sqrt{34}}\right)$

---

22. What is the value of $\sqrt{\dfrac{3}{2}}-\sqrt{\dfrac{2}{3}}$?

(A) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{6}}$

(B) $\dfrac{1}{\sqrt{6}}$

(C) $\frac{\sqrt{3}}{3}$

(D) $\frac{\sqrt{3}}{2}$

(E) $\frac{\sqrt{5}}{\sqrt{6}}$

23. $\frac{4}{5}$ of the women and $\frac{3}{4}$ of the men in a group speak Spanish. If there are 40% as many men as women in the group, what fraction of the group speaks Spanish?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

$abcd \neq 0$

	Quantity A	**Quantity B**
24.	$\frac{a^2b}{cd^2} \times \frac{d^3}{abc}$	$\frac{d^2}{bc} \times \frac{ab^2}{bd}$

$m \neq 0$

	Quantity A	**Quantity B**
25.	$\left(\frac{1}{2}+\frac{1}{m}\right)(m+2)$	$\frac{(m+2)^2}{2m}$

---

26. Which two of the following numbers, when added together, yield a sum between 1 and 2?

Indicate <u>two</u> such numbers.

- ☐ $\frac{7(2^3)}{3^3-7}$
- ☐ $\frac{2^4}{1+2+3+4}$
- ☐ $\frac{3}{11} \div \frac{6}{11}$
- ☐ $\frac{-2^3 3^2}{2^2 5^2}$
- ☐ $\frac{-11^2-11^3}{(30)(44)}$

27. Which three of the following numbers, when multiplied by each other, yield a product less than –1?

Indicate <u>three</u> such numbers.

- ☐ $\frac{-15}{17}$
- ☐ $\frac{-18}{19}$

☐ $\frac{23}{-22}$

☐ $\frac{17}{-16}$

28. What is the value of $(3 - \frac{1}{3})^2 + (3 + \frac{1}{3})^2$?

(A) $\frac{122}{9}$

(B) $\frac{164}{9}$

(C) 36

(D) $\frac{164}{3}$

(E) 162

29. If $\dfrac{3}{\dfrac{m+1}{m}+1}$ = 1, what is the value of *m*?

(A) –2

(B) –1

(C) 0

(D) 1

(E) 2

$$rs = \sqrt{3}$$

	Quantity A	**Quantity B**
30.	$\dfrac{2r\sqrt{12}}{r^2s\sqrt{72}}$	$\dfrac{14rs^2}{42s}$

	Quantity A	**Quantity B**
31.	$\dfrac{\sqrt{10}}{\sqrt{8}} \div \dfrac{\sqrt{9}}{\sqrt{10}}$	$\dfrac{\sqrt{11}}{\sqrt{9}} \div \dfrac{\sqrt{10}}{\sqrt{11}}$

---

32. Which of the following fractions has the greatest value?

(A) $\dfrac{7}{(16)(25)}$

(B) $\dfrac{5}{(32)(5^4)}$

(C) $\dfrac{30}{(512)(5^3)}$

(D) $\dfrac{5}{(4^6)(5)}$

(E) $\dfrac{4}{(2^{11})(5^2)}$

$$\frac{m}{p} > \frac{n}{p}$$

	Quantity A	**Quantity B**
33.	*m*	*n*

---

34. If $2x \neq y$ and $5x \neq 4y$, then what is the value of $\dfrac{\dfrac{5x-4y}{2x-y}}{\dfrac{3y}{y-2x}+5}$?

(A) $\dfrac{1}{2}$

(B) $\dfrac{3}{2}$

(C) $\dfrac{5}{2}$

(D) $\dfrac{7}{2}$

(E) $\dfrac{9}{2}$

35. What is the value of $\dfrac{39^2}{2^4} \div \dfrac{13^3}{4^2}$?

(A) $\dfrac{13}{2}$

(B) $\dfrac{9}{2}$

(C) $\dfrac{3}{2}$

(D) $\dfrac{3}{13}$

(E) $\frac{9}{13}$

Fractions and Decimals Answers

1. **(A).** In Quantity A, use a common denominator to add:

$$-\frac{3}{4}+\frac{2}{3}=-\frac{9}{12}+\frac{8}{12}=-\frac{1}{12}$$

In Quantity B, multiply straight across both top and bottom (common denominators are only needed for addition and subtraction). Cancel where possible:

$$-\frac{3}{4}\times\frac{2}{3}=-\frac{\cancel{3}}{4}\times\frac{2}{\cancel{3}}=-\frac{2}{4}=-\frac{1}{2}$$

Quantity A is greater. (Be careful with negatives! The closer to 0 a negative is, the greater it is.)

2. **(D).** This is a common GRE setup—the question presents several fractions and one actual number. First find what fraction of the whole that number represents, then solve for the total (call the total *m*). Notice that all the denominators are primes, so they don't share any factors. Therefore, the common denominator is their product: 5 × 7 × 3 = 105.

$$\frac{2}{5}+\frac{1}{7}+\frac{1}{3}=\frac{42}{105}+\frac{15}{105}+\frac{35}{105}=\frac{92}{105}$$

The remaining 26 monsters represent $\frac{13}{105}$ of the total monsters at the convention:

$$26 = \frac{13}{105}m$$

$$\frac{105}{13}\times 26 = m$$

$$105\times 2 = m$$

$210 = m$

3. $\mathbf{\frac{17}{24}}$ **(or any equivalent fraction).** The textbook costs $\frac{1}{4}$ of Devora's money. The notebook costs $\frac{1}{6}$ *of that amount*, or $\frac{1}{6}\left(\frac{1}{4}\right) = \frac{1}{24}$ of Devora's money. Thus, Devora has spent $\frac{1}{4} + \frac{1}{24} = \frac{6}{24} + \frac{1}{24} = \frac{7}{24}$ of her money. Subtract from 1 to get the fraction she has left: $1 - \frac{7}{24} = \frac{24}{24} - \frac{7}{24} = \frac{17}{24}$.

Alternatively, pick a value for Devora's money. (Look at the denominators in the problem—4 and 6—and pick a value that both numbers go into evenly.) For instance, say Devora has $120. She would spend $\frac{1}{4}$, or $30, on the textbook. She would spend $\frac{1}{6}$ *of that* amount, or $5, on the notebook. She would have spent $35 and would have $85 left, and thus $\frac{85}{120}$ of her money left. Reduce $\frac{85}{120}$ to get $\frac{17}{24}$, or enter $\frac{85}{120}$ in the boxes.

4. **0.3482×10^{-2}, 34.82×10^{-4}, and $3{,}482 \times 10^{-6}$ only.** Note that the first answer is negative, so it cannot be correct. For the second answer, move the decimal 2 places to the left: $0.3482 \times 10^{-2} = 0.003482$ (correct). For the third answer, move the decimal 4 places to the *right* (since the exponent is positive)—this move makes the number much greater and cannot be correct. For the fourth answer, move the decimal 4 places to the left: $34.82 \times 10^{-4} = 0.003482$ (correct). For the fifth answer, move the decimal 6 places to the left: $3{,}482 \times 10^{-6} = 0.003482$ (correct).

5. **0.00001212×10^3 only.** First, simplify $12.12 \times 10^{-3} = 0.01212$. Now, test which answers are equal to this value. The first answer is negative, so it cannot be correct. The second answer is 0.012 and is therefore incorrect (the end has been "chopped off," so the number is not the same value). The third answer is $0.00001212 \times 10^3 = 0.01212$ and is correct. The fourth answer is $0.01212 \times 10^3 = 12.12$ and is not correct.

6. **(A).** Translate the words into math. If x means "how many," then "how many fifths" is $\frac{x}{5}$:

$$5 = \frac{x}{5} \times 10$$

$$5 = \frac{10x}{5}$$

$$25 = 10x$$

$$\frac{25}{10} = x$$

$$x = 2.5$$

7. **(A).** Simplify each quantity by breaking down to primes and canceling factors:

Quantity A:

$$\frac{75}{4^2} \times \frac{3^2}{45} \times \frac{2^4}{45} = \frac{3 \times 5 \times 5}{\left(2^2\right)^2} \times \frac{3^2}{3 \times 3 \times 5} \times \frac{2^4}{3 \times 3 \times 5} = \frac{2^4 \times 3^3 \times 5^2}{2^4 \times 3^4 \times 5^2} = \frac{1}{3}$$

Quantity B:

$$\frac{3^2}{4^2} \times \frac{2^2}{5^2} \times \frac{10}{3} = \frac{3^2}{\left(2^2\right)^2} \times \frac{2^2}{5^2} \times \frac{2 \times 5}{3} = \frac{2^3 \times 3^2 \times 5}{2^4 \times 3 \times 5^2} = \frac{3}{2 \times 5} = \frac{3}{10}$$

Since $\frac{1}{3} > \frac{3}{10}$, Quantity A is greater. You can compare these fractions by making a common denominator, by cross-multiplying, or by comparing the decimal equivalents 0.333 (repeating infinitely) and 0.3.

If there are identical factors in each quantity in the same position (e.g., 3^2 on top or 4^2 on bottom), then you can save time by canceling those factors first from both quantities.

8. **(D).** This question is *not* asking for $\frac{1}{4}$ of $\frac{5}{12}$. Rather, $\frac{1}{4}$ and $\frac{5}{12}$ are fractions of the same number (the number of students in the whole class). A good way to avoid this confusion is to plug in a number for the class. Pick 12, as it is divisible by both 4 and 12 (the denominators of the given fractions).

Class = 12
Girls = 5

Girls who take Spanish = 3 (which is $\frac{1}{4}$ of all the students in the class)

The question asks for the number of girls who take Spanish over the number of girls. Thus, the answer is $\frac{3}{5}$.

9. $\mathbf{\frac{13}{15}}$ **(or any equivalent fraction).** If $\frac{1}{5}$ of all the cars are red, and $\frac{2}{3}$ of *those* are convertibles, then the fraction of all the cars that are red convertibles $= \left(\frac{1}{5}\right)\left(\frac{2}{3}\right) = \frac{2}{15}$. Since you want all of the cars that are NOT red convertibles, subtract $\frac{2}{15}$ from 1 to get $\frac{13}{15}$.

10. **(B).** If two pies are cut into 16 parts, each pie is cut into eighths. Thus, $\frac{1}{8}$ of a pie is was divided among three people. "One-third of one-eighth" = $\left(\frac{1}{3}\right)\left(\frac{1}{8}\right) = \frac{1}{24}$.

11. **(C).** Transform Quantity B by splitting the numerator:

$$\frac{2xy+1}{xy} = \frac{2xy}{xy} + \frac{1}{xy}$$

Next, cancel the common factor *xy* from top and bottom of the first fraction:

$\frac{2xy}{xy}+\frac{1}{xy}=2+\frac{1}{xy}$, which is the same as Quantity A.

Alternatively, you can transform Quantity A by turning 2 into a fraction with the same denominator (xy) as the second term.

$2+\frac{1}{xy}=\frac{2xy}{xy}+\frac{1}{xy}=\frac{2xy+1}{xy}$, which is the same as Quantity B. Thus, the quantities are equal.

12. **(B).** Simplify each quantity from the inside out.

Quantity A:

$$\frac{\frac{1}{4}}{\frac{2}{3}-\left(\frac{1-2}{\frac{1}{3}}\right)}=\frac{\frac{1}{4}}{\frac{2}{3}-\left(\frac{-1}{\frac{1}{3}}\right)}=\frac{\frac{1}{4}}{\frac{2}{3}-(-3)}=\frac{\frac{1}{4}}{\left(\frac{2}{3}+3\right)}=\frac{\frac{1}{4}}{\left(\frac{2}{3}+\frac{9}{3}\right)}=\frac{\frac{1}{4}}{\frac{11}{3}}=\frac{1}{4}\times\frac{3}{11}=\frac{3}{44}$$

Quantity B:

$$\frac{\frac{1}{3}}{\frac{1}{4}-\left(\frac{3-4}{\frac{2}{3}}\right)}=\frac{\frac{1}{3}}{\frac{1}{4}-\left(\frac{-1}{\frac{2}{3}}\right)}=\frac{\frac{1}{3}}{\frac{1}{4}-\left(\frac{-3}{2}\right)}=\frac{\frac{1}{3}}{\left(\frac{1}{4}+\frac{3}{2}\right)}=\frac{\frac{1}{3}}{\left(\frac{1}{4}+\frac{6}{4}\right)}=\frac{\frac{1}{3}}{\frac{7}{4}}=\frac{1}{3}\times\frac{4}{7}=\frac{4}{21}$$

Since Quantity B has a greater numerator *and* a smaller denominator, it is greater than Quantity A. This rule works for any positive fractions. You could also use the calculator to compute the decimal equivalents.

13. **(C).** Whether you choose fractions or decimals, put $\frac{3}{4}$ and 0.375 in the same form to more easily compare. Either way, 0.75 is double 0.375 (or $\frac{3}{4}$ is double $\frac{3}{8}$). Since store *B* has twice as many apples, 0.375 of store B's apples is the same number as 0.75 of store A's apples.

Alternatively, pick numbers such that store B has twice as many apples. If store A has 4 apples and store B has 8 apples, then store A would have $\left(\frac{3}{4}\right)$ (4) = 3 red apples and store B would have (0.375)(8) = 3 red apples. The two quantities are equal.

14. $\mathbf{\frac{13}{40}}$ **(or any equivalent fraction).** Manuel ate $\frac{1}{4}$ of one-half of the entire pot of soup and then $\frac{2}{5}$ of the other half of the entire pot of soup. As math:

$$\frac{1}{4}\left(\frac{1}{2}\right)+\frac{2}{5}\left(\frac{1}{2}\right)=\frac{1}{8}+\frac{1}{5}=\frac{13}{40}$$

Alternatively, pick numbers, ideally a large number with many factors. For example, say there are 120 ounces of soup. Each bowl would then have 60 ounces. Manuel ate $\frac{1}{4}$ of one bowl (15 ounces) and $\frac{2}{5}$ of the other bowl (24 ounces). In total, he ate 39 ounces out of 120. While $\frac{39}{120}$ would be counted as correct, reducing $\frac{39}{120}$ (by dividing both numerator and denominator by 3) yields $\frac{13}{40}$, the answer reached via the other method above.

15. **(D).** To divide by a fraction, multiply by its reciprocal:

$$\frac{\frac{ab}{c}}{\frac{cd}{a}}=\frac{ab}{c}\times\frac{a}{cd}=\frac{a^2b}{c^2d}$$

16. **(C).** Pull squares out of the square roots and cancel common factors:

$$\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right)=\frac{2\sqrt{3}}{5}\times\frac{2\sqrt{15}}{2^4}\times\frac{3\sqrt{5}}{3^2}=\frac{\sqrt{3}}{5}\times\frac{\sqrt{15}}{2^2}\times\frac{\sqrt{5}}{3}$$

Since $\sqrt{15}=\sqrt{3}\sqrt{5}$,

$$\frac{\sqrt{3}}{5}\times\frac{\sqrt{15}}{2^2}\times\frac{\sqrt{5}}{3}=\frac{\sqrt{3}}{5}\times\frac{\sqrt{3}\sqrt{5}}{2^2}\times\frac{\sqrt{5}}{3}=\frac{3\times5}{5\times2^2\times3}=\frac{1}{2^2}=\frac{1}{4}$$

Alternatively, in the calculator multiply 12 × 60 × 45 = 32,400, then take the square root to get 180 for the numerator. The denominator is 5 × 2 × 2 × 2 × 2 × 3 × 3 = 720. Finally, calculate $\frac{180}{720}$ = 0.25, which is $\frac{1}{4}$.

17. **(C).** Combine the four fractions by finding a common denominator (8*xy*, which is also suggested by the answer choices):

$$\frac{-1}{2x}-\frac{1}{4y}+\frac{1}{xy}+\frac{1}{8}=\frac{-1(4y)}{2x(4y)}-\frac{1(2x)}{4y(2x)}+\frac{1(8)}{xy(8)}+\frac{1(xy)}{8(xy)}$$
$$=\frac{-4y}{8xy}-\frac{2x}{8xy}+\frac{8}{8xy}+\frac{xy}{8xy}=\frac{xy-4y-2x+8}{8xy}$$

Now the key is to factor the top expression correctly:

$xy - 4y - 2x + 8 = (x - 4)(y - 2)$

It is a good idea to FOIL the expression on the right to make sure it matches the left-hand side. Or, FOIL the numerators of the choices to see which matches the distributed form of the numerator above.

Finally, $\frac{xy-4y-2x+8}{8xy}=\frac{(x-4)(y-2)}{8xy}$.

18. **(A).** Since the decimal rounds to 12.16, the thousandths digit *x* must be 5, 6, 7, 8, or 9. All of these possibilities are greater than 4.

19. $\mathbf{\frac{1}{3}}$ **(or any equivalent fraction).** One option is to punch the whole numerator and the whole denominator into the calculator and submit each

product. If you’re very careful, that will work. However, it might be wise to try canceling some common factors out of the fraction, to save time and to avoid errors. It’s fine to switch to the calculator whenever the cancelations aren’t obvious:

$$\frac{(17^2)\overset{2}{\cancel{(22)}}(38)(41)(91)}{(19)(34)(123)\cancel{(11)}(119)(26)} = \frac{(17^2)(2)(\overset{2}{\cancel{38}})(41)(91)}{\cancel{(19)}(34)(123)(119)(26)}$$

$$= \frac{(17^{\cancel{2}})(2)(2)(41)(91)}{(\underset{2}{\cancel{34}})(123)(119)(26)} = \frac{(17)(2)(2)\cancel{(41)}(91)}{(2)(\underset{3}{\cancel{123}})(119)(26)}$$

$$= \frac{(17)(2)(2)(\overset{7}{\cancel{91}})}{(2)(3)(119)(\underset{2}{\cancel{26}})} = \frac{\cancel{(17)}(2)(2)\cancel{(7)}}{(2)(3)(\cancel{17}\times\cancel{7})(2)}$$

$$= \frac{\cancel{(2)}\cancel{(2)}}{\cancel{(2)}(3)\cancel{(2)}} = \frac{1}{3}$$

20. $\mathbf{\frac{91}{12}}$ **(or any equivalent fraction).** First, turn the decimal into a sum of two pieces, to separate the repeating portion:

$7.58\overline{3} = 7.58 + 0.00\overline{3}$

Deal with each piece in turn. Like any other terminating decimal, 7.58 can be written as a fraction with a power of 10 in the denominator:

$$7.58 = \frac{758}{100}$$

The repeating portion is similar to $0.\overline{3} = 0.3333\ldots = \frac{1}{3}$

So $0.00\overline{3}$ is just $\frac{1}{3}$, moved by a couple of decimal places: $0.00\overline{3} = (0.\overline{3})(0.01)$ $= \left(\frac{1}{3}\right)\left(\frac{1}{100}\right) = \frac{1}{300}$.

Finally, write the original decimal as a sum of fractions, and then combine those fractions:

$$7.58\overline{3} = 7.58 + 0.00\overline{3} = \frac{758}{100} + \frac{1}{300} = \frac{758 \times 3}{300} + \frac{1}{300} = \frac{2,275}{300}$$

Enter $\frac{2,275}{300}$ unreduced, or you can reduce it to $\frac{91}{12}$.

21. **(A).** Both quantities are positive square roots, so just compare the underlying numbers.

Quantity A: $\frac{(25)(8)}{(10)(15)} = 1.3\overline{3}$

Quantity B: $\frac{(51)(23)}{(46)(34)} = 0.75$

The square root of $1.3\overline{3}$ (or $\frac{4}{3}$) is greater than the square root of 0.75 (or $\frac{3}{4}$).

22. **(B).** The square root of a fraction is the square root of the top over the square root of the bottom:

$$\sqrt{\frac{3}{2}} - \sqrt{\frac{2}{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3}}$$

Then make a common denominator: $\sqrt{3}\sqrt{2} = \sqrt{6}$.

$$\frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{2}} - \frac{\sqrt{2}\sqrt{2}}{\sqrt{3}\sqrt{2}} = \frac{3}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

23. $\frac{11}{14}$ **(or any equivalent fraction).** If a question refers to fractions of different numbers that are *also* related by a fraction or percent, try plugging in numbers. Since there are 40% as many men as women, some convenient numbers are:

Men = 40

Women = 100

Women who speak Spanish = $\frac{4}{5}(100) = 80$

Men who speak Spanish = $\frac{3}{4}(40) = 30$

The group has 140 total people and 110 Spanish speakers. The answer is $\frac{110}{140} = \frac{11}{14}$ (you are not required to reduce, as long as your answer is correct and fits in the box).

24. **(D).** Cancel factors on top and bottom of each product:

Quantity A: $\frac{a^2b}{cd^2} \times \frac{d^3}{abc} = \frac{a^2bd^3}{abc^2d^2} = \frac{ad}{c^2}$

Quantity B: $\frac{d^2}{bc} \times \frac{ab^2}{bd} = \frac{ab^2d^2}{b^2cd} = \frac{ad}{c}$

The two quantities differ in the denominators: Quantity A has c^2, while Quantity B has c. It cannot be determined which quantity is greater, because for some values (e.g., $c = 2$) c^2 is greater than c, and for others (e.g., $c = 0.5$) c^2 is less than c.

25. **(C).** Multiply out Quantity A by FOILing.

$$\left(\frac{1}{2}+\frac{1}{m}\right)(m+2)=\frac{1}{2}(m)+\frac{1}{2}(2)+\frac{1}{m}(m)+\frac{1}{m}(2)$$

$$=\frac{m}{2}+1+1+\frac{2}{m}=\frac{m}{2}+2+\frac{2}{m}$$

Make a common denominator (2*m*) to sum these terms (also, note that this makes Quantity A similar in form to Quantity B):

$$\frac{m}{2}+2+\frac{2}{m}=\frac{m(m)}{2(m)}+2\left(\frac{2m}{2m}\right)+\frac{(2)2}{(2)m}=\frac{m^2+4m+4}{2m}$$

Since the quantities now have the same denominators and $(m+2)^2 = m^2 + 4m + 4$, the two quantities are equal.

26. $\mathbf{\frac{7(2^3)}{3^3-7}}$ **and** $\mathbf{\frac{-11^2-11^3}{(30)(44)}}$ **only.** To start, compute each value:

$$\frac{7(2^3)}{3^3-7}=\frac{7\times 8}{27-7}=\frac{56}{20}=2.8$$

$$\frac{2^4}{1+2+3+4}=\frac{16}{10}=1.6$$

$$\frac{3}{11}\div\frac{6}{11}=\frac{3}{11}\times\frac{11}{6}=\frac{3}{6}=\frac{1}{2}=0.5$$

$$\frac{-2^3 3^2}{2^2 5^2}=\frac{-8\times 9}{10^2}=\frac{-72}{100}=-0.72$$

$$\frac{-11^2-11^3}{(30)(44)}=\frac{-11^2(1+11)}{(30)(44)}=\frac{-121(12)}{(30)(44)}=\frac{-1{,}452}{1{,}320}=-1.1$$

The question asks for exactly two values that sum to a number between 1 and 2.

No two of the positive numbers sum to a number between 1 and 2. So the answers must be a positive and a negative. The only two possibilities that work are 2.8 and –1.1.

27. $\mathbf{\frac{-18}{19}}$**,** $\mathbf{\frac{23}{-22}}$**, and** $\mathbf{\frac{17}{-16}}$ **only.** The product of three of the numbers must be less than –1. You can brute-force the calculation by trying all possible products, but use the relative size of the numbers to reduce the effort.

Notice that the four answer choices are all very close to –1, but some are greater than –1, and others are less than –1. To get the exact order, you can use the calculator, or you can think about the difference between each fraction and –1:

$$\frac{-15}{17}=\frac{-17}{17}+\frac{2}{17}=-1+\frac{2}{17}$$

$\frac{-18}{19}=\frac{-19}{19}+\frac{1}{19}=-1+\frac{1}{19}$, which is less than the previous number (since $\frac{2}{17}>\frac{1}{19}$)

$$\frac{23}{-22}=\frac{-23}{22}=\frac{-22}{22}-\frac{1}{22}=-1-\frac{1}{22}$$

$\frac{17}{-16}=\frac{-17}{16}=\frac{-16}{16}-\frac{1}{16}=-1-\frac{1}{16}$, a greater decrease from –1 than the previous number

So the order of the original numbers relative to each other and to –1 is this: $\frac{17}{-16} < \frac{23}{-22} < -1 < \frac{-18}{19} < \frac{-15}{17}$.

Try multiplying the three lowest numbers first, since they will produce the lowest product. Only *one* product of the three numbers can be less than –1 (or there would be more than one right answer), so the three numbers must be as follows, as you can check on the calculator:

$$\frac{17}{-16} \times \frac{23}{-22} \times \frac{-18}{19} \approx -1.052 \ldots < -1$$

28. **(B).** First, simplify inside the parentheses. Then, square and add:

$$\left(\frac{8}{3}\right)^2 + \left(\frac{10}{3}\right)^2$$

$$\frac{64}{9} + \frac{100}{9}$$

The answer is $\frac{164}{9}$.

29. **(D).** If the left-hand side of the equation is equal to 1, then the numerator and denominator must be equal. Thus, the denominator must also be equal to 3:

$$\frac{m+1}{m} + 1 = 3$$

$$\frac{m+1}{m} = 2$$

$m + 1 = 2m$
$1 = m$

Alternatively, plug in each answer choice (into both instances of *m* in the original equation), and stop as soon as one of them works.

30. **(B).** Cancel common factors in each quantity and substitute in for *rs*:

Quantity A:

$$\frac{2r\sqrt{12}}{r^2s\sqrt{72}} = \frac{2\sqrt{12}}{rs\sqrt{72}} = \frac{2\sqrt{12}}{\sqrt{3}\sqrt{72}} = \frac{2\sqrt{4}}{\sqrt{72}} = \frac{2\times 2}{\sqrt{36}\sqrt{2}} = \frac{4}{6\sqrt{2}} = \frac{2}{3\sqrt{2}}$$

Quantity B: $\frac{14rs^2}{42s} = \frac{14rs}{42} = \frac{14\sqrt{3}}{3\times 14} = \frac{\sqrt{3}}{3}$

At this point, use the calculator, or compare the two quantities with an "invisible inequality":

$$\frac{2}{3\sqrt{2}} \quad ?? \quad \frac{\sqrt{3}}{3}$$

Since everything is positive, it is safe to cross-multiply:

2×3 ?? $3\sqrt{2}\sqrt{3}$

Now square both sides. Since everything is positive, the invisible inequality is unaffected:

$(2 \times 3)^2$?? $3^2 \times 2 \times 3$

36 ?? 54

Since 36 < 54, Quantity B is greater.

31. **(A).** To divide fractions, multiply by the reciprocal:

Quantity A:

$$\frac{\sqrt{10}}{\sqrt{8}} \div \frac{\sqrt{9}}{\sqrt{10}} = \frac{\sqrt{10}}{\sqrt{8}} \times \frac{\sqrt{10}}{\sqrt{9}} = \frac{10}{\sqrt{72}} = \frac{10}{6\sqrt{2}} = \frac{5}{3\sqrt{2}}$$

Quantity B: $\frac{\sqrt{11}}{\sqrt{9}} \div \frac{\sqrt{10}}{\sqrt{11}} = \frac{\sqrt{11}}{\sqrt{9}} \times \frac{\sqrt{11}}{\sqrt{10}} = \frac{11}{3\sqrt{10}}$

Square both quantities to get rid of the square roots:

Quantity A: $\left(\frac{5}{3\sqrt{2}}\right)^2 = \frac{5^2}{3^2 2} = \frac{25}{18}$

Quantity B: $\left(\frac{11}{3\sqrt{10}}\right)^2 = \frac{11^2}{3^2 10} = \frac{121}{90}$

At this point, use the calculator. Quantity A is approximately 1.389, whereas Quantity B is approximately 1.344.

32. **(A).** To determine which fraction is greatest, cancel common terms from all five fractions until the remaining values are small enough for the calculator. Note that every choice has at least one 5 on the bottom, so cancel 5^1 from all of the denominators.

Note also that every fraction has a power of 2 on the bottom, so convert 16, 32, 512, 4^6, and 2^{11} to powers of 2. Since $16 = 2^4$, $32 = 2^5$, $512 = 2^9$, and $4^6 = (2^2)^6 = 2^{12}$, the modified choices are:

(A) $\frac{7}{(2^4)(5)}$

(B) $\dfrac{5}{(2^5)(5^3)}$

(C) $\dfrac{30}{(2^9)(5^2)}$

(D) $\dfrac{5}{(2^{12})}$

(E) $\dfrac{4}{(2^{11})(5)}$

Since every choice has at least 2^4 on the bottom, cancel 2^4 from all 5 choices:

(A) $\frac{7}{5}$

(B) $\frac{5}{(2)(5^3)}$

(C) $\frac{30}{(2^5)(5^2)}$

(D) $\frac{5}{2^8}$

(E) $\frac{4}{(2^7)(5)}$

Note that the numerators also have some powers of 2 and 5 that will cancel out with the bottoms of each of the fractions. In choice (C), 30 = (2)(3)(5):

(A) $\frac{7}{5}$

(B) $\frac{1}{(2)(5^2)}$

(C) $\frac{3}{(2^4)(5)}$

(D) $\frac{5}{2^8}$

(E) $\frac{1}{(2^5)(5)}$

These values are now small enough for the calculator. Note that the GRE calculator does not have an exponent button—to get 2^8, you must multiply 2 by itself 8 times. This is why you should memorize powers of 2 up to 2^{10}, and powers of 3, 4, and 5 up to about the 4th power.

(A) 1.4

(B) 0.02

(C) 0.0375

(D) 0.01953125

(E) 0.00625

Alternatively, you might notice in the previous step that only the choice (A) simplified fraction is greater than 1; in all the others, the denominator is greater than the numerator.

33. **(D).** Without knowing the signs of the variables, do not assume that m is greater than n. While it certainly *could* be (e.g., $m = 4$, $n = 2$, and $p = 1$), if p is negative, the reverse will be true (e.g., $m = 2$, $n = 4$, and $p = -1$).

34. **(A).** This expression is complicated, but the answer choices are just numbers, so the variables must cancel. This, and the relative lack of constraints on the variables, suggests that you can plug in values for *x* and *y* and then solve.

Try $x = 2$ and $y = 3$. For these numbers, $2x \neq y$ and $5x \neq 4y$ as required. Any other numbers that also follow those constraints would yield the same result below:

$$\frac{\frac{5x-4y}{2x-y}}{\frac{3y}{y-2x}+5}=\frac{\frac{5(2)-4(3)}{2(2)-(3)}}{\frac{3(3)}{(3)-2(2)}+5}=\frac{\frac{10-12}{4-3}}{\frac{9}{3-4}+5}=\frac{\frac{-2}{1}}{\frac{9}{-1}+5}=\frac{-2}{-9+5}=\frac{-2}{-4}=\frac{1}{2}$$

35. **(E).** To divide fractions, multiply by the reciprocal of the divisor:

$$\frac{39^2}{2^4}\div\frac{13^3}{4^2}=\frac{39^2}{2^4}\times\frac{4^2}{13^3}$$

Now break down to primes and cancel common factors:

$$\frac{39^2}{2^4}\times\frac{4^2}{13^3}=\frac{(3\times13)^2\times\left(2^2\right)^2}{2^4\times13^3}=\frac{3^2\times13^2\times2^4}{2^4\times13^3}=\frac{3^2}{13}=\frac{9}{13}$$

Chapter 12

Percents

In This Chapter…

Percents

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	**Quantity B**
1.	50 as a percent of 30	The percent increase from 30 to 80

2. If Ken’s salary were 20% higher, it would be 20% less than Lorena’s. If Lorena’s salary is $60,000, what is Ken’s salary?

   (A) $36,000

   (B) $40,000

   (C) $42,500

   (D) $42,850

   (E) $45,000

Greta’s salary was *x* thousand dollars per year, then she received a *y*% raise. Annika’s salary was *y* thousand dollars per year, then she received an

x% raise. x and y are positive integers.

	Quantity A	Quantity B
3.	The dollar amount of Greta’s raise	The dollar amount of Annika’s raise

Roselba's annual income exceeds twice Jane's annual income and both pay the same positive percent of their respective incomes in transportation fees.

	Quantity A	**Quantity B**
4.	The annual amount Jane pays in transportation fees	Half the annual amount Roselba pays in transportation fees

An item's price was discounted by 16%. Later, the discounted price was increased by 16%.

	Quantity A	**Quantity B**
5.	The original price	The price after the discount and increase

6. 12 is 5% of what number?

7. 7% of 9 is what percent of 7?

%

8. What percent of 13 is 20% of 195?

%

9. 25% of 30 is 75% of what number?

10. What is the percent increase from 50 to 60?

% increase

11. If *x* were reduced by 30%, the resulting number would be 63. What is the

value of x?

12. What is 230% of 15% of 400?

13. 45% of 80 is x% more than 24. What is the value of x?

$\boxed{}$

14. 10% of 30% of what number is 200% of 6?

$\boxed{}$

15. If $y \neq 0$, what percent of y% of 50 is 40% of y?

$\boxed{}$ %

16. If $a \neq 0$, 200% of 4% of a is what percent of $\frac{a}{2}$?

$\boxed{}$ %

17. If positive integer m were increased by 20%, decreased by 25%, and then increased by 60%, the resulting number would be what percent of m?

$\boxed{}$ %

	Quantity A	Quantity B
18.	The price of an item after five consecutive 10% discounts are applied	50% of the price of the item

---

19. Raymond borrowed $450 at 0% interest. If he pays back 0.5% of the total amount every 7 days, beginning exactly 7 days after the loan was disbursed, and has thus far paid back $18, with the most recent payment made today, how many days ago did he borrow the money?

(A) 6

(B) 8

(C) 25

(D) 42

(E) 56

At a warehouse, an order was shipped out, reducing the number of parts in inventory by half. Then a shipment of parts was received, increasing the current number of parts in inventory by 50%.

	Quantity A	**Quantity B**
20.	The number of parts in inventory before the two shipments	The number of parts in inventory after the two shipments

---

A house valued at $200,000 two years ago lost 40% of its value in the first year and a further 20% of that reduced value during the second year.

	Quantity A	Quantity B
21.	The current value of the house	$100,000

---

22. 1% of 200% of 360 is what percent of 0.1% of 60?

$\boxed{}$ %

23. If Mary has half as many cents as Nora has dollars, then Nora has what percent more cents than Mary does? (100 cents = 1 dollar)

(A) 100%

(B) 200%

(C) 1,990%

(D) 19,900%

(E) 20,000%

24. The number that is 50% greater than 60 is what percent less than the number that is 20% less than 150?

(A) 5%

(B) 10%

(C) 15%

(D) 20%

(E) 25%

25. A cockroach population doubles every 3 days. In 30 days, by what percent would a cockroach population increase?

(A) 900%

(B) 1,000%

(C) 9,999%

(D) 102,300%

(E) 102,400%

26. After a 15% discount, the price of a computer was $612. What was the price of the computer before the discount?

(A) $108.00

(B) $520.20

(C) $703.80

(D) $720.00

(E) $744.00

At the end of April, the price of fuel was 40% greater than the price at the beginning of the month. At the end of May, the price of fuel was 30% greater than the price at the end of April.

	Quantity A	**Quantity B**
27.	The price increase in April	The price increase in May

---

28. Aloysius spends 50% of his income on rent, utilities, and insurance, and 20% on food. If he spends 30% of the remainder on video games and has no other expenditures, what percent of his income is left after all of the expenditures?

(A) 30%

(B) 21%

(C) 20%

(D) 9%

(E) 0%

29. In 1970, company X had 2,000 employees, 15% of whom were women, and 10% of these women were executives. In 2012, the company had 12,000 employees, 45% of whom were women. If 40% of those women were executives, what was the percent increase in the number of women executives from 1970 to 2012?

$\boxed{}$ %

30. 75% of all the boys and 48% of all the girls at Smith High School take civics. If there are 20% fewer boys than there are girls in the school, what percent of all the students take civics?

%

Airline A and airline B both previously charged \$400 for a certain flight. Airline A then reduced its price by 25%. Airline B responded by reducing its price by 55% but adding \$150 in fees. Then, airline A increased its reduced price by 10%.

	Quantity A	**Quantity B**
31.	The final price of the flight on airline A	The final price of the flight on airline B

p is 75% of q and p equals $2r$.

	Quantity A	**Quantity B**
32.	$0.375q$	r

$0 < x < 100$

	Quantity A	**Quantity B**
33.	x% of 0.5% of 40,000	0.05% of 2,000% of $40x$

Profit Per Student (in Dollars) at Dan’s Dojo, 2000–2004

2000	60
2001	80
2002	80
2003	100
2004	162

34. At Dan’s Dojo, the percent increase from 2004 to 2005 (not shown) was the same as the percent increase from 2000 to 2001. What was the profit per student for 2005?

\$ [ ]

35. If x is 0.5% of y, then y is what percent of x?

(A) 199%

(B) 200%

(C) 2,000%

(D) 19,900%

(E) 20,000%

Bill pays 20% tax on his gross salary of $5,000 each month and spends 25% of the remaining amount on rent.

	Quantity A	**Quantity B**
36.	The monthly tax paid on Bill's salary	The rent paid monthly by Bill

37. Four people shared a dinner with an $80 bill and tipped the waiter 15% of this amount. If each person contributed equally to paying the bill and tip, how much did each person pay?

(A) $20.00

(B) $23.00

(C) $23.75

(D) $24.00

(E) $25.00

The price of a certain stock rose by 25% and then decreased by y%. After the decrease, the stock was back to its original price.

	Quantity A	**Quantity B**
38.	y	25

39. A chemist is mixing a solution of acetone and water. She currently has 30 ounces mixed, 10 of which are acetone. How many ounces of acetone should she add to her current mixture to attain a 50/50 mixture of acetone and water if no additional water is added?

(A) 2.5

(B) 5

(C) 10

(D) 15

(E) 20

By the end of July, a certain baseball team had played 80% of the total games to be played that season and had won 50% of those games. Of the remaining games for the season, the team won 60%.

	Quantity A	Quantity B
40.	Percent of total games won for the season	52%

	Quantity A	Quantity B
41.	0.4% of 4% of 1.25	0.002

42. Jane has a 40-ounce mixture of apple juice and seltzer that is 30% apple juice. If she pours 10 more ounces of apple juice into the mixture, what percent of the mixture will be seltzer?

(A) 33%

(B) 44%

(C) 50%

(D) 56%

(E) 67%

Half of the shirts in a closet are white and 30% of the remaining shirts are gray.

	Quantity A	Quantity B
43.	The percent of the shirts in the closet that are not white or gray.	20%

The length and width of a painted rectangle were each increased by 10%.

	Quantity A	Quantity B
44.	The percent increase in the area of the painted rectangle	10%

---

45. If 35% of x equals 140, what is 20% of x?

(A) 9.8

(B) 39.2

(C) 80

(D) 320

(E) 400

46. A population of a colony of bacteria increases by 20% every 3 minutes. If at 9:00am the colony had a population of 144,000, what was the population of the colony at 8:54am?

(A) 100,000

(B) 112,000

(C) 120,000

(D) 121,000

(E) 136,000

The price of an item is greater than \$90 and less than \$150.

	Quantity A	**Quantity B**
47.	The price of the item after a 10%-off discount and then a \$20-off discount	The price of the item after a \$10-off discount and then a 20%-off discount

48. The number that is 20% less than 300 is what percent greater than 180?

(A) 25

(B) $33\frac{1}{3}$

(C) 50

(D) $66\frac{2}{3}$

(E) 75

49. A tank that was 40% full of oil was emptied into a 20-gallon bucket. If the oil fills 35% of the bucket's volume, then what is the total capacity of the tank, in gallons?

(A) 8.75

(B) 15

(C) 16

(D) 17.5

(E) 19

50. If 150 were increased by 60% and then decreased by *y*%, the result would be 192. What is the value of *y*?

(A) 20

(B) 28

(C) 32

(D) 72

(E) 80

51. If x is 150% greater than 200, x is what percent greater than 50% of 500?

(A) 0

(B) 20

(C) 50

(D) 100

(E) 200

52. 16 ounces of birdseed mix contains 10% sesame seed by weight. How much sesame seed must be added to produce a mix that is 20% sesame seed by weight?

(A) 1 ounce

(B) 1.6 ounces

(C) 2 ounces

(D) 2.4 ounces

(E) 4 ounces

---

a, b, and c are positive.

	Quantity A	Quantity B
53.	$(a + b)$% of c	c% of $(a + b)$

Conference Ticket Advance Discounts	
5-29 days in advance	15%
30-59 days in advance	30%
60-89 days in advance	40%

54. Helen paid $252 for a conference ticket. If she had purchased the ticket one day later, she would have paid $306. How many days in advance did she purchase the ticket?

(A) 5

(B) 30

(C) 59

(D) 60

(E) 89

Percents Answers

1. **(C).** 50 as a percent of 30 is $\left(\frac{50}{30}\times 100\right)\% = 166.\overline{6}\%$. (Note: it's incorrect to calculate "50% of 30," which is 15. This asked for 50 *as a percent* of 30, which is equivalent to asking, "What percent of 30 is 50?")

To find the percent increase from 30 to 80, use the percent change formula:

$$\text{Percent Change} = \left(\frac{Difference}{Original}\times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{80-30}{30}\times 100\right)\% = 166.\overline{6}\%$$

The two quantities are equal. Note that doing the final calculation in each quantity is not necessary, because both equal $\frac{50}{30} \times 100$.

2. **(B).** The question asks for Ken's salary, so set a variable: call Ken's salary *k*. Lorena's salary is $60,000. Now, translate the equation in the first sentence.

"If Ken's salary were 20% higher" can be translated as Ken's salary + 20% of Ken's salary, or $k + 0.2k$. "It would be 20% less than Lorena's" can be translated as (Lorena's salary – 20% of Lorena's salary), or 60,000 – (0.2)(60,000). This is equivalent to (0.8)(60,000). Now solve:

$$\begin{aligned} 1.2k &= 0.8(60{,}000) \\ 1.2k &= 48{,}000 \\ k &= 40{,}000 \end{aligned}$$

Ken's salary is $40,000.

3. **(C).** Because the problem never indicates real values, pick your own smart numbers. If $x = 100$ and $y = 50$, then:

Greta's salary was $100,000 and she received a 50% raise. Greta's raise, therefore, was $50,000.

> Annika’s salary was \$50,000 and she received a 100% raise. Annika’s raise, therefore, was \$50,000.

The two quantities are equal. This holds true for any positive numbers chosen for x and y, because x% of y = y% of x. Thus, any two numbers can be used—just as 50% of 100 = 100% of 50, it is also true that 1% of 2,000 = 2,000% of 1, or a% of b = b% of a.

4. **(B).** Roselba’s income is more than twice as great as Jane’s income. If both pay the same percent of income in transportation fees, that means Roselba must pay *more* than twice as much as Jane in transportation fees. Therefore, half of Roselba’s fees will still be greater than Jane’s fees. Quantity B is greater.

Alternatively, use smart numbers. Call Jane’s income \$100. Roselba’s income, then, is greater than \$200. If both pay 10% in transportation fees, then Jane pays \$10 and Roselba pays more than \$20. Half of Roselba’s amount equals more than \$10.

5. **(A).** The problem doesn’t indicate any specific values, so pick a smart number. Because this is a percent problem, call the original price \$100. Quantity A equals \$100.

Decreasing a value by 16% is the same as taking (100 – 16)% = 84% of the number: so (0.84)(100) = \$84. To increase the value by 16%, take 116% of the number, or multiply by 1.16: (1.16)(84) = \$97.44.

Quantity A is greater.

6. **240.** Translate the question as $12 = 0.05x$ and solve on the calculator: $x = 240$. Alternatively, translate the question as $12 = \frac{5}{100}x$ and solve on paper:

$$\begin{aligned} 12 &= \frac{1}{20}x \\ (12)(20) &= x \\ x &= 240 \end{aligned}$$

7. **9**. Always translate the phrase “what percent” as $\frac{x}{100}$. Translate the question as:

$$\begin{aligned} 0.07(9) &= \frac{x}{100}(7) \\ 0.63 &= \frac{7x}{100} \end{aligned}$$

$$63 = 7x$$
$$9 = x$$

Incidentally, the pattern "x% of y = y% of x" always holds true! Here, 7% of 9 = 9% of 7, but it is also true that 2% of 57 = 57% of 2, etc. This works with any two numbers. If you notice this, then you can "fill in the blank" on the answer immediately: "what percent" must be 9%.

Finally, notice that the answer is 9 and not 0.09 or 9%. The question asks "what percent," so the percent is already incorporated into the sentence—the "what" by itself represents only the number itself, 9.

8. **300.** Always translate the phrase “what percent” as $\frac{x}{100}$. Translate the question as:

$$\begin{aligned} \frac{x}{100}(13) &= 0.2(195) \\ \frac{13x}{100} &= 39 \\ 13x &= 3{,}900 \\ x &= 300 \end{aligned}$$

Alternatively, take 20 percent of 195 (0.2 × 195 = 39) and rephrase the question: “What percent of 13 is 39?” Since 39 is three times as big as 13, the answer is 300.

9. **10.** Translate the question as 0.25(30) = 0.75x and solve on the calculator: x = 10.

Alternatively, write the percents in simplified fraction form and solve on paper:

$$\begin{aligned} \frac{1}{4}(30) &= \frac{3}{4}x \\ 30 &= 3x \\ x &= 10 \end{aligned}$$

10. **20% increase.** Use the percent change formula:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{60-50}{50} \times 100\right)\% = \left(\frac{10}{50} \times 100\right)\% = (0.2 \times 100)\% = 20\%$$

11. **90.** Because 30% less than x is the same as 70% of x, translate as follows: $0.7x = 63$. Use the calculator to get $x = 90$. Alternatively, solve on paper:

$$\begin{aligned}
\frac{7}{10}x &= 63 \\
x &= (63)\left(\frac{10}{7}\right) \\
x &= (9)(10) \\
x &= 90
\end{aligned}$$

12. **138.** Translate into decimals (for the percents, move the decimal two places to the left) and use the calculator to solve:

$$x = 2.3(0.15)(400)$$

$$x = 138$$

Alternatively, translate into fractions and solve on paper:

$$\frac{230}{100} \times \frac{15}{100} \times 400 =$$

$$\frac{23}{10} \times \frac{15}{1} \times 4 =$$

$$\frac{23}{2} \times \frac{3}{1} \times 4 =$$

$$\frac{23}{1} \times \frac{3}{1} \times 2 = 138$$

13. **50.** The left-hand side of the equation is given: 45% of 80 is (0.45)(80) = 36. The problem then becomes: "36 is *x*% more than 24." From this step, there are two possible approaches.

One approach is to translate the equation and solve:

$$36 = 24 + \frac{x}{100}(24)$$

$$12 = \frac{24x}{100}$$

$$12\left(\frac{100}{24}\right) = x$$

$$50 = x$$

Alternatively, the increase (36 – 24) is 12, so rephrase the statement as "12 is *x*% of 24.” Recognizing that 12 is half of 24, *x* must be 50. Or, translate and solve:

$$12 = \frac{x}{100}(24)$$

$$12\left(\frac{100}{24}\right) = x$$

$$50 = x$$

14. **400.** Translate as decimals and use the calculator to solve, keeping in mind that taking 200% of a number is the same as doubling it, or multiplying by 2:

$$\begin{aligned} 0.10(0.30)x &= 2(6) \\ 0.03x &= 12 \\ x &= 400 \end{aligned}$$

Alternatively, translate as fractions and solve on paper:

$$\left(\frac{1}{10}\right)\left(\frac{3}{10}\right)x = 2(6)$$

$$x = 12\left(\frac{100}{3}\right)$$

$$x = 400$$

15. **80.** The question already contains a variable (y). Use another variable to represent the desired value. Represent "what" with the variable x, and isolate x to solve. Notice that by the end, the y variables cancel out:

$$\left(\frac{x}{100}\right)\left(\frac{y}{100}\right)50 = \left(\frac{40}{100}\right)y$$

At this point, there are *many* options for simplifying, but do simplify before multiplying anything. Here is one way to simplify:

$$\left(\frac{x}{100}\right)\left(\frac{y}{2}\right) = \left(\frac{2}{5}\right)y$$

$$x = \frac{2y(100)(2)}{5y}$$

$$x = 80$$

16. **16.** 200% of 4% is the same as 2 × 4% (note that 200% equals the plain number 2), or 8%. Rephrase the question as “8% of *a* is what percent of $\frac{a}{2}$?” Without translating to an equation, this can be simplified by multiplying both sides of the “equation” by 2 (remember that “is” means “equals”):

8% of *a* is what percent of $\frac{a}{2}$?

16% of *a* is what percent of *a*?

Thus, the answer is 16.

Alternatively, translate the words into math:

$$\left(\frac{200}{100}\right)\left(\frac{4}{100}\right)a = \left(\frac{x}{100}\right)\left(\frac{a}{2}\right)$$

$$\left(\frac{2}{25}\right)a = \frac{xa}{200}$$

$$\left(\frac{2}{25}\right)a\left(\frac{200}{a}\right) = x$$

$$16 = x$$

17. **144.** If *m* were increased by 20%, decreased by 25%, and then increased by 60%, it would be multiplied by 1.2, then 0.75, then 1.6. Since (1.2)(0.75)(1.6) = 1.44, doing these manipulations is the same as increasing by 44%, or taking 144% of a number (this is true regardless of the value of *m*).

Alternatively, pick a real value for *m*. Because this is a percent problem, 100 is a good number to pick. First, 100 is increased by 20%: (100)(1.2) = 120. Next, 120 is decreased by 25%, which is the same as multiplying by 75%: (120)(0.75) = 90. Finally, 90 is increased by 60%: (90)(1.6) = 144. The new number is 144 and the starting number was 100, so the new number is $\left(\frac{144}{100}\right)$% of the original number, or 144%.

18. **(A).** Say the item costs \$100. After the first 10% discount, the item costs \$90. After the second, the item costs \$81 (the new discount is only \$9, or 10% of 90). After the third discount, the item costs \$81 – \$8.10 = \$72.90. What is the trend here? The cost goes down with each discount, yes, but the discount itself also gets smaller each time; it is only a \$10 discount the very first time. The total of the five discounts, then, will be less than \$50.

If the item costs \$100 to start, then the value for Quantity B will be \$50, or a total discount of \$50. This is greater than the total discount described for Quantity A.

Finally, make sure to answer (A) for the higher price—don't accidentally pick (B) for the "better deal"!

19. **(E).** 1% of \$450 is \$4.50, so 0.5% is \$2.25. That's the amount Raymond

pays back every week. Because he has paid back $18 in total, divide 18 by 2.25 to determine the total number of payments: $\frac{\$18}{\$2.25}$ = 8.

So Raymond has made 8 payments, once every 7 days. The payments themselves spread over only a 7-week period (in the same way that 2 payments spread over only a 1-week period). Raymond waited 1 week to begin repayment, however, so a total of 8 weeks, or 56 days, have passed since he borrowed the money.

20. **(A).** The number of parts in inventory first decreased by 50%, then increased by 50%. If the initial number of parts in inventory was x, the number after both shipments was $x(0.50)(1.5) = 0.75x$. The number of parts after the shipments was 75% of the number before, which is fewer. Quantity A is greater.

Alternatively, choose a smart number to test. If $x = 100$, then the inventory first decreased to 50, and then increased from 50 to 75. Quantity A is 100 and Quantity B is 75.

Finally, it is possible to solve this question using logic. The 50% decrease is taken as a percent of the original number. The 50% increase, however, is taken as a percent of the new, *smaller* number. The increase, therefore, must be smaller than the decrease, making the final value smaller than the original.

21. **(B).** To reduce \$200,000 by 40%, multiply by 0.6 (reducing by 40% is the same as keeping 60%): \$200,000(0.6) = \$120,000.

To reduce \$120,000 by 20%, multiply by 0.8 (reducing by 20% is the same as keeping 80%): \$120,000(0.8) = \$96,000. Quantity B is greater.

22. **12,000%.** Translate the statement into an equation. Since one of the percents is a variable, fractions are preferable to decimals:

$$\frac{1}{100} \times \frac{200}{100} \times 360 = \frac{x}{100} \times \frac{0.1}{100} \times 60$$

Because 100 appears twice on the bottom of both sides of the equation, multiply each side of the equation by 10,000 (or 100 twice) to cancel the 100's out:

$$\frac{1}{\cancel{100}} \times \frac{200}{\cancel{100}} \times 360 = \frac{x}{\cancel{100}} \times \frac{0.1}{\cancel{100}} \times 60$$

$$200 \times 360 = x(0.1)(60)$$

$$\frac{200 \times 360}{60} = x\left(\frac{1}{10}\right)$$

$$200 \times 6 \times 10 = x$$

$$x = 12{,}000$$

The answer is 12,000%. (The phrase "what percent" translates into math as $\frac{x}{100}$. Additionally, $\frac{12{,}000}{100}$ is the same thing as 12,000%, just as $\frac{50}{100}$ is equal to 50%. While 12,000% may seem quite large, it is correct.)

Alternatively, use decimals, while still writing "what percent" as a fraction.

Then, use the calculator to solve:

$$(0.01)(2)(360) = \frac{x}{100}(0.001)(60)$$

$$7.2 = \frac{x}{100}(0.06)$$

$$120 = \frac{x}{100}$$

$$12{,}000 = x$$

23. **(D).** Because no actual amounts of money are stated in the question, use smart numbers to solve this problem. If Mary has half as many cents as Nora has dollars, then, as an example, if Nora had \$10, Mary would have 5 cents. Nora's \$10 equals 1,000 cents. To determine what *percent more* cents Nora has, use the percent change formula:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{1{,}000 - 5}{5} \times 100\right)\% = 19{,}900\%$$

Any example in which "Mary has half as many cents as Nora has dollars" will yield the same result. Note that the percent change formula is required—a percent *more* (or percent increase) is not the same as a percent *of* something.

To do the problem algebraically (which is more difficult than using a smart number, as above), use M for Mary's cents and N for Nora's cents. Divide N by 100 in order to convert from cents to dollars, $\frac{N}{100}$, and set up an equation to reflect that Mary has half as many cents as Nora has dollars:

$$M = \frac{1}{2}\left(\frac{N}{100}\right)$$

$$M = \frac{N}{200}$$

$$200M = N$$

Therefore, Nora has 200 times as many cents. 200 times *as many* is 199 times *more*. To convert 199 times *more* to a percent, add two zeros to get 19,900%.

24. **(E).** Rather than trying to write out the whole statement as math, note that "the number that is 50% greater than 60" can be calculated: 1.5(60) = 90. Similarly, "the number that is 20% less than 150" is 0.8(150) = 120. The question can be rephrased as "90 is what percent less than 120?" Use the percent change formula. Since the question specifies a "percent *less*," the "original" number is 120:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\% = \left(\frac{30}{120} \times 100\right)\% = 25\%$$

25. **(D).** The percent increase is the difference between the amounts divided by the original, converted to a percent. If the population doubles, mathematically the increase can be written as a power of 2. In the 30-day interval, if the original population is 1, it will double to 2 after three days—so, 2^1 represents the population after the first increase, the second increase would then be 2^2, and so on. Since there are ten increases, the final population would be 2^{10} or 1,024. Therefore, the difference, 1,024 – 1, is 1,023. Use the percent change formula to calculate percent increase:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\% = \left(\frac{1{,}023}{1} \times 100\right)\% = 102{,}300\%$$

Note that the new number *is* 102,400% of the original, but that was not the question asked—the percent *increase* is 102,300%.

26. **(D).** Call the original price x. That price is discounted by 15% to get 612:

$$\begin{aligned} 0.85x &= \$612 \\ x &= \$720 \end{aligned}$$

Do not add 15% of $612 to $612. The 15% figure is a percent of the unknown original number, not of $612.

27. **(B).** Call the original price x. At the end of April, the total price was $1.4x$. The price increase in April was $1.4x - 1x = 0.4x$.

In May, the price increased an additional 30% over April's final price of $1.4x$. Thus, the price at the end of May was $(1.3)(1.4)x$, or $1.82x$. The price increase in May was $1.82x - 1.4x = 0.42x$.

Since x is positive, $0.42x$ (42% of x) is greater than $0.4x$ (40% of x). Quantity B is greater.

Alternatively, use smart numbers. If the original price was $100, April's increase would result in a price of $140 and May's increase would be (1.3)(140) = $182. Thus, April's increase was $40 and May's increase was $42. May's increase will be greater no matter what number is used as the starting price (it is reasonable in GRE problems to assume that a price must be a positive number).

28. **(B).** The 50% spent on rent, utilities, and insurance and the 20% spent on food are both percents of the total, so sum the percents: 50% + 20% = 70%. After these expenditures, Aloysius has 30% left. He then spends 30% *of the remaining 30%* on video games. 30% of 30% = $0.30 \times 0.30 = 0.09$, or 9% of the total, so 30% – 9% = 21% of his income remains.

Alternatively, use smart numbers. If Aloysius's income is $100, he would spend $50 on rent, utilities, and insurance, and $20 on food, for a total of $70. Of his remaining $30, he would spend 30%, or $9, on video games, leaving $21, or 21%, of the original amount.

29. **7,100%.** In 1970, company X had 0.15(2,000) = 300 female employees. Of those, 0.10(300) = 30 were female executives.

In 2012, company X had 0.45(12,000) = 5,400 female employees. Of those, 0.40(5,400) = 2,160 were female executives.

Percent Change = $$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

Percent Change =

$$\text{Percent Change} = \left(\frac{2130}{30} \times 100\right)\% = 7{,}100\%$$

30. **60%.** Use smart numbers. There are 20% fewer boys than girls, so choose 100 for the number of girls (100 is a good number to pick for percent problems). Thus, there are (100)(0.8) = 80 boys in the school. If 75% of all the boys take civics, then 0.75(80) = 60 boys take civics. If 48% of all the girls take civics, then 0.48(100) = 48 girls take civics.

Therefore, 60 + 48 = 108 students take civics and there are 180 total students:

$$\left(\frac{108}{180}\times 100\right)\% = 60\%$$

31. **(C).** Airline A reduced its price by 25% to ($400)(0.75) = $300, but then increased that price by 10% to ($300)(1.1) = $330. Airline B reduced its fare to ($400)(0.45) = $180, but added $150 in fees, bringing the total price to $180 + $150 = $330. The two quantities are equal.

32. **(C).** Write an equation from the first part of the given information: $p = 0.75q$. Since $p = 2r$, substitute $2r$ for p in the first equation:

$$\begin{aligned} 2r &= 0.75q \\ r &= 0.375q \end{aligned}$$

The two quantities are equal.

Alternatively, use smart numbers. If q is 8, then p is (8)(0.75) = 6. (Note: because you have to multiply q by 0.75, or $\frac{3}{4}$, try to pick something divisible by 4 for q, so that p will be an integer.) Therefore, r is $\frac{6}{2} = 3$.

Since $0.375q = (0.375)(8) = 3$, the value for r is also 3. The two quantities are equal.

33. **(A).** When a percent contains a variable, use fractions to translate. Quantity A is:

$$\frac{x}{100}\times\frac{0.5}{100}\times\frac{40{,}000}{1} = x(0.5)(4) = 2x$$

Quantity B is:

$$\frac{0.05}{100}\times\frac{2{,}000}{100}\times\frac{40x}{1} = (0.05)(2)(4x) = 0.4x$$

Since x is positive, Quantity A is greater (this is true even if x is a fraction).

Alternatively, use smart numbers. If x = 50, then Quantity A equals:

$$\frac{50}{100} \times \frac{0.5}{100} \times \frac{40{,}000}{1} = (0.5)(0.5)(400) = 100$$

Quantity B equals:

$$\frac{0.05}{100} \times \frac{2{,}000}{100} \times \frac{(40)(50)}{1} = (0.05)(2)(4)(50) = 20$$

Quantity A is greater.

34. **216.** The percent increase from 2000 to 2001 is:

$$\text{Percent Change} = \left(\frac{Difference}{Original} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{20}{60} \times 100\right)\% = 33.\overline{3}\%$$

Now, apply a $33.\overline{3}\%$, or $\frac{1}{3}$, increase to 2004’s figure. The GRE calculator cannot accept a repeating decimal; instead, divide 162 by 3 to get the amount of increase, and then add 162 to get the new profit per student in 2005: 162 ÷ 3 + 162 = 216.

35. **(E).** First, write “*x* is 0.5% of *y*” as math. Make sure you don’t accidentally interpret 0.5% as 50%!

$$x = \frac{0.5}{100} \times y$$

The question asks “*y* is what percent of *x*?” so solve for *y*:

$$\begin{aligned} 100x &= 0.5y \\ 200x &= y \end{aligned}$$

If *y* is 200 times *x*, multiply by 100 to convert to a percent:

$$\frac{200x}{1} \times \frac{100}{100} = \frac{20{,}000x}{100}$$

The answer is 20,000%. (For reference, if one number is 2 times as big as the other, it is 200% the size—add two zeros. So, 200 times as big = 20,000%.)

Alternatively, use smart numbers. If $y = 100$, then $x = \frac{0.5}{100}(100) = 0.5$. Next, answer the question, “100 is what percent of 0.5?” Pick a new variable to translate the “what percent” portion of the sentence:

$$100 = \frac{n}{100} \times 0.5$$

$$10{,}000 = 0.5n$$

$$20{,}000 = n$$

(In translating percents problems to math, always translate “what percent” as a variable over 100.)

36. **(C).** Bill’s tax is (0.20)($5,000) = $1,000. Thus, his remaining salary is $4,000. His rent is therefore (0.25)($4000) = $1,000. The two quantities are equal.

37. **(B).** If four people shared the $80 bill equally, then each person paid for one-quarter of the bill, or $\frac{\$80}{4}$ = $20.

The tip is calculated as a percent of the bill. Because the question asks about the amount that each (one) person paid, calculate the 15% tip based solely on one person's portion of the bill ($20): (0.15)(20) = $3.

In total, each person paid $20 + $3 = $23.

Alternatively, find the total of the bill plus tip and take one-fourth of that for the total contribution of each person. The total of bill and tip is $80 + (0.15)($80) = $80 + $12 = $92. One-fourth of this is $\frac{\$92}{4}$ = $23.

38. **(B).** Use a smart number for the price of the stock; for a percent problem, $100 is a good choice. The price of the stock after a 25% increase is (1.25) × $100 = $125.

Next, find the percent decrease (*y*) needed to reduce the price back to the original $100. Because $125 – $25 = $100, rephrase the question: 25 is what percent of 125?

$$25 = \frac{x}{100}(125)$$

$$\frac{2{,}500}{125} = x$$

$$x = 20$$

You have to reduce 125 by 20% in order to get back to $100. Therefore, Quantity A is 20%, so Quantity B is greater.

39. **(C).** The chemist now has 10 ounces of acetone in a 30-ounce mixture, so she must have 20 ounces of water. The question ask how many ounces of acetone must be added to make this mixture a 50% solution. No additional water is added, so the solution must finish with 20 ounces of water. Therefore, she also needs a total of 20 ounces of acetone, or 10 more ounces than the mixture currently contains.

Note that one trap answer is (B), or 5. This answer is not correct because the final number of ounces in the solution is *not* 30; when the chemist adds acetone, the amount of total solution also increases—adding 5 ounces acetone would result in a solution that is $\frac{15}{(30+5)}$ acetone, which is not equivalent to a 50% mixture.

40. **(C).** Choose a smart number for the total number of games; for a percent problem, 100 is a good number to pick. If the total number of games for the season is 100 and the team played 80% of them by July, then the team played (100)(0.8) = 80 games. The team won 50% of these games, or (80)(0.5) = 40 games.

Next, the team won 60% of its *remaining* games. As there were 100 total games and the team has played 80 of them, there are 20 games left to play. Of these, the team won 60%, or (20)(0.6) = 12 games.

Therefore, the team has won a total of 40 + 12 = 52 games out of 100, or 52% of its total games. The two quantities are equal.

Alternatively, this problem could be done using weighted averages, where the total percent of games won is equal to the sum of all of the individual percents multiplied by their weightings. In this case:

Total Percent Won = (50%)(80%) + (60%)(100% – 80%) × 100%
Total Percent Won = [(0.5)(0.8) + (0.6)(0.2)] × 100%
Total Percent Won = [(0.4) + (0.12)] × 100%
Total Percent Won = 0.52 × 100%
Total Percent Won = 52%

41. **(B).** In order to compare, use the calculator to find 0.4% of 4% of 1.25 (be careful with the decimals!):

$$0.004 \times 0.04 \times 1.25 = 0.0002$$

Or, as fractions:

$$\frac{0.4}{100} \times \frac{4}{100} \times 1.25 = \frac{2}{1{,}000} = 0.0002$$

Quantity B is greater.

42. **(D).** Originally, Jane had a 40-ounce mixture of apple and seltzer that was 30% apple. Since 0.30(40) = 12, 12 ounces were apple and 28 ounces were seltzer.

When Jane pours 10 more ounces of apple juice into the mixture, it yields a mixture that is 50 ounces total, still with 28 ounces of seltzer. Now, the percent of seltzer in the final mixture is $\frac{28}{50} \times 100 = 56\%$.

43. **(A).** Choose a smart number for the total number of shirts in the closest; this is a percent problem, so 100 is a good number to pick. Out of 100 shirts, half, or 50, are white.

You know 30% of the *remaining* shirts are gray. If there are 50 white shirts, there are also 50 remaining shirts and so (0.3)(50) = 15 gray shirts. Therefore, there are 50 + 15 = 65 total shirts that are white or gray, and 100 – 65 = 35 shirts that are neither white nor gray. Since 35 out of 100 shirts are neither white nor gray, exactly 35% of the shirts are neither white nor gray.

Alternatively, use algebra, though that is trickier on a problem such as this one. Set a variable, such as x, for the total number of shirts. The number of white shirts is $0.5x$ and the remaining shirts would equal $x - 0.5x = 0.5x$. The number of gray shirts, then, is $(0.5x)(0.3) = 0.15x$. Thus, there are $0.5x + 0.15x = 0.65x$ white or gray shirts, and $x - 0.6x = 0.35x$ shirts that are neither white nor gray. Therefore, $0.35x \div x = 0.35$, or 35%.

44. **(A).** Choose smart numbers for the dimensions of the rectangle—for instance, length = 20 and width = 10.

The original area of the rectangle = length × width = 200

After a 10% increase for both the length and the width, the area becomes 22 × 11 = 242.

Use the formula for percent change:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\left(\frac{242-200}{200} \times 100\right)\% = \left(\frac{42}{200} \times 100\right)\% = \left(\frac{21}{100} \times 100\right)\% = 21\%$$

Quantity A is greater.

Alternatively, use logic. The formula for area requires multiplying the length and the width. If just one side is increased by 10%, then the overall area will increase by 10%. If two sides are increased by 10%, then the overall area will increase by more than 10%.

45. **(C).** Translate the given information into math:

$$\frac{35}{100}x = 140$$

$$x = 140 \times \frac{100}{35}$$

$$x = 400$$

Next, find 20% of *x*, or 0.20(400) = 80.

46. **(A).** Every 3 minutes, the population increases by 20% (which is the same as multiplying by 1.2). Beginning at 8:54am, this change would occur at 8:57am and again at 9:00am. Use the variable *x* to represent the original quantity. Note that the 20% increase occurs twice:

$$x(1.2)(1.2) = 144{,}000$$

$$x = 100{,}000$$

Note that you cannot just reduce 144,000 by 20% twice, because 20% is not a percent of 144,000—it is a percent of the unknown, original number.

Alternatively, begin from 144,000 and calculate “backwards”:

From 8:57am to 9:00am: $y(1.2) = 144{,}000$, so $y = \frac{144{,}000}{1.2} =$ 120,000.

From 8:54am to 8:57am: $z(1.2) = 120{,}000$, so $z = \frac{120{,}000}{1.2} =$ 100,000.

47. **(D).** Reducing a number by a percent involves multiplication; reducing a number by a fixed amount involves subtraction. The order of operations (PEMDAS) will make a difference.

One possible value for the item is \$100. In this case, the value of Quantity A = (\$100)(0.9) – \$20 = \$70. The value of Quantity B = (\$100 – \$10)(0.80) = \$72. Here, Quantity B is greater.

However, a greater starting value may change the result, because a 20% discount off a greater starting value can result in a much greater decrease. For a \$140 item, the value of Quantity A = (\$140)(0.9) – \$20 = \$106. The value of Quantity B = (\$140 – \$10)(0.80) = \$104. Here, Quantity A is greater. The relationship cannot be determined from the information given.

48. **(B).** 20% less than 300 is the same as 80% of 300, or 0.80(300) = 240. The question is “240 is what percent greater than 180?”

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

Percent Change =

$$\text{Percent Change} = \left(\frac{60}{180} \times 100\right)\% = 33.\overline{3}\%$$

49. **(D).** First find the volume of oil in the bucket. The oil fills 35% of the bucket’s 20-gallon volume, or (20)(0.35) = 7 gallons of oil.

These 7 gallons originally filled 40% of the tank. If T is the volume of the tank, T(0.4) = 7, so T = 17.5 gallons.

50. **(A).** First, find the value of 150 increased by 60%: (150)(1.6) = 240. If 240 were then decreased by y%, the result would be 192. Because 240 is decreased by 48 to get 192, the question can be rephrased: 48 is what percent of 240?

$$48 = \frac{x}{100}(240)$$

$$48\left(\frac{10}{24}\right) = x$$

$$x = 20$$

51. **(D).** “150% greater than 200” means 150% of 200, or 300, *added back to* 200. This is the not the same figure as 150% *of* 200. Thus, 150% greater than 200 is 200 + (200)(1.5) = 500.

50% of 500 = 250. Translate the question as “500 is what percent greater than 250?” Since 500 is twice 250, it is 100% greater than 250.

Alternatively, use the percent change formula.

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{500 - 250}{250} \times 100\right)\% = 100\%$$

52. **(C).** A 16-ounce mix that contains 10% sesame by weight has 1.6 ounces of sesame. It might be tempting to think that adding another 1.6 ounces would make a mixture that is 20% sesame. However, this is incorrect—adding 1.6 ounces of sesame will also add 1.6 ounces to the total amount of seed in the jar, reducing the concentration of sesame in the mix:

$$\left(\frac{3.2 \text{ ounces sesame}}{17.6 \text{ ounces total}} \times 100\right)\% = 18.18\%.$$

Instead, write an equation expressing the ratio of sesame to the total mixture, where *x* is the amount of sesame to add; this equals the desired 20% (or $\frac{1}{5}$) figure:

$$\frac{1.6+x}{16+x}=\frac{1}{5}$$

Cross-multiply and solve for *x*:

$$\begin{aligned} 5(1.6+x) &= 16+x \\ 8+5x &= 16+x \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

53. **(C).** It is always the case that, for two positive quantities, *M*% of *N* = *N*% of *M*. In this case, (*a* + *b*) makes the problem appear more complicated, but the principle still applies. Algebraically:

Quantity A	**Quantity B**
$\frac{(a+b)}{100}\times c$	$\frac{c}{100}\times(a+b)$

Both quantities can be simplified to $\frac{c(a+b)}{100}$. The two quantities are equal.

54. **(B).** Helen bought a ticket for $252; if she had bought it one day later, she would have paid $54 more. There are three possibilities that represent the dividing lines between the given discount levels:

Possibility 1: She bought the ticket 60 days in advance for a 40% discount (if she’d bought it 1 day later, or 59 days in advance, she would have received a 30% discount instead).

Possibility 2: She bought the ticket 30 days in advance for a 30% discount (if she’d bought it 1 day later, or 29 days in advance, she would have received a 15% discount instead).

Possibility 3: She bought the ticket 5 days in advance for a 15% discount (if she’d bought it 1 day later, or 4 days in advance, she would not have received any kind of discount).

This question is harder than it looks, do not calculate a percent change between \$252 and \$306. The discounts are *percents of the full-price ticket*, which is an unknown value. Call it *x*.

Note that the only three possible answers are 5, 30, and 60 (answers (A), (B), and (D), respectively); 59 days ahead and 89 days ahead do not represent days for which the next day (58 and 88 days ahead, respectively) results in a change in the discount.

Possibility 1 (60 days in advance): \$252 would represent a 40% discount from the original price, so the original price would be \$252 = 0.6*x*, and *x* would be \$420.

If the full ticket price is \$420, then buying the ticket 1 day later would result in a 30% discount instead, or (\$420)(0.7) = \$294. The problem indicates, however, that Helen would have paid \$306, so Possibility 1 is not correct.

Possibility 2 (30 days in advance): \$252 would represent a 30% discount from the original price, so the original price would be \$252 = $0.7x$ and x would be \$360.

If the full ticket price is \$360, then buying the ticket 1 day later would result in a 15% discount instead, or (\$360)(0.85) = \$306. This matches the figure given in the problem, so Possibility 2 is correct; you do not need to test Possibility 3. Helen bought the ticket 30 days in advance.

Chapter 13

Divisibility and Primes

In This Chapter…

Divisibility and Primes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. For how many positive integer values of x is $\frac{65}{x}$ an integer?

2. If x is a number such that $0 < x \leq 20$, for how many values of x is $\frac{20}{x}$ an integer?

   (A) Four

   (B) Six

   (C) Eight

   (D) Ten

(E) More than ten

	Quantity A	Quantity B
3.	The number of distinct positive factors of 10	The number of distinct prime factors of 210

	Quantity A	Quantity B
4.	The least common multiple of 22 and 6	The greatest common factor of 66 and 99

5. The number of students who attend a school could be divided among 10, 12, or 16 buses, such that each bus transports an equal number of students. What is the minimum number of students that could attend the school?

(A) 120

(B) 160

(C) 240

(D) 320

(E) 480

6.

Quantity A	Quantity B
The number of distinct prime factors of 27	The number of distinct prime factors of 18

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7. How many factors greater than 1 do 120, 210, and 270 have in common?

(A) One

(B) Three

(C) Six

(D) Seven

(E) Thirty

8. Company H distributed $4,000 and 180 pencils evenly among its employees, with each employee getting an equal integer number of dollars and an equal integer number of pencils. What is the greatest number of employees that could work for company H?

(A) 9

(B) 10

(C) 20

(D) 40

(E) 180

9. *n* is divisible by 14 and 3. Which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ 12 is a factor of *n*.

- ☐ 21 is a factor of n.
- ☐ n is a multiple of 42.

10. Positive integers a and b each have exactly four positive factors. If a is a one-digit number and $b = a + 9$, what is the value of a?

$\boxed{}$

11. Ramon wants to cut a rectangular board into identical square pieces. If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?

(A) 4

(B) 6

(C) 9

(D) 12

(E) 15

12. When the positive integer x is divided by 6, the remainder is 4. Each of the following could also be an integer EXCEPT

(A) $\frac{x}{2}$

(B) $\frac{x}{3}$

(C) $\frac{x}{7}$

(D) $\frac{x}{11}$

(E) $\frac{x}{17}$

13. If $x^y = 64$ and x and y are positive integers, which of the following could be the value of $x + y$?

Indicate <u>all</u> such values.

☐ 2

☐ 6

- ☐ 7
- ☐ 8
- ☐ 10
- ☐ 12

14. If k is a multiple of 24 but not a multiple of 16, which of the following cannot be an integer?

(A) $\frac{k}{8}$

(B) $\frac{k}{9}$

(C) $\frac{k}{32}$

(D) $\frac{k}{36}$

(E) $\frac{k}{81}$

15. If $a = 16b$ and b is a prime number greater than 2, how many positive distinct factors does a have?

$\boxed{}$

16. If a and b are integers such that $a > b > 1$, which of the following cannot be a multiple of either a or b?

(A) $a - 1$

(B) $b + 1$

(C) $b - 1$

(D) $a + b$

(E) ab

17. 616 divided by 6 yields remainder p, and 525 divided by 11 yields remainder q. What is $p + q$?

$\boxed{}$

18. If x is divisible by 18 and y is divisible by 12, which of the following statements must be true?

Indicate all such statements.

- ☐ $x + y$ is divisible by 6.
- ☐ xy is divisible by 48.
- ☐ $\frac{x}{y}$ is divisible by 6.

19. If p is divisible by 7 and q is divisible by 6, pq must have at least how many factors greater than 1?

(A) One

(B) Three

(C) Six

(D) Seven

(E) Eight

20. If r is divisible by 10 and s is divisible by 9, rs must have at least how many positive factors?

(A) Two

(B) Four

(C) Twelve

(D) Fourteen

(E) Sixteen

21. If t is divisible by 12, what is the least possible integer value of a for which $\frac{t^2}{2^a}$ might not be an integer?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

22. If a, b, and c are multiples of 3 such that $a > b > c > 0$, which of the following values must be divisible by 3?

Indicate <u>all</u> such values.

☐ $a + b + c$

☐ $a - b + c$

☐ $\frac{abc}{9}$

23. New cars leave a car factory in a repeating pattern of red, blue, black, and gray cars. If the first car to exit the factory was red, what color is the 463rd car to exit the factory?

(A) Red

(B) Blue

(C) Black

(D) Gray

(E) It cannot be determined from the information given.

24. Jason deposits money at a bank on a Tuesday and returns to the bank 100 days later to withdraw the money. On what day of the week did Jason withdraw the money from the bank?

(A) Monday

(B) Tuesday

(C) Wednesday

(D) Thursday

(E) Friday

25. x and h are both positive integers. When x is divided by 7, the quotient is h with a remainder of 3. Which of the following could be the value of x?

(A) 7

(B) 21

(C) 50

(D) 52

(E) 57

26. When x is divided by 10, the quotient is y with a remainder of 4. If x and y are both positive integers, what is the remainder when x is divided by 5?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

27. What is the remainder when $13^{17} + 17^{13}$ is divided by 10?

$\boxed{}$

28. If n is an integer and n^3 is divisible by 24, what is the largest number that must be a factor of n?

(A) 1

(B) 2

(C) 6

(D) 8

(E) 12

10! is divisible by 3^x5^y, where x and y are positive integers.

	Quantity A	Quantity B
29.	The greatest possible value for x	Twice the greatest possible value for y

	Quantity A	Quantity B
30.	The number of distinct prime factors of 100,000	The number of distinct prime factors of 99,000

31. Which of the following values times 12 is not a multiple of 64?

Indicate all such values.

- ☐ 6^6
- ☐ 12^2
- ☐ 18^3
- ☐ 30^3
- ☐ 222

32. If $3^x(5^2)$ is divided by $3^5(5^3)$, the quotient terminates with one decimal digit. If $x > 0$, which of the following statements must be true?

(A) x is even

(B) x is odd

(C) $x < 5$

(D) $x \geq 5$

(E) $x = 5$

33. *abc* is a three-digit number in which a is the hundreds digit, b is the tens digit, and c is the units digit. Let &(*abc*)& = $(2^a)(3^b)(5^c)$. For example, &(203)& = $(2^2)(3^0)(5^3) = 500$. For how many three-digit numbers *abc* does the function &(*abc*)& yield a prime number?

(A) Zero

(B) One

(C) Two

(D) Three

(E) Nine

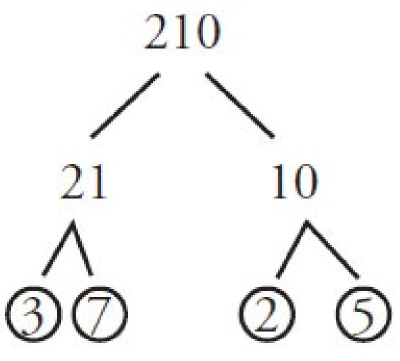
Divisibility and Primes Answers

1. **4.** If x is a positive integer such that $\frac{65}{x}$ is also an integer, then x must be a factor of 65. The factors of 65 are 1, 5, 13, and 65. Thus, there are four positive integer values of x such that $\frac{65}{x}$ is an integer.

2. **(E).** Notice that the problem did *not* say that x had to be an integer. Therefore, the factors of 20 will work (1, 2, 4, 5, 10, 20), but so will 0.5, 0.1, 0.25, 2.5, etc. It is possible to divide 20 into fractional parts—for instance, something 20 inches long could be divided evenly into quarter inches (there would be 80 of them, as $\frac{20}{0.25}$ = 80). There are an infinite number of x values that would work (it is possible to divide 20 into thousandths, millionths, etc.), so the answer is (E). It is very important on the GRE to notice whether there is an integer constraint on a variable or not! Any answer like “More than 10” should be a clue that this problem may be less straightforward than it seems.

3. **(C).** The *positive factors* of 10 are 1 & 10, and 2 & 5. Since there are four positive factors, Quantity A is 4.

The *prime factors* of 210 are 2, 3, 5, and 7:



Because 210 has four prime factors, Quantity B is also 4. The two quantities are equal.

4. **(A).** The least common multiple of 22 and 6 is 66. One way to find the least common multiple is to list the larger number’s multiples (it is more efficient to begin with the larger number) until reaching a multiple that the other number goes into. The multiples of 22 are 22, 44, 66, 88, etc. The smallest of

these that 6 goes into is 66.

The greatest common factor of 66 and 99 is 33. One way to find the greatest common factor is to list all the factors of one of the numbers, and then pick the greatest one that also goes into the other number. For instance, the factors of 66 are 1 & 66, 2 & 33, 3 & 22, and 6 & 11. The greatest of these that also goes into 99 is 33. Thus, Quantity A is greater.

5. **(C).** The number of students must be divisible by 10, 12, and 16. So the question is really asking, "What is the least common multiple of 10, 12, and 16?" Since all of the answer choices end in 0, each is divisible by 10. Just use the calculator to test which choices are also divisible by 12 and 16. Because the question asks for the minimum, start by checking the smallest choices. Since $\frac{120}{16}$ and $\frac{160}{12}$ are not integers, the smallest choice that works is 240.

6. **(B).** "Distinct" means different from each other. To find distinct prime factors, make a prime factor tree, and then disregard any repeated prime factors. The integer 27 breaks down into 3 × 3 × 3. Thus, 27 has only 1 distinct prime factor. The integer 18 breaks down into 2 × 3 × 3. Thus, 18 has 2 distinct prime factors.

7. **(D).** Pick one of the numbers and list all of its factors. The factors of 120 are: 1 & 120, 2 & 60, 3 & 40, 4 & 30, 5 & 24, 6 & 20, 8 & 15, 10 & 12. Since the problem specifically asks for factors “greater than 1,” eliminate 1 now. Now cross off any factors that do *not* go into 210:

~~120~~, 2 & ~~60~~, 3 & ~~40~~, ~~4~~ & 30, 5 & ~~24~~, 6 & ~~20~~, ~~8~~ & 15, 10 & ~~12~~

Now cross off any factors remaining that do *not* go into 270. Interestingly, all of the remaining factors (2, 3, 5, 6, 10, 15, 30) *do* go into 270. There are seven shared factors.

8. **(C).** In order to distribute \$4,000 and 180 pencils evenly, the number of employees must be a factor of each of these two numbers. Because the question asks for the greatest number of employees possible, start by checking the greatest choices:

> (E) \$4,000 could not be evenly distributed among 180 employees (although 180 pencils could).
> (D) \$4,000 could be evenly divided among 40 people, but 180 pencils could not.
> (C) is the greatest choice that works—\$4,000 and 180 pencils could each be evenly distributed among 20 people.

Alternatively, find the greatest common factor (GCF) of the two numbers. Factor: $4{,}000 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^5 \times 5^3$ and $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$. These numbers have $2 \times 2 \times 5$ in common, so 20 is the GCF.

9. **“21 is a factor of *n*” and “*n* is a multiple of 42” only.** Since n is divisible by 14 and 3, n contains the prime factors of both 14 and 3, which are 2, 7, and 3. Thus, any numbers that can be constructed using only these prime factors (no additional factors) are factors of n. Since $12 = 2 \times 2 \times 3$, you *cannot* make 12 by multiplying the prime factors of n (you would need one more 2). However, you *can* construct 21 by multiplying two of the known prime factors of n ($7 \times 3 = 21$), so the second statement is true. Finally, n must be at least 42 ($= 2 \times 7 \times 3$, the *least common multiple* of 14 and 3), so n is definitely a multiple of 42. That is, n can only be 42, 84, 126, etc.

10. **6.** Start by considering integer a, which is the most constrained variable. It is a positive one-digit number (between 1 and 9, inclusive) and it has four positive factors. Prime numbers have exactly two positive factors. Prime numbers have exactly two factors: themselves and one, so only look at non-prime one-digit positive integers. That’s a short enough list:

1 has just one positive factor
4 has three positive factors: 1, 2, and 4
6 has four positive factors: 1, 2, 3, and 6
8 has four positive factors: 1, 2, 4, and 8
9 has three positive factors: 1, 3, and 9

So the two possibilities for a are 6 and 8. Now apply the two constraints for b. It is 9 greater than a, and it has exactly four positive factors. Check the possibilities:

If $a = 6$, then $b = 15$, which has four factors: 1, 3, 5, and 15.
If $a = 8$, then $b = 17$, which is prime, so it has only has two factors: 1 and 17.
Only $b = 15$ works, so a must be 6.

11. **(E).** Cutting a rectangular board into square pieces means that Ramon needs to cut pieces that are equal in length and width. “Without wasting any of the board” means that he needs to choose a side length that divides evenly into both 18 and 30. “The least number of square pieces” means that he needs to choose the largest possible squares. With these three stipulations, choose the largest integer that divides evenly into 18 and 30, or the greatest common factor, which is 6. This would give Ramon 3 pieces going one way and 5 pieces going the other. He would cut 3 × 5 = 15 squares of dimension 6” × 6”. Note that this solution ignored squares with non-integer side length for the sake of convenience, a potentially dangerous thing to do. (After all, identical squares of 1.5” by 1.5” could be cut without wasting any of the board.) However, to cut squares any larger than 6” × 6”, Ramon could only cut 2 squares of 9” or 1 square of 18” from the 18” dimension of the rectangle, neither of which would evenly divide the 30” dimension of the rectangle. The computed answer is correct.

12. **(B).** When dealing with remainder questions on the GRE, the best thing to do is test a few real numbers:

Multiples of 6 are 0, 6, 12, 18, 24, 30, 36, etc.

Numbers with a remainder of 4 when divided by 6 are those 4 greater than the multiples of 6:

x could be 4, 10, 16, 22, 28, 34, 40, etc.

You could keep listing numbers, but this is probably enough to establish a pattern.

(A) $\frac{x}{2}$ → ALL of the listed *x* values are divisible by 2. Eliminate (A).

(B) $\frac{x}{3}$ → NONE of the listed *x* values are divisible by 3, but continue checking.

(C) $\frac{x}{7}$ → 28 is divisible by 7.

(D) $\frac{x}{11}$ → 22 is divisible by 11.

(E) $\frac{x}{17}$ → 34 is divisible by 17.

The question is "Each of the following could also be an integer EXCEPT." Since four of the choices could be integers, (B) must be the answer.

13. **7, 8, and 10 only.** If $x^y = 64$ and x and y are positive integers, perhaps the most obvious possibility is that $x = 8$ and $y = 2$. However, "all such values" implies that other solutions are possible. One shortcut is noting that only an even base, when raised to a power, could equal 64. So you only have to worry about even possibilities for x. Here are all the possibilities:

$2^6 = 64 \rightarrow x + y = 8$

$4^3 = 64 \rightarrow x + y = 7$

$8^2 = 64 \rightarrow x + y = 10$

$64^1 = 64 \rightarrow x + y = 65$

The only possible values of $x + y$ listed among the choices are 7, 8, and 10.

14. **(C).** If k is a multiple of 24, it contains the prime factors of 24: 2, 2, 2, and 3. (It could also contain other prime factors, but the only ones for certain are the prime factors contained in 24.)

If k were a multiple of 16, it would contain the prime factors of 16: 2, 2, 2, and 2.

Thus, if k is a multiple of 24 but *not* of 16, k must contain 2, 2, and 2, but *not* a fourth 2 (otherwise, it would be a multiple of 16).

Thus: k definitely has 2, 2, 2, and 3. It could have any other prime factors (including more 3's) *except* for more 2's.

An answer choice in which the denominator contains more than three 2's would guarantee a non-integer result. Only choice (C) works. Since k has fewer 2's than 32, $\frac{k}{32}$ can never be an integer.

Alternatively, list multiples of 24 for k: 24, 48, 72, 96, 120, 144, 168, etc.

Then, eliminate multiples of 16 from this list: 24, ~~48~~, 72, ~~96~~, 120, ~~144~~, 168, etc.

A pattern emerges: k = (an odd integer) × 24:

(A) $\frac{k}{8}$ can be an integer, for example when $k = 24$.

(B) $\frac{k}{9}$ can be an integer, for example when $k = 72$.

(C) $\frac{k}{32}$ is correct by process of elimination.

(D) $\frac{k}{36}$ can be an integer, for example when $k = 72$.

(E) $\frac{k}{81}$ can be an integer, for example when $k = 81 \times 24$.

15. **10.** Because this is a numeric entry question, there can be only one correct answer. So, plugging in any prime number greater than 2 for b must yield the

same result. Try $b = 3$.

If $a = 16b$ and $b = 3$, then a is 48. The factors (*not* prime factors) of 48 are: 1 & 48, 2 & 24, 3 & 16, 4 & 12, and 6 & 8. There are 10 distinct factors.

16. **(C).** Since a positive multiple must be greater than or equal to the number it is a multiple of, answer choice (C) cannot be a multiple of a or b, as it is smaller than both integers a and b.

Alternatively, try testing numbers such that a is larger than b:

(A) If $a = 3$ and $b = 2$, $a - 1 = 2$, which is a multiple of b.

(B) If $a = 3$ and $b = 2$, $b + 1 = 3$, which is a multiple of a.

(C) Is the correct answer by process of elimination.

(D) If $a = 4$ and $b = 2$, $a + b = 6$, which is a multiple of b.

(E) If $a = 3$ and $b = 2$, $ab = 6$, which is a multiple of both a and b.

17. **12.** Remember, remainders are always whole numbers, so dividing 616 by 6 in the GRE calculator won't yield the answer. Rather, find the largest number less than 616 that 6 *does* go into (not 615, not 614, not 613…). That number is 612. Since 616 – 612 = 4, the remainder p is equal to 4.

Alternatively, divide 616 by 6 in your calculator to get 102.66…. Since 6 goes into 616 precisely 102 whole times, multiply 6 × 102 to get 612, then subtract from 616 to get the remainder 4.

This second method might be best for finding q. Divide 525 by 11 to get 47.7272…. Since 47 × 11 = 517, the remainder is 525 – 517 = 8.

Therefore, $p + q = 4 + 8 = 12$.

18. **"$x + y$ is divisible by 6" only.** To solve this problem with examples, make a short list of possibilities for each of x and y:

x = 18, 36, 54…

y = 12, 24, 36…

Now try to *disprove* the statements by trying several combinations of x and y above. In the first statement, $x + y$ could be 18 + 12 = 30, 54 + 12 = 66, 36 + 24 = 60, or many other combinations. All of those combinations are multiples of 6. This makes sense, as x and y individually are multiples of 6, so their sum is, too. The first statement is true.

To test the second statement, xy could be 18(12) = 216, which is *not* divisible by 48. Eliminate the second statement.

As for the third statement, $\frac{x}{y}$ could be $\frac{18}{12}$, which is not even an integer (and therefore not divisible by 6), so the third statement is not necessarily true.

19. **(D).** This problem is most easily solved with an example. If $p = 7$ and $q = 6$, then $pq = 42$, which has the factors 1 & 42, 2 & 21, 3 & 14, and 6 & 7. That's 8 factors, but read carefully! The question asks how many factors *greater than 1*, so the answer is 7. Note that choosing the smallest possible examples ($p = 7$ and $q = 6$) was the right move here, since the question asks "at least how many factors … ?" If testing $p = 70$ and $q = 36$, many, many more factors would have resulted. The question asks for the minimum.

20. **(C).** This problem is most easily solved with an example. If $r = 10$ and $s = 9$, then $rs = 90$. The positive factors of 90 are 1 & 90, 2 & 45, 3 & 30, 5 & 18,

6 & 15, and 9 & 10. Count to get a minimum of 12 positive factors.

21. **(D).** If t is divisible by 12, then t^2 must be divisible by 144 or $2 \times 2 \times 2 \times 2 \times 3 \times 3$. Therefore, t^2 can be divided evenly by 2 at least four times, so a must be at least 5 before $\frac{t^2}{2^a}$ might not be an integer.

Alternatively, test values. If $t = 12$, $\frac{t^2}{2^a} = \frac{144}{2^a}$. Plug in the choices as possible a values, starting with the smallest choice and working up:

(A) Since $\frac{144}{2^2} = 36$, eliminate.

(B) Since $\frac{144}{2^3} = 18$, eliminate.

(C) Since $\frac{144}{2^4}$ = 9, eliminate.

(D) $\frac{144}{2^5}$ = 4.5. The first choice for which $\frac{t^2}{2^a}$ might not be an integer is (D).

22. ***a* + *b* + *c*, *a* – *b* + *c*, and** $\boldsymbol{\frac{abc}{9}}$. Since *a*, *b*, and *c* are all multiples of 3, *a* = 3*x*, *b* = 3*y*, *c* = 3*z*, where *x* > *y* > *z* > 0 and all are integers. Substitute these new expressions into the statements.

First statement: *a* + *b* + *c* = 3*x* + 3*y* + 3*z* = 3(*x* + *y* + *z*). Since (*x* + *y* + *z*) is an integer, this number must be divisible by 3.

Second statement: *a* – *b* + *c* = 3*x* – 3*y* + 3*z* = 3(*x* – *y* + *z*). Since (*x* + *y* + *z*) is an integer, this number must be divisible by 3.

Third statement: $\frac{abc}{9} = \frac{3x3y3z}{9} = \frac{27xyz}{9}$ = 3*xyz*. Since *xyz* is an integer, this number must be divisible by 3.

23. **(C).** Pattern problems on the GRE often include a very large series of items that would be impossible (or at least unwise) to write out on paper. Instead, try to recognize and exploit the pattern. In this case, after every fourth car, the color pattern repeats. By dividing 463 by 4, you find that there will be 115 cycles through the 4 colors of cars—red, blue, black, gray—for a total of 460 cars to exit the factory. The key to solving these problems is the remainder. Because there are 463 – 460 = 3 cars remaining, the first such car will be red, the second will be blue, and the third will be black.

24. **(D).** This is a pattern problem. An efficient method is to recognize that the 7th day after the initial deposit would be Tuesday, as would the 14th day, the 21st day, etc. Divide 100 by 7 to get 14 full weeks comprising 98 days, plus 2 days left over. For the 2 leftover days, think about when they would fall. The first day after the deposit would be a Wednesday, as would the first day after waiting 98 days. The second day after the deposit would be a Thursday, and so would the 100th day.

25. **(D).** Division problems can be interpreted as follows: dividend = divisor × quotient + remainder. This problem is dividing *x* by 7, or distributing *x* items

equally to 7 groups. After the items are distributed among the 7 groups, there are 3 items left over, the remainder. This means that the value of *x* must be some number that is 3 larger than a multiple of 7, such as 3, 10, 17, 24, etc. The only answer choice that is 3 larger than a multiple of 7 is 52.

26. **(E).** This is a bit of a trick question—any number that yields remainder 4 when divided by 10 will also yield remainder 4 when divided by 5. This is because the remainder 4 is less than both divisors, and all multiples of 10 are also multiples of 5. For example, 14 yields remainder 4 when divided either by 10 or by 5. This also works for 24, 34, 44, 54, etc.

27. **0.** The remainder when dividing an integer by 10 always equals the units digit. You can also ignore all but the units digits, so the question can be rephrased as: *What is the units digit of* $3^{17} + 7^{13}$*?*

The pattern for the units digits of 3 is [3, 9, 7, 1]. Every fourth term is the same. The 17th power is 1 past the end of the repeat: 17 – 16 = 1. Thus, 3^{17} must end in 3.

The pattern for the units digits of 7 is [7, 9, 3, 1]. Every fourth term is the same. The 13th power is 1 past the end of the repeat: 13 – 12 = 1. Thus, 7^{13} must end in 7. The sum of these units digits is 3 + 7 = 10. Thus, the units digit is 0.

28. **(C).** Start by considering the relationship between n and n^3. Because n is an integer, for every prime factor n has, n^3 must have three of them. Thus, n^3 must have prime numbers in multiples of 3. If n^3 has one prime factor of 3, it must actually have two more, because n^3's prime factors can only come in triples.

The question says that n^3 is divisible by 24, so n^3's prime factors must include at least three 2's and a 3. But since n^3 is a cube, it must contain at least three 3's. Therefore, n must contain at least one 2 and one 3, or 2 × 3 = 6.

29. **(C).** First, expand 10! as 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1.

(Do *not* multiply all of those numbers together to get 3,628,800—it's true that 3,628,800 is the value of 10!, but analysis of the prime factors of 10! is easier in the current form.)

Note that 10! is divisible by 3^x5^y, and the question asks for the greatest possible values of x and y, which is equivalent to asking, "What is the maximum number of times you can divide 3 and 5, respectively, out of 10! while still getting an integer answer?"

In the product 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1, only the multiples of 3 have 3 in their prime factors, and only the multiples of 5 have 5 in their prime factors. Here are all the primes contained in 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 and therefore in 10!:

10	=	5 × 2
9	=	3 × 3
8	=	2 × 2 × 2
7	=	7
6	=	2 × 3
5	=	5
4	=	2 × 2
3	=	3

2 = 2

1 = no primes

There are four 3’s and two 5’s total. The maximum values are $x = 4$ and $y = 2$. Therefore, the two quantities are equal.

30. **(B).** Since only the number of *distinct* prime factors matter, not what they are or how many times they are present, it is possible to tell on sight that Quantity A has only two distinct prime factors, because 100,000 is a power of 10. (Any prime tree for 10, 100, or 1,000, etc. will contain only the prime factors 2 and 5, occurring in pairs.)

In Quantity B, 99,000 breaks down as 99 × 1,000. Since 1,000 also contains 2’s and 5’s, and 99 contains even more factors (specifically 3, 3, and 11), Quantity B is greater. It is not necessary to make prime factor trees for each number.

31. **18^3, 30^3, and 222 only.** Because $64 = 2^6$, multiples of 64 would have at least six 2’s among their prime factors.

Since 12 (which is $2 \times 2 \times 3$) has two 2's already, a number that could be multiplied by 12 to generate a multiple of 64 would need to have, at minimum, the *other* four 2's needed to generate a multiple of 64.

Since you want the choices that don't multiply with 12 to generate a multiple of 64, select only the choices that have *fewer than four 2's* within their prime factors.

6^6	$= (2 \times 3)^6$	six 2's	INCORRECT
12^2	$= (2^2 \times 3)^2$	four 2's	INCORRECT
18^3	$= (2 \times 3^2)^3$	three 2's	CORRECT
30^3	$= (2 \times 3 \times 5)^3$	three 2's	CORRECT
222	$= (2 \times 3 \times 37)$	one 2	CORRECT

32. **(D).** When a non-multiple of 3 is divided by 3, the quotient does not terminate (for instance, $\frac{1}{3} = 0.333\ldots$).

Since $\frac{3^x(5^2)}{3^5(5^3)}$ does *not* repeat forever, x must be large enough to cancel out the 3^5 in the denominator. Thus, x must be at least 5. Note that the question asks what *must* be true. Choice (D) must be true. Choice (E), $x = 5$, represents one value that would work, but this choice does not *have* to be true.

33. **(B).** Since a prime number has only two factors, 1 and itself, $(2^a)(3^b)(5^c)$ cannot be prime unless the digits a, b, and c are such that two of the digits are 0 and the third is 1. For instance, $(2^0)(3^1)(5^0) = (1)(3)(1) = 3$ is prime. Thus, the only three values of $\underline{abc}$ that would result in a prime number &($\underline{abc}$)& are 100, 010, and 001. However, only one of those three numbers (100) is a three-digit number.

Chapter 14

Exponents and Roots

In This Chapter…

Exponents and Roots

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

	Quantity A	Quantity B
1.	25^7	5^{15}

$$216 = 2^x 3^y$$

x and y are integers.

	Quantity A	Quantity B
2.	x	y

	Quantity A	Quantity B
3.	$\sqrt{9}\sqrt{25}$	$\sqrt{15}$

---

4. If $5{,}000 = 2^x 5^y$ and x and y are integers, what is the value of $x + y$?

80 is divisible by 2^x.

	Quantity A	Quantity B
5.	x	3

6. If $17\sqrt[3]{m} = 34$, what is the value of $6\sqrt[3]{m}$?

$\boxed{}$

7. $\dfrac{\frac{1}{1}}{\frac{1}{5^{-2}}}$ is equal to which of the following?

(A) $\dfrac{1}{25}$

(B) $\dfrac{1}{5}$

(C) 1

(D) 5

(E) 25

8. $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}}$ is equal to which of the following?

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) 4

(E) $4\sqrt{2}$

	Quantity A	Quantity B
9.	$10^6 + 10^5$	$10^7 + 10^4$

10. For which of the following positive integers is the square of that integer divided by the cube root of the same integer equal to nine times that integer?

(A) 4

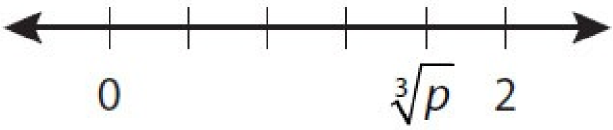
(B) 8

(C) 16

(D) 27

(E) 125

11.



If the hash marks above are equally spaced, what is the value of p?

(A) $\frac{3}{2}$

(B) $\frac{8}{5}$

(C) $\frac{24}{15}$

(D) $\frac{512}{125}$

(E) $\frac{625}{256}$

12. What is the greatest prime factor of $2^{99} - 2^{96}$?

13. If $2^k - 2^{k+1} + 2^{k-1} = 2^k m$, what is the value of m?

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) 2

14. If $5^{k+1} = 2{,}000$, what is the value of $5^k + 1$?

(A) 399

(B) 401

(C) 1,996

(D) 2,000

(E) 2,001

15. If $3^{11} = 9^x$, what is the value of x?

16. If $\sqrt[5]{x^6} = x^{\frac{a}{b}}$, what is the value of $\frac{a}{b}$?

Give your answer as a fraction.

17. Which of the following is equal to $\dfrac{10^{-8}25^{7}2^{16}}{20^{6}8^{-1}}$?

(A) $\frac{1}{5}$

(B) $\frac{1}{2}$

(C) 2

(D) 5

(E) 10

18. If $\dfrac{5^7}{5^{-4}} = 5^a$, $\dfrac{2^{-3}}{2^{-2}} = 2^b$, and $3^8(3) = 3^c$, what is the value of $a + b + c$?

19. If 12^x is odd and x is an integer, what is the value of x^{12}?

20. What is the value of $\dfrac{44^{\frac{5}{2}}}{\sqrt{11^3}}$?

$\boxed{}$

$$\frac{(10^3)(0.027)}{(900)(10^{-2})} = (3)(10^m)$$

	Quantity A	**Quantity B**
21.	m	3

---

22. Which of the following equals $\dfrac{2^2 + 2^2 + 2^3 + 2^4}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$?

(A) 2

(B) 4

(C) 8

(D) 16

(E) 32

23. If $\dfrac{0.000027 \times 10^x}{900 \times 10^{-4}} = 0.03 \times 10^{11}$, what is the value of x?

(A) 13

(B) 14

(C) 15

(D) 16

(E) 17

24. Which of the following equals $\left(\sqrt[2]{x}\right)\left(\sqrt[3]{x}\right)$?

(A) $\sqrt[5]{x}$

(B) $\sqrt[6]{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[5]{x^6}$

(E) $\sqrt[6]{x^5}$

$n = 0.00025 \times 10^4$ and $m = 0.005 \times 10^2$

	Quantity A	Quantity B
25.	$\frac{n}{m}$	0.5

26. If $2^2 < \frac{x}{2^6 - 2^4} < 2^3$, which of the following could be the value of x?

Indicate all such values.

- ☐ 24
- ☐ 64
- ☐ 80
- ☐ 128
- ☐ 232
- ☐ 256

27. Which of the following is equal to $x^{\frac{3}{2}}$?

(A) $x^2\sqrt{x}$

(B) $x\sqrt{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[3]{x}$

(E) $(x^3)^2$

28. If $125^{14}48^8$ were expressed as an integer, how many consecutive zeros would that integer have immediately to the left of its decimal point?

(A) 22

(B) 32

(C) 42

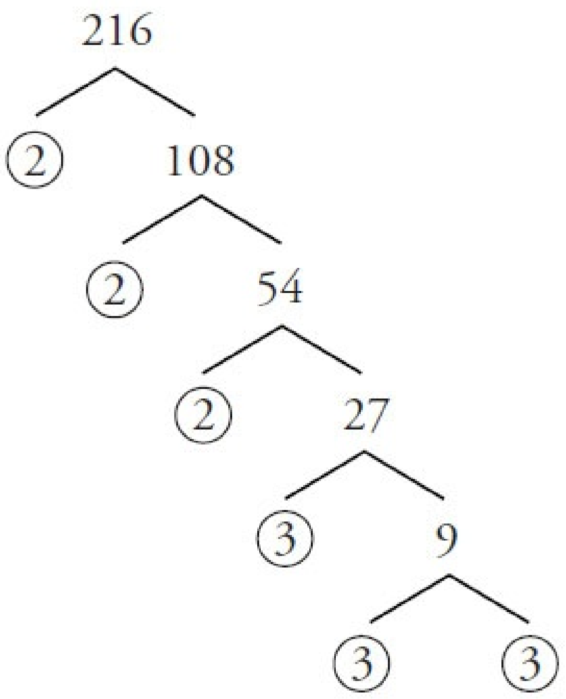
(D) 50

(E) 112

Exponents and Roots Answers

1. **(B).** If a problem combines exponents with different bases, convert to the same base if possible. Since $25 = 5^2$, Quantity A is equal to $(5^2)^7$. Apply the appropriate exponent formula: $(a^b)^c = a^{bc}$. Quantity A is equal to 5^{14}, thus Quantity B is greater.

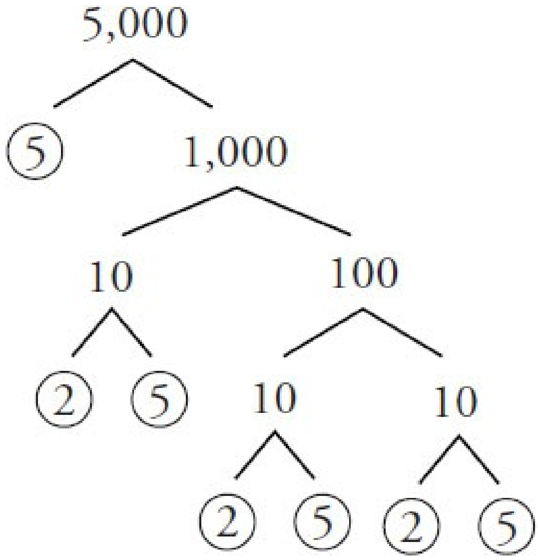
2. **(C).** Construct a prime factor tree for 216:



$216 = 2^3 3^3$, so $x = 3$ and $y = 3$.

3. **(A).** In Quantity A, $\sqrt{9}\sqrt{25} = 3 \times 5 = 15$. Since 15 is greater than $\sqrt{15}$, Quantity A is greater.

4. **7.** Construct a prime factor tree for 5,000:



Thus, 5,000 = $2^3 5^4$, therefore $x = 3$ and $y = 4$, and the answer is 3 + 4 = 7.

5. **(D).** Construct a prime factor tree for 80; it has four factors of 2 and one factor of 5.

That doesn't mean x is 4, however! The problem does not say "80 is equal to 2^x". Rather, it says "divisible by."

80 is divisible by 2^4, and therefore also by 2^3, 2^2, 2^1, and 2^0 (any non-zero number to the 0th power equals 1). Thus, x could be 0, 1, 2, 3, or 4, and could therefore be less than, equal to, or greater than 3. Thus, the relationship cannot be determined.

6. **12.** This question looks much more complicated than it really is—note that $\sqrt[3]{m}$ is in both the given equation and the question. Just think of $\sqrt[3]{m}$ as a very fancy variable that you don't have to break down:

$$17\sqrt[3]{m} = 34$$

$$\sqrt[3]{m} = \frac{34}{17}$$

$$\sqrt[3]{m} = 2$$

Therefore, $6\sqrt[3]{m} = 6(2) = 12$.

7. **(A).** This question requires recognizing that a negative exponent in the denominator turns into a positive exponent in the numerator. In other words, the lowermost portion of the fraction, $\frac{1}{5^{-2}}$, is equal to 5^2. The uppermost portion of the fraction, $\frac{1}{1}$, is just equal to 1.

Putting these together, the original fraction can be simplified.

$\dfrac{\frac{1}{1}}{\frac{1}{5^{-2}}} = \dfrac{1}{5^2} = \dfrac{1}{25}$, which is the final answer.

8. **(B).** To solve, start at the “inner core”—that is, the physically smallest root sign:

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$$
$$\sqrt{2+\sqrt{2+\sqrt{2+2}}} =$$
$$\sqrt{2+\sqrt{2+2}} =$$
$$\sqrt{2+2} = 2$$

9. **(B).** Be careful! These quantities are not equal! When *multiplying* exponents with the same base, it is correct to add the exponents:

$10^6 \times 10^5 = 10^{11}$

However, numbers raised to powers cannot be directly combined by addition or subtraction. Instead, sum this way:

Quantity A = $10^6 + 10^5 = 1{,}000{,}000 + 100{,}000 = 1{,}100{,}000$

Quantity B = $10^7 + 10^4 = 10{,}000{,}000 + 10{,}000 = 10{,}010{,}000$

Thus, Quantity B is greater.

Alternatively, you can do some fancy factoring. The distributive property is a big help here: $ab + ac = a(b + c)$. In other words, factor out the a.

Factor out 10^5 in Quantity A:

$10^6 + 10^5 = 10^5(10^1 + 1) = 10^5(11) \cong 10^6$

Factor out 10^4 in Quantity B:

$10^7 + 10^4 = 10^4(10^3 + 1) = 10^4(1{,}001) \cong 10^7$

The approximation in the last step is just to make the point that you don’t have to be too precise: Quantity B is about 10 times greater than Quantity A.

10. **(D).** To solve this question, translate the text into an equation. Call “the square of that integer” x^2, “the cube root of the same integer” $\sqrt[3]{x}$, and “nine times that integer” $9x$:

$$\frac{x^2}{\sqrt[3]{x}} = 9x$$

Test the answers; doing so shows that choice (D) is correct:

$$\frac{27^2}{\sqrt[3]{27}} = 9(27)$$

$$\frac{27^2}{3} = 9(27)$$

$$27^2 = 9(27)(3)$$

$$27 = 9(3)$$

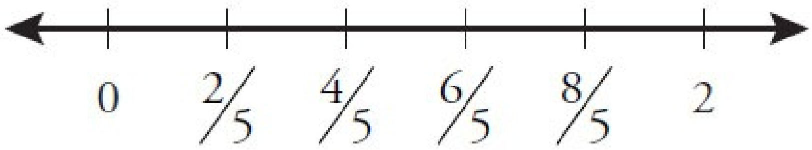
Choices (A) and (C) are not likely to be correct because the cube roots of 4 and 16, respectively, are not integers; test the others first:

Choice (B): $\frac{64}{2} = 9(8)$? No.

Choice (D): Correct as shown above.

Choice (E): $\frac{15,626}{5} = 9(125)$? No.

11. **(D).** To determine the distance between hash marks, divide 2 (the distance from 0 to 2) by 5 (the number of segments the number line has been divided into). The result is $\frac{2}{5}$. Therefore:



Note that 2 is equal to $\frac{10}{5}$, so the number line is labeled correctly.

Since $\sqrt[3]{p}$ marks the same hash mark on the number line as $\frac{8}{5}$:

$$\sqrt[3]{p} = \frac{8}{5}$$

$$p = \left(\frac{8}{5}\right)^3$$

$$p = \frac{512}{125}$$

The answer is (D). Watch out for trap answer choice (B), which represents $\sqrt[3]{p}$, not *p*.

12. **7**. You cannot subtract $2^{99} – 2^{96}$ to get 2^3! You cannot directly combine numbers raised to powers when adding or subtracting. (As it turns out, the difference between 2^{99} and 2^{96} is much, much greater than 2^3.) Instead, factor out the greatest common factor of 2^{99} and 2^{96} :

$$2^{99} – 2^{96} = 2^{96} (2^3 – 1) = 2^{96} (7)$$

Since $2^{99} – 2^{96}$ is equal to $2^{96}\ 7^1$, its greatest prime factor is 7.

13. **(B).** First, factor 2^{k+1} into $2^k 2^1$ and 2^{k-1} into $2^k 2^{-1}$:

$$2^k - 2^k 2^1 + 2^k 2^{-1} = 2^k m$$

Factor out 2^k from the left, then cancel 2^k from both sides:

$$\begin{aligned} 2^k(1 - 2^1 + 2^{-1}) &= 2^k m \\ 1 - 2^1 + 2^{-1} &= m \\ 1 - 2 + \frac{1}{2} &= m \\ -\frac{1}{2} &= m \end{aligned}$$

14. **(B).** The key to solving this problem is to understand that 5^{k+1} can be factored into $5^k 5^1$. (Exponents are added when multiplying numbers with the same base, so the process can also be reversed; thus, any expression with the form x^{a+b} can be split into $x^a x^b$.) Thus:

$$5^{k+1} = 2{,}000$$

$$5^k 5^1 = 2{,}000$$

Now divide both sides by 5:

$$5^k = 400$$

So, $5^k + 1 = 401$.

Notice that you can't solve for k itself—k is not an integer, since 400 is not a "normal" power of 5. But you don't need to solve for k. You just need 5^k.

15. **5.5.** Begin by converting 9 to a power of 3:

$$\begin{aligned} 3^{11} &= (3^2)^x \\ 3^{11} &= 3^{2x} \end{aligned}$$

Thus, $11 = 2x$ and $x = 5.5$.

16. $\frac{6}{5}$. The square root of a number equals that number to the $\frac{1}{2}$ power, so

too is a fifth root the same as a $\frac{1}{5}$ exponent. Thus:

$$\sqrt[5]{x^6} = (x^6)^{\frac{1}{5}} = x^{\frac{6}{5}}$$

Since $x^{\frac{6}{5}} = x^{\frac{a}{b}}$, $\frac{a}{b} = \frac{6}{5}$.

17. **(B).** Since $10^{-8} = \frac{1}{10^8}$ and $\frac{1}{8^{-1}} = 8^1$, first substitute to convert any term with negative exponents to one with a positive exponent:

$$\frac{10^{-8}25^7 2^{16}}{20^6 8^{-1}} = \frac{25^7 2^{16} 8^1}{10^8 20^6}$$

Then, convert the non-prime terms to primes, combining and canceling where possible:

$$\frac{25^7 2^{16} 8^1}{10^8 20^6} = \frac{\left(5^2\right)^7 2^{16}\left(2^3\right)^1}{\left(2^1 5^1\right)^8\left(2^2 5^1\right)^6} = \frac{5^{14}2^{16}2^3}{2^8 5^8 2^{12} 5^6} = \frac{5^{14}2^{19}}{2^{20}5^{14}} = \frac{1}{2}$$

18. **19.** To solve this problem, you need to know that to divide numbers with the same base, subtract the exponents, and to multiply them, add the exponents. Thus:

$\frac{5^7}{5^{-4}} = 5^{7-(-4)} = 5^{11}$, so $a = 11$.

$\frac{2^{-3}}{2^{-2}} = 2^{-3-(-2)} = 2^{-1}$, so $b = -1$.

$3^8(3) = 3^8(3^1) = 3^9$, so $c = 9$.

Therefore, $a + b + c = 11 + (-1) + 9 = 19$.

19. **0.** This is a bit of a trick question. 12^x is odd? How strange! 12^1 is 12, 12^2 is 144, 12^3 is 1,728 … every "normal" power of 12 is even. (An even number such as 12 multiplied by itself any number of times will yield an even answer.) These normal powers are 12 raised to a positive integer. What about negative integer exponents? They are all fractions of this form:

$$\frac{1}{12^{\text{positive integer}}}.$$

The only way for 12^x to be odd is for x to equal 0. Any non-zero number to the 0th power is equal to 1. Since $x = 0$ and the question asks for x^{12}, the answer is 0.

20. **352.** A square root is the same as a $\frac{1}{2}$ exponent, so

$$\frac{44^{\frac{5}{2}}}{\sqrt{11^3}} = \frac{44^{\frac{5}{2}}}{\left(11^3\right)^{\frac{1}{2}}} = \frac{44^{\frac{5}{2}}}{11^{\frac{3}{2}}}.$$

The common factor of 44 and 11 is 11, so factor the numerator:

$$\frac{44^{\frac{5}{2}}}{11^{\frac{3}{2}}} = \frac{11^{\frac{5}{2}}4^{\frac{5}{2}}}{11^{\frac{3}{2}}}$$

When dividing exponential expressions that have a common base, subtract the exponents:

$$\left(\frac{11^{\frac{5}{2}}}{11^{\frac{3}{2}}}\right)4^{\frac{5}{2}} = \left(11^{\frac{5}{2}-\frac{3}{2}}\right)4^{\frac{5}{2}} = \left(11^{\frac{2}{2}}\right)4^{\frac{5}{2}} = \left(11^{1}\right)4^{\frac{5}{2}}$$

Now simplify the 4 term, again noting that a $\frac{1}{2}$ exponent is the same as a square root:

$$\left(11^{1}\right)4^{\frac{5}{2}} = (11)\left(4^{\frac{1}{2}}\right)^{5} = (11)\left(\sqrt{4}\right)^{5} = (11)\left(2^{5}\right) = 352$$

21. **(B).** Since $(10^3)(0.027)$ is 27 and $(900)(10^{-2})$ is 9:

$$\frac{27}{9} = (3)(10^m)$$

$$3 = 3(10^m)$$

$$1 = 10^m$$

You might be a little confused at this point as to how 10^m can equal 1. However, you can still answer the question correctly. If m were 3, as in Quantity B, 10^m would equal 1,000. However, 10^m actually equals 1. So m must be less than 3.

As it turns out, the only way 10^m can equal 1 is if $m = 0$. Any non-zero number to the 0th power is equal to 1.

22. **(D).** You could factor 2^2 out of the numerator, but the numbers are small enough that you might as well just say that the numerator is 4 + 4 + 8 + 16 = 32.

FOIL the denominator:

$$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$$

$$\sqrt{25}+\sqrt{3}\sqrt{5}-\sqrt{3}\sqrt{5}-\sqrt{9}$$

$$\sqrt{25}-\sqrt{9}$$

5 – 3 = 2

$\frac{32}{2}$ = 16 is the final answer.

23. **(A).** One good approach is to convert 0.000027, 900, and 0.03 to powers of 10:

$$\frac{27\times10^{-6}\times10^{x}}{9\times10^{2}\times10^{-4}}=3\times10^{-2}\times10^{11}$$

Now combine the exponents from the terms with base 10:

$$\frac{27 \times 10^{-6+x}}{9 \times 10^{-2}} = 3 \times 10^9$$

Since $\frac{27}{9}$ = 3, cancel the 3 from both sides, then combine powers of 10:

$$\frac{10^{-6+x}}{10^{-2}} = 10^9$$

$$10^{-6+x-(-2)} = 10^9$$

$$10^{-4+x} = 10^9$$

Thus, $-4 + x = 9$, and $x = 13$.

24. **(E).** A good first step is to convert to fractional exponents. A square root is the same as the $\frac{1}{2}$ power and a cube root is the same as the $\frac{1}{3}$ power:

$$x^{\frac{1}{2}}x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\left(\frac{3}{6}+\frac{2}{6}\right)} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

25. **(A).** To simplify 0.00025×10^4, move the decimal in 0.00025 four places to the right to get 2.5. To simplify 0.005×10^2, move the decimal in 0.005 two places to the right to get 0.5. Thus, $n = 2.5$, $m = 0.5$, and $\frac{n}{m} = \frac{2.5}{0.5}$ = 5.

26. **232 and 256 only.** The inequality could be simplified using exponent rules, but all the numbers are small enough either to have memorized or to quickly calculate:

$$2^2 < \frac{x}{2^6 - 2^4} < 2^3$$

$$4 < \frac{x}{64 - 16} < 8$$

$$4 < \frac{x}{48} < 8$$

To isolate *x*, multiply all three parts of the inequality by 48:

$$192 < x < 384$$

The only choices in this range are 232 and 256.

27. **(B).** Since a number to the $\frac{1}{2}$ power equals the square root of that number, $x^{\frac{3}{2}}$ could also be written as $\sqrt{x^3}$. This, however, does not appear in the choices. Note, however, that $\sqrt{x^3}$ can be simplified:

$$\sqrt{x^2 \times x}$$

$$\sqrt{x^2} \times \sqrt{x}$$

$$x\sqrt{x}$$

This matches choice (B). Alternatively, convert the answer choices. For instance, in incorrect choice (A), $x^2\sqrt{x} = x^2x^{\frac{1}{2}} = x^{\frac{5}{2}}$. Since this is not equal to $x^{\frac{3}{2}}$, eliminate (A). Correct choice (B) can be converted as such: $x\sqrt{x} = x^1x^{\frac{1}{2}} = x^{\frac{3}{2}}$.

28. **(B).** Exponents questions usually involve prime factorization, because you always want to find common bases, and the fundamental common bases are prime numbers. Test some values to see what leads to zeros at the end of an integer.

$10 = 5 \times 2$
$40 = 8 \times 5 \times 2$
$100 = 10 \times 10 = 2 \times 5 \times 2 \times 5$
$1{,}000 = 10 \times 10 \times 10 = 2 \times 5 \times 2 \times 5 \times 2 \times 5$

Ending zeros are created by 10's, each of which is the product of one 2 and one 5. So, to answer this question, determine how many pairs of 2's and 5's are in the expression:

$$125^{14}48^8 = (5^3)^{14} \times (2^4 \times 3)^8 = 5^{42} \times 2^{32} \times 3^8$$

Even though there are 42 powers of 5, there are only 32 powers of 2, so you can only form 32 pairs of one 5 and one 2.

Chapter 15

Number Properties

In This Chapter…

Number Properties

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

On a number line, the distance from A to B is 4 and the distance from B to C is 5.

	Quantity A	Quantity B
1.	The distance from A to C	9

---

a, b, c, and d are consecutive integers such that $a < b < c < d$.

	Quantity A	Quantity B
2.	The average (arithmetic mean) of a, b, c, and d	The average (arithmetic mean) of b and c

---

3. w, x, y, and z are consecutive odd integers such that $w < x < y < z$. Which of

the following statements must be true?

Indicate <u>all</u> such statements.

- ☐ *wxyz* is odd
- ☐ $w + x + y + z$ is odd
- ☐ $w + z = x + y$

	Quantity A	**Quantity B**
4.	The sum of all the odd integers from 1 to 100, inclusive	The sum of all the even integers from 1 to 100, inclusive

---

$$x > 0 > y$$

5.

Quantity A	Quantity B
$x - y$	$(x + y)^2$

$$a < b < c < d < 0$$

6.

Quantity A	Quantity B
$a - d$	bc

---

7. If set S consists of all positive integers that are multiples of both 2 and 7, how many numbers in set S are between 140 and 240, inclusive?

$\boxed{}$

$$ab > 0$$
$$bc < 0$$

8.

Quantity A	Quantity B
ac	0

$$abc < 0$$
$$b^2c > 0$$

9.

Quantity A	Quantity B
ab	0

---

a, b, and c are integers such that $a < b < c$.

10.

Quantity A	Quantity B
$\dfrac{a+b+c}{3}$	b

---

11. If $0 < a < \frac{1}{b} < 1$, then which of the following must be true?

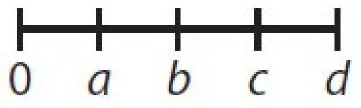
(A) $a^2 > a > b > b^2$

(B) $b > a > a^2 > b^2$

(C) $b^2 > a > a^2 > b$

(D) $b^2 > a^2 > b > a$

(E) $b^2 > b > a > a^2$



	Quantity A	**Quantity B**
12.	$a \times c$	$b \times d$

	Quantity A	**Quantity B**
13.	The number of distinct positive factors of 32	The number of distinct positive factors of 20

---

14. If $y^2 = 4$ and $x^2y = 18$, which of the following values could equal $x + y$? Indicate <u>two</u> such values.

- ☐ –5
- ☐ –1
- ☐ 1
- ☐ 5
- ☐ 6

	Quantity A	**Quantity B**
15.	The remainder when 10^{11} is divided by 2	The remainder when 3^{13} is divided by 3

---

q is odd.

	Quantity A	Quantity B
16.	$(-1)^q$	$(-1)^{q+1}$

n is a positive integer.

	Quantity A	Quantity B
17.	$(-1)^{4n} \times (-1)^{202}$	$(3)^3 \times (-5)^5$

18. If x is a positive integer, which one of the following could be the remainder when 73^x is divided by 10?

Indicate all such remainders.

- ☐ 0
- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ 5
- ☐ 6
- ☐ 7
- ☐ 8
- ☐ 9

19. If x, y, and z are integers, $y + z = 13$, and $xz = 9$, which of the following must be true?

(A) x is even

(B) $x = 3$

(C) y is odd

(D) $y > 3$

(E) $z < x$

	Quantity A	**Quantity B**
20.	The least prime number greater than 13	The greatest prime number less than 16

21. The average (arithmetic mean) of 11 integers is 35. What is the sum of all the integers?

☐

22. What is the sum of all the integers from 1 to 80, inclusive?

(A) 3,200

(B) 3,210

(C) 3,230

(D) 3,240

(E) 3,450

23. If p is the sum of all the integers from 1 to 150, inclusive, and q is the sum of all the integers from 1 to 148, inclusive, what is the value of $p - q$?

$\boxed{}$

24. If m is the product of all the integers from 2 to 11, inclusive, and n is the product of all the integers from 4 to 11, inclusive, what is the value of $\frac{n}{m}$?

Give your answer as a fraction.

$\dfrac{\boxed{}}{\boxed{}}$

---

a, b, and c are positive even integers such that $8 > a > b > c$.

	Quantity A	Quantity B
25.	The range of a, b, and c	The average (arithmetic mean) of a, b, and c

---

26. If x is a non-zero integer and $0 < y < 1$, which of the following must be greater than 1?

(A) x

(B) $\frac{x}{y}$

(C) xy^2

(D) x^2y

(E) $\frac{x^2}{y}$

a, b, and c are consecutive integers such that $a < b < c < 4$.

	Quantity A	**Quantity B**
27.	The range of a, b, and c	The average of a, b, and c

$\sqrt{xy}$ is a prime number, xy is even, and $x > 4y > 0$.

	Quantity A	**Quantity B**
28.	y	1

x is even, $\sqrt{x}$ is a prime number, and $x + y = 11$.

	Quantity A	**Quantity B**
29.	x	y

The product of positive integers f, g, and h is even and the product of integers f and g is odd.

	Quantity A	**Quantity B**
30.	The remainder when f is divided by 2	The remainder when h is divided by 2

$x^2 > 25$ and $x + y < 0$

	Quantity A	**Quantity B**
31.	x	y

p and w are single-digit prime numbers such that $p + w < 6$. p^2 is odd.

	Quantity A	Quantity B
32.	w	3

$x^2 > y^2$ and $x > -|y|$

	Quantity A	Quantity B
33.	x	y

The sum of four consecutive integers is –2.

	Quantity A	Quantity B
34.	The smallest of the four integers	–2

---

35. If g is an integer and x is a prime number, which of the following must be an integer?

Indicate <u>all</u> such expressions.

- ☐ $\dfrac{g^2x + 5gx}{x}$
- ☐ $g^2 - x^2\left(\dfrac{1}{3}\right)$
- ☐ $6\left(\dfrac{g}{2}\right) - 100\left(\dfrac{g}{2}\right)^2$

Number Properties Answers

1. **(D).** Whenever a question looks this straightforward ($4 + 5 = 9$, so the quantities initially appear equal), be suspicious. Draw the number line described. If the points *A*, *B*, and *C* are in alphabetical order from left to right, then the distance from *A* to *C* will be 9. However, alphabetical order is not required. If the points are in the order *C*, *A*, and *B* from left to right, then the distance from *A* to *C* is $5 – 4 = 1$. Therefore, the relationship cannot be determined.

2. **(C).** When integers are consecutive (or just evenly spaced), the average equals the median. Since the median of this list is the average of the two middle numbers, Quantity A and Quantity B both equal the average of *b* and *c*. Alternatively, try this with real numbers. If the set is 2, 3, 4, 5, both quantities equal 3.5. No matter what consecutive integers are tested, the two quantities are equal.

3. ***wxyz* is odd and $w + z = x + y$ only.** This question tests the properties of odd numbers as well as of consecutives.

The first choice is TRUE, as multiplying only odd integers together (and no evens) always yields an odd answer.

However, when adding, the rule is “an odd number of odds makes an odd.” Summing an even number of odds produces an even, so the second choice is FALSE.

The third choice is TRUE. Since *w*, *x*, *y*, and *z* are consecutive odd integers, all can be defined in terms of *w*:

$w = w$
$x = w + 2$
$y = w + 4$
$z = w + 6$

Thus, $w + z = w + (w + 6) = 2w + 6$, and $x + y = (w + 2) + (w + 4) = 2w + 6$.

Therefore, $w + z = x + y$. Alternatively, try real numbers, such as 1, 3, 5, and 7. It is true that $1 + 7 = 3 + 5$. This would hold true for any set of four consecutive, ordered odd numbers tested.

4. **(B).** No math is required to solve this problem. Note that the numbers from 1 to 100 include 50 even integers and 50 odd integers. The first few odds are 1, 3, 5, etc. The first few evens are 2, 4, 6, etc. Every even is 1 greater than its counterpart (2 is 1 greater than 1, 4 is 1 greater than 3, 6 is 1 greater than 5, etc.). Not only is Quantity B greater, it's greater by precisely 50.

5. **(D).** From the constraint, x is positive and y is negative. So Quantity A is definitely positive: $x - y$ = positive – negative = positive. Quantity B is the square of a number, which cannot be negative. Quantity B could be zero, if, for example, $x = 2$ and $y = -2$: $(x + y)^2 = (2 + -2)^2 = (0)^2$. In this case, Quantity A is greater. But if $x = 100$ and $y = -1$, Quantity A is $100 - (-1) = 101$ and Quantity B is $(100 + -1)^2 = 99^2$, which is much greater (close to 10,000). The relationship cannot be determined from the information given.

6. **(B).** This problem can be approached either conceptually or by picking values. For the former, anytime a greater number is subtracted from a smaller one, the result will be negative. Thus, $a - d < 0$. Conversely, since the product of two negatives is positive, $bc > 0$. Because any positive value is greater than all negative values, Quantity B must be greater. Alternatively, picking simple values for the variables would also lead to the same result.

7. **8.** A positive integer that is a multiple of both 2 and 7 is a multiple of 14. Since 140 is a multiple of 14, start listing there and count the terms in the range: 140, 154, 168, 182, 196, 210, 224, 238.

Alternatively, note that 140 is the 10th multiple of 14, and $240/14 \approx 17.143$ (use the calculator). Therefore, the 10th through the 17th multiples of 14, inclusive, are in this range. The number of terms is $17 - 10 + 1 = 8$ ("add one before you are done" for an inclusive list).

8. **(B).** If $ab > 0$, then a and b have the same sign. If $bc < 0$, then b and c have opposite signs. Therefore, a and c must have opposite signs. Therefore, ac is negative, so Quantity B is greater.

If you find the logic difficult (a and b are same sign, b and c are opposite signs, therefore a and c are opposite signs), you could make a quick chart of the possibilities using plus and minus signs:

a	*b*	*c*	
+	+	–	⟵ First possibility, *a* and *c* have different signs.
–	–	+	⟵ Second possibility, *a* and *c* have different signs.

9. **(B).** If abc is negative, then either exactly 1 of or all 3 of the values a, b, and c are negative:

–	–	–	⟵ First possibility, all are negative.
–	+	+	⟵ Second possibility, 1 negative and 2 positives (order can vary).

If b^2c is positive, then c must be positive, since b^2 cannot be negative. If c is positive, eliminate the first possibility since all three variables cannot be negative. Thus, only one of a, b, and c are negative, but the one negative cannot be c. Either a or b is negative, and the other is positive. It doesn't matter which one of a or b is negative—that's enough to know that ab is negative and Quantity B is greater.

10. **(D).** Note that $\frac{a+b+c}{3}$ is just another way to express "the average of a, b, and c." The average of a, b, and c would equal b if the numbers were evenly spaced (such as 1, 2, 3 or 5, 7, 9), but that is not specified. For instance, the integers could be 1, 2, 57 and still satisfy the $a < b < c$ constraint. In that case, the average is 20, which is greater than $b = 2$. The relationship cannot be determined from the information given.

11. **(E).** The goal in this question is to order a, a^2, b, and b^2 by magnitude. Based on the original inequality $0 < a < \frac{1}{b} < 1$, several things are true. First, a and $\frac{1}{b}$ are positive, and thus b itself is positive. If $\frac{1}{b} < 1$ and b is positive, multiply both sides of the inequality by b to get $1 < b$, and then again to get $b < b^2$. (Multiplying by a positive value both times meant there was no need to flip the inequality sign.) Two of the expressions in the answer choices have been ordered: $b < b^2$. Eliminate choices that contradict this fact: choices (A) and (B) are wrong.

Second, note that $a < 1$ in the given inequality. Since a is a positive number less than 1, $a^2 < a$. Show this either by multiplying both sides of $a < 1$ by a (again, no need to flip the inequality sign when multiplying by a positive value) or by number properties (squaring positive fractions less than 1 always yields a smaller fraction). Eliminate choices that contradict the fact that $a^2 < a$: choices (A) and (D) are wrong.

Now, what is the relationship between the terms with a and the terms with b? From the first paragraph above, $1 < b$. From the second paragraph (and the given inequality), $a < 1$. Put these together: $a < 1 < b$, or just $a < b$. Eliminate the choices that contradict this fact: choices (A) and (C) are wrong.

At this point, choice (A) has been eliminated for three reasons, and (B), (C), and (D) for one reason each. The only choice remaining is (E), so it must be right by elimination.

(E) can be proven right by putting together the three separate inequalities ($b < b^2$) and ($a^2 < a$) and ($a < b$) into a single inequality: $a^2 < a < b < b^2$. This is equivalent to choice (E): $b^2 > b > a > a^2$.

12. **(B).** The exact values of a, b, c, and d are unknown, as is whether they are evenly spaced (do not assume that they are, just because the figure looks that way). However, it is known that all of the variables are positive such that 0 <

$a < b < c < d$.

Because $a < b$ and $c < d$ and all the variables are positive, $a \times c < b \times d$. In words, the product of the two smaller numbers is less than the product of the two greater numbers. Quantity B is greater.

You could also try this with real numbers. You could try $a = 1$, $b = 2$, $c = 3$, and $d = 4$, or you could mix up the spacing, as in $a = 0.5$, $b = 7$, $c = 11$, $d = 45$. For any scenario that matches the conditions of the problem, Quantity B is greater.

13. **(C).** This question asks for the greater number of distinct positive factors, not prime factors. The approach to determine the number of distinct positive factors is to create a chart that systematically lists all the combinations of two positive integers that equal the number in question. When you find the same pair in reverse order, the chart is done.

Quantity A (32)	Quantity B (20)
1 × 32	1 × 20
2 × 16	2 × 10
4 × 8	4 × 5

Each has six distinct positive factors and the two quantities are equal.

14. –**1 and 5 only.** From the first equation, it seems that y could equal either 2 or –2, but if $x^2y = 18$, then y must equal only 2 (otherwise, x^2y would be negative). Still, the squared x indicates that x can equal 3 or –3. So the possibilities for $x + y$ are:

$3 + 2 = 5$
$(-3) + 2 = -1$

15. **(C).** It is not necessary to calculate 10^{11} or 3^{13}. Because 10 is an even number, so is 10^{11}, and 0 is the remainder when any even is divided by 2. Similarly, 3^{13} is a multiple of 3 (it has 3 among its prime factors), and 0 is the remainder when any multiple of 3 is divided by 3. Therefore, the quantities are equal.

16. **(B).** The negative base –1 to any odd power is –1, and the negative base –1 to any even power is 1. Since q is odd, Quantity A = –1 and Quantity B = 1.

17. **(A).** Before doing any calculations on a problem with negative bases raised to integer exponents, check to see whether one quantity is positive and one quantity is negative, in which case no further calculation is necessary. Note that a negative base to an even exponent is positive, while a negative base to an odd exponent is negative.

Since n is an integer, $4n$ is even. Thus, in Quantity A, $(-1)^{4n}$ and $(-1)^{202}$ are both positive, so Quantity A is positive. In Quantity B, $(3)^3$ is positive but $(-5)^5$ is negative, and thus Quantity B is negative. Since a positive is by definition greater than a negative, Quantity A is greater.

18. **1, 3, 7, and 9 only.** As with multiplication, when an integer is raised to a power, the units digit is determined solely by the product of the units digits. Those products will form a repeating pattern. Here, $3^1 = \underline{3}$, $3^2 = \underline{9}$, $3^3 = 2\underline{7}$, $3^4 = 8\underline{1}$, and $3^5 = 24\underline{3}$. Here the pattern returns to its original value of 3 and any larger power of 3 will follow this same pattern: 3, 9, 7, and then 1. Thus, the units digit of 73^x must be 1, 3, 7, or 9. When dividing by 10, the remainder is the units digits, so those same values are the complete list of possible remainders.

19. **(D).** If $xz = 9$ and x and z must both be integers, then they are 1 and 9 (or –1 and –9) or 3 and 3 (or –3 and –3). Therefore, they are both odd. More generally, the product of two integers will only be odd if the component integers themselves are both odd. Because z is odd, and $y + z$ equals 13 (an odd), y must be even.

(A): x is NOT even. Eliminate.

(B): x could be 3 but doesn't have to be. Eliminate.

(C): y is NOT odd. Eliminate.

(E): z does not have to be less than x (for instance, they could both be 3). Eliminate.

At this point, only (D) remains, so it must be the answer. To prove it, consider the constraint that limits the value of y: $y + z = 13$. Since z could be –1, 1, –3, 3, –9, or 9, the maximum possible value for z is 9, so y must be at least 4. All values that are at least 4 are also greater than 3, so (D) must be true.

20. **(A).** This question draws upon knowledge of the smaller prime numbers. It might be helpful to list out the first few prime numbers on your paper: 2, 3, 5, 7, 11, 13, 17, 19…

The smallest prime number greater than 13 is 17, and the greatest prime number less than 16 is 13. Therefore, Quantity A is greater.

21. **385.** To find the sum of a set of numbers, given the average and number of terms, use the average formula. Average = $\frac{\text{Sum}}{\text{Number of Terms}}$, so Sum = Average × Number of Terms = 35 × 11 = 385.

22. **(D).** To find the sum of a set of evenly spaced numbers, multiply the median (which is also the average) by the number of terms in the set. The median of the numbers from 1 to 80 inclusive is 40.5 (the first 40 numbers are 1 through 40, and the second 40 numbers are 41 through 80, so the middle is 40.5). You can also use the formula $\frac{\text{First} + \text{Last}}{2}$ to calculate the median of an evenly spaced set: $\frac{1+80}{2}$ = 40.5. Multiply 40.5 times 80 to get the answer: 3,240.

23. **299.** *p* is a large number, but it consists entirely of *q* + 149 + 150. Thus, *p* – *q* is what's left of *p* once the common terms are subtracted: 149 + 150 = 299.

24. $\boldsymbol{\frac{1}{6}}$**.** There is a trick to this problem—all of the integers in the product *n* will be canceled out by the same integers appearing in the product *m*:

$$\frac{n}{m}=\frac{\cancel{4\times5\times6\times7\times8\times9\times10\times11}}{2\times3\times\cancel{4\times5\times6\times7\times8\times9\times10\times11}}=\frac{1}{2\times3}=\frac{1}{6}$$

25. **(C).** Integers *a*, *b*, and *c* must be 6, 4, and 2, respectively, as they are positive even integers less than 8 and ordered according to the given inequality. The range of *a*, *b*, and *c* is 6 – 2 = 4. The average of *a*, *b*, and *c* is $\frac{6+4+2}{3}$ = $\frac{12}{3}$ = 4. The two quantities are equal.

26. **(E).** Find the choices that do not have to be greater than 1. It is possible

that x could be negative, which eliminates (A), (B), and (C). For choice (D), if $x^2 = 1$, that times the positive fraction y would be less than 1. In choice (E), x^2 must be positive and at least 1, so dividing by the positive fraction y increases the value.

27. **(D)**. If the variables were also constrained to be positive, they would have to be 1, 2, and 3, making the quantities both equal to 2. However, the variables could be negative, for example, $a = –10$, $b = –9$, $c = –8$. The range of a, b, and c will always be 2 because the integers are consecutive, but the average can vary depending on the specific values. There is not enough information to determine the relationship.

28. **(B).** If $\sqrt{xy}$ is a prime number, $\sqrt{xy}$ could be 2, 3, 5, 7, 11, 13, etc. Square these possibilities to get a list of possibilities for *xy:* 4, 9, 25, 49, 121, 169, etc. However, *xy* is even, so *xy* must equal 4.

Finally, $x > 4y > 0$, which implies that both *x* and *y* are positive. Solve $xy = 4$ for *x*, then substitute to eliminate the variable *x* and solve for *y*:

If $xy = 4$, then $x = \frac{4}{y}$.

If $x > 4y$, then $\frac{4}{y} > 4y$.

Because *y* is positive, you can multiply both sides of the inequality by *y* and you don't have to flip the sign of the inequality: $4 > 4y^2$.

Finally, divide both sides of the inequality by 4: $1 > y^2$.

Thus, *y* is a positive fraction less than 1 (it was already given that $y > 0$). Quantity B is greater.

29. **(B).** If $\sqrt{x}$ is a prime number, $x = \left(\sqrt{x}\right)\left(\sqrt{x}\right)$ is the square of a prime number. Squaring a number does not change whether it is odd or even (the square of an odd number is odd and the square of an even number is even). Since *x* is even, it must be the square of the only even prime number. Thus, $\sqrt{x} = 2$ and $x = 4$. Since $x + y = 11$, $y = 7$ and Quantity B is greater.

30. **(A).** If *fg* is odd and both *f* and *g* are positive integers, both *f* and *g* are odd. The remainder when odd *f* is divided by 2 is 1. Since *fgh* is even and *f* and *g* are odd, integer *h* must be even. Thus, when *h* is divided by 2, the remainder is 0. Quantity A is greater.

31. **(D).** If $x^2 > 25$, then $x > 5$ OR $x < -5$. For instance, *x* could be 6 or –6.

If $x = 6$:

$$6 + y < 0$$
$$y < -6$$

x is greater than *y*.

If $x = -6$:

$$-6 + y < 0$$

$y < 6$

y could be less than *x* (e.g., $y = –7$) or greater than *x* (e.g., $y = 4$). Therefore, you do not have enough information.

32. **(B).** If the sum of two primes is less than 6, either the numbers are 2 and 3 (the two smallest unique primes), or both numbers are 2 (just because the variables are different letters doesn't mean that *p* cannot equal *w*). Both numbers cannot equal 3, though, or $p + w$ would be too great. If p^2 is odd, *p* is odd, and therefore $p = 3$, so *w* can only be 2.

33. **(A).** If $x^2 > y^2$, *x* must have a greater absolute value than *y*. For instance:

	x	y
Example 1	3	2
Example 2	–3	2
Example 3	3	–2
Example 4	–3	–2

If $x > -|y|$ must also be true, which of the examples continue to be valid?

	x	y	$x > -\|y\|$?	
Example 1	3	2	$3 > -\|2\|$	TRUE
Example 2	–3	2	$-3 > -\|2\|$	FALSE
Example 3	3	–2	$3 > -\|-2\|$	TRUE
Example 4	–3	–2	$-3 > -\|-2\|$	FALSE

Only Example 1 and Example 3 remain.

	x	y
Example 1	3	2
Example 3	3	–2

Thus, either x and y are both positive and x has a greater absolute value (Quantity A is greater) or x is positive and y is negative (Quantity A is greater). In either case, Quantity A is greater.

34. **(C).** Write an equation: $x + (x + 1) + (x + 2) + (x + 3) = -2$. Now solve:

$$\begin{aligned} 4x + 6 &= -2 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

Thus, the integers are –2, –1, 0, and 1. The smallest of the four integers equals –2, so the quantities are equal.

35. $\dfrac{g^2x+5gx}{x}$ **and** $6\left(\dfrac{g}{2}\right)-100\left(\dfrac{g}{2}\right)^2$ **=only.** In the first choice, x can be factored out and canceled:

$$\frac{g^2+5gx}{x}=\frac{x\left(g^2+5g\right)}{x}=g^2+5g$$

Since g is an integer, so too is $g^2 + 5g$.

In the second choice, g^2 is certainly an integer, but $x^2\left(\dfrac{1}{3}\right)$ is only an integer if $x = 3$ (since 3 is the only prime number divisible by 3), so the second choice is not necesarily an integer.

When the third choice is simplified, $6\left(\frac{g}{2}\right) - 100\left(\frac{g}{2}\right)^2 = 3g - \frac{100g^2}{4} = 3g - 25g^2$ results; since g is an integer, $3g - 25g^2$ is also an integer.

Chapter 16

Word Problems

In This Chapter…

Word Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If a taxi charges $8.00 for the first mile, and $1.00 for each additional quarter mile, how much does the taxi charge for a 4.5 mile ride?

   (A) $16.00

   (B) $18.00

   (C) $22.00

   (D) $24.00

   (E) $26.00

2. If Nash had 12 grandchildren and three times as many granddaughters as grandsons, how many granddaughters did he have?

   (A) 3

   (B) 4

   (C) 6

(D) 8

(E) 9

3. If Deepak pays 30% of his income in taxes and his take-home pay after taxes is $2,800 per month, how much does Deepak make per month, before taxes?

$ $\boxed{}$

4. A movie theater charges $6 per ticket, and pays $1,750 of expenses each time a movie is shown. How many tickets must be sold each time a movie is shown for the theater to make $1 of profit per ticket?

   (A) 300

   (B) 325

   (C) 350

   (D) 375

   (E) 400

5. Arnaldo earns $11 for each ticket that he sells, and a bonus of $2 per ticket for each ticket he sells beyond the first 100 tickets. If Arnaldo was paid $2,400, how many tickets did he sell?

   (A) 120

   (B) 160

   (C) 180

   (D) 200

   (E) 250

6. Attendees at a charity dinner each gave at least $85 to the charity. If $6,450 was collected, what is the maximum number of people who could have attended?

   (A) 73

   (B) 74

   (C) 75

   (D) 76

   (E) 77

7. Eva meditates for 20 minutes at a time, with a 5-minute break in between sessions. If she begins meditating at 10:10, what time will it be when she completes her third 20-minute meditation session?

   (A) 11:20

   (B) 11:25

   (C) 11:50

   (D) 11:55

(E) 12:25

8. A washing machine takes 35 minutes to wash one load of laundry, and in between washing different loads of laundry it takes Derek 2 minutes to unload and another 4 minutes to reload the machine. If the washing machine begins washing one load of laundry at 12:30pm, how many loads of laundry can Derek wash and unload before 6:35pm?

(A) 8

(B) 9

(C) 10

(D) 14

(E) 15

Kendra is more than 5 years old.

	Quantity A	**Quantity B**
9.	Five years less than twice Kendra's age	Twice what Kendra's age was five years ago

---

10. Each day that the drama club washes cars to raise money, the club's only expense is a fixed amount for supplies. If the club charged $12 for each car washed and earned a total profit of $190 in one day by washing 20 cars, how much did the club pay for supplies?

$ [ ]

11. A store owner pays her assistant $22 per hour for every hour the store is open. If all other expenses for the store are $160 per day, and the store is open for 8 hours on Monday and sells $720 worth of merchandise on that day, what is the store's profit for the day?

(A) $384

(B) $396

(C) $530

(D) $538

(E) $560

12. Regular gas costs $3.00 a gallon and is consumed at a rate of 25 miles per

gallon. Premium costs $4.00 a gallon and is consumed at a rate of 30 miles per gallon. How much more will it cost to use premium rather than regular for a 300-mile trip?

(A) $ 1

(B) $ 4

(C) $ 5

(D) $36

(E) $40

13. A retailer sold toys at a regular selling price of 25% greater than the retailer's cost to buy the toys. If the retailer reduces the regular selling price by 80%, what is then the loss on each toy sold as a percent of the retailer's cost?

(A) 25%

(B) 30%

(C) 40%

(D) 75%

(E) 80%

14. Mr. Choudury's fourth-grade class consists of 20 students: 12 boys and 8 girls. If the boys weigh an average of 80 pounds each and the girls weigh an average of 70 pounds each, what is the average weight, in pounds, of all 20 students?

(A) 71

(B) 74

(C) 75

(D) 76

(E) 79

15. It costs a certain bicycle factory a fixed amount of $11,000 to operate for one month, plus $300 for each bicycle produced during the month. Each of the bicycles sells for a retail price of $700. What is the minimum number of bicycles that the factory must sell in one month to make a profit?

(A) 26

(B) 27

(C) 28

(D) 29

(E) 30

16. The yoga company Yoga for Life offers 45-minute classes at $12 per class. If the number of minutes Randolf spent doing yoga this month was 132 greater than the number of dollars he paid, how many classes did he attend?

(A) 3

(B) 4

(C) 5

(D) 6

(E) 8

17. An online merchant sells wine for $20 for an individual bottle or $220 for a case of 12. Either way, the merchant's cost for the wine is $10 per bottle. Shipping costs the merchant $5 for a bottle and $40 for a case. If 12 cases and 60 individual bottles were sold, and there were no other revenue or expenses, the merchant's profit was equal to which of the following?

(A) $ 780

(B) $1,020

(C) $2,160

(D) $2,640

(E) $3,840

18. If every donor to a charity drive contributed at least $14 and $237 was collected, what is the maximum number of donors?

(A) 13

(B) 14

(C) 15

(D) 16

(E) 17

In a certain barter system, 1 sack of rice can be traded for 2.5 pounds of beans or $\frac{1}{3}$ of a bushel of tomatoes.

	Quantity A	**Quantity B**
19.	The number of sacks of rice equivalent to 1 pound of beans	The number of sacks of rice equivalent to 1 bushel of tomatoes

---

20. Francisco's MP3 player with a capacity of 64 gigabytes (GB) was three-quarters full. He then deleted 25% of the data saved on the device before saving another 20 GB of new data to the device. The resulting amount of data saved is what percent of the capacity of Francisco's MP3 player?

(A) 62.5 %

(B) 70 %

(C) 75 %

(D) 87.5 %

(E) 95 %

21. Last year, a magazine charged a $50 subscription fee. This year, the price will be increased by $10. If the magazine could lose 4 subscribers this year and still collect the same revenue as it did last year, how many subscribers did the magazine have last year?

(A) 20

(B) 21

(C) 22

(D) 23

(E) 24

22. A rectangular public park has an area of 3,600 square feet. It is surrounded on three sides by a chain link fence. If the entire length of the fence measures 180 feet, how many feet long could the unfenced side of the rectangular park be?

Indicate <u>all</u> such lengths.

☐ 30
☐ 40
☐ 60
☐ 90
☐ 120

23. The perimeter of a rectangular patio is 268 feet and its length is 168% of its width. What is the area of the patio, in square feet?

(A) 4,000

(B) 4,200

(C) 4,320

(D) 4,600

(E) 4,760

24. Randall purchased a shirt for \$19.44 using a \$20 bill. If his correct change was returned in only dimes (\$0.10) and pennies (\$0.01), how many coins could Randall have received?

(A) 9

(B) 21

(C) 29

(D) 37

(E) 44

25. In the modern era, the global population of humans has increased by 1 billion people approximately every 13 years. If that rate were to continue, approximately how many years would it take for the Earth's population to double from its current population of 7 billion people?

(A) 26

(B) 52

(C) 91

(D) 104

(E) 169

Gerald bought a used motorcycle for \$1,200 and spent \$305 repairing it. He then sold the motorcycle for 20% more than the total amount he spent for purchase and repairs.

	Quantity A	**Quantity B**
26.	The final selling price of the motorcycle	\$1,800

A turbine salesman earns a commission of *x*% of the purchase price of every turbine he sells, where *x* is a constant. His commission for a \$300,000 turbine was \$1,500.

	Quantity A	**Quantity B**
27.	The commission earned on a turbine that sold for \$180,000.	\$800

Word Problems Answers

1. **(C).** Break the trip into two parts: the first mile and the final 3.5 miles. The first mile costs \$8, and the final 3.5 miles cost \$1 per $\frac{1}{4}$ mile, or \$4 per mile.

The total cost is 8 + 3.5(4) = 8 + 14 = \$22.

2. **(E).** Rather than assigning separate variables to the granddaughters and grandsons, define them both in terms of the same unknown multiplier, based on the ratio given:

Number of granddaughters = 3*m*
Number of grandsons = *m*

Note that you are solving for 3*m*, not for *m*!

$$\begin{aligned} 3m + m &= 12 \\ 4m &= 12 \\ m &= 3 \\ 3m &= 9 \end{aligned}$$

Alternatively, suppose that Nash had exactly one grandson and three granddaughters. That would sum to four grandchildren altogether. Triple the number of grandsons and granddaughters to triple the number of grandchildren.

3. **\$4,000.** If Deepak pays 30% in taxes, his take-home pay after taxes is 70%. Since this amount is equal to \$2,800:

$$\begin{aligned} 0.70x &= 2{,}800 \\ x &= 4{,}000 \end{aligned}$$

4. **(C).** This problem requires the knowledge that profit equals revenue minus cost. Memorize the formula: Profit = Revenue – Cost (or Profit = Revenue – Expenses), but you could just think about it logically—a business has to pay its expenses out of the money it makes: the rest is profit.

The cost each time a movie is shown is \$1,750. If the theater charges \$6 per ticket and t is the number of tickets, the revenue is $6t$. In order for the profit to be \$1 per ticket, the profit must be t dollars.

Plug these values into the equation Profit = Revenue – Cost:

$$
\begin{aligned}
t &= 6t - 1{,}750 \\
-5t &= -1{,}750 \\
t &= 350
\end{aligned}
$$

5. **(D).** Let x = the total number of tickets sold. Therefore, $(x - 100)$ = the number of tickets Arnaldo sold beyond the first 100. Using the information given, set up an equation and solve:

$$\begin{aligned} 11x + 2(x - 100) &= 2{,}400 \\ 11x + 2x - 200 &= 2{,}400 \\ 13x &= 2{,}600 \\ x &= 200 \end{aligned}$$

6. **(C).** Divide \$6,450 by \$85 to get 75.88… But don't just round up! Each person gave at least \$85. If 76 people attended and each gave the minimum of \$85, then \$6,460 would have been collected. Since only \$6,450 was collected, that 76th person could not have attended. Instead, round down to 75. (This means at least one person gave more than the minimum.)

7. **(A).** List Eva's meditation sessions and breaks:

10:10–10:30	session 1
10:30–10:35	break
10:35–10:55	session 2
10:55–11:00	break
11:00–11:20	session 3

Note that the question asks for the time when she will complete her third session, so do not add a third break!

A quicker way to do this problem would be to add 20(3) + 5(2) to get 70 minutes, and 70 minutes after 10:10 is 11:20.

8. **(B).** You *could* list Derek's activities:

12:30–1:05	load 1
1:05–1:11	unload/reload
1:11–1:46	load 2
1:46–1:52	unload/reload
1:52–2:27	load 3

Etc.

However, completing this rather tedious list all the way up to 6:35pm is not a good expenditure of time on the GRE. A better approach would be to determine how many minutes are available for Derek to do laundry. From 12:30 to 6:35 is 6 hours and 5 minutes, or 365 minutes.

It takes 41 minutes to do one load of laundry and then switch to the next one (35 + 4 + 2 minutes).

Divide 365 minutes by 41 to get 8.9… So, Derek can definitely do 8 total loads of laundry plus switching time.

What about that extra 0.9…? You need to figure out whether Derek can fit in one more laundry load. Importantly, for this last load he needs only 2 extra minutes to unload, since he will not be reloading the machine.

Multiply 8 (the total number of loads Derek can definitely do) by 41 minutes to get 328 minutes. Subtract 328 from the 365 available minutes to get 37 minutes. That is *exactly* how much time it takes Derek to do one load of laundry (35 minutes) and then unload it (2 minutes). So, Derek can wash and unload 9 total loads of laundry.

9. **(A).** This is an algebraic translation, meaning you need to translate the text into algebra. Use k to represent Kendra's age, and the problem states that $k > 5$ (this is important only because Quantity B requires you to consider Kendra's age five years ago, and if she were younger than 5 years old, that would create an impossible negative age!).

Quantity A = $2k - 5$
Quantity B = $2(k - 5) = 2k - 10$

Since $2k$ is common to both quantities it can be subtracted from both without affecting their relative values.

Quantity A = -5
Quantity B = -10

Quantity A is less negative, so it is greater.

10. **$50.** Since Profit = Revenue – Expenses, and $12 for a car wash multiplied by 20 car washes = $240:

$$190 = 240 - E$$

$$-50 = -E$$

$$\$50 = E$$

11. **(A).** Since Profit = Revenue – Expenses, and revenue = $720:

$$P = 720 - E$$

Expenses are equal to $22 per hour times 8 hours, plus a fixed $160, or 22(8) + 160 = $336. Thus:

$$P = 720 - 336$$

$$P = \$384$$

12. **(B).** 12 gallons of regular gas are needed to go 300 miles (300 divided by 25 miles per gallon), costing \$36 (12 gallons × \$3 per gallon). 10 gallons of premium would be needed to go 300 miles (300 divided by 30 miles per gallon), costing \$40 (10 gallons × \$4 per gallon). The question asks for the difference, which is \$40 – \$36 = \$4.

13. **(D).** For problems that ask for percents and use no real numbers, it is almost always possible to use 100 as a starting number. Suppose the retailer buys each toy for \$100, and thus sells it for a regular price of \$125. A reduction of this regular selling price by 80% drops the price to \$25. The loss on each toy sold as a percent of the retailer's cost is:

$$\left[\frac{(100-25)}{100}\right]\times 100 = 75\%$$

14. **(D).** The most straightforward approach is to determine the total weight of all 20 students and divide that total by 20:

12 boys × 80 pounds per boy = 960 pounds
8 girls × 70 pounds per girl = 560 pounds
Total = 1,520 pounds

$$\frac{1,520}{20} = 76$$

Alternatively, many or even most GRE multiple-choice weighted average problems have the same five answers:

Much closer to the lesser value
A little closer to the lesser value
The unweighted average of the two values
A little closer to the greater value
Much closer to the greater value

Any of these five choices *could* be correct, but the correct answer is usually "a little closer to the lesser value" or "a little closer to the greater value." In this case, because there are a few more boys than girls, the average for the whole class will be a little closer to the boys' average weight than to the girls'.

15. **(C).** The question asks how many bicycles the factory must sell to make a profit. One way of phrasing that is to say the profit must be greater than 0. Since Profit = Revenue – Cost, you can rewrite the equation to say:

Revenue – Cost > 0

Let b equal the number of bicycles sold. Each bike sells for $700, so the total revenue is $700b$. The cost is equal to $11,000 plus $300 for every bicycle sold.

$(700b) – (11,000 + 300b) > 0$

Isolate b on one side of the inequality:

$700b – 11,000 + 300b > 0$
$400b – 11,000 > 0$
$400b > 11,000$
$b > 27.5$

If b must be greater than 27.5, then the factory needs to sell at least 28 bicycles to make a profit.

16. **(B).** The typical way to do this problem would be to assign variables and

set up equations, using x to represent the number of classes Randolf took, $12x$ to represent the amount he paid, and $45x$ to represent the number of minutes he spent.

A quicker way might be to notice that with every class Randolf takes, the difference between the number of minutes he spends and the amount he pays increases by 33. If Randolf takes 1 class, then the number of minutes he spends is 33 greater than the number of dollars he pays. If he takes 2 classes, the number of minutes is 66 greater than the number of dollars, and so on. Since 132 = 4 × 33, Randolf must have taken 4 classes.

17. **(B).** Profit is equal to Revenue – Expenses. First, calculate revenue:

12 cases sold for \$220 each = \$2,640
60 bottles sold for \$20 each = \$1,200

Total Revenue = \$3,840

Now, calculate expenses. How many total bottles of wine were sold? 12 cases × 12 bottles, plus 60 individual bottles = 204 bottles. Note that the bottles sold individually versus those sold in cases have the same purchase cost (\$10), but different shipping costs. Thus:

204 bottles at \$10 each = \$2,040
Shipping on bottles = 60 × \$5 = \$300
Shipping on cases = 12 × \$40 = \$480

Total Expenses = \$2,820

Profit = Revenue – Expenses
Profit = \$3,840 – \$2,820
Profit = \$1,020

18. **(D).** This is a maximization question. To solve maximization questions, you often have to minimize something else. In order to find the maximum number of donors, minimize the donation per person. In this case, everyone could pay exactly \$14:

$$\frac{237}{14} = 16.92$$

Rounding up to 17 is not right, because it is not possible that 17 people donated \$14 each (the total contributions would be \$238, which is greater than \$237). The answer is (D), or 16.

19. **(B).** If 1 sack of rice is worth $\frac{1}{3}$ of a bushel of tomatoes, buying the whole bushel would require 3 sacks of rice. Quantity B is equal to 3. The math is a bit tougher in Quantity A, but no calculation is really required—if a

sack trades for 2.5 pounds of beans, a single pound of beans is worth less than a sack of rice. Quantity A is less than 1.

20. **(D).** If Francisco's MP3 player is three-quarters full, the current content equals $\frac{3}{4}$ × 64 GB = 48 GB. He then deleted 25%, or 12 GB, of that data, reducing the amount of data saved to 48 – 12 = 36 GB. After saving 20 GB of new data to the device, it holds 36 + 20 = 56 GB. This is $\left(\frac{56}{64}\times 100\right)\%$ = 87.5% of the total capacity.

21. **(E).** Assign the variable s for the number of subscribers last year:

Last year:
\$50 per subscription
s subscribers

This year:
\$60 per subscription
$s - 4$ subscribers

The question states that the magazine “could” lose 4 subscribers and that the magazine would then collect the same revenue as last year—don’t let the “could” throw you off. Calculate using this hypothetical situation:

$$\begin{aligned} 50s &= 60(s - 4) \\ 50s &= 60s - 240 \\ -10s &= -240 \\ s &= 24 \end{aligned}$$

22. **60 and 120 only**. The two values given are the area of the park and three out of the four sides of the perimeter of the park. If the side without fencing is a length, the equation for the overall length of the existing fence is $180 = 2W + L$, so $L = 180 - 2W$. The equation for the area of the park is $LW = 3{,}600$. With two variables and two equations, it is now possible to solve for the possible values of L:

$L \times W = 3{,}600$
$L = 180 - 2W$

$$\begin{aligned} (180 - 2W)W &= 3{,}600 \\ 180W - 2W^2 &= 3{,}600 \\ 90W - W^2 &= 1{,}800 \\ 0 &= W^2 - 90W + 1{,}800 \\ 0 &= (W - 60)(W - 30) \end{aligned}$$

So $W = 30$ or 60. Plug each value back into either of the original two equations to solve for the corresponding length, which is 120 or 60, respectively.

23. **(B)**. This rectangle problem requires applying the perimeter and area formulas. The area of a rectangle is equal to length times width ($A = LW$) and the perimeter is $2L + 2W = 268$. The question states that the length equals 168% of the width, $L = 1.68W$.

$$\begin{aligned} 2L + 2W &= 268 \\ L + W &= 134 \\ 1.68W + W &= 134 \\ 2.68W &= 134 \\ W &= 50 \end{aligned}$$

Solve for L by plugging 50 in for W in either equation:

$$\begin{aligned} L + 50 &= 134 \\ L &= 84 \\ A &= 84(50) = 4{,}200 \end{aligned}$$

24. **(C)**. From the first sentence, calculate Randall's change (20.00 – 19.44 = 0.56). Then it's a matter of systematic tests to determine the various combinations of dimes and pennies that Randall could have received, stopping when one matches an answer choice listed:

5 dimes (0.50) + 6 pennies (0.06) = 11 coins	Not an option in the choices
4 dimes (0.40) + 16 pennies (0.16) = 20 coins	Not an option in the choices
3 dimes (0.30) + 26 pennies (0.26) = 29 coins	Correct answer

25. **(C)**. To double from the current population of 7 billion people, the population would need to increase by 7 billion. If the population increases by 1 billion every 13 years, an increase of 7 billion would take 7 × 13 = 91 years.

26. **(A)**. Gerald spent \$1,200 + \$305 = \$1,505 total on purchase and repairs. The selling price was 20% more, or 1.2 times this amount. Either plug 1.2 × 1,505 into the calculator to get \$1,806, or recognize that 1.2 of \$1,500 is exactly \$1,800, so Quantity A is a little more than that.

27. **(A)**. There are a few options for solving the given problem. First, you could find out exactly what x equals by setting up an equation: "1,500 is x percent of 300,000" translates algebraically to 1,500 = $\left(\frac{x}{100}\right)$ (300,000).

Solving the equation will reveal that x = 0.5, so the commission is 0.5%. Taking 0.5% of \$180,000 gives \$900 for Quantity A, which is greater than Quantity B.

Alternatively, you could reason by proportion: $\frac{1500}{300{,}000} = \frac{c}{180{,}000}$.

This works because the commission the salesman earns represents the same proportion of the total in all cases, so any changes to the total will be reflected in changes to the commission. This gives the same value for Quantity A, \$900, and is still greater than Quantity B.

Chapter 17

Two-Variable Word Problems

In This Chapter…

Two-Variable Word Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box , you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box , you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Two parking lots can hold a total of 115 cars. The Green lot can hold 35 fewer cars than the Red lot. How many cars can the Red lot hold?

   (A) 35

   (B) 40

   (C) 70

   (D) 75

   (E) 80

2. Three friends ate 14 slices of pizza. If two of the friends ate the same number of slices, and the third ate 2 more slices than each of the other two, how many slices were eaten by the third friend?

   (A) 3

   (B) 4

(C) 5

(D) 6

(E) 7

In 8 years, Polly's age, which is currently p, will be twice Quan's age, which is currently q.

	Quantity A	**Quantity B**
3.	$p - 8$	$2q$

---

4. Pens cost 70 cents each and pencils cost 40 cents each. If Iris spent $5.20 on a total of 10 pens and pencils, how many pencils did she purchase? ($1 = 100 cents)

(A) 4

(B) 6

(C) 8

(D) 10

(E) 13

5. Jack downloaded 10 songs and 2 books for $48, Jill downloaded 15 songs and 1 book for $44. How much did Jack spend on books, if all songs are the same price and all books are the same price?

(A) $14

(B) $20

(C) $28

(D) $29

(E) $30

6. Marisa has $40 more than Ben, and Ben has one-third as much money as Marisa. How many dollars does Ben have?

$

7. Norman is 12 years older than Michael. In 6 years, he will be twice as old as Michael. How old is Michael now?

(A) 3

(B) 6

(C) 12

(D) 18

(E) 24

8. Krunchy Kustard sells only two kinds of doughnuts: glazed and cream-filled. A glazed doughnut has 200 calories, and a cream-filled doughnut has 360 calories. If Felipe ate 5 doughnuts totaling 1,640 calories, how many were glazed?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

9. The “aspect ratio” of a computer monitor is the ratio of the monitor’s width to its height. If a particular monitor has an aspect ratio of 16 : 9, and a perimeter of 100 inches, how many inches wide is the monitor?

(A) 18

(B) 25

(C) 32

(D) 36

(E) 64

10. Cindy bought 48 containers of soda, all either 12-ounce cans or 20-ounce bottles. If the number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles, how many bottles of soda did Cindy buy?

(A) 18

(B) 21

(C) 24

(D) 27

(E) 30

11. Two runners’ race times sum to 170 seconds and one of the race times is 10 seconds less than twice the other. What is the faster race time, in seconds?

(A) 40

(B) 50

(C) 60

(D) 70

(E) 110

12. Beth is 12 years younger than Alan. In 20 years, Beth will be 80% of Alan’s age. How old is Beth now?

$\boxed{}$ years old

13. Rey is 12 years younger than Sebastian. Five years ago, Rey was half Sebastian’s age. How old will Sebastian be next year?

(A) 15

(B) 20

(C) 25

(D) 30

(E) 35

14. During a sale, the local outlet of the Chasm sold three times as many jeans as chinos. If they made twice as much profit for a pair of chinos as for a pair of jeans, and sold no other items, what percent of their profits during the sale came from chinos?

(A) $16\frac{2}{3}\%$

(B) 20%

(C) 40%

(D) 60%

(E) $83\frac{1}{3}\%$

15. Marisol is twice as old as Vikram. Eight years ago, Marisol was 6 years younger than three times Vikram's age at that time. How old will Marisol be in 5 years?

16. The length of a rectangle is 2 more than twice its width, and the area of the rectangle is 40. What is the rectangle's perimeter?

17. Marcy bought one pair of jeans at 70% off and one blouse at 40% off. If she paid $12 more for the blouse than for the jeans, and she spent a total of $84, what was the original price of the jeans?

(A) $76

(B) $96

(C) $100

(D) $120

(E) $124

18. Wall-to-wall carpeting is installed in a certain hallway. The carpeting costs $4.25 per square foot. If the perimeter of the hallway (in feet) is equal to 44% of the area of the hallway (in square feet) and the hallway is 50 feet long, how much did the carpeting cost?

(A) $182.50

(B) $212.50

(C) $505.25

(D) $1,062.50

(E) $1,100.00

19. Jamal got three monthly electric bills over the course of three months. If his average monthly bill over these three months was \$44 more than the median bill, and the sum of the largest and the smallest bills was \$412, what was the total amount of the three electric bills Jamal got over the course of the three months?

(A) \$456

(B) \$552

(C) \$600

(D) \$824

(E) \$1,000

20. A certain dog kennel houses only collies, labs, and golden retrievers. If the ratio of collies to labs is 5 : 9, there are 66 golden retrievers, and 12 more golden retrievers than labs in the kennel, what percent of the dogs in the kennel are collies?

(A) 5%

(B) 9%

(C) 12%

(D) 20%

(E) 25%

21. If Mason is now twice as old as Gunther was 10 years ago, and *G* is Gunther's current age in years, which of the following represents the sum of Mason and Gunther's ages 4 years from now?

(A) $\frac{3G}{2} + 3$

(B) $3G + 28$

(C) $3G – 12$

(D) $8 – G$

(E) $14 – \frac{3G}{2}$

22. A baker made a combination of chocolate chip cookies and peanut butter cookies for a school bake sale. His recipes only allow him to make chocolate chip cookies in batches of 7, and peanut butter cookies in

batches of 6. If he made exactly 95 cookies for the bake sale, what is the minimum possible number of chocolate chip cookies that he made?

(A) 7

(B) 14

(C) 21

(D) 28

(E) 35

23. Anke has 5 fewer candies than Conrad. If Anke gives Conrad 5 candies, Conrad will then have 4 times as many candies as Anke. How many candies does Anke have?

(A) 5

(B) 10

(C) 15

(D) 20

(E) 25

24. Lou has three daughters: Wen, Mildred, and Tyla. Three years ago, when Lou was twice as old as Tyla, he was 30 years older than Mildred. Now, he is 47 years older than Wen. In 4 years, Wen will be half as old as Tyla. What is the sum of the current ages of Lou and his three daughters?

(A) 138

(B) 144

(C) 154

(D) 166

(E) 181

25. Dwayne planted 70 acres with two types of beans: navy and pinto. Each acre of navy beans yielded 27 bushels and each acre of pinto beans yielded 36 bushels. If Dwayne grew twice as many bushels of pinto beans as navy beans, how many acres of pinto beans did he plant?

(A) 28

(B) 30

(C) 35

(D) 40

(E) 42

Two-Variable Word Problems Answers

1. **(D).** Let g = the number of cars that the Green lot can hold. Let r = the number of cars that the Red lot can hold.

The first two sentences can be translated into two equations:

$$\begin{aligned} g + r &= 115 \\ g &= r - 35 \end{aligned}$$

The question asks for r, so substitute (r – 35) for g in the first equation:

$$\begin{aligned} (r - 35) + r &= 115 \\ 2r - 35 &= 115 \\ 2r &= 150 \\ r &= 75 \end{aligned}$$

2. **(D).** Let P = the number of slices of pizza eaten by each of the two friends who eat the same amount. Let T = the number of slices of pizza eaten by the third friend.

$$\begin{aligned} T &= P + 2 \\ P + P + T &= 14 \end{aligned}$$

Substitute (P + 2) for T in the second equation:

$$\begin{aligned} P + P + (P + 2) &= 14 \\ 3P + 2 &= 14 \\ 3P &= 12 \\ P &= 4 \end{aligned}$$

Solve for T:

$$T = P + 2 = 4 + 2 = 6$$

3. **(C).** This is an algebraic translation question, so start by translating the given information into equations. Remember to add 8 to both Polly and Quan's ages, because they will *both* be 8 years older in 8 years!

$$
\begin{aligned}
p + 8 &= 2(q + 8) \\
p + 8 &= 2q + 16 \\
p &= 2q + 8 \\
p - 8 &= 2q
\end{aligned}
$$

The two quantities are equal.

4. **(B)**. Assign one variable to the pencils and another variable to the pens:

Number of pencils = x
Number of pens = y

$x + y = 10$
$70y + 40x = 520$

The question asks for the number of pencils, x, so isolate y in the first equation and substitute into the second:

$$y = 10 - x$$

$$\begin{aligned} 70(10 - x) + 40x &= 520 \\ 700 - 70x + 40x &= 520 \\ 700 - 30x &= 520 \\ 180 &= 30x \\ x &= 6 \end{aligned}$$

Alternatively, test the answer choices. Starting with the middle choice, if Iris bought 8 pencils and therefore 2 pens, she spent $(8 \times 40) + (2 \times 70) = 320 + 140 = 460$. That's 60 cents too little, so Iris must have bought fewer pencils and more pens. Try 6 pencils and 4 pens: $(6 \times 40) + (4 \times 70) = 240 + 280 = 520$. (You might also have noticed that every time Iris swaps a pencil for a pen, she spends an extra 30 cents.)

5. **(C).** The equations are $10s + 2b = 48$ and $15s + b = 44$. The easiest next move would be to solve the second equation for b:

$$b = 44 - 15s$$

Substitute that into the first equation:

$$\begin{aligned} 10s + 2(44 - 15s) &= 48 \\ 10s + 88 - 30s &= 48 \\ -20s + 88 &= 48 \\ -20s &= -40 \\ s &= 2 \end{aligned}$$

Plug $s = 2$ back into either original equation to get that $b = 14$, and thus the two books that Jack bought cost $28.

6. **20.** Translate the given information. Let *M* equal Marisa's money and *B* equal Ben's:

$$M = B + 40$$

$$B = \frac{1}{3} M$$

The question asks for *B*, so solve the second equation for *M* and substitute into the first equation:

$3B = M$

$3B = B + 40$
$2B = 40$
$B = 20$

Check the answer. If Ben has \$20 and Marisa has \$40 more than Ben, she has \$60. It is true that Ben has one-third as much money as Marisa.

7. **(B).** Let N = Norman's age now; $(N + 6)$ = Norman's age in 6 years.
Let M = Michael's age now; $(M + 6)$ = Michael's age in 6 years.

Translate the first two sentences into equations. Note that the second equation deals with Norman and Michael's ages in 6 years:

$$N = M + 12$$

$$(N + 6) = 2(M + 6)$$

The question asks for M, so substitute $(M + 12)$ for N in the second equation:

$$(M + 12) + 6 = 2(M + 6)$$

$$M + 18 = 2M + 12$$

$$M + 6 = 2M$$

$$6 = M$$

8. **(A).** Assign variables to the two types of doughnuts, and write equations based on the given information:

Number of glazed = G
Number of cream-filled = C

$G + C = 5$ (the number of doughnuts)

$1{,}640 = 200G + 360C$ (the number of calories)

The question asks for G, so isolate C in first equation and substitute into the second equation:

$C = 5 - G$

$$1{,}640 = 200G + 360(5 - G)$$

$$1{,}640 = 200G + 1{,}800 - 360G$$

$$1{,}640 = 1{,}800 - 160G$$

$$-160 = -160G$$
$$G = 1$$

Check the answer. If Felipe at 1 glazed doughnut, he ate 4 cream-filled doughnuts. He ate $(1 \times 200) + (4 \times 360) = 200 + 1{,}440 = 1{,}640$ calories.

9. **(C).** Rather than assigning separate variables to the width and height, define them both in terms of the same unknown multiplier, based on the ratio given:

Width = $16m$
Height = $9m$

Remember that the question asks for the width, so answer for $16m$, not for m!

The perimeter of a rectangle is equal to 2(length + width), or in this case 2(width + height):

$$\begin{aligned} 100 &= 2 \times (16m + 9m) \\ 100 &= 50m \\ m &= 2 \\ 16m &= 32 \end{aligned}$$

An alternative method depends on the same underlying logic, but forgoes the algebra. Suppose the dimensions were 16 inches and 9 inches. This would yield a perimeter of 50 inches. Double the width and height to double the perimeter.

10. **(A)**. Define variables and translate equations from the given information:

Number of bottles = b
Number of cans = c

$b + c = 48$	(Number of containers purchased)
$12c = 20b$	(Equal number of ounces in bottles purchased and cans purchased)

The question asks for b, so isolate c in the first equation and substitute into the second equation:

$c = 48 – b$

$12(48 – b) = 20b$
$576 – 12b = 20b$
$576 = 32b$
$b = 18$

Check the answer. If Cindy bought 18 bottles, she bought 30 cans. The number of ounces in the bottles was $18 \times 20 = 360$. The number of ounces in the cans was $30 \times 12 = 360$.

11. **(C).** Call the race times x and y. The question provides a sum: $x + y = 170$.

One of the race times is 10 seconds less than twice the other: $x = 2y – 10$.

Since the second equation is already solved for x, plug $(2y – 10)$ in for x in the first equation:

$$\begin{aligned} 2y - 10 + y &= 170 \\ 3y - 10 &= 170 \\ 3y &= 180 \\ y &= 60 \end{aligned}$$

If $y = 60$ and the times sum to 170, then $x = 110$.

Note that the question asks for the *faster* race time—that means the smaller number! The answer is 60.

12. **28.** Since Beth is 12 years younger than Alan, you can write:

$$B = A - 12$$

To translate "in 20 years, Beth will be 80% of Alan's age," make sure that Beth becomes $B + 20$ and Alan becomes $A + 20$ (if you will be doing other operations to these values, put parentheses around them to make sure the rules of PEMDAS are not violated):

$$\begin{aligned} B + 20 &= 0.8(A + 20) \\ B + 20 &= 0.8A + 16 \\ B + 4 &= 0.8A \end{aligned}$$

Since the first equation is already solved for B, plug $(A - 12)$ into the simplified version of the second equation in place of B:

$$\begin{aligned} B + 4 &= 0.8A \\ (A - 12) + &= 0.8A \\ A - 8 &= 0.8A \\ 0.2A - 8 &= 0 \\ 0.2A &= 8 \\ A &= 40 \end{aligned}$$

Alan is 40. Since $B = A - 12$, Beth is 28.

Check the answer. In 20 years, Beth will be 48 and Alan will be 60, and it is true that 48 is 80% of 60.

13. **(D).** Assign variables and translate equations from the given information:

r = Rey's age NOW
s = Sebastian's age NOW

$r = s - 12$

$(r - 5) = \frac{1}{2}(s - 5)$

Multiply the second equation by 2 to eliminate the fraction and simplify:

$2r - 10 = s - 5$
$2r = s + 5$

Since the question asks for Sebastian's age next year and r is already isolated in the first equation, substitute for r in the adjusted second equation and solve:

$$2(s - 12) = s + 5$$
$$2s - 24 = s + 5$$
$$2s = s + 29$$
$$s = 29$$

If Sebastian is 29 now, he will be 30 next year.

Check the answer. If Sebastian is 29 now, Rey is 17 now. Five years ago, they were 24 and 12, respectively, and 12 is half of 24.

14. **(C).** If all the values given in a problem and its answers are *percents, ratios*, or *fractions of* some unknown, then the problem will probably be easiest to solve by stipulating values for the unknowns. In this problem, the two ratios given are 3 : 1 (jeans sold : chinos sold) and 2 : 1 (profits per pair of chinos : profits per pair of jeans). The easiest numbers to stipulate are:

3 pairs of jeans sold
1 pair of chinos sold
$2 profit/pair of chinos
$1 profit/pair of jeans

This yields $2 profit from the chinos out of a total $5 in profit: 2/5 = 40%.

15. **49.** Write each sentence as its own equation:

$$M = 2V$$
$$(M - 8) = 3(V - 8) - 6$$

Simplify the second equation before substituting for M from the first equation into the second:

$$M - 8 = 3V - 24 - 6$$
$$M - 8 = 3V - 30$$
$$(2V) + 22 = 3V$$
$$22 = V$$

Thus, $M = 44$, and Marisol will be 49 years old in 5 years.

Check the answer. Eight years ago, Marisol was 36 and Vikram was 14. Three times Vikram's age at that time was 42, and Marisol was 6 years younger than that.

16. **28.** Convert this word problem into two equations with two variables. “The length is two more than twice the width” can be written as:

$L = 2W + 2$

Since the area is 40 and area is equal to length × width:

$LW = 40$

Since the first equation is already solved for L, plug $(2W + 2)$ in for L into the second equation:

$(2W + 2)W = 40$
$2W^2 + 2W = 40$

Since this is now a quadratic (there are both a W^2 and a W term), get all terms on one side to set the expression equal to zero:

$2W^2 + 2W - 40 = 0$

Simplify as much as possible—in this case, divide the entire equation by 2—before trying to factor:

$$\begin{aligned} W^2 + W - 20 &= 0 \\ (W - 4)(W + 5) &= 0 \\ W &= 4 \text{ or } -5 \end{aligned}$$

Since a width cannot be negative, the width is equal to 4. Since LW is equal to 40, the length must be 10. Now use the equation for perimeter to solve:

Perimeter = $2L + 2W$
Perimeter = $2(10) + 2(4)$
Perimeter = 28

Note that it might have been possible for you to puzzle out that the sides were 4 and 10 just by trying values. However, if you did this, you got lucky—no one said that the values even had to be integers! The ability to translate into equations and solve is very important for the GRE.

17. **(D).** To solve this problem, establish the following variables:

J = original jean price
B = original blouse price

Next, establish a system of equations, keeping in mind that “70% off” is the same as 100% – 70% = 30%, or 0.3, of the original price:

$$\begin{aligned} 0.3J + 12 &= 0.6B \\ 0.3J + 0.6B &= 84 \end{aligned}$$

Now use whatever strategy you're most comfortable with to solve a system of equations—for example, aligning the equations and then subtracting them:

$$\begin{array}{r} 0.3J + 12 = 0.6B \\ \underline{0.3J - 84 = -0.6B} \\ 0 + 96 = 1.2B \end{array}$$

$$B = \frac{96}{1.2}$$

$$B = 80$$

You can plug the price of the blouse back into the original equation to get the price of the jeans:

$$\begin{aligned} 0.3J + 12 &= 0.6B \\ 0.3J + 12 &= 48 \\ 0.3J &= 36 \\ J &= 120 \end{aligned}$$

Alternatively, you could first figure out the price of the discounted jeans, x, with this equation:

$$\begin{aligned} x + (x + 12) &= 84 \\ 2x + 12 &= 84 \\ 2x &= 72 \\ x &= 36 \end{aligned}$$

Then plug that discounted price into the equation *discounted price = original price × (100% – percent discount):*

$$\begin{aligned} 36 &= 0.3P \\ 360 &= 3P \\ 120 &= P \end{aligned}$$

18. **(D).** The equation for the perimeter of a space is $2W + 2L = P$, where W is width and L is length.

The equation for the area is $A = W \times L$. Thus:

$$\begin{aligned} 0.44(W \times L) &= 2W + 2L \\ 0.44(50W) &= 2W + 2(50) \\ 22W &= 2W + 100 \\ 20W &= 100 \\ W &= 5 \end{aligned}$$

If $W = 5$ and $L = 50$, then the area of the hallway is 250 sq. ft., and the total cost is: \$4.25 × 250 = \$1,062.50.

19. **(B).** Call the smallest bill *S*, the middle bill *M*, and the largest bill *L*.

M is the same as the median, since there are only three values. The equation for average is:

$$\frac{\text{Sum of bills}}{\text{Number of bills}} = \text{Average}$$

Incorporate the equation for averages into the following equation:

$$\frac{S+M+L}{3} = M + 44$$
$$S+M+L = 3M + 132$$
$$S+L = 2M + 132$$

While the individual values of *S* and *L* are not given, their sum is:

$$412 = 2M + 132$$
$$280 = 2M$$
$$140 = M$$

Finally, add *M* to the sum of *S* and *L*:

$140 + 412 = 552$

20. **(D).** Start by assigning variables:

C = Number of collies
L = Number of labs
G = Number of golden retrievers

According to the given information:

$$G = 66$$
$$L = 66 - 12$$
$$L = 54$$

Ratios work like fractions, and you can set them up accordingly:

$$\frac{5}{9} = \frac{C}{54}$$

Cross-multiplying and simplifying, you get:

$C = 30$

Now take the number of collies and express it as a percent of the total number of dogs:

Total # of Dogs = 30 + 54 + 66 = 150

$$\left(\frac{30}{150}\times 100\right)\% = 20\%$$

21. **(C).** The sum of Mason and Gunther's ages 4 years from now requires adding 4 to both ages.

The question asks for the following, the sum of Mason and Gunther's ages 4 years from now:

$$(M + 4) + (G + 4) = ?$$
$$M + G + 8 = ?$$

Since Mason is twice as old as Gunther was 10 years ago, put $(G - 10)$ in parentheses and build the second equation from there (the parentheses are crucial):

$$M = 2(G - 10)$$
$$M = 2G - 20$$

Note that the answer choices ask for the sum of the ages 4 years from now, in terms of G, so substitute for M (the variable you substitute for is the one that drops out).

Substituting from the second equation into the first:

$$(2G - 20) + G + 8 = ?$$
$$3G - 12 = ?$$

This matches choice (C).

Alternatively, you could write the second equation, $M = 2(G - 10)$, and then come up with two values that "work" in this equation for M and G. The easiest way to do this is to make up G, which will then tell you M. For instance, set $G = 12$ (use any number you want, as long as it's over 10, since the problem strongly implies that Gunther has been alive for more than 10 years):

$$M = 2(12 - 10)$$
$$M = 4$$

If Gunther is 12, then Mason is 4. In four years, they will be 16 and 8, respectively. Add these together to get 24.

Now, plug $G = 12$ into each answer choice to see which yields the correct answer (for this example), 24. Only choice (C) works.

22. **(E).** The equation for the situation described is $7x + 6y = 95$, where x stands for the number of batches of chocolate chip cookies and y stands for the number of batches of peanut butter cookies.

It looks as though this equation is not solvable, because there are two variables and only one equation. However, since the baker can only make whole batches, x and y must be integers, which really limits the possibilities.

Furthermore, the question asks for the *minimum* number of chocolate chip cookies the baker could have made. So, try 1 for x and see if you get an integer for y (use your calculator when needed!):

$$\begin{aligned} 7(1) + 6y &= 95 \\ 6y &= 88 \\ y &= 14.6\ldots \end{aligned}$$

Since this did not result in an integer number of batches of peanut butter cookies, this situation doesn't work. Try 2, 3, 4, etc. for x. (Don't try values out of order—remember, there might be more than one x value that works, but you need to be sure that you have the smallest one!)

The smallest value that works for x is 5:

$$\begin{aligned} 7(5) + 6y &= 95 \\ 6y &= 60 \\ y &= 10 \end{aligned}$$

Remember that you need the minimum number of chocolate chip *cookies*, not *batches of cookies*. Since the minimum number of batches is 5 and there are 7 cookies per batch, the minimum number of chocolate chip cookies is 35.

23. **(B).** First, translate the problem into two equations, writing "Anke after she gave Conrad 5 candies" as (A – 5) and "Conrad after receiving 5 more candies" as (C + 5):

$$\begin{aligned} A &= C - 5 \\ 4(A - 5) &= C + 5 \end{aligned}$$

Since $A = C - 5$, plug $C - 5$ in for A in the second equation:

$$\begin{aligned} 4(C - 5 - 5) &= C + 5 \\ 4(C - 10) &= C + 5 \\ 4C - 40 &= C + 5 \\ 4C &= C + 45 \\ 3C &= 45 \\ C &= 15 \end{aligned}$$

If $C = 15$, then, since Anke has 5 fewer candies, she has $A = 10$.

Check the answer. If Anke starts with 10 candies, after giving 5 to Conrad she has 5. If Conrad starts with 15 candies, he has 20 after receiving 5 from Anke. It is true that 20 is 4 times 5.

24. **(A).** The key to this tricky-sounding problem is setting up variables correctly and ensuring that you subtract or add appropriately for these variables when representing their ages at different points in time:

L = Lou's age now
W = Wen's age now
M = Mildred's age now
T = Tyla's age now

Two equations come from the second sentence of the problem:

Equation 1: $(L - 3) = 2(T - 3)$
Equation 2: $(L - 3) = (M - 3) + 30$

Another two equations come from the third sentence of the problem:

Equation 3: $L = W + 47$

Equation 4: $(W + 4) = \dfrac{(T + 4)}{2}$

In order to solve this problem effectively, look for ways to get two of the equations to have the same two variables in them. If you have two equations with only two variables, you can solve for both of those variables. Equation 4 has a *W* and a *T*; the only other equation with a *T* is Equation 1. If you substitute the *L* in Equation 1 with the *W* from Equation 3, you will have two equations with just *W*'s and *T*'s.

Equation 1: $(L - 3) = 2(T - 3)$

$$\begin{aligned} (W + 47) - 3 &= 2(T - 3) \\ W + 44 &= 2T - 6 \\ W + 50 &= 2T \end{aligned}$$

Equation 4: $(W + 4) = \dfrac{(T + 4)}{2}$

$2W + 8 = T + 4$
$2W + 4 = T$

Now combine the equations to solve for *W*.

$$\begin{aligned} W + 50 &= 2(2W + 4) \\ W + 50 &= 4W + 8 \\ W + 42 &= 4W \\ 42 &= 3W \\ 14 &= W \end{aligned}$$

Now that you know Wen's age, you can solve for the rest.

Equation 3: $L = W + 47$

$L = 14 + 47$
$L = 61$

Equation 1: $(L - 3) = 2(T - 3)$

$$\begin{aligned} (61 - 3) &= 2(T - 3) \\ 58 &= 2T - 6 \end{aligned}$$

$$64 = 2T$$

$$32 = T$$

Equation 2: $(L - 3) = (M - 3) + 30$

$$(61 - 3) = (M - 3) + 30$$

$$58 = M + 27$$

$$31 = M$$

Now that you know that $L = 61$, $W = 14$, $M = 31$, and $T = 32$, sum them to find the answer:

$$61 + 14 + 31 + 32 = 138$$

25. **(E).** This question is difficult to translate. Begin by finding two things that are equal, and build an equation around that equality. *Dwayne grew twice as many bushels of pinto beans as navy beans*:

2(bushels of navy beans) = (bushels of pinto beans)

Break that down further:

bushels of navy beans = acres of navy beans × bushels per acre of navy beans
bushels of pinto beans = acres of pinto beans × bushels per acre of pinto beans

So:

2(acres of navy beans × bushels per acre of navy beans) =
(acres of pinto beans × bushels per acre of pinto beans)

"Each acre of navy beans yielded 27 bushels and each acre of pinto beans yielded 36 bushels":

2 × 27 × (acres of navy beans) = 36 × (acres of pinto beans)

Number of acres planted with pinto beans = p

Number of acres planted with navy beans = $70 - p$

$$2 \times 27(70 - p) = 36p$$

$$54(70 - p) = 36p$$

$$3{,}780 - 54p = 36p$$

$$
\begin{aligned}
3{,}780 &= 90p \\
p &= 42
\end{aligned}
$$

Check the answer. If Dwayne planted 42 acres of pinto beans, he planted 28 acres of navy beans. The yield of pinto beans was 42 × 36 = 1,512 bushels. The yield of navy beans was 28 × 27 = 756 bushels, which was half the yield of pinto beans.

Chapter 18

Rates and Work

In This Chapter…

Rates and Work

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Running on a 10-mile loop in the same direction, Sue ran at a constant rate of 8 miles per hour and Rob ran at a constant rate of 6 miles per hour. If they began running at the same point on the loop, how many hours later did Sue complete exactly 1 more lap than Rob?

   (A) 3

   (B) 4

   (C) 5

   (D) 6

   (E) 7

2. Svetlana ran the first 5 kilometers of a 10-kilometer race at a constant rate of 12 kilometers per hour. If she completed the entire 10-kilometer race in 55 minutes, at what constant rate did she run the last 5 kilometers of the race, in kilometers per hour?

(A) 15

(B) 12

(C) 11

(D) 10

(E) 8

3. A standard machine fills paint cans at a rate of 1 gallon every 4 minutes. A deluxe machine fills gallons of paint at twice the rate of a standard machine. How many hours will it take a standard machine and a deluxe machine, working together, to fill 135 gallons of paint?

(A) 1

(B) 1.5

(C) 2

(D) 2.5

(E) 3

4. Wendy can build a birdhouse in 15 hours and Miguel can build an identical birdhouse in 10 hours. How many hours will it take Wendy and Miguel, working together at their respective constant rates, to build a birdhouse? (Assume that they can work on the same birdhouse without changing each other's work rate.)

(A) 5

(B) 6

(C) 7

(D) 8

(E) 9

5. Machine A, which produces 15 golf clubs per hour, fills a production lot in 6 hours. Machine B fills the same production lot in 1.5 hours. How many golf clubs does machine B produce per hour?

$\boxed{}$ golf clubs per hour

Davis drove from Amityville to Beteltown at 50 miles per hour, and returned by the same route at 60 miles per hour.

	Quantity A	**Quantity B**
6.	Davis's average speed for the round trip, in miles per hour	55

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7. If a turtle traveled $\frac{1}{30}$ of a mile in 5 minutes, what was its speed in miles

per hour?

(A) 0.02

(B) $0.1\overline{6}$

(C) 0.4

(D) 0.6

(E) 2.5

Akilah traveled at a rate of x miles per hour for $2x$ hours.

	Quantity A	Quantity B
8.	The number of miles Akilah traveled	$3x$

9. Claudette traveled the first $\frac{2}{3}$ of a 60-mile trip at 20 miles per hour (mph) and the remainder of the trip at 30 mph. How many minutes later would she have arrived if she had completed the entire trip at 20 mph?

$\boxed{}$ minutes

10. Rajesh traveled from home to school at 30 miles per hour. Then he returned home at 40 miles per hour, and finally he went back to school at 60 miles per hour, all along the same route. What was his average speed for the entire trip, in miles per hour?

(A) 32

(B) 36

(C) 40

(D) 45

(E) 47

11. Twelve workers pack boxes at a constant rate of 60 boxes in 9 minutes. How many minutes would it take 27 workers to pack 180 boxes, if all workers pack boxes at the same constant rate?

(A) 12

(B) 13

(C) 14

(D) 15

(E) 16

12. To service a single device in 12 seconds, 700 nanorobots are required, with all nanorobots working at the same constant rate. How many hours would it take for a single nanorobot to service 12 devices?

(A) $\frac{7}{3}$

(B) 28

(C) 108

(D) 1,008

(E) 1,680

13. If a baker made 60 pies in the first 5 hours of his workday, by how many pies per hour did he increase his rate in the last 3 hours of the workday in order to complete 150 pies in the entire 8-hour period?

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

14. Nine identical machines, each working at the same constant rate, can stitch 27 jerseys in 4 minutes. How many minutes would it take 4 such machines to stitch 60 jerseys?

(A) 8

(B) 12

(C) 16

(D) 18

(E) 20

15. Brenda walked a 12-mile scenic loop in 3 hours. If she then reduced her walking speed by half, how many hours would it take Brenda to walk the same scenic loop two more times?

(A) 6

(B) 8

(C) 12

(D) 18

(E) 24

16. A gang of criminals hijacked a train heading due south. At exactly the same time, a police car located 50 miles north of the train started driving south toward the train on an adjacent roadway parallel to the train track. If the train traveled at a constant rate of 50 miles per hour, and the police car traveled at a constant rate of 80 miles per hour, how long after the hijacking did the police car catch up with the train?

(A) 1 hour

(B) 1 hour and 20 minutes

(C) 1 hour and 40 minutes

(D) 2 hours

(E) 2 hours and 20 minutes

Each working at a constant rate, Rachel assembles a brochure every 10 minutes and Terry assembles a brochure every 8 minutes.

Quantity A	Quantity B
The number of minutes it will take Rachel and Terry, working together, to assemble 9 brochures	40

17.

18. With 4 identical servers working at a constant rate, a new Internet search provider processes 9,600 search requests per hour. If the search provider adds 2 more identical servers, and server work rate never varies, the search provider can process 216,000 search requests in how many hours?

(A) 15

(B) 16

(C) 18

(D) 20

(E) 24

19. If Sabrina can assemble a tank in 8 hours, and Janis can assemble a tank in 13 hours, then Sabrina and Janis working together at their constant respective rates can assemble a tank in approximately how many hours?

(A) 21

(B) 18

(C) 7

(D) 5

(E) 2

20. Phil collects virtual gold in an online computer game and then sells the virtual gold for real dollars. After playing 10 hours a day for 6 days, he collected 540,000 gold pieces. If he immediately sold this virtual gold at a rate of $1 per 1,000 gold pieces, what were his average earnings per hour, in real dollars?

(A) $5

(B) $6

(C) $7

(D) $8

(E) $9

21. After completing a speed training, Alyosha translates Russian literature into English at a rate of 10 more than twice as many words per hour as he was able to translate before the training. If he was previously able to translate 10 words per minute, how many words can he now translate in an hour?

(A) 30

(B) 70

(C) 610

(D) 1,210

(E) 1,800

22. Jenny takes 3 hours to sand a picnic table; Laila can do the same job in $\frac{1}{2}$ hour. Working together at their respective constant rates, Jenny and Laila can sand a picnic table in how many hours?

(A) $\frac{1}{6}$

(B) $\frac{2}{9}$

(C) $\frac{1}{3}$

(D) $\frac{3}{7}$

(E) $\frac{5}{6}$

23. Riders board the Jelly Coaster in groups of 4 every 15 seconds. If there are 200 people in front of Kurt in line, in approximately how many minutes will Kurt board the Jelly Coaster?

(A) 5

(B) 8

(C) 10

(D) 13

(E) 20

A team of 8 chefs produce 3,200 tarts in 5 days. All chefs produce tarts at the same constant rate.

	Quantity A	Quantity B
24.	The number of chefs needed to produce 3,600 tarts in 3 days	The number of days that 4 chefs need to produce 4,800 tarts

25. Working together at their respective constant rates, robot A and robot B polish 88 pounds of gemstones in 6 minutes. If robot A's rate of polishing is $\frac{3}{5}$ that of robot B, how many minutes would it take robot A alone to polish 165 pounds of gemstones?

(A) 15.75

(B) 18

(C) 18.75

(D) 27.5

(E) 30

26. Car A started driving north from point *X* traveling at a constant rate of 40 miles per hour. One hour later, car B started driving north from point *X* at a constant rate of 30 miles per hour. Neither car changed direction of travel. If each car started with 8 gallons of fuel, which is consumed at a rate of 30 miles per gallon, how many miles apart were the two cars when car A ran out of fuel?

(A) 30

(B) 60

(C) 90

(D) 120

(E) 150

27. One robot, working independently at a constant rate, can assemble a doghouse in 12 minutes. What is the maximum number of <u>complete</u> doghouses that can be assembled by 10 such identical robots, each working on <u>separate</u> doghouses at the same rate for $2\frac{1}{2}$ hours?

(A) 20

(B) 25

(C) 120

(D) 125

(E) 150

28. Working continuously 24 hours a day, a factory bottles Soda Q at a rate of 500 liters per second and Soda V at a rate of 300 liters per second. If twice as many bottles of Soda V as of Soda Q are filled at the factory each day, what is the ratio of the volume of a bottle of Soda Q to a bottle of Soda V?

(A) $\frac{3}{10}$

(B) $\frac{5}{6}$

(C) $\frac{6}{5}$

(D) $\frac{8}{3}$

(E) $\frac{10}{3}$

Rates and Work Answers

1. **(C).** If Sue completed exactly one more lap than Rob, she ran 10 more miles than Rob. If Rob ran *d* miles, then Sue ran *d* + 10 miles. Rob and Sue began running at the same time, so they ran for the same amount of time. Let *t* represent the time they spent running. Fill out a chart for Rob and Sue, using the formula Distance = Rate × Time (D = RT):

	D (miles)	=	R (miles per hour)	×	T (hours)
Rob	d	=	6	×	t
Sue	$d + 10$	=	8	×	t

There are two equations:

$d = 6t \quad d + 10 = 8t$

Substitute 6*t* for *d* in the second equation and then solve for *t*:

$$\begin{aligned} 6t + 10 &= 8t \\ 10 &= 2t \\ 5 &= t \end{aligned}$$

2. **(D).** To calculate Svetlana's speed during the second half of the race, first calculate how long it took her to run the first half of the race. Svetlana ran the first 5 kilometers at a constant rate of 12 kilometers per hour. These values can be used in the *D* = *RT* formula:

D (km)	=	R (km per hr)	×	T (hr)
5	=	12	×	t

Svetlana's time for the first part of the race is $\frac{5}{12}$ hours, or 25 minutes.

She completed the entire 10-kilometer race in 55 minutes, so she ran the last 5 kilometers in 55 – 25 = 30 minutes, or 0.5 hours. Now create another chart to find the rate at which she ran the last 5 kilometers:

D (km)	=	R (km per hr)	$\times$	T (hr)
5	=	r	$\times$	0.5

$$5 = 0.5r$$

$$10 = r$$

Svetlana ran the second half of the race at a speed of 10 kilometers per hour.

3. **(E).** The question asks for the amount of time in hours, convert the work rates from gallons per minute to gallons per hour. First, calculate the rate of the standard machine:

$$\frac{1 \text{ gallon}}{4 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{60 \text{ gallons}}{4 \text{ hours}} = 15 \text{ gallons per hour}$$

Since the deluxe machine's rate is twice the standard machine's rate, the deluxe machine can fill 15 × 2 = 30 gallons of paint per hour. Together, the machines can fill 15 + 30 = 45 gallons of paint per hour. Now apply the formula for work, $W = RT$:

$$135 = 45 \times T$$
$$3 = T$$

4. **(B).** Use two separate lines in a $W = RT$ chart, one for Wendy and one for Miguel, to calculate their respective rates. Building 1 birdhouse equals doing 1 unit of work:

	W (birdhouses)	=	R (birdhouses per hour)	×	T (hours)
Wendy	1	=	R_W	×	15
Miguel	1	=	R_M	×	10

Thus, Wendy's rate is $\frac{1}{15}$ birdhouses per hour, and Miguel's rate is $\frac{1}{10}$ birdhouses per hour. Since Wendy and Miguel are working together, add their rates:

	W (birdhouses)	=	R (birdhouses per hour)	×	T (hours)
Wendy + Miguel	1	=	$\frac{1}{15}+\frac{1}{10}$	×	t

Now solve for t by first combining the fractions:

$$1 = \left(\frac{1}{15} + \frac{1}{10} \right) t$$

$$1 = \left(\frac{2}{30} + \frac{3}{30} \right) t$$

$$1 = \left(\frac{5}{30} \right) t$$

$$\frac{30}{5} = t$$

$$6 = t$$

5. **60 golf clubs per hour.** First, calculate the size of a production lot. Machine A works at a rate of 15 golf clubs per hour and completes a production lot in 6 hours. Plug this information into the *W* = *RT* formula:

W (clubs)	$=$	R (clubs per hour)	$\times$	T (hours)
w	$=$	15	$\times$	6

w = 15 clubs per hour × 6 hours = 90 clubs

Therefore, a production lot consists of 90 golf clubs. Since machine *B* can complete the lot in 1.5 hours, use the *W* = *RT* chart a second time to calculate the rate for machine B:

W (clubs)	$=$	R (clubs per hour)	$\times$	T (hours)
90	$=$	r	$\times$	1.5

Make the calculation easier by converting 1.5 hours to $\frac{3}{2}$ hours:

$$90 = \frac{3}{2}r$$
$$\frac{2}{3} \times 90 = r$$
$$2 \times 30 = r$$
$$60 = r$$

6. **(B).** Never take an average speed by just averaging the two speeds (50 mph and 60 mph). Instead, use the formula Average Speed = Total Distance ÷ Total Time. Fortunately, for Quantitative Comparisons, you can often sidestep actual calculations.

Davis's average speed can be thought of as an average of the speed he was traveling at every single moment during his journey—for instance, imagine that Davis wrote down the speed he was going during every second he was driving, then he averaged all the seconds. Since Davis spent more *time* going 50 mph than going 60 mph, the average speed will be closer to 50 than 60, and Quantity B is greater. If the distances are the same, average speed is always weighted towards the *slower* speed.

To actually do the math, pick a convenient number for the distance between Amityville and Beteltown—for instance, 300 miles (divisible by both 50 and 60). If the distance is 300 miles, it took Davis 6 hours to drive there at 50 mph, and 5 hours to drive back at 60 mph. Using Average Speed = Total Distance ÷ Total Time (and a total distance of 600 miles, for both parts of the journey), you get the following:

$$\text{Average Speed} = \frac{600\text{ miles}}{11\text{ hours}}$$

Average Speed = 54.54 … (which is less than 55)

The result will be the same for any value chosen. Quantity B is greater.

7. **(C).** The turtle traveled $\frac{1}{30}$ of a mile in 5 minutes, which is $\frac{1}{12}$ of an hour. Using the $D = RT$ formula, solve for r:

D (mile)	=	R (miles per hour)	×	T (hours)
$\frac{1}{30}$	=	r	×	$\frac{1}{12}$

$\frac{1}{30} = \frac{1}{12}r$

$\frac{12}{30} = r$

$0.4 = r$

8. **(D).** Use $D = RT$:

Distance = $x(2x)$

Distance = $2x^2$

Which is greater, $2x^2$ or $3x$? If $x = 1$, then $3x$ is greater. But if $x = 2$, then $2x^2$ is greater.

Without information about the value of x, the relationship cannot be determined.

9. **20 minutes.** First, figure out how long it took Claudette to travel 60 miles under the actual conditions. The first leg of the trip was $\frac{2}{3}$ of 60 miles, or 40 miles. To travel 40 miles at a rate of 20 miles per hour, Claudette spent $\frac{40}{20}$ = 2 hours = 120 minutes. The second leg of the trip was the remaining 60 – 40 = 20 miles. To travel that distance at a rate of 30 miles per hour, Claudette spent $\frac{20}{30} = \frac{2}{3}$ hour = 40 minutes. In total, Claudette traveled for 120 + 40 = 160 minutes.

Now consider the hypothetical trip. If Claudette had traveled the whole distance of 60 miles at 20 miles per hour, the trip would have taken $\frac{60}{20}$ = 3 hours = 180 minutes.

Finally, compare the two trips. The real trip took 160 minutes, so the hypothetical trip would have taken 180 – 160 = 20 minutes longer.

10. **(C).** Do not just average the three speeds, as Rajesh spent more time at slower rates than at higher rates, weighting the average toward the slower rate(s). To compute the average speed for a trip, figure out the total distance and divide by the total time.

Pick a convenient distance from home to school, one that is divisible by 30, 40, and 60—say 120 miles (tough for Rajesh, but easier for you).

The first part of the journey (from home to school) took $\frac{120}{30}$ = 4 hours. The second part of the journey took $\frac{120}{40}$ = 3 hours. The third part of the journey took $\frac{120}{60}$ = 2 hours.

The total distance Rajesh traveled is 120 + 120 + 120 = 360 miles. The total time was 4 + 3 + 2 = 9 hours. Finally, his average speed for the entire trip was $\frac{360}{9}$ = 40 miles per hour.

11. **(A).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Use the first sentence to solve for an individual worker's rate. Plug in the fact that 12 workers pack boxes at a constant rate of 60 boxes in 9 minutes:

Work = *Individual Rate* × *Number of Workers* × *Time*

60 boxes = (R)(12)(9 minutes)

$R = \frac{5}{9}$ boxes per minute

In other words, each worker can pack $\frac{5}{9}$ of a box per minute. Plug that rate back into the formula, but use the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*

$$\begin{aligned} 180 &= \left(\frac{5}{9}\right)(27)(T) \\ 180 &= 15T \\ 12 &= T \end{aligned}$$

12. **(B).** To solve a Rates & Work problem with multiple workers, modify the

standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual nanorobot's rate, using the fact that 700 nanorobots can service 1 device in 12 seconds. Notice that the "work" here is 1 device:

Work = *Individual Rate* × *Number of Workers* × *Time*

$$1 \text{ device} = (R)(700)(12 \text{ seconds})$$

$$R = \frac{1}{8{,}400} \text{ devices per second}$$

That is, each nanorobot can service $\frac{1}{8,400}$ of a device in 1 second. Plug that rate back into the formula, but using the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*

$$12 \text{ devices} = \left(\frac{1}{8,400} \text{ devices per second}\right)(1)(T)$$

$$T = 100,800 \text{ seconds}$$

The answer is 100,800 seconds. Divide by 60 to convert this time to 1,680 minutes; divide by 60 again to get 28 hours.

13. **(D)**. The question asks by how many pies per hour the baker's rate of pie-making increased, so determine his rate for the first 5 hours and his rate in the last 3 hours. The difference is the ultimate answer:

Rate for last 3 hours – Rate for first 5 hours = Increase

The rate for the first 5 hours was 60 pies ÷ 5 hours = 12 pies per hour.

In the last 3 hours, the baker made 150 – 60 = 90 pies. The rate in the last 3 hours of the workday was thus 90 pies ÷ 3 hours = 30 pies per hour.

Now find the difference between the two rates of work:

30 pies per hour – 12 pies per hour = 18 pies per hour

14. **(E).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual machine's rate, using the fact that 9 machines can stitch 27 jerseys in 4 minutes:

Work = *Individual Rate* × *Number of Workers* × *Time*

27 jerseys = (R)(9)(4 minutes)

$R = \frac{3}{4}$ jersey per minute

That is, each machine can stitch $\frac{3}{4}$ of a jersey in 1 minute. Plug that rate back into the formula, but using the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*

$$6 = \left(\frac{3}{4}\right)(4)(T)$$

$$T = 20$$

15. **(C)**. This question compares an actual scenario with a hypothetical one. Start by figuring out the rate (speed) for Brenda's actual walk. Since she walked 12 miles in 3 hours, she walked at a rate of 12 ÷ 3 = 4 miles per hour.

Now, in the hypothetical situation, she would walk the loop two more times, for a total additional distance of 12 × 2 = 24 miles. Her hypothetical speed would be half of 4 miles per hour, or 2 miles per hour.

Walking 24 miles at a rate of 2 miles per hour would take Brenda 24 ÷ 2 = 12 hours.

Alternatively, note that both of the changes—doubling the distance and halving the rate—have the same effect: Each change makes the trip take twice as long as it would have before. So the time required for this hypothetical situation is multiplied by four: 3 × 4 = 12 hours.

16. **(C).** In this "chase" problem, the two vehicles are moving in the same direction, with one chasing the other. To determine how long it will take the rear vehicle to catch up, *subtract* the rates to find out how quickly the rear vehicle is gaining on the one in front.

The police car gains on the train at a rate of 80 – 50 = 30 miles per hour. Since the police car needs to close a gap of 50 miles, plug into the $D = RT$ formula to find the time:

$$50 = 30t$$

$$\frac{5}{3} = t$$

The time it takes to catch up is $\frac{5}{3}$ hours, or 1 hour and 40 minutes.

17. **(C).** "Cheat" off the easier quantity. In 40 minutes (from Quantity B), Rachel would assemble 40 ÷ 10 = 4 brochures and Terry would assemble 40 ÷ 8 = 5 brochures, for a total of 4 + 5 = 9 brochures. Thus, Quantity A is also 40, and the two quantities are equal.

18. **(A).** If the search provider adds 2 identical servers to the original 4, there are now 6 servers. Because 6 ÷ 4 = 1.5, the rate at which all 6 servers work is 1.5 times the rate at which 4 servers work:

9,600 searches per hour × 1.5 = 14,400 searches per hour

Now apply this rate to the given amount of work (216,000 searches), using the $W = RT$ formula:

$216{,}000 = 14{,}400 \times T$
$216{,}000 \div 14{,}400 = 15$ hours

19. **(D).** Since Sabrina and Janis are working together, add their rates. Sabrina completes 1 tank in 8 hours, so she works at a rate of $\frac{1}{8}$ tank per hour. Likewise, Janis works at a rate of $\frac{1}{13}$ tank per hour. Now, add these fractions:

$\frac{1}{8} + \frac{1}{13} = \frac{13}{104} + \frac{8}{104} = \frac{21}{104}$ tanks per hour, when working together.

Next, plug this combined rate into the $W = RT$ formula to find the time. You might also notice that since the work is equal to 1, the time will just be the reciprocal of the rate:

	Work (tank)	=	*Rate* (tanks per hour)	×	*Time* (min)
Sabrina & Janis:	1	=	$\frac{21}{104}$	×	$\frac{104}{21}$

At this point, you do not need to do long division or break out the calculator! Just approximate: $\frac{104}{21}$ is about 100 ÷ 20 = 5.

Alternatively, use some intuition to work the answer choices and avoid setting up this problem at all! You can immediately eliminate (A) and (B), since these times exceed either worker's individual time. Also, since Sabrina is the faster worker, Janis's contribution will be less than Sabrina's. The two together won't work twice as fast as Sabrina, but they will work *more* than twice as fast as Janis. Therefore, the total time should be more than half of Sabrina's individual time, and less than half of Janis's individual time. $4 < t < 6.5$, which leaves (D) as the only possible answer.

20. **(E).** To solve for average earnings, fill in this formula:

Total earnings ÷ Total hours = Average earnings per hour

Since the gold-dollar exchange rate is \$1 per 1,000 gold pieces, Phil's real dollar earnings for the 6 days were 540,000 ÷ 1,000 = \$540. His total time worked was 10 hours per day × 6 days = 60 hours. Therefore, his average hourly earnings were \$540 ÷ 60 hours = \$9 per hour.

21. **(D).** To find the new rate in words per hour, start by setting up an equation to find this value:

New words per hour = 10 + 2(Old words per hour)

The old rate was given in words per minute, so convert to words per hour:

10 words per min × 60 min per hour = 600 words per hour

Now plug into the equation:

New words per hour = 10 + 2(600) = 1,210

Note that it would be dangerous to start by working with the rate per minute. If you did so, you might calculate 10 + 2(10) = 30 words per minute, then 30 × 60 = 1,800 words per hour. This rate is inflated because you added an additional 10 words per minute instead of per hour. Perform the conversions right away!

22. **(D).** Since the two women are working together, add their rates. To find their individual rates, divide work by time. Never divide time by work! (Also, be careful when dividing the work by $\frac{1}{2}$. The rate is the reciprocal of $\frac{1}{2}$, or 2 tables per hour.)

Find Jenny and Laila's combined rate, then divide the work required (1 table) by this rate: 1 table ÷ $\frac{7}{3}$ table per hour = $\frac{3}{7}$ hour.

	Work (tables)	=	*Rate* (table per hour)	×	*Time* (hours)
Jenny	1	=	$\frac{1}{3}$	×	3
Laila	1	=	2	×	$\frac{1}{2}$
Jenny & Laila	1	=	$\frac{1}{3}+2=\frac{7}{3}$	×	$\frac{3}{7}$

23. **(D).** To find Kurt's wait time, determine how long it will take for 200 people to board the Jelly Coaster. The problem states that 4 people board every 15 seconds. Since there are four 15-second periods in one minute, this rate converts to 16 people per minute. To find the time, divide the "work" (the people) by this rate:

200 people ÷ 16 people per minute = 200 ÷ 16 = 12.5 minutes.

The question asks for an approximation, and the closest answer is (D). In theory there may be an additional 15 seconds while Kurt's group is boarding (the problem doesn't really say), but Kurt's total wait time would still be approximately 13 minutes.

24. **(C).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual chef's rate, using the fact that 8 chefs produce 3,200 tarts in 5 days:

Work = *Individual Rate* × *Number of Workers* × *Time*
3,200 tarts = (*R*)(8)(5 days)
R = 80 tarts per day

That is, each chef can produce 80 tarts per day. Plug that rate back into the formula for each of the quantities:

<u>Quantity A</u>
Work = *Individual Rate* × *Number of Workers* × *Time*
3,600 = (80)(*Number of Workers*)(3)
Number of Workers = 15

<u>Quantity B</u>
Work = *Individual Rate* × *Number of Workers* × *Time*
4,800 = (80)(4)(Time)
Time = 15 days

The number of chefs in Quantity A is equal to the number of days in Quantity B.

25. **(E).** When rate problems involve multiple situations, it can help to set up an initial "skeleton" *W* = *RT* chart for the solution. That way, you can determine what data is needed and fill in that data as you find it. Since the question asks how long robot A will take alone, the chart will look like this:

	Work (pounds)	=	*Rate* (pounds per min)	×	*Time* (min)
Robot A	165	=	*A*'s rate	×	*t*

Work is known and the question asks for time, so robot *A*'s rate is needed. Call the rates *a* and *b*. Now set up another chart representing what you know about the two robots working together.

	Work (pounds)	=	*Rate* (pounds per min)	×	*Time* (min)
Robot A	$6a$	=	a	×	6
Robot B	$6b$	=	b	×	6
A & B together	$6(a + b) = 88$	=	$a + b$	×	6

Now, $6(a + b) = 88$ and, from the question stem, robot A's rate is $\frac{3}{5}$ of B's rate. This can be written as $a = \frac{3}{5}b$. To solve for a, substitute for b:

$$a = \left(\frac{3}{5}\right)b$$

$$\left(\frac{5}{3}\right)a = b$$

$$6\left(a + \left(\frac{5}{3}\right)a\right) = 88$$

$$6\left(\frac{8}{3}\right)a = 88$$

$$\left(\frac{48}{3}\right)(a) = 88$$

$$a = 88\left(\frac{3}{48}\right)$$

$$a = 88\left(\frac{1}{16}\right) = \left(\frac{88}{16}\right) = \frac{11}{2}$$

So A's rate is $\frac{11}{2}$ pounds per minute. Now just plug into the original chart:

	Work (pounds)	=	*Rate* (pounds per min)	×	*Time* (min)
Robot A	165	=	$\frac{11}{2}$	×	30

The time robot A takes to polish 165 pounds of gems is $165/\frac{11}{2} = \frac{330}{165} = 30$

minutes.

26. **(C)**. The question asks (indirectly) how far the two cars traveled, as those distances are necessary to find the distance between them. Since the cars go in the same direction, the skeleton equation is as follows:

> Car A's distance – Car B's distance = Distance between cars
> All distances refer to the time when car A ran out of fuel.

Since the limiting factor in this case is A's fuel supply, calculate how far the car is able to drive before running out of fuel. This in itself is a rate problem of sorts:

> 30 miles per gallon × 8 gallons = 240 miles

So car A will end up 240 miles north of its starting point, which happens 240 ÷ 40 = 6 hours after it started. What about car B? It started an hour later and thus traveled (30 miles per hour)(6 hours – 1 hour) = 150 miles by that time.

Therefore, the two cars were 240 – 150 = 90 miles apart when car A ran out of fuel.

27. **(C)**. Note that choice (D) is a trap. This issue is relatively rare, but it's worthwhile to be able to recognize it if you see it. In this case, each robot is *independently* assembling complete doghouses. Since the question asks for the number of *completed* doghouses after $2\frac{1}{2}$ hours, any *incomplete* doghouses must be removed from the calculations.

Since one robot completes a doghouse in 12 minutes, the individual hourly rate is 60 ÷ 12 = 5 doghouses per hour.

Therefore, each robot produces 5 × 2.5 = 12.5 doghouses in $2\frac{1}{2}$ hours. (Or just divide the 150 total minutes by 12 minutes per doghouse to get the same result.)

However, the questions asks about *completed* doghouses, and the robots are working independently, so drop the decimal. Each robot completes only 12 complete doghouses in the time period, for a total of 12 × 10 = 120 doghouses.

28. **(E)**. If twice as many bottles of Soda V as of Soda Q are filled at the factory each day, then twice as many bottles of Soda V as of Soda Q are filled at the factory each second.

Use smart numbers for the number of bottles filled each second. Since twice as many bottles of Soda V are produced, the output in one second could be 100 bottles of Soda V and 50 bottles of Soda Q. Using these numbers, the volume of the Q bottles is 500 liters ÷ 50 bottles = 10 liters per bottle and the volume of the V bottles is 300 liters ÷ 100 bottles = 3 liters per bottle. The ratio of the volume of a bottle of Q to a bottle of V is 10 liters ÷ 3 liters = $\frac{10}{3}$

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Chapter 19

Variables-in-the-Choices Problems

In This Chapter…

Variables-in-the-Choices Problems

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If Josephine reads b books per week and each book has, on average, 100,000 words, which best approximates the number of words Josephine reads per day?

   (A) $100{,}000b$

   (B) $\dfrac{100{,}000b}{7}$

   (C) $\dfrac{700{,}000}{b}$

   (D) $\dfrac{7b}{100{,}000}$

(E) $\frac{100{,}000b}{(7)(24)}$

2. The width of a rectangle *w* is twice the length of the rectangle. Which of the following equals the area of the rectangle in terms of *w*?

(A) w

(B) $2w^2$

(C) $3w^2$

(D) $\frac{w^2}{2}$

(E) $\frac{w^2}{4}$

3. A clothing store bought 100 shirts for $\$x$. If the store sold all of the shirts at the same price for a total of \$50, what is the store's profit per shirt, in dollars, in terms of x?

(A) $50 - \dfrac{x}{100}$

(B) $50 - x$

(C) $5 - x$

(D) $0.5 - x$

(E) $0.5 - \dfrac{x}{100}$

4. Two trees have a combined height of 60 feet, and the taller tree is x times the height of the shorter tree. How tall is the shorter tree, in terms of x?

(A) $\dfrac{60}{1+x}$

(B) $\dfrac{60}{x}$

(C) $\dfrac{30}{x}$

(D) $60 - 2x$

(E) $30 - 5x$

5. Louise is three times as old as Mary. Mary is twice as old as Natalie. If Louise is L years old, what is the average (arithmetic mean) age of the three women, in terms of L?

(A) $\dfrac{L}{3}$

(B) $\dfrac{L}{2}$

(C) $\dfrac{2L}{3}$

(D) $\dfrac{L}{4}$

(E) $\frac{L}{6}$

6. Toshi is four times as old as Kosuke. In x years Toshi will be three times as old as Kosuke. How old is Kosuke, in terms of x?

(A) $2x$

(B) $3x$

(C) $4x$

(D) $8x$

(E) $12x$

7. A shirt that costs k dollars is increased by 30%, then by an additional 50%. What is the new price of the shirt in dollars, in terms of k?

(A) $0.2k$

(B) $0.35k$

(C) $1.15k$

(D) $1.8k$

(E) $1.95k$

8. Carlos runs a lap around the track in x seconds. His second lap is five seconds slower than the first lap, but the third lap is two seconds faster than the first lap. What is Carlos's average (arithmetic mean) number of minutes per lap, in terms of x?

(A) $x - 1$

(B) $x + 1$

(C) $\frac{x-1}{60}$

(D) $\frac{x+1}{60}$

(E) $\frac{x+3}{60}$

9. Andrew sells vintage clothing at a flea market at which he pays \$150 per day to rent a table plus \$10 per hour to his assistant. He sells an average of \$78 worth of clothes per hour. Assuming no other costs, which of the functions below best represents profit per day, P, in terms of hours, h, that the flea market table is open for business?

(A) $P(h) = 238 - 10h$

(B) $P(h) = 72 - 10h$

(C) $P(h) = 68h - 150$

(D) $P(h) = 78h - 160$

(E) $P(h) = -160h + 78$

10. If a, b, c, and d are consecutive integers and $a < b < c < d$, what is the average (arithmetic mean) of a, b, c, and d in terms of d?

(A) $d - \frac{5}{2}$

(B) $d - 2$

(C) $d - \frac{3}{2}$

(D) $d + \frac{3}{2}$

(E) $\frac{4d - 6}{7}$

11. Cheese that costs c cents per ounce costs how many dollars per pound? (16 ounces = 1 pound and 100 cents = 1 dollar)

(A) $\frac{4c}{25}$

(B) $\frac{25c}{4}$

(C) $\frac{25}{4c}$

(D) $\frac{c}{1{,}600}$

(E) $1{,}600c$

12. A bag of snack mix contains 3 ounces of pretzels, 1 ounce of chocolate chips, 2 ounces of mixed nuts, and x ounces of dried fruit by weight. What percent of the mix is dried fruit, by weight?

(A) $\frac{x}{600}$

(B) $\frac{100}{6x}$

(C) $\frac{100x}{6}$

(D) $\frac{100x}{6+x}$

(E) $\frac{x}{100(6+x)}$

13. At her current job, Mary gets a 1.5% raise twice per year. Which of the following choices represents Mary's current income y years after starting the job at a starting salary of s?

(A) $s(1.5)^{2y}$

(B) $s(0.015)^{2y}$

(C) $s(1.015)^{2y}$

(D) $s(1.5)^{\frac{y}{2}}$

(E) $s(1.015)^{\frac{y}{2}}$

14. Phone plan A charges $1.25 for the first minute and $0.15 for every minute thereafter. Phone plan B charges a $0.90 connection fee and $0.20 per minute. Which of the following equations could be used to find the length, in minutes, of a phone call that costs the same under either plan?

(A) $1.25 + 0.15x = 0.90x + 0.20$

(B) $1.25 + 0.15x = 0.90 + 0.20x$

(C) $1.25 + 0.15(x - 1) = 0.90 + 0.20x$

(D) $1.25 + 0.15(x - 1) = 0.90 + 0.20(x - 1)$

(E) $1.25 + 0.15x + 0.90x + 0.20 = x$

15. If powdered drink mix costs c cents per ounce and p pounds of it are purchased by a supplier who intends to resell it, what will be the total revenue, in dollars, in terms of c and p if all of the drink mix is sold at a price per ounce equivalent to three times what the supplier paid? (16 ounces = 1 pound and 100 cents = 1 dollar)

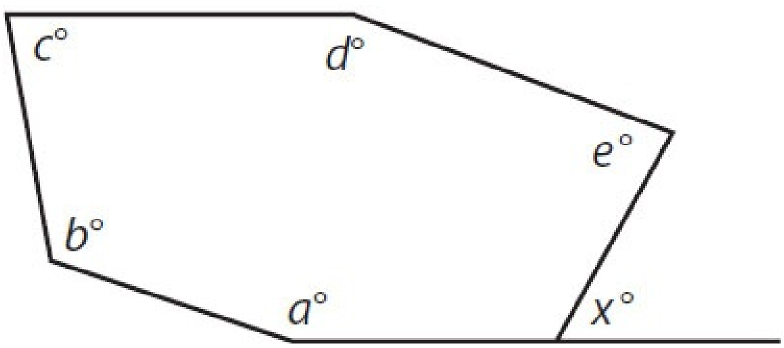
(A) $48cp$

(B) $\frac{32cp}{100}$

(C) $\frac{100(32)}{cp}$

(D) $\frac{12cp}{25}$

(E) $\frac{25cp}{12}$



16. If $d = 2c$ and $e = \frac{1}{2}a$, what is x in terms of a, b, and c?

(A) $\frac{3}{2}a + b + 3c - 540$

(B) $\frac{3}{2}a + b + 3c$

(C) $720 - \frac{3}{2}a - b - 3c$

(D) $720 - \frac{1}{2}a - b - 2c$

(E) $540 - \frac{1}{2}a - b - \frac{3}{2}c$

17. a, b, and c are three consecutive odd integers such that $a < b < c$. If a is halved to become m, b is doubled to become n, c is tripled to become p, and $k = mnp$, which of the following is equal to k in terms of a?

(A) $3a^3 + 18a^2 + 24a$

(B) $3a^3 + 9a^2 + 6a$

(C) $\frac{11}{2}a + 16$

(D) $6a^2 + 36a + 24$

(E) $a^3 + 6a^2 + 4a$

18. If m pencils cost the same as n pens, and each pencil costs 20 cents, what is the cost, in dollars, of 10 pens, if each pen costs the same amount? (100 cents = 1 dollar)

(A) $\frac{200n}{m}$

(B) $\frac{2n}{100m}$

(C) $\frac{2m}{n}$

(D) $\frac{2n}{m}$

(E) $200mn$

19. Randi sells forklifts at a dealership where she makes a base salary of \$2,000 per month, plus a commission equal to 5% of the selling price of the first 10 forklifts she sells that month, and 10% of the value of the selling price of any forklifts after that. If all forklifts have the same sale price, s, which of the choices below represents Randi's monthly pay, P, as a function of number of forklifts sold, f, in months in which she sells more than 10 forklifts? (Assume Randi's pay is made up entirely of base salary and commission, and no deductions are taken from this pay.)

(A) $P = 2{,}000 + 0.05sf + 0.10sf$

(B) $P = 2{,}000 + 0.05sf + 0.10s(f - 10)$

(C) $P = 2{,}000 + 0.05s + 0.10\text{s}(f - 10)$

(D) $P = 2{,}000 + 0.5s + 0.10sf - 10$

(E) $P = 2{,}000 + 0.5s + 0.10s(f - 10)$

20. If the width of a rectangle is w, the length is l, the perimeter is p, and $w = 2l$, what is the area in terms of p?

(A) $\frac{p^2}{18}$

(B) $\frac{p^2}{36}$

(C) $\frac{p}{9}$

(D) $\frac{p^2}{9}$

(E) $\frac{p}{6}$

Variables-in-the-Choices Problems Answers

1. **(B).** Since Josephine reads b books per week and each book has an average of 100,000 words, she reads 100,000b words per week. However, the question asks for words per *day,* so divide this quantity by 7.

Alternatively, you could try picking numbers. Notice that the question talks about weeks and days, so think about a number that is divisible by the 7 days in the week. If $b = 14$, for instance, then Josephine would read 14 books per week, or 2 books per day. This is equivalent to reading 200,000 words each day. Plug 14 in for b in each answer choice, and only (B) results in 200,000.

2. **(D).** Since width is twice length, write $w = 2L$. However, the question requires an answer in terms of w, so solve for L:

$$L = \frac{w}{2}$$

Since area is $L \times W$ and $L = \frac{w}{2}$:

$$A = \frac{w}{2} \times W$$

Therefore, $A = \frac{w^2}{2}$, or choice (D).

Alternatively, pick values. If width were 4, length would be 2. The area would therefore be $4 \times 2 = 8$. Plug in 4 for w to see which answer choice yields 8. Only (D) works.

3. **(E).** This problem requires the knowledge that profit equals revenue minus cost. You could memorize the formula Profit = Revenue – Cost (or Profit = Revenue – Expenses), or just think about it logically—a business has to pay its expenses out of the money it makes: the rest is profit.

The revenue for all 100 shirts was \$50, and the cost to purchase all 100 shirts was \$$x$. Therefore:

Total profit = $50 - x$

The question does not ask for the total profit, but for the profit per shirt. The store sold 100 shirts, so divide the total profit by 100 to get the profit per shirt:

Profit per shirt = $\frac{50 - x}{100}$

None of the answer choices match this number, so you need to simplify the fraction. Split the numerator into two separate fractions:

$$\frac{50 - x}{100} = \frac{50}{100} - \frac{x}{100} = 0.5 - \frac{x}{100}$$

4. **(A).** First, define variables. Let s = the height of the shorter tree. Let t = the height of the taller tree.

If the combined height of the trees is 60 feet, then:

$$s + t = 60$$

The question also states that the height of the taller tree is x times the height of the shorter tree:

$$t = xs$$

In order to solve for the height of the shorter tree, substitute (xs) for t in the first equation:

$$s + (xs) = 60$$

Then isolate s by factoring it out of the left side of the equation:

$$s(1 + x) = 60$$

$$s = \frac{60}{1+x}$$

5. **(B).** First, express all three women's ages in terms of L. If Louise is three times as old as Mary, then Mary's age is $\frac{L}{3}$.

You also know that Mary is twice as old as Natalie. If Mary's age is $\frac{L}{3}$, then Natalie's age is $\frac{1}{2}$ of that, or $\frac{L}{6}$.

Now plug those values into the average formula. The average of the three ages is:

$$\text{Average} = \frac{L + \frac{L}{3} + \frac{L}{6}}{3}$$

To eliminate the fractions in the numerator, multiply the entire fraction by $\frac{6}{6}$:

$$\frac{6}{6}\times\left(\frac{L+\frac{L}{3}+\frac{L}{6}}{3}\right)=\frac{6L+2L+L}{18}=\frac{9L}{18}=\frac{L}{2}$$

6. **(A).** Let T = Toshi's age; (T + x) = Toshi's age in x years
Let K = Kosuke's age; (K + x) = Kosuke's age in x years

If Toshi is four times as old as Kosuke, then T = 4K.

To translate the second sentence correctly, remember to use (T + x) and (K + x) to represent their ages:

$$(T + x) = 3(K + x)$$

The question asks for Kosuke's age in terms of x, so replace T with $(4K)$ in the second equation:

$$\begin{aligned}(4K) + x &= 3K + 3x \\ K + x &= 3x \\ K &= 2x\end{aligned}$$

7. **(E).** If the cost of the shirt is increased 30%, then the new price of the shirt is 130% of the original price. If the original price was k, then the new price is $1.3k$.

Remember that it is this new price that is increased by 50%. Multiply $1.3k$ by 1.5 (150%) to get the final price of the shirt:

$$1.3k \times 1.5 = 1.95k$$

8. **(D).** Carlos's lap times can be expressed as x, $x + 5$, and $x - 2$. (Remember, *slower* race times are *greater* numbers, so "five seconds slower" means *plus* 5, not *minus* 5!) Average the lap times:

$$\frac{x+(x+5)+(x-2)}{3} = \frac{3x+3}{3} = x+1$$

His average time is $x + 1$ *seconds*. But the question requires *minutes*. Since there are 60 seconds in a minute, divide by 60 to get $\frac{x+1}{60}$, or choice (D).

Alternatively, pick values. If x were 60 seconds, for example, Carlos's lap times would be 60, 65, and 58. His average time would be 61 seconds, or 1 minute and 1 second, or $1\frac{1}{60}$ minutes, or $\frac{61}{60}$ minutes. Plug in $x = 60$ to see which value yields $\frac{61}{60}$. Only (D) works.

9. **(C).** For every hour Andrew's business is open, he sells \$78 worth of clothes but pays \$10 to his assistant. Thus, he is making \$68 an hour after paying the assistant. He also must pay \$150 for the whole day.

Using Revenue – Expenses = Profit and h for hours he is open, you get the following equation:

$$\text{Profit} = 68h - 150$$

Written as a function of profit in terms of hours, this is $P(h) = 68h - 150$, or choice (C).

Be careful that you are reading the answer choices as *functions*. P is not a variable that is being multiplied by h! P is the *name* of the function and h is the variable on which the output of the function depends.

Note that (D) is a very good trap—this formula represents what the profit would be if Andrew only had to pay the assistant $10 *total*. However, he pays the assistant $10 *per hour.*

Alternatively, pick numbers. If Andrew were open for an 8-hour day (that is, test $h = 8$), he would make \$68 an hour (\$78 of sales minus \$10 to the assistant), or \$544 total. Subtract the \$150 rental fee to get \$394.

Then, plug 8 into the answer choices in place of h to see which answer yields 394. Only (C) works.

10. **(C).** Since a, b, c, and d are consecutive and d is largest, you can express c as $d - 1$, b as $d - 2$, and a as $d - 3$. Therefore, the average is:

$\frac{(d-3)+(d-2)+(d-1)+d}{4} = \frac{4d-6}{4} = d - \frac{6}{4}$ or $d - \frac{3}{2}$, which matches choice (C).

Alternatively, plug in numbers. Say a, b, c, and d are 1, 2, 3, and 4. (Generally, you want to avoid picking the numbers 0 and 1, lest *several* of the choices appear to be correct and you have to start over, but since only d appears in the choices, it's no problem that a is 1 in this example.)

Thus, the average would be 2.5. Plug in 4 for d to see which choice yields an answer of 2.5. Only (C) works.

11. **(A).** If cheese costs c cents per ounce, it costs $16c$ cents per pound. To convert from cents to dollars, divide by 100:

$\frac{16c}{100} = \frac{4c}{25}$, or choice (A).

Alternatively, pick numbers. If $c = 50$, a cheese that costs 50 cents per ounce would cost 800 cents, or \$8, per pound. Plug in $c = 50$ and select the answer that gives the answer 8. Only (A) works.

12. **(D).** To figure out what *fraction* of the mix is fruit, put the amount of fruit over the total amount of the mix: $\frac{x}{6+x}$. To convert a fraction to a percent, multiply by 100: $\frac{x}{6+x}(100) = \frac{100x}{6+x}$, or answer choice (D).

Alternatively, pick smart numbers. For instance, say $x = 4$. In that case, the total amount of mix would be 10 ounces, 4 of which would be dried fruit. Since $\frac{4}{10} = 40\%$, the answer to the question for your example would be 40%.

Now, plug $x = 4$ into each answer choice to see which yields 40%. Only choice (D) works: $\frac{100(4)}{6+(4)} = \frac{400}{10} = 40$. This will work for any number you choose for x, provided that you correctly calculate what percent of the mix would be dried fruit in your particular example.

13. **(C).** To increase a number by 1.5%, first convert 1.5% to a decimal by dividing by 100 to get 0.015.

Do *not* multiply the original number by 0.015—this approach would be very inefficient, because multiplying by 0.015 would give you only the increase, not the new amount (you would then have to add the increase back to the original amount, a process so time-wasting and inefficient that it would not likely appear in a formula in the answer choices).

Instead, multiply by 1.015. Multiplying by 1 keeps the original number the same; multiplying by 1.015 gets you the original number plus 1.5% more.

Finally, if you want to multiply by 1.015 twice per year, you will need to do it $2y$ times. This $2y$ goes in the exponent spot to give you $s(1.015)^{2y}$, or choice (C).

14. **(C).** Write an equation to find the cost of a call under plan A, using x as the number of minutes:

$$\text{Cost} = 1.25 + 0.15(x - 1)$$

Note that you need to use $x - 1$ because the caller does *not* pay $0.15 for every single minute—the first minute was already paid for by the $1.25 charge.

Now write an equation to find the cost of a call under plan *B*, using x as the number of minutes:

$$\text{Cost} = 0.90 + 0.20x$$

Note that here you do *not* use $x - 1$ because the connection fee does not "buy" the first minute—the plan costs $0.20 for every minute.

To find the length of a call that would cost the same under either plan, set the two equations equal to one another:

$$1.25 + 0.15(x - 1) = 0.90 + 0.20x$$

This is choice (C). Note that you are not required to solve this equation, but you might be required to solve a similar equation in a different problem on this topic:

$$\begin{aligned} 1.25 + 0.15x - 0.15 &= 0.90 + 0.20x \\ 1.1 + 0.15x &= 0.90 + 0.20x \\ 0.20 &= 0.05x \\ 20 &= 5x \\ 4 &= x \end{aligned}$$

A 4-minute call would cost the same under either plan. To test this, calculate the cost of a 4-minute call under both plans: it's $1.70 either way.

15. **(D).** The mix costs c cents per ounce. Since you want the final answer in dollars, convert right now:

$$c \text{ cents per ounce} = \frac{c}{100} \text{ dollars per ounce}$$

The supplier then purchases *p* pounds of mix. You cannot just multiply *p* by $\frac{c}{100}$, because *p* is in pounds and $\frac{c}{100}$ is in dollars per *ounce*. Since there are 16 ounces in a pound, it makes sense that a pound would cost 16 times more than an ounce:

$$\frac{c}{100} \text{ dollars per ounce} = \frac{16c}{100} \text{ dollars per pound}$$

Reduce to get $\frac{4c}{25}$ dollars per pound.

Multiply by *p*, the number of pounds, to get what the supplier paid: $\frac{4cp}{25}$ dollars.

Now, the supplier is going to sell the mix for three times what he or she paid. (Don't worry that the problem says three times the "price per ounce"—whether you measure in ounces or pounds, this stuff just got three times more expensive.)

Thus, $\frac{4cp}{25} \times 3 = \frac{12cp}{25}$, or answer choice (D).

Note: Make sure to calculate for revenue, not profit! The question did not require subtracting expenses (what the supplier paid) from the money he or she will be making from selling the mix.

Alternatively, plug in smart numbers. An easy number to pick when working with cents is 50 (or 25—whatever is easy to think about and convert to dollars). Write a value on your paper along with what the value means in words:

$c = 50$ mix costs 50¢ per ounce

Now, common sense (and the fact that 16 ounces = 1 pound) will allow you to convert:

50¢ per ounce = $8.00 per pound

The supplier bought *p* pounds. Pick any number you want. For example:

$p = 2$ bought 2 pounds, so spent $16

Notice that no one asked for this $16 figure, but when calculating with smart numbers, it's best to write down next steps in the reasoning process.

Finally, the supplier is going to sell the mix for three times what he or she paid, so the supplier will sell it for $48.

Plug in $c = 50$ and $p = 2$ to see which answer choice generates 48. Only (D) works.

16. **(A).** Since the figure has six sides, use the formula $(n - 2)(180)$, where *n* is the number of sides, to figure out that the sum of the angles inside the

figure is equal to (6 – 2)(180) = 720.

The angle supplementary to x can be labeled as 180 – x (since two angles that make up a straight line must sum to 180). Thus:

$$a + b + c + d + e + 180 - x = 720$$

$$a + b + c + d + e - x = 540$$

Solve for x. Since x is being subtracted from the left side, it would be easiest to add x to both sides, and get everything else on the opposite side.

$$a + b + c + d + e - x = 540$$
$$a + b + c + d + e = 540 + x$$
$$a + b + c + d + e - 540 = x$$

Since $d = 2c$ and $e = \frac{1}{2}a$ and the answers are in terms of *a, b,* and *c*, you need to make the *d* and *e* drop out of $a + b + c + d + e - 540 = x$.

Fortunately, $d = 2c$ and $e = \frac{1}{2}a$ are already solved for *d* and *e,* the variables that need to drop out. Substitute:

$$a + b + c + 2c + \frac{1}{2}a - 540 = x$$
$$\frac{3}{2}a + b + 3c - 540 = x$$

This is a match with answer choice (A).

Alternatively, pick numbers. To do this, use the formula $(n - 2)(180)$, where *n* is the number of sides, to figure out that the sum of the angles inside the figure: $(6 - 2)(180) = 720$. Then, pick values for *a, b, c, d,* and *e,* so that $d = 2c$ and $e = \frac{1}{2}a$:

$a = 100$

$b = 110$

$c = 120$

$d = 240$ (This is twice the value picked for *c*.)

$e = 50$ (This is $\frac{1}{2}$ the value picked for *a*.)

Subtract all of these values from 720 to get that the unlabeled angle, for this example, is equal to 100. This makes *x* equal to $180 - 100 = 80$.

Now plug $a = 100$, $b = 110$, and $c = 120$ into the answers to see which formula yields a value of 80. (A) is the correct answer.

17. **(A).** One algebraic solution involves defining all three terms in terms of *a*. Since the terms are consecutive odd integers, they are 2 apart from each other, as such:

$$a$$

$$b = a + 2$$

$$c = a + 4$$

Then, *a* is halved to become *m*, *b* is doubled to become *n*, and *c* is tripled to become *p*, so:

$$\frac{1}{2}a = m$$

$$2b = n$$

$$2(a + 2) = n$$

$$2a + 4 = n$$

$$3c = p$$

$$3(a + 4) = p$$

$$3a + 12 = p$$

Since $k = mnp$, multiply the values for m, n, and p:

$$k = \left(\frac{1}{2}a\right)(2a + 4)(3a + 12)$$

$$k = \left(\frac{1}{2}a\right)(6a^2 + 24a + 12a + 48)$$

$$k = \left(\frac{1}{2}a\right)(6a^2 + 36a + 48)$$

$$k = 3a^3 + 18a^2 + 24a$$

This is a match with answer choice (A).

A smart numbers solution would be to pick three consecutive odd integers for a, b, and c. When picking numbers for a Variables-in-the-Choices problem, avoid picking 0, 1, or any of the numbers in the problem (this can sometimes cause more than one answer to appear to be correct, thus necessitating starting over with another set of numbers). So:

$a = 3$
$b = 5$
$c = 7$

Then, a is halved to become m, b is doubled to become n, and c is tripled to become p, so:

$1.5 = m$
$10 = n$

$21 = p$

Since $k = mnp$, multiply the values for m, n, and p:

$k = (1.5)(10)(21)$
$k = 315$

Now, plug $a = 3$ (the value originally selected) into the answer choices to see which choice equals 315. Only (A) works.

Because the correct answer is a mathematical way of writing the situation described in the problem, this will work for any value you pick for a, provided that a, b, and c are consecutive odd integers and you calculate k correctly.

18. **(C).** The phrase "*m* pencils cost the same as *n* pens" can be written as an equation, using *x* for the cost per pencil and *y* for the cost per pen:

$$mx = ny$$

Keep in mind here that *m* stands for the *number* of pencils and *n* for the *number* of pens (not the cost). Now, since pencils cost 20 cents, or $0.2 (the answer needs to be in dollars, so convert to dollars now), substitute in for *x*:

$$0.2m = ny$$

Solve for *y* to get the cost of 1 pen:

$$y = \frac{0.2m}{n}$$

Since *y* is the cost of 1 pen and $y = \frac{0.2m}{n}$, multiply by 10 to get the cost of 10 pens:

$$10y = 10\left(\frac{0.2m}{n}\right)$$

$$10y = \frac{2m}{n}$$

Thus, the answer is $\frac{2m}{n}$, or (C).

Alternatively, plug in smart numbers. Since pencils cost 20 cents, maybe pens cost 40 cents (you can arbitrarily pick this number). The question states that "*m* pencils cost the same as *n* pens"—pick a number for one of these variables, and then determine what the other variable would be for the example you've chosen. For instance, if $m = 10$, then 10 pencils would cost $2.00. Since 5 pens can be bought for $2.00, *n* would be 5. Now, answer the final question as a number: the cost of 10 pens in this example is $4.00, so the final answer is 4. Plug in $m = 10$ and $n = 5$ to all of the answer choices to see which yields an answer of 4. Only (C) works. For any working system you choose in which "*m* pencils cost the same as *n* pens," choice (C) will work.

19. **(E).** One way to do this problem is to construct a formula. Randi's pay is equal to $2,000 plus commission:

$P = 2000 + \ldots$

The question only asks about Randi's pay in months in which she sells more than 10 forklifts, so she will definitely be receiving 5% commission on 10 forklifts that each cost s. Since the revenue from the forklifts would then be $10s$, Randi's commission would be $0.05(10s)$, or $0.5s$:

$P = 2{,}000 + 0.5s + \ldots$

Now, add the commission for the forklifts she sells above the first 10. Since these first 10 forklifts are already accounted for, denote the forklifts at this commission level by writing $f - 10$. Since each forklift still costs s, the revenue from these forklifts would be $s(f - 10)$. Since Randi receives 10% of this as commission, the amount she receives would be $0.10s(f - 10)$:

$$P = 2{,}000 + 0.5s + 0.10s(f - 10)$$

It is possible to simplify further by distributing $0.10s(f - 10)$, but before doing more work, check the answers—answer choice (E) is already an exact match.

Alternatively, plug in numbers. Say forklifts cost \$100 (so, $s = 100$). Randi makes \$5 each for the first 10 she sells, so \$50 total. Then she makes \$10 each for any additional forklifts. Pick a value for f (make sure the value is more than 10, since the question asks for a formula for months in which Randi sells more than 10 forklifts). So, in a month in which she sells, for example, 13 forklifts (so, $f = 13$), she would make \$2,000 + \$50 + 3(\$10) = \$2,080.

In this example:

$s = 100$
$f = 13$

Plug in these values for s and f to see which choice yields \$2,080. Only choice (E) works:

$$P = 2{,}000 + 0.5(100) + 0.10(100)(13 - 10)$$
$$P = 2{,}000 + 50 + 10(3)$$
$$P = 2{,}080$$

20. **(A).** This question can be solved either with smart numbers or algebra. First, consider plugging in smart numbers.

Set $l = 2$, so $w = 4$. The perimeter will be $2l + 2w = 2(2) + 2(4) = 12$. The answer is the area, which is $wl = (2)(4) = 8$ based on these numbers. Now plug $p = 12$ into the choices to see which choice equals 8:

(A) $\frac{144}{18} = 8$

(B) $\frac{144}{36} = 4$

(C) $\frac{12}{9} = \frac{4}{3}$

(D) $\frac{144}{9}$ = 16

(E) $\frac{12}{6}$ = 2

The correct answer is (A).

Though smart numbers are easier and faster here, an algebra solution is also possible. If $w = 2l$:

$a = l \times w = l \times 2l = 2l^2$
$p = 2l + 2w = 2l + 4l = 6l$

Solve the second equation for *l*:

$$l = \frac{p}{6}$$

And plug back into the first equation:

$$2\left(\frac{p}{6}\right)^2 = 2\left(\frac{p^2}{36}\right) = \frac{2p^2}{36} = \frac{p^2}{18}$$

Chapter 20

Ratios

In This Chapter…

Ratios

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

The ratio of men to women in a garden club is 5 to 4.

	Quantity A	**Quantity B**
1.	The smallest possible number of garden club members	20

---

2. A pantry holds x cans of beans, twice as many cans of soup, and half as many cans of tomato paste as there are cans of beans. If there are no other cans in the pantry, which of the following could be the total number of cans in the pantry?

Indicate <u>two</u> such numbers.

- ☐ 6
- ☐ 7
- ☐ 36

- ☐ 45
- ☐ 63

3. If there are 20 birds and 6 dogs in a park, which of the following represents the ratio of dogs to birds in the park?

(A) 3 : 13

(B) 3 : 10

(C) 10 : 3

(D) 13 : 3

(E) 1 : 26

4. If there are 7 whole bananas, 14 whole strawberries, and no other fruit in a basket, what is the ratio of strawberries to the total pieces of fruit in the basket?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

5. The ratio of cheese to sauce for a single pizza is 1 cup to $\frac{1}{2}$ cup. If Bob used 15 cups of sauce to make a number of pizzas, how many cups of cheese did he use on those pizzas?

$\boxed{}$ cups of cheese

6. Laura established a new flower garden, planting 4 tulip plants for every 1 rose plant, and no other plants. If she planted a total of 50 plants in the garden, how many of those plants were tulips?

$\boxed{}$ tulip plants

7. A certain automotive dealer sells only cars and trucks, and the ratio of cars to trucks on the lot is 1 to 3. If there are currently 51 trucks for sale, how many cars does the dealer have for sale?

(A) 17

(B) 34

(C) 68

(D) 153

(E) 204

8. A steel manufacturer combines 98 ounces of iron with 2 ounces of carbon to make one sheet of steel. How many ounces of iron were used to manufacture $\frac{1}{2}$ of a sheet of steel?

(A) 1

(B) 49

(C) 50

(D) 198

(E) 200

9. Maria uses a recipe for 36 cupcakes that requires 8 cups of flour, 12 cups of milk, and 4 cups of sugar. How many cups of milk would Maria require for a batch of 9 cupcakes?

(A) 2

(B) 3

(C) 4

(D) 6

(E) 8

10. In a certain orchestra, each musician plays only one instrument and the ratio of musicians who play either the violin or the viola to musicians who play neither instrument is 5 to 9. If 7 members of the orchestra play the viola and four times as many play the violin, how many play neither?

(A) 14

(B) 28

(C) 35

(D) 63

(E) 72

11. The ratio of 0.4 to 5 equals which of the following ratios?

(A) 4 to 55

(B) 5 to 4

(C) 2 to 25

(D) 4 to 5

(E) 4 to 45

12. On a wildlife preserve, the ratio of giraffes to zebras is 37 : 43. If there are 300 more zebras than giraffes, how many giraffes are on the wildlife preserve?

(A) 1,550

(B) 1,850

(C) 2,150

(D) 2,450

(E) 2,750

13. At a certain company, the ratio of male to female employees is 3 to 4. If there are 5 more female employees than male employees, how many male employees does the company have?

(A) 12

(B) 15

(C) 18

(D) 21

(E) 24

14. On Monday, a class has 8 girls and 20 boys. On Tuesday, a certain number of girls joined the class just as twice that number of boys left the class, changing the ratio of girls to boys to 7 to 4. How many boys left the class on Tuesday?

(A) 5

(B) 6

(C) 11

(D) 12

(E) 18

15. If a dak is a unit of length and 14 daks = 1 jin, how many squares with a side length of 2 daks can fit in a square with a side length of 2 jins?

(A) 14

(B) 28

(C) 49

(D) 144

In a group of adults, the ratio of women to men is 5 to 6, while the ratio of left-handed people to right-handed people is 7 to 9. Everyone is either left- or right-handed; no one is both.

	Quantity A	**Quantity B**
16.	The number of women in the group	The number of left-handed people in the group

---

Party Cranberry is 3 parts cranberry juice and 1 part seltzer. Fancy Lemonade is 1 part lemon juice and 2 parts seltzer. An amount of Party Cranberry is mixed with an equal amount of Fancy Lemonade.

	Quantity A	**Quantity B**
17.	The fraction of the resulting mix that is cranberry juice	The fraction of the resulting mix that is seltzer

---

The ratio of 16 to g is equal to the ratio of g to 49.

	Quantity A	**Quantity B**
18.	g	28

---

19. In a parking lot, $\frac{1}{3}$ of the vehicles are black and $\frac{1}{5}$ of the remainder are white. How many vehicles could be parked on the lot?

(A) 8

(B) 12

(C) 20

(D) 30

(E) 35

20. Oil, vinegar, and water are mixed in a 3 to 2 to 1 ratio to make salad dressing. If Larry has 8 cups of oil, 7 cups of vinegar, and access to any amount of water, what is the maximum number of cups of salad dressing he can make with the ingredients he has available, if fractional cup measurements are possible?

(A) 12

(B) 13

(C) 14

(D) 15

(E) 16

21. With *y* dollars, 5 oranges can be bought. If all oranges cost the same, how many dollars do 25 oranges cost, in terms of *y*?

(A) $\frac{y}{5}$

(B) y

(C) $y + 5$

(D) $5y$

(E) $25y$

22. A woman spent $\frac{5}{8}$ of her weekly salary on rent and $\frac{1}{3}$ of the remainder on food, leaving $40 available for other expenses. What is the woman's weekly salary?

(A) $160

(B) $192

(C) $216

(D) $240

(E) $256

23. In a certain rectangle, the ratio of length to width of a rectangle is 3 : 2 and the area is 150 square centimeters. What is the perimeter of the rectangle, in centimeters?

(A) 10

(B) 15

(C) 25

(D) 40

(E) 50

24. In a certain country, 8 rubels are worth 1 schilling, and 5 schillings are worth 1 lemuw. In this country, 6 lemuws are equivalent in value to how many rubels?

(A) $\frac{20}{3}$

(B) 30

(C) 40

(D) 48

(E) 240

25. Team A and team B are raising money for a charity event. The ratio of money collected by team A to money collected by team B is 5 : 6. The ratio of the number of students on team A to the number of students on team B is 2 : 3. What is the ratio of money collected per student on team A to money collected per student on team B?

(A) 4 : 5

(B) 5 : 4

(C) 5 : 6

(D) 5 : 9

(E) 9 : 5

26. Jarod uses $\frac{2}{3}$ of an ounce of vinegar in every 2 cups of sushi rice that he prepares. To prepare 7 cups of sushi rice in the same proportion, how many ounces of vinegar does Jarod need?

(A) $\frac{3}{2}$

(B) $\frac{4}{3}$

(C) $\frac{7}{3}$

(D) $\frac{7}{2}$

(E) $\frac{14}{3}$

The total cost of 3 bananas, 2 apples, and 1 mango is \$3.50. The total cost of 3 bananas, 2 apples, and 1 papaya is \$4.20. The ratio of the cost of a mango to the cost of a papaya is 3 : 5.

	Quantity A	**Quantity B**
27.	The cost of a papaya	\$2.00

---

28. In a certain town, $\frac{2}{5}$ of the total population is employed. Among the unemployed population, the ratio of males to females is 5 : 7. If there are 40,000 employed people in the town, how many females are unemployed?

(A) 16,000

(B) 25,000

(C) 35,000

(D) 65,000

(E) 75,000

29. On a certain map of the United States, $\frac{3}{5}$ of an inch represents a distance of 400 miles. If Oklahoma City and Detroit are separated on the map by approximately $\frac{3}{2}$ of an inch, what is the approximate distance between them in miles?

(A) 240

(B) 360

(C) 600

(D) 800

(E) 1,000

30. A machine can manufacture 20 cans per hour, and exactly 10 such cans fit into every box. Maria packs cans in boxes at a constant rate of 3 boxes per hour. If the machine ran for 2 hours and was then turned off before Maria started packing the cans in boxes, how many minutes would it take Maria to pack all the cans that the machine had made?

(A) 40

(B) 45

(C) 80

(D) 160

(E) 800

31. If Beth has $\frac{1}{4}$ more money than Ari, and each person has an integer number of dollars, which of the following could be the combined value of Beth and Ari's money?

Indicate all such values.

- ☐ $12
- ☐ $54
- ☐ $72
- ☐ $200

32. If salesperson A sold 35% more motorcycles than salesperson B, which of the following could be the total number of motorcycles sold by both salespeople?

Indicate all such total numbers of motorcycles.

- ☐ 47
- ☐ 70
- ☐ 135
- ☐ 235

33. A zoo has twice as many zebras as lions and four times as many monkeys as zebras. Which of the following could be the total number of zebras, lions, and monkeys at the zoo?

Indicate all such totals.

- ☐ 14
- ☐ 22
- ☐ 28
- ☐ 55
- ☐ 121

34. In nation Z, 10 terble coins equal 1 galok. In nation Y, 6 barbar coins equal 1 murb. If a galok is worth 40% more than a murb, what is the ratio of the value of 1 terble coin to the value of 1 barbar coin?

(A) $\frac{3}{5}$

(B) $\frac{11}{13}$

(C) $\frac{3}{7}$

(D) $\frac{21}{23}$

(E) $\frac{21}{25}$

35. Autolot has a 2 : 1 ratio of blue cars to red cars and a 6 : 1 ratio of red cars to orange cars on the lot. What could be the total number of blue, red, and orange cars on the lot?

(A) 38

(B) 39

(C) 40

(D) 41

(E) 42

Ratios Answers

1. **(B).** The ratio of men to women is 5 to 4. Since both 5 and 4 are whole numbers, they could actually *be* the number of men and women, respectively. These are also the *lowest* possible numbers of men and women, because reducing the ratio of 5 to 4 any further is impossible without making one part a non-integer (e.g., 2.5 to 2) or both parts negative, and the numbers of men and women must be positive integers.

Therefore, the smallest possible number of garden club members is 5 + 4 = 9. Quantity B is greater.

2. **7 and 63 only.** Write the number of each type of can:

Cans of beans = x Cans of soup = $2x$ Cans of tomato paste = $0.5x$

Since each number of cans must be an integer, x must be even or there would be partial cans of tomato paste. The total number of cans is $x + 2x + 0.5x = 3.5x$ and since x must be even, the total number of cans could be 7, 14, 21, etc. Thus, the total number of cans must be a multiple of 7. Of the answer choices, only 7 and 63 are multiples of 7.

3. **(B).** If there are 6 dogs and 20 birds in the park, the ratio of dogs to birds is 6 : 20, which reduces to 3 : 10.

4. $\mathbf{\frac{2}{3}}$ **or any equivalent fraction.** If there are 7 bananas and 14 strawberries, then there are 7 + 14 = 21 total pieces of fruit. The ratio of strawberries to the total is therefore 14 : 21. Write this ratio as a fraction and cancel the common factor of 7 from top and bottom: $\frac{14}{21} = \frac{2 \times 7}{3 \times 7} = \frac{2}{3}$. The original ratio of $\frac{14}{21}$ would also be counted as correct if entered as is.

5. **30 cups of cheese.** Bob used cheese to sauce in a 1 : 1/2 ratio, which could be multiplied by 2 to yield the equivalent cheese to sauce ratio of 2 : 1. In words, there are twice as many cups of cheese as there are cups of sauce in the pizzas. Bob actually used 15 cups of sauce, so he used 2 × 15 = 30 cups of cheese.

6. **40 tulip plants.** To solve for the number of tulips, work with the Part : Part

: Whole ratio. The ratio of tulips to roses is 4 : 1, so the Tulip : Rose : Total relationship is 4 : 1 : 5. This ratio can be written as $4x : 1x : 5x$, with x as the unknown integer multiplier. There are 50 total plants in the garden, so set $5x$ equal to 50 and solve for x:

$$\begin{aligned} 5x &= 50 \\ x &= 10 \end{aligned}$$

Now plug this value into the expression for the actual number of tulips: $4x = 4(10) = 40$. Laura planted 40 tulip plants in the garden.

7. **(A).** Focus on the given Part : Part ratio. The ratio of cars to trucks is 1 : 3, or $x : 3x$ with x as the unknown multiplier. Since there are 51 trucks for sale, set $3x$ equal to 51 and solve for x.

$$3x = 51$$
$$x = 17$$

Since x also represents the number of cars, the dealer has 17 cars for sale.

8. **(B).** Iron and carbon combine to make steel in a specific given ratio. The ratio of iron (ounces) to carbon (ounces) to steel (sheets) is 98 : 2 : 1. Because there are different units (ounces and sheets), the Part numbers do not sum to the Whole number as they typically do, but don't be concerned.

This ratio can be written as $98x : 2x : x$, with x as the unknown multiplier, which is also the number of sheets. To make $\frac{1}{2}$ a sheet of steel, set x equal to $\frac{1}{2}$.

Now plug this value into the expression for the number of iron ounces: $98x = (98)(1/2) = 49$. To make $\frac{1}{2}$ a sheet of steel, 49 ounces of iron are required.

9. **(B).** As a ratio, Flour : Milk : Sugar : Cupcakes is equal to 8 : 12 : 4 : 36, where the first three numbers are in cups. Because there are different units (cups and cupcakes), the Part numbers do not sum to the Whole number, but don't be concerned.

This ratio can be written as $8x : 12x : 4x : 36x$, with x as the unknown multiplier. To make 9 cupcakes, set $36x$ equal to 9 and solve for x.

$$36x = 9$$
$$x = \frac{1}{4}$$

In words, for a batch of 9 cupcakes, Maria would make $\frac{1}{4}$ of the original recipe.

Now plug this value into the expression for cups of milk: $12x = (12)\left(\frac{1}{4}\right) =$

3. Maria would need 3 cups of milk.

10. **(D).** Since 7 members of the orchestra play the viola and four times as many play the violin, then (7)(4) = 28 people must play the violin. Altogether, 7 + 28 = 35 musicians in the orchestra play either the viola or the violin.

The ratio of *either* to *neither* is 5 : 9, or $5x : 9x$ using the unknown multiplier. Since 35 people play either instrument, set $5x$ equal to 35 and solve for x.

$$\begin{aligned} 5x &= 35 \\ x &= 7 \end{aligned}$$

Now plug this value into the expression for *neither*: $9x = 9(7) = 63$. There are 63 people in the orchestra who play neither instrument.

11. **(C).** You can rewrite ratios as fractions and then multiply or divide top and bottom by the same number, keeping the ratio (or fraction) the same.

First, multiply top and bottom by 10, to remove the decimal:

$$\frac{0.4}{5} = \frac{0.4 \times 10}{5 \times 10} = \frac{4}{50}$$

Next, cancel the common factor of 2: $\frac{4}{50} = \frac{2 \times 2}{25 \times 2} = \frac{2}{25}$

Finally, the fraction $\frac{2}{25}$ is the same as the ratio of 2 to 25, which is therefore equivalent to the original ratio of 0.4 to 5.

12. **(B).** The ratio of giraffes to zebras is 37 : 43. Introduce the unknown multiplier x: the number of giraffes is $37x$, and the number of zebras is $43x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 300 more zebras than giraffes” becomes Zebras – Giraffes = 300, or $43x - 37x = 300$. Solve for x:

$$\begin{aligned} 43x - 37x &= 300 \\ 6x &= 300 \\ x &= 50 \end{aligned}$$

Finally, substitute into the expression for the number of giraffes: $37x = 37(50) = 1{,}850$. There are 1,850 giraffes.

Alternatively, the right answer must be a multiple of 37, because the giraffe number in the ratio is 37, and there must be a positive whole number of giraffes. Test the answer choices in the calculator to find that only 1,850 is divisible by 37. This shortcut doesn’t always work this well, of course!

13. **(B).** The ratio of male to female employees is 3 : 4. Introduce the unknown multiplier x, making the number of males $3x$ and the number of females $4x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 5 more female employees than male employees” becomes Females – Males = 5, or $4x - 3x = 5$. Solve for x:

$$4x - 3x = 5$$

$$x = 5$$

Finally, substitute into the expression for the number of male employees: $3x = 3(5) = 15$. There are 15 male employees.

14. **(D).** Call the number of girls who joined the class x, so the new number of girls in the class is $8 + x$. Twice as many boys left the class, so the number of boys who left the class is $2x$, and the new number of boys in the class is $20 - 2x$.

The resulting ratio of boys to girls is 7 to 4. Since there is already a variable in the problem, don't use an unknown multiplier. Rather, set up a proportion and solve for x:

$$\frac{\text{Girls}}{\text{Boys}} = \frac{8+x}{20-2x} = \frac{7}{4}$$

$4(8 + x) = 7(20 - 2x)$
$32 + 4x = 140 - 14x$
$18x = 108$
$x = 6$

Finally, the question asks for the number of boys who left the class. This is $2x = 2(6) = 12$ boys.

Check: There were 8 girls in the class, then 6 joined for a total of 14 girls. There were 20 boys in the class until 12 left the class, leaving 8 boys in the class. The resulting ratio of girls to boys is $\frac{14}{8} = \frac{7\times 2}{4\times 2} = \frac{7}{4}$, as given.

15. **(E).** Since 14 daks = 1 jin, a length measured in daks is 14 times the same length measured in jins. In other words, the ratio of the length in daks to the length in jins is 14 to 1.

Write this relationship as a fraction: $\frac{14 \text{ daks}}{1 \text{ jin}}$. You can also write $\frac{1 \text{ jin}}{14 \text{ daks}}$. You can convert a measurement from one unit to the other by multiplying by one of these unit conversion factors.

Side of big square: $(2 \text{ jins})\left(\frac{14 \text{ daks}}{1 \text{ jin}}\right) = 28$ daks. Since the small square has a side length of 2 daks, the number of small sides that will fit along a big side is $28 \div 2 = 14$.

However, 14 is not the right answer; 14 is the number of small squares that will fit along *one wall* of the big square, in one row. There will be 14 rows, so in all there will be $(14)(14) = 196$ small squares that fit inside the big square.

16. **(A).** Write two different Part : Part : Whole relationships. In each relationship, the two parts sum to the whole.

Women : Men : Total = 5 : 6 : 11, so Women : Total = 5 : 11.

Left-handed : Right-handed : Total = 7 : 9 : 16, so Left-handed : Total = 7 : 16.

In other words, women account for $\frac{5}{11}$ = 45.$\overline{45}$% of the group, left-handed people for $\frac{7}{16}$ = 43.75% of the group.

Since the total number of people is the same (it's the same group, whether divided by gender or handedness), the percents can be compared directly. There must be more women than left-handed people in the group. Quantity A is greater.

17. **(B).** Be careful—don't just add the "parts" from the different mixtures, because the parts will generally not be the same size! Start by writing Part : Part : Whole relationships for each glass. In each relationship, the whole is the sum of the parts:

For Party Cranberry, *Cranberry* : *Seltzer* : *Whole* = 3 : 1 : 4

For Fancy Lemonade, *Lemon* : *Seltzer* : *Whole* = 1 : 2 : 3

Since the two amounts that are mixed are the same size, choose a smart number to represent the total volume for both Party Cranberry and Fancy Lemonade. This number should be a multiple of both 4 and 3, according to the ratios above, so it is convenient to say that the amount of each is 12 ounces. Multiply the Party Cranberry ratio by 3 and the Fancy Lemonade ratio by 4, in both cases to get 12 total ounces:

For Party Cranberry, *Cranberry* : *Seltzer* : *Whole* = 9 : 3 : 12

For Fancy Lemonade, *Lemon* : *Seltzer* : *Whole* = 4 : 8 : 12

Finally, when the two glasses are mixed, the resulting total is 24 ounces, of which 9 ounces are cranberry juice but 3 + 8 = 11 ounces are seltzer. There is more seltzer in the resulting mix, so its fraction of the mix is greater than cranberry juice's fraction of the mix. Quantity B is greater.

18. **(D).** Write the ratios as fractions and set them equal to each other:

$$\frac{16}{g} = \frac{g}{49}$$

Cross-multiply to get $16 \times 49 = g^2$.

Remember that when "unsquaring" an equation with a squared variable, you must account for the negative possibility. The value of g could be *either* $4 \times 7 = 28$ *or negative* 28. Nothing in the problem indicates that g must be positive. Since Quantity A might equal Quantity B *or* be less than Quantity B, the relationship cannot be determined from the information given.

19. **(D).** Since vehicles must be counted with whole numbers and $\frac{1}{3}$ of the cars are black, the total number of cars must be divisible by 3. Otherwise, $\frac{1}{3}$ of the total would not be a whole number. The answer must be (B) or (D).

The remainder of the cars is $1 - \frac{1}{3} = \frac{2}{3}$ of the total. Of these, $\frac{1}{5}$ are white, so

$\frac{1}{5}$ of $\frac{2}{3}$, or $\frac{2}{15}$ of the total number of vehicles are white. Again, because the white cars must be countable with whole numbers, $\frac{2}{15}$ of the total must be an integer. You can write the equation using fractions:

$$\left(\frac{2}{15}\right)\text{(Total)} = \text{Integer}$$

To get an integer outcome, the total must be divisible by 15. Of the answer choices, only (D) is divisible by 15.

20. **(E).** Since the ratio of ingredients is 3 : 2 : 1 in the recipe, imagine that Larry works in cups. Then a recipe makes 3 + 2 + 1 = 6 cups of dressing. To figure out the "limiting factor," take each available amount of ingredient and figure out how many times he could make the recipe, permitting fractions, if he had more than enough of the other ingredients.

Oil: 8 cups available ÷ 3 cups needed per recipe = $\frac{8}{3}$ recipes (in other words, $2\frac{2}{3}$ times the recipe). There is no need to round down, because fractional cups of ingredients are allowed.

Vinegar: 7 cups available ÷ 2 cups needed per recipe = $\frac{7}{2}$ recipes (in other words, $3\frac{1}{2}$ times through the recipe).

Water availability is not limited, so ignore it.

Oil is the limiting factor, because Larry can make the fewest recipes with it. Thus, he can only make $\frac{8}{3}$ recipes. To find the total cups of salad dressing, multiply this fraction by the total number of cups that a recipe makes:

$$\frac{8}{3} \text{ recipe} \times 6 \text{ cups per recipe} = 16 \text{ cups}$$

21. **(D).** Create a unit conversion factor using the given ratio of oranges to dollars. The conversion factor will look like either $\frac{5 \text{ oranges}}{y \text{ dollars}}$ or $\frac{y \text{ dollars}}{5 \text{ oranges}}$. Which one to use depends on how you want to convert the units.

The question asks how many dollars, in terms of *y*, 25 oranges cost. Since the given is oranges and the question asks for dollars, choose the conversion unit that cancels oranges and leaves dollars on top: $\frac{y \text{ dollars}}{5 \text{ oranges}}$. Then multiply:

$$(25 \text{ oranges})\left(\frac{y \text{ dollars}}{5 \text{ oranges}}\right) = \frac{25y}{5} \text{ dollars} = 5y \text{ dollars}$$

Intuitively, a total of 25 oranges is the same as 5 sets of 5 oranges each. Each set costs *y* dollars. Therefore, the total cost for 5 sets of oranges is $5 \times y = 5y$.

22. **(A).** The total amount of money left over after paying rent and buying food is $40. From this number, you can find the woman’s total weekly salary

by determining what fraction this is of her total salary.

Since the woman spent $\frac{5}{8}$ of her salary on rent, she had $1 - \frac{5}{8} = \frac{3}{8}$ of her salary remaining. Of the remainder, she spent $\frac{1}{3}$ on food and had $\frac{2}{3}$ left over. So, $\frac{2}{3}$ of $\frac{3}{8}$ of her total weekly salary was left over for other expenses:

$$\left(\frac{2}{3}\right)\left(\frac{3}{8}\right) = \frac{2}{8} = \frac{1}{4}$$

One-quarter of her salary was the \$40 left over. Now you can find T, her total weekly salary:

$$\left(\frac{1}{4}\right)T = \$40$$

$$T = \$160$$

23. **(E).** Rewrite the given ratio using the unknown multiplier x, so that the length of the rectangle is $3x$, while the width is $2x$. Now express the area of the rectangle in these terms, set it equal to 150 square centimeters, then solve for x:

$$\begin{aligned} \text{Area} &= (\text{Length})(\text{Width}) \\ 150 &= (3x)(2x) \\ 150 &= 6x^2 \\ 25 &= x^2 \\ 5\text{ cm} &= x \end{aligned}$$

In this case, you don't need to worry about the negative possibility for the square root, since lengths cannot be less than zero. The length is $3x = 15$ centimeters, while the width is $2x = 10$ centimeters.

Finally, the perimeter of a rectangle is twice the length, plus twice the width:

$$\begin{aligned} &\text{Perimeter} = 2 \times \text{length} + 2 \times \text{width} \\ &\text{Perimeter} = 2 \times 15\text{ cm} + 2 \times 10\text{ cm} \\ &\text{Perimeter} = 30\text{ cm} + 20\text{ cm} \\ &\text{Perimeter} = 50\text{ cm} \end{aligned}$$

24. **(E).** The question requires converting an amount of money in “lemuws” to “rubels.” Conceptually, there are two steps: first convert lemuws to schillings, then convert schillings to rubels. The fast way to do this two-step conversion is to multiply the money by the right conversion factors, which express identities (such as 8 rubels = 1 schilling) in the form of ratios: $\frac{8 \text{ rubels}}{1 \text{ schilling}}$ or $\frac{1 \text{ schilling}}{8 \text{ rubels}}$. If you make sure that the units cancel correctly, then you can always be sure under pressure whether to multiply or divide by 8.

Here is the conversion, done all in one line:

$$(6 \text{ lemuws})\left(\frac{5 \text{ schillings}}{1 \text{ lemuw}}\right)\left(\frac{8 \text{ rubels}}{1 \text{ schilling}}\right) = 240 \text{ rubels}$$

Both lemuws and schillings cancel on the left, leaving rubels. 6 lemuws are worth 240 rubels.

25. **(B).** To solve this ratios problem, choose smart numbers for the money collected for each team and the number of students on each team. Choose multiples of the ratios given, such as the following:

Money collected by team A = \$10
Money collected by team B = \$12
Number of students in team A = 2
Number of students in team B = 3

Then compute the money per student:

Money per student in team A = \$10 ÷ 2 = \$5 per student
Money per student in team B = \$12 ÷ 3 = \$4 per student

Thus, the ratio of money per student in team A to money per student in team B is 5 : 4.

26. **(C).** To find how much vinegar Jarod needs, think about how many multiples of his original recipe Jarod wants to make. The original recipe makes 2 cups of sushi rice, so 7 cups of rice is $\frac{7}{2}$ times his original recipe.

Since Jarod is scaling proportionally, to make $\frac{7}{2}$ times the usual amount of rice, he must also use $\frac{7}{2}$ times as much vinegar. Therefore, Jarod must use:

$$\left(\frac{7}{2}\right)\left(\frac{2}{3}\text{ ounces}\right)=\frac{7}{3}\text{ ounces of vinegar}$$

Alternatively, you can start with 7 cups of rice and multiply by the recipe's ratio of vinegar to rice, canceling cups of rice and producing ounces of vinegar:

$$(7 \text{ cups of rice})\left(\frac{\frac{2}{3} \text{ ounces of vinegar}}{2 \text{ cups of rice}}\right) = \frac{7}{3} \text{ ounces of vinegar}$$

27. **(B).** Solve for the cost of a papaya by translating the information given into mathematical statements. The first sentence states that 3 bananas, 2 apples, and 1 mango cost \$3.50. Letting B represent the cost of a banana, A the cost of an apple, and M the cost of a mango, set up an equation:

$3B + 2A + M = \$3.50$

Similarly, for the second sentence, you get the following equation:

$3B + 2A + P = \$4.20$ (where P is the cost of a papaya)

The problem requires finding the cost of a papaya and provides the ratio of the costs of a mango and papaya. To use this information, remove bananas and apples from the list of unknowns. Here's how: try elimination. Specifically, subtract the first equation from the second:

$$\begin{array}{r} 3B + 2A + P = \$4.20 \\ -\,(3B + 2A + M = \$3.50) \\ \hline P - M = \$0.70 \end{array}$$

Now, since the ratio of the cost of a mango to a papaya is 3 : 5, write a proportion:

$\frac{M}{P} = \frac{3}{5}$, which becomes $M = \frac{3}{5}P$ if you isolate M. Now substitute back into the equation above, to eliminate M and solve for P:

$$P - M = \$0.70$$

$$P - \frac{3}{5}P = \$0.70$$

$$= \$0.70$$

$$\frac{5}{5}P - \frac{3}{5}P$$

$$\frac{2}{5}P = \$0.70$$

$$P = \frac{5}{2}(\$0.70) = \$1.75$$

Quantity B is greater.

28. **(C).** To solve for the number of unemployed females, first compute the total number of people who are unemployed. You need to represent the total number of people in the town. Call this number *x*. Since $\frac{2}{5}$ of the town is employed, a total of 40,000 people, write the ratio:

$$\frac{\text{Employed}}{\text{Total population}} = \frac{40{,}000}{x} = \frac{2}{5}$$

Cross-multiply and solve for *x*:

$$\begin{aligned} 5(40{,}000) &= 2x \\ 200{,}000 &= 2x \\ 100{,}000 &= x \end{aligned}$$

If 40,000 people in the town are employed, then 100,000 – 40,000 = 60,000 people are unemployed.

Finally, the ratio of unemployed males to females is 5 : 7. In other words, out of every 5 + 7 = 12 unemployed people, there are 7 unemployed females. Therefore, the fraction of unemployed females in the total unemployed population is 7 out of 12, or $\frac{7}{12}$. Set *y* as the number of unemployed females:

$$\frac{\text{Unemployed females}}{\text{Total unemployed}} = \frac{y}{60{,}000} = \frac{7}{12}$$

To get the number of unemployed females, solve for *y: y* = $\frac{7 \times 60{,}000}{12}$ = 7 × 5,000 = 35,000

29. **(E).** According to the problem, $\frac{3}{5}$ of an inch on the map is equivalent to 400 miles of actual distance. So you can set up a ratio of these two measurements to use as a conversion factor: $\frac{{}^{3}\!/_{5}\text{ inch}}{400\text{ miles}}$ or $\frac{400\text{ miles}}{{}^{3}\!/_{5}\text{ inch}}$.

Which one you use depends on which way you're converting: from miles to inches or vice versa.

The question states that Oklahoma City is separated from Detroit by approximately $\frac{3}{2}$ inches on the map, and asks how many real miles, approximately, lie between the two cities. To go from inches to miles, multiply the given measurement $\left(\frac{3}{2}\text{ inches}\right)$ by the conversion factor that will cancel out inches and leave miles:

$$\left(\frac{3}{2}\text{ inches}\right)\left(\frac{400\text{ miles}}{3/5\text{ inch}}\right)=\left(\frac{3}{2}\right)(400)\left(\frac{5}{3}\right)\text{miles}=1{,}000\text{ miles}$$

30. **(C).** First, figure out how many boxes worth of cans the machine produced in the 2 hours that it was on. The first step is to find the number of cans produced in 2 hours. Use the formula Work = Rate × Time. The question tells you 20 cans per hour is the rate and 2 hours is the time:

Work = (20 cans per hour) × (2 hours) = 40 cans

Now, since there are 10 cans per box, compute the number of boxes:

$$\text{Number of boxes} = 40 \text{ cans} \times \left(\frac{1 \text{ box}}{10 \text{ cans}}\right) = 4 \text{ boxes}$$

So Maria must pack 4 whole boxes to accommodate all the cans that the machine had made.

One more time, use the formula Work = Rate × Time. Maria's rate is 3 boxes per hour, while the total work as 4 boxes. Rearrange and plug in:

$$\text{Time} = \frac{\text{Work}}{\text{Rate}} = \frac{4 \text{ boxes}}{3 \text{ boxes per hour}} = \frac{4}{3} \text{ hours}$$

Finally, convert from hours to minutes as the question requires:

$$\text{Time} = \frac{4}{3} \text{ hours} \times \left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right) = 80 \text{ minutes}$$

31. **\$54 and \$72 only.** If Beth has $\frac{1}{4}$ more money than Ari, their money is in a ratio of 5 : 4 (because 5 is $\frac{1}{4}$ more than 4). Another way to see this result is with algebra:

$$B = A + \frac{1}{4}A = \frac{5}{4}A, \text{ so } \frac{B}{A} = \frac{5}{4}.$$

As a result, for every \$9 total, Beth has \$5 and Ari has \$4. To keep both Ari and Beth in integer dollar values, the answer needs to be a multiple of 9. Among the answer choices, only 54 and 72 are multiples of 9.

32. **47 and 235 only.** Since salesperson A sold 35% more motorcycles than salesperson B, their sales are in a ratio of 135 : 100. You can reduce this ratio to 27 : 20 by canceling a common factor of 5.

As a result, for every 47 motorcycles sold, salesperson A sold 27 and salesperson B sold 20. The number of motorcycles sold must be integer multiples of these numbers (because you can't sell partial motorcycles—not legally anyway), so the total needs to be a multiple of 47. Among the answer choices, only 47 and 235 are multiples of 47.

33. **22, 55, and 121 only.** First, figure out which animal there are fewest of. “Twice as many zebras as lions” means Zebras > Lions and “four times as many monkeys as zebras” means Monkeys > Zebras. So lions are found at the zoo in smallest numbers. To make the calculation straightforward, pick 1 lion as a smart number. Since there are twice as many zebras, there are 2 zebras. Finally, there are four times as many monkeys as zebras, so there are $4 \times 2 = 8$ monkeys. Putting all of that together:

Lions : Zebras : Monkeys = 1 : 2 : 8

So, for every 11 animals ($1 + 2 + 8$), there are 1 lion, 2 zebras, and 8 monkeys. To preserve integer numbers of lions, zebras, and monkeys, the total number of animals could only be a multiple of 11. Among the answer choices, only 22, 55, and 121 fit the bill.

34. **(E).** To tackle this question, rewrite all these ridiculously named currencies in terms of just one currency, ideally a real currency. Use whatever real currency you like, but here's an example with dollars.

Say that 1 murb is worth $1.

A galok is worth 40% more than a murb, or 40% more than $1. A galok is worth $1.40.

Since 10 terble coins equal 1 galok, then 10 terble coins are worth a total of $1.40. Each terble coin, therefore, is worth $1.40 ÷ 10 = $0.14, or 14 cents.

Since 6 barbar coins equal 1 murb, then 6 barbar coins equal $1. Each barbar coin is worth $\$\frac{1}{6}$ or $\frac{100}{6}$ cents.

The ratio of the value of 1 terble coin to the value of 1 barbar coin:

$$\frac{1\text{ terble}}{1\text{ barbar}} = \frac{14\text{ cents}}{100/6\text{ cents}} = 14 \times \frac{6}{100} = \frac{21}{25}$$

35. **(A).** Manipulate the given ratios to create one ratio that includes all three colors. You might use a table:

R	B	O
1	2	
6		1

The problem here is the red car: that column contains both a 1 and a 6. In order to fix this issue, create a common term. Multiply the entire first ratio (the first row) by 6:

R	B	O
6	12	
6		1

Now that the same number is in both rows of the red column, you can combine the two rows into a single ratio:

R : B : O = 6 : 12 : 1

For every 19 cars (6 + 12 + 1), there are 6 red cars, 12 blue cars, and 1 orange car. To maintain whole numbers of cars in each color, the correct answer has to be a multiple of 19. Only 38 is a multiple of 19.

Chapter 21

Averages, Weighted Averages, Median, and Mode

In This Chapter…

Averages, Weighted Averages, Median, and Mode

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Husain and Dino had an average (arithmetic mean) of $20 each. Dino then won a cash prize, which increased the average amount of money they had to $80. If no other changes occurred, how many dollars did Dino win?

$ $\boxed{}$

Janani is 6 centimeters taller than Preeti, who is 10 centimeters taller than Rey.

	Quantity A	**Quantity B**
2.	The average (arithmetic mean)height of the three people	The median height of the three people

---

The average (arithmetic mean) of x and y is 55.

The average of y and z is 75.

	Quantity A	Quantity B
3.	$z - x$	40

4. What is the average (arithmetic mean) of x, $x - 6$, and $x + 12$?

(A) x

(B) $x + 2$

(C) $x + 9$

(D) $3x + 6$

(E) It cannot be determined from the information given.

$ab < 0$

	Quantity A	Quantity B
5.	$\frac{a+b}{2}$	0

---

6. If x is negative, what is the median of the list 20, x, 7, 11, 3?

(A) 3

(B) 7

(C) 9

(D) 11

(E) 15.5

7. If the average (arithmetic mean) of n and 11 is equal to $2n$, what is the average of n and $\frac{13}{3}$?

(A) 4

(B) 8

(C) 11

(D) 14

(E) 19

	Quantity A	Quantity B
8.	The average (arithmetic mean) of $x - 3$, x, $x + 3$, $x + 4$, and $x + 11$	The median of $x - 3$, x, $x + 3$, $x + 4$, and $x + 11$

---

9. John bought 5 books with an average (arithmetic mean) price of \$12. If John then buys another book with a price of \$18, what is the average price of all 6 books?

(A) \$12.50

(B) \$13

(C) \$13.50

(D) \$14

(E) \$15

10. Every week, Renee is paid \$40 per hour for the first 40 hours she works, and \$80 per hour for each hour she works after the first 40 hours. How many hours would Renee have to work in one week to earn an average (arithmetic mean) of \$60 per hour that week?

(A) 60

(B) 65

(C) 70

(D) 75

(E) 80

At a certain school, all 118 juniors have an average (arithmetic mean) final exam score of 88 and all 100 seniors have an average final exam score of 92.

	Quantity A	**Quantity B**
11.	The average (arithmetic mean) final exam score for all of the juniors and seniors combined	90

	Quantity A	**Quantity B**
12.	The average (arithmetic mean) of x, y, and z	The average (arithmetic mean) of $0.5x$, $0.5y$, and $0.5z$

---

13. Aaron's first three quiz scores were 75, 84, and 82. If his score on the fourth quiz reduced his average (arithmetic mean) quiz score to 74, what was his score on the fourth quiz?

$\boxed{}$

Four people have an average (arithmetic mean) age of 18, and none of the people are older than 30.

	Quantity A	**Quantity B**
14.	The range of the four people's ages	25

Dataset A consists of 5 numbers, which have an average (arithmetic mean) value of 43. Dataset B consists of 5 numbers.

	Quantity A	**Quantity B**
15.	The value of x if the average of x and the 5 numbers in dataset A is 46.	The average of dataset B if the average of the 10 numbers in datasets A and B combined is 52.

16. The average (arithmetic mean) of 7 numbers in a certain list is 12. The average of the 4 smallest numbers in this list is 8, while the average of the 4 greatest numbers in this list is 17. How much greater is the sum of the 3 greatest numbers in the list than the sum of the 3 smallest numbers in the list?

(A) 4

(B) 14

(C) 28

(D) 36

(E) 52

17. If the average (arithmetic mean) of a, b, c, 5, and 6 is 6, what is the average of a, b, c, and 13?

(A) 8

(B) 8.5

(C) 9

(D) 9.5

(E) It cannot be determined from the information given.

18. A group consists of both men and women. The average (arithmetic mean) height of the women is 66 inches, and the average (arithmetic mean) height of the men is 72 inches. If the average (arithmetic mean) height of all the people in the group is 70 inches, what is the ratio of women to men in the group?

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 2 : 3

(E) 3 : 2

19. The average (arithmetic mean) of 13 numbers is 70. If the average of 10 of these numbers is 90, what is the average of the other 3 numbers?

(A) –130

(B) $\frac{10}{3}$

(C) 30

(D) 90

(E) 290

20. Town A has 6,000 citizens and an average (arithmetic mean) of 2 radios per citizen. Town B has 10,000 citizens and an average of 4 radios per citizen. What is the average number of radios per citizen in both towns combined?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

21. Fiber X cereal is 55% fiber. Fiber Max cereal is 70% fiber. Sheldon combines an amount of the two cereals in a single bowl of mixed cereal that is 65% fiber. If the bowl contains a total of 12 ounces of cereal, how much of the cereal, in ounces, is Fiber X?

(A) 3

(B) 4

(C) 6

(D) 8

(E) 9

22. The average (arithmetic mean) population in town X was recorded as 22,455 during the years 2000–2010, inclusive. However, an error was later uncovered: the figure for 2009 was erroneously recorded as 22,478, but should have been correctly recorded as 22,500. What was the average population in town X during the years 2000–2010, inclusive, once the error was corrected?

While driving from city A to city B, a car got 22 miles per gallon and while returning on the same road, the car got 30 miles per gallon.

	Quantity A	**Quantity B**
23.	The car’s average gas mileage for the entire trip, in miles per gallon	26

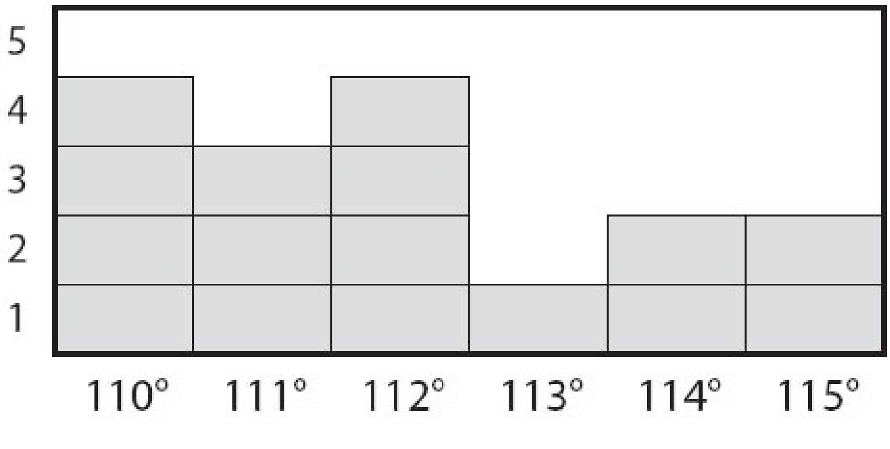
$S_n = 3n + 3$

Sequence S is defined for each integer n such that $0 < n < 10{,}000$.

	Quantity A	**Quantity B**
24.	The median of sequence S	The mean of sequence S

---

25. The bar graph below displays the number of temperature readings at each value from a sample, measured in degrees Fahrenheit. What was the average (arithmetic mean) temperature reading?



degrees Fahrenheit

26. Score results on a college mathematics proficiency exam:

	Freshmen	Sophomores
Number of students taking the exam	120	80
Average (arithmetic mean) score on the exam	78 points	84 points

What was the average (arithmetic mean) score for all the freshmen and sophomores taking the exam? Give your answer rounded to the nearest 0.1 points.

[ ] points

27. Set A: 1, 3, 5, 7, 9
Set B: 6, 8, 10, 12, 14

For the sets of numbers above, which of the following statements are true?

Indicate all such statements.

- ☐ The mean of set B is greater than the mean of set A.
- ☐ The median of set B is greater than the median of set A.
- ☐ The standard deviation of set B is greater than the standard deviation of set A.
- ☐ The range of set B is greater than the range of set A.

28. Three people have \$32, \$72, and \$98, respectively. If they pool their money then redistribute it among themselves, what is the maximum possible value for the median amount of money?

(A) \$72

(B) \$85

(C) \$98

(D) \$101

(E) \$202

29.

Weekly Revenue per Product Category at Office Supply Store *X*

Product Category	Weekly Revenue
Pens	\$164
Pencils	\$111
Legal pads	\$199
Erasers	\$38
Average (arithmetic mean) of categories above	\$128

According to the chart above, the average (arithmetic mean) revenue per week per product category is \$128. However, there is an error in the chart; the revenue for Pens is actually \$176, not \$164. What is the new, correct average revenue per week per product category?

(A) \$130

(B) \$131

(C) \$132

(D) \$164

(E) \$176

Set *M* consists of 20 evenly spaced integers, 10 numbers of which are positive and 10 of which are negative.

	Quantity A	Quantity B
30.	The average (arithmetic mean) of all the numbers in set *M*	0

The average (arithmetic mean) of $3x$, x, and y is equal to $2x$.

	Quantity A	Quantity B
31.	$2x$	y

---

32. The average (arithmetic mean) age of the buildings on a certain city block is greater than 40 years old. If four of the buildings were built 2 years ago and none of the buildings are more than 80 years old, which of the following could be the number of buildings on the block?

Indicate all such numbers.

- ☐ 4
- ☐ 6
- ☐ 8
- ☐ 11
- ☐ 40

33. Four students contributed to a charity drive, and the average (arithmetic mean) amount contributed by each student was $20. If no student gave more than $25, what is the minimum amount that any student could have contributed?

$ ☐

The average (arithmetic mean) of seven distinct integers is 12, and the least of these integers is –15.

	Quantity A	**Quantity A**
34.	The maximum possible value of the greatest of these integers	84

---

35. The average (arithmetic mean) of 15 consecutive integers is 88. What is the greatest of these integers?

☐

Three numbers have a range of 2 and a median of 4.4.

	Quantity A	**Quantity A**
36.	The greatest of the numbers	5.4

Averages, Weighted Averages, Median, and Mode Answers

1. **$120.** If the two people had an average of $20 each, they held a sum of 2($20) = $40. After Dino won a cash prize, the new sum is 40 + *p* and the new average is 80. Plug into the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$80 = \frac{40 + p}{2}$$

$$160 = 40 + p$$

$$120 = p$$

Dino won $120.

2. **(B).** Pick numbers that agree with the given height constraints. Rey is the shortest person, and if Rey is 100 cm tall, Preeti is 110 cm tall, and Janani is 116 cm tall. The average height is $\frac{100+110+116}{3}$ = 108.67 cm (rounded to nearest 0.01 cm). The median height is the middle height, which is 110 cm. Quantity B is greater.

Alternatively, note that Preeti's height is the median. Preeti's height is closer to Janani's than to Rey's. Since the average of Janani and Rey's heights would be midway between those heights, and Preeti's height is greater than that middle, the median is greater than the average.

3. **(C).** Since the average of *x* and *y* is 55, $\frac{x+y}{2}$ = 55, so $x + y = 110$.

Since the average of *y* and *z* is 75, $\frac{y+z}{2}$ = 75, so $y + z = 150$.

Stack the two equations and subtract to cancel the *y*'s and get $z - x$ directly:

$$z + y = 150$$

$$\begin{array}{rcl} -(x + y & = & 110) \\ \hline z - x & = & 40 \end{array}$$

The two quantities are equal.

4. **(B).** The average formula can be applied to algebraic expressions, just as to arithmetic ones:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$\text{Average} = \frac{(x) + (x - 6) + (x + 12)}{3}$$

$$\text{Average} = \frac{3x + 6}{3} = x + 2$$

5. **(D).** The best way to solve Quantitative Comparisons problems with variables is to plug in multiple values for the variables, trying to prove (D).

If $ab < 0$, then one of the variables is positive and the other negative.

Try $a = 2$ and $b = -3$:

$\frac{2+-3}{2} = -\frac{1}{2}$ and Quantity B is greater. Therefore, the answer cannot be (A) or (C).

Try $a = 3$ and $b = -2$:

$\frac{3+-2}{2} = \frac{1}{2}$ and now Quantity A is greater. Therefore, the answer cannot be (B) or (C). If you plug in two different sets of numbers and get two different results for which quantity is greater, the answer must be (D).

6. **(B).** The easiest way to start thinking about a question like this is to plug in a value and see what happens. If $x = -1$, the list looks like this when ordered from least to greatest:

–1, 3, 7, 11, 20

The median is 7. Because any negative x used will be the least term in the list, the order of the list won't change, so the median will always be 7.

7. **(A).** This question can be solved with the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of terms}}$$

$$2n = \frac{n+11}{2}$$

$$4n = n + 11$$

$$3n = 11$$

$$n = \frac{11}{3}$$

Since $n = \frac{11}{3}$, the average of n and $\frac{13}{3}$ is:

$$\frac{\frac{11}{3}+\frac{13}{3}}{2}=\frac{\frac{24}{3}}{2}=\frac{8}{2}=4$$

Alternatively, just notice that the midpoint between $\frac{11}{3}$ and $\frac{13}{3}$ is $\frac{12}{3}$, just as 12 is the midpoint between 11 and 13. The average is $\frac{12}{3}$ = 4.

8. **(C).** To find the median of the numbers, notice that they are already in order from least to greatest: $x - 3$, x, $x + 3$, $x + 4$, $x + 11$.

The median is the middle, or third, term: $x + 3$.

Now find the average of the numbers:

$$\frac{(x-3)+(x)+(x+3)+(x+4)+(x+11)}{5} = \frac{5x+15}{5} = x+3$$

The median and the mean are both ($x + 3$), therefore, the two quantities are equal.

9. **(B).** First, calculate the cost of the first 5 books.

Sum = (Average cost)(Number of books) = ($12)(5) = $60

Total cost of all 6 books = $60 + $18 = $78 Total number of books = 6

Average = $78/6 = $13 per book.

10. **(E).** Let h = number of hours Renee would have to work. The average rate Renee gets paid is equal to the total wages earned divided by the total number of hours worked. Rene earns $40 per hour for the first 40 hours, so she makes 40 × 40 = $1,600 in the first 40 hours. She also earns $80 for every hour after 40 hours, for additional pay of $80($h$ – 40). Therefore, her total pay can be calculated as:

$$\frac{1{,}600 + 80(h-40)}{h} = 60$$

$$1{,}600 + 80h - 3{,}200 = 60h$$

Now isolate h:

$$\begin{aligned} 80h - 1{,}600 &= 60h \\ -1{,}600 &= -20h \\ 80 &= h \end{aligned}$$

You could also notice that 60 is exactly halfway between 40 and 80. Therefore, Renee needs to work an equal number of hours at $40 per hour and $80 per hour. If she works 40 hours at $40 per hour, she also needs to work 40 hours at $80 per hour, yielding 80 hours of total work.

11. **(B).** This is a weighted average problem. Because the number of juniors is

greater than the number of seniors, the overall average will be closer to the juniors' average than the seniors' average. Since 90 is halfway between 88 and 92, and the weighted average is closer to 88, Quantity B is greater.

It is not necessary to do the math because this is a Quantitative Comparison question with a very convenient number as Quantity B. However, you can actually calculate the overall average by summing up all 218 scores and dividing by the number of people: $\frac{118(88)+100(92)}{118+100}$ = 89.83…

12. **(D).** The average of *x*, *y*, and *z* is $\frac{x+y+z}{3}$. Calculated similarly, the average of 0.5*x*, 0.5*y*, and 0.5*z* is exactly half that. If the sum of the variables is positive, Quantity A is greater. However, if the sum of the variables is negative, Quantity B is greater. If the sum of the variables is zero, the two quantities are equal.

13. **55.** To find Aaron's fourth quiz score, set up an equation:

$$\begin{aligned} \frac{75+84+82+x}{4} &= 74 \\ 241 + x &= 296 \\ x &= 55 \end{aligned}$$

14. **(D).** If 4 people have an average age of 18, then the sum of their ages is 4 × 18 = 72. Since the question is about range, try to minimize and maximize the range. Minimizing the range is easy—if everyone were exactly 18, the average age would be 18 and the range would be 0. So clearly, the range can be smaller than 25.

To maximize the range, make the oldest person the maximum age of 30, and see whether the youngest person could be just 1 year old while still obeying the other rules of the problem: the sum of the ages is 72 and, of course, no one can be a negative age.

One such set: 1, 20, 21, 30

This is just one example that would work. In this case, the range is 30 – 1 = 29, which is greater than 25.

15. **(C).** If the average of the 5 numbers in dataset A is 43, the sum of dataset A is (5)(43) = 215.

For Quantity A, use the average formula and sum all 6 numbers and divide by 6:

$$\frac{\text{Sum of the 5 numbers in dataset A} + x}{6} = 46$$

$$\frac{215 + x}{6} = 46$$

$$215 + x = 276$$

$$x = 61$$

For Quantity B, use the average formula again:

$$\frac{\text{Sum of dataset A} + \text{Sum of dataset B}}{10} = 52$$

$$215 + \text{Sum of dataset B} = 520$$

$$\text{Sum of dataset B} = 305$$

The average of the 5 numbers in dataset B is $\frac{305}{5}$ = 61.

Alternatively, note that each dataset of 5 numbers has the same "weight" in the average of all 10 numbers. The average of dataset A is 43, which is 52 – 43 = 9 below the average of all 10 numbers. The average of dataset B must be 9 above the average of all 10 numbers: 52 + 9 = 61.

16. **(D).** Using the average formula, Average = $\frac{\text{Sum}}{\text{Number of terms}}$, build three separate equations:

All 7 numbers:

$$12 = \frac{\text{Sum of all 7 numbers}}{7}$$

Sum of all 7 numbers = 84

The 4 smallest numbers:

$$8 = \frac{\text{Sum of the 4 smallest numbers}}{4}$$

Sum of the 4 smallest numbers = 32

The 4 greatest numbers:

$$17 = \frac{\text{Sum of the 4 greatest numbers}}{4}$$

Sum of the 4 greatest numbers = 68

There are only 7 numbers, yet information is given about the 4 smallest and the 4 greatest, which is a total of 8 numbers! The middle number has been counted twice—it is included in both the 4 greatest and the 4 smallest.

The sum of all 7 numbers is 84, but the sum of the 4 greatest and 4 smallest is 68 + 32 = 100. The difference can only be attributed to the double counting of the middle number in the set of 7: 100 – 84 = 16.

The middle number is 16, so subtract it from the sum of the 4 smallest numbers to get the sum of the 3 smallest numbers: 32 – 16 = 16.

Now subtract the middle number from the sum of the 4 greatest numbers to get the sum of the 3 greatest numbers: 68 – 16 = 52.

The difference between the sum of the 3 greatest numbers and the sum of the 3 smallest numbers is 52 – 16 = 36.

17. **(A).** Since Average = $\frac{\text{Sum}}{\text{Number of terms}}$:

$$6 = \frac{a+b+c+5+6}{5}$$

$$30 = a + b + c + 11$$

$$19 = a + b + c$$

It is not necessary, or possible, to determine the values of *a*, *b*, and *c* individually. The second average includes all three variables, so the values will be summed again anyway.

$$\text{Average} = \frac{a+b+c+13}{4}$$

$$\text{Average} = \frac{19+13}{4}$$

$$\text{Average} = \frac{32}{4} = 8$$

18. **(B).** Use the weighted average formula:

Average height of all =

$$\frac{(\text{Total height of all women})+(\text{Total height of all men})}{(\text{Number of women})+(\text{Number of men})}$$

From the average formula, Sum = Average × Number of terms. If *w* is the number of women and *m* is the number of men, the total heights of women and men respectively are:

Total height of all women = 66*w*
Total height of all men = 72*m*

Plug into the average formula, recalling that the average height for the entire group is 70 inches:

$$\text{Avgerage height of all} = \frac{66w+72m}{w+m} = 70$$

Cross-multiply and simplify:

$$\begin{aligned} 66w + 72m &= 70(w + m) \\ 66w + 72m &= 70w + 70m \\ 72m &= 4w + 70m \\ 2m &= 4w \\ m &= 2w \end{aligned}$$

At this point, it might be tough to determine whether the answer is (B) or (C). This is an ideal time to plug in numbers. For instance, if $w = 3$, then $m = 6$. Now, the ratio of women to men is 3 : 6 or 1 : 2, answer choice (B).

Alternatively, continue with the algebra, solving for the $\frac{w}{m}$ ratio:

$$m = 2w$$

$$\frac{m}{2m} = \frac{2w}{2m}$$

$$\frac{1}{2} = \frac{w}{m}$$

19. **(B).** Remember the Average formula, Average = $\frac{\text{Sum}}{\text{Number of terms}}$, can also be rewritten as: Sum = Average × Number of Terms.

The average of 13 numbers is 70, so:

Sum of all 13 terms = 70 × 13 = 910

The average of 10 of these numbers is 90, so:

Sum of 10 of these numbers = 90 × 10 = 900

Subtract to find the sum of “the other 3 numbers”: 910 – 900 = 10.

Average of the other 3 numbers = $\frac{\text{Sum}}{\text{Number of terms}} = \frac{10}{3}$.

20. $\mathbf{\frac{13}{4}}$ **(or any equivalent fraction).** To find this weighted average, first find the sum of all the radios in towns A and B, and then divide by the total number of people in both towns:

$$\text{Average} = \frac{6{,}000(2) + 10{,}000(4)}{16{,}000}$$

Cancel three zeros from each term:

$$\text{Average} = \frac{6(2) + 10(4)}{16}$$

Average = $\frac{52}{16}$

This reduces to $\frac{13}{4}$, though reduction is not required.

21. **(B).** Use the weighted average formula to get the ratio of Fiber X to Fiber Max:

$\frac{0.55x + 0.70m}{x + m}$ = 0.65, where *x* is the amount of Fiber X and *m* is the amount of Fiber Max.

This is not that different from the regular average formula—on the top, there is the total amount of fiber (55% of Fiber X and 70% of Fiber Max), which is divided by the total amount of cereal ($x + m$) to get the average. Simplify by multiplying both sides by ($x + m$):

$0.55x + 0.70m = 0.65(x + m)$
$0.55x + 0.70m = 0.65x + 0.65m$

To simplify, multiply both sides of the equation by 100 to eliminate all the decimals:

$55x + 70m = 65x + 65m$
$55x + 5m = 65x$
$5m = 10x$

$\frac{m}{x} = \frac{10}{5}$ or $\frac{2}{1}$

Since *m* and *x* are in a 2 to 1 ratio, $\frac{2}{3}$ of the total is *m* and $\frac{1}{3}$ of the total is *x*.

Since the total is 12 ounces, Fiber X accounts for $\frac{1}{3}(12) = 4$ ounces of the mixed cereal.

One shortcut to this procedure is to note that the weighted average (65%) is 10% away from Fiber X’s percent and 5% away from Fiber Max’s percent. Since 10 is twice as much as 5, the ratio of the two cereals is 2 to 1. However, it is a 2 to 1 ratio of Fiber Max to Fiber X, not the reverse! Whichever number is closer to the weighted average (in this case, 70% is closer to 65%) gets the larger of the ratio parts. Since the ratio is 2 to 1 (Fiber Max to Fiber X), again, $\frac{1}{3}$ of the cereal is Fiber X and $\frac{1}{3}(12) = 4$.

22. **22,457.** There is a simple shortcut for a change to an average. The figure for 2009 was recorded as 22,478, but actually should have been recorded as 22,500, meaning 22 people in that year were not counted. Thus, the sum should have been 22 higher when the average was originally calculated.

2000–2010, inclusive, is 11 years (subtract low from high and then add 1 to count an inclusive list of consecutive numbers). When taking an average, divide the sum by the number of things being averaged (in this case, 11). So the shortcut is to take the change to the sum and "spread it out" over all of the values being averaged by dividing the change by the number of things being averaged.

Divide 22 by 11 to get 2. The average should have been 2 greater. Thus, the correct average for the 11-year period is 22,457.

Alternatively, use the traditional method: 22,455 × 11 years = 247,005, the sum of all 11 years' recorded populations. Add the 22 uncounted people, making the corrected sum 247,027. Divide by 11 to get the corrected average: 22,457. (Note that while the traditional method is faster to explain, the shortcut is faster to actually execute!)

23. **(B).** One trap is to mistakenly pick (C), thinking that the car got a simple average of 22 and 30 miles per gallon. However, the trip from *A* to *B* required *more* gallons of gas; therefore, the average will be "weighted" more to the side of 22 (same number of miles, but *more* gallons) and wind up less than 26 mpg. Quantity (B) is therefore greater.

To show this explicitly, use a smart number of miles from city A to B, for example, a multiple of 22 and 30, such as 660 miles. Set up a chart using the formula:

(Miles per gallon) × (Gallons) = (Miles)

	Miles per gallon	×	Gallons	=	Miles
A to *B*	22	×	30	=	660
B to *A*	30	×	22	=	660
Total	x	×	52	=	1,320

The average miles per gallon for the whole trip is the total number of miles (1,320) divided by the total number of gallons (52):

$\frac{1,320}{52}$ = 25.38… , which is less than 26.

Quantity B is greater.

24. **(C).** Sequence *S* is an evenly spaced set, which can be seen by plugging in a few *n* values:

$S_1 = 3(1) + 3 = 6$
$S_2 = 3(2) + 3 = 9$
$S_3 = 3(3) + 3 = 12\ldots$

Terms increase by 3 every time *n* increases by 1; this meets the definition of an evenly spaced set. For *any* evenly spaced set, the median equals the mean.

25. **112.** This is a weighted average problem. *Do not* simply average 110, 111, 112, 113, 114, and 115. Instead, take into account how many times each number appears. The chart is really another way of writing:

110, 110, 110, 110
111, 111, 111
112, 112, 112, 112
113
114, 114
115, 115

In other words, the average temperature reading is really an average of 16 numbers. The easiest way to do this is:

$$\frac{4(110)+3(111)+4(112)+1(113)+2(114)+2(115)}{16}$$

Use the calculator—the correct answer is 112.

26. **80.4.** In order to determine the average score for *all* the freshman and sophomores combined, compute the total points for everyone, and divide by the total number of students.

Because Average = $\frac{\text{Sum}}{\text{Number of terms}}$, it can also be written as the Sum = Average × Number of terms. Use the formula to compute the total number of points for all the freshmen and all the sophomores as individual groups:

Freshman total points = 78 points × 120 = 9,360
Sophomore total points = 84 points × 80 = 6,720

Combined, the freshmen and sophomores scored 9,360 + 6,720 = 16,080 points.

The total number of students is 120 + 80 = 200.

Now, apply the average formula, Average = $\frac{\text{Sum}}{\text{Number of terms}}$, to the combined group:

$$\text{Average} = \frac{16{,}080}{200} = 80.4$$

27. **1st and 2nd only.** In both sets, the numbers are evenly spaced. Moreover, both sets are evenly spaced by the same amount (adjacent terms increase by 2) and have the same number of terms (5 numbers in each set). The difference is that each term in set *B* is 5 greater than the corresponding term in set *A* (i.e., 6 – 1 = 5, 8 – 3 = 5, etc.).

In evenly spaced sets, the mean = median. Also, if an evenly spaced set has an odd number of numbers, the mean and median both equal the middle number. (When such a set has an even number of numbers, the mean and median both equal the average of the two middle numbers.)

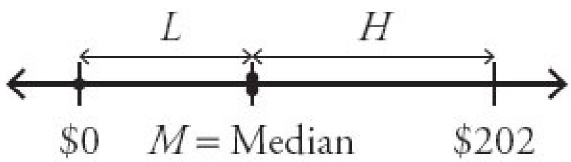
So, set *A* has mean and median of 5 and set B has mean/median of 10. The first and second statements are true.

Since sets *A* and *B* are equally spaced and have the same number of elements, their standard deviations are equal (that is, set *A* is exactly as spread out from its own mean as set *B* is from its own mean), so the third statement is false.

Since 9 – 1 = 8 and 14 – 6 = 8, the ranges are equal and the fourth statement is false.

28. **(D).** The pool of money is \$32 + \$72 + \$98 = \$202. After the redistribution, each person will have an amount between \$0 and \$202, inclusive. Call the amounts *L*, *M,* and *H* (low, median, high). To maximize *M*, minimize *L* and *H*.

The minimum value for *H* is *M*. The "highest" of the three values can be equal to the median (if *H* were lower than *M*, the term order and therefore which number is the median would change, but if *H* = *M*, *M* can still be the median). Draw it out:



Minimum *L* = \$0
Minimum *H* = *M*

Maximum *M* = Total pool of money – Minimum *L* – Minimum *H*
M = \$202 – \$0 – *M*
2*M* = \$202
M = \$101

The correct answer is (D).

29. **(B).** The chart provides the average and the number of product categories. If the incorrectly calculated average was $128 for the 4 categories, then the sum was $4 \times 128 = \$512$. Since the revenue for pens was actually $176, not $164, the sum should have been $12 higher. Thus, the correct sum is $524. Divide by 4 to get $131, the answer.

Alternatively, notice that the $128 average given in the question stem actually does a lot of work for you. If $164 jumps up to $176, that's an increase of $12. Distributed over the four categories, it will bring the overall average up by $3, from $128 to $131.

30. **(D).** “Evenly spaced” means ascending by some regular increment (each number greater than the next by some value). If 10 of the integers are positive and 10 are negative, then none of the numbers in the set are 0. Therefore, the 10th number must be less than 0 and the 11th greater than 0. To better understand this, try listing values for set *M*, starting in the middle. If the middle numbers are

…–1, +1…

then the spacing between the numbers is 2 and set M would look like:

–19, –17, –15, –13, –11, –9, –7, –6, –5, –4, –3, –2, –1, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

The sum and the average of all the numbers in this set are both 0, so (A) and (B) cannot be the answer. However, is the average always 0? If the middle two numbers were offset a little:

…–1, 4…

then the spacing between the numbers is 5 and set M would look like:

–46, –41, –36, –31, –26, –21, –16, –11, –6, –1, 4, 9, 14, 19, 24, 29, 34, 39, 44, 49

The first and the last terms sum to –46 + 49 = 3. The second term and penultimate term sum to –41 + 44 = 3, and so on. Each term can be paired with another for a sum of 3. There are 10 such pairs, so the total sum is 3 × 10 = 30 and the average is 30/20 = 1.5.

The relationship cannot be determined from the information given.

31. **(C).** Write “the average of 3*x*, *x*, and *y* is equal to 2*x*” as an equation and solve:

$$\frac{3x + x + y}{3} = 2x$$
$$4x + y = 6x$$
$$y = 2x$$

The two quantities are equal.

32. **8, 11, and 40 only.** Because Average = $\frac{\text{Sum}}{\text{Number of terms}}$, this question about averages depends both on *x*, the total number of buildings on the block, and on the sum of the building ages. The 4 buildings that are 2 years old have

a total age of 4(2), and the (x – 4) other buildings have a total age of (x – 4) (no more than 80). Set up an equation to find the average age:

$$\text{Average age} = \frac{4(2)+(x-4)(\text{no more than } 80)}{x}$$

Having many 80-year-old buildings on the block would raise the average much closer to 80. (For instance, if there were a million 80-year-old buildings and four 2-year-old buildings, the average would be very close to 80 years old.) So, there is some minimum number of older buildings that could raise the average above 40.

Ignore the “greater than” 40 years old constraint on the average building age for a moment. What is the minimum *x* needs to be to make the average age exactly 40 when the age of the other buildings is maximized at 80?

$$\begin{aligned} 40x &= 8 + (x - 4)(80) \\ 40x &= 8 + 80x - 320 \\ -40x &= -320 \\ x &= \frac{312}{40} = 7.8 \end{aligned}$$

Because there can’t be a partial building, and the age of the buildings can’t be greater than 80, *x* must be at least 8 to bring the average age up over 40. (More buildings would be required to bring the average above 40 if those older buildings were only between 50 and 70 years old, for example.)

Alternatively, test the answer choices. Try the first choice, 4 buildings. Since 4 of the buildings on the block are only 2 years old, this choice can’t work—the average age of the buildings would be 2.

Try the second choice. With 6 total buildings, there would be four 2-year-old buildings, plus two others. To maximize the average age, maximize the ages of the two other buildings by making them both 80 years old:

$$\frac{4(2) + 2(80)}{6} = 28$$

Since the average is less than 40 years old, this choice is not correct.

Try the third choice. With eight total buildings, there would be the four 2-year-old buildings, plus four others. To maximize the average age, maximize the ages of the four other buildings by making them each 80 years old:

$$\frac{4(2) + 4(80)}{8} = 41$$

Since the average age is greater than 40 years old, this choice is correct. Since the other, greater choices allow the possibility of even more 80-year-old buildings, increasing the average age further, those choices are also correct.

33. **$5.** The average of four values is $20. Thus, the sum of the four values is $80. To determine the minimum contribution one student could have given, maximize the contributions of the other three students. If the three other students each gave the maximum of $25, the fourth student would only have to give $5 to make the sum equal to $80.

34. **(A).** If the average of 7 integers is 12, then their sum must be 7 × 12 = 84. To maximize the largest of the numbers, minimize the others.

The smallest number is –15. The integers are distinct (that is, different from each other), so the minimum values for the smallest 6 integers are –15, –14, –13, –12, –11, and –10. To find the maximum value for the 7th integer, sum –15, –14, –13, –12, –11, –10, and *x*, while setting that sum equal to 84:

$$
\begin{aligned}
-15 + (-14) + (-13) + (-12) + (-11) + (-10) + x &= 84 \\
-75 + x &= 84 \\
x &= 159
\end{aligned}
$$

Quantity A is greater.

35. **95.** In any evenly spaced set, the average equals the median. Thus, 88 is the middle number in the set. Since the set has 15 elements, the 8th element is the middle one:

Lowest seven integers:	81	82	83	84	85	86	87
Middle integer:	88						
Greatest seven integers:	89	90	91	92	93	94	95

The largest integer in the list is 95. Confidence in this process allows you to skip the counting process. Instead, reason that to go from 8th integer to the 15th integer, simply add 7: 88 + 7 = 95.

36. **(D).** If the set has an odd number of terms, then the median is the middle number, so the middle number is 4.4. The set has a range of 2. The other two numbers could be 2 apart and also equally distributed around 4.4:

Example 1: 3.4, 4.4, 5.4

Here, the two quantities are equal.

Or, the two other numbers could be 2 apart but both a bit higher or both a bit lower.

Example 2: 4.3, 4.4, 6.3
Example 3: 2.5, 4.4, 4.5

Thus, Quantity A could be equal to, less than, or greater than Quantity B. The relationship cannot be determined from the information given.

Chapter 22

Standard Deviation and Normal Distribution

In This Chapter…

Standard Deviation and Normal Distribution

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Set *S*: {5, 10, 15}

   If the number 15 were removed from set *S* and replaced with the number 1,000, which of the following values would change?

   Indicate <u>all</u> such values.

   - ☐ The mean of the set
   - ☐ The median of the set
   - ☐ The standard deviation of the set

Dataset *W*: –9, –3, 3, 9
Dataset *X*: 2, 4, 6, 8
Dataset *Y*: 100, 101, 102, 103
Dataset *Z*: 7, 7, 7, 7

2. Which of the following choices lists the four datasets above in order from least standard deviation to greatest standard deviation?

(A) W, X, Y, Z

(B) W, Y, X, Z

(C) W, X, Z, Y

(D) Z, Y, X, W

(E) Z, X, Y, W

Set N is a set of x distinct positive integers where $x > 2$.

	Quantity A	Quantity B
3.	The standard deviation of set N	The standard deviation of set N if every number in the set were multiplied by –3

The 75th percentile on a test corresponded to a score of 700, while the 25th percentile corresponded to a score of 450.

	Quantity A	Quantity B
4.	A 95th percentile score	800

A species of insect has an average mass of 5.2 grams and a standard deviation of 0.6 grams. The mass of the insects follows a normal distribution.

	Quantity A	Quantity B
5.	The percent of the insects that have a mass between 5.2 and 5.8 grams	The percent of the insects that have a mass between 4.9 and 5.5 grams

The lengths of a certain population of earthworms are normally distributed with a mean length of 30 centimeters and a standard deviation of 3 centimeters. One of the worms is picked at random.

	Quantity A	Quantity B
6.	The probability that the worm is between 24 and 30 centimeters, inclusive	The probability that the worm is between 27 and 33 centimeters, inclusive

Home values among the 8,000 homeowners of Town X are normally distributed, with a standard deviation of \$11,000 and a mean of \$90,000.

Quantity A	Quantity B
The number of homeowners in Town X whose home value is	300

7. greater than $112,000

Exam grades among the students in Ms. Harshman’s class are normally distributed, and the 50th percentile is equal to a score of 77.

	Quantity A	Quantity B
8.	The number of students who scored less than 80 on the exam	The number of students who scored greater than 74 on the exam

9. The length of bolts made in factory Z is normally distributed, with a mean length of 0.1630 meters and a standard deviation of 0.0084 meters. The probability that a randomly selected bolt is between 0.1546 meters and 0.1756 meters long is between

(A) 54% and 61%

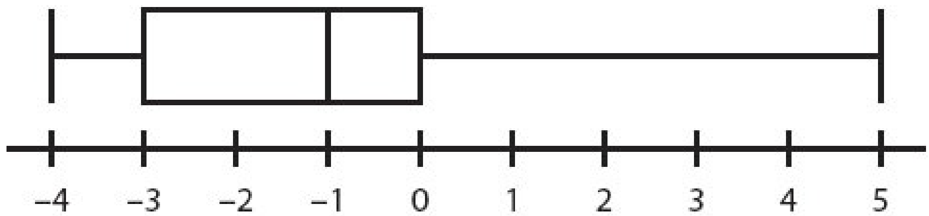
(B) 61% and 68%

(C) 68% and 75%

(D) 75% and 82%

(E) 82% and 89%

10. Which of the following sets of data applies to this box-and-whisker plot?



(A) –4, –4, –2, 0, 0, 5

(B) –4, 1, 1, 3, 4, 4

(C) –4, –4, –3, 1, 5

(D) –5, 3, 4, 5

(E) –4, –4, –2, –2, 0, 0, 0, 5

11. If a set of data consists of only the first ten positive multiples of 5, what is the interquartile range of the set?

(A) 15

(B) 25

(C) 27.5

(D) 40

(E) 45

12. On a given math test with a maximum possible score of 100 points, the vast majority of the 149 students in a class scored either a perfect score or a zero, with only one student scoring within 5 points of the mean. Which of the following logically follows about dataset *T*, made up of the scores on the test?

Indicate <u>all</u> such statements.

- ☐ Dataset *T* is not normally distributed.
- ☐ The range of dataset *T* would be significantly smaller if the scores had been more evenly distributed.
- ☐ The mean of dataset *T* is not equal to the median.

---

13. Jane scored in the 68th percentile on a test, and John scored in the 32nd percentile.

Quantity A	**Quantity B**
The proportion of the class that received a score less than John's score	The proportion of the class that scored equal to or greater than Jane's score

---

14. In a class with 20 students, a test was administered and was scored only in whole numbers from 0 to 10. At least 1 student got every possible score, and the average score was 7.

Quantity A	**Quantity B**
The lowest score that could have been received by more than 1 student	4

---

15. A test is scored out of 100 and the scores are divided into five quintile groups. Students are not told their scores, but only their quintile group.

Quantity A	**Quantity B**
The scores of two students in the bottom quintile group, chosen at random and added together	The score of a student in the top quintile group, chosen at random.

---

16. In a set of 10 million numbers, one percentile would represent what percent of the total number of terms?

(A) 1,000,000

(B) 100,000

(C) 10,000

(D) 100

(E) 1

17. What is the range of the dataset of numbers comprised entirely of {1, 6, x, 17, 20, y} if all terms in the dataset are positive integers and $xy = 18$?

(A) 16

(B) 17

(C) 18

(D) 19

(E) It cannot be determined from the information given.

18. On a particular test whose scores are distributed normally, the 2nd percentile is 1,720, while the 84th percentile is 1,990. What score, rounded to the nearest 10, most closely corresponds to the 16th percentile?

(A) 1,750

(B) 1,770

(C) 1,790

(D) 1,810

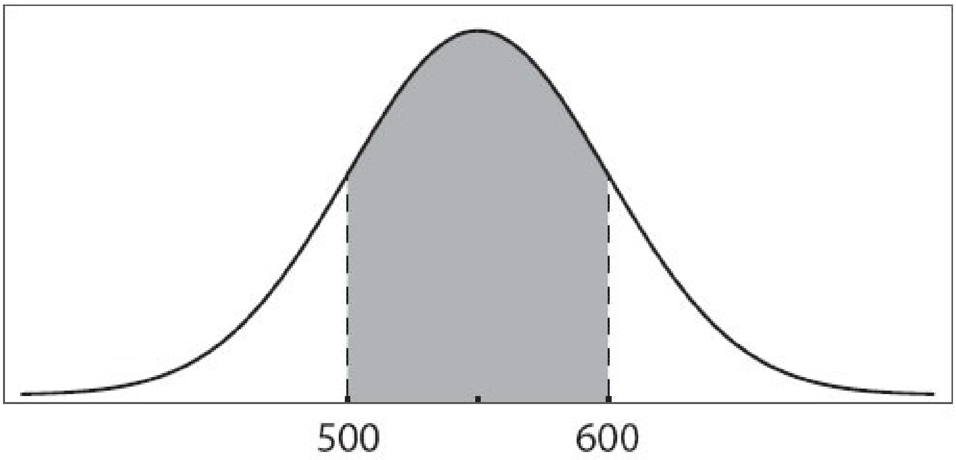
(E) 1,830

A dataset contains at least two different integers.

	Quantity A	Quantity B
19.	The range of the dataset	The interquartile range of the dataset

Some rock samples are weighed, and their weights are determined to be normally distributed. One standard deviation below the mean is 250 grams and one standard deviation above the mean is 420 grams.

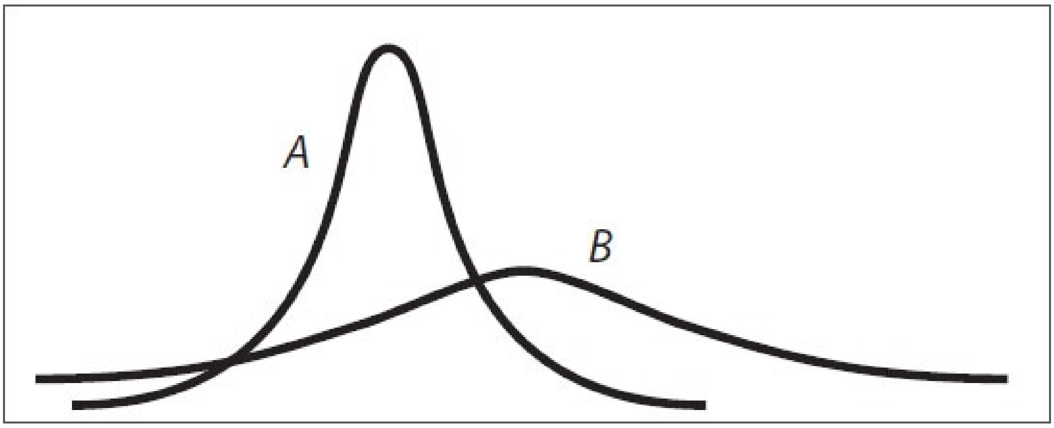
	Quantity A	Quantity B
20.	The median weight, in grams	335 grams



The graph represents the normally distributed scores on a test. The shaded area represents approximately 68% of the scores.

	Quantity A	**Quantity B**
21.	The mean score on the test	550

22.



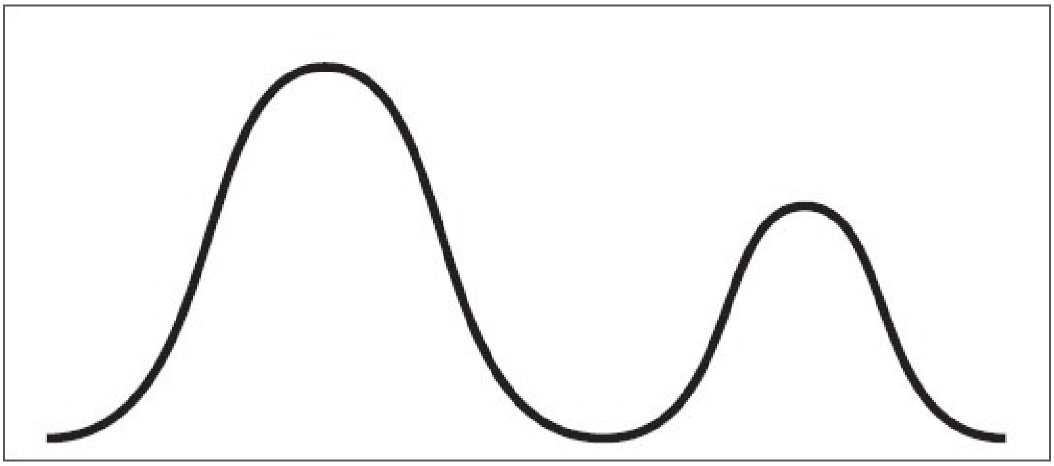
A and *B* are graphical representations of normally distributed random variables *X* and *Y*, respectively, with relative positions, shapes, and sizes as shown. Which of the following must be true?

Indicate <u>all</u> such statements.

- ☐ *Y* has a greater standard deviation than *X*.
- ☐ The probability that *Y* falls within 2 standard deviations of its mean is greater than the probability that *X* falls within 2 standard deviations of its mean.
- ☐ *Y* has a greater mean than *X*.

The outcome of a standardized test is an integer between 151 and 200, inclusive. The percentiles of 400 test scores are calculated, and the scores are divided into corresponding percentile groups.

	Quantity A	Quantity B
23.	The minimum number of integers between 151 and 200, inclusive, that include more than one percentile group	The minimum number of percentile groups that correspond to a score of 200



24. Which of the following would the data pattern shown above best describe?

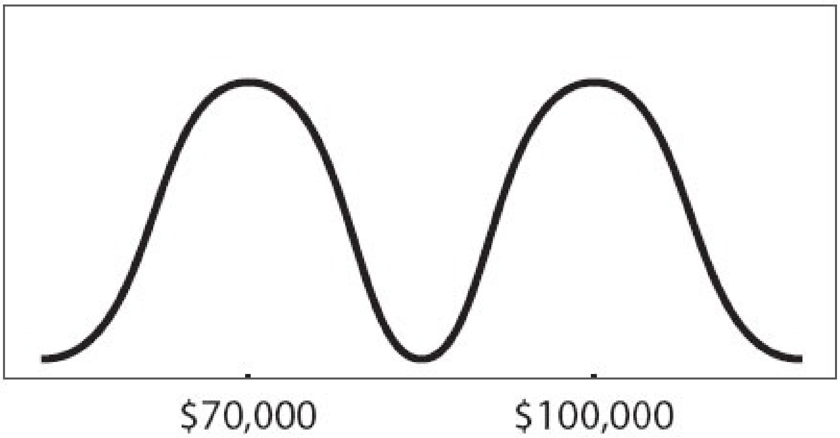
(A) The number of grams of sugar in a selection of drinks is normally distributed.

(B) A number of male high school principals and a larger number of female high school principals have normally distributed salaries, distributed around the same mean.

(C) A number of students have normally distributed heights and a smaller number of taller, adult teachers also have normally distributed heights.

(D) The salary distribution for biologists skews to the left of the median.

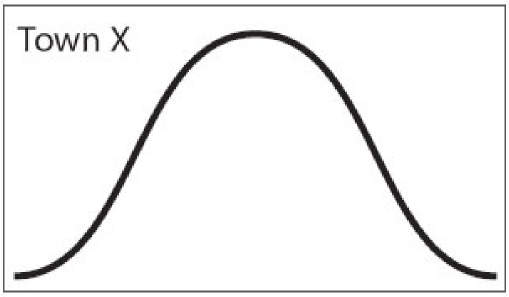
(E) The maximum-weight bench presses for a number of male athletes are normally distributed and the maximum-weight bench presses for a smaller number of female athletes are also normally distributed, although around a smaller mean.

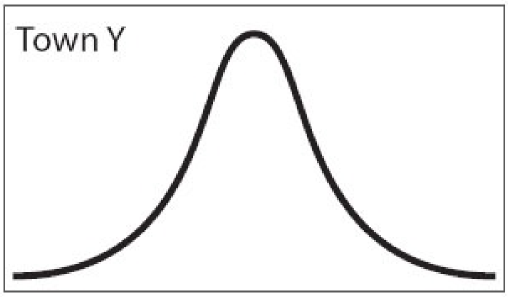


25. A number of scientists' salaries were reported; physicists' salaries clustered around a mean of $100,000 and biologists' clustered around a mean of $70,000. Which of the following statements could be true, according to the graph above?

Indicate all such statements.

- ☐ Some biologists earn more than some physicists.
- ☐ Both biologists' and physicists' salaries are normally distributed.
- ☐ The range of salaries is greater than $150,000.



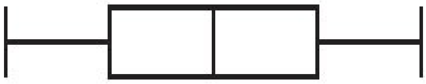


26. The graph on the left above represents the number of family members per family in Town X, while the graph on the right above represents the number of family members per family in Town Y. The median family size for Town X is equal to the median family size for Town Y. The horizontal and vertical dimensions of the boxes above are identical and correspond to the same measurements. Which of the following statements *must* be true?

Indicate all such statements.

- ☐ The range of family sizes measured as the number of family members is larger in Town X than in Town Y.
- ☐ Families in Town *Y* are more likely to have sizes within 1 family member of the mean than are families in Town X.

☐ The data for Town X has a larger standard deviation than the data for Town Y.



27. The box-and-whisker plot shown above could be a representation of which of the following sets?

(A) –2, 0, 2, 4

(B) 3, 3, 3, 3, 3, 3

(C) 1, 25, 100

(D) 2, 4, 8, 16, 32

(E) 1, 13, 14, 17

28. Which of the following statements must be true about the data described by the box-and-whisker plot above?

Indicate all such statements.

- ☐ The median of the whole set is closer to the median of the lower half of the data than it is to the median of the upper half of the data.
- ☐ The data is normally distributed.
- ☐ The set has a standard deviation greater than zero.

29. The earthworms in sample A have an average length of 2.4 inches, and the earthworms in sample B have an average length of 3.8 inches. The average length of the earthworms in the combined samples is 3.0 inches. Which of the following must be true?

Indicate all such statements.

- ☐ There are more earthworms in sample A than in sample B.
- ☐ The median length of the earthworms is 3.2 inches.
- ☐ The range of lengths of the earthworms is 1.4.

Standard Deviation and Normal Distribution Answers

1. **“The mean of the set” and “The standard deviation of the set.”** The word *mean* is a synonym for the average. Because an average is calculated by taking the sum of the terms in the set and dividing by the number of numbers in the set, changing *any* one number in a set (without adjusting the others) will change the sum and, therefore, the average. The median is the middle number in a set, so making the biggest number even bigger won’t change that (the middle number is still 10). Standard deviation is a measure of how *spread out* the numbers in a set are—the more spread out the numbers, the larger the standard deviation—so making the biggest number *really far away* from the others would greatly increase the standard deviation.

2. **(D).** Standard deviation is a measure of how “spread out” the numbers in a set are—in other words, how far are the individual data points from the average of all of the data points? The GRE will not ask you to calculate standard deviation—in problems like this one, you will be able to eyeball which sets are more spread out and which are less spread out.

Since dataset *Z*’s members are identical, the standard deviation is zero. Zero is the smallest possible standard deviation for any set, so it must be the smallest here. You can eliminate answer choices (A), (B), and (C). Dataset *Y*’s members have a *spread* of 1 between each number, dataset *X*’s members are 2 away from each other, and dataset *W*’s members are 6 away from each other, so dataset *Y* has the next-smallest standard deviation (note that this is enough to eliminate answer choice (E) and choose answer choice (D)). The correct answer is (D) *Z, Y, X, W*.

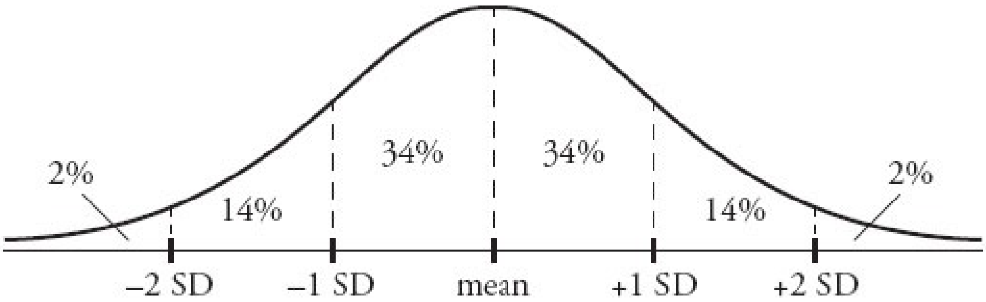
3. **(B).** “Set *N* is a set of *x* distinct positive integers where $x > 2$” just means that the members of the set are all positive integers different from each other and that there are at least 3 of them. Nothing is given about the standard deviation of the set other than that it is not zero. (Because the numbers are different from each other, they are at least a little spread out, which means the standard deviation must be greater than zero. The only way to have a standard deviation of zero is to have a “set” of identical numbers, which would be referred to as a list or a dataset because all of the elements of a proper set (in math) must be different).

In Quantity B, multiplying each of the distinct integers by –3 would definitely spread out the numbers and thus increase the standard deviation. For instance, if the set had been 1, 2, 3, it would become –3, –6, –9. The negatives are irrelevant—multiplying any set of *different* integers by 3 will spread them out more.

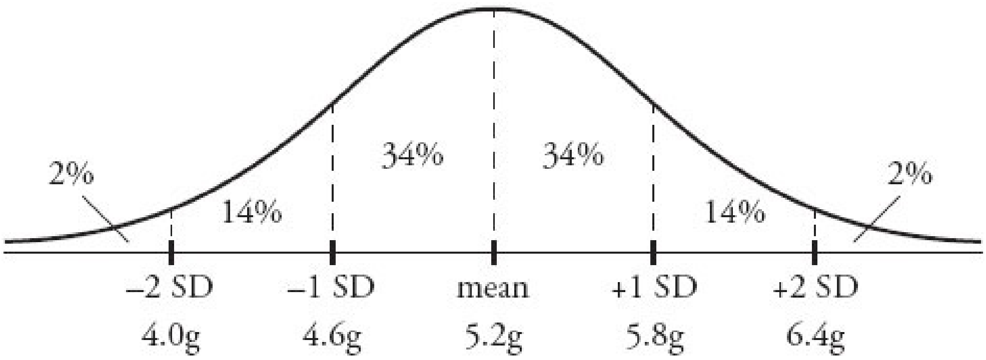
Thus, whatever the standard deviation is for the set in Quantity A, Quantity B must represent a larger standard deviation because the numbers in that set are more spread out.

4. **(D).** Scoring scales on a test are not necessarily linear, so do not line up the difference in percentiles with the difference in score; it is not possible to make any predictions about *other* percentiles. For all you know, 750 could be the 95th percentile score—or 963 could be. All that is certain is that 25% of the scores are ≤ 450, while 50% of the scores are > 450 and ≤ 700, and 25% of the scores are > 700.

5. **(B).** Whenever the words "Normal distribution" appear on the GRE, draw a bell-curve diagram that approximates the one below. Memorize the numbers 34 : 14 : 2.



The middle of the bell curve is the average, or mean, so place 5.2 underneath the 0 in the center; 34%, 14%, and 2% represent the approximate percentages that fall between the standard deviation lines. For instance, 14% of the population falls between 1 and 2 standard deviations below the mean. Now, use the standard deviation of 0.6 grams to figure out the exact dividing lines between the marked regions of the normal curve. The mass of an insect that is exactly 1 standard deviation above the mean is 5.2 + 0.6 = 5.8, and the mass of one that is 1 standard deviation below the mean is 5.2 – 0.6 = 4.6. Similarly, the mass at exactly 2 standard deviations above the mean is 6.4 and at 2 below is 4.0.



Quantity A, the percent between 5.2 and 5.8 grams, is 34%.

However, Quantity B will require some estimating. Note that 4.9 is halfway between 4.6 and 5.2, while 5.5 is halfway between 5.2 and 5.8. Therefore, the area between 4.9 and 5.5, while still a range of 0.6, is under the bigger part of the bell curve in the center. Since the area under the center is bigger than the area between 0 and 1 standard deviations, the percentage of the area under the center must also be greater. Therefore, Quantity B is greater.

6. **(B).** Normal distributions are always centered on and symmetrical around the mean, so the chance that the worm’s length will be within a certain 6-centimeter range (or any specific range) is highest when that range is centered on the mean, which in this case is 30 centimeters.

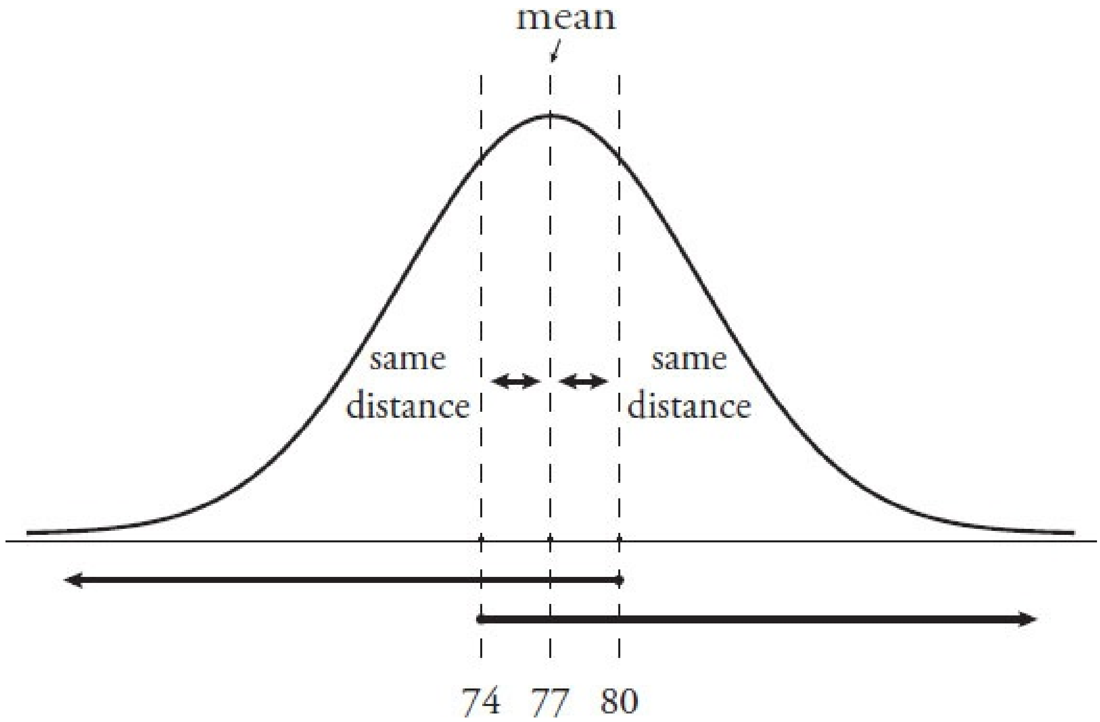
More specifically, Quantity A equals the area between –2 standard deviations and the mean of the distribution. In a normal distribution, roughly 34 + 34 + 14 + 14 = 96% of the sample will fall within 2 standard deviations above or below the mean. Limit yourself only to the 2 standard deviations below the mean, then half of that, or 96% ÷ 2 = 48%, falls in this range. In contrast, Quantity B equals the area between –1 standard deviation and +1 standard deviation. In a normal distribution, roughly 34 + 34 = 68% of the sample falls within 1 standard deviation above or below the mean. Since 68% is greater than 48%, Quantity B is greater.

Note that exact figures are not required to answer this question! Picture any bell curve—the area under the “hump” (that is, centered around the middle) is bigger! Thus, it has more members of the dataset (in this case, worms) in it.

7. **(B).** How many standard deviations above $90,000 is $112,000? The difference between the two numbers is $22,000, which is two times the standard deviation of $11,000. So Quantity A is really the number of home values greater than 2 standard deviations above the mean.

In any normal distribution, roughly 2% will fall more than 2 standard deviations above the mean (this is something to memorize). The value of Quantity A is roughly 8,000 × 0.02 = 160, so Quantity B is greater.

8. **(C).** The normal distribution is symmetrical around the mean. For any symmetrical distribution, the mean equals the median (also known as the 50th percentile). Thus, the number of students who scored *less* than 3 points *above* the mean (77 + 3 = 80) must be the same as the number of students who scored *greater* than 3 points *below* the mean (77 – 3 = 74). As long as the boundary scores (80 and 74) are placed symmetrically around the mean, the distribution will have equal proportions. Draw the normal distribution plot if it is at all confusing:



Notice that the two conditions overlap and are perfectly symmetrical. Each number consists of a short segment between it and the 50th percentile mark, as well as half of the students (either above or below the 50th percentile mark). That is, the "less than 80" category consists of the segment between 80 and 77, as well as all students below the 50th percentile mark (below 77). The "greater than 74" category consists of the segment between 74 and 77, as well as all students above the 50th percentile mark (above 77). Therefore, the quantities are equal.

9. **(D).** First, make the numbers easier to use. Either multiply every number by the same constant or move the decimal the same number of places for each number. In the case of moving the decimal four places, the mean becomes 1,630, the standard deviation becomes 84, and the two other numbers become 1,546 and 1,756.

Next, "normalize" the boundaries. That is, take 1,546 meters (the lower boundary) and 1,756 meters (the upper boundary) and convert each of them to a number of standard deviations away from the mean. To do so, subtract the mean. Then divide by the standard deviation.

Lower boundary: 1,546 – 1,630 = –84

–84 ÷ 84 = –1

So the lower boundary is –1 standard deviation (that is, 1 standard deviation less than the mean).

Upper boundary: 1,756 – 1,630 = 126

126 ÷ 84 = 1.5

So the upper boundary is 1.5 standard deviations above the mean.

You need to find the probability that a random variable distributed according to the standard normal distribution falls between –1 and 1.5.

Use the approximate areas under the normal curve. Approximately 34 + 34 = 68% falls within 1 standard deviation above or below the mean, so 68% accounts for the –1 to 1 portion of the standard normal distribution. What about the portion from 1 to 1.5?

Approximately 14% of the bolts fall between 1 and 2 standard deviations above the mean. You are not expected to know the exact area between 1 and 1.5; however, since a normal distribution has its hump around 0, *more* than half of the area between 1 and 2 must fall closer to 0 (between 1 and 1.5). So the area under the normal curve between 1 and 1.5 must be greater than half of the area, or greater than 7%, but less than the full area, 14%.

Put it all together. The area under the normal curve between –1 and 1.5 is approximately 68% + (something between 7% and 14%). The lower estimate is 68% + 7% = 75% and the upper estimate is 68% + 14% = 82%.

10. **(E).** In a box-and-whisker plot, the middle line in the box represents the median, or middle, of the dataset. The outsides of the box are the medians of the data below and above the median, respectively, which mark the first and third quartile boundaries, or Q_1 and Q_3.

The median is –1; now check the medians of the answer choices. The median of (A) is the average of 0 and –2, which is –1; (A) could be the right answer. The median of (B) is 2, of (C) is –3, and of (D) is 3.5, so none of these are the correct answers. The median of (E) is between –2 and 0, which is –1; (E) could also be the right answer.

Q_1 is –3; check Q_1 for both (A) and (E). The median of the smaller three numbers (–4, –4, –2) for (A) is –4, which is wrong; you want Q_1 to be –3. Choice (E) is the only answer choice left; choose it without checking if you're confident in your previous work. Here's the actual proof: the median of the smaller four numbers (–4, –4, –2, –2) is –3.

11. **(B).** The interquartile range of a dataset is the distance between Q_1 (quartile marker 1, the median of the first half of the dataset) and Q_3 (quartile marker 3, the median of the second half of the dataset).

The first ten positive multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. Q_1 is the median of the first five terms, or 15. Q_3 is the median of the last five terms, or 40.

Take the difference between Q_3 and Q_1: 40 – 15 = 25.

12. **"Dataset *T* is not normally distributed" only.** The definition of "normally distributed" is that about two-thirds of the data falls within 1 standard deviation of the mean. If only one person scored close to the mean (and most people were at the top or bottom of the curve), that dataset is not normally distributed, so the first statement is true.

The second statement is false—the range of the data would not necessarily change if the dataset were more evenly distributed. For instance, as long as one person still had a zero and one person still had a score of 100, the other scores could fall anywhere without changing the range.

The third statement is also false. The mean of dataset *T* might or might not be equal to the median. For instance, the one student within 5 points of the mean could have a score equal to the mean; of the remaining 148 students, half could have scores of 0 and half could have scores of 100. In this case, the mean would equal the median. However, the same scenario with *unequal* numbers of students scoring 0 and 100 would result in the mean *not* equaling the median.

13. **(C).** Percentiles define the proportion of a group that scores below a particular benchmark. Since John scored in the 32nd percentile, by definition, 32% of the class scored worse than John. Quantity A is equal to 32%.

Jane scored in the 68th percentile, so 68% of the class scored worse than she did. Since 100 – 68 = 32, 32% of the class scored equal to or greater than Jane. Quantity B is also equal to 32%.

14. **(A).** Since the average is 7, use the average formula to find the sum of the scores in the class:

$$\begin{aligned} \text{Average} &= \text{Sum} \div (\text{\# of terms}) \\ 7 &= \text{Sum} \div 20 \\ \text{Sum} &= 140 \end{aligned}$$

At least one student got every possible score. There are 11 possible scores: 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. This is an evenly spaced set, so calculate the sum by multiplying the average of the set by the number of terms in the set. The average is (10 + 0)/2 = 5 and the number of terms is 11, so the sum of the set is 5 × 11 = 55. Subtract this from the earlier sum; the remaining 9 students had to score 140 – 55 = 85 points.

Quantity A is the lowest score that could have been received by more than 1 student. If 9 students scored a total of 85 points, and any 1 student could not score more than 10 points, then what is the lowest possible score that one of these 9 students could have received? In order to minimize that number, maximize the numbers for the other students. If 8 students scored 10 points each, for a total of 80 points, then the 9th student must have scored exactly 5.

Quantity A must be greater than Quantity B. Notice that the average score of 7 forces a lot of the scores to be 10 in order to to balance out the very low scores of 0, 1, 2, etc., that are required in the class (at least one of each). The lowest score that could have been received by 2 students is 5, so Quantity A is 5.

15. **(D).** Quintiles ("fifths" of the data) define relative scores, not absolute scores. Imagine two possible score distributions:

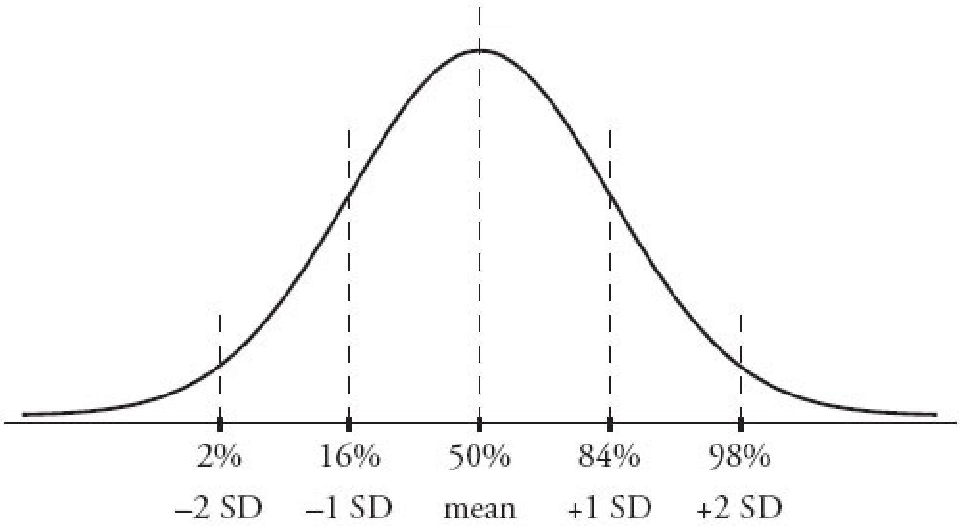
> Example 1: The class's scores are 1, 2, 3, 4, 5 (20% of the class scored each of these). In this case, adding up the two lowest quintile students would be 1 + 1 = 2, which is less than 5, the score of a top quintile student.
>
> Example 2: The class's scores are 10, 11, 12, 13, 14 (20% of the class scored each of these). In this case, adding up the two lowest quintile students would be 10 + 10 = 20, which is greater than 14, the score of a top quintile student.

The relationship cannot be determined from the information given.

16. **(E).** A percentile *always* represents 1% of a set of data. If the question had asked how many *terms* one percentile represented, that would be a different question (with a different answer).

17. **(D).** The values of x or y are unknown, but since they are both positive integers, they can only be 1 and 18, 2 and 9, or 3 and 6 (because they have a product of 18). So the smallest number in the set is 1 and the greatest is 20. Since 20 – 1 = 19, the range is 19.

18. **(D).** The diagram below shows the standard distribution curve for any normally distributed variable. The percent figures correspond roughly to the standard percentiles both 1 and 2 standard deviations (SD) away from the mean:



The 2nd percentile is 1,720, roughly corresponding to 2 standard deviations below the mean. Therefore, the mean –2 standard deviations = 1,720.

Likewise, the 84th percentile is 1,990: 84% of a normally distributed set of data falls below the mean + 1 standard deviation, so the mean + 1 standard deviation = 1,990.

Call the mean M and the standard deviation S. Solve for these variables:

$$M - 2S = 1{,}720$$
$$M + S = 1{,}990$$

Subtract the first equation from the second equation:

$$3S = 270$$

$S = 90$

The question asks for the 16th percentile, which is the mean – 1 standard deviation or $M - S$. (It's a fact to memorize that approximately 2% of normally distributed data falls below $M - 2S$, and approximately 14% of normally distributed data falls between $M - 2S$ and $M - S$.)

Since $M - 2S = 1{,}720$, add another S to get $M - S$:

$$(M - 2S) + S = 1{,}720 + 90 = 1{,}810$$

Notice that the percentiles are *not* linearly spaced. The normal distribution is hump-shaped, so percentiles are bunched up around the hump and spread out farther away.

19. **(D).** In most datasets, the range is larger than the interquartile range because the interquartile range ignores the smallest and largest data points. That's actually the purpose of interquartile range—to get a good picture of where *most* of the data is (think of the "big hump" on a bell curve). For instance:

Example set *A*: 1, 2, 3, 4, 5, 6, 7, 100

Here, the range is 100 – 1 = 99.

The interquartile range is $Q_3 - Q_1$, or the median of the upper half of the data minus the median of the lower half of the data: 6.5 – 2.5 = 4.

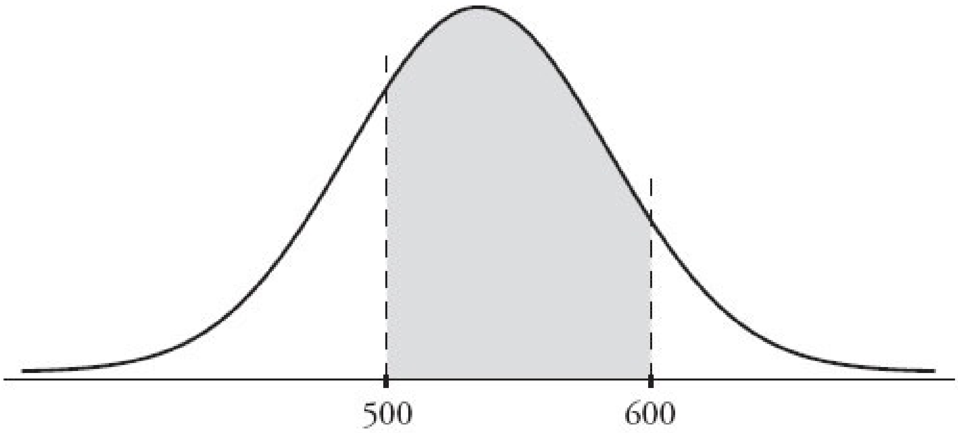
In this example, the range is much larger. However, consider this set:

Example set *B*: 4, 4, 4, 4, 5, 5, 5, 5

In this set, the range is 5 – 4 = 1. The interquartile range is also 5 – 4 = 1. While the interquartile range can never be *greater* than the range, they can certainly be equal.

20. **(C).** Since 1 standard deviation below the mean is 250 and 1 standard deviation above the mean is 420, the mean/median must be halfway in between. Since 420 – 250 = 170 and half of 170 is 85, add 85 to 250 (or subtract it from 420) to get the mean/median of 335. (Note that in a normal distribution, the mean is equal to the median, so the two terms can be used interchangeably.)

21. **(D).** While the shaded area may appear to be evenly located on either side of the mean, it isn't necessarily. For example, the 68% could be more lopsided, like so:



This area could still represent 68% of the scores, even if it's not 1 standard deviation to either side of the mean. In order to determine that the mean is 550, the problem would need to state explicitly that 500 and 600 each represent 1 standard deviation from the mean (or at least that 500 and 600 are equally far from the mean).

The fact that 68% of the data is located between 500 and 600 is a trick implying that 500 and 600 are –1 and +1 standard deviation from the mean, but this is not necessarily true. While it is always true that, in a normal distribution, about 68% (some people memorize the approximation as two-thirds) of the data is within 1 standard deviation of the mean, the reverse is not true: do not assume that any chunk of data that is about 68% of the whole is therefore within 1 standard deviation of the mean.

22. **1st and 3rd only.** The first statement is true. Standard deviation describes how much a set of data diverges from the mean. Curve *B* is more widely spread than curve *A*, and thus *Y* has a greater standard deviation than *X*.

The second statement is not true. The probability that *any* normally distributed variable falls within 2 standard deviations of its mean is the same, approximately 0.14 + 0.34 + 0.34 + 0.14 = 0.96, or 96%. Memorize this value for the GRE.

The third statement is true. The mean of a normal curve is the point along the horizontal axis below the "peak" of the curve. The highest point of curve *B* is clearly to the right of the highest point of curve *A*, so the mean of *Y* is larger than the mean of *X*. Notice that the mean has nothing to do with the *height* of the normal curve, which only corresponds to how tightly the variable is gathered around the mean (i.e., how small the standard deviation is).

23. **(A).** There are 400 test scores distributed among 50 possible outcomes (integers between 151 and 200, inclusive, which number 200 – 151 + 1 = 50

integers). There is an average of 400 ÷ 50 = 8 scores per integer outcome, and there are 400 ÷ 100 = 4 scores in each percentile. So, if all the scores were completely evenly distributed with exactly 8 scores per integer, there would be two percentile groups per integer outcome (0th and 1st percentiles at 151, 2nd and 3rd percentiles at 152, etc.). In that case, all 50 integers from 151 to 200 would correspond to more than one percentile group.

Reduce the number of integers corresponding to more than one percentile group by bunching up the scores. Imagine that everyone gets a 157. Then that integer is the only one that corresponds to more than one percentile group (it corresponds to all 100 groups, in fact). However, don’t reduce further this way. This gives exactly 1 integer, so the minimum number of integers corresponding to more than one percentile group is 1, which is Quantity A.

As for Quantity B, though, a particular integer may have *no* percentile groups corresponding to it. In the previous example, if everyone gets a 157, then no one gets a 158, or a 200 for that matter. So the minimum number of percentile groups corresponding to a score of 200 (or to any other particular score) is 0, which is Quantity B.

24. **(C).** A two-humped shape could come from two overlapping normal distributions with different averages. Since the hump on the right is smaller, the distribution with a higher average should contain less data. Of the possible answer choices, only (C) describes such a scenario.

25. **1st, 2nd, and 3rd.** The first statement could be true. Although biologists' salaries cluster around a lower number than physicists' salaries do, do not assume that *every* biologist's salary is lower than *every* physicist's salary. Some biologists' salaries can be high, and some physicists' salaries can be low. The graph shows a small area of overlap between the two means.

The second statement could be true. Normal distributions are consistent with the hump shapes you see in the graph. While it's not possible to *prove* that they're normal, it's also not possible to prove that they're definitely not—they certainly *could* be normal.

The third statement could be true. From real-world normal distributions of an unknown amount of data, there's no way to tell the maximum or minimum values of the data. So the range certainly could be more than $150,000.

26. **2nd and 3rd only.** The first statement is not necessarily true. Range is calculated this way: *Largest value – Smallest value.* From the graphs as shown (assuming that they do not continue "off screen" left and right), it is possible that the two distributions have the *same* range, because the distributions are above zero on both the far left and the far right. (In the real world, the graphs might even continue off screen, leading to even less confidence about the range of each distribution.)

The second statement is true. The graph on the right (Town Y) has a smaller standard deviation (it is less spread out around its mean). So families in Town Y are more likely to be within 1 family member of the mean than families in Town X are.

The third statement is true. The graph on the left is more spread out, so it has a larger standard deviation.

27. **(A).** The plot is symmetrical, so you can eliminate any non-symmetrical datasets (such as (C), (D), and (E)). In (B), all the data points are the same, so

there would be no width to the box-and-whisker plot. Choice (A) is the only remaining possibility: the data is evenly spaced, leading to equal widths for each segment of the plot, as shown.

28. **3rd only.** The first statement is not true. The median of the whole set is the line in the middle of the box. As shown, it is closer to the *right* side of the box (the median of the upper half of the data) than to the *left* side of the box (the median of the lower half of the data)—the opposite of what this statement claims.

The second statement is not true. This non-symmetrical plot could never represent a symmetrical distribution such as the normal distribution. In fact, a *true* normal distribution cannot be represented by a box-and-whisker plot at all, because such a distribution stretches infinitely to the right and to the left, in theory.

The third statement is true. Any set represented by a box-and-whisker plot has a standard deviation greater than zero, because the plot displays some spread in the data. The only set that has a zero standard deviation is a set containing identical data points with zero spread between them, such as {3, 3, 3, 3}.

29. **1st only.** Since the overall average length of all the earthworms is closer to the average length of earthworms in sample A than to the average for sample B, there are more earthworms in sample A.

However, without individual values, the mean and the range of the dataset are still unknown. For instance, the lengths of all the worms in sample *A* could be exactly 2.4, or they could be spread out quite a bit from 2.4. Similarly, the worms in sample B could measure exactly 3.8, or they could have a variety of different lengths that average to 3.8. Thus, the median and range could vary quite a bit.

Chapter 23

Probability, Combinatorics, and Overlapping Sets

In This Chapter…

Probability, Combinatorics, and Overlapping Sets

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A number is randomly chosen from a list of 10 consecutive positive integers. What is the probability that the number selected is greater than the average (arithmetic mean) of all 10 integers?

   (A) $\frac{3}{10}$

   (B) $\frac{2}{5}$

   (C) $\frac{1}{2}$

   (D) $\frac{7}{10}$

(E) $\frac{4}{5}$

2. A number is randomly chosen from the first 100 positive integers. What is the probability that it is a multiple of 3?

(A) $\frac{32}{100}$

(B) $\frac{33}{100}$

(C) $\frac{1}{3}$

(D) $\frac{34}{100}$

(E) $\frac{2}{3}$

3. A restaurant menu has several options for tacos. There are 3 types of shells, 4 types of meat, 3 types of cheese, and 5 types of salsa. How many distinct tacos can be ordered assuming that any order contains exactly one of each of the above choices?

$\boxed{}$

4. A history exam features five questions. Three of the questions are multiple-choice with four options each. The other two questions are true or false. If Caroline selects one answer for every question, how many different ways can she answer the exam?

$\boxed{}$

5. The probability is $\frac{1}{2}$ that a certain coin will turn up heads on any given toss and the probability is $\frac{1}{6}$ that a number cube with faces numbered 1 to 6 will turn up any particular number. What is the probability of turning up a heads and a 6?

(A) $\frac{1}{36}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

(E) $\frac{2}{3}$

6. An integer is randomly chosen from 2 to 20 inclusive. What is the probability that the number is prime?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

7. An Italian restaurant boasts 320 distinct pasta dishes. Each dish contains exactly 1 pasta, 1 meat, and 1 sauce. If there are 8 pastas and 4 meats available, how many sauces are there to choose from?

$\boxed{}$

8. A 10-student class is to choose a president, vice president, and secretary from the group. If no person can occupy more than one post, in how many ways can this be accomplished?

$\boxed{}$

9. BurgerTown offers many options for customizing a burger. There are 3 types of meats and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard, and special sauce. A burger must include meat, but may include as many or as few condiments as the customer wants. How many different burgers are possible?

(A) 8!

(B) (3)(7!)

(C) (3)(8!)

(D) $(8)(2^7)$

(E) $(3)(2^7)$

10. The probability of rain is $\frac{1}{6}$ for any given day next week. What is the probability that it will rain on both Monday and Tuesday?

(A) $\frac{1}{36}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) $\frac{2}{3}$

11. How many five-digit numbers can be formed using the digits 5, 6, 7, 8, 9, 0 if no digits can be repeated?

(A) 64

(B) 120

(C) 240

(D) 600

(E) 720

12. A bag contains 3 red, 2 blue, and 7 white marbles. If a marble is randomly chosen from the bag, what is the probability that it is <u>not</u> blue?

Give your answer as a fraction.

13. A man has 3 different suits, 4 different shirts, 2 different pairs of socks, and 5 different pairs of shoes. If an outfit consists of exactly 1 suit, 1 shirt, 1 pair of socks, and 1 pair of shoes, how many different outfits can be made with the man's clothing?

A state issues automobile license plates that begin with two letters selected from a 26-letter alphabet, followed by four numerals selected from the digits 0 through 9, inclusive. Repeats are permitted. For example, one possible license plate combination is GF3352.

	Quantity A	Quantity B
14.	The number of possible unique license plate combinations	6,000,000

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15. A bag contains 6 black chips numbered 1–6 respectively and 6 white chips numbered 1–6 respectively. If Pavel reaches into the bag of 12 chips and removes 2 chips, one after the other, without replacing them, what is the probability that he will pick black chip #3 and then white chip #3?

Give your answer as a fraction.

Tarik has a pile of 6 green chips numbered 1 through 6 respectively and another pile of 6 blue chips numbered 1 through 6 respectively. Tarik will randomly pick 1 chip from the green pile and 1 chip from the blue pile.

16.

Quantity A	**Quantity B**
The probability that both chips selected by Tarik will display a number less than 4	$\frac{1}{2}$

17. A bag contains 6 red chips numbered 1 through 6, respectively, and 6 blue chips numbered 1 through 6, respectively. If 2 chips are to be picked sequentially from the bag of 12 chips, without replacement, what is the probability of picking a red chip and then a blue chip with the same number?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

In a school of 150 students, 75 study Latin, 110 study Spanish, and 11 study neither.

18.

Quantity A	**Quantity B**
The number of students who study only Latin	46

19. How many 10-digit numbers can be formed using only the digits 2 and 5?

(A) 2^{10}

(B) (22)(5!)

(C) (5!)(5!)

(D) $\frac{10!}{2}$

(E) 10!

20. A 6-sided cube has faces numbered 1 through 6. If the cube is rolled twice, what is the probability that the sum of the two rolls is 8?

(A) $\frac{1}{9}$

(B) $\frac{1}{8}$

(C) $\frac{5}{36}$

(D) $\frac{1}{6}$

(E) $\frac{7}{36}$

21. A certain coin with heads on one side and tails on the other has a $\frac{1}{2}$ probability of landing on heads. If the coin is flipped 5 times, how many distinct outcomes are possible if the last flip must be heads? Outcomes are distinct if they do not contain exactly the same results in exactly the same order.

$\boxed{}$

In a class of 25 students, each student studies either Spanish, Latin, or French, or two of the three, but no students study all three languages. 9 study Spanish, 7 study Latin, and 5 study exactly two languages.

	Quantity A	**Quantity B**
22.	The number of students who study French	14

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23. Pedro has a number cube with 24 faces and an integer between 1 and 24 on each face. Every number is featured exactly once. When he rolls, what is the probability that the number showing is a factor of 24?

Give your answer as a fraction.

24. A baby has x total toys. If 9 of the toys are stuffed animals, 7 of the toys were given to the baby by her grandmother, 5 of the toys are stuffed animals given to the baby by her grandmother, and 6 of the toys are neither stuffed animals nor given to the baby by her grandmother, what is the value of x?

$\boxed{}$

25. How many integers between 2,000 and 3,999 have a ones digit that is a prime number?

$\boxed{}$

26. A group of 12 people who have never met are in a classroom. How many handshakes are exchanged if each person shakes hands exactly once with each of the other people in the room?

(A) 12

(B) 22

(C) 66

(D) 132

(E) 244

27. A class consists of 12 girls and 20 boys. One quarter of the girls in the class have blue eyes. If a child is selected at random from the class, what is the probability that the child is a girl who does not have blue eyes?

(A) $\frac{3}{32}$

(B) $\frac{9}{32}$

(C) $\frac{3}{8}$

(D) $\frac{23}{32}$

(E) $\frac{29}{32}$

28. A certain coin with heads on one side and tails on the other has a $\frac{1}{2}$ probability of landing on heads. If the coin is flipped 3 times, what is the probability of flipping 2 tails and 1 head, in any order?

(A) $\frac{1}{8}$

(B) $\frac{1}{3}$

(C) $\frac{3}{8}$

(D) $\frac{5}{8}$

(E) $\frac{2}{3}$

29. A number cube has six faces numbered 1 through 6. If the cube is rolled twice, what is the probability that at least one of the rolls will result in a number greater than 4?

(A) $\frac{2}{9}$

(B) $\frac{1}{3}$

(C) $\frac{4}{9}$

(D) $\frac{5}{9}$

(E) $\frac{2}{3}$

30. 100 tiles are labeled with the integers from 1 to 100 inclusive; no numbers are repeated. If Alma chooses 1 tile at random, replaces it in the group, and

chooses another tile at random, what is the probability that the product of the two integer values on the tiles is odd?

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) $\frac{3}{4}$

31. If the word “WOW” can be rearranged in exactly 3 ways (WOW, OWW, WWO), how many different arrangements of the letters in “MISSISSIPPI” are possible?

The probability of rain is $\frac{1}{2}$ on any given day next week.

	Quantity A	Quantity B
32.	The probability that it rains on at least one of the 7 days next week	$\frac{127}{128}$

33. Two number cubes with six faces numbered with the integers from 1 through 6 are tossed. What is the probability that the sum of the exposed faces on the cubes is a prime number?

Give your answer as a fraction.

34. Jan and 5 other children are in a classroom. The principal of the school will choose 2 of the children at random. What is the probability that Jan will be chosen?

(A) $\frac{4}{5}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{7}{15}$

(E) $\frac{1}{2}$

The probability that Maria will eat breakfast on any given day is 0.5. The probability that Maria will wear a sweater on any given day is 0.3. The two probabilities are independent of each other.

	Quantity A	Quantity B
35.	The probability that Maria eats breakfast or wears a sweater	0.8

The probability of rain in Greg’s town on Tuesday is 0.3. The probability that Greg’s teacher will give him a pop quiz on Tuesday is 0.2. The events occur independently of each other.

	Quantity A	**Quantity B**
36.	The probability that either or both events occur	The probability that neither event occurs

37. A certain city has a $\frac{1}{3}$ chance of rain occurring on any given day. In any given 3-day period, what is the probability that the city experiences rain?

(A) $\frac{1}{3}$

(B) $\frac{8}{27}$

(C) $\frac{2}{3}$

(D) $\frac{19}{27}$

(E) 1

38. Five students, Adnan, Beth, Chao, Dan, and Edmund are to be arranged in a line. How many such arrangements are possible if Beth is not allowed to stand next to Dan?

(A) 24

(B) 48

(C) 72

(D) 96

(E) 120

39. A polygon has 12 edges. How many different diagonals does it have? (A diagonal is a line drawn from one vertex to any other vertex inside the given shape. This line cannot touch or cross any of the edges of the shape. For example, a triangle has zero diagonals and a rectangle has two.)

(A) 54

(B) 66

(C) 108

(D) 132

(E) 144

An inventory of coins contains 100 different coins.

	Quantity A	Quantity B
40.	The number of possible collections of 56 coins that can be selected (the order of the coins does not matter)	The number of possible collections of 44 coins that can be selected (the order of the coins does not matter)

An office supply store carries an inventory of 1,345 different products, all of which it categorizes as "business use," "personal use," or both. There are 740 products categorized as "business use" only and 520 products categorized as both "business use" and "personal use."

	Quantity A	Quantity B
41.	The number of products characterized as "personal use"	600

42. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) $\frac{1}{32}$

(B) $\frac{1}{4}$

(C) $\frac{2}{5}$

(D) $\frac{7}{15}$

(E) $\frac{8}{15}$

43. At Lexington High School, each student studies at least one language—Spanish, French, or Latin—and no student studies all three languages. If 100 students study Spanish, 80 study French, 40 study Latin, and 22 study exactly two languages, how many students are there at Lexington High School?

(A) 198

(B) 220

(C) 242

(D) 264

(E) 286

Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be either migratory or not migratory.)

	Quantity A	**Quantity B**
44.	The number of the 60 birds that are neither migratory nor songbirds	16

Probability, Combinatorics, and Overlapping Sets Answers

1. **(C).** In a list of 10 consecutive integers, the mean is the average of the 5th and 6th numbers. Therefore, the 6th through 10th integers (5 total integers) is greater than the mean. Since probability is determined by the number of desired items divided by the total number of choices, the probability that the number chosen is greater than the average of all 10 integers is $\frac{5}{10} = \frac{1}{2}$.

Another approach to this problem is to create a set of 10 consecutive integers; the easiest such list contains the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The mean is one-half the sum of the first element plus the last element, or $\frac{1+10}{2}$ = 5.5. Therefore, there are 5 numbers greater than the mean in the list: 6, 7, 8, 9, and 10. Again, the probability of choosing a number greater than the average of all 10 integers is $\frac{5}{10} = \frac{1}{2}$.

2. **(B).** The first 100 positive integers comprise the set of numbers containing the integers 1 to 100. Of these numbers, the only ones that are divisible by 3 are {3, 6, 9, …, 96, 99}, which adds up to exactly 33 numbers. This can be determined in several ways. One option is to count the multiples of 3, but that's a bit slow. Alternatively, compute $\frac{99}{3}$ = 33 and realize that there are 33 multiples of 3 up to and including 99. The number 100 is not divisible by 3, so the correct answer is $\frac{33}{100}$.

Alternatively, use the "add one before you're done" trick, subtracting the first multiple of 3 from the last multiple of 3, dividing by 3 and then adding 1: $\frac{(99-3)}{3}$ + 1 = 33. Then, since probability is determined by the number of desired options divided by the total number of options, the probability that the number chosen is a multiple of 3 is $\frac{33}{100}$.

3. **180.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Therefore, the total number of tacos is (3)(4)(3)(5) = 180 tacos.

4. **256.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The five separate test questions means there are five independent choices. For the three multiple-choice questions there are four options each, whereas for the two true/false questions there are two options each. Multiplying the independent choices yields (4)(4)(4)(2)(2) = 256 different ways to answer the exam.

5. **(B).** The probability of independent events A *and* B occurring is equal to the product of the probability of event A and the probability of event B. In this case, the probability of the coin turning up heads is $\frac{1}{2}$ and the probability of rolling a 6 is $\frac{1}{6}$. Therefore, the probability of heads *and* a 6 is equal to $\left(\frac{1}{2}\right)\left(\frac{1}{6}\right)=\frac{1}{12}$. Alternatively, list all the possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6. There are 12 total outcomes and only 1 with heads and a 6. Therefore, the desired outcome divided by the total number of outcomes is equal to $\frac{1}{12}$.

6. $\frac{8}{19}$ **(or any equivalent fraction).** Among the integers 2 through 20, inclusive, there are 8 primes: 2, 3, 5, 7, 11, 13, 17, and 19. From 2 to 20, inclusive, there are exactly 20 – 2 + 1 = 19 integers; remember to "add one before you're done" to include both endpoints. Alternatively, there are 20 integers from 1 to 20, inclusive, so there must be 19 integers from 2 to 20, inclusive. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the number chosen is prime is $\frac{8}{19}$.

7. **10.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Let the number of sauces be represented by the variable S. The total number of possible pasta dishes can be represented by each separate choice multiplied together: $(8)(4)(S)$, or $32S$. The problem also indicates that the total number of pasta dishes must be equal to 320. Therefore, $32S = 320$, so $S = 10$.

8. **720.** One possible approach is to ask, "How many choices do I have for each of the class positions?" Begin by considering the president of the class. Since no one has been chosen yet, there are 10 students from whom to choose. Then, for the vice president there are 9 options because now 1 student has already been chosen as president. Similarly, there are 8 choices for the secretary. Using the fundamental counting principle, the total number of possible selections is $(10)(9)(8) = 720$.

Alternatively, use factorials. In this case, order matters because people are selected for specific positions. This problem is synonymous to asking, "How many different ways can you line up 3 students as first, second, and third from a class of 10?" The number of ways to arrange the entire class in line is 10!. However, the problem is only concerned with the first 3 students in line, so exclude rearrangements of the last 7. The way in which these "non-chosen" 7 students can be ordered is 7!. Thus, the total number of arrangements for 3 students from a class of 10 is $\frac{10!}{7!} = (10)(9)(8) = 720$ choices.

9. **(E).** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The key to this problem is realizing how many choices there are for each option. For the meat, there are 3 choices. For each of the condiments

there are exactly 2 choices: yes or no. The only real choice regarding each condiment is whether to include it at all. As there are 7 condiments, the total number of choices is (3)(2)(2)(2)(2)(2)(2)(2) = $(3)(2^7)$.

Note: the condiment options cannot be counted as 8! (0 through 7 = 8 options) because, in this case, the order in which the options are chosen does not matter; a burger with lettuce and pickles is the same as a burger with pickles and lettuce.

10. **(A).** For probability questions, always begin by separating out the probabilities of each individual event. Then, if all the events happen (an "*and* question"), multiply the probabilities together. If only one of the multiple events happens (an "*or* question"), add the probabilities together.

In this case, there are two events: rain on Monday and rain on Tuesday. The question asks for the probability that it will rain on Monday *and* on Tuesday, so multiply the individual probabilities together:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

11. **(D).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. The problem asks how many five-digit numbers can be created from the digits 5, 6, 7, 8, 9, and 0. For the first digit, there are only 5 options (5, 6, 7, 8, and 9) because a five-digit number must start with a non-zero integer. For the second digit, there are 5 choices again, because now zero can be used but one of the other numbers has already been used, and numbers cannot be repeated. For the third number, there are 4 choices, for the fourth number there are 3 choices, and for the fifth number there are 2 choices. Thus, the total number of choices is (5)(5)(4)(3)(2) = 600.

Alternatively, use the same logic and realize there are 5 choices for the first digit. (Separate out the first step because you have to remove the zero from consideration.) The remaining five digits all have an equal chance of being chosen, so choose four out of the remaining five digits to complete the number. The number of ways in which this second step can be accomplished is $\frac{(5!)}{(1!)}$ = (5)(4)(3)(2). Thus, the total number of choices is again equal to (5)(5)(4)(3)(2) = 600.

12. $\frac{5}{6}$ **(or any equivalent fraction).** In the bag of marbles, there are 3 red marbles and 7 white marbles, for a total of 10 marbles that are not blue. There are a total of 3 + 7 + 2 = 12 marbles in the bag. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the marble chosen is *not* blue is $\frac{10}{12}=\frac{5}{6}$.

13. **120.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Since the man must choose one suit, one shirt, one pair of socks, and one pair of shoes, the total number of outfits is the number of suits times the number of shirts times the number of socks times the number of shoes: (3)(4)(2)(5) = 120.

14. **(A).** This is a combinatorics problem. The license plates have 2 letters followed by 4 numbers, so make 6 “slots” and determine how many possibilities there are for each slot. There are 26 letters in the alphabet and 10 digits to pick from, so:

$$\underline{26} \quad \underline{26} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10}$$

Multiply 26 × 26 on the calculator to get 676. Add four zeros for the four 10's to get 6,760,000. Quantity A is greater.

15. $\mathbf{\frac{1}{132}}$ **(or any equivalent fraction).** The probability of picking black chip #3 is $\frac{1}{12}$. Once Pavel has removed the first chip, only 11 chips remain, so the probability of picking white chip #3 is $\frac{1}{11}$. Multiply $\frac{1}{12}\times\frac{1}{11}=\frac{1}{132}$.

16. **(B).** In this problem, Tarik is *not* picking 1 chip out of all 12. Rather, he is picking 1 chip out of 6 green ones, and then picking another chip out of 6 blue ones. There are 3 green chips with numbers less than 4, so Tarik has a $\frac{3}{6}$ chance of selecting a green chip showing a number less than 4. Likewise, Tarik has a $\frac{3}{6}$ chance of selecting a blue chip showing a number less than 4. Therefore, Quantity A is equal to $\frac{3}{6}\times\frac{3}{6}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. Quantity B is greater.

17. $\frac{1}{22}$ **(or any equivalent fraction).** The trap answer in this problem is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. This is *not* the answer to the question being asked—rather, this is the answer to the question, "What is the probability of picking a red chip and then a blue chip that both have #3?" (or any other specific number). This is a more specific question than the one actually asked. In the question, asked, there are six possible ways to fulfill the requirements of the problem, not one, because the problem does not specify whether the number should be 1, 2, 3, 4, 5, or 6.

Thus, *any* of the 6 red chips is acceptable for the first pick. However, on the second pick, only the blue chip with the same number as the red one that was just picked is acceptable (the chip must "match" the first one picked). Thus:

$$\frac{6}{12} \times \frac{1}{11} = \frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$$

18. **(B).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 – Both + Neither. (Adding the two groups—in this case Latin and Spanish—double-counts the students who study both languages, so the formula subtracts the "both" students.) Set up your equation:

$$150 = 75 + 110 - B + 11$$
$$150 = 196 - B$$
$$46 = B$$

Careful! This is not the value of Quantity A. Since 46 students study both Latin and Spanish, subtract 46 from the total who study Latin to find those who study only Latin:

$$75 - 46 = 29$$

Thus, Quantity A is 29. Therefore, Quantity B is greater.

19. **(A).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. For any digit of the 10-digit number there are exactly two options, a 2 or a 5. Thus, since there are two choices for each digit and it is a 10-digit number, there are (2)(2)(2)(2)(2)(2)(2)(2)(2)(2) = 2^{10} total choices.

20. **(C).** The probability of any event equals the number of ways to get the

desired outcome divided by the total number of ways for the event to happen. Starting with the denominator, use the fundamental counting principle to compute the total number of ways to roll a cube twice. There are 6 possibilities (1, 2, 3, 4, 5, or 6) for the first roll and 6 for the second, giving a total of (6)(6) = 36 possibilities for the two rolls. For the numerator, determine the number of possible combinations that will sum to 8. For example, rolling a 2 the first time and a 6 the second time. The full set of options is (2, 6), (3, 5), (4, 4), (5, 3), and (6, 2). Thus, there are 5 possible combinations that sum to 8, yielding a probability of $\frac{5}{36}$.

21. **16.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. For the first flip, there are 2 options: heads or tails. Similarly, for the second flip, there 2 options; for the third, there are 2 options; for the fourth, there are 2 options; and for the fifth there is only 1 option because the problem restricts this final flip to heads. Therefore, the total number of outcomes

is (2)(2)(2)(2)(1) = 16. A good rephrasing of this question is, “How many different outcomes are there if the coin is flipped 4 times?” The fifth flip, having been restricted to heads, is irrelevant. Therefore, the total number of ways to flip the coin five times with heads for the fifth flip is equal to the total number of ways to flip the coin four times; either way, the answer is 16.

22. **(C).** The problem specifies that no one studies all three languages. In addition, a total of 5 people study two languages. Thus, 5 people have been double-counted. Since the total number of people who have been double-counted (5) and triple-counted (0) is known, use the standard overlapping sets formula:

Total = Spanish + French + Latin – (Two of the three) – 2(All three)

25 = 9 + French + 7 – 5 – 2(0)
25 = 11 + French
14 = French

The two quantities are equal.

23. $\mathbf{\frac{1}{3}}$ **(or any equivalent fraction).** Probability equals the number of desired outcomes divided by the total number of possible outcomes. Among the integers 1 through 24, there are four factor pairs of 24: (1, 24), (2, 12), (3, 8), and (4, 6), for a total of 8 factors. The total number of possible outcomes when rolling the cube once is 24. The probability that the number chosen is a factor of 24 is $\frac{8}{24}=\frac{1}{3}$.

24. **17.** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 – Both + Neither. Here, the groups are “stuffed animal” and “given by the baby’s grandmother.” The problem indicates that the “both” category is equal to 5 and that the “neither” number is 6. The total is x.

Total = Group 1 + Group 2 – Both + Neither

$x = 9 + 7 – 5 + 6$
$x = 17$

25. **800.** This is a combinatorics problem. Make four “slots” (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

____ ____ ____ ____

Since the number must begin with 2 or 3, there are two possibilities for the first slot. Because the ones digit must be prime and there are only four prime one-digit numbers (2, 3, 5, and 7), there are four possibilities for the last slot:

$\underline{\ \ 2\ \ }\quad \underline{\qquad}\quad \underline{\qquad}\quad \underline{\ \ 4\ \ }$

The other slots have no restrictions, so put 10 in them, since there are ten digits from 0–9:

$$\underline{\ 2\ }\quad\underline{\ 10\ }\quad\underline{\ 10\ }\quad\underline{\ 4\ }$$

Multiply to get 800.

Alternatively, figure out the pattern and add up the number of qualifying four-digit integers. In the first ten numbers, 2000–2009, there are exactly four numbers that have a prime units digit: 2002, 2003, 2005, and 2007. The pattern then repeats in the next group of ten numbers, 2010–2020, and so on. In any group of ten numbers, then, four qualify. Between 2,000 and 3,999 there are 3,999 – 2,000 + 1 = 2,000 numbers, or $\frac{2,000}{10}$ = 200 groups of ten numbers, so there are a total of 400 × 2 = 800 numbers that have a prime units digit.

26. **(C).** Multiple approaches are possible here. One way is to imagine the scenario and count up the number of handshakes. How many hands does everyone need to shake? There are 11 other people in the room, so the first person needs to shake hands 11 times. Now, move to the second person: how many hands must he shake? He has already shaken one hand, leaving him 10 others with whom to shake hands. The third person will need to shake hands with 9 others, and so on. Therefore, there are a total of 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 handshakes. The fastest way to find the sum of a group of consecutive numbers is to take the average of the first and last terms and multiply it by the number of terms. The average is $\frac{(11+1)}{2}$ = 6 and there are 11 – 1 + 1 = 11 terms (find the difference between the terms and “add one before you’re done”). The sum is 6 × 11 = 66.

Alternatively, rephrase the question as “How many different ways can any 2 people be chosen from a group of 12?” (This works because the problem ultimately asks you to “choose” each distinct pair of 2 people one time.) The key here is to realize that handshakes are independent of order, that is, it doesn’t matter if A shakes hands with B or if B shakes hands with A; it’s the same outcome. Thus, it only matters how many pairs you can make. Any time a question presents a group of order-independent items selected from a larger set, apply the formula $\frac{\text{total!}}{\text{in!out!}}$ to arrive at the total number of combinations.

Thus: $\frac{12!}{2!10!} = \frac{12 \times 11}{2}$ = 66.

27. **(B).** The probability of any outcome is equal to the number of desired outcomes divided by the total number of outcomes. There are 12 girls and 20 boys in the classroom. If one-quarter of the girls have blue eyes, then there are $(12)\left(\frac{1}{4}\right)$ = 3 girls with blue eyes. Therefore, there are 12 – 3 = 9 girls who do *not* have blue eyes. The total number of ways in which a child could be chosen is the total number of children in the class, namely 12 + 20 = 32. Therefore, the probability of choosing a girl who does not have blue eyes equals the number of girls without blue eyes divided by the total number of children, which is $\frac{9}{32}$.

28. **(C).** There are only 2 possible outcomes for each flip and only 3 flips total. The most straightforward approach is to list all of the possible outcomes: {HHH, HHT, HTH, HTT, TTT, TTH THH, THT}. Of these 8 possibilities, 3 of the outcomes have 1 head and 2 tails, so the probability of this event is $\frac{3}{8}$.

Alternatively, count the total number of ways of getting 1 head without listing all the possibilities. If the coin is flipped 3 times and you want only 1 head, then there are 3 possible positions for the single head: on the first flip alone, on the second flip alone, or on the third flip alone. Since there are 2 possible outcomes for each flip, heads or tails, there are (2)(2)(2) = 8 total outcomes. Again, the probability is $\frac{3}{8}$.

Finally, another alternative is to compute the probability directly. The probability of flipping heads is $\frac{1}{2}$ and the probability of flipping tails is also $\frac{1}{2}$. The probability of getting heads in the first position alone, or HTT, is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$, multiplied because the coin was flipped heads *and* tails *and* tails. This represents the probability of heads in position 1, but heads could also be in position 2 alone or in position 3 alone. Since there are 3 possible positions for the heads, multiply by 3 to get the total probability $(3)\left(\frac{1}{8}\right)=\frac{3}{8}$.

29. **(D).** Because this problem is asking for an "at least" solution, use the 1 – *x* shortcut. The probability that at least one roll results in a number greater than 4 is equal to 1 minus the probability that both of the rolls result in numbers 4 or lower. For one roll, there are 6 possible outcomes (1 through 6) and 4 ways in which the outcome can be 4 or lower, so the probability is $\frac{4}{6}=\frac{2}{3}$. Thus, the probability that both rolls result in numbers 4 or lower is $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)=\frac{4}{9}$. This is the result that you do *not* want; subtract this from 1 to get the probability that you do want. The probability that at least one of the rolls results in a number greater than 4 is 1 – $\left(\frac{4}{9}\right)=\frac{5}{9}$.

Alternatively, write out the possibilities. The total number of possibilities for two rolls is (6)(6) = 36. Here are the ways in which at least one number greater than 4 can be rolled:

51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66
15, 25, 35, 45 (note: 55 and 65 have already been counted above)
16, 26, 36, 46 (note: 56 and 66 have already been counted above)

There are 20 elements (be careful not to double-count any options). The probability of at least one roll resulting in a number greater than 4 is $\frac{20}{36} = \frac{5}{9}$.

30. **(B).** Use both probability and number properties concepts in order to answer this question. First, in order for two integers to produce an odd integer, the two starting integers must be odd. An odd times an odd equals an odd. An even times an odd, by contrast, produces an even, as does an even times an even.

Within the set of tiles, there are 50 even numbers (2, 4, 6, …, 100) and 50 odd numbers (1, 3, 5, …, 99). One randomly chosen tile will have a $\frac{50}{100} = \frac{1}{2}$ probability of being even and a $\frac{1}{2}$ probability of being odd. The probability of choosing an odd tile first is $\left(\frac{1}{2}\right)$ and the probability of choosing an odd tile second is also $\left(\frac{1}{2}\right)$, so the probability of "first odd *and* second odd" is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

Alternatively, recognize that there are only four options for odd/even pairs if two tiles are chosen: OO, OE, EO, EE. The only one of these combinations that yields an odd product is OO. Since all of these combinations are equally likely, and since OO is exactly one out of the four possibilities, the probability of choosing OO is $\frac{1}{4}$.

31. **34,650.** This is a combinatorics problem, and the WOW example is intended to make it clear that any W is considered identical to any other W—switching one W with another would *not* result in a different combination, just as switching one S with another in MISSISSIPPI would not result in a different combination.

Therefore, solve this problem using the classic combinatorics formula for accounting for subgroups among which order does not matter:

$$\frac{\text{Total number of items!}}{\text{First group! Second group! Etc.}}$$

Because MISSISSIPPI has 11 letters, including 1 M, 4 S's, 4 I's, and 2 P's:

$$\frac{11!}{1!4!4!2!}$$

Now expand the factorials and cancel; use the calculator for the last step of the calculation:

$$\frac{11\times10\times9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{1!}\cancel{4!}(4\times3\times2\times1)(2\times1)}=\frac{11\times10\times9\times\cancel{8}\times7\times\cancel{6}\times5}{(\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})(\cancel{2}\times\cancel{1})}=11\times10\times9\times7\times5=34{,}650$$

32. **(C).** Since Quantity A is an "at least" problem, use the 1 – *x* shortcut. Rather than calculate the probability of rain on exactly 1 day next week, and then the probability of rain on exactly 2 days next week, and so on (after which you would still have to add all of the probabilities together!), instead calculate the probability of no rain at all on any day, and then subtract that number from 1. That will give the combined probabilities for any scenarios that include rain on at least 1 day.

Probability of NO rain for any of the 7 days =

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{128}$$

Subtract this probability from 1:

$$1-\frac{1}{128}=\frac{128}{128}-\frac{1}{128}=\frac{127}{128}$$

Quantities A and B are equal.

33. $\frac{5}{12}$**.** First think about the prime numbers less than 12, the maximum sum

of the numbers on the cube. These primes are 2, 3, 5, 7, 11.

The probability of rolling 2, 3, 5, 7, or 11 is equal to the number of ways to roll any of these sums divided by the total number of possible rolls. The total number of possible cube rolls is $6 \times 6 = 36$. Make a list:

Sum of 2 can happen 1 way: 1 + 1.
Sum of 3 can happen 2 ways: 1 + 2 or 2 + 1.
Sum of 5 can happen 4 ways: 1 + 4, 2 + 3, 3 + 2, 4 + 1.
Sum of 7 can happen 6 ways: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1.

Sum of 11 can happen 2 ways: 5 + 6, 6 + 5.

That's a total of 1 + 2 + 4 + 6 + 2 = 15 ways to roll a prime sum.

Thus, the probability is $\frac{15}{36} = \frac{5}{12}$.

34. **(B).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of outcomes.

Start with the denominator, which is the total number of ways that the principal can choose 2 children from the classroom. Use the fundamental counting principle. There are 6 possible options for the first choice and 5 for the second, giving (6)(5) = 30 possibilities. However, this double-counts some cases; for example, choosing Jan and then Robert is the same as choosing Robert and then Jan. Divide the total number of pairs by 2: $\frac{(6)(5)}{2}$ = 15.

Alternatively, use the formula for a set in which the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. In this case: $\frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{(2)(4!)} = \frac{6 \times 5}{2} = 15$.

Now compute the numerator, which is the number of pairs that include Jan. Since the pair only includes 2 children and 1 is already decided (Jan), there are exactly 5 options for the other child. Thus, there are 5 total pairs that include Jan: Jan with each of the other students.

The probability of choosing a pair with Jan is $\frac{5}{15} = \frac{1}{3}$.

Alternatively, you can calculate the probability of not choosing Jan and use the 1 – *x* shortcut. The probability of not choosing Jan as the first student is $\frac{5}{6}$, and the probability of choosing again and not choosing Jan is $\frac{4}{5}$ so 1 – $\left(\frac{5}{6} \times \frac{4}{5}\right)$ = 1 – $\frac{20}{30}$ = $\frac{10}{30}$ = $\frac{1}{3}$.

As a final alternative, list all the pairs of students and count how many of them include Jan. Label the students in the class as J, 1, 2, 3, 4, and 5, where J is Jan. Then all the pairs can be listed as (J1), (J2), (J3), (J4), (J5), (12), (13), (14), (15), (23), (24), (25), (34), (35), and (45). (Be careful not to include repeats.) There are 15 total elements in this list and 5 that include Jan, yielding a probability of $\frac{5}{15} = \frac{1}{3}$.

35. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, the first step is to calculate the “or” situation, but don’t stop there. Adding 0.5 + 0.3 = 0.8 double counts the occurrences when both events occur. To compensate, subtract out the probability of both events occurring in order to get rid of the “double counted” occurrences.

Notice that this is a Quantitative Comparison. Because the 0.8 figure includes at least one “both” occurrence, the real figure for Quantity A must be less than 0.8. Therefore, Quantity B must be greater.

To do the actual math, find the probability of both events occurring (breakfast *and* sweater): (0.5)(0.3) = 0.15. Subtract the “and” occurrences from the total “or” probability: 0.8 – 0.15 = 0.65.

Thus, Quantity B is greater.

36. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, do not just add both events, even though it is an "or" situation. Adding 0.3 + 0.2 = 0.5 is incorrect because the probability that both events occur is counted twice. (Only add probabilities in an "or" situation when the probabilities are mutually exclusive.)

While Quantity A's value should include the probability that both events occur, make sure to count this probability only once, not twice. Since the probability that both events occur is 0.3(0.2) = 0.06, subtract this value from the "or" probability.

Quantity A: Add the two probabilities (rain *or* pop quiz) and subtract *both* scenarios (rain *and* pop quiz):

$$0.3 + 0.2 - (0.3)(0.2) = 0.44$$

Quantity B: Multiply the probability that rain does *not* occur (0.7) and the probability that the pop quiz does *not* occur (0.8):

$$0.7(0.8) = 0.56$$

Alternatively, note that the two quantities, collectively, include every possibility and are mutually exclusive of one another (Quantity A includes "rain and no quiz," "quiz and no rain," and "both rain and quiz," and Quantity B includes "no rain and no quiz"). Therefore, the values of Quantities A and B must sum to 1. Calculating the value of either Quantity A or Quantity B would automatically indicate the value for the other quantity.

If you do this, calculate Quantity B first (because it's the easier of the two quantities to calculate) and then subtract Quantity B from 1 in order to get Quantity A's value. That is, 1 – 0.56 = 0.44.

37. **(D).** In essence, the question is asking, "What is the probability that one or more days are rainy days?" since any single rainy day would mean the city experiences rain. In this case, employ the 1 – *x* shortcut, where the probability of rain on one or more days is equal to 1 minus the probability of no rain on any day. Since the probability of rain is $\frac{1}{3}$ on any given day, the probability of no rain on any given day is $1 - \frac{1}{3} = \frac{2}{3}$. Therefore, the probability of no rain on three consecutive days is $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$. Finally, subtract from 1 to find the probability that it rains on one or more days: P(1 or more days) =

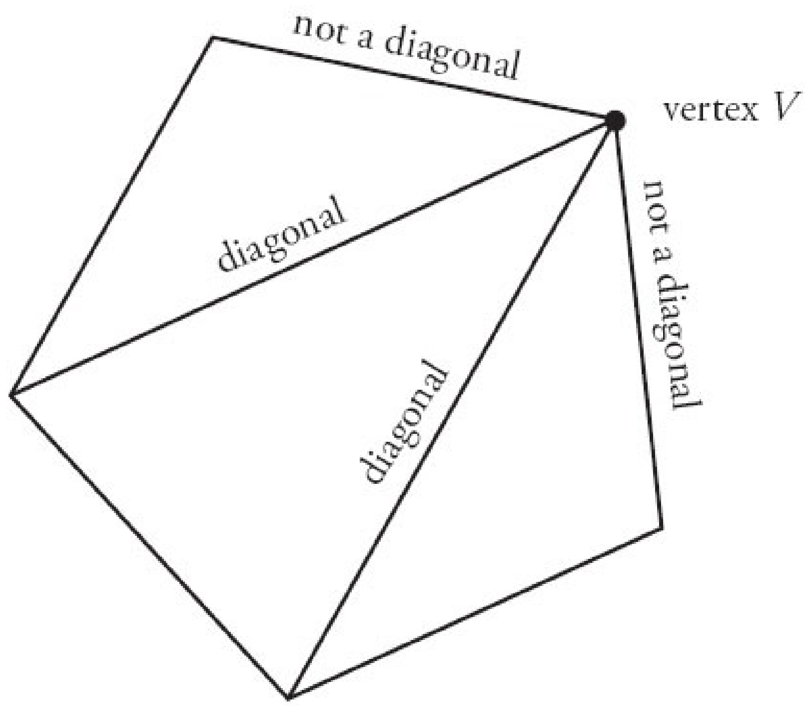
1 – P(no rain) = 1 – $\frac{8}{27} = \frac{19}{27}$.

38. **(C).** The number of ways in which the students can be arranged with Beth and Dan separated is equal to the total number of ways in which the students can be arranged minus the number of ways they can be arranged with Beth and Dan together. The total number of ways to arrange 5 students in a line is 5! = 120. To compute the number of ways to arrange the 5 students such that Beth and Dan are together, group Beth and Dan as “one” person, since they must be lined up together. Then the problem becomes one of lining up 4 students, which gives 4! possibilities. However, remember that there are actually two options for the Beth and Dan arrangement: Beth first and then Dan or Dan first and then Beth. Therefore, there are (4!)(2) = (4)(3)(2)(1)(2) = 48 total ways in which the students can be lined up with Dan and Beth together. Finally, there are 120 – 48 = 72 arrangements in which Beth is separated from Dan.

Alternatively, compute the number of ways to arrange the students directly by considering individual cases. In this case, investigate how many ways there are to arrange the students if Beth occupies each spot in line and sum them to find the total. If Beth is standing in the first spot in line, then there are 3 options for the second spot (since Dan cannot occupy this position), 3 options for the next spot, 2 options for the next spot, and finally 1 option for the last spot. This yields (3)(3)(2)(1) = 18 total possibilities if Beth is first. If Beth is second, then there are 3 options for the first person (Dan cannot be this person), 2 options for the third person (Dan cannot be this person either), 2 options for the fourth person, and 1 option for the fifth. This yields (3)(2)(2)(1) = 12 possibilities. In fact, if Beth is third or fourth in line, the situation is the same as when Beth is second. Thus, there are 12 possible arrangements whether Beth is 2nd, 3rd, or 4th in line, yielding 36 total arrangements for these 3 cases. Using similar logic, the situation in which Beth is last in line is exactly equal to the situation in which she is first in line. Thus, there are (18)(2) = 36 possibilities in which Beth is first or last. In total, this yields 36 + 36 = 72 possible outcomes when considering all of the possible placements for Beth.

39. **(A).** A diagonal of a polygon is an internal line segment connecting any 2 unique vertices; this line segment does not lie along an edge of the given shape. Consider a polygon with 12 vertices. Construct a diagonal by choosing any two vertices and connecting them with a line. Remember that this is order independent; the line is the same regardless of which is the starting vertex. Therefore, this is analogous to choosing any 2 elements from a set of 12, and can be written as $\frac{12!}{10!2!} = \frac{12 \times 11 \times 10!}{10!(2)(1)} = \frac{12 \times 11}{2}$ = 6 × 11 = 66. However, this method includes the vertices connected to their adjacent vertices, which form edges instead of diagonals. In order to account for this, subtract the number of edges on the polygon from the above number: 66 – 12 = 54.

Alternatively, choose a random vertex of the 12-sided shape. There are 12 – 1 = 11 lines that can be drawn to other vertices since no line can be drawn from the vertex to itself. However, the lines from this vertex to the 2 adjacent vertices will lie along the edges of the polygon and therefore cannot be included as diagonals (see the figure of a pentagon below for an example):



Thus, there are 12 – 1 – 2 = 9 diagonals for any given vertex. Since there are 12 vertices, it is tempting to think that the total number of diagonals is equal to (12)(9) = 108. However, this scheme counts each diagonal twice, using each side of the diagonal once as the starting point. Therefore, there are half this many different diagonals: $\frac{108}{2}$ = 54.

40. **(C).** This is a classic combinatorics problem in which *order doesn't matter* —in fact, the problem states that explicitly. Use the standard "order doesn't matter" formula:

$$\frac{\text{total!}}{\text{in! out!}}$$

For Quantity A:

$$\frac{100!}{56!44!}$$

Because the numbers are so large, there must be a way to solve the problem without actually simplifying (even with a calculator, this is unreasonable under GRE time limits). Try Quantity B and compare:

$$\frac{100!}{44!56!}$$

The quantities are equal. Note that this will always work—when order doesn't matter, the number of ways to pick 56 and leave out 44 is the same as the number of ways to pick 44 and leave out 56. Either way, it's one group of 56 and one group of 44. What actually happens to those groups (being part of a collection, being left out of the collection, etc.) is irrelevant to the ultimate solution.

41. **(A).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 – Both + Neither. But first, add 740 ("business use" *only*) + 520 ("business use" and "personal use") to get 1,260, the total number of products categorized as "business use."

Also note that the problem indicates that *all* of the products fall into one or both of the two categories, so "neither" in this formula is equal to zero:

$$\begin{aligned} \text{Total} &= \text{Business} + \text{Personal} - \text{Both} + \text{Neither} \\ 1{,}345 &= 1{,}260 + P - 520 + 0 \\ 1{,}345 &= 740 + P \\ 605 &= P \end{aligned}$$

Quantity A is greater. Note that the question asked for the number of products characterized as "personal use" (which includes products in the "both" group). If the problem had asked for the number of products characterized as "personal use" *only*, you would have had to subtract the "both" group to get 605 – 520 = 85. In this problem, however, Quantity A equals 605.

42. **(E).** Because this is an "at least" question, use the $1 - x$ shortcut:

(The probability of picking at least one man) + (The probability of picking no men) = 1

The probability of picking no men is an *and* setup: woman *and* woman *and* woman.

For the first choice, there are 8 women out of 10 people: $\frac{8}{10} = \frac{4}{5}$.

For the second choice, there are $\frac{7}{9}$ (because one woman has already been chosen).

For the third choice, there are $\frac{6}{8} = \frac{3}{4}$.

Multiply the three probabilities together to find the probability that the committee will be comprised of woman *and* woman *and* woman:

$$\frac{4}{5} \times \frac{7}{9} \times \frac{3}{4} = \frac{1}{5} \times \frac{7}{3} \times \frac{1}{1} = \frac{7}{15}$$

To determine the probability of picking at least one man, subtract this result from 1:

$$1 - \frac{7}{15} = \frac{8}{15}$$

43. **(A).** This overlapping sets question can be solved with the following equation:

> Total # of people = Group 1 + Group 2 + Group 3 – (# of people in two groups) – (2)(# of people in all three groups) + (# of people in no groups)

The problem indicates that everyone studies at least one language, so the number of people in no groups is zero. The problem also indicates that nobody studies all three languages, so that value is also zero:

> Total # of students = 100 + 80 + 40 – 22 – (2)(0) + 0 = 198.

44. **(A).** It is not possible to solve for a single value for Quantity A, but it is possible to tell that Quantity A will always be greater than 16. Since 20 birds are songbirds and 23 are migratory, the total of these groups is 43, which is less than 60. It is possible for the overlap (the number of migratory songbirds) to be as little as 0, which would result in 20 songbirds, 23 non-songbird migratory birds, and 60 – 20 – 23 = 17 birds that are neither songbirds nor migratory.

It is also possible that there could be as many as 20 birds that overlap the two categories. (Find this figure by taking the number of birds in the smaller group; in this case, there are 20 songbirds). In the case that there are 20 migratory songbirds, there would also be 3 migratory birds that are not songbirds, in which case there would be 60 – 20 – 3 = 37 birds that are neither migratory nor songbirds.

Thus, the number of birds that are neither migratory nor songbirds is at least 17 and at most 37. No matter where in the range that number may be, it is greater than Quantity B, which is only 16.

Chapter 24

Data Interpretation

In This Chapter…

Data Interpretation

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

Problem Set A

9th Grade Students at Millbrook High School

	Boys	Girls
Enrolled in Spanish	12	13
Not Enrolled in Spanish	19	16

1. Approximately what percent of the 9th grade girls at Millbrook High School are enrolled in Spanish?

   (A) 21%

   (B) 37%

   (C) 45%

(D) 50%

(E) 57%

2. What fraction of the students in 9th grade at Millbrook High School are boys who are enrolled in Spanish?

(A) $\frac{1}{5}$

(B) $\frac{19}{60}$

(C) $\frac{5}{12}$

(D) $\frac{12}{31}$

(E) $\frac{12}{25}$

3. What is the ratio of 9th grade girls not enrolled in Spanish to all 9th grade students at Millbrook High School?

(A) 1 : 16

(B) 13 : 60

(C) 4 : 15

(D) 19 : 60

(E) 16 : 29

4. If x percent more 9th grade students at Millbrook High School are not enrolled in Spanish than are enrolled in Spanish, what is the value of x?

(A) 20

(B) 25

(C) 30

(D) 40

(E) 50

5. If two of the 9th grade boys at Millbrook High School who are not enrolled in Spanish decided to enroll in Spanish, and then eight additional girls and seven additional boys attended the 9th grade at Millbrook High School and also enrolled in Spanish, what percent of 9th grade students at Millbrook would then be enrolled in Spanish?

(A) 52%

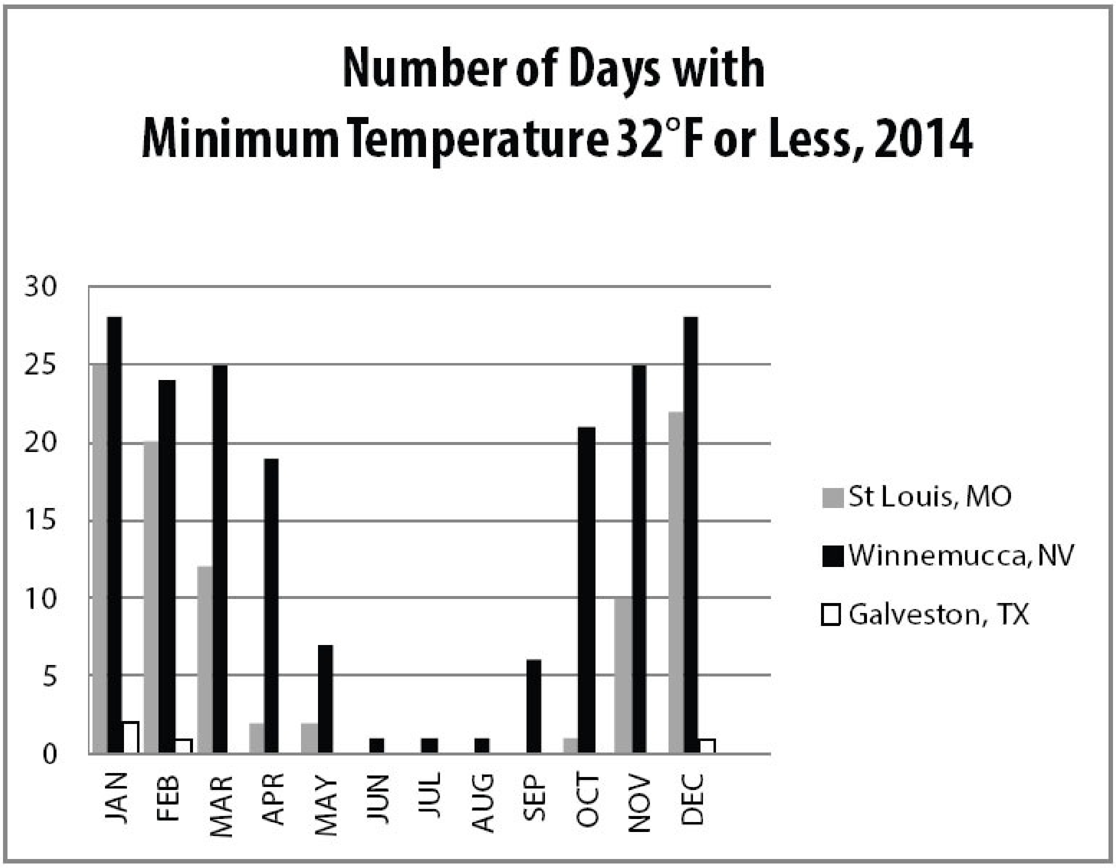
(B) 53%

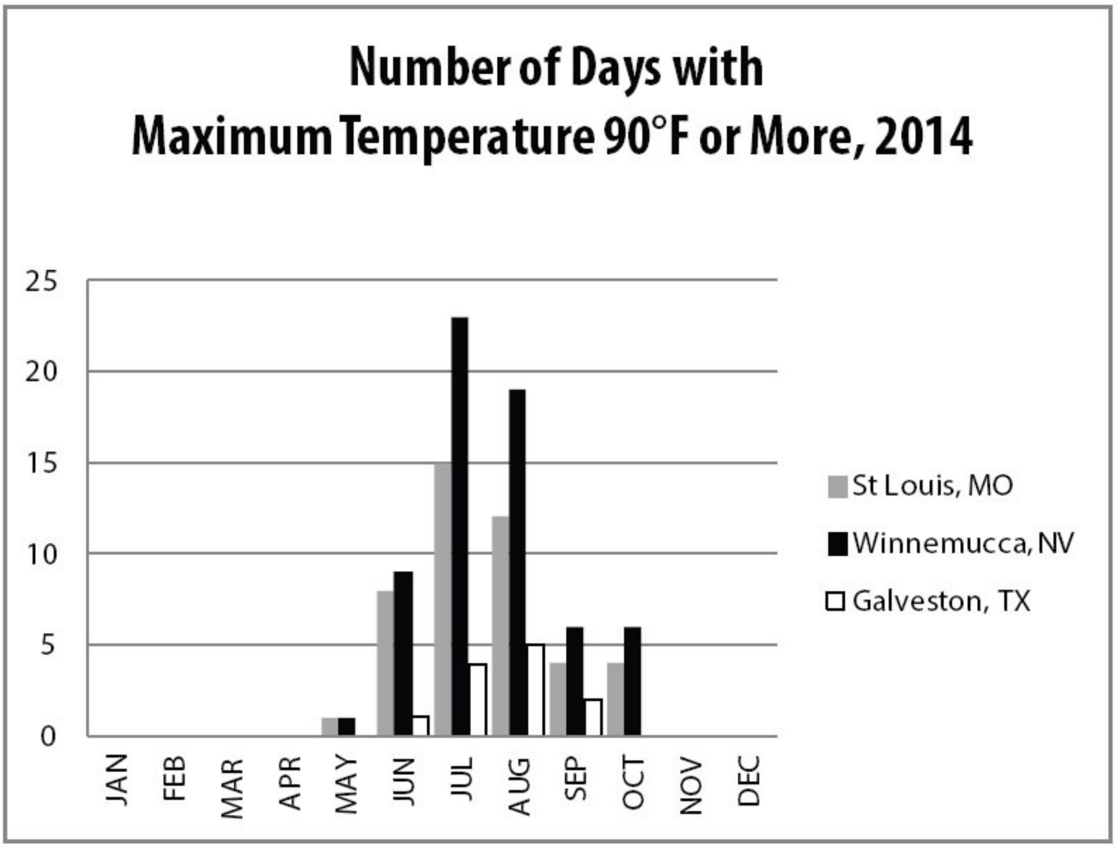
(C) 54%

(D) 55%

(E) 56%

Problem Set B


Number of Days with
Minimum Temperature 32°F or Less, 2014
30
25
20
15
10
5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
St Louis, MO
Winnemucca, NV
Galveston, TX


Number of Days with
Maximum Temperature 90°F or More, 2014
25
20
15
10
5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
St Louis, MO
Winnemucca, NV
Galveston, TX

6. In how many months of the year were there more than 20 days with temperatures 32°F or less in Winnemucca?

(A) 2

(B) 3

(C) 4

(D) 6

(E) 7

7. On how many days in the entire year did the temperature in Galveston rise to at least 90°F or fall at least as low as 32°F?

(A) 11

(B) 16

(C) 28

(D) 42

(E) 59

8. Approximately what percent of the days with maximum temperature of 90°F or more in St. Louis occurred in July?

(A) 6%

(B) 15%

(C) 17%

(D) 34%

(E) 44%

9. The number of freezing January days in Winnemucca was approximately what percent more than the number of freezing January days in St. Louis? (A "freezing" day is one in which the minimum temperature is 32°F or less.)

(A) 3%

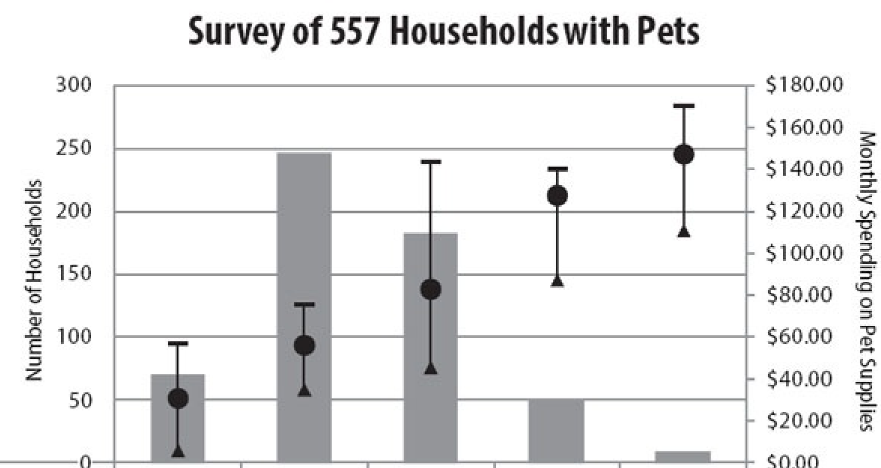
(B) 6%

(C) 12%

(D) 24%

(E) 28%

Problem Set C



Number of Pets	1	2	3	4	5
■ Number of Households with This Number of Pets	70	246	183	49	9
▬ Maximum Monthly Spending on Pet Supplies	$57.32	$75.98	$143.57	$140.48	$170.23
▲ Minimum Monthly Spending on Pet Supplies	$6.34	$35.47	$45.84	$87.46	$111.20
● Average Monthly Spending on Pet Supplies	$31.25	$56.42	$83.11	$127.74	$147.38

10. Approximately what percent of the surveyed households have more than three pets?

    (A) 10%

    (B) 20%

    (C) 30%

    (D) 40%

    (E) 50%

11. Which of the following is the median number of pets owned by the households in the survey?

    (A) 1

    (B) 2

    (C) 3

    (D) 4

    (E) 5

12. What is the range of monthly spending on pet supplies for the household group with the largest such range?

(A) $69.03

(B) $97.73

(C) $116.13

(D) $138.98

(E) $170.23

13. The household group with which number of pets had the greatest average (arithmetic mean) monthly spending per pet?

(A) 1 pet

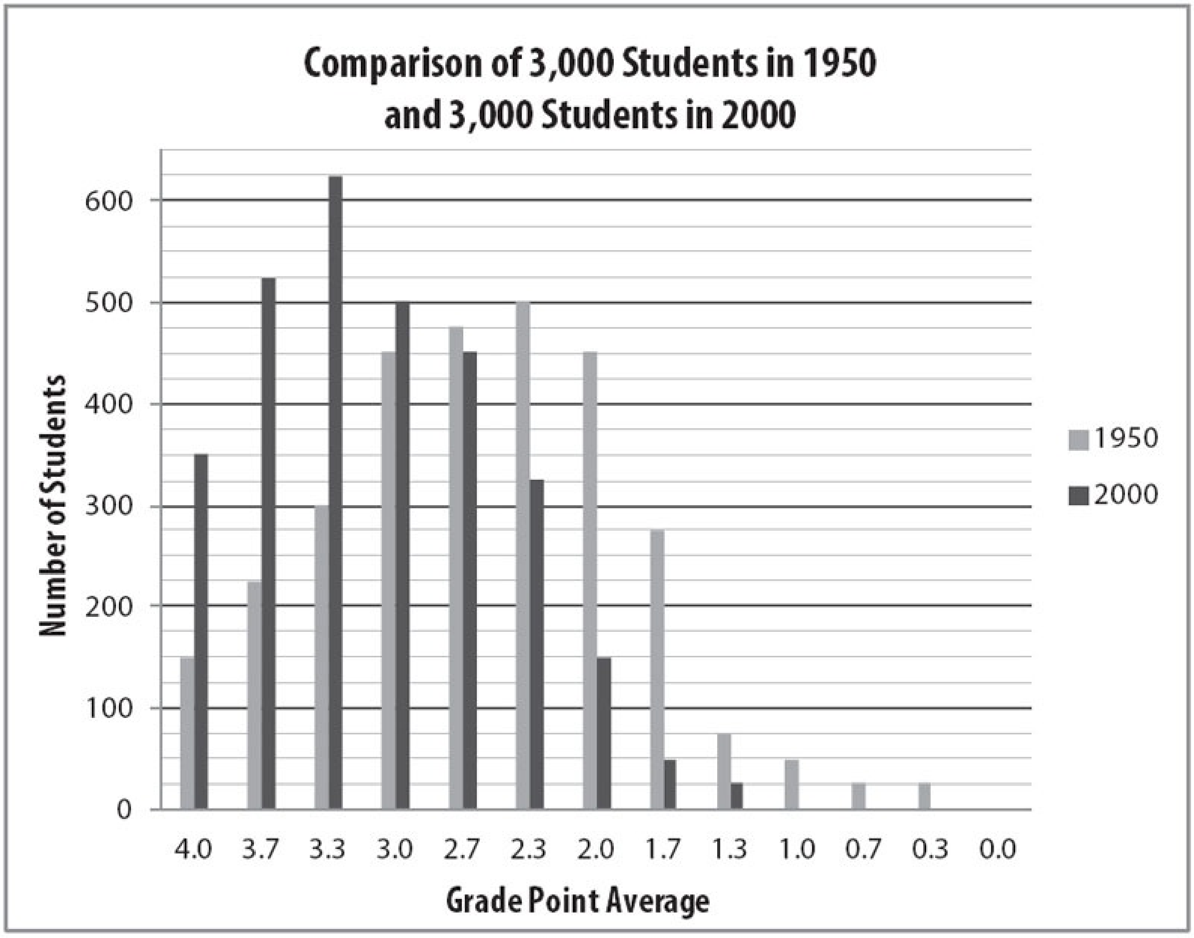
(B) 2 pets

(C) 3 pets

(D) 4 pets

(E) 5 pets

Problem Set D



14. What was the mode for grade point average of the 3,000 students in 2000?

(A) 3.7

(B) 3.3

(C) 3.0

(D) 2.7

(E) 2.3

15. What was the median grade point average of the 3,000 students in 1950?

(A) 3.7

(B) 3.3

(C) 3.0

(D) 2.7

(E) 2.3

16. Approximately what percent of the students in 2000 earned at least a 3.0 grade point average?

(A) 25%

(B) 50%

(C) 67%

(D) 80%

(E) 97.5%

17. Approximately what percent of the students in 1950 earned a grade point average less than 3.0?

(A) 33%

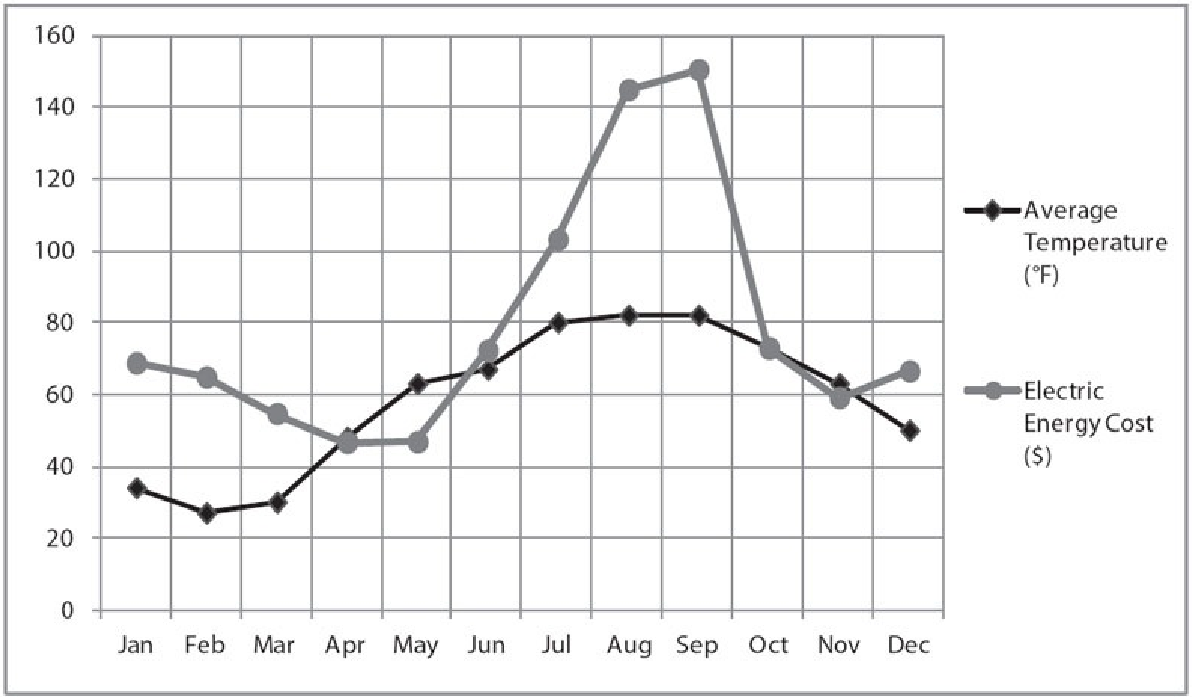
(B) 37.5%

(C) 50%

(D) 62.5%

(E) 75%

Problem Set E



18. According to the chart, which two-month period had the greatest increase in electric energy cost?

(A) Between January and February

(B) Between May and June

(C) Between June and July

(D) Between July and August

(E) Between November and December

19. According to the chart, in which two-month period did electric energy cost increase the least?

(A) Between January and February

(B) Between April and May

(C) Between May and June

(D) Between June and July

(E) Between November and December

20. Approximately what was the average (arithmetic mean) electric energy cost per month for the first half of the year?

(A) $45

(B) $50

(C) $60

(D) $70

(E) $75

21. In which month was the electric energy cost per Fahrenheit degree (°F) of average temperature the least?

(A) April

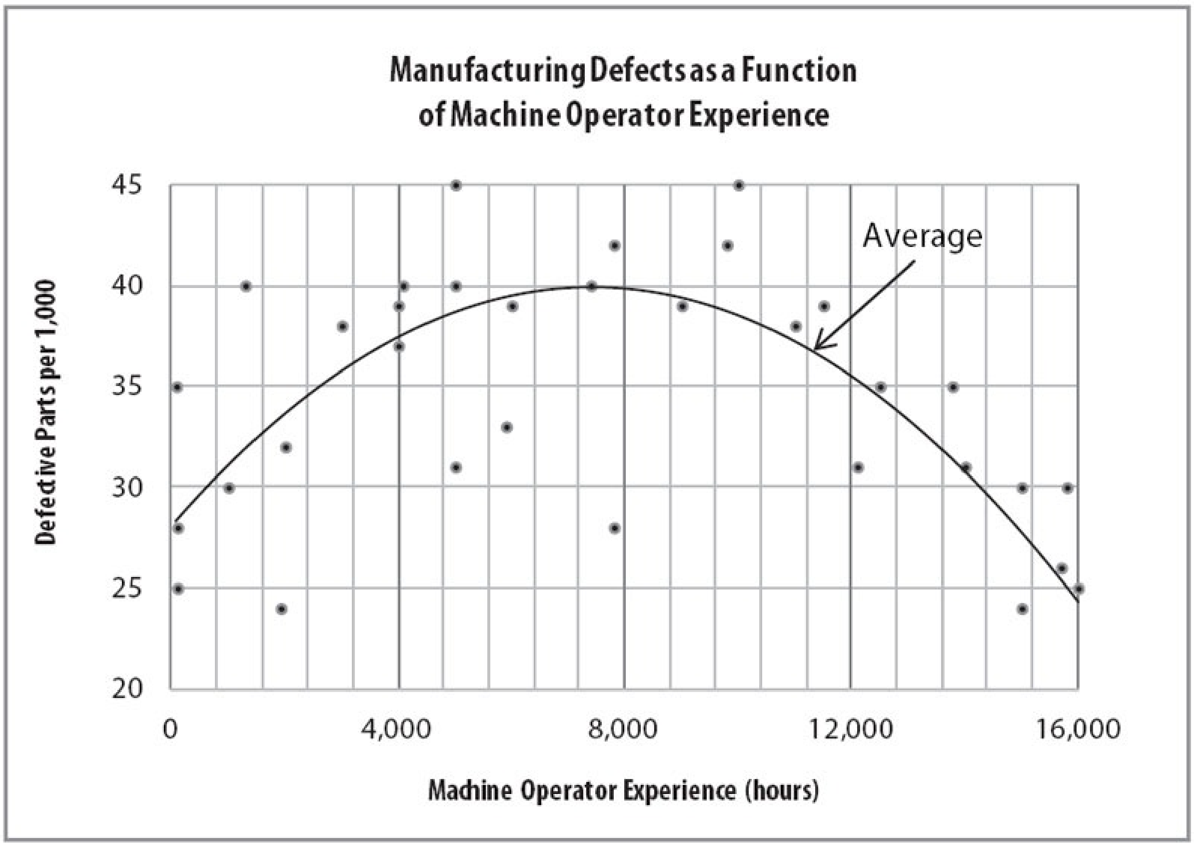
(B) May

(C) October

(D) November

(E) December

Problem Set F



22. On average, the machine operators that produce the fewest defective parts per 1,000 have how many hours of experience?

    (A) 40

    (B) 4,000

    (C) 8,000

    (D) 12,000

    (E) 16,000

23. On average, the machine operators with approximately how many hours of experience have the same defective part rate as those with 12,000 hours of experience?

    (A) 2,000

    (B) 2,700

    (C) 4,400

    (D) 8,400

    (E) 12,800

24. On average, approximately how many hours of experience do machine operators who produce the most defective parts per 1,000 have?

(A) 40

(B) 4,000

(C) 8,000

(D) 12,000

(E) 16,000

25. Of the individual machine operators who recorded a defective part rate of 4.2%, approximately how many hours of experience did the least experienced operator have?

(A) 2,300

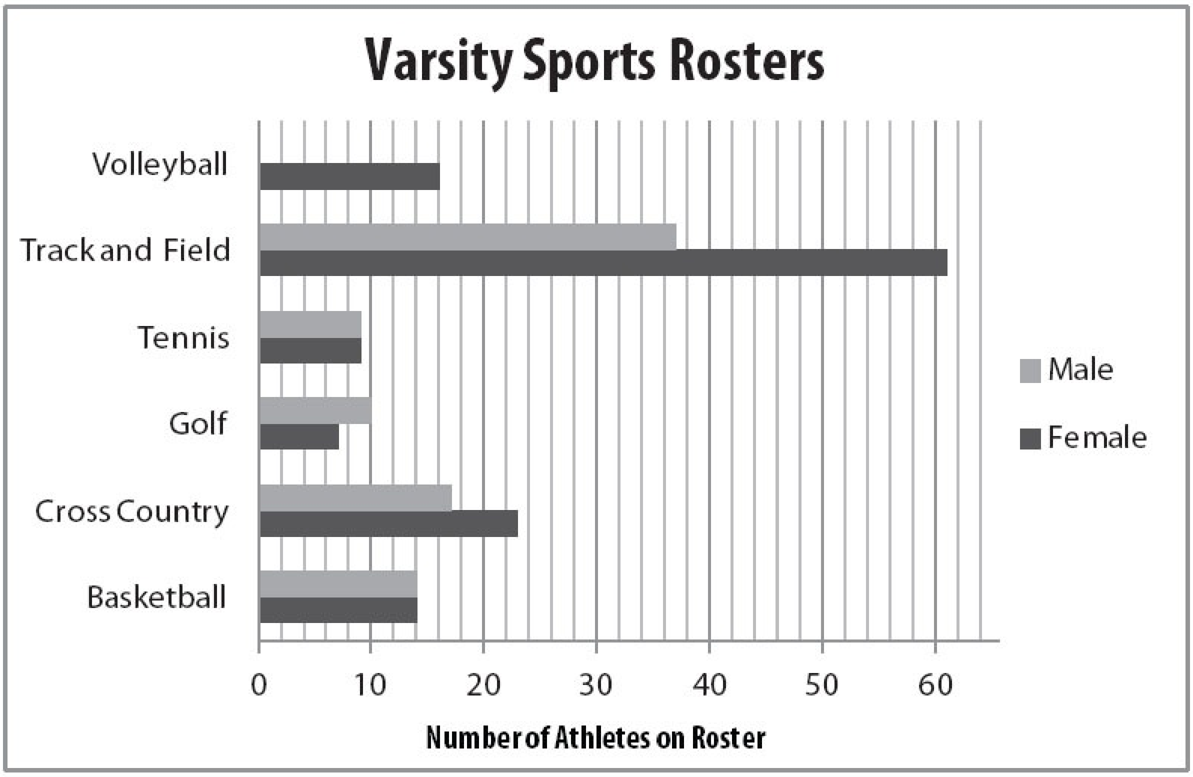
(B) 5,000

(C) 7,700

(D) 9,800

(E) 15,100

Problem Set G



26. What is the ratio of male athletes to female athletes on the track and field roster?

(A) $\frac{37}{61}$

(B) $\frac{9}{14}$

(C) $\frac{17}{23}$

(D) $\frac{14}{9}$

(E) $\frac{61}{37}$

27. All athletes are on only one varsity sports roster EXCEPT those who are on both the Track and Field team and the Cross Country team. If there are 76 male athletes in total on the varsity sports rosters, how many male

athletes are on both the Track and Field team and the Cross Country team?

(A) 11

(B) 17

(C) 37

(D) 54

(E) 76

28. On which varsity sports rosters do male athletes outnumber female athletes?

Indicate <u>all</u> such rosters.

- ☐ Volleyball
- ☐ Track and Field
- ☐ Tennis
- ☐ Golf
- ☐ Cross Country
- ☐ Basketball

29. What is the ratio of female tennis players to male basketball players on the varsity sports rosters?

(A) $\frac{5}{12}$

(B) $\frac{9}{14}$

(C) $\frac{7}{8}$

(D) $\frac{14}{9}$

(E) $\frac{12}{5}$

Problem Set H

Population and GDP for 50 African Countries

Gross Domestic Product	Population: More Than 50 Million	20–50 Million	10–20 Million	2–10 Million	Less Than 2 Million	Total
More Than $100 Billion	3	2	0	0	0	5
$20–100 Billion	1	7	1	1	0	10
$10–20 Billion	1	3	3	3	3	13
Less Than $10 Billion	0	0	7	8	7	22
Total	5	12	11	12	10	50

30. Among the 50 African countries represented in the chart above, how many countries have a population between 10 million and 50 million people and a GDP between $10 billion and $20 billion?

(A) 6

(B) 7

(C) 13

(D) 16

(E) 23

31. Among the 50 African countries represented in the chart above, what percent of the countries have a population of less than 20 million people and a GDP of less than $20 billion?

(A) 38%

(B) 44%

(C) 62%

(D) 68%

(E) 90%

32. Approximately what percent of the African countries in the chart above that have a GDP between $10 billion and $20 billion also have a population between 10 million and 20 million?

(A) 6%

(B) 23%

(C) 26%

(D) 30%

(E) 51%

33. According to the chart above, which of the following is greatest?

(A) The number of countries with more than $10 billion of GDP and a population of less than 20 million

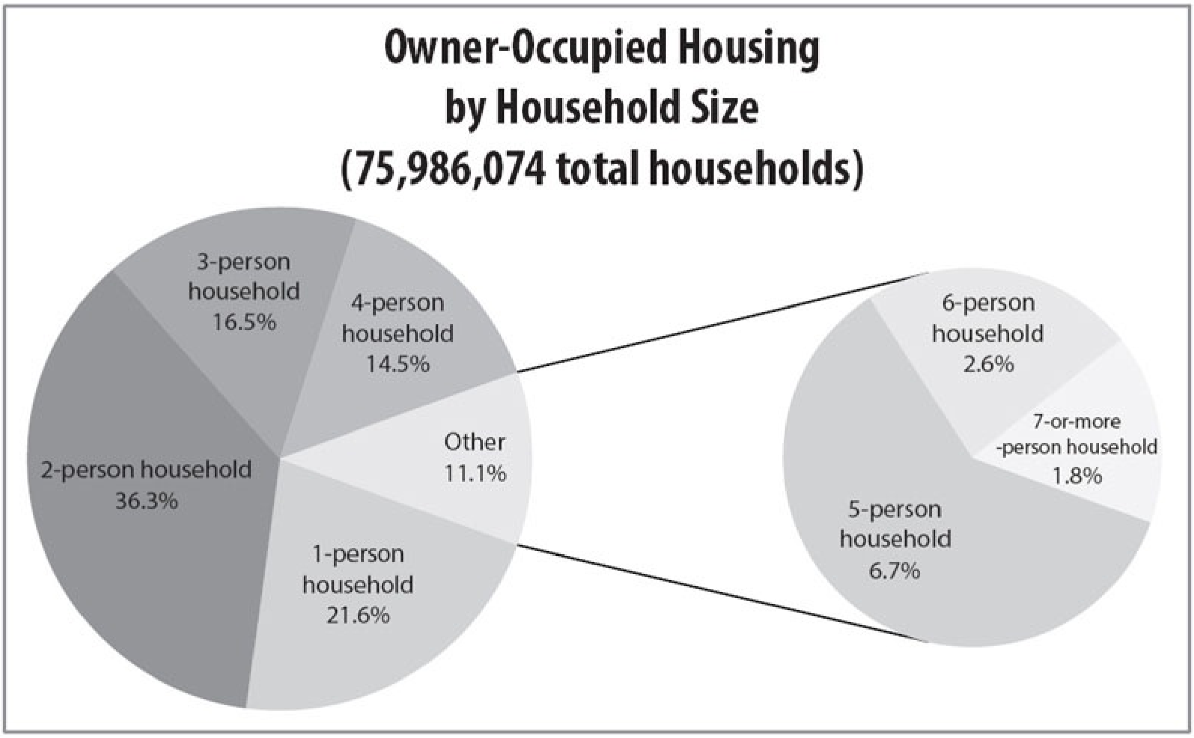
(B) The number of countries with less than $20 billion of GDP and a population of more than 10 million

(C) The number of countries with more than $20 billion of GDP

(D) The number of countries with less than $100 billion of GDP and a population of less than 10 million

(E) The number of countries with less than $100 billion of GDP and a population between 10 million and 50 million

Problem Set I



34. What percent of owner-occupied housing units are households with fewer than 4 people?

    (A) 11.1%

    (B) 14.5%

    (C) 25.6%

    (D) 74.4%

    (E) 88.9%

35. Among the owner-occupied housing units represented in the chart above, approximately how many households are 5-person households?

    (A) 1 million

    (B) 2 million

    (C) 3 million

    (D) 4 million

    (E) 5 million

36. Based on the total number of people living in all such households, which of the following is a correct ordering, from least to greatest, of 1-person households, 3-person households, and 5-person households?

(A) 1-person households, 3-person households, 5-person households

(B) 1-person households, 5-person households, 3-person households

(C) 3-person households, 1-person households, 5-person households

(D) 3-person households, 5-person households, 1-person households

(E) 5-person households, 3-person households, 1-person households

37. Which combination of household sizes accounts for more than 50% of all owner-occupied housing units?

(A) 2- and 3-person

(B) 3- and 4-person

(C) 4- and 5-person

(D) 5- and 6-person

(E) 6- and 7-person

Data Interpretation Answers

Problem Set A: The title of the chart indicates that the total population is the total number of 9th graders at Millbrook High School.

When given a chart that depends on addition (boys + girls = total students, and also those enrolled in Spanish + those not enrolled in Spanish = total students), it can be helpful to sketch a quick version of the chart and add a total column. For example:

	Boys	Girls	TOTAL
Enrolled in Spanish	12	13	
Not Enrolled in Spanish	19	16	
TOTAL			

Now add down and across:

	Boys	Girls	TOTAL
Enrolled in Spanish	12	13	25
Not Enrolled in Spanish	19	16	35
TOTAL	31	29	60

1. **(C).** There are 29 total girls and 13 are enrolled in Spanish. The fraction of girls enrolled in Spanish is $\frac{13}{29}$. Convert to a percent: $\left(\frac{13}{29}\times 100\right)\%$ = 44.827…%, or about 45%.

2. **(A).** There are 60 total students and 12 boys enrolled in Spanish. The answer is $\frac{12}{60}$, which reduces to $\frac{1}{5}$. (Read carefully! "What fraction of the students … are boys who are enrolled in Spanish?" is *not* the same as "What fraction of the boys are enrolled in Spanish?")

3. **(C).** There are 16 girls not enrolled in Spanish and 60 total students. The ratio is $\frac{16}{60}$, which reduces to $\frac{4}{15}$ or 4 : 15.

4. **(D).** There are 35 students not enrolled in Spanish and 25 who are. The question can be rephrased as, "35 is what percent greater than 25?" Using the percent change formula:

$$\text{Percent Change} = \left(\frac{Difference}{Original} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{10}{25} \times 100\right)\% = 40\%$$

Thus, *x* is 40.

5. **(E)**. Sketch a new chart to reflect the changes. Switch 2 of the boys from “not enrolled” to “enrolled.” Then, add 8 new girls and 7 new boys to the “enrolled” groups:

	Boys	Girls	TOTAL
Enrolled in Spanish	12 + 2 + 7 = 21	13 + 8 = 21	42
Not Enrolled in Spanish	19 – 2 = 17	16	33
TOTAL	38	37	75

Update the Total rows and columns as well. Both “Boys” and “Girls,” as well as “Enrolled in Spanish” and “Not Enrolled in Spanish,” now sum to 75.

What percent of 9th grade students at Millbrook would then be taking Spanish? Since 42 out of 75 students would be enrolled in Spanish, calculate $\frac{42}{75}$ (with the calculator) and multiply by 100 to convert it to a percent. The answer is 56%.

Problem Set B: The two charts show how often daily temperature extremes occurred in each month of the year for three cities. For the sake of simplicity, you can think of the top chart as "cold" and the bottom chart as "hot."

Note that there is no information about exactly how hot or how cold the days tallied are: a day with a minimum temperature of 27°F counts as a "cold" day, just as a day with minimum temperature of –10°F would. Therefore, it is likely that questions will just reference one or both of the two temperature categories broadly (≥ 90 and ≤ 32).

6. **(D).** From the "cold" chart, the black bar referring to Winnemucca rises above 20 in Jan, Feb, Mar, Oct, Nov, and Dec, for a total of 6 months.

7. **(B).** This question asks about number of days with both temperature extremes in Galveston. Galveston had 1 "hot" day in June, 4 in July, 5 in August, and 2 in September, for a total of 12. It had 2 "cold" days in January and 1 each in February and December, for a total of 4. The total number of days with either extreme temperature is 16 days.

8. **(D).** From the grey bars on the "hot" day chart, St. Louis had a total of 1 + 8 + 15 + 12 + 4 + 4 = 44 days when the temperature reached at least 90°F, and 15 of those were in July. These July days account for $\left(\frac{15}{44} \times 100\right)\% \approx 34\%$ of all the hot days in St. Louis (approximately).

9. **(C).** In January, Winnemucca had 28 freezing days, while St. Louis had 25. So the question is asking, "28 is what percent more than 25?" Use the percent change formula: $\left(\frac{Difference}{Original} \times 100\right)\% = \left(\frac{3}{25} \times 100\right)\% = 12\%$.

Problem Set C: This table tallies the number of households, according to number of pets in the household, and each column captures information about these households. For example, the left most column with numbers indicates that there are 70 households that have one pet, and these households spend an average of $31.25 per month on pet supplies. In that group, the households that spent the least spent $6.34 on pet supplies, while the households that spent the most spent $57.32. Notice that the bars and the max/min/average range lines duplicate the information in the table. For exact calculations, rely on the chart numbers. For broader questions, such as "which is greater," the more visual representation of the data can often provide a quick answer.

10. **(A).** There are 557 households, of which 49 have four pets and 9 have five

pets. Thus, a total of 58 households have more than three pets. To express this as a percent, divide this number by the total number of households:

$$\frac{58}{557} \approx 10\%$$

11. **(B).** Since there are 557 households, the median household would be midway between the 1st and 557th households on the list if the households are ranked by how many pets they own. The midpoint between the 1st and 557th households is the $\frac{(1+557)}{2}$ = 279th household. Check: There are 278 households below this one, and 278 households above (because 279 + 278 = 557). Ranked by number of pets, households 1 through 70 (the first 70 households) have one pet, which means households 71 through 316 (in other words, the next 246 households) have two pets. Since the 279th household falls in this interval, the median household owns two pets.

12. **(B).** The group with the largest range of monthly spending is the group of households that own three pets. This can be seen by looking at the length of the vertical line between the maximum spending bar and the minimum spending triangle. Within this group, the maximum amount spent is \$143.57 and the minimum is \$45.84, so the range is \$143.57 – \$45.84 = \$97.73.

13. **(D).** The group with one pet spent an average of \$31.25 per pet, as indicated in the chart. The group with two pets spent an average of \$56.42 on two pets, which is \$28.21 per pet $\left(\frac{\$56.42}{2}\right)$. The 3rd group spent an average of \$83.11 on three pets, or $\frac{\$83.11}{3}$ = \$27.70 per pet (approximately). The 4th group spent an average of \$127.74 on four pets, or $\frac{\$127.74}{4}$ = \$31.94 per pet (approximately). The 5th group spent an average of \$147.38 on five pets, or $\frac{\$147.38}{5}$ = \$29.47 per pet (approximately). The highest average is among the group that has four pets.

Problem Set D: The dark gray bars indicate the number of students with various grade point averages in 2000, and the light gray bars indicate number of students in the same categories in 1950. The title states that the surveys consist of 3,000 students.

Note the general contrast between students in the two years. Connecting the top of each light gray bar with a smooth line, the result would be a sort-of bell curve that peaks at grade point average of 2.3. Similarly, the dark gray bars form a similar bell curve, but its peak is at grade point average of 3.3, so the grades in general are clustered at the higher end of the scale in 2000.

14. **(B).** The mode of a list of numbers is the number that occurs most frequently in the list. In the bar graph for grade point average, dark gray bars represent the students in 2000, and the mode of that dataset is indicated by the tallest dark gray bar. This is at grade point average of 3.3. There were 625 students with a grade point average of 3.3 in the year 2000.

15. **(D).** The median is the middle value of an ordered list of numbers. For the 3,000 students in 1950, the median grade point average is the average of the 1,500th highest grade point average and the 1,501st highest grade point average. The students in 1950 are represented by the light gray bars. From the chart, you know the following:

> 150 students had a 4.0 grade point average.
> 225 students had a 3.7 grade point average. (Total with this GPA and higher = 150 + 225 = 375)
> 300 students had a 3.3 grade point average. (Total with this GPA and higher = 375 + 300 = 675)
> 450 students had a 3.0 grade point average. (Total with this GPA and higher = 675 + 450 = 1,125)
> 475 students had a 2.7 grade point average. (Total with this GPA and higher = 1,125 + 475 = 1,600)

The 1,500th and 1,501st students fall between the 1,125th and 1,600th students. Thus, the 1,500th and 1,501st highest grade point averages are both 2.7.

16. **(C).** The students in 2000 are represented by the dark gray bars:

> 350 students had a 4.0 grade point average.
> 525 students had a 3.7 grade point average.
> 625 students had a 3.3 grade point average.
> 500 students had a 3.0 grade point average.

There were 350 + 525 + 625 + 500 = 2,000 students who earned at least a 3.0

grade point average in the year 2000, out of a total of 3,000 students. This is $\frac{2}{3}$ of the students, or about 67% of the students.

17. **(D).** The students in 1950 are represented by the light gray bars:

150 students had a 4.0 grade point average.
225 students had a 3.7 grade point average.
300 students had a 3.3 grade point average.
450 students had a 3.0 grade point average.

In 1950, 150 + 225 + 300 + 450 = 1,125 students had a grade point average of 3.0 or higher. Thus, 3,000 – 1,125 = 1,875 students earned a grade point average *less than* 3.0. As a percent of the class, this is equal to

$\left(\frac{1,875}{3,000}\times 100\right)\%$ = 62.5%.

Problem Set E: The vertical number scale on the left side of the graph applies to both datasets, but for Average Temperature the units are °F and for Electric Energy Cost the units are dollars ($). For example, in January the average temperature was between 30°F and 40°F and the electric energy cost was about $70. Be careful to read data from the correct set.

18. **(D).** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two circular data points. The greater the slope of the light gray line segment, the greater the cost increase between those two months. There was an increase each month between May and September, and again between November and December. But the steepest positive slope is between July and August.

The cost increase from July to August was approximately $145 – $103 = $42. For comparison, the cost increase from June to July was only about $103 – $70 = $33. The correct answer is between July and August.

19. **(B).** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two data points, and a cost decrease would mean a negative slope. The steeper the slope of the line segment, the greater the cost change between two consecutive months. A cost change of $0 would mean the line segment has a slope of 0 (i.e., it is horizontal).

To find the two consecutive months with the smallest electric energy cost change, look for the light gray line segment that is most horizontal. The line segment between April and May is nearly horizontal. The correct answer is between April and May.

20. **(C).** There are two ways to approximate average electric energy cost per month in the first half of the year.

One way is to use the electric energy costs on the chart and compute the average for the first six months, using the light gray circular data points:

Approximate average cost =

$$\frac{\$70 + \$65 + \$55 + \$47 + \$47 + \$70}{6} = \frac{\$354}{6} = \$59$$

Answer choice (C) $60 is closest.

The other method is more visual. Consider choice (A), $45, and imagine a horizontal line at $45. All six cost data points for the first half of the year are above this horizontal line, so the average must be more than $45. Similarly, imagine a horizontal line at $75 for choice (E). All six cost data points for the first half of the year are below this horizontal line, so the average must be less than $75. When a horizontal line at $60 is considered, the six cost data points “balance”: three are above the line and three are below, by approximately the same amount.

21. **(B).** To minimize $\frac{\text{Electric Energy Cost }(\$)}{\text{Average Temperature }({}^\circ\text{F})}$, minimize cost (light gray circular data points) while maximizing average temperature (black diamond data points). Only in April, May, October, and November is the black data point equal to or greater than the gray data point (i.e., the $\frac{\text{Electric Energy Cost }(\$)}{\text{Average Temperature }({}^\circ\text{F})}$ ratio is equal to or less than 1). In April, October, and November, this ratio is close to 1. In May, the difference between the cost and the average temperature is greatest, so the electric energy cost per °F of average temperature is least. The correct answer is May.

Problem Set F: The chart shows defective parts per 1,000 as a function of machine operator experience. The dots indicate individual machine operators, and there is quite a bit of variance by individual. The line labeled "Average" shows the average performance of the group as a whole. A trend emerges: inexperienced machine operators and very experienced machine operators make fewer mistakes than those with medium level of experience. Also, certain individual machine operators produce defective parts at a lower rate than others with similar levels of experience.

22. **(E).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. At the lowest point on this average curve, operators with 16,000 hours of experience produce slightly fewer than 25 defective parts per 1,000. Another low point is for operators with minimal experience, but even they produce between 25 and 30 defective parts per 1,000. In contrast, the defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

23. **(B).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. Machine operators with 12,000 hours of experience produce an average of about 36 defective parts per 1,000.

The other group of machine operators that produces about 36 defective parts per 1,000 has a little less than 3,200 hours of experience. (Note that there are 5 grid lines for every 4,000 hours, so each vertical grid line is 800 hours apart. The grid mark to the left of the 4,000 mark represents 4,000 – 800 = 3,200 hours.) Choice (B) is close to and less than 3,200.

Alternatively, check the average defective part rate for machine operators with the hours of experience listed in the choices:

(A) 2,000 hours (around 33 or 34 defective parts per 1,000)
(B) 2,700 hours (a bit over 35 defective parts per 1,000) CORRECT
(C) 4,400 hours (around 38 defective parts per 1,000)
(D) 8,400 hours (a bit less than 40 defective parts per 1,000)
(E) 12,800 (around 34 defective parts per 1,000)

24. **(C).** Because the question specifies “on average,” refer to the curve marked “Average” rather than the individual data points. The defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

25. **(C).** Because the question refers to "individual machine operators," refer to the individual data points rather than the curve marked "Average."

A defective part rate of 4.2% equates to $\frac{4.2}{100} \times 1{,}000 = 42$ defective parts per 1,000. The chart has only two data points at approximately 42 defective parts per 1,000. The less experienced of these two machine operators had just under 8,000 hours of experience.

Problem Set G: Note that there are five vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people.

26. **(A).** On the Track and Field roster, there are between 36 and 38 men (therefore 37) represented by the light gray bar. On the Track and Field roster, there are between 60 and 62 women (therefore 61) represented by the dark gray bar. In fraction form, the "ratio of men to women" is $\frac{\text{men}}{\text{women}}$. The correct answer is $\frac{37}{61}$.

27. **(A).** Male athletes are represented by the light gray bars for each sport. Sum the male athletes on each of the separate varsity sports rosters:

Males on Volleyball roster: 0
Males on Track and Field roster: between 36 and 38 (therefore 37)
Males on Tennis roster: between 8 and 10 (therefore 9)
Males on Golf roster: 10
Males on Cross Country roster: between 16 and 18 (therefore 17)
Males on Basketball roster: 14

There are 0 + 37 + 9 + 10 + 17 + 14 = 87 male names on all of the rosters combined, but there are only 76 male athletes total. Since tennis, golf, and basketball players are all on only one roster, there must be 87 – 76 = 11 male athletes who are counted twice, on both the Track and Field team and the Cross Country team. The correct answer is 11.

28. **Golf only.** Male athletes are represented by the light gray bars, female athletes by the dark gray bars. A sport in which male athletes outnumber female athletes will have a shorter dark gray bar than light gray bar.

This is only the case for Golf; there are 10 male athletes and 7 female athletes. Volleyball only has female athletes, so they outnumber the zero male athletes on the roster. In Tennis and Basketball, there are equal numbers of men and women. Female athletes outnumber male athletes on the Cross Country and Track and Field rosters.

29. **(B).** There are between 8 and 10 female tennis players (therefore 9) represented by the dark gray bar. There are 14 male basketball players represented by the light gray bar. In fraction form, the "ratio of female tennis players to male basketball players" is $\frac{\text{female tennis players}}{\text{male basketball players}}$. Thus, the answer is $\frac{9}{14}$.

Problem Set H: The table categorizes 50 African countries according to GDP (rows) and population (columns). Notice that each row sums to a subtotal number of countries in that GDP range, and each column sums to a subtotal number of countries in that population range. Both the subtotal row and subtotal column sum to 50, the grand total. Moreover, notice that both population and GDP are shown in descending order: high population/high GDP countries are in the upper left corner of the table, while low population/low GDP countries are in the lower right corner of the table.

30. **(A).** GDP between \$10 billion and \$20 billion is a single row in the table. Population between 10–50 million people includes two columns in the table. Look at the intersections between this row and two columns. There are three countries with populations of 10–20 million and GDPs of \$10 billion to \$20 billion. There are also three countries with populations of 20–50 million and GDPs of \$10 billion to \$20 billion GDP, for a total of six countries.

31. **(C).** Adding the entries that are in both the bottom two rows (less than \$20 billion GDP) and the last three columns (population less than 20 million), the number of countries is 3 + 3 + 3 + 7 + 8 + 7 = 31. Out of 50 countries, 31 fit this description, so the percent is $\left(\frac{31}{50}\times 100\right)\%$, or 62%.

32. **(B).** There are 13 countries with GDPs between \$10–\$20 billion, and of these, 3 have populations between 10–20 million. Thus, the percent is $\left(\frac{3}{13}\times 100\right)\%$, or approximately 23%.

33. **(D).** For each choice, carefully find the row(s)/column(s) that fit the description, and sum all table entries that apply.

> (A) More than \$10 billion GDP (the top three rows) and a population of less than 20 million (the three columns on right, before the subtotal column): 0 + 0 + 0 + 1 + 1 + 0 + 3 + 3 + 3 = 11.
> (B) Less than \$20 billion GDP (the bottom two rows above the subtotal row) and a population of more than 10 million (the three columns on left): 1 + 3 + 3 + 0 + 0 + 7 = 14.
> (C) More than \$20 billion GDP (the entire top two rows): 5 + 10 = 15.
> (D) Less than \$100 billion GDP (the bottom three rows above the subtotal row) and a population of less than 10 million (the two columns on the right before the subtotal column): 1 + 0 + 3 + 3 + 8 + 7 = 22.
> (E) Less than \$100 billion GDP (the bottom three rows above the subtotal row) and a population between 10–50 million (the second and third columns): 7 + 1 + 3 + 3 + 0 + 7 = 21. *Greatest.*

Choice (D), 22, is the greatest.

Problem Set I: This pie chart represents about 76 million owner-occupied housing units, categorized by household population. The smaller pie chart on the right further subdivides the households with at least 5 people. These categories could have been shown as small slivers in the pie chart on the left in the "Other" slice (notice that 6.7% + 2.6% + 1.8% = 11.1%).

34. **(D).** Sum the households with one, two, or three people (i.e., "fewer than four people"). Together these account for 21.6% + 36.3% + 16.5% = 74.4% of the total.

35. **(E).** According to the chart, 6.7% of the 75,986,074 households are 5-person households. Multiply 0.067 by 76 (keep "million" in mind). The result is about 5, so the answer is 5 million households.

36. **(B).** Approximate the total number of households as 76 million (close enough to 75,986,074).

One-person households are 21.6% of the total, or approximately 16.4 million. Since each such household has only 1 person, this represents about 16.4 million people.

Three-person households are 16.5% of the total, or approximately 12.5 million. Since each of these households has 3 people, that is about 37.5 million people.

Five-person households are 6.7% of the total, or approximately 5.1 million.

Since each of these households has 5 people, that is about 25.5 million people.

Since 16.4 million < 25.5 million < 37.5 million, the correct ranking is 1-person households, 5-person households, 3-person households.

37. **(A).** The 2- and 3-person households account for 36.3% + 16.5% = 52.8% of all households, so this is the correct answer. Quickly rule out the other choices as a check:

> (B) The 3- and 4-person households account for 16.5% + 14.5% = 31.0% of all households.
> (C) The 4- and 5-person households account for 14.5% + 6.7% = 21.2% of all households.
> (D) The 5- and 6-person households account for 6.7% + 2.6% = 9.3% of all households.
> (E) The 6- and 7-person households account for at most 2.6% + 1.8% = at most 4.4% of all households (remember that some of the 1.8% could consist of households with more than 7 people).

All of the choices other than (A) are less than 50%.

Chapter 25

Polygons and Rectangular Solids

In This Chapter…

Polygons and Rectangular Solids

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

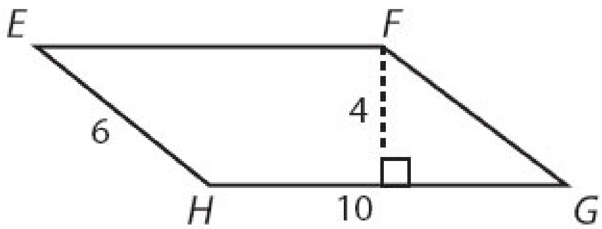
(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

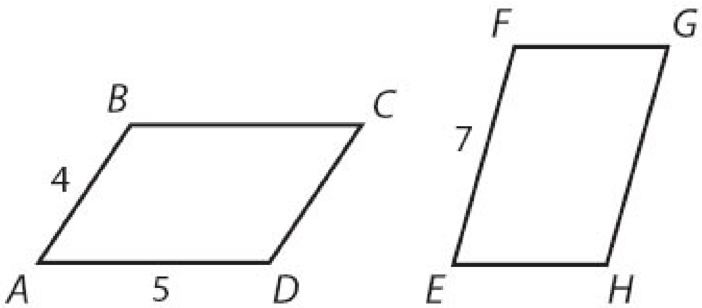
Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

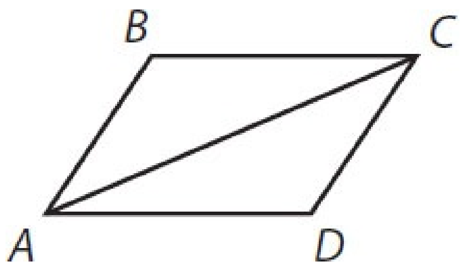
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.



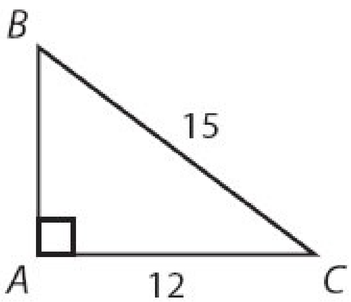
1. What is the area of parallelogram *EFGH*?

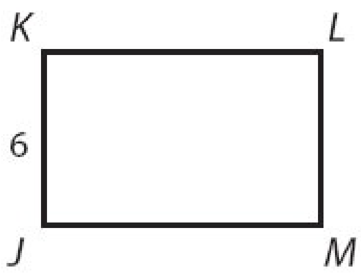


2. The two parallelograms pictured above have the same perimeter. What is the length of side *EH*?



3. In parallelogram *ABCD*, triangle *ABC* has an area of 12. What is the area of triangle *ACD*?



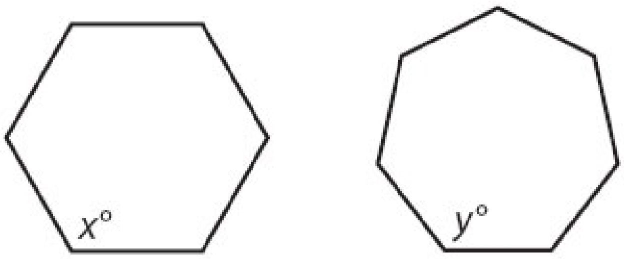


4. Triangle *ABC* and rectangle *JKLM* have equal areas. What is the perimeter of rectangle *JKLM*?

	Quantity A	**Quantity B**
5.	The area of a rectangle with perimeter 20	30

6. What is the area of a square with a diagonal measuring $6\sqrt{2}$?

	Quantity A	Quantity B
7.	The area of a parallelogram with a base of length 4 and height of 3.5	The area of a trapezoid with two parallel sides of lengths 5 and 9 and a height of 2



	Quantity A	Quantity B
8.	x	y

The perimeter of square *W* is 50% of the perimeter of square *D*.

	Quantity A	Quantity B
9.	The ratio of the area of square *W* to the area of square *D*	$\frac{1}{4}$

10. A 10-inch by 15-inch rectangular picture is displayed in a 16-inch by 24-inch rectangular frame. What is the area, in inches, of the part of the frame not covered by the picture?

(A) 150

(B) 234

(C) 244

(D) 264

(E) 384

A rectangular box has edges of length 2, 3, and 4.

	Quantity A	Quantity B
11.	Twice the volume of the box	The surface area of the box

12. What is the maximum number of 2-inch by 2-inch by 2-inch solid cubes that can be cut from six solid cubes that are 1 foot on each side? (12 inches = 1 foot)

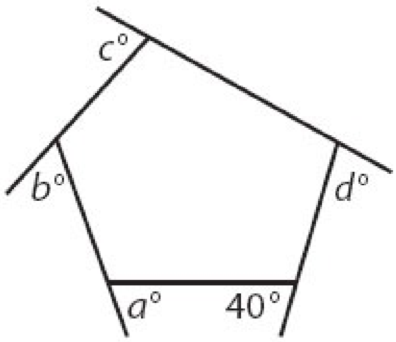
(A) 8

(B) 64

(C) 216

(D) 1,296

(E) 1,728



13. What is the value of $a + b + c + d$?

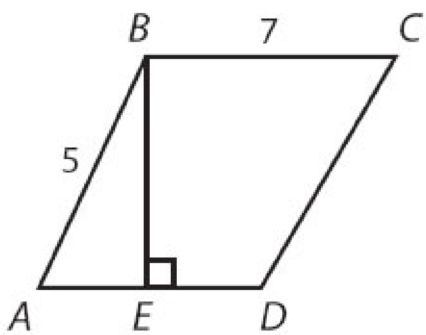
(A) 240

(B) 320

(C) 360

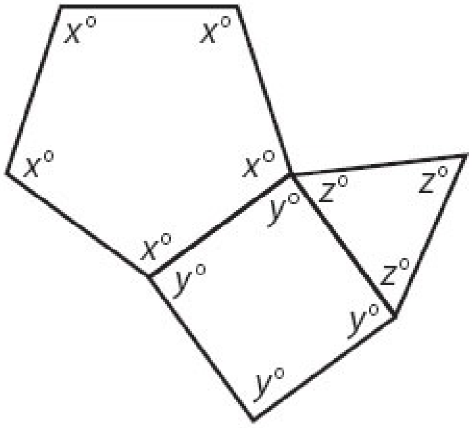
(D) 500

(E) 540



In the trapezoid above, $AE = ED = 3$ and BC is parallel to AD.

	Quantity A	**Quantity B**
14.	The area of the trapezoid	35



	Quantity A	Quantity B
15.	$x + y + z$	270

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16. A 2-meter by 2-meter sheet of paper is to be cut into 2-centimeter by 10-centimeter rectangles. What is the maximum number of such rectangles that can be cut from the sheet of paper? (1 meter = 100 centimeters)

A parallelogram has two sides with length 10 and two sides with length 5.

	Quantity A	Quantity B
17.	The area of the parallelogram	30

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18. What is the area of a regular hexagon of side length 4?

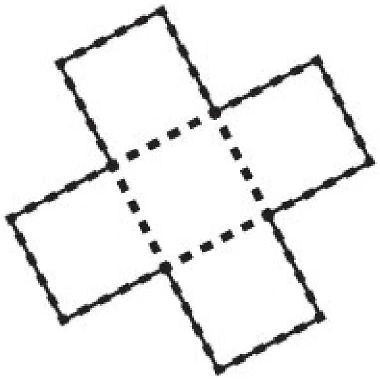
(A) $4\sqrt{3}$

(B) $6\sqrt{3}$

(C) $12\sqrt{3}$

(D) $24\sqrt{3}$

(E) $36\sqrt{3}$



The figure above is composed of 5 squares of equal area, as indicated by the dotted lines. The total area of the figure is 45.

	Quantity A	**Quantity B**
19.	The perimeter of the figure	48

20. A 2-foot by 2-foot by 2-foot solid cube is cut into 2-inch by 2-inch by 4-inch rectangular solids. What is the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube? (1 foot = 12 inches)

(A) 2 : 1

(B) 4 : 1

(C) 5 : 1

(D) 8 : 1

(E) 10 : 1

21. If a cube has the same volume (in cubic units) as surface area (in square units), what is the length of one side?

(A) 1

(B) 3

(C) $\frac{5}{3}$

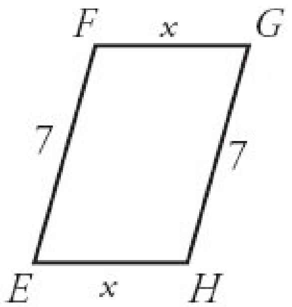
(D) 6

(E) No such cube is possible.

Polygons and Rectangular Solids Answers

1. **40.** The area of a parallelogram is base × height. In this parallelogram, the base is 10 and the height is 4 (remember, base and height need to be perpendicular). So the area is $10 \times 4 = 40$.

2. **2.** First find the perimeter of parallelogram *ABCD*. If two sides have a length of 4, and two sides have a length of 5, the perimeter is $2(4 + 5) = 8 + 10 = 18$. That means parallelogram *EFGH* also has a perimeter of 18. Because *EF* is labeled 7, *GH* also is 7. The lengths of the other two sides are unknown but equal to each other, so for now say the length of each side is x. The parallelogram now looks like this:



Therefore, the perimeter is:

$$2(7 + x) = 18$$
$$2x + 14 = 18$$
$$2x = 4$$
$$x = 2$$

The length of side *EH* is 2.

3. **12.** One property that is true of any parallelogram is that the diagonal will split the parallelogram into two equal triangles. If triangle *ABC* has an area of 12, then triangle *ACD* must also have an area of 12.

4. **30.** To find the area of right triangle *ABC*, use the Pythagorean theorem to first find the length of side *AB*:

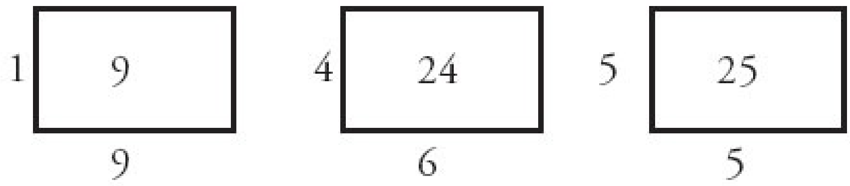
$$12^2 + AB^2 = 15^2$$
$$144 + AB^2 = 225$$
$$AB^2 = 81$$
$$AB = 9$$

(A 9–12–15 triangle is a 3–4–5 triangle, with all the measurements tripled.)

The area of triangle ABC is $\frac{1}{2}(12)(9) = \frac{108}{2} = 54$.

Rectangle *JKLM* also has an area of 54. One side of the rectangle is labeled, so solve for the other: $6 \times JM = 54$. The length of side *JM* is 9, so the perimeter of *JKLM* is $2(6 + 9) = 12 + 18 = 30$.

5. **(B).** While a rectangle with perimeter 20 could have many different areas, all of these areas are less than 30:



How can you be sure this will always be the case? It would be helpful to know the rule that the area of a rectangle with constant perimeter increases as length and width become more similar, and is maximized when the rectangle is a square. Thus, the 5-by-5 version of the rectangle represents the maximum possible area, which is still less than 30.

6. **36.** When a square is cut by a diagonal, two 45–45–90 triangles are created. Use the 45–45–90 formula (sides in the ratio $1 : 1 : \sqrt{2}$) to determine that the sides are equal to 6, and thus the area is $6 \times 6 = 36$. Alternatively, label each side of the square x (since they're the same) and use the Pythagorean theorem:

$$x^2 + x^2 = (6\sqrt{2})^2$$
$$2x^2 = 72$$
$$x^2 = 32$$
$$x = 16$$

If each side of the square is 6, the area is $6 \times 6 = 36$.

7. **(C).** The formula for area of a parallelogram is *base* × *height*, so Quantity A is $4 \times 3.5 = 14$.

The formula for area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides, so Quantity B is $\frac{(5+9)}{2} \times 2 = 14$.

The two quantities are equal.

8. **(D).** Do not assume that any polygon is a regular figure unless the problem explicitly or implicitly says so. (For instance, if *every* angle in the hexagon were labeled with the same variable, you could be sure the hexagon was regular.)

Using the formula $(n - 2)(180°)$ where n is the number of sides, calculate that the sum of the angles in the 6-sided figure is 720° and the sum of the angles in the 7-sided figure is 900°. However, those totals could be distributed any number of ways among the interior angles, so either x or y could be greater.

9. **(C).** If one square has twice the perimeter, it has twice the side length, so it will have four times the area. Why is this? Doubling only the length doubles the area. Then, doubling the width doubles the area *again*.

Alternatively, prove this with real numbers. Say square *W* has perimeter 8 and square *D* has perimeter 16. Thus, square *W* has side 2 and square *D* has side 4. The areas are 4 and 16, respectively. As a ratio, $\frac{4}{16}$ reduces to $\frac{1}{4}$. The two quantities are equal.

10. **(B).** The area of the picture is 10 × 15 = 150. The area of the frame is 16 × 24 = 384. Subtract to get the answer: 384 – 150 = 234.

11. **(B).** The volume of a rectangular box is length × width × height. Therefore: 2 × 3 × 4 = 24. Quantity A is double this volume, or 48.

The surface area of a rectangular box is 2(length × width) + 2(width × height) + 2(length × height). Therefore:
2(6) + 2(12) + 2(8) = 52.

Quantity B is greater.

12. **(D).** Each large solid cube is 12 inches × 12 inches × 12 inches. Each dimension (length, width, and height) is to be cut identically at 2-inch increments, creating 6 smaller cubes in each dimension. Thus, 6 × 6 × 6 small cubes can be cut from each large cube. There are 6 large cubes to be cut this way, though, so the total number of small cubes that can be cut is 6(6 × 6 × 6) = 6 × 216 = 1,296.

13. **(B).** The interior figure shown is a pentagon, although an irregular one. The sum of the interior angles of any polygon can be determined using the formula $(n – 2)(180°)$, where n is the number of sides:

$$(5 - 2)(180°) = (3)(180°) = 540°$$

Using the rule that angles forming a straight line sum to 180°, the interior angles of the pentagon (starting at the top and going clockwise) are $180 - c$, $180 - d$, 140, $180 - a$, and $180 - b$. The sum of these angles can be set equal to 540:

$$\begin{aligned} 540 &= (180 - c) + (180 - d) + 140 + (180 - a) + (180 - b) \\ 540 &= 140 + 4(180) - a - b - c - d \\ 540 - 140 - 720 &= -(a + b + c + d) \\ -320 &= -(a + b + c + d) \end{aligned}$$

So, $a + b + c + d = 320$.

14. **(B).** While the figure may *look* like a parallelogram, it is actually a trapezoid, as it has two parallel sides of unequal length (*AD* = *AE* + *ED* = 6 and *BC* = 7). The two parallel sides in a trapezoid are referred to as the bases.

The formula for the area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides and *h* is the height, which is the distance between the parallel sides (*BE* in this figure).

Triangle *ABE* is a 3–4–5 special right triangle, so *BE* is 4. (Alternatively, use the Pythagorean theorem to determine this.)

Thus, the area is $\frac{(6+7)}{2} \times 4 = 26$. Quantity B is greater.

15. **(B).** Each angle in the pentagon is labeled with the same variable, so this is a regular pentagon. Using the formula (n – 2)(180°), where n is the number of sides, the sum of all the interior angles of the pentagon is (3)(180°) = 540°. Divide by 5 to get x = 108.

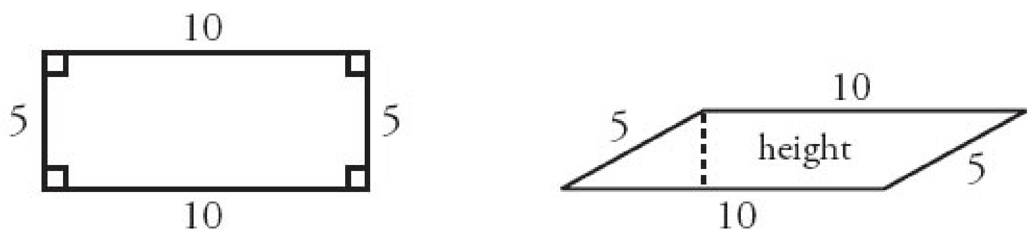
Now, the quadrilateral. All four-sided figures have interior angles that sum to 360°. Alternatively, use the formula (n – 2)(180°) to determine this. Divide 360 by 4 to get y = 90.

Now, the triangle. It is equilateral, so z = 60. (The sum of angle measures in a triangle is always 180°; if the angles are equal, they will each equal 60°.)

Thus, $x + y + z = 108 + 90 + 60 = 258$. Quantity B is greater.

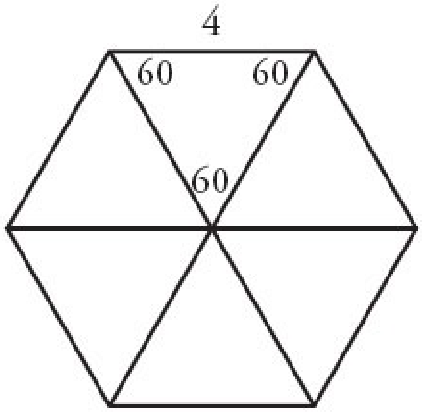
16. **2,000.** Since the sheet of paper is measured in meters and the small rectangles in centimeters, first convert the measures of the sheet of paper to centimeters. The large sheet of paper measures 200 centimeters by 200 centimeters. The most efficient way to cut 2-centimeter by 10-centimeter rectangles is to cut vertically every 2 centimeter and horizontally every 10 centimeter (or vice versa; the idea is that all the small rectangles should be oriented the same direction on the larger sheet). Doing so creates a grid of $\frac{200}{2} \times \frac{200}{10}$ = 100 × 20 = 2,000 small rectangles.

17. **(D).** The formula for the area of a parallelogram is base × height, where height is the perpendicular distance between the parallel bases, not necessarily the other side of the parallelogram. However, if the parallelogram is actually a rectangle, the height *is* the other side of the parallelogram and is thereby maximized. So, if the parallelogram is actually a rectangle, the area would be equal to 50, but if the parallelogram has more extreme angle measures, the height could be very, very small, making the area much less than 30.



18. **(D).** Divide the hexagon with three diagonals (running through the center) to get six triangles. Since the sum of the angles in any polygon is (n – 2)(180°), the sum for a hexagon is 720°. Divide by 6 to get that each angle in the original hexagon is 120°. When the hexagon is divided into triangles, each 120° angle is halved, creating two 60° angles for each triangle. Any triangle

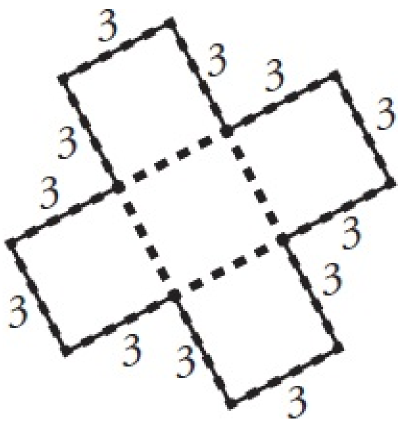
that has two angles of 60° must have a third angle of 60° as well, since triangles always sum to 180°. Thus, all six triangles are equilateral. Therefore, all three sides of each triangle are equal to 4.



For any equilateral triangle, the height equals half the side times $\sqrt{3}$. Therefore, the height is $2\sqrt{3}$. Since $A = \frac{1}{2}bh$, the area of each equilateral triangle is $A = \frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$. Since there are six such triangles, the answer is $24\sqrt{3}$.

19. **(B).** If a figure with area of 45 is composed of 5 equal squares, divide to get that the area of each square is 9 and thus the side of each square is 3.

Don't make the mistake of adding up *every* side of every square to get the perimeter—only count lengths that are actually part of the perimeter of the overall figure. (Note that the central square does not have any lengths that are part of the perimeter), as shown below:



The perimeter is made of 12 segments, each with length 3. The perimeter is 36.

Incorrect choice (A) comes from reasoning that 5 squares have 20 total sides, each of length 3, and thus the combined length would be 60. Do not just subtract the four dotted line lengths, as each of these was actually counted twice, as part of the central square and one of the others. This mistake would incorrectly yield choice (C). The best approach here is to make a quick sketch of the figure, label the sketch with the given information, and count up the perimeter.

20. **(E).** To find the surface area of the original cube, first convert the side lengths to inches (it is NOT okay to find surface area or volume and then convert using 1 foot = 12 inches; this is only true for straight-line distances). The equation for surface area is $6s^2$, so, the surface area of the large original cube is $6(24 \text{ inches})^2 = 3{,}456$ square inches.

Each large solid cube is 24 inches × 24 inches × 24 inches. To cut the large cube into 2-inch by 2-inch by 4-inch rectangular solids, two dimensions (length and width, say) will be sliced every 2 inches, while one dimension (height, say) will be sliced every 4 inches. Thus, $\frac{24}{2} \times \frac{24}{2} \times \frac{24}{4}$ = 12 × 12 × 6 = 864 small rectangular solids can be cut from the large cube.

The equation for the surface area of a rectangular solid is $2lw + 2wh + 2lh$. In this case, that is 2(2 × 2) + 2(2 × 4) + 2(2 × 4) = 8 + 16 + 16 = 40 square inches per small rectangular solid. There are 864 small rectangular solids, so the total surface area is 40 × 864 = 34,560 square inches.

Finally, the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube is the ratio of 34,560 to 3,456. This ratio reduces to 10 to 1.

21. **(D).** To solve this question, use the equations for the volume and the surface area of a cube:

Volume = s^3 Surface area = $6s^2$

If a cube has the same volume as surface area, set these equal:

$$s^3 = 6s^2$$
$$s = 6$$

Chapter 26

Circles and Cylinders

In This Chapter…

Circles and Cylinders

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A circle has an area of 16π. What is its circumference?

    (A) 4π

    (B) 8π

    (C) 16π

    (D) 32π

    (E) It cannot be determined from the information given.

2. A circle has a diameter of 5. What is its area?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi^2}{2}$

(D) 10π

(E) 25π

3. A circle's area equals its circumference. What is its radius?

(A) 1

(B) 2

(C) 4

(D) 8

(E) 16

Circle C has a radius r such that $1 < r < 5$.

	Quantity A	**Quantity B**
4.	The area of circle C	The circumference of circle C

5. A circle has radius 3.5. What is its area?

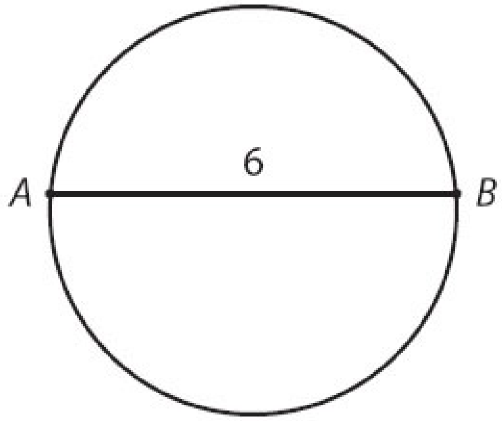
(A) $\frac{7}{2}\pi$

(B) 9.5π

(C) 10.5π

(D) $\frac{49}{4}\pi$

(E) $\frac{49}{2}\pi$



AB is <u>not</u> a diameter of the circle.

	Quantity A	Quantity B
6.	The area of the circle	9π

7. A circle has radius 0.001. What is its area?

(A) $\pi \times 10^{-2}$

(B) $\pi \times 10^{-3}$

(C) $\pi \times 10^{-4}$

(D) $\pi \times 10^{-6}$

(E) $\pi \times 10^{-9}$

8. A circle has an area of 4π. If the radius were doubled, the new area of the circle would be how many times the original area?

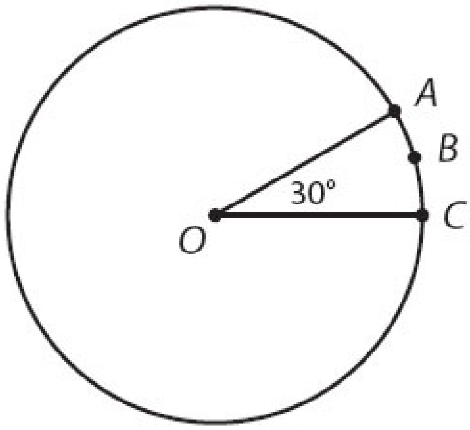
(A) 2

(B) 3

(C) 4

(D) 5

(E) It cannot be determined from the information given.



The radius of the circle with center *O* is 6.

	Quantity A	**Quantity B**
9.	The length of arc *ABC*	3

10. A sector of a circle has a central angle of 120°. If the circle has a diameter of 12, what is the area of the sector?

(A) 4π

(B) 8π

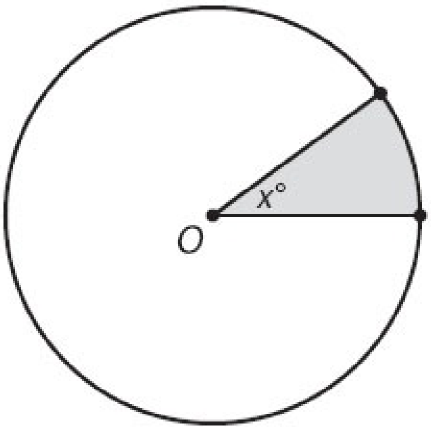
(C) 12π

(D) 18π

(E) 36π

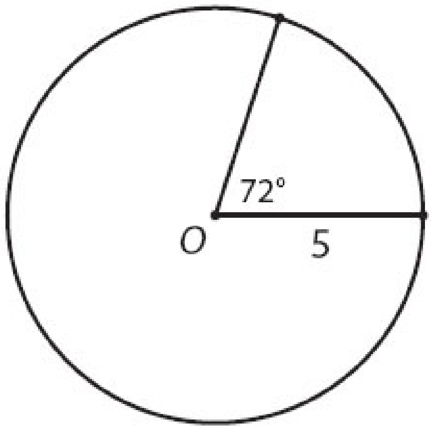
Within a circle with radius 12, a sector has an area of 24π.

	Quantity A	**Quantity B**
11.	The measure of the central angle of the sector, in degrees	90



In the circle with center O, the area of the shaded sector is $\frac{1}{10}$ of the area of the full circle.

	Quantity A	**Quantity B**
12.	$2x$	75



13. If O is the center of the circle, what is the perimeter of the sector with central angle 72°?

(A) $5 + 2\pi$

(B) $10 + 2\pi$

(C) $10 + 4\pi$

(D) $10 + 5\pi$

(E) $20 + 2\pi$

14. A sector of a circle has a radius of 10 and an area of 20π. What is the arc length of the sector?

(A) π

(B) 2π

(C) 4π

(D) 5π

(E) 10π

Sector A and sector B are sectors of two different circles.
Sector A has a radius of 4 and a central angle of 90°.
Sector B has a radius of 6 and a central angle of 45°.

	Quantity A	**Quantity B**
15.	The area of sector A	The area of sector B

---

16. What is the height of a right circular cylinder with radius 2 and volume 32π?

A right circular cylinder has volume 24π.

	Quantity A	**Quantity B**
17.	The height of the cylinder	The radius of the cylinder

18. If a half-full 4-inch by 2-inch by 8-inch box of soymilk is poured into a right circular cylindrical glass with radius 2 inches, how many inches high will the soymilk reach? (Assume that the capacity of the glass is greater than the volume of the soymilk.)

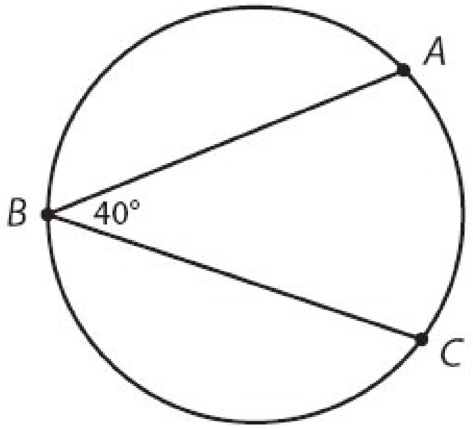
(A) 8

(B) 16

(C) $\frac{4}{\pi}$

(D) $\frac{8}{\pi}$

(E) $\frac{16}{\pi}$



19. If the diameter of the circle is 36, what is the length of arc *ABC*?

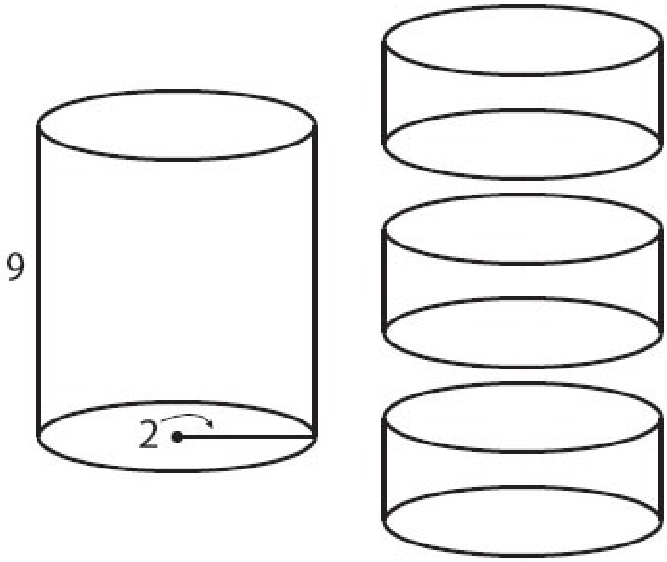
(A) 8

(B) 8π

(C) 28π

(D) 32π

(E) 56π



20. If a solid right circular cylinder with height 9 and radius 2 is cut as shown into three new cylinders, each of equal and uniform height, how much new surface area is created?

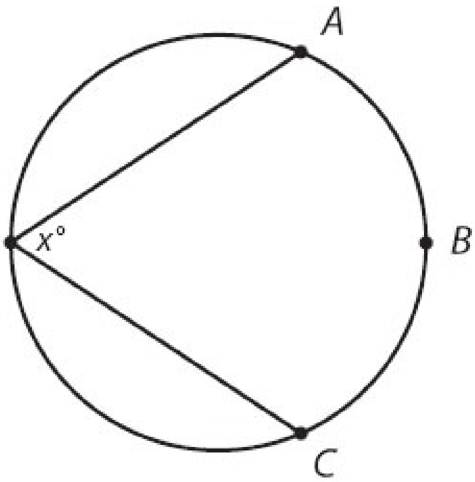
(A) 4π

(B) 12π

(C) 16π

(D) 24π

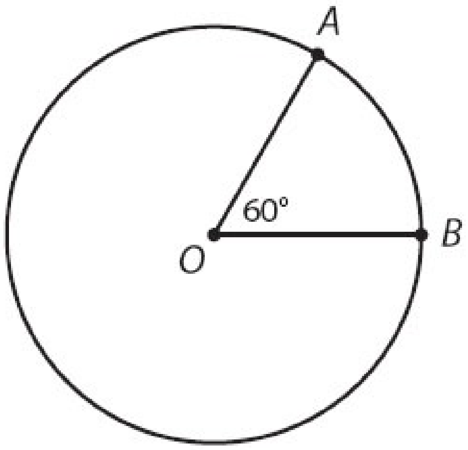
(E) 36π



$x > 60°$

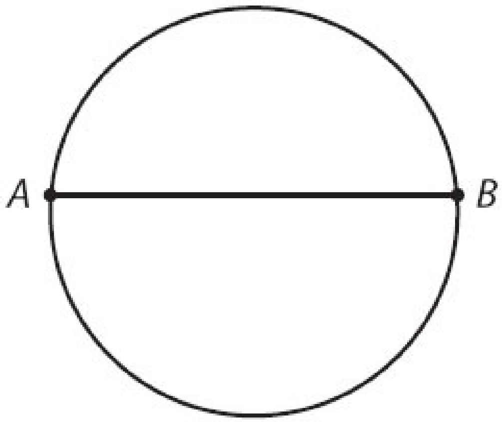
Quantity A	**Quantity B**
The ratio of the length of arc *ABC*	

21. to the circumference of the circle $\frac{1}{3}$



Point O is the center of the circle above.

	Quantity A	**Quantity B**
22.	The ratio of the length of minor arc AB to major arc AB	$\frac{1}{6}$



The circle above has area 25.

	Quantity A	**Quantity B**
23.	The length of chord AB	10

Circles and Cylinders Answers

1. **(B).** Since the area formula for a circle is $A = \pi r^2$:

$$16\pi = \pi r^2$$
$$16 = r^2$$
$$4 = r$$

Since the circumference formula is $C = 2\pi r$ and $r = 4$:

$$C = 2\pi(4)$$
$$C = 8\pi$$

2. **(A).** If a circle's diameter is 5, its radius is $\frac{5}{2}$. Plug this into the area formula:

$$A = \pi\left(\frac{5}{2}\right)^2$$
$$A = \pi \times \frac{25}{4}$$
$$A = \frac{25\pi}{4}$$

3. **(B).** To find the radius that would make the area and the circumference of a circle equal, set the area and circumference formulas equal to one another:

$$\pi r^2 = 2\pi r$$

Since both sides have both r and π, divide both sides by πr:

$$r = 2$$

4. **(D).** Picking numbers is the easiest way to prove (D). If the radius is 3, the area is 9π and the circumference is 6π, so Quantity A is greater. If the radius is 4, the area is 16π and the circumference is 8π, so once again Quantity A is greater. But if the radius is 2, both the area and the circumference equal 4π. Therefore, Quantity A is not always greater. Note also that r is not required to be an integer. If you try a radius close to the minimum, such as 1.1, Quantity B would be greater.

5. **(D).** The area formula for a circle is $A = \pi r^2$, so plug the radius in. Since the decimal will be unwieldy, it is easier to plug the fractional version of 3.5 $\left(\text{i.e., } \frac{7}{2}\right)$ into the formula:

$$A = \pi\left(\frac{7}{2}\right)^2$$

$$A = \frac{49\pi}{4}$$

6. **(A).** Since a diameter is the longest straight line you can draw from one point on a circle to another (that is, a diameter is the longest chord in a circle), the actual diameter must be *greater* than 6.

If the diameter were exactly 6, the radius would be 3, and the area would be:

$$A = \pi(3)^2$$
$$A = 9\pi$$

However, since the diameter must actually be greater than 6, the area must be greater than 9π. Do *not* make the mistake of picking (D) for Quantitative Comparison geometry questions in which you cannot "solve." There is often still a way to determine which quantity is greater.

7. **(D).** The formula for the area of a circle is $A = \pi r^2$, so plug radius 0.001 into the formula. However, since the answers are in exponential form, it would be easier to first convert 0.001 to 1×10^{-3}, or just 10^{-3}, and use that in

the formula:

$$A = \pi(10^{-3})^2$$
$$A = \pi(10^{-6})$$

8. **(C).** To begin, find the original radius of the circle: Area = $\pi r^2 = 4\pi$, so r = 2. Once doubled, the new radius is 4. A circle with a radius of 4 has an area of 16π. The new area of 16π is 4 times the old area of 4π.

9. **(A).** If the sector has a central angle of 30°, then it is $\frac{1}{12}$ of the circle, because $\frac{30}{360} = \frac{1}{12}$. To find the arc length of the sector, first find the circumference of the entire circle. The radius of the circle is 6, so the circumference is $2\pi(6) = 12\pi$. That means that the arc length of the sector is $\frac{1}{12}(12\pi) = \pi$. Since π is about 3.14, Quantity A is greater.

10. **(C).** The sector is $\frac{1}{3}$ of the circle, because $\frac{120}{360} = \frac{1}{3}$. To find the area of the sector, first find the area of the whole circle. The diameter of the circle is 12, so the radius of the circle is 6, and the area is $\pi(6)^2 = 36\pi$. That means the area of the sector is $\frac{1}{3}(36\pi) = 12\pi$.

11. **(B).** First find the area of the whole circle. The radius is 12, which means the area is $\pi(12)^2 = 144\pi$. Since the sector has an area of 24π and $\frac{24\pi}{144\pi} = \frac{1}{6}$, the sector is $\frac{1}{6}$ of the entire circle. That means that the central angle is $\frac{1}{6}$ of 360, or 60°. Quantity B is greater.

12. **(B).** If the area of the sector is $\frac{1}{10}$ of the area of the full circle, then the central angle is $\frac{1}{10}$ of the degree measure of the full circle, or $\frac{1}{10}$ of 360° = 36° = x°. Thus, Quantity A = 2(36) = 72, so Quantity B is greater.

13. **(B).** To find the perimeter of a sector, first find the radius of the circle and the arc length of the sector. Begin by determining what fraction of the circle the sector is. The central angle of the sector is 72°, so the sector is $\frac{72}{360} = \frac{1}{5}$ of the circle. The radius is 5, so the circumference of the circle is $2\pi(5) = 10\pi$. The arc length of the sector is $\frac{1}{5}$ of the circumference: $\frac{1}{5}(10\pi) = 2\pi$. The

perimeter of the sector is this 2π plus the two radii that make up the straight parts of the sector: $10 + 2\pi$.

14. **(C).** Compare the given area of the sector to the calculated area of the whole circle. The radius of the circle is 10, so the area of the whole circle is $\pi(10)^2 = 100\pi$. The area of the sector is 20π, or $\frac{20\pi}{100\pi} = \frac{1}{5}$ of the circle. The radius is 10, so the circumference of the whole circle is $2\pi(10) = 20\pi$. Since the sector is $\frac{1}{5}$ of the circle, the arc length is $\left(\frac{1}{5}\right)(20\pi) = 4\pi$.

15. **(B).** Sector A is $\frac{90}{360} = \frac{1}{4}$ of the circle with radius 4. The area of this circle is $\pi(4)^2 = 16\pi$, so the area of sector A is $\frac{1}{4}$ of 16π, or 4π.

Sector B is $\frac{45}{360} = \frac{1}{8}$ of the circle with radius 6. The area of this circle is $\pi(6)^2 = 36\pi$, so the area of sector B is $\frac{1}{8}$ of 36π, or 4.5π.

Since 4.5π is greater than 4π, Quantity B is greater.

16. **8.** Use the formula for the volume of a right circular cylinder, $V = \pi r^2 h$:

$$\begin{aligned} 32\pi &= \pi(2)^2 h \\ 32 &= 4h \\ 8 &= h \end{aligned}$$

17. **(D).** Plugging into the formula for volume of a right circular cylinder, $V = 24\pi = \pi r^2 h$. However, there are many combinations of r and h that would make the volume 24π. For instance, $r = 1$ and $h = 24$, or $r = 4$ and $h = 1.5$. Keep in mind that the radius and height don't even have to be integers, so there truly are an infinite number of possibilities, some for which h is greater and some for which r is greater.

18. **(D).** A box is a rectangular solid whose volume formula is V = length × width × height. Thus, the volume of the box is 4 inches × 2 inches × 8 inches = 64 inches^3. Since the box is half full, there are 32 inches^3 of soymilk. This volume will not change when the soymilk is poured from the box into the cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$, so:

$$32 = \pi(2)^2 h, \text{ where } r \text{ and } h \text{ are in units of inches.}$$

$$\begin{aligned} \frac{32}{4\pi} &= h \\ \frac{8}{\pi} &= h \end{aligned}$$

The height is $\frac{8}{\pi}$ inches. Note that the height is "weird" (divided by π) because the volume of the cylinder did *not* have a π.

19. **(C).** Note that a *minor* arc is the "short way around" the circle from one point to another, and a *major* arc is the "long way around." Arc *ABC* is thus the same as major arc *AC*.

For a given arc, an inscribed angle is always half the central angle, which would be 80° in this case. The minor arc *AC* is thus $\frac{80}{360} = \frac{2}{9}$ of the circle.

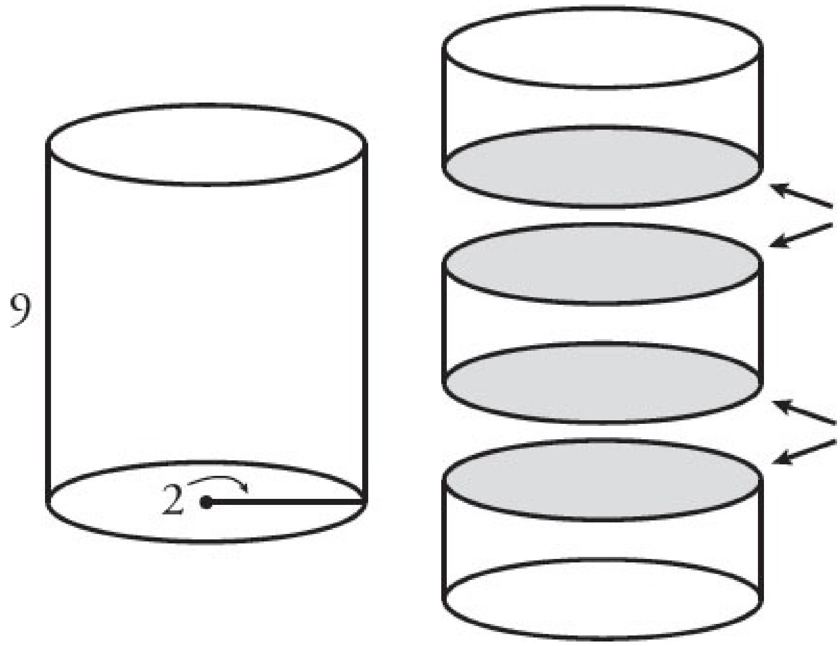
Since the circumference is 36π:

minor arc $AC = \frac{2}{9}(36\pi) = 8\pi$

Arc *ABC*, or major arc *AC*, is the entire circumference minus the minor arc:

$$36\pi - 8\pi = 28\pi$$

20. **(C).** One method is to find the surface area of the large cylinder, then the surface areas of the three new cylinders, then subtract the surface area of the large cylinder from the combined surface areas of the three new cylinders. However, there is a much faster way. When the large cylinder is cut into three smaller ones, only a few *new* surfaces are created—the bottom base of the top cylinder, the top and bottom bases of the middle cylinder, and the top surface of the bottom cylinder.



Thus, these four circular bases represent the new surface area created. Since the radius of each base is 2, use the area formula for a circle, $A = \pi r^2$:

$$A = \pi(2)^2$$
$$A = 4\pi$$

Since there are 4 such bases, multiply by 4 to get 16π.

21. **(A).** If $x°$ were equal to 60°, arc *ABC* would have a central angle of 120°. (Inscribed angles, with the vertex at the far side of the circle, are always half the central angle.) A 120° arc is $\frac{120}{360} = \frac{1}{3}$ of the circumference of the circle.

Since x is actually greater than 60°, the arc is actually greater than $\frac{1}{3}$ of the circumference. Thus, the ratio of the arc length to the circumference is greater than $\frac{1}{3}$.

22. **(A).** Since the angle that determines the arc is equal to 60 and $\frac{60}{360} = \frac{1}{6}$, minor arc *AB* is $\frac{1}{6}$ of the circumference of the circle. (There are always 360° in a circle. Minor arc *AB* is the “short way around” from *A* to *B*, while major arc *AB* is the “long way around.”)

Since minor arc *AB* is $\frac{1}{6}$ of the circumference, major arc *AB* must be the other $\frac{5}{6}$. Therefore, the ratio of the minor arc to the major arc is 1 to 5 (*not* 1 to 6). You could calculate this as $\frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{6} \times \frac{6}{5} = \frac{1}{5}$, or you could just reason the ratio of 1 of *anything* (such as sixths) to 5 of the same thing (again, sixths) is a 1 to 5 ratio.

The trap answer here is (C). This is a common mistake: $\frac{1}{6}$ of the total is not the same as a 1 to 6 ratio of two parts.

23. **(B).** The equation for the area of a circle is $A = \pi r^2$. Note that the given area is just 25, *not* 25π! So:

$$\begin{aligned} \pi r^2 &= 25 \\ r^2 &= \frac{25}{\pi} \approx 8 \\ r &= \text{a bit less than 3.} \end{aligned}$$

So the diameter of the circle is a bit less than 6. The diameter is the chord with maximum length, so wherever *AB* is on this circle, it’s significantly shorter than 10.

Chapter 27

Triangles

In This Chapter…

Triangles

For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

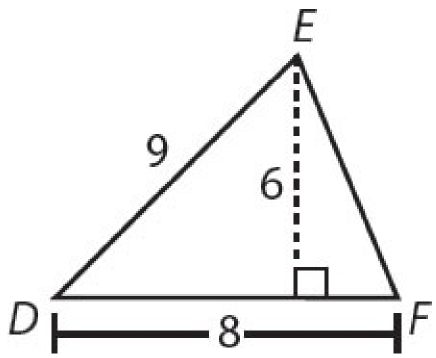
(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.



1. What is the area of triangle *DEF*?

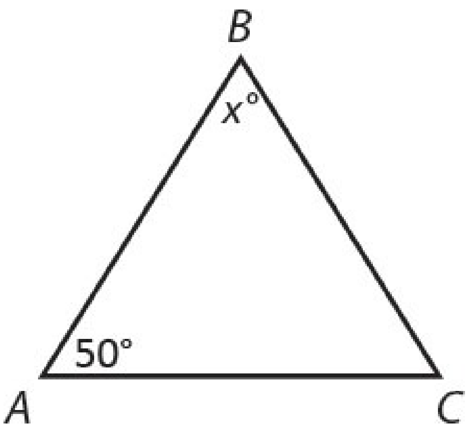
   (A) 23

   (B) 24

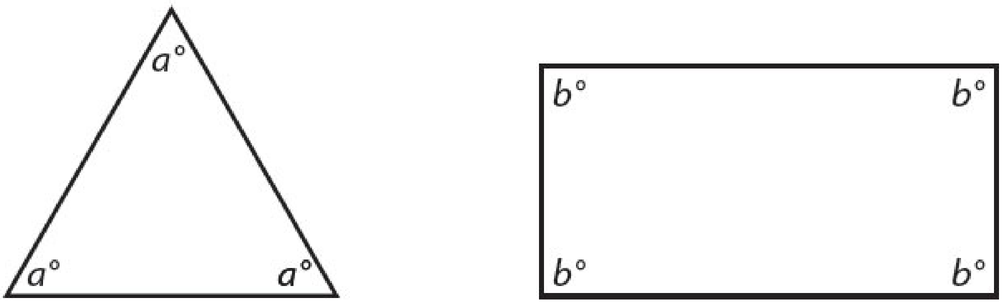
   (C) 48

   (D) 56

   (E) 81

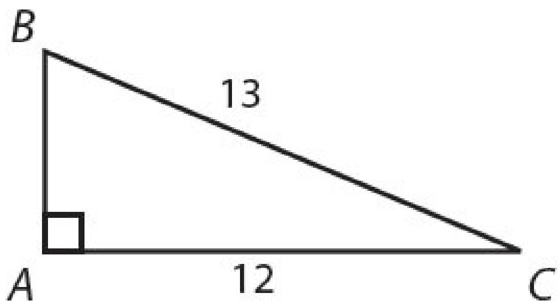


2. If *AB* and *BC* have equal lengths, what is the value of *x*?

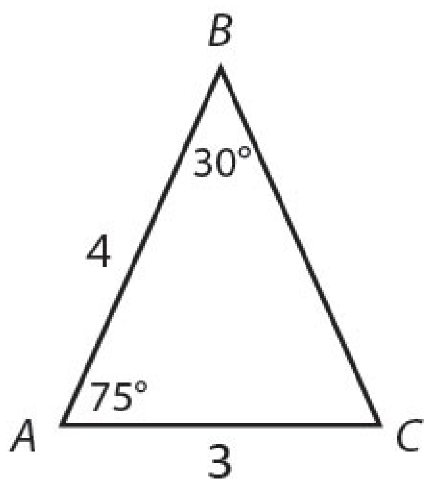


Quantity A	**Quantity B**
$2a + b$	$3a + \frac{b}{3}$

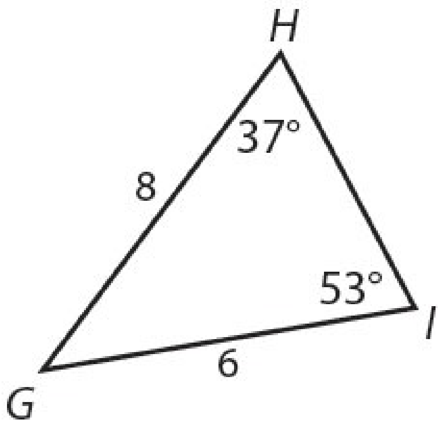
3.



4. What is the area of right triangle *ABC*?



5. What is the perimeter of triangle *ABC*?



6. What is the length of side *HI*?

7. If the hypotenuse of an isosceles right triangle is $8\sqrt{2}$, what is the area of the triangle?

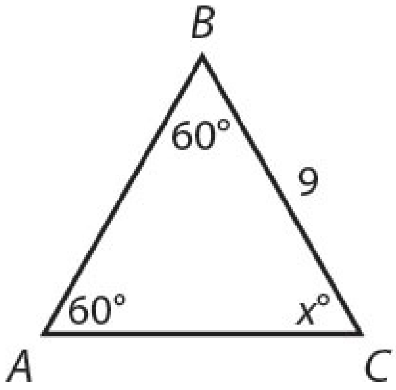
(A) 18

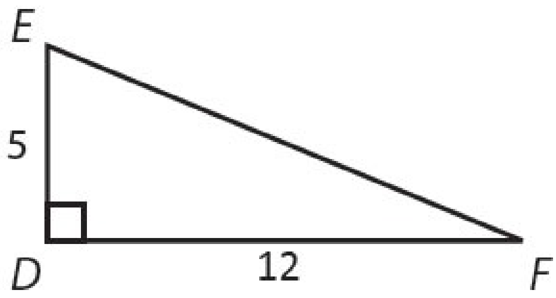
(B) 24

(C) 32

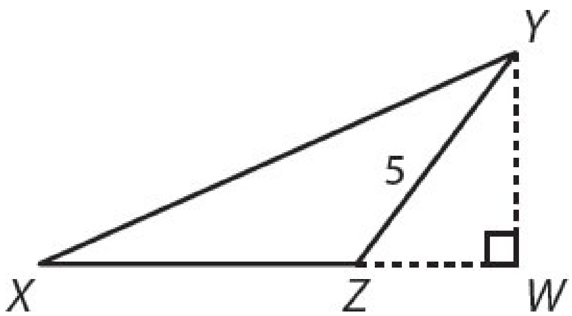
(D) 48

(E) 64

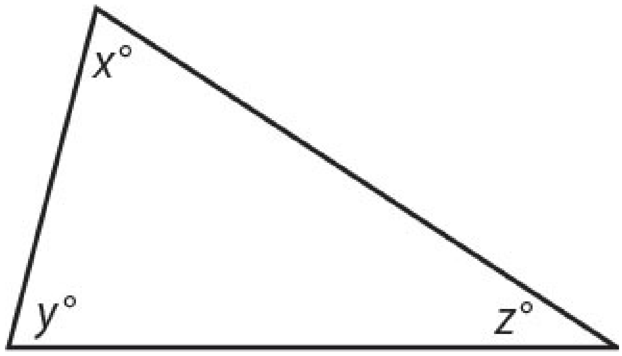




	Quantity A	**Quantity B**
8.	Perimeter of triangle *ABC*	Perimeter of triangle *DEF*



9. *ZW* has a length of 3 and *XZ* has a length of 6. What is the area of triangle *XYZ*?



In the figure above, $x + z$ equals 110.

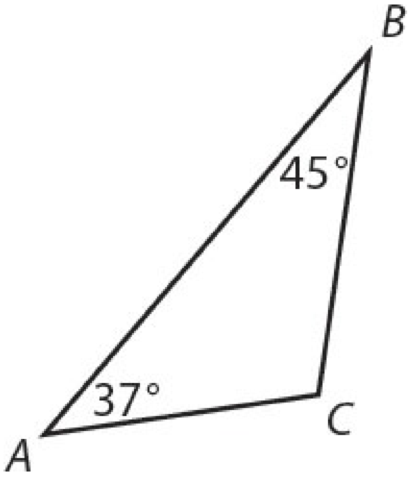
	Quantity A	Quantity B
10.	x	y

Two sides of an isosceles triangle are 8 and 5 in length, respectively.

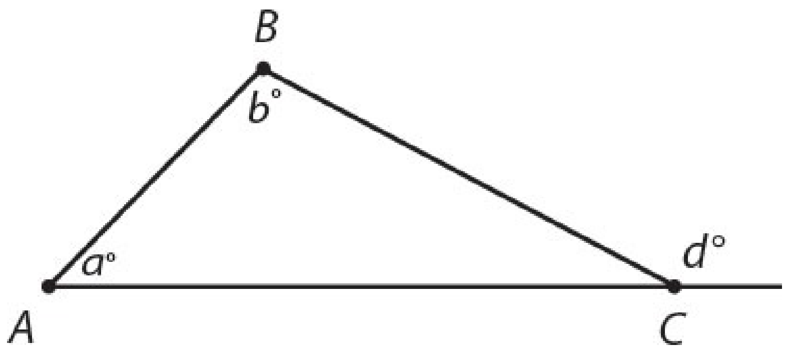
	Quantity A	Quantity B
11.	The length of the third side	8

Two sides of an isosceles triangle are 2 and 11 in length, respectively.

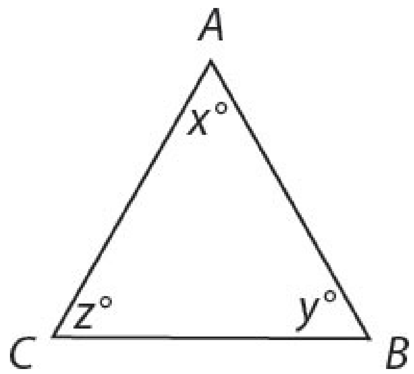
	Quantity A	Quantity B
12.	The length of the third side	11



	Quantity A	Quantity B
13.	AC	BC

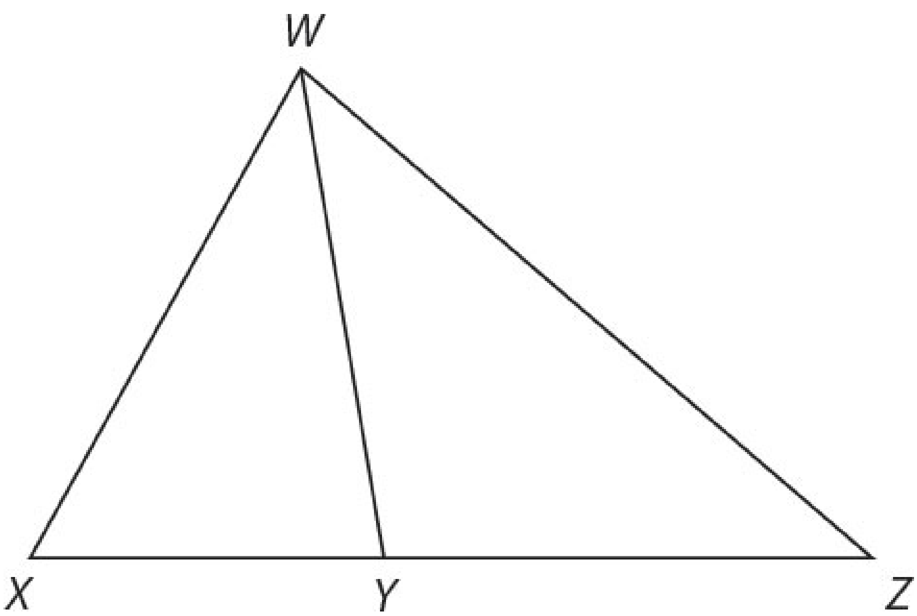


	Quantity A	**Quantity B**
14.	$a + b$	d



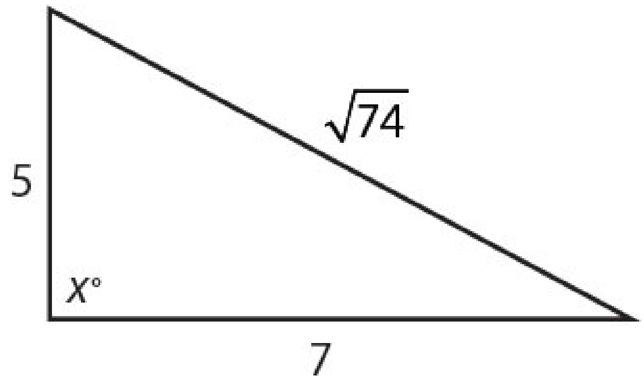
$y < z$
$y > 60$

	Quantity A	**Quantity B**
15.	The length of side *AC*	The length of side *BC*



$XY = YZ$

	Quantity A	Quantity B
16.	The area of *XWY*	The area of *YWZ*



	Quantity A	**Quantity B**
17.	x	90

---

18. If p is the perimeter of a triangle with one side of 7 and another side of 9, what is the range of possible values for p?

(A) $2 < p < 16$

(B) $3 < p < 17$

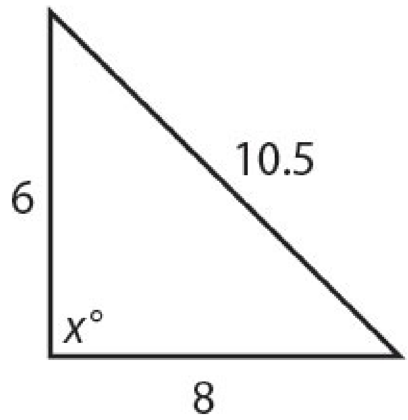
(C) $18 < p < 32$

(D) $18 < p < 33$

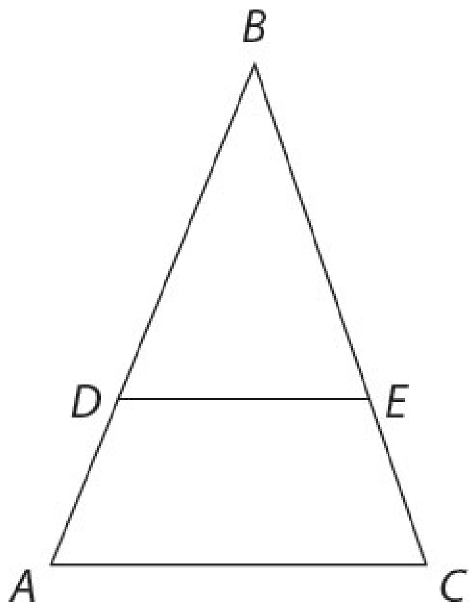
(E) $17 < p < 63$

A right triangle has a hypotenuse of 12 and legs of 9 and y.

	Quantity A	**Quantity B**
19.	y	15



	Quantity A	**Quantity B**
20.	x	90



In the figure above, DE is parallel to AC.

$BE = 2EC$

$DE = 12$

	Quantity A	Quantity B
21.	AC	18

Two sides of a triangle are 8 and 9 long.

	Quantity A	Quantity B
22.	The length of the third side of the triangle	$\sqrt{290}$

---

23. What is the area of an equilateral triangle with side length 6?

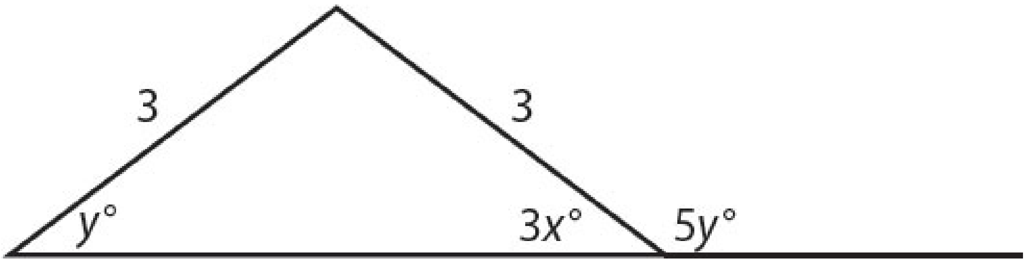
(A) $4\sqrt{3}$

(B) $6\sqrt{2}$

(C) $6\sqrt{3}$

(D) $9\sqrt{2}$

(E) $9\sqrt{3}$



24. What is the value of x in the figure above?

(A) 5

(B) 10

(C) 18

(D) 30

(E) 54

25. An isosceles right triangle has an area of 50. What is the length of the hypotenuse?

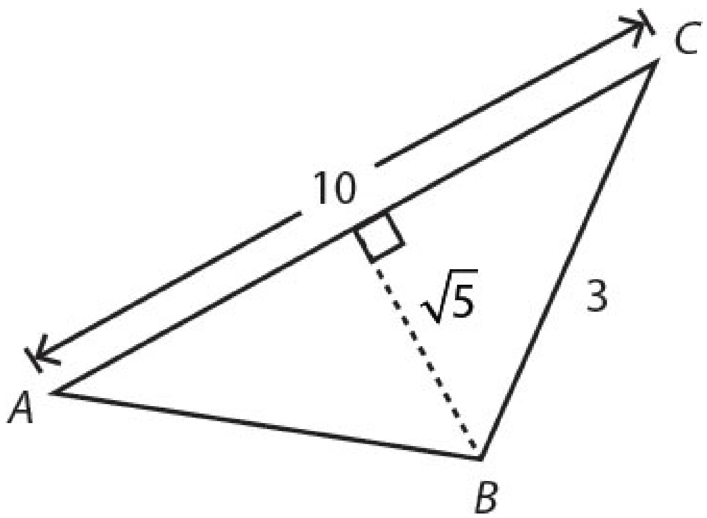
(A) 5

(B) $5\sqrt{2}$

(C) $5\sqrt{3}$

(D) 10

(E) $10\sqrt{2}$



26. In the figure above, what is the length of side *AB*?

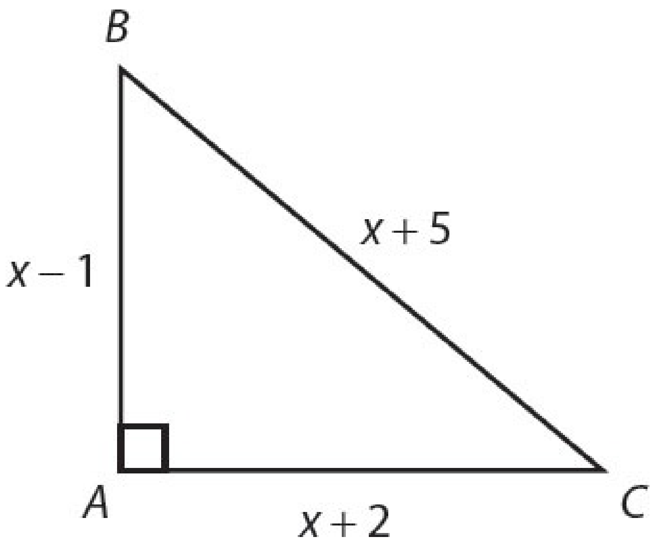
(A) 5

(B) $\sqrt{30}$

(C) $5\sqrt{2}$

(D) 8

(E) $\sqrt{69}$



27. In the right triangle above, what is the length of AC?

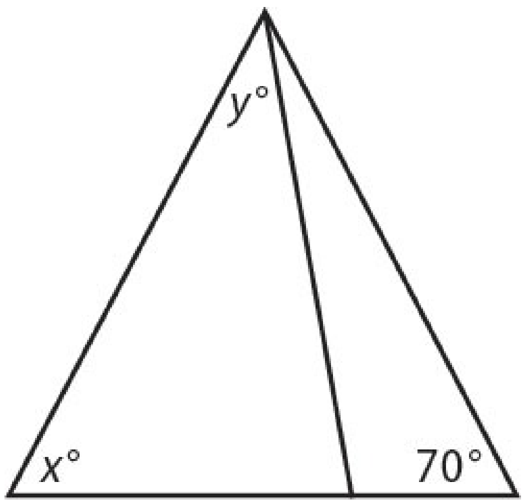
(A) 9

(B) 10

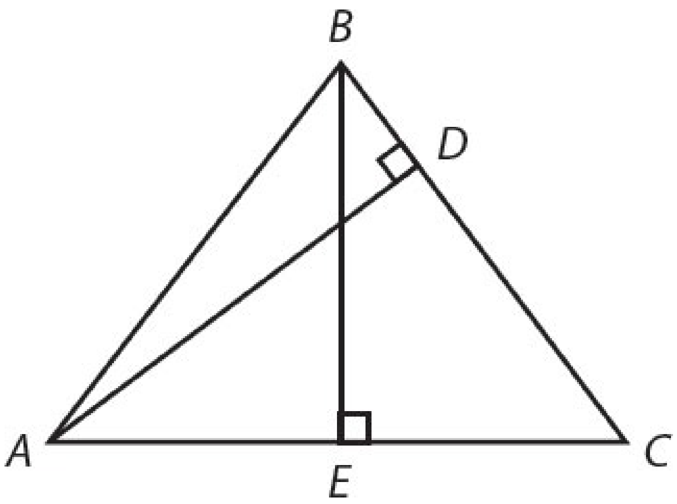
(C) 12

(D) 13

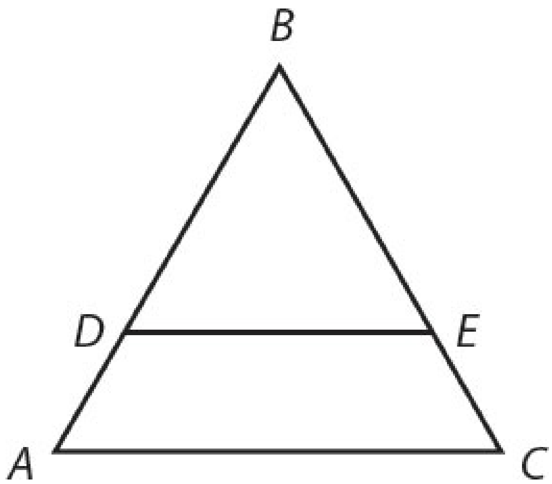
(E) 15



	Quantity A	**Quantity B**
28.	$x + y$	110



	Quantity A	**Quantity B**
29.	The product of *BE* and *AC*	The product of *BC* and *AD*



30. In the figure above, *DE* and *AC* are parallel lines. If *AC* = 12, *DE* = 8, and *AD* = 2, what is the length of *AB*?

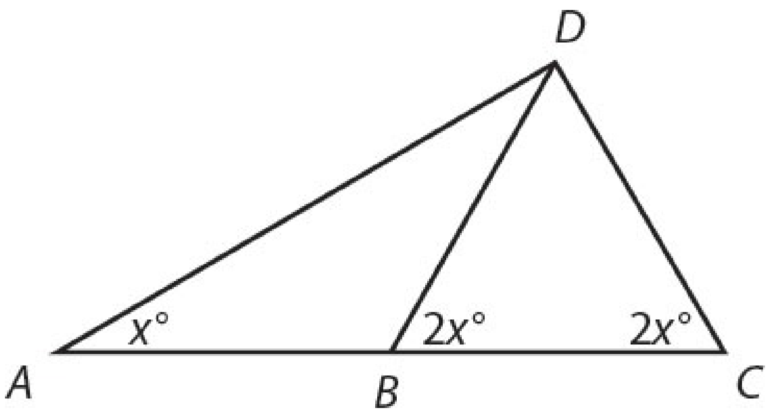
(A) 2

(B) 3

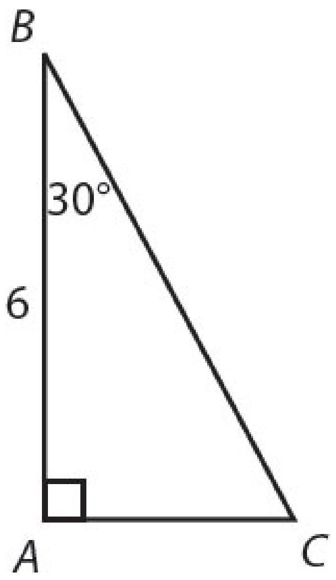
(C) 4

(D) 5

(E) 6



	Quantity A	**Quantity B**
31.	*DC*	*AB*



32. What is the perimeter of right triangle *ABC* above?

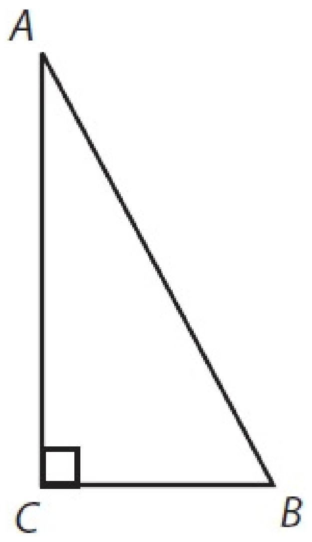
(A) $6 + 4\sqrt{3}$

(B) $6 + 6\sqrt{3}$

(C) $6 + 8\sqrt{3}$

(D) $9 + 6\sqrt{3}$

(E) $18 + 6\sqrt{3}$



33. Triangle *ABC* has an area of 9. If *AC* is three times as long as *CB*, what is the length of *AB*?

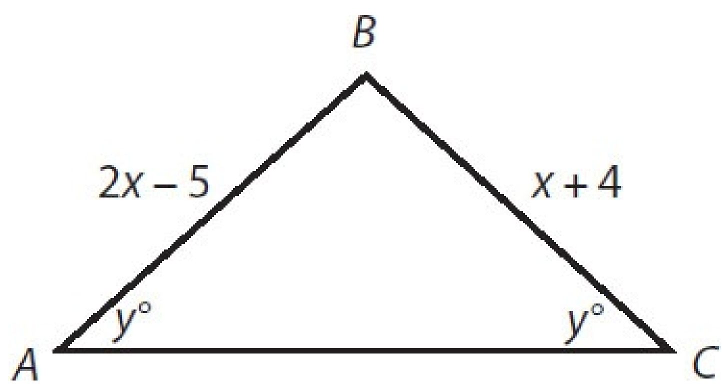
(A) 6

(B) $3\sqrt{6}$

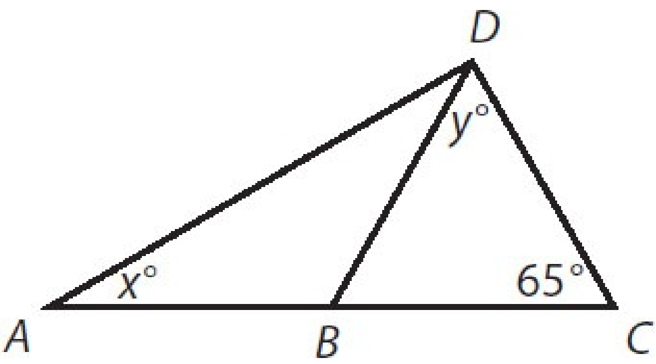
(C) $2\sqrt{15}$

(D) $4\sqrt{15}$

(E) 15

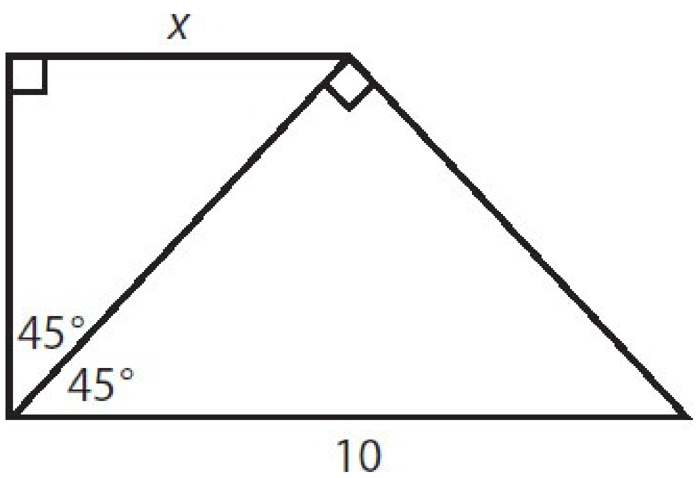


	Quantity A	**Quantity B**
34.	*CB*	7



In the figure above, side lengths *AB*, *BD*, and *DC* are all equal.

	Quantity A	**Quantity B**
35.	x	y



36. In the figure above, what is the value of *x*?

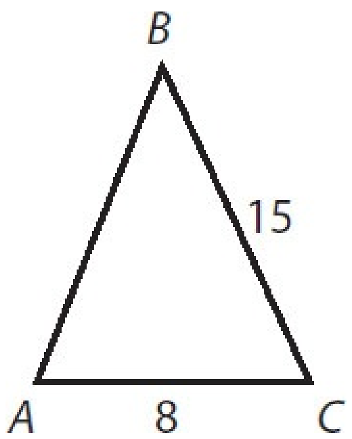
(A) 2.5

(B) $\frac{5}{\sqrt{2}}$

(C) 5

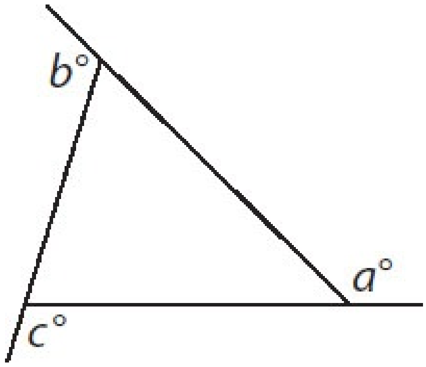
(D) $5\sqrt{2}$

(E) $\frac{10}{\sqrt{2}}$

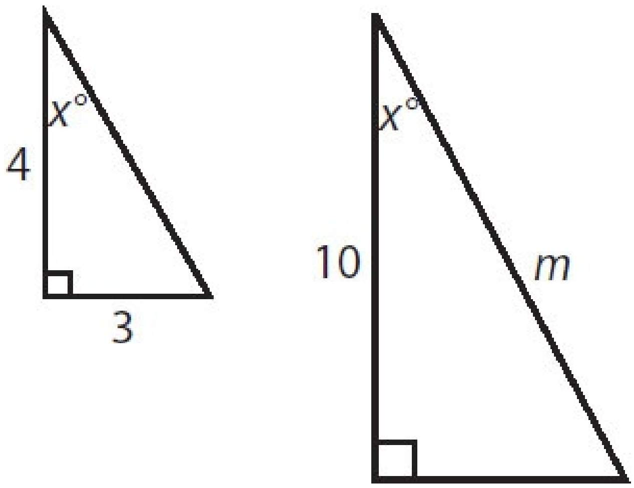


37. Which of the following statements, considered independently, provide sufficient information to calculate the area of triangle *ABC*?

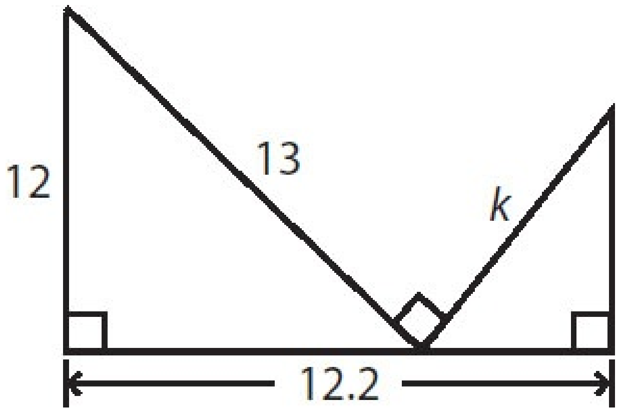
- ☐ Angle *ACB* equals 90°
- ☐ *AB* = 17
- ☐ *ABC* is a right triangle



	Quantity A	**Quantity B**
38.	$a + b + c$	180



	Quantity A	**Quantity B**
39.	m	15

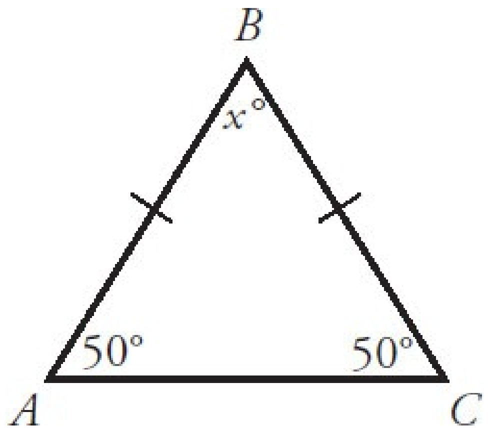


40. What is the length of hypotenuse *k*?

Triangles Answers

1. **(B).** The area of a triangle is equal to $\frac{bh}{2}$. Base and height must always be perpendicular. Use 8 as the base and 6 as the height: $A = \frac{(8)(6)}{2} = 24$.

2. **80.** If two of the angles in a triangle are known, the third can be found because all three angles must sum to 180°. In triangle *ABC*, sides *AB* and *BC* are equal. That means their opposite angles are also equal, so angle *ACB* is 50°.



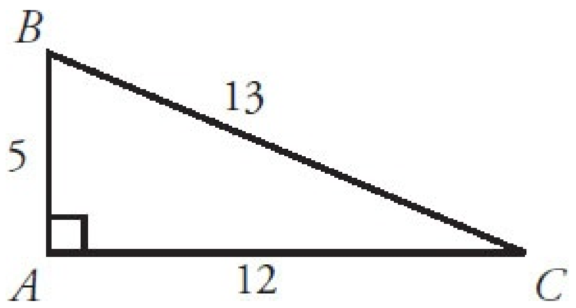
Because $50 + 50 + x = 180$, $x = 80$.

3. **(C).** The three angles in a triangle must sum to 180°, so $3a = 180$ and $a = 60$ (the triangle is equilateral). The four angles in a quadrilateral must sum to 360°, so $4b = 360$ and $b = 90$ (the angles are right angles, so the figure is a rectangle).

Substitute the values of *a* and *b* into Quantity A to get $2(60) + 90 = 120 + 90 = 210$. Likewise, substitute into Quantity B to get $3(60) + \frac{90}{3} = 180 + 30 = 210$. The two quantities are equal.

4. **30.** To find the area of a triangle, a base and height are needed. If the length of *AB* can be determined, then *AB* can be the height and *AC* can be the base, because the two sides are perpendicular to each other.

Use the Pythagorean theorem to find the length of side *AB*: $(a)^2 + (12)^2 = (13)^2$, so $a^2 + 144 = 169$, which means that $a^2 = 25$, and finally $a = 5$. Alternatively, recognize that the triangle is a Pythagorean triple 5–12–13.

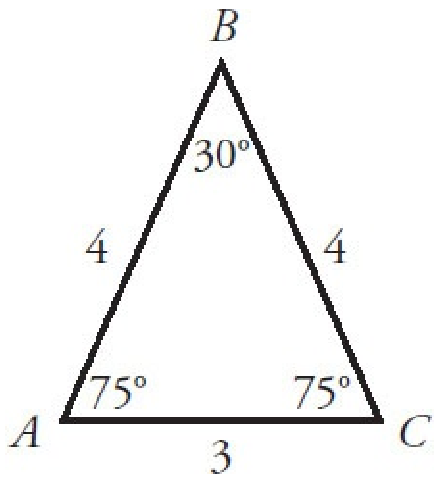


Base and height are known, so Area = $\frac{(12)(5)}{2}$ = 30.

5. **11.** To find the perimeter of triangle *ABC*, sum the lengths of all three sides. Side *BC* is not labeled, so inferences must be made from the given in the question.

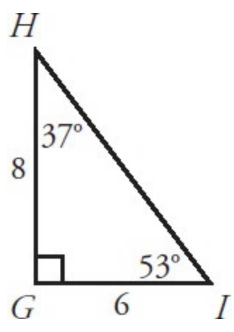
Given the degree measures of two of the angles in triangle *ABC*, the degree measure of the third can be determined. If the third angle is x°, then 30 + 75 + x = 180 and therefore x = 75.

Angle *BAC* and angle *BCA* are both 75°, which means triangle *ABC* is an isosceles triangle. If those two angles are equal, their opposite sides are also equal. Side *AB* has a length of 4, so *BC* also has a length of 4:



To find the perimeter, sum the lengths of the three sides: 4 + 4 + 3 = 11.

6. **10.** Side *HI* is not labeled, so inferences will have to be drawn from other information provided in the figure. Two of the angles of triangle *GHI* are labeled, so if the third angle is x°, then 37 + 53 + x = 180. That means x = 90, and the triangle really looks like this:



You should definitely redraw once you discover the triangle is a right triangle!

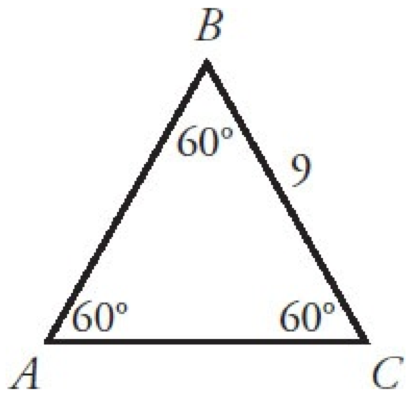
Now you can use the Pythagorean theorem to find the length of *HI*. *HI* is the hypotenuse, so $(6)^2 + (8)^2 = c^2$, which means $36 + 64 = 100 = c^2$, so $c = 10$. The length of *HI* is 10.

Alternatively, recognize the Pythagorean triple: triangle *GHI* is a 6–8–10 triangle.

7. **(C).** All isosceles right triangles (or 45–45–90 triangles) have sides in the ratio of 1 : 1 : $\sqrt{2}$. Thus, an isosceles right triangle with hypotenuse $8\sqrt{2}$ has sides of 8, 8, and $8\sqrt{2}$. Use the two legs of 8 as base and height of the triangle in the formula for area:

$$A = \frac{bh}{2} = \frac{(8)(8)}{2} = 32$$

8. **(B).** To determine which triangle has the greater perimeter, find all three side lengths of both triangles. Begin with triangle *ABC,* in which two of the angles are labeled, so the third can be calculated. If the unknown angle is x°, then 60 + 60 + x = 180 and, therefore, x = 60.



All three angles in triangle *ABC* are 60°. If all three angles are equal, all three sides are equal and every side of triangle *ABC* has a length of 9. The perimeter of *ABC* is 9 + 9 + 9 = 27.

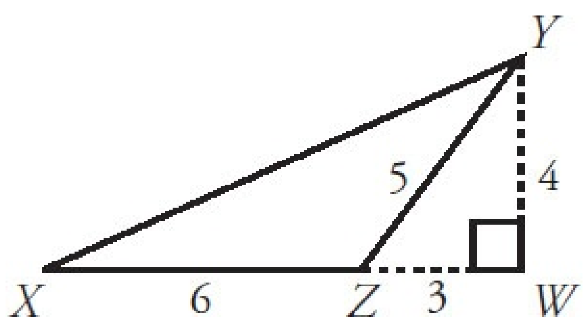
Now look at triangle *DEF,* which is a right triangle. Use the Pythagorean theorem to find the length of side *EF,* which is the hypotenuse, so $(5)^2 + (12)^2 = c^2$, which means $25 + 144 = 169 = c^2$ and, therefore, $c = 13$. The perimeter of *DEF* is 5 + 12 + 13 = 30. Alternatively, 5–12–13 is a Pythagorean triple.

Because 30 > 27, triangle *DEF* has a greater perimeter than triangle *ABC.* Quantity B is greater.

9. **12.** Start by redrawing the figure, filling in all the information given in the text. To find the area of triangle *XYZ,* a base and a height are required. If side *XZ* is a base, then *YW* can act as a height, as these two are perpendicular.

Because triangle *ZYW* is a right triangle with two known sides, the third can be determined using the Pythagorean theorem: *ZY* is the hypotenuse, so $(a)^2 + (3)^2 = (5)^2$, meaning that $a^2 + 9 = 25$ and $a^2 = 16$, so $a = 4$.

Alternatively, recognize the Pythagorean triple: *ZYW* is a 3–4–5 triangle:



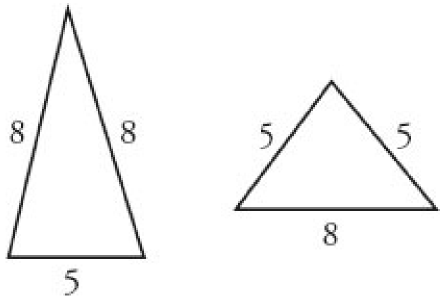
The area of triangle *XYZ* is $\frac{bh}{2} = \frac{(6)(4)}{2} = 12$.

10. **(D).** The problem indicates that $x + z = 110$. Since the angles of a triangle must sum to 180°, $x + y + z = 180$. Substitute 110 for $x + z$ on the left side:

$$y + 110 = 180$$
$$y = 70$$

The problem compares *x* and *y*. Although *y* is known, the exact value of *x* is not known, only that it must be greater than 0 and less than 110. The relationship cannot be determined from the information given.

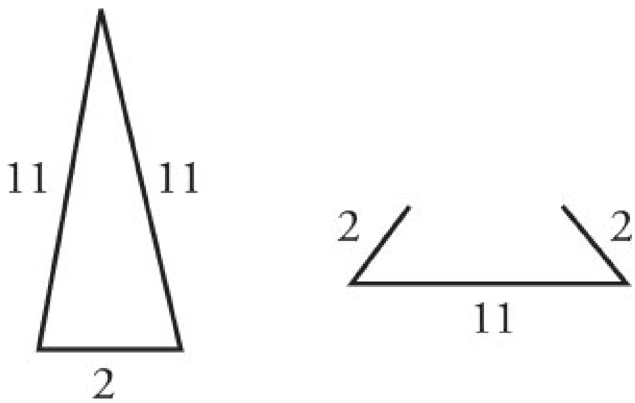
11. **(D).** An isosceles triangle has two equal sides, so this triangle must have a third side of either 8 or 5. Use the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to check whether both options are actually possible.



Since 8 – 5 = 3 and 8 + 5 = 13, the third side has to be greater than 3 and less than 13. Therefore, that side could indeed be either 5 or 8. The two quantities could be equal, or Quantity A could be less than Quantity B, so the relationship cannot be determined from the information given.

12. **(C).** An isosceles triangle has two equal sides, so this triangle must have a third side of either 2 or 11. Because one side is so long and the other so short, it is worth testing via the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to see whether both possibilities are really possible.

From the Third Side Rule, a triangle with sides of 2 and 11 must have a third side greater than 11 – 2 = 9 and less than 11 + 2 = 13. Since 2 is not between 9 and 13, it is just not possible to have a triangle with sides of length 2, 2, and 11. However, a 2–11–11 triangle is possible. So the third side must be 11.



The two quantities are equal.

13. **(A).** Within any triangle, the following is true: the larger the angle, the longer the side opposite that angle.

The side opposite the 45° angle (*AC*) must be longer than the side opposite the 37° angle (BC): $AC > BC$. Quantity A is greater.

14. **(C).** By definition, the exterior angle d is equal to the sum of the two opposite interior angles. Thus, $d = a + b$.

Alternatively, label the interior angle at vertex C as c°. The sum of the angles in a triangle is 180°, so $a + b + c = 180$. The sum of angles that form a line is also 180, so $c + d = 180$, or $c = 180 – d$. Substitute into the first equation:

$$a + b + c = 180$$

$$a + b + (180 – d) = 180$$

$$a + b – d = 0$$

$$a + b = d$$

(This, incidentally, is the proof of the rule stated in the first line of this explanation.)

15. **(A).** Putting the constraints together, $60 < y < z$. That means that $y + z > 120$, leaving less than 60° for the remaining angle x. The angles can now be ordered by size: $x < y < z$.

The shortest side is across from the smallest angle, which is x, so the shortest side must be *BC*. The median length side is across from the median angle, which is y, so the median length side must be *AC*. Since none of the angles are equal, none of the sides are equal, and the length of *AC* is greater than the length of *BC*. Quantity A is greater.

16. **(C).** The area of a triangle is equal to $\frac{bh}{2}$. The two triangles have equal bases, since $XY = YZ$. They also have the same height, since they both have the same height as the larger triangle *XWZ*. The two quantities are equal.

17. **(C).** Do not *assume* that $x = 90$. Instead, since all three side lengths are labeled, *test* whether the triangle is a right triangle by plugging into the Pythagorean theorem and seeing whether the result is a true statement:

$$5^2 + 7^2 = (\sqrt{74})^2$$
$$25 + 49 = 74$$
$$74 = 74$$

Since 74 equals 74, the Pythagorean theorem does apply to this triangle. So the triangle is a right triangle. Notice also that the side across from x was used as the hypotenuse. It must be that $x = 90$. The two quantities are equal.

18. **(C).** From the Third Side Rule, any side of a triangle must be greater than the difference of the other two sides and less than their sum. Since $9 - 7 = 2$ and $9 + 7 = 16$, the unknown third side must be between 2 and 16, not inclusive. To get the lower boundary for the perimeter, add the lower boundary of the third side to the other two sides: $2 + 7 + 9 = 18$. To get the upper boundary for the perimeter, add the upper boundary for the third side to the other two sides: $16 + 7 + 9 = 32$. Thus, p must be between 18 and 32, not inclusive—in other words, $18 < p < 32$.

19. **(B).** You may have memorized the 3–4–5 Pythagorean triple, of which 9–12–15 is a multiple. This question is trying to exploit this—don't be tricked into thinking that $y = 15$. In a 9–12–15 triangle, 15 would have to be the hypotenuse. In any right triangle, the hypotenuse must be the longest side.

Since the given triangle has 12 as the hypotenuse, the leg of length y must be less than 12, and thus less than 15. At this point, it is safe to choose (B). Although unnecessary, to get the actual value of y, apply the Pythagorean theorem:

$$9^2 + y^2 = 12^2$$

$$81 + y^2 = 144$$

$$y^2 = 63$$

So y is a little less than 8, which is definitely less than 15. Quantity B is greater.

20. **(A).** One good approach here is to test the value in Quantity B. If angle x equals 90°, then this is a right triangle. Use the legs of 6 and 8 to find the hypotenuse using the Pythagorean theorem: $6^2 + 8^2 = c^2$ will tell you that c equals 10 in this case. (Or, memorize the 6–8–10 multiple of the 3–4–5 Pythagorean triple, since it appears often on the GRE.) Since the hypotenuse is slightly longer than 10, the angle across from the "hypotenuse" must actually be slightly larger than 90°. Therefore, x is greater than 90.

21. **(C).** If AC is parallel to DE, then triangles DBE and ABC are similar. If $BE = 2EC$ then if EC is set to equal x, BE would equal $2x$ and BC would equal $x + 2x$, or $3x$. That means that the big triangle is in a 3 : 2 ratio with the small triangle (since $BC : BE = 3x : 2x$).

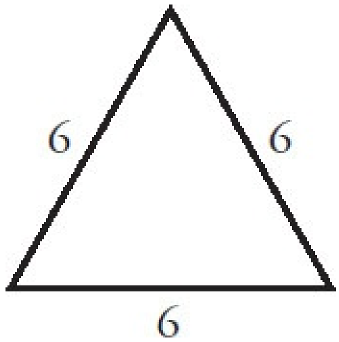
Set up the proportion for the bottom sides of these triangles, both of which are opposite the shared vertex at B:

$$\begin{aligned} \frac{2}{3} &= \frac{DE}{AC} \\ \frac{2}{3} &= \frac{12}{AC} \\ 2(AC) &= 36 \\ AC &= 18 \end{aligned}$$

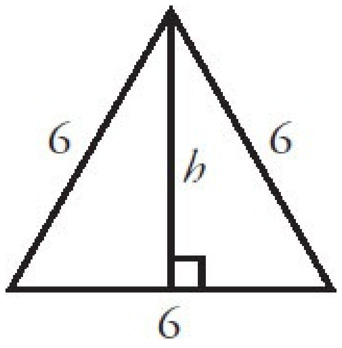
The two quantities are equal.

22. **(B).** From the Third Side Rule, a triangle with sides of 8 and 9 must a have a third side greater than $9 - 8 = 1$ and less than $8 + 9 = 17$. Since 17^2 is 289, which is less than 290, the measure of the third side is definitely less than $\sqrt{290}$. Quantity B is greater.

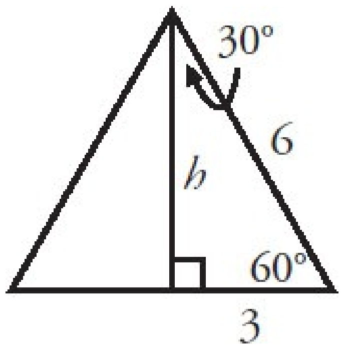
23. **(E).** An equilateral triangle with side length 6 can be drawn as:



In order to find the area, recall that the area of a triangle is $A = \frac{bh}{2}$. The base of the triangle is already known to be 6, so find the height in order to solve for area. The height is the straight line from the highest point on the triangle dropped down perpendicular to the base:



The angle opposite *h* must be 60°, since it is one of the three angles of the original equilateral triangle. Thus, the triangle formed by *h* is a 30–60–90 triangle as shown below.



Using the properties of 30–60–90 triangles, *h* is equal to the shortest side multiplied by $\sqrt{3}$. Thus, $h = 3\sqrt{3}$ and the area is:

$$A = \frac{bh}{2} = \frac{6 \times 3\sqrt{3}}{2} = 9\sqrt{3}$$

24. **(B).** Since there are two unknowns, look for two equations to solve. The first equation comes from the fact that $3x$ and $5y$ make a straight line, so they must sum to 180:

$$3x + 5y = 180$$

A triangle with at least two sides of equal length is called an isosceles triangle. In such a triangle, the angles opposite the two equal sides are themselves equal in measure. In this case, the two sides with length 3 are equal, so the angles opposite them (y and $3x$) must also be equal:

$$y = 3x$$

Substitute for *y* in the first equation:

$$\begin{aligned} 3x + 5(3x) &= 180 \\ 3x + 15x &= 180 \\ 18x &= 180 \\ x &= 10 \end{aligned}$$

25. **(E).** If the area of the triangle is 50, then $\frac{bh}{2} = 50$. In an isosceles right triangle, the base and height are the two perpendicular legs, which have equal length. Since base = height, substitute another *b* in for *h*:

$$\frac{b^2}{2} = 50$$

$$b^2 = 100$$

$$b = 10$$

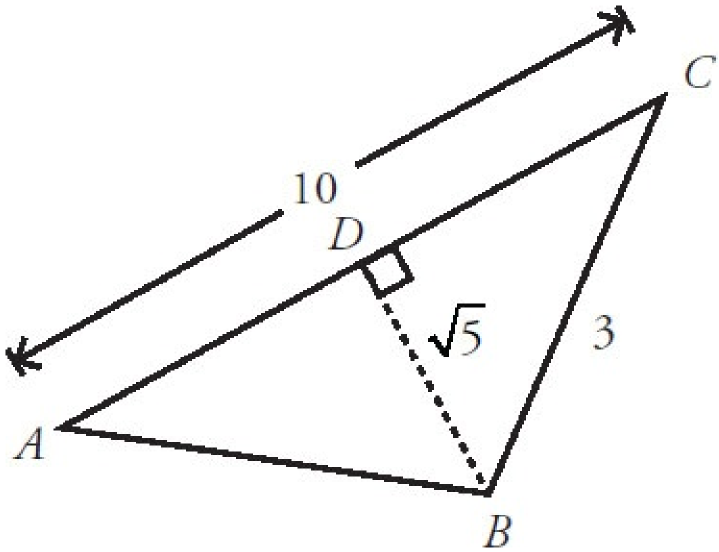
An isosceles right triangle follows the 45–45–90 triangle formula, so the hypotenuse is $10\sqrt{2}$.

Alternatively, use the Pythagorean theorem to find the hypotenuse:

$$10^2 + 10^2 = c^2$$
$$200 = c^2$$

Thus, $c = \sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$.

26. **(E).** For convenience, put the letter *D* on the point at the right angle between *A* and *C*, as shown:



Solve this multi-step problem by working backwards from the goal. To find the length of *AB*, use the Pythagorean theorem on triangle *ADB*, since angle *ADB* must be a right angle. In order to use the Pythagorean theorem, find the lengths of the two legs. *BD* is known, so *AD* needs to be determined. Since *AD* and *DC* sum to a line segment of length 10, *AD* = 10 – *DC*.

Finally, to find *DC*, apply the Pythagorean theorem to triangle *BDC*:

$$(\sqrt{5})^2 + (DC)^2 = 3^2$$
$$5 + (DC)^2 = 9$$
$$(DC)^2 = 4$$
$$DC = 2$$

Therefore, *AD* = 10 – *DC* = 10 – 2 = 8. Now apply the Pythagorean theorem to *ADB*:

$(\sqrt{5})^2 + 8^2 = (AB)^2$
$5 + 64 = (AB)^2$
$69 = (AB)^2$
$AB = \sqrt{69}$

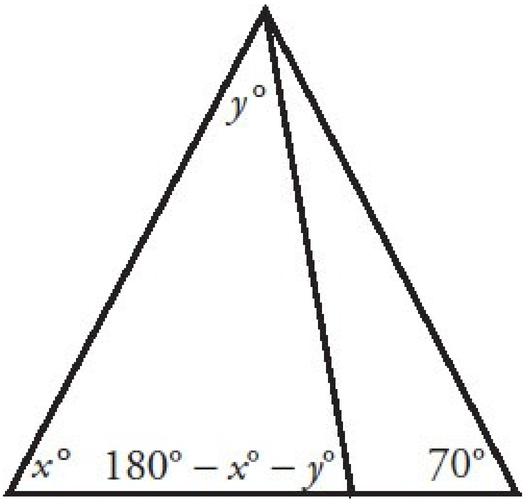
27. **(C).** Because this is a right triangle, the Pythagorean theorem applies to the lengths of the sides. The Pythagorean theorem states that $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the legs of a right triangle. Plug the expressions into the theorem and simplify:

$(x - 1)^2 + (x + 2)^2 = (x + 5)^2$
$(x^2 - 2x + 1) + (x^2 + 4x + 4) = x^2 + 10x + 25$
$2x^2 + 2x + 5 = x^2 + 10x + 25$
$x^2 - 8x - 20 = 0$
$(x - 10)(x + 2) = 0$
$x = 10$ or $x = -2$

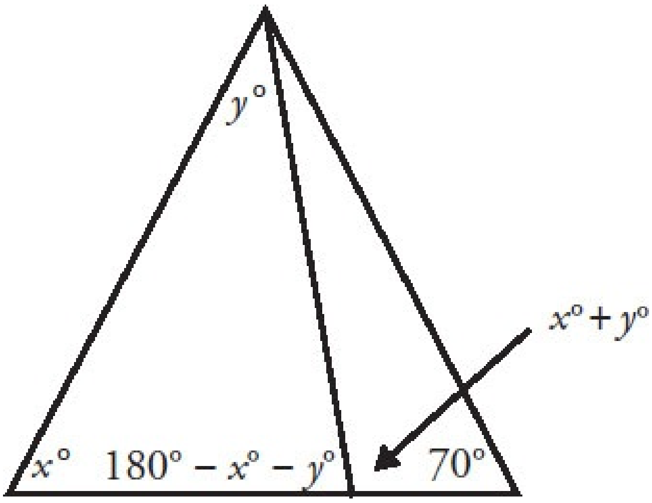
However, $x = –2$ is not an option; side lengths can't be negative. So x must equal 10. This is *not* the final answer, however. The question asks for side length AC:

$$AC = x + 2 = 10 + 2 = 12$$

28. **(B).** The question compares $x + y$ with 110. To do so, fill in the missing angles on the triangles. In the triangle on the left, all three angles must sum to 180°. Therefore, the missing angle must be $(180 – x – y)$, as shown here:



Now consider the angle next to the one you just solved for. These two angles sum to 180°, forming a straight line. So the adjacent angle must be $x + y$:



Alternatively, notice also that $x + y$ is the exterior angle to the triangle on the left, so it must be the sum of the two opposite interior angles (namely, x and y).

Now, the three angles of a triangle must sum to 180°, and no angle can equal 0. So any *two* angles in a triangle must sum to *less* than 180°. Consider the triangle on the right side, which contains angles of $x + y$ and 70°. Their sum is less than 180°:

$$(x + y) + 70 < 180$$

Subtract 70 from both sides:

$$x + y < 110$$

Quantity B is greater.

29. **(C).** First determine how the quantities relate to the triangle. For instance, examine Quantity A, the product of *BE* and *AC*. Notice that *BE* is the height of the largest triangle *ABC*, while *AC* is the base. This product should remind you of the formula for area: $A = \frac{bh}{2}$.

Plug in $b = AC$ and $h = BE$, then move the 2:

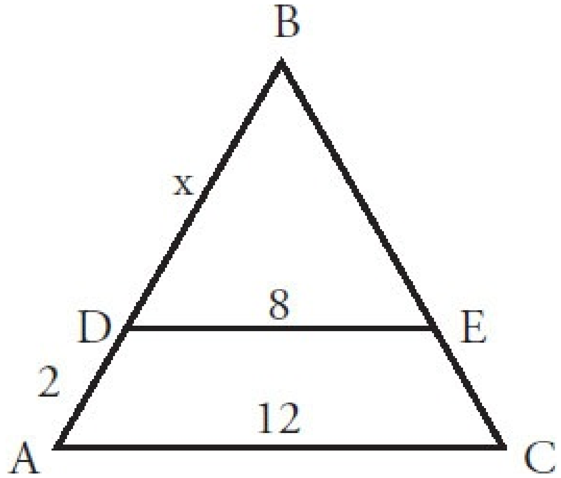
$$2 \times \text{Area} = (AC)(BE)$$

What about Quantity B, the product of*BC* and *AD*? If you consider *BC* the base, then *AD* is the height to that base. So you can put $b = BC$ and $h = AD$ into the area formula, as before:

$$2 \times \text{Area} = (BC)(AD)$$

Both Quantity A and Quantity B are twice the area of the big triangle *ABC*. The two quantities are equal.

30. **(E).** If *DE* and *AC* are parallel lines, triangles *ABC* and *DBE* are similar. That means that there is a fixed ratio between corresponding sides of the two triangles. Since $AC = 12$ and $DE = 8$, that ratio is 12 : 8 or 3 : 2. This means that each side of triangle *ABC* (the larger triangle) is 1.5 times the corresponding side of triangle *DBE* (the smaller triangle), as shown below:



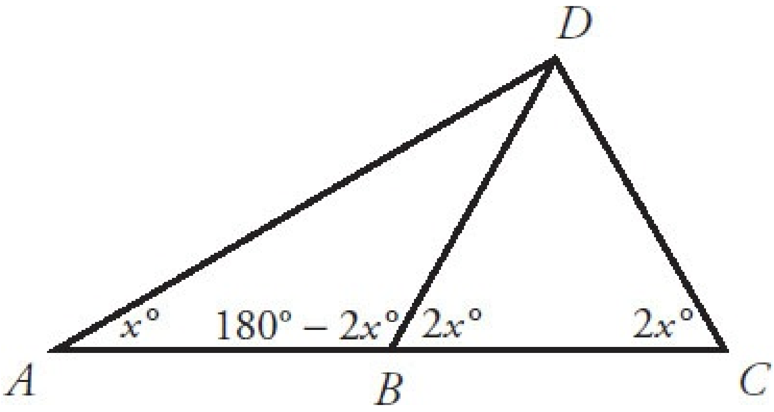
Assign a value of *x* to line *DB*, in order to set up the following proportion:

$$\frac{AB}{DB} = \frac{3}{2} = \frac{2+x}{x}$$

$3x = 2(2 + x)$
$3x = 4 + 2x$
$x = 4$

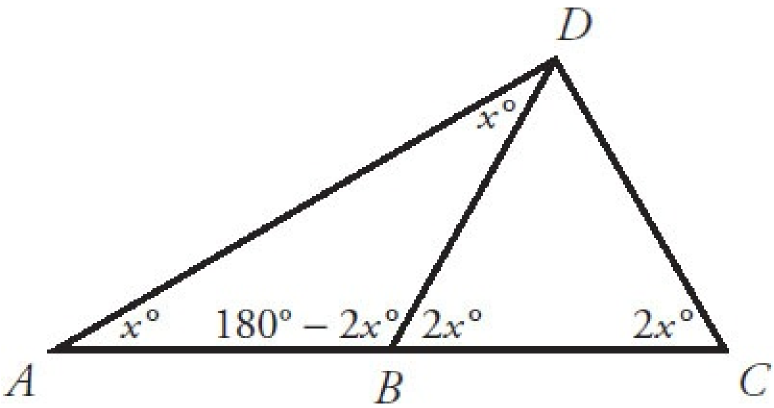
Since the question is asking for the value of *AB*, the answer is $2 + 4 = 6$.

31. **(C).** To compare *DC* and *AB*, first solve for the unlabeled angles in the diagram. The two angles at point *B* make a straight line, so they sum to 180°, and the unlabeled angle is 180 – 2*x*, as shown:



Now ensure that the angles of triangle *ADB* on the left sum to 180°. The top vertex of triangle *ADB* must measure 180 – *x* – (180 – 2*x*) = 180 – *x* – 180 + 2*x* = *x*.

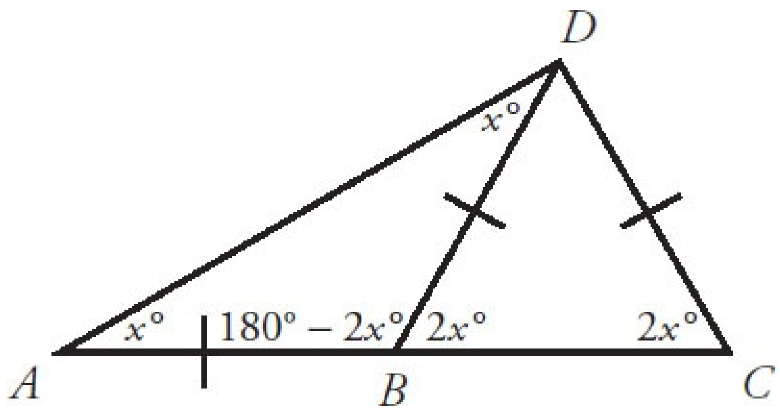
Therefore, the figure becomes:



Alternatively, notice that angle *DBC* (equal to 2*x*) is the exterior angle to the triangle on the left, and so it equals the sum of the two opposite interior angles in that triangle on the left. One of those angles is *x*, so the other one must be *x* as well.

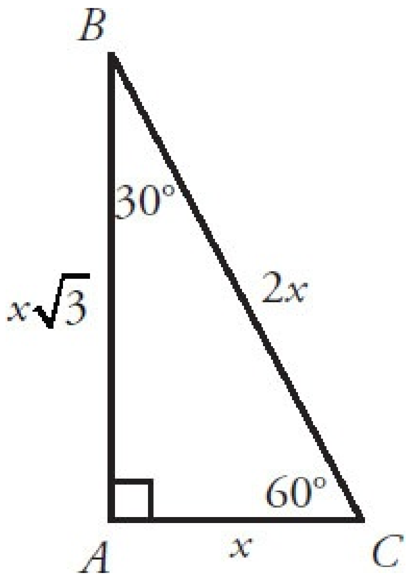
Now apply the properties of isosceles triangles. The two angles labeled *x* are equal, so the triangle that contains them (triangle *ADB*) is isosceles, and the sides opposite those equal angles are also equal. Put a slash through those sides (*AB* and *BD*) to mark them as the same length.

Likewise, the two angles labeled 2*x* are equal, so the triangle that contains them (triangle *BDC*) is isosceles, and the sides opposite those angles (*BD* and *DC*) are equal. Add one more slash through *DC* in the figure:



Thus, sides *AB* and *DC* have the same length. The two quantities are equal.

32. **(B).** To compute the perimeter of this triangle, sum the lengths of all three sides. Because one angle is labeled as a right angle and another as 30°, right triangle *ABC* is a 30–60–90 triangle. For any 30–60–90 triangle, the sides are in these proportions:



Match up this universal 30–60–90 triangle to the given triangle, in order to find *x* in this particular case. The only labeled side in the given triangle (6) matches the $x\sqrt{3}$ side in the universal triangle (they're both opposite the 60° angle), so set them equal to each other:

$$6 = x\sqrt{3}$$

$$x = \frac{6}{\sqrt{3}}$$

Rationalize the denominator by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$ (which does not change the value of *x*, as $\frac{\sqrt{3}}{\sqrt{3}}$ is just a form of 1):

$$x = \frac{6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$x = \frac{6\sqrt{3}}{3}$$

$x = 2\sqrt{3}$

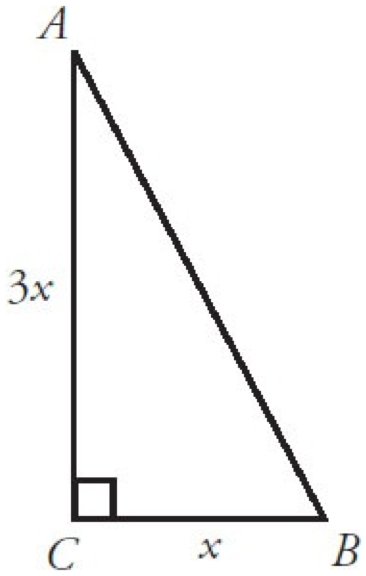
Now figure out all the sides in the given triangle. The length of side *AC* is $x = 2\sqrt{3}$, the length of side *AB* is given as 6, and the length of side *BC* is $2x = 2(2\sqrt{3}) = 4\sqrt{3}$.

Finally, sum all the sides to get the perimeter:

Perimeter = $6 + 2\sqrt{3} + 4\sqrt{3}$

Perimeter = $6 + 6\sqrt{3}$

33. **(C).** Draw a diagram and label the sides of the triangle with the information given. Since *AC* is three times as long as *CB*, label *CB* as *x* and *AC* as 3*x*, as shown:



Use the area of the triangle, which is given as 9, to find the base *x* and the height 3*x*. The formula for area is $A = \frac{bh}{2}$. Plug in and solve for *x*:

$$9 = \frac{x(3x)}{2} = \frac{3x^2}{2}$$

$18 = 3x^2$

$6 = x^2$

$\sqrt{6} = x$

So $CB = \sqrt{6}$ and $AC = 3\sqrt{6}$. Use the Pythagorean theorem to find *AB*:

$(AB)^2 = (\sqrt{6})^2 + (3\sqrt{6})^2$

$(AB)^2 = 6 + 54 = 60$

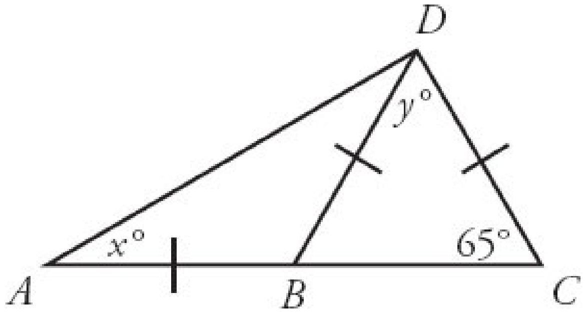
$AB = \sqrt{60} = 2\sqrt{15}$

34. **(A).** Since the bottom left and bottom right angles are both equal to $y°$, the triangle is isosceles, and the sides opposite those angles (*AB* and *BC*) must also be equal:

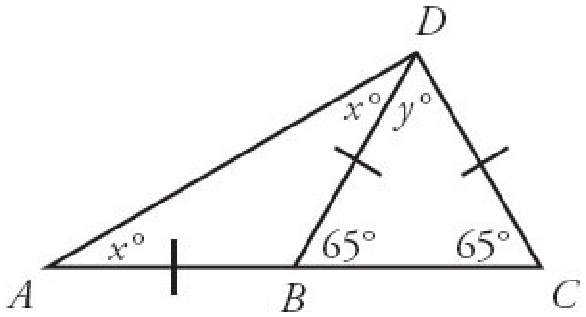
$2x - 5 = x + 4$
$x - 5 = 4$
$x = 9$

BC is therefore equal to 9 + 4 = 13. Quantity A is greater.

35. **(B).** Redraw the figure and label the equal sides:



The small triangle on the left (*ADB*) is isosceles as it has two sides of equal length. Likewise for the small triangle on the right (*BDC*) within it. In an isosceles triangle, the angles opposite the two equal sides are themselves equal in measure. Accordingly, label more angles on the figure:



The three angles in the triangle on the right must sum to 180°:

$$65 + 65 + y = 180$$
$$130 + y = 180$$
$$y = 50$$

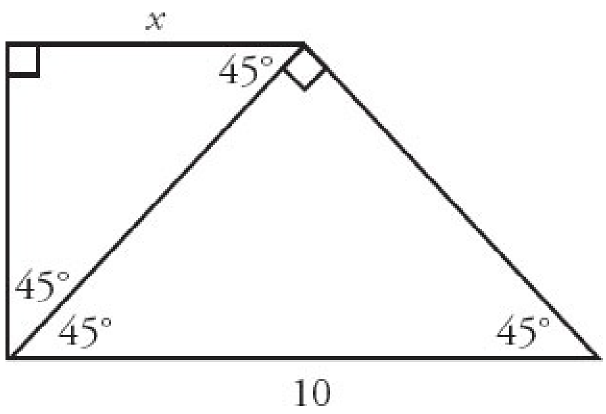
The two angles at point *B* make a straight line, so they sum to 180°. So the unlabeled angle must be 180 – 65 = 115°.

Finally, the three angles in the triangle on the left must sum to 180°:

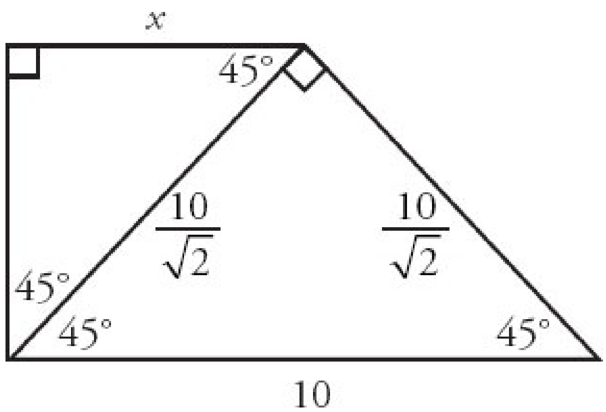
$x + x + 115 = 180$
$2x = 65$
$x = 32.5$

So *y* is greater than *x*. Quantity B is greater.

36. **(C).** Redraw the figure and label all angles, applying the rule that the angles in a triangle sum to 180°:



These are two separate 45–45–90 triangles. In a 45–45–90 triangle, the sides are in a $1 : 1 : \sqrt{2}$ ratio. Thus, the length of each leg equals the length of the hypotenuse divided by $\sqrt{2}$. The hypotenuse of the larger triangle is 10, so each leg of that triangle is $\frac{10}{\sqrt{2}}$.



The hypotenuse of the smaller triangle is $\frac{10}{\sqrt{2}}$. Divide by $\sqrt{2}$ again according to the 45–45–90 triangle ratio ($1 : 1 : \sqrt{2}$) to see that $x = \frac{10}{\sqrt{2}\sqrt{2}} = \frac{10}{2} = 5$.

37. **Angle *ACB* equals 90° and *AB* = 17 only.** If angle *ACB* = 90°, then 8 and 15 are the base and height, and you can calculate the area. The first statement is sufficient.

If *AB* = 17, you can plug 8, 15, and 17 into the Pythagorean theorem to see whether you get a true statement. Use 17 as the hypotenuse in the Pythagorean theorem because 17 is the longest side:

$8^2 + 15^2 = 17^2$
$64 + 225 = 289$
$289 = 289$

Since this is true, the triangle is a right triangle with the right angle at C. If angle $ACB = 90°$, then 8 and 15 are the base and height, and you can calculate the area. (Since 8–15–17 is a Pythagorean triple, if you had that fact memorized, you could skip the step above.) The second statement is sufficient.

Knowing that ABC is a right triangle (the third statement) is *not* sufficient to calculate the area because it's not specified which angle is the right angle. A triangle with sides of 8 and 15 could have hypotenuse 17, but another scenario is possible: perhaps 15 is the hypotenuse. In this case, the third side is shorter than 15, and the area is smaller than in the 8–15–17 scenario.

38. **(A).** The three interior angles of the triangle sum to 180°. Try an example: say each interior angle is 60°. In that case, a, b, and c would each equal 120° (since two angles that make up a straight line sum 180), and Quantity A would equal 360°.

It is possible to prove this result in general by expressing each interior angle in terms of a, b, and c, and then setting their sum equal to 180°:

$(180 - a) + (180 - b) + (180 - c) = 180$
$540 - a - b - c = 180$
$360 = a + b + c$

Quantity A is greater.

39. **(B).** Since both triangles have a 90° angle and an angle $x°$, the third angle of each is the same as well (because the three angles in each triangle sum to 180°). All the corresponding angles are equal, so the triangles are similar, and the ratio of corresponding sides is constant.

The smaller triangle is a 3–4–5 Pythagorean triple (the missing hypotenuse is 5). Set up a proportion that includes two pairs of corresponding sides. The words "4 is to 10 as 5 is to m" become this equation:

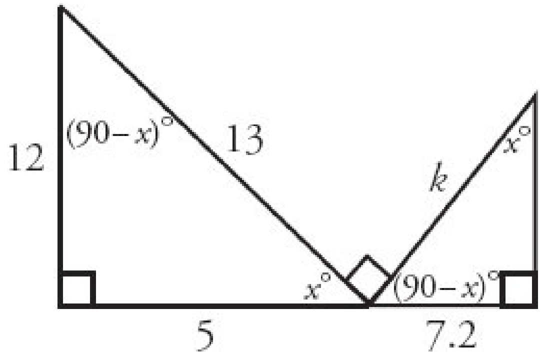
$\frac{4}{10} = \frac{5}{m}$
$4m = 50$
$m = 12.5$

Quantity B is greater.

40. **7.8.** Begin by noting that the triangle on the left is a 5–12–13 Pythagorean triple, so the bottom side is 5. Subtract 12.2 – 5 = 7.2 to get the bottom side of the triangle on the right.

Next, the two unmarked angles that "touch" at the middle must sum to 90°, because they form a straight line together with the right angle of 90° between them, and all three angles must sum to 180°. Mark the angle on the left x. The angle on the right must then be $90 - x$.

Now the other angles that are still unmarked can be labeled in terms of *x*. Using the rule that the angles in a triangle sum to 180°, the angle between 12 and 13 must be 90 – *x*, while the last angle on the right must be *x*, as shown:



Since each triangle has angles of 90, *x*, and 90 – *x*, the triangles are similar. This observation is the key to the problem. Now you can make a proportion, carefully tracking which side corresponds to which. The 7.2 corresponds to 12, since each side is across from angle *x*. Likewise, *k* corresponds to 13, since each side is the hypotenuse. Write the equation and solve for *k*:

$$\frac{7.2}{12} = \frac{k}{13}$$

$$\frac{(13)7.2}{12} = k$$

$$7.8 = k$$

Chapter 28

Coordinate Geometry

In This Chapter…

Coordinate Geometry

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

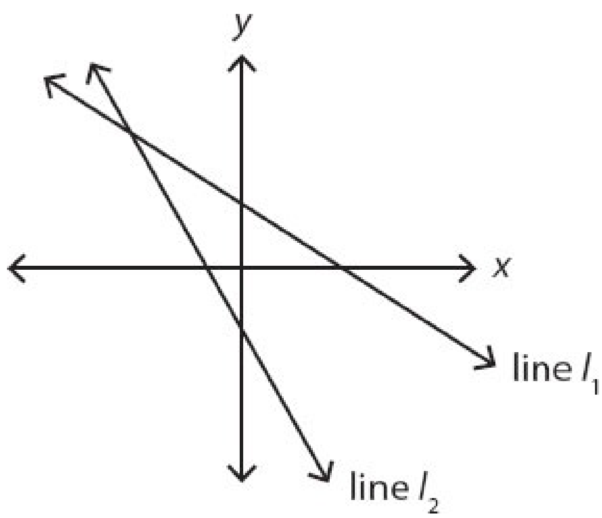
(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

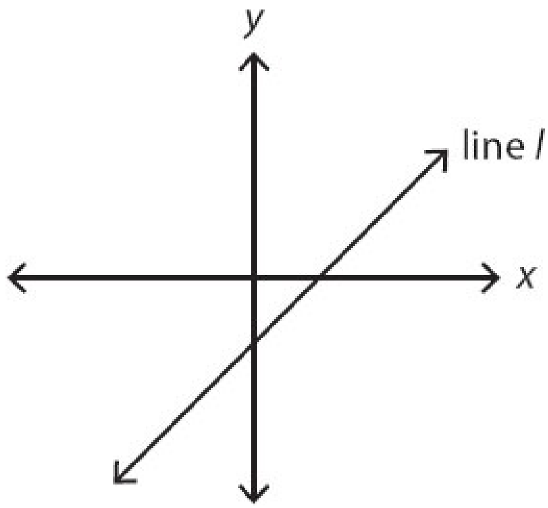
Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

For questions followed by a numeric entry box , you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box , you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.



	Quantity A	**Quantity B**
1.	The slope of line l_1	The slope of line l_2



2. If the figure above is drawn to scale, which of the following could be the equation of line l?

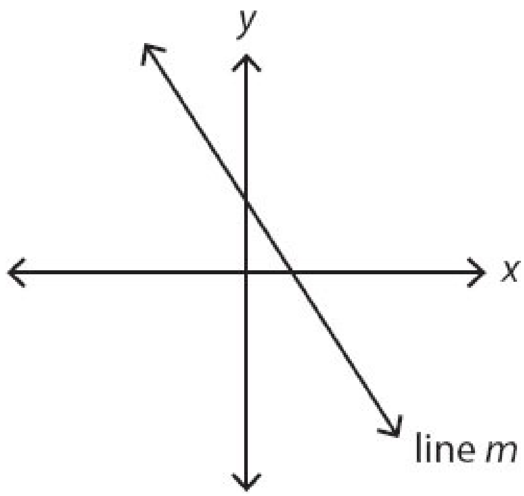
   (A) $y = 4x + 4$

   (B) $y = 4x – 4$

   (C) $y = x – 6$

   (D) $y = x + \frac{1}{2}$

   (E) $y = –x – 3$



3. If the figure above is drawn to scale, which of the following could be the equation of line m?

   (A) $6y + 6x = 7$

   (B) $3y = –4x – 3$

   (C) $5y + 10 = –4x$

(D) $y = 2$

(E) $x = -2$

4. What is the slope of a line that passes through the points (–4, 5) and (1, 2)?

(A) $-\frac{3}{5}$

(B) –1

(C) $-\frac{5}{3}$

(D) $-\frac{7}{3}$

(E) –3

5. Which of the following could be the slope of a line that passes through the point (–2, –3) and crosses the *y*-axis above the origin?

Indicate all such slopes.

☐ $-\frac{2}{3}$

☐ $\frac{3}{7}$

☐ $\frac{3}{2}$

☐ $\frac{5}{3}$

☐ $\frac{9}{4}$

☐ 4

6. If a line has a slope of –2 and passes through the points (4, 9) and (6, *y*), what is the value of *y*?

☐

7. What is the distance between the points (–2, –2) and (4, 6)?

(A) 6

(B) 7

(C) 8

(D) 10

(E) $8\sqrt{2}$

8. Which of the following points in the coordinate plane lies on the line $y = 2x - 8$?

Indicate all such points.

- ☐ (3, –2)
- ☐ (–8, 0)
- ☐ ($\frac{1}{2}$, –7)

9. Which of the following points in the coordinate plane does not lie on the curve $y = x^2 - 3$?

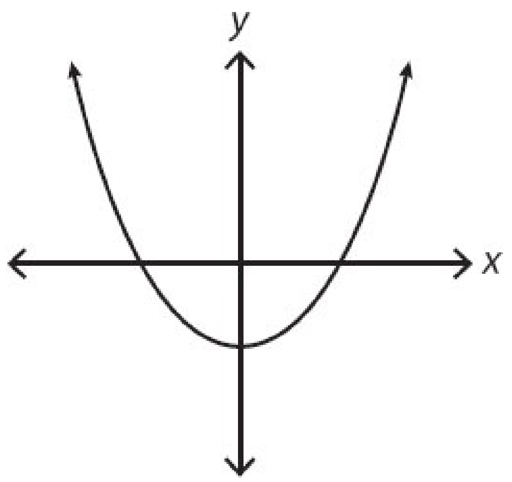
(A) (3, 6)

(B) (–3, 6)

(C) (0, –3)

(D) (–3, 0)

(E) (0.5, –2.75)



10. Which of the following could be the equation of the figure above?

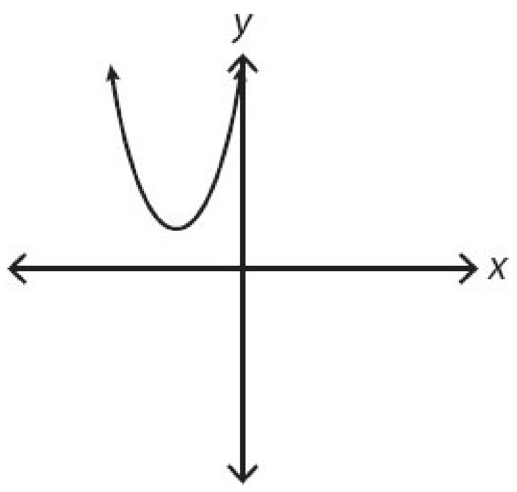
(A) $y = x - 3$

(B) $y = 2x^2 - x$

(C) $y = x^2 - 3$

(D) $y = x^2 + 3$

(E) $y = x^3 - 3$



11. Which of the following could be the equation of the parabola in the coordinate plane above?

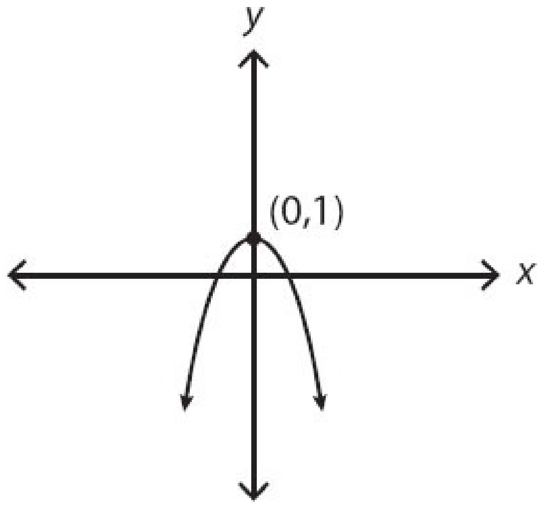
(A) $y = x^2 + 3$

(B) $y = (x - 3)^2 + 3$

(C) $y = (x + 3)^2 - 3$

(D) $y = (x - 3)^2 - 3$

(E) $y = (x + 3)^2 + 3$



12. Which of the following could be the equation of the parabola in the coordinate plane above?

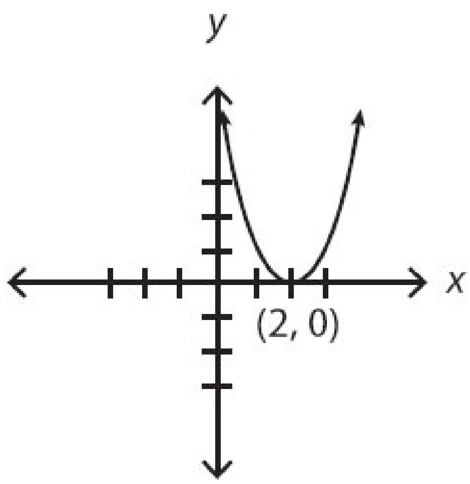
(A) $y = -x - 1$

(B) $y = x^2 + 1$

(C) $y = -x^2 - 1$

(D) $y = -x^2 + 1$

(E) $y = -(x - 1)^2$



13. If the equation of the parabola in the coordinate plane above is $y = (x - h)^2 + k$ and $(-3, n)$ is a point on the parabola, what is the value of n?

In the coordinate plane, the equation of line p is $3y - 9x = 9$.

	Quantity A	Quantity B
14.	The slope of line p	The x-intercept of line p

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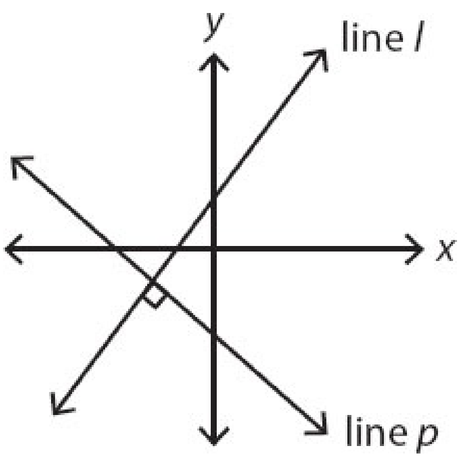
15. In the xy-coordinate plane, lines j and k intersect at point (1, 3). If the equation of line j is $y = ax + 10$, and the equation of k is $y = bx + a$, where a and b are constants, what is the value of b?

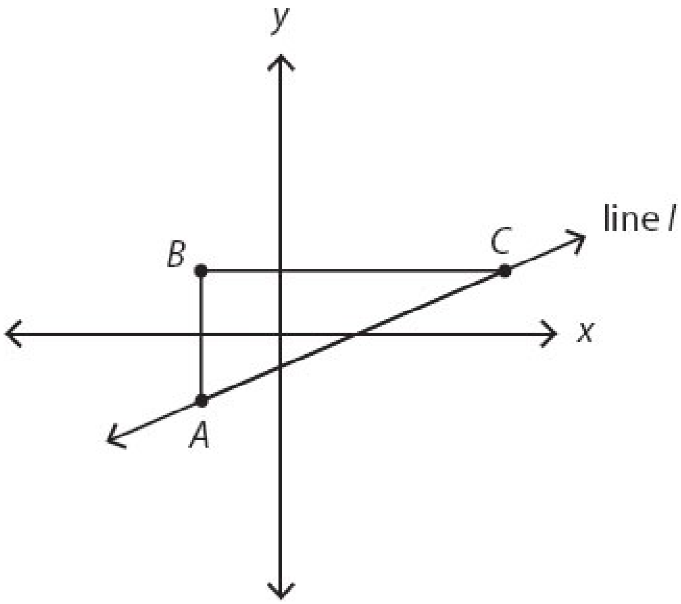


The slope of line l is greater than 1.

	Quantity A	**Quantity B**
16.	The slope of line p	-1

Lines l_1 and l_2 are parallel, and their respective slopes sum to less than 1.

	Quantity A	Quantity B
17.	The slope of a line perpendicular to lines l_1 and l_2	$-\frac{1}{2}$



18. In the coordinate system above, the slope of line l is $\frac{1}{3}$ and the length of line segment *BC* is 4, how long is line segment *AB*?

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) 3

(D) 4

(E) 12

19. What is the area of a triangle with vertices (–2, 4), (2, 4), and (–6, 6) in the coordinate plane?

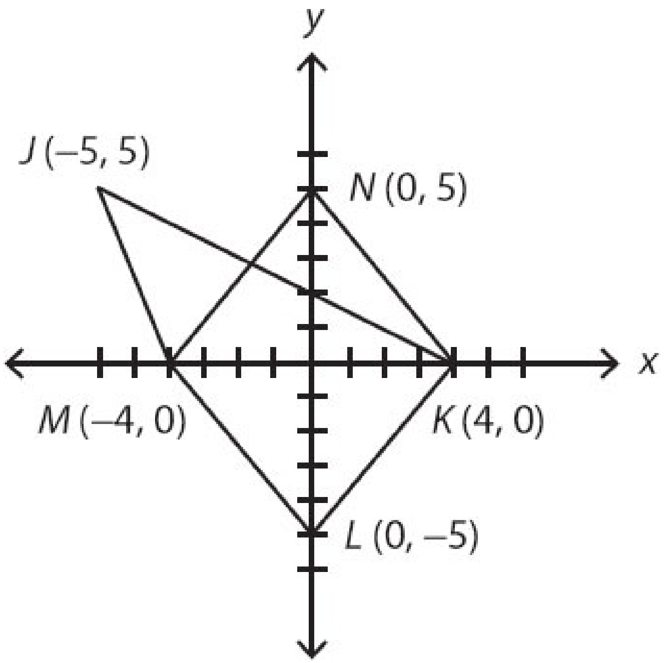
Lines *m* and *n* are perpendicular, neither line is vertical, and line *m* passes through the origin.

	Quantity A	Quantity B
20.	The product of the slopes of lines *m* and *n*	The product of the *x*-intercepts of lines *m* and *n*

In the coordinate plane, points (a, b) and (c, d) are equidistant from the origin.

$$|a| > |c|$$

	Quantity A	**Quantity B**
21.	$|b|$	$|d|$



	Quantity A	**Quantity B**
22.	The area of parallelogram *KLMN*	The area of quadrilateral *JKLM*

23. Which of the following could be the equation of a line parallel to the line $3x + 2y = 8$?

(A) $y = \frac{2}{3}x + 7$

(B) $y = -\frac{2}{3}x + 7$

(C) $y = \frac{3}{2}x + 7$

(D) $y = -\frac{3}{2}x + 7$

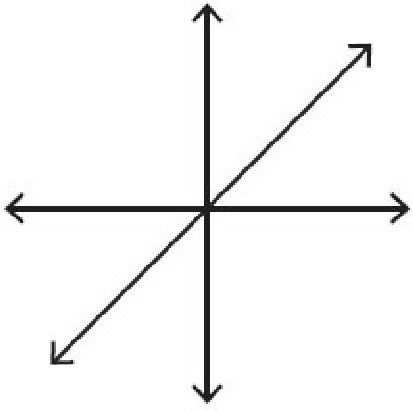
(E) $y = \frac{3}{2}x - 7$

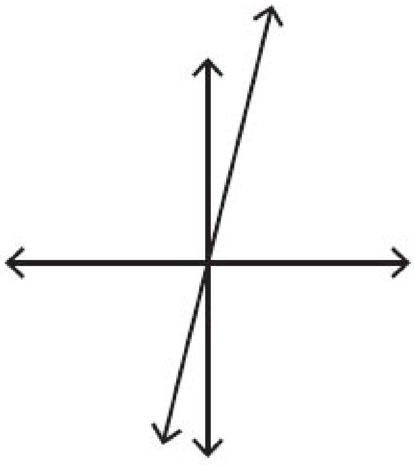
Coordinate Geometry Answers

1. **(A).** Both slopes are negative (pointing down when reading from left to right), and line l_2 is steeper than line l_1. Thus, the slope of l_2 has a greater *absolute value*. But since the values are both negative, the slope of l_1 is a greater number. For instance, the slope of l_1 could be –1 and the slope of l_2 could be –2. Whatever the actual numbers are, the slope of l_1 is closer to 0 and therefore greater.

2. **(C).** Since there are no numbers on the graph, the exact equation of the line cannot be determined, but the line has a positive slope (it points upward when reading from left to right) and a negative y-intercept (it crosses the y-axis below the origin). All of the answers are already in slope-intercept form ($y = mx + b$, where m = slope and b = y-intercept). Choices (A), (B), (C), and (D) have positive slope. Of those, only choices (B) and (C) have a negative y-intercept.

Now, is the slope closer to positive 4 or positive 1? A slope of 1 makes 45° angles when it cuts through the x and y axes, and this figure looks very much like it represents a slope of 1. A slope of 4 would look much steeper than this picture. Note that xy-planes are drawn to scale on the GRE, and units on the x-axis and on the y-axis are the same, unless otherwise noted.





The correct answer is (C). Note that the GRE would only give questions in which the answers are far enough apart that you can determine the intended answer.

3. **(A).** Since there are no numbers on the graph, the exact equation of the line

cannot be determined, but the line has a negative slope (it points down when reading from left to right) and a positive *y*-intercept (it crosses the *y*-axis above the origin).

Change the answers to slope-intercept form ($y = mx + b$, where m = slope and b = *y*-intercept). First note that (D) and (E) cannot be the answers—choice (D) represents a horizontal line crossing through (0, 2), and choice (E) represents a vertical line passing through (–2, 0).

Choice (A):

$$\begin{aligned} 6y + 6x &= 7 \\ 6y &= -6x + 7 \\ y &= -x + \frac{7}{6} \end{aligned}$$

This line, choice (A), has a slope of –1 and *y*-intercept of $\frac{7}{6}$.

Choice (B):

$$3y = -4x - 3$$

$$y = -\frac{4}{3}x - 1$$

This line, choice (B), has a slope of $-\frac{4}{3}$ and *y*-intercept of –1.

Choice (C):

$$5y + 10 = -4x$$

$$5y = -4x - 10$$

$$y = -(-\frac{4}{5})x - 2$$

This line, choice (C), has a slope of $-\frac{4}{5}$ and *y*-intercept of –2.

The only choice with a negative slope and a positive *y*-intercept is choice (A).

4. **(A).** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. It doesn't matter which point is first; just be consistent. Using (–4, 5) as x_1 and y_1 and (1, 2) as x_2 and y_2:

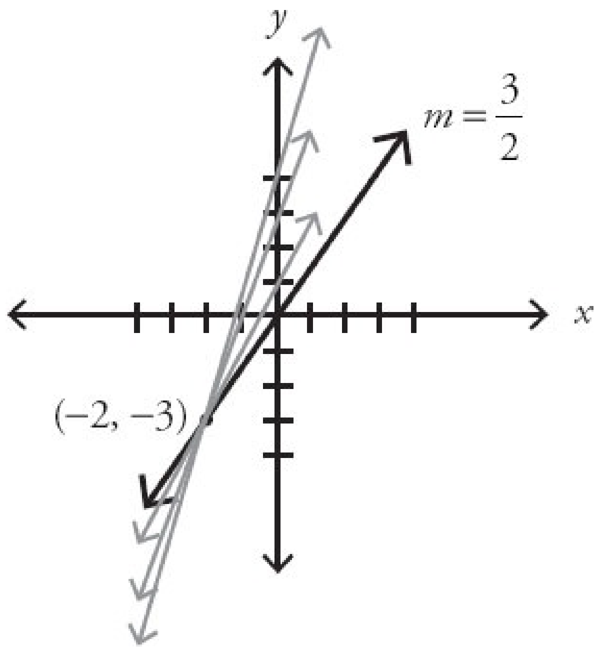
$$m = \frac{2-5}{1-(-4)} = -\frac{3}{5}$$

5. $\mathbf{\frac{5}{3}}$**,** $\mathbf{\frac{9}{4}}$**, and 4 only.** The line must hit a point on the *y*-axis above (0, 0). That means the line could include (0, 0.1), (0, 25), or even (0, 0.00000001). Since the *y*-intercept could get very, very close to (0, 0), use the point (0, 0) to calculate the slope—and then reason that since the line can't *actually* go through (0, 0), the slope will actually have to be steeper than that.

The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (0, 0) as x_1 and y_1 and (–2, –3) as x_2 and y_2 (you can make either pair of points x_1 and y_1, so make whatever choice is most convenient):

$$m = \frac{-3 - 0}{-2 - 0} = \frac{-3}{-2} = \frac{3}{2}$$

Since the slope is positive and the line referenced in the problem needs to hit the *y*-axis above (0, 0), the slope of that line will have to be greater than $\frac{3}{2}$, as in the gray lines below:



Select all answers with a slope greater than $\frac{3}{2}$. Thus, only $\frac{5}{3}$, $\frac{9}{4}$, and 4 are correct.

6. **5.** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (4, 9) as x_1 and y_1 and (6, *y*) as x_2 and y_2, and plugging in –2 for the slope:

$$-2 = \frac{y-9}{6-4}$$

$$-2 = \frac{y-9}{2}$$

$$-4 = y - 9$$

$$5 = y$$

7. **(D).** Use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$d = \sqrt{(4-(-2))^2 + (6-(-2))^2}$$

$$d = \sqrt{(6)^2 + (8)^2}$$

$d = \sqrt{36 + 64}$

$d = \sqrt{100}$

$d = 10$

Alternatively, recognize the Pythagorean triple 6–8–10.

8. **(3, –2) and ($\frac{1}{2}$, –7) only.** For the point (3, –2) to lie on the line $y = 2x - 8$, y needs to equal –2 when 3 is plugged in for x:

$y = 2(3) - 8$
$y = 6 - 8 = -2$

Since y does equal –2 when x equals 3, the point (3, –2) does lie on the line. However, when –8 is plugged in for x, y does not equal 0, so (–8, 0) is not a point on the line. When $\frac{1}{2}$ is plugged in for x, y equals –7, so point $\left(\frac{1}{2}, -7\right)$, lies on the line.

9. **(D).** The problem asks for the point that does <u>not</u> lie on the curve. $y = x^2 - 3$ is the equation of a parabola, but you don't need to know that fact in order to answer this question. For each choice, plug in the coordinates for x and y. For instance, try choice (A):

$$6 = (3)^2 - 3$$
$$6 = 6$$

Since this is a true statement, choice (A) lies on the curve. The only choice that yields a false statement when plugged in is choice (D), the correct answer.

For the point (–3, 0) to lie on the curve $y = x^2 - 3$, y needs to equal 0 when –3 is plugged in for x:

$$y = (-3)^2 - 3$$
$$y = 9 - 3 = 6$$

y does not equal 0 when x equals –3, so the point does not lie on the curve.

10. **(C).** The graph is of a parabola, so its equation must be in the general form of $y = ax^2 + bx + c$. That eliminates choices (A) and (E). Of the remaining answer choices, only answer choice (C) gives a negative y value when $x = 0$ is plugged in. Also, it should be noted that when a parabola lacks a bx term, that is $b = 0$, it will be centered around the y-axis, just as this graph is.

11. **(E).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Here is the vertex of the parabola described by each answer choice:

(A)	(0, 3)	On the axis
(B)	(3, 3)	Incorrect quadrant
(C)	(–3, –3)	Incorrect quadrant
(D)	(3, –3)	Incorrect quadrant

(E)	(–3, 3)	Correct

Only choice (E) places the vertex in the correct quadrant.

12. **(D).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Eliminate choice (A), as it is not the equation of a parabola. Here is the vertex of the parabola described by each remaining answer choice:

(B)	(0, 1)	Correct
(C)	(0, –1)	Incorrect
(D)	(0, 1)	Correct
(E)	(1, 0)	Incorrect

Both (B) and (D) have the correct vertex. However, choice (B) describes a parabola pointing upward from that vertex, because the x^2 term is positive. The negative in front of choice (D) indicates a parabola pointing downward from that vertex.

13. **25.** The equation of the given parabola is $y = (x - h)^2 + k$. The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). (Since the equation of this particular parabola does not have constant a, a must be equal to 1.)

Using $y = (x - h)^2 + k$ and the vertex (2, 0) shown in the graph:

$$y = (x - 2)^2 + 0$$
$$y = (x - 2)^2$$

Since $(-3, n)$ is a point on the parabola, plug in -3 and n for x and y, respectively:

$$n = (-3 - 2)^2$$
$$n = (-5)^2$$
$$n = 25$$

14. **(A).** In slope intercept form ($y = mx + b$, where m is the slope and b is the y-intercept):

$$\begin{aligned} 3y - 9x &= 9 \\ 3y &= 9x + 9 \\ y &= 3x + 3 \end{aligned}$$

The slope is 3. The y-intercept is also 3, but the problem asks for the x-intercept. To get an x-intercept, substitute 0 for y:

$$0 = 3x + 3$$
$$-3 = 3x$$
$$-1 = x$$

Thus, the slope is 3 and the x-intercept is -1. Quantity A is greater.

15. **10.** If lines k and m intersect at the point (1, 3), then 1 can be plugged in for x and 3 plugged in for y in either line equation.

For line j:

$$y = ax + 10$$
$$3 = a(1) + 10$$

$-7 = a$

For line k, plug in not only $x = 1$ and $y = 3$, but also the fact that $a = -7$:

$$
\begin{aligned}
y &= bx + a \\
3 &= (b)(1) + (-7) \\
10 &= b
\end{aligned}
$$

16. **(A).** If the slope of line l is greater than 1 and line p is perpendicular (because of the right angle symbol on the figure), then line p has a negative slope between –1 and 0, because perpendicular lines have negative reciprocal slopes—that is, the product of the two slopes is –1.

Try a few examples to better illustrate this: line l could have a slope of 2, in which case line p would have a slope of $-\frac{1}{2}$. Line l could have a slope of $\frac{3}{2}$, in which case line p would have a slope of $-\frac{2}{3}$. Or line l could have a slope of 100, in which case line p would have a slope of $-\frac{1}{100}$.

All of these values ($-\frac{1}{2}$, $-\frac{2}{3}$, and $-\frac{1}{100}$) are greater than –1. This will work with any example you try. Since line l has a slope greater than 1, line p has a slope with an absolute value less than 1. Because that value will also be negative, it will always be the case that $-1 <$ the slope of line $p < 0$.

17. **(D).** Since lines l_1 and l_2 are parallel, they have the same slope. Call that slope m. Since the slopes sum to less than 1:

$$
\begin{aligned}
m + m &< 1 \\
2m &< 1 \\
m &< \frac{1}{2}
\end{aligned}
$$

Thus, lines l_1 and l_2 each have the same slope that is less than $\frac{1}{2}$. A line perpendicular to those lines would have a negative reciprocal slope. However, there isn't much more you can do here. Lines l_1 and l_2 could have slopes of

$\frac{1}{4}$ (in which case a perpendicular line would have slope = –4) or slopes of –100 (in which case a perpendicular line would have slope = $\frac{1}{100}$). Thus, the slope of the perpendicular line could be less than or greater than $-\frac{1}{2}$.

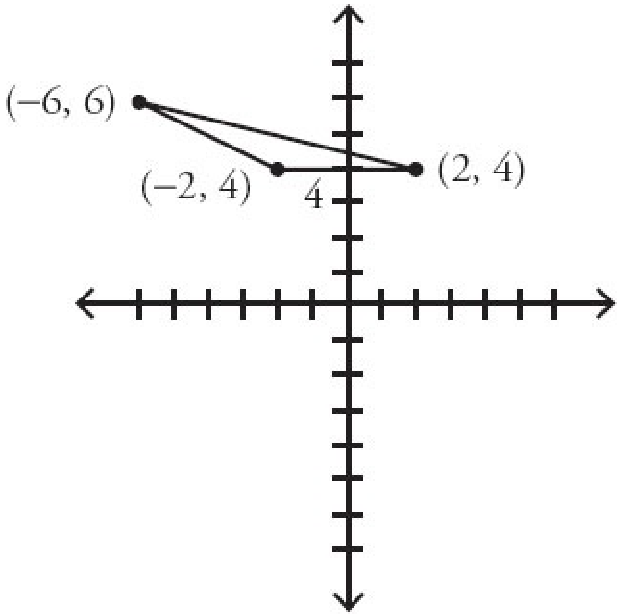
18. **(B).** The slope of line *l* is $\frac{1}{3}$. Since slope = $\frac{\text{rise}}{\text{run}}$ (or "change in *y*" divided by "change in *x*"), for every 1 unit the line moves up, it will move 3 units to the right.

Since the *actual* move to the right is equal to 4, create a proportion:

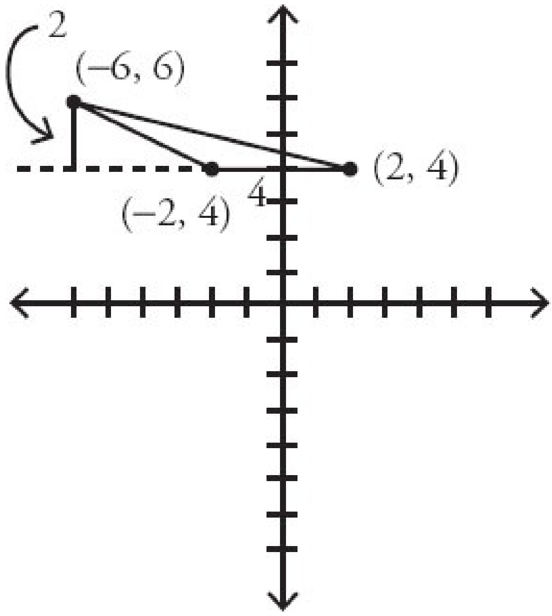
$$\frac{1}{3} = \frac{AB}{4}$$

Cross-multiply to get $3AB = 4$ or $AB = \frac{4}{3}$.

19. **4.** Make a quick sketch of the three points, joining them to make a triangle. Since (–2, 4) and (2, 4) make a horizontal line, use this line as the base. Since these two points share a *y*-coordinate, the distance between them is the distance between their *x*-coordinates: 2 – (–2) = 4, as shown below:



The height of a triangle is always perpendicular to the base. Drop a height vertically from (–6, 6). Subtract the *y*-coordinates to get the distance: 6 – 4 = 2.

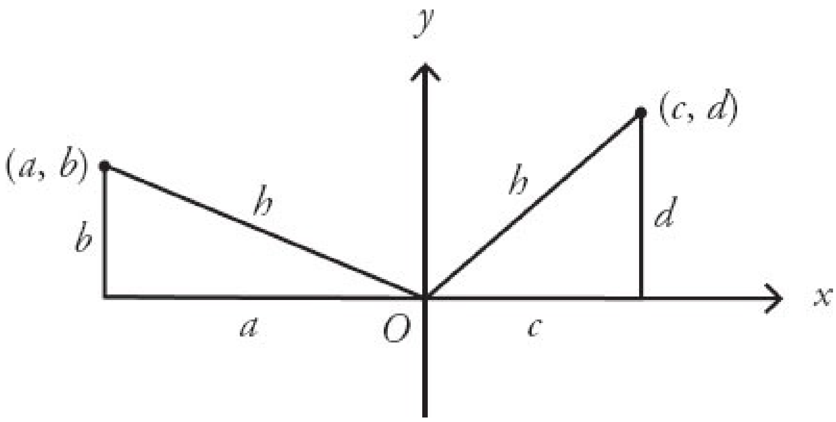


The formula for area of a triangle is $\frac{bh}{2}$. Thus, the area is $\frac{(4)(2)}{2}$, or 4.

20. **(B).** The slopes of perpendicular lines are the negative inverse of each other, so their product is –1. For example, perpendicular lines could have

slopes of 2 and $-\frac{1}{2}$, or $-\frac{5}{7}$ and $\frac{7}{5}$. In all of these cases, Quantity A is –1. (The only exception is when one of the lines has an undefined slope because it's vertical, but that case has been specifically excluded.) If line *m* passes through the origin, its *x*-intercept is 0, so regardless of the *x*-intercept of line *n*, Quantity B is 0. Quantity B is greater.

21. **(B).** A point's distance from the origin can be calculated by constructing a right triangle in which the legs are the vertical and horizontal distances. Sketch a diagram in which you place (*a*, *b*) and (*c*, *d*) anywhere in the coordinate plane that you wish; then construct two right triangles using (0, 0) as a vertex.



Both hypotenuses are labeled *h*, since the points are equidistant from the origin. Set up two Pythagorean equations:

$a^2 + b^2 = h^2$
$c^2 + d^2 = h^2$

So $a^2 + b^2 = c^2 + d^2$.

Since $|a| > |c|$, it is true that $a^2 > c^2$. (Try it with test numbers.) To make the equation $a^2 + b^2 = c^2 + d^2$ true, you must have $b^2 < d^2$. This means that $|b| < |d|$, and Quantity B is greater.

22. **(C).** Both figures share triangle *MLK*, so there is no need to calculate anything for this part of the figure. Parallelogram *KLMN* and quadrilateral *JKLM* each have a “top” (the part above the *x*-axis) that is a triangle with base *MK* (= 8) and height 5. If two triangles have the same base and equal heights, their areas are equal. No calculation is needed to pick (C).

23. **(D).** Rearrange the equation to get it into $y = mx + b$ format, where m is the slope:

$$3x + 2y = 8$$
$$2y = -3x + 8$$
$$y = -\frac{3}{2}x + 4$$

The slope is $-\frac{3}{2}$. Parallel lines have the same slope, so only choice (D) is parallel.

Chapter 29

Mixed Geometry

In This Chapter…

Mixed Geometry

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.

(B) Quantity B is greater.

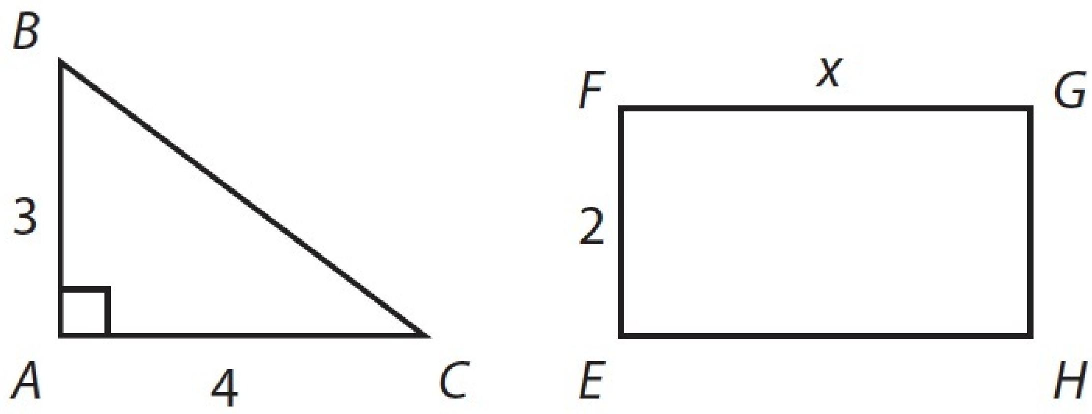
(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

Where answer choices do not appear on Quantitative Comparison questions in this book, you should choose A, B, C or D based on the above.

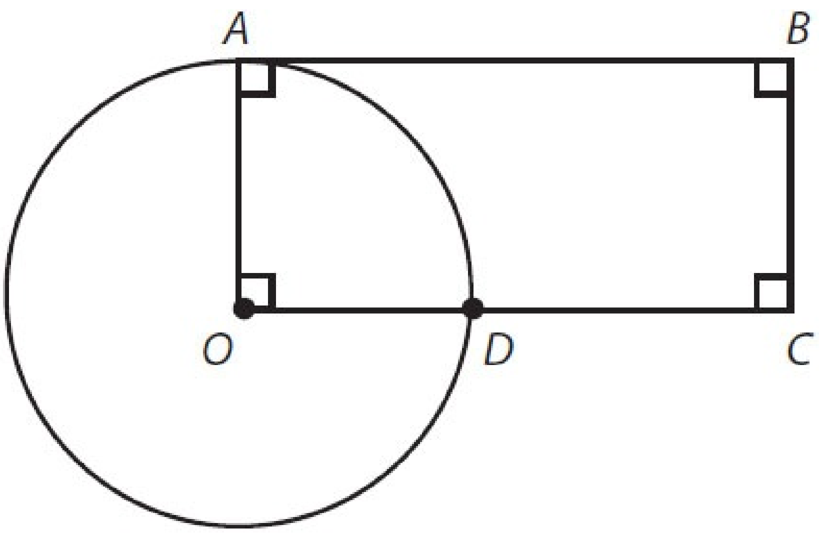
For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by a fraction-style numeric entry box $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is $\frac{1}{4}$, you may enter $\frac{25}{100}$ or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations, such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.



1. Right triangle *ABC* and rectangle *EFGH* have the same perimeter. What is the value of *x*?

$\boxed{}$



Point O is the center of the circle.

2. If the area of the circle is 36π and the area of the rectangle is 72, what is the length of DC?

$\boxed{}$

3. The center of a circle is (10, –3). The point (10, 9) is outside the circle, and the point (6, –3) is inside the circle; neither point is on the circle. If the radius, r, is an integer, how many possible values are there for r?

(A) Seven

(B) Eight

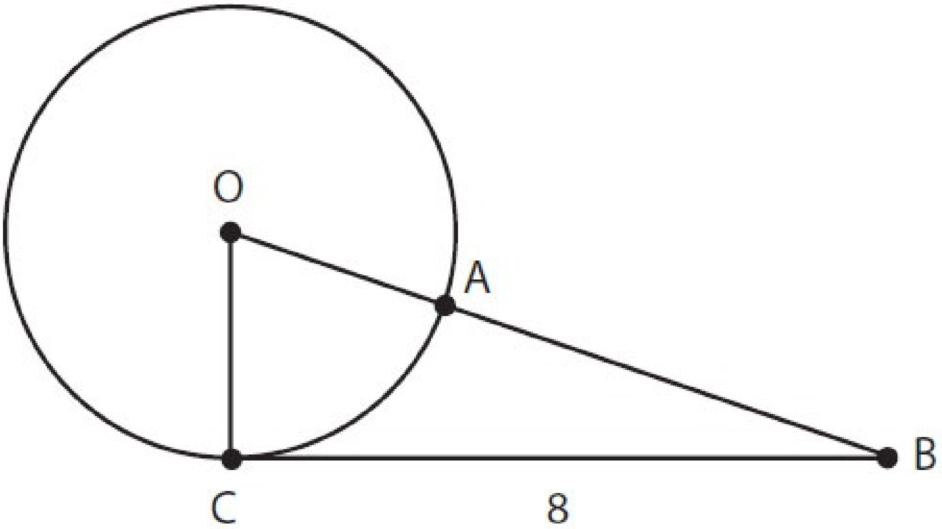
(C) Nine

(D) Ten

(E) Eleven

A square's perimeter in inches is equal to its area in square inches.
A circle's circumference in inches is equal to its area in square inches.

	Quantity A	**Quantity B**
4.	The side length of the square	The diameter of the circle



5. In the figure above, point *O* is the center of the circle, points *A* and *C* are located on the circle, and line segment *BC* is tangent to the circle. If the area of triangle *OBC* is 24, what is the length of *AB*?

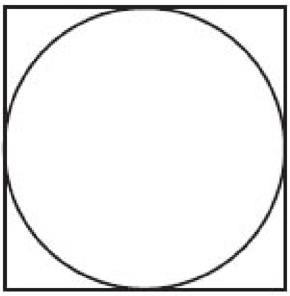
(A) 2

(B) 4

(C) 6

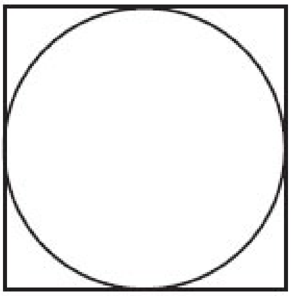
(D) 8

(E) 10



In the figure above, the circle is inscribed in the square.
The area of the circle is 9π.

	Quantity A	Quantity B
6.	The area of the square	30



7. In the figure above, the circle is inscribed in a square that has an area of 50. What is the area of the circle?

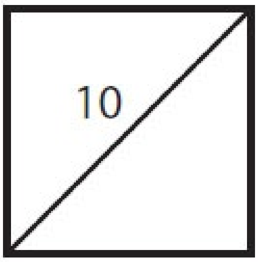
   (A) $\dfrac{25\pi}{4}$

   (B) $\dfrac{25\pi}{2}$

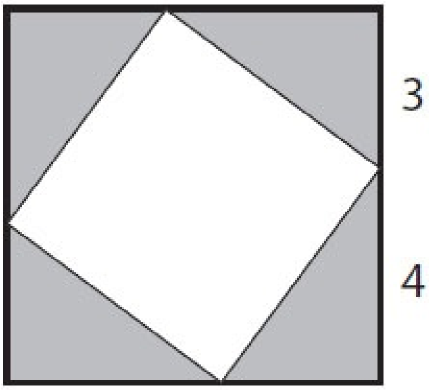
   (C) 25π

   (D) 50π

   (E) $\dfrac{625\pi}{16}$



8. What is the area of the square in the figure above?



9. In the 7-inch square above, another square is inscribed. What fraction of the larger square is shaded?

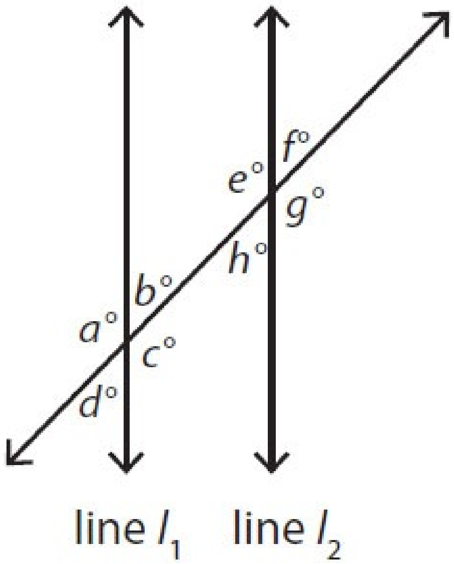
(A) $\frac{3}{12}$

(B) $\frac{24}{49}$

(C) $\frac{1}{2}$

(D) $\frac{25}{49}$

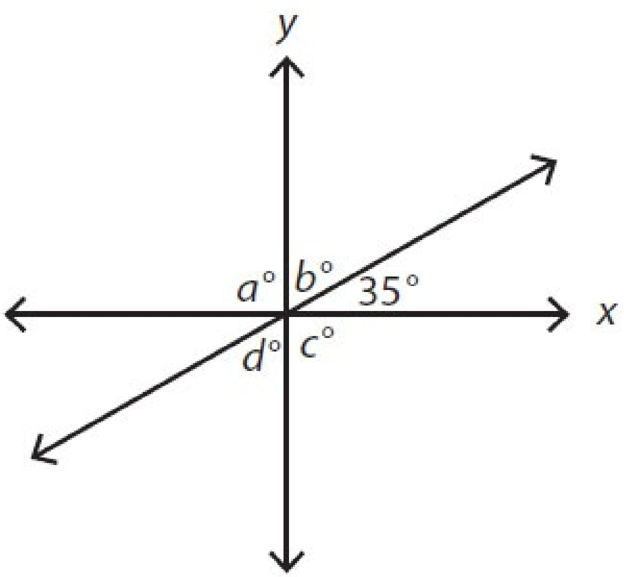
(E) $\frac{7}{12}$



Lines l_1 and l_2 are parallel.

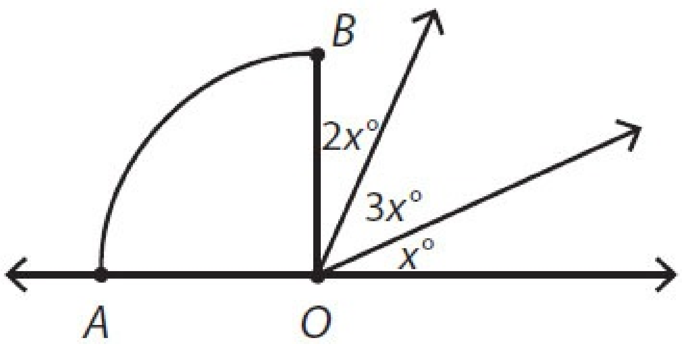
$a > 90$

	Quantity A	Quantity B
10.	$a + f + g$	$b + e + h$



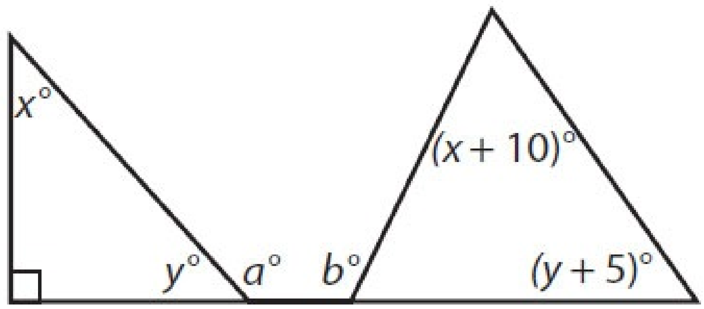
11. What is the value of $a + b + c + d$?

$\boxed{}$



Sector *OAB* is a quarter-circle.

	Quantity A	Quantity B
12.	x	15



13. What is a in terms of b and y?

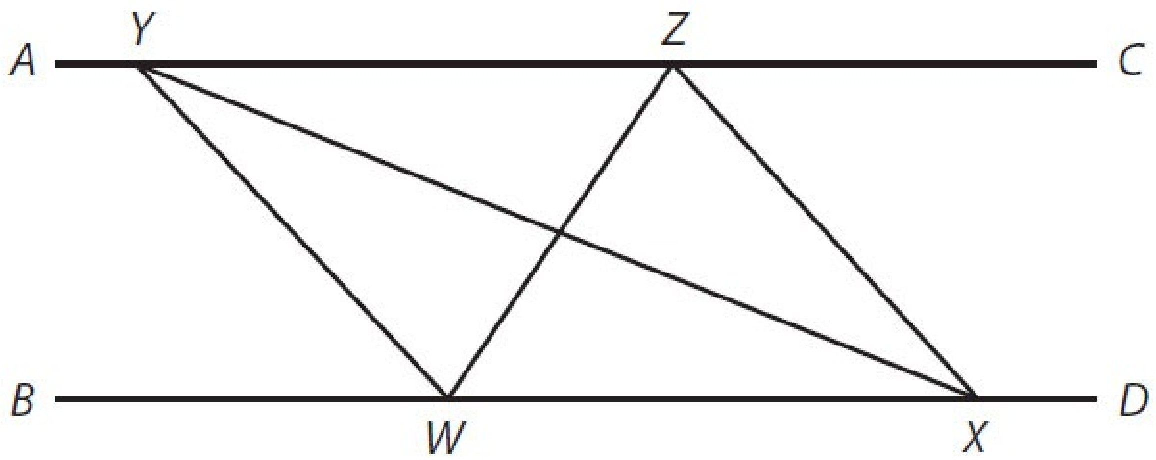
(A) $b + y + 65$

(B) $b - y + 65$

(C) $b + y + 75$

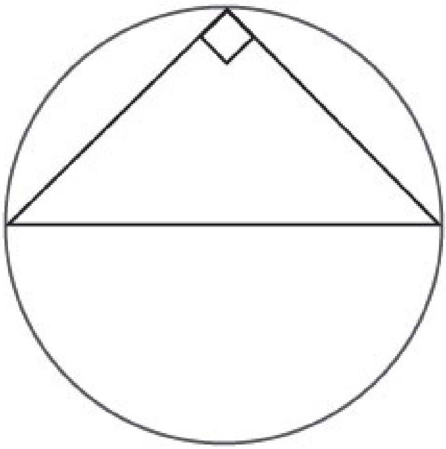
(D) $b - 2y + 45$

(E) $b - y + 75$



In the figure above, line segments *AC* and *BD* are parallel.

	Quantity A	Quantity B
14.	The area of triangle *WYX*	The area of triangle *WZX*



In the figure above, a right triangle is inscribed in a circle with an area of 16π cm^2.

	Quantity A	**Quantity B**
15.	The hypotenuse of the triangle, in centimeters	8

16. A rectangular box has a length of 6 centimeters, a width of 8 centimeters, and a height of 10 centimeters. What is the length of the diagonal of the box, in centimeters?

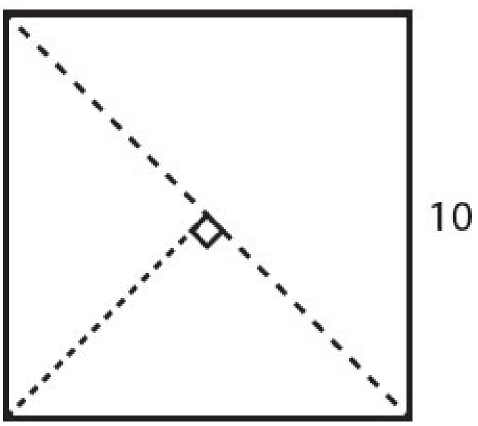
    (A) 10

    (B) 12

    (C) $10\sqrt{2}$

    (D) $10\sqrt{3}$

    (E) 24



17. Julian takes a 10-inch by 10-inch square piece of paper and cuts it in half

along the diagonal. He then takes one of the halves and cuts it in half again from the corner to the midpoint of the opposite side. All cuts are represented in the figure with dotted lines. What is the perimeter of one of the smallest triangles, in inches?

(A) 10

(B) $10\sqrt{2}$

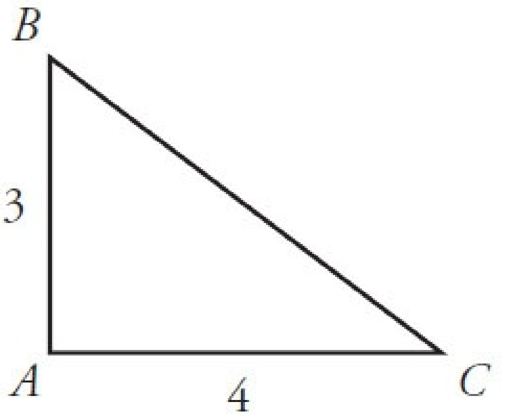
(C) 20

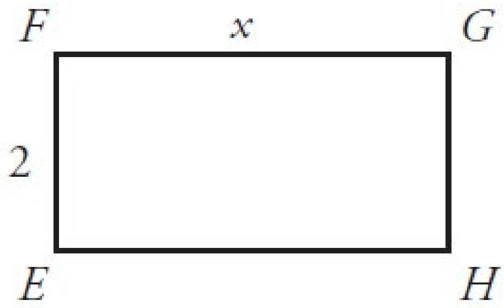
(D) 10 + $10\sqrt{2}$

(E) 10 + $20\sqrt{2}$

Mixed Geometry Answers

1. **4.** Triangle *ABC* is a right triangle, so this must be a 3–4–5 triangle, and the length of side *BC* is 5. That means the perimeter of triangle *ABC* is 3 + 4 + 5 = 12.





Thus, the perimeter of rectangle *EFGH* is also 12. Using that information, find x:

$$2 \times (2 + x) = 12.$$
$$4 + 2x = 12$$
$$2x = 8$$
$$x = 4$$

2. **6.** The area of this circle is 36π and the area of any circle is πr^2, so the radius of this circle is 6. Label both radii (*OA* and *OD*) as 6. Because *ABCO* is a rectangle, its area is equal to base times height, where radius *OA* is the height.

$$\begin{aligned} \text{Area of a rectangle} &= bh \\ 72 &= b(6) \\ 12 &= b \end{aligned}$$

Since *OC* is a base of the rectangle, it is equal to 12. Subtract radius *OD* from base *OC* to get the length of segment *DC*: 12 – 6 = 6.

3. **(A).** This problem does not actually require any special formulas regarding circles. Calculate the distance between the center point (10, –3) and point (10, 9). Since the x-coordinates are the same and 9 – (–3) = 12, the two points are 12 apart. Because (10, 9) is outside of the circle, the radius must be less than

12. Similarly, calculate the distance between the center point (10, –3) and point (6, –3). Since the y-coordinates are the same, the distance is 10 – 6 = 4. Because (6, –3) is inside the circle, the radius must be more than 4. The radius must be an integer that is greater than 4 and less than 12, so it can only be 5, 6, 7, 8, 9, 10, or 11. Thus, there are seven possible values for r.

4. **(C).** The perimeter of a square is $4s$ and the area of a square is s^2 (where s is a side length). If the square's perimeter equals its area, set the two expressions equal to each other and solve:

$$\begin{aligned} 4s &= s^2 \\ 0 &= s^2 - 4s \\ 0 &= s(s - 4) \\ 4 \text{ or } 0 &= s \end{aligned}$$

Only $s = 4$ would result in an actual square, so $s = 0$ is not a valid solution.

The circumference of a circle is $2\pi r$ and the area of a circle is πr^2 (where r is the radius). If the circle's circumference equals its area, set the two expressions equal to each other and solve:

$$\begin{aligned} 2\pi r &= \pi r^2 \\ 2r &= r^2 \\ 0 &= r^2 - 2r \\ 0 &= r(r - 2) \\ 2 \text{ or } 0 &= r \end{aligned}$$

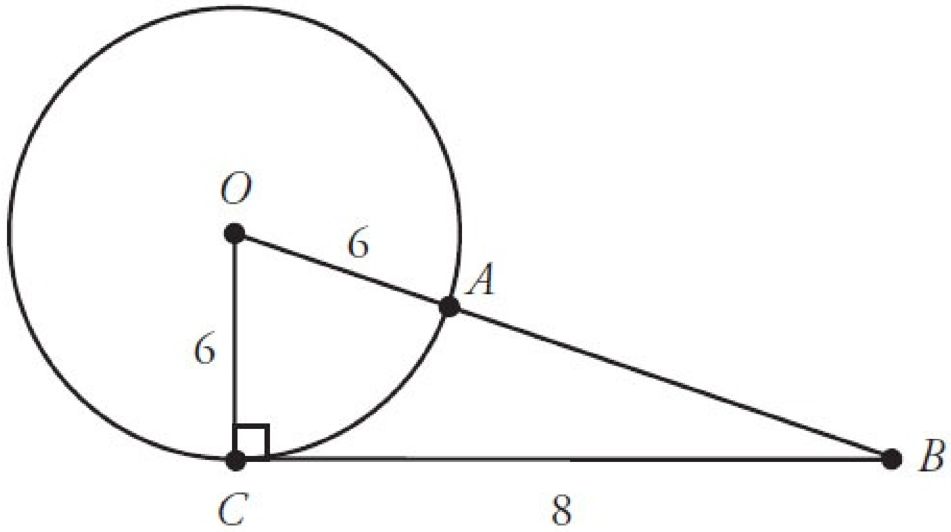
Only $r = 2$ would result in an actual circle, so $r = 0$ is not a valid solution.

If the radius of the circle is 2, then the diameter is 4. Thus, the two quantities are equal.

5. **(B).** Because BC is tangent to the circle, angle OCB is a right angle. Thus, radius OC is the height of the triangle. If the area of the triangle is 24, use the area formula for a triangle (and 8 as the base, from the figure) to determine the height:

$$\begin{aligned} A &= \frac{bh}{2} \\ 24 &= \frac{8 \times OC}{2} \\ 48 &= 8 \times OC \\ 6 &= OC \end{aligned}$$

Thus, the radius of the circle is 6 (note that there are two radii on the diagram, OC and OA). Since two sides of the right triangle are known, use the Pythagorean theorem to find the third:



$$\begin{aligned} 6^2 + 8^2 &= (OB)^2 \\ 36 + 64 &= (OB)^2 \\ 100 &= (OB)^2 \\ 10 &= OB \end{aligned}$$

(The 6–8–10 triangle is one of the special right triangles you should memorize for the GRE!)

Since the hypotenuse *OB* is equal to 10 and the radius *OA* is equal to 6, subtract to get the length of *AB*. The answer is 10 – 6 = 4.

6. **(A).** If the area of the circle is $9\pi = \pi r^2$, the radius must be 3. The radius represents half the side of the square, so the square is 6 on each side. The area of the square is thus $A = s^2 = 36$, which is greater than 30. Quantity A is greater.

7. **(B).** If the area of the square is 50, the sides of the square are

$\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$.

If the square is $5\sqrt{2}$ "tall," so is the circle. That is, the side of the square is equal to the diameter of the circle. Since the diameter of the circle is $5\sqrt{2}$, the radius is $\frac{5\sqrt{2}}{2}$. Using the formula for the area of a circle, $A = \pi r^2$:

$$A = \pi\left(\frac{5\sqrt{2}}{2}\right)^2$$

$$A = \pi\left(\frac{25\times 2}{4}\right)$$

$$A = \frac{25\pi}{2}$$

Note that even if you got a bit lost in the math, you could estimate quite reliably! The square is a bit larger than the circle, so the circle area should be a bit less than 50. Put all the answers in the calculator, using 3.14 as an approximate value for π, and you will quickly see that choice (A) is equal to 19.625, which is too small, and choice (B) is equal to 39.25, while the other three choices are much too large (larger than the square).

8. **50.** One way to solve this problem is by using the Pythagorean theorem. All sides of a square are equal to s, so:

$$s^2 + s^2 = 10^2$$

$$2s^2 = 100$$

$$s^2 = 50$$

Note that you *could* solve for *s* ($s = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$), but the area of the square is s^2, which is already calculated above. The area of the square is 50.

9. **(B).** Each of the shaded triangles is a 3–4–5 Pythagorean triple. (Or, just note that each shaded triangle has legs of 3 and 4; the Pythagorean theorem will tell you that each hypotenuse is equal to 5).

Since each hypotenuse is also a side of the square, the square has an area of 5 × 5 = 25.

The larger square (the overall figure) has an area of 7 × 7 = 49.

Subtract to find the area of the shaded region: 49 – 25 = 24.

The fraction of the larger square that is shaded is therefore $\frac{24}{49}$.

10. **(A).** While the exact measures of any of the angles are not given, when parallel lines are cut by a transversal, only two angle measures are created: all the “big” angles are the same and all the “small” angles are the same. Further, the sum of a “small” and a “big” angle is 180° (These angles are said to “form a line.”) Use this fact to simplify both quantities.

Quantity A: $a + f + g = a + (f + g) = a + (180)$
Quantity B: $b + e + h = b + (e + h) = b + (180)$

Subtract 180 from each quantity, and the question is really asking for the comparison of a and b. Since $a > 90$ and $a + b = 180$, $b = 180 - a = 180 -$ (greater than 90) = less than 90. If a is greater than 90 and b is less than 90, Quantity A is greater.

11. **290.** Angles that “go around in a circle” sum to 360°. It may be tempting to just subtract 35 from 360 and answer 325, but don’t overlook the unlabeled angle, which is opposite and therefore equal to 35°. Therefore, subtract 35 + 35 = 70 from 360 to get the answer, 290.

12. **(C).** If sector *OAB* is a quarter-circle, then the angle inside the quarter-circle at *O* measures 90°. Since angles that make up a straight line must sum to 180, $2x + 3x + x$ must sum to 90:

$2x + 3x + x = 90$
$6x = 90$
$x = 15$

The two quantities are equal.

13. **(E).** An exterior angle of a triangle is equal to the sum of the two opposite interior angles. From the left triangle, $a = x + 90$. From the right triangle, $b = (x + 10) + (y + 5) = x + y + 15$.

Alternatively, you could use the facts that the interior angles of a triangle sum to 180, as do angles that form a straight line. From the left triangle, $x + y + 90$ and $y + a$ both equal 180, so $x + y + 90 = y + a$, or $x + 90 = a$. From the right triangle, $180 = (x + 10) + (y + 5) + (180 – b)$, or $b = x + y + 15$.

The question asks for a in terms of b and y, so x is the variable that needs to be eliminated. Eliminate the variable by solving one equation for x, and substituting this expression for x in the other equation. From the right triangle:

$$b = x + y + 15$$
$$x = b – y – 15$$

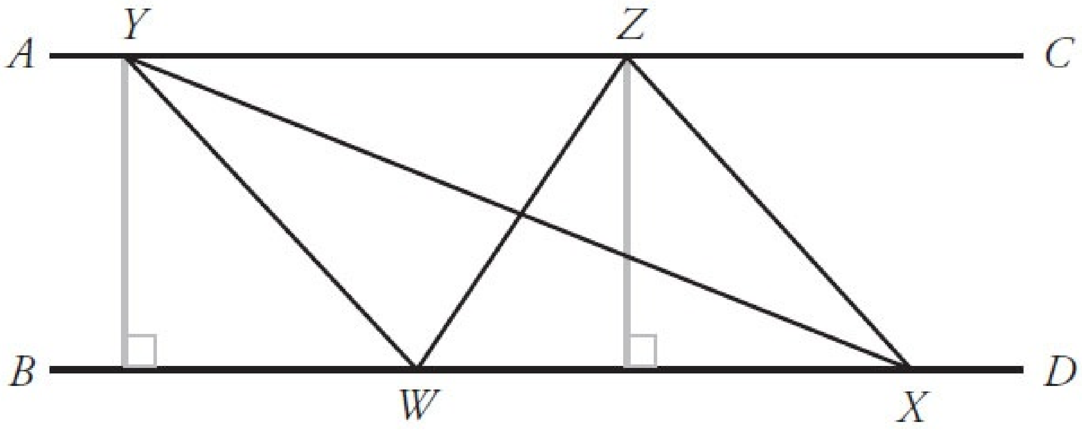
From the left triangle:

$$a = x + 90$$
$$a = (b – y – 15) + 90$$
$$a = b – y + 75$$

14. **(C).** Both triangles, *WYX* and *WZX*, share a common base of segment *WX*. Consider the formula for the area of a triangle:

$$\text{Area} = \left(\frac{1}{2}\right)(\text{base})(\text{height})$$

If two triangles have equal bases, the triangle with the greater area is the one with the greater height. The height is a perpendicular line drawn from the highest point on the triangle to the base. In this case, the heights would be the gray lines below:



By the definition of parallel lines, *AC* and *BD* are uniform distance apart. Therefore, the heights shown are the same. Because these triangles have equal bases and heights, they must have equal areas.

15. **(C).** To solve this problem, recall that a triangle inscribed in a semicircle will be a right triangle *if and only if* one side of the triangle is the diameter (i.e., the center of the circle must lie on one side of the triangle). Because this is a right triangle, the hypotenuse must be the diameter of the circle.

To find the diameter of the circle, recall the formula for area, Area = πr^2, and set up an equation:

$$\begin{aligned} 16\pi \text{ cm}^2 &= \pi r^2 \\ 16 \text{ cm}^2 &= r^2 \\ r &= 4 \text{ cm} \end{aligned}$$

Given that diameter is twice the radius, the diameter (i.e., the hypotenuse of the triangle) is 8 cm. Quantity A is 8, so the two quantities are equal.

16. **(C).** A fast approach to solving this problem is to use the “Super

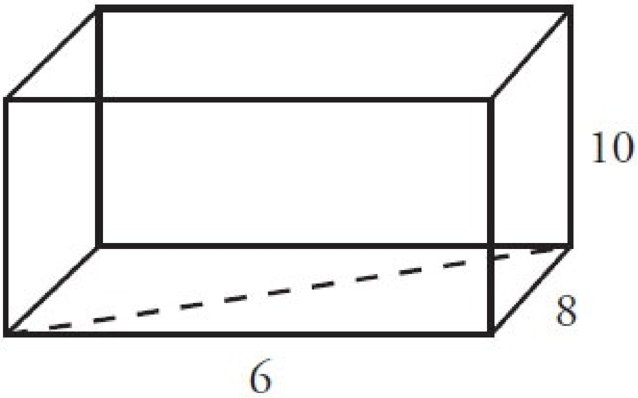
Pythagorean theorem," which states that the diagonal of any rectangular box is d in the following formula:

$$d^2 = l^2 + w^2 + h^2$$

where l, w, and h are the length, width, and height of the box, respectively. Plugging in the given information yields:

$$d^2 = 6^2 + 8^2 + 10^2$$
$$d^2 = 36 + 64 + 100$$
$$d^2 = 200$$
$$d = 10\sqrt{2}$$

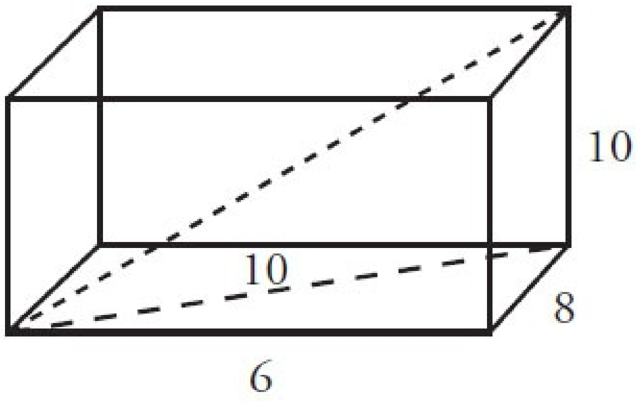
Alternatively, if you don't remember the Super Pythagorean, apply the normal Pythagorean theorem twice. To find the diagonal of the box, first find the diagonal of one of the sides. Use the bottom side of the figure below as the base:



where the dashed line represents the diagonal of the base. Applying the Pythagorean theorem:

$c^2 = a^2 + b^2$
$c^2 = 6^2 + 8^2$
$c^2 = 36 + 64$
$c^2 = 100$
$c = 10$

From here, draw the diagonal of the box and apply the Pythagorean theorem again to the vertically oriented triangle with legs 10 and 10 again as shown:



$d^2 = 10^2 + 10^2$
$d^2 = 100 + 100$
$d^2 = 200$
$d = 10\sqrt{2}$

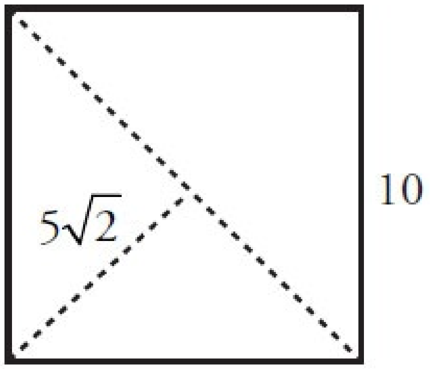
17. **(D).** In order to compute the perimeter of one of the smaller triangles, first

compute the length of the diagonal. For a square with side length 10 inches, the length of the diagonal can be computed by the Pythagorean theorem, $(\text{diagonal})^2 = (\text{side})^2 + (\text{side})^2$:

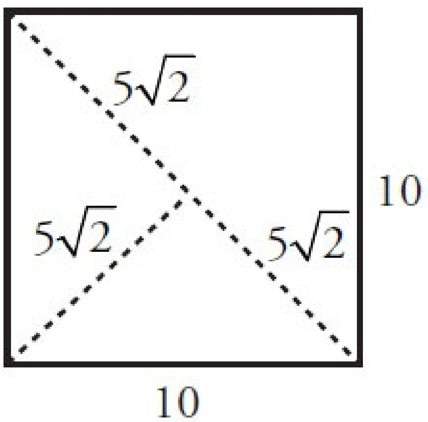
$$d^2 = 10^2 + 10^2$$
$$d^2 = 200$$
$$d = 10\sqrt{2}$$

Alternatively, recognize that the diagonal of a square is always $\sqrt{2}$ times the side length.

The second cut goes from the corner to the midpoint of the diagonal, so that slice is half as long as the diagonal of the square: $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$. This can be seen as:



Similarly, because the remaining line in each of the smaller triangles is half of a diagonal, each is of length $5\sqrt{2}$ inches:



Summing the lengths of the sides, the perimeter of the smallest triangle is:

Perimeter = $10 + 5\sqrt{2} + 5\sqrt{2}$

Perimeter = $10 + 10\sqrt{2}$

Chapter 30

Advanced Quant

In This Chapter…

Advanced Quant

The following questions are *extremely* advanced for the GRE. We have included them by popular demand—students who are aiming for perfect math GRE scores often wish to practice on problems that may be even harder than any seen on the real GRE. We estimate that a GRE test-taker who does well on the first math section and therefore is given a difficult second section might see one or two problems, at most, of this level of difficulty.

If you are *not* aiming for a perfect math score, we absolutely recommend that you skip these problems!

If you are taking the GRE for business school or another quantitative program, you may wish to attempt some of these problems. For instance, you might do one or two of these problems—think of them as "brain teasers"—to cap off a study session from elsewhere in the book. (For reference, getting 50% of these problems correct would be a pretty incredible performance!)

Even if you *are* aiming for a perfect math score, though, please make sure you are *flawless* at the types of math problems in the *rest* of this book before you work on these. You will gain far more points by reducing minor mistakes (through practice, steady pacing, and good organization) on easy and medium questions than you would by focusing on ultra-hard questions.

For more such problems, visit the Manhattan Prep GRE blog for our monthly Challenge Problem. (Access to the archive of over fifteen dozen Challenge Problems is available to our course and Guided Self-Study students for free and to anyone else for a small fee.)

That said, attempt these Advanced Quant problems—if you dare!

1. 21 people per minute enter a previously empty train at a station beginning at 7:00:01 pm (7 o'clock and one second). Every 9 minutes beginning at 7:04:00 pm, the train departs and takes with it everyone who has entered the train in the last 9 minutes. If the last train departs at 8:25:00 pm, what is the average number of people who get on each of the trains leaving from 7:00:00 pm to 8:25:00 pm, inclusive?

   (A) 84

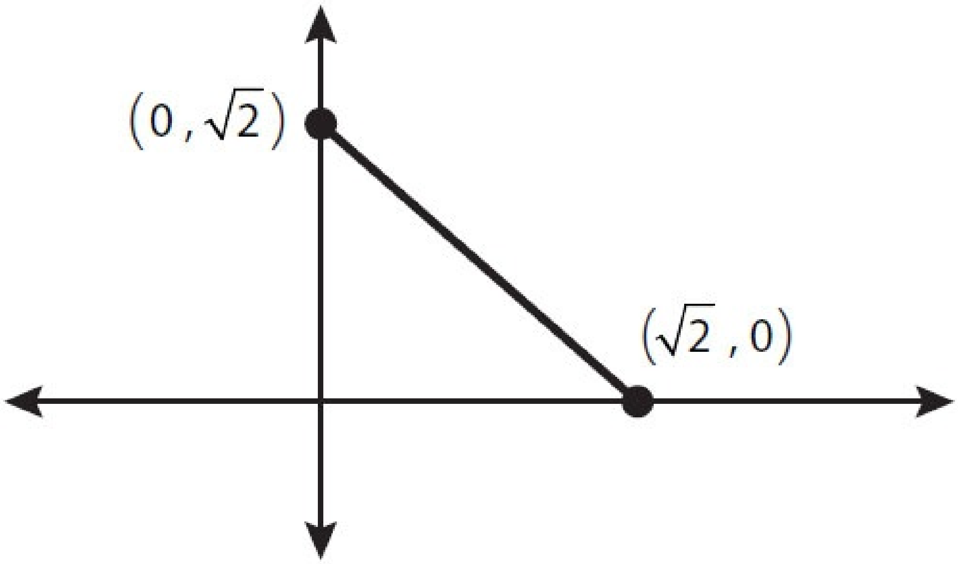
   (B) 136.5

   (C) 178.5

   (D) 189

   (E) 198.5

2. The random variable x has the following continuous probability distribution in the range $0 \le x \le \sqrt{2}$, as shown in the coordinate plane with x on the horizontal axis:



The probability that $x < 0$ = the probability that $x > \sqrt{2} = 0$.

What is the median of x?

   (A) $\frac{\sqrt{2}-1}{2}$

   (B) $\frac{\sqrt{2}}{4}$

   (C) $\sqrt{2} - 1$

(D) $\dfrac{\sqrt{2}+1}{4}$

(E) $\dfrac{\sqrt{2}}{2}$

$x < 0$

	Quantity A	**Quantity B**
3.	$x^2 - 5x + 6$	$x^2 - 9x + 20$

---

4. If x is a positive integer, what is the units digit of $(24)^{5+2x}(36)^6(17)^3$?

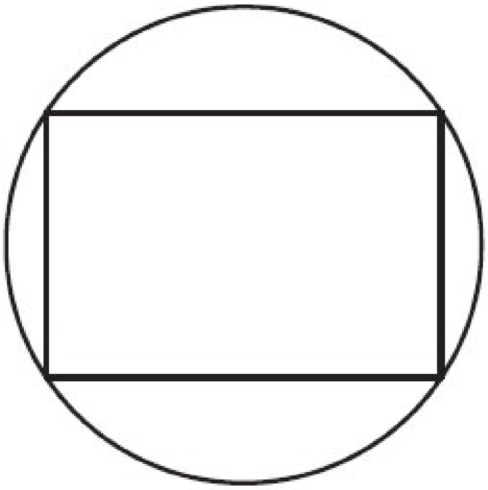
(A) 2

(B) 3

(C) 4

(D) 6

(E) 8



5. In the figure above, the circumference of the circle is 20π. Which of the following is the maximum possible area of the rectangle?

(A) 80

(B) 200

(C) 300

(D) $100\sqrt{2}$

(E) $200\sqrt{2}$

6. The length of each edge of a cube equals 6. What is the distance between the center of the cube to one of its vertices?

(A) $3\sqrt{2}$

(B) $6\sqrt{2}$

(C) $3\sqrt{3}$

(D) $4\sqrt{3}$

(E) $6\sqrt{3}$

7. If c is randomly chosen from the integers 20 to 99, inclusive, what is the probability that $c^3 - c$ is divisible by 12?

Give your answer as a fraction.

8. The remainder when 120 is divided by single-digit integer m is positive, as is the remainder when 120 is divided by single-digit integer n. If $m > n$, what is the remainder when 120 is divided by $m - n$?

9. A circular microchip with a radius of 2 centimeters is manufactured following a blueprint scaled such that a measurement of 1 centimeter on the blueprint corresponds to a measurement of 0.8 millimeters on the microchip. What is the diameter of the blueprint representation of the microchip, in centimeters? (1 centimeter = 10 millimeters)

centimeters

For a certain quantity of a gas, pressure P, volume V, and temperature T are related according to the formula $PV = kT$, where k is a constant.

	Quantity A	Quantity B
10.	The value of P if $V = 20$ and $T = 32$	The value of T if $V = 10$ and $P = 78$

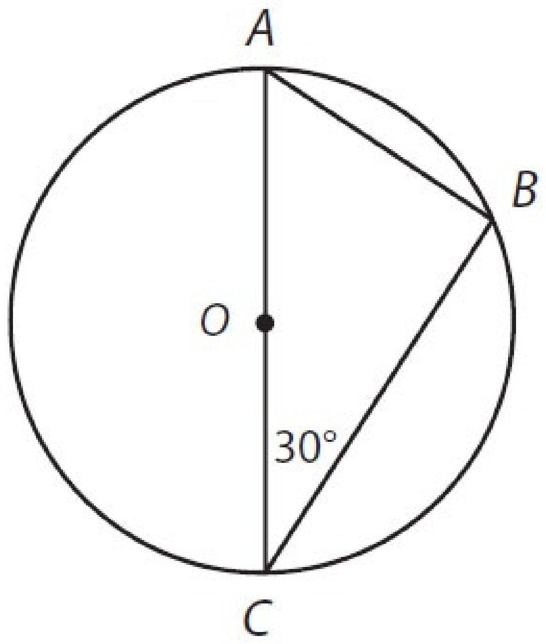


Figure not drawn to scale.

11. The circle with center O has a circumference of $12\pi\sqrt{3}$. If AC is a diameter of the circle, what is the length of line segment AB?

    (A) $3\sqrt{2}$

    (B) 6

    (C) $6\sqrt{3}$

    (D) 18

    (E) $18\sqrt{3}$

12. A batch of widgets costs $p + 15$ dollars for a company to produce and each batch sells for $p(9 - p)$ dollars. For which of the following values of p does the company make a profit?

    (A) 3

    (B) 4

    (C) 5

    (D) 6

    (E) 7

13. If k is the sum of the reciprocals of the consecutive integers from 41 to 60 inclusive, which of the following are less than k?

Indicate all such statements.

- ☐ $\frac{1}{4}$
- ☐ $\frac{1}{3}$
- ☐ $\frac{1}{2}$

14. Half an hour after car A started traveling from Newtown to Oldtown, a distance of 62 miles, car B started traveling along the same road from Oldtown to Newtown. The cars met each other on the road 15 minutes after car B started its trip. If car A traveled at a constant rate that was 8 miles per hour greater than car B's constant rate, how many miles had car B driven when they met?

(A) 14

(B) 12

(C) 10

(D) 9

(E) 8

15.

x and y are positive integers such that $x^2 5^y = 10{,}125$

Quantity A	**Quantity B**
x^2	5^y

---

16. Which of the following is equal to $\dfrac{-2}{\sqrt{n-1}-\sqrt{n+1}}$ for all values of $n >$ 1?

(A) –1

(B) 1

(C) $2\left(\sqrt{n-1}+\sqrt{n+1}\right)$

(D) $\sqrt{n-1}+\sqrt{n+1}$

(E) $\dfrac{\sqrt{n-1}}{\sqrt{n+1}}$

17. Bank account A contains exactly x dollars, an amount that will decrease by 10% each month for the next two months. Bank account B contains exactly y dollars, an amount that will increase by 20% each month for the next two months. If A and B contain the same amount at the end of two

months, what is the ratio of $\sqrt{x}$ to $\sqrt{y}$?

(A) 4 : 3

(B) 3 : 2

(C) 16 : 9

(D) 2 : 1

(E) 9 : 4

Body Mass Index (BMI) is calculated by the formula $\dfrac{703w}{h^2}$, where w is weight in pounds and h is height in inches.

	Quantity A	**Quantity B**
18.	The number of pounds gained by a 74-inch-tall person whose BMI increased by 1.0	The number of pounds lost by a 65-inch-tall person whose BMI decreased by 1.2

19. How many times does the digit grouping “57” (in that order) appear in all of the five-digit positive integers? For instance, “57” appears once in 12,357, twice in 57,057, and does not appear in 24,675.

(A) 279

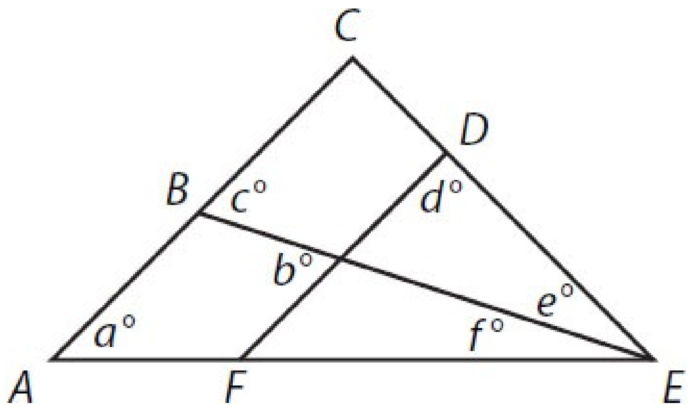
(B) 3,000

(C) 3,471

(D) 3,700

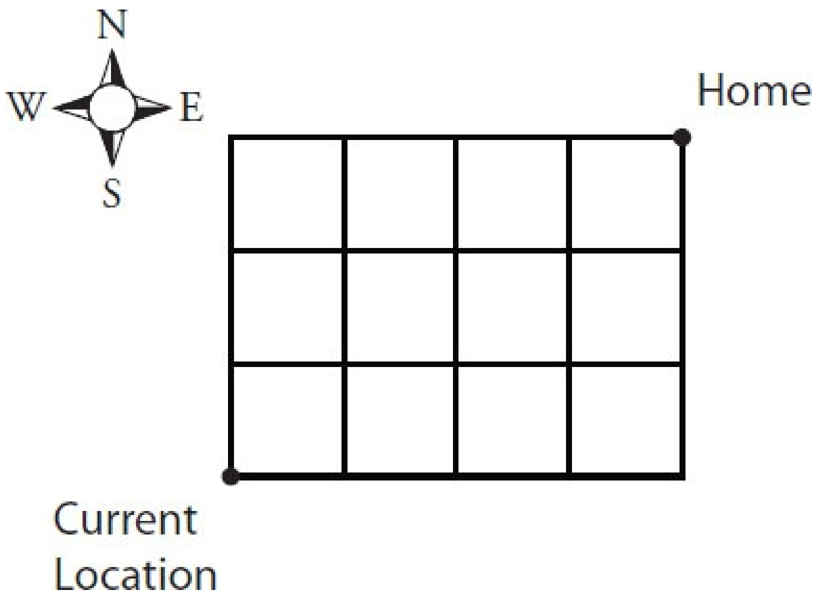
(E) 4,029

	Quantity A	**Quantity B**
20.	The average of all the multiples of 5 between 199 and 706	The average of all the multiples of 10 between 199 and 706



$AC \parallel FD$

	Quantity A	**Quantity B**
21.	$a + d - c - 90$	$90 - e - b - f$



22. A man travels to his home from his current location on the rectangular grid shown above. If he may choose to travel north or east at any corner, but may never travel south or west, how many different paths can the man take to get home?

    (A) 12

    (B) 24

    (C) 32

    (D) 35

    (E) 64

23. A bag contains 3 white, 4 black, and 2 red marbles. Two marbles are drawn from the bag. If replacement is not allowed, what is the probability that the second marble drawn will be red?

    (A) $\frac{1}{36}$

    (B) $\frac{1}{12}$

    (C) $\frac{7}{36}$

    (D) $\frac{2}{9}$

(E) $\frac{7}{9}$

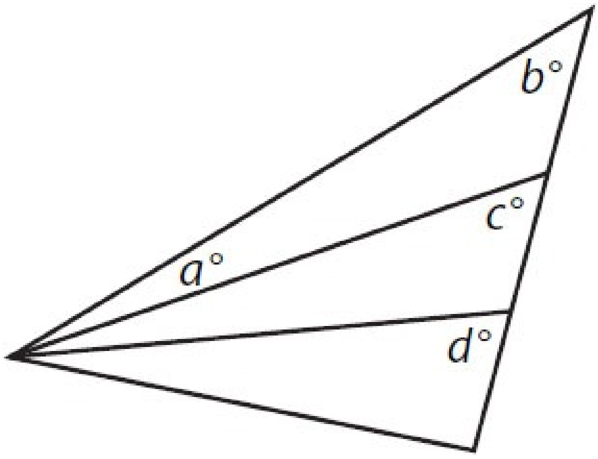
$x < 0$

	Quantity A	**Quantity B**
24.	$((25^{x})^{-2})^{3}$	$((5^{-3})^{2})^{-x}$

	Quantity A	**Quantity B**
25.	The sum of all the multiples of 6 between –126 and 342, inclusive	8,502

x is an integer.

	Quantity A	Quantity B
26.	$(-1)^{x^2} + (-1)^{x^3} + (-1)^{x^4}$	$(-1)^{x} + (-1)^{2x} + (-1)^{3x} + (-1)^{4x}$

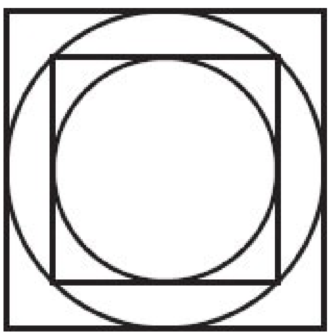


$d > c$

	Quantity A	Quantity B
27.	a	$d - b$

The circumference of a circle is $\frac{7}{8}$ the perimeter of a square.

	Quantity A	Quantity B
28.	The area of the square	The area of the circle



Quantity A	Quantity B
The ratio of the area of the larger square to the area of the smaller	Twice the ratio of the area of the smaller circle to the area of the

	Quantity A	Quantity B
29.	square	larger circle

$$m = 2^{16}3^{17}4^{18}5^{19}$$
$$n = 2^{19}3^{18}4^{17}5^{16}$$

	Quantity A	**Quantity B**
30.	The number of zeros at the end of *m* when written in integer form	The number of zeros at the end of *n* when written in integer form

The sequence of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$ is defined by

$a_n = 2^n - \dfrac{1}{2^{n-33}}$ for each integer $n \geq 1$.

	Quantity A	**Quantity B**
31.	The sum of the first 32 terms of this sequence	The sum of the first 31 terms of this sequence

---

32. Each of 100 balls has an integer value from 1 to 8, inclusive, painted on the surface. The number n_x of balls representing integer x is defined by the formula $n_x = 18 - (x - 4)^2$. What is the interquartile range of the 100 integers?

(A) 1.5

(B) 2.0

(C) 2.5

(D) 3.0

(E) 3.5

The operator ! is defined such that $a!b = a^b \times b^{-a}$.

	Quantity A	**Quantity B**
33.	$\dfrac{(x!4)}{(4!x)}$	$\dfrac{x^8}{16^x}$

---

34. What is the ratio of the sum of the odd positive integers between 1 and 100, inclusive, and the sum of the even positive integers between 100 and 150, inclusive?

(A) 2 to 3

(B) 5 to 7

(C) 10 to 13

(D) 53 to 60

(E) 202 to 251

35. For integer $n \geq 3$, a sequence is defined as $a_n = (a_{n-1})^2 - (a_{n-2})^2$ and $a_n > 0$ for all positive integers n. The first term a_1 is 2, and the fourth term is equal to the first term multiplied by the sum of the second and third terms. What is the third term, a_3?

(A) 0

(B) 3

(C) 5

(D) 10

(E) 16

36. In a certain sequence, each term beyond the second term is equal to the average of the previous two terms. If a_1 and a_3 are positive integers, which of the following is <u>not</u> a possible value of a_5?

(A) $-\frac{9}{4}$

(B) 0

(C) $\frac{9}{4}$

(D) $\frac{75}{8}$

(E) $\frac{41}{2}$

37. The operator @ is defined by the following expression: $a@b = \left|\frac{a+1}{a}\right| - \frac{b+1}{b}$ where $ab \neq 0$. What is the sum of the solutions to the equation $x@2 = \frac{x@(-1)}{2}$?

(A) –1

(B) –0.75

(C) –0.25

(D) 0.25

(E) 0.75

x is a non-negative number and the square root of $(10 – 3x)$ is greater than x.

	Quantity A	**Quantity B**
38.	$\lvert x \rvert$	2

The area of an equilateral triangle is greater than $25\sqrt{3}$ but less than $36\sqrt{3}$.

	Quantity A	**Quantity B**
39.	The length of one of the sides of the triangle	9

40. The inequality $|8 - 2x| < 3y - 9$ is equivalent to which of the following?

(A) $2x < \dfrac{(17-3y)}{2}$

(B) $3y + 2x > 1$

(C) $6y - 2 < 2x$

(D) $1 - y < 2x < 17 + y$

(E) $3y - 1 > 2x > 17 - 3y$

In the sport of mixed martial arts, more than 30% of all fighters are skilled in both the Muy Thai and Brazilian Jiu Jitsu styles of fighting. 20% of the fighters who are not skilled in Brazilian Jiu Jitsu are skilled in Muy Thai. 60% of all fighters are skilled in Brazilian Jiu Jitsu.

	Quantity A	**Quantity B**
41.	The percent of fighters who are skilled in Muy Thai	37%

The rate of data transfer, *r*, over a particular network is directly proportional to the bandwidth, *b*, and inversely proportional to the square of the number of networked computers, *n*.

	Quantity A	**Quantity B**
42.	The resulting rate of data transfer if the bandwidth is quadrupled and the number of networked computers is more than tripled	$\frac{4}{9}r$

Advanced Quant Answers

1. **(C).** From 7:00:01 pm to 7:03:01, 84 people enter the first train (21 per minute). These 84 people will depart at 7:04.

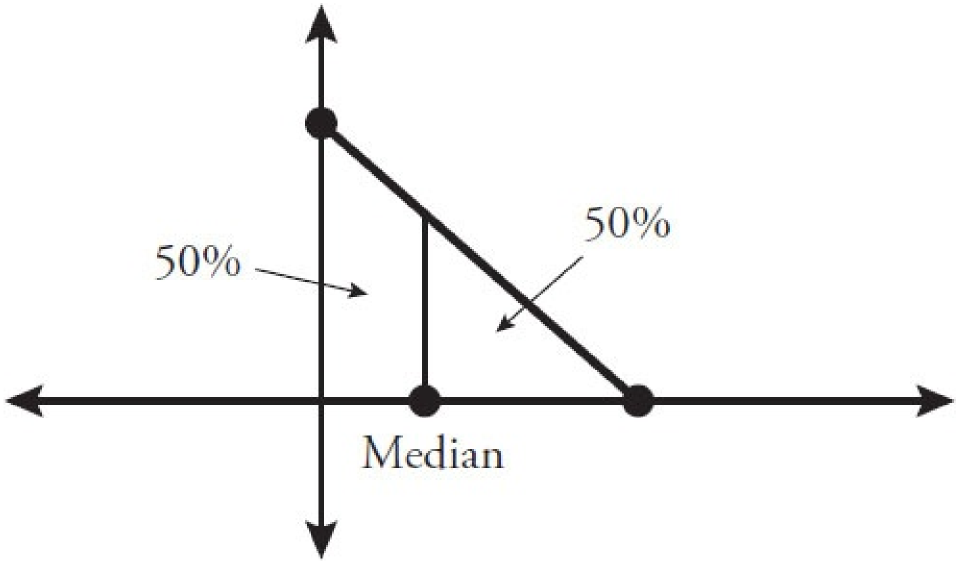
After that, for each 9-minute period, 9(21) = 189 people will enter the next train. These trains will leave at 7:13, 7:22, 7:31, 7:40, 7:49, 7:58, 8:07, 8:16, and 8:25.

Since 9 trains each have 189 people and the first train has 84 people, the average is:

$$\frac{9(189)+1(84)}{10}=178.5$$

Note that the strange time format (minutes and seconds) doesn't make the problem any harder—the problem is actually clearer because people board the train at 1 second after each minute, whereas the train leaves at the beginning of each minute.

2. **(C).** A continuous probability distribution has a total area of 100%, or 1, underneath the entire curve. The median of such a distribution splits the area into two equal halves, with 50% of the area to the left of the median and the other 50% to the right of the median:



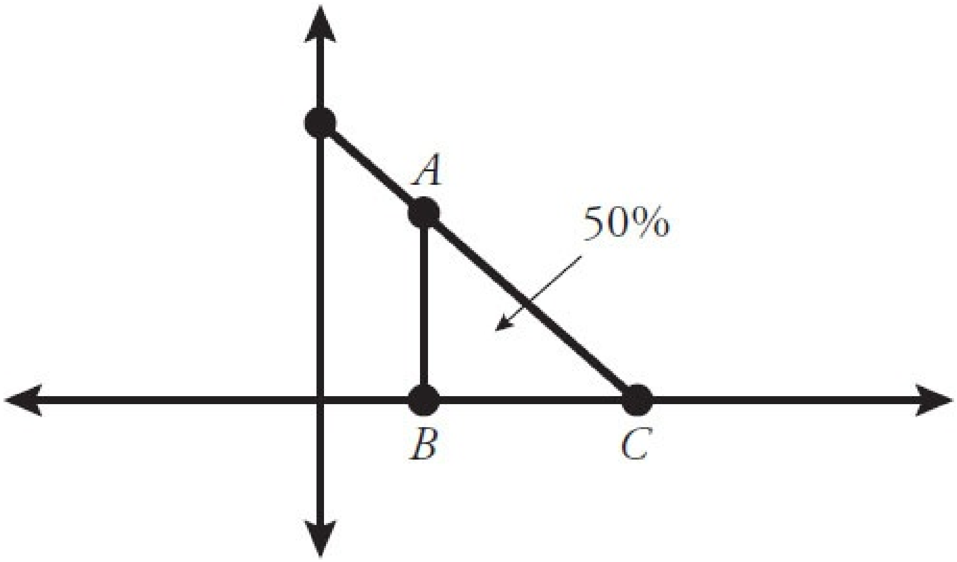
In simpler terms, the random variable *x* has a 50% chance of being above the median and a 50% chance of being below the median. You can ignore the regions to the right or the left of this triangle, since the probability that *x* could fall in either of those regions is zero. So the question becomes this:

what point on the *x*-axis will divide the large right triangle into two equal areas?

One shortcut is to note that the area of the large isosceles right triangle must be 1, which equals the total area under any probability distribution curve. Confirm by means of the area formula for this right triangle:

$$\frac{1}{2}bh = \frac{1}{2}\left(\sqrt{2}\right)\left(\sqrt{2}\right) = \frac{2}{2} = 1.$$

The quickest way to find the median is to consider the *small* isosceles right triangle, *ABC*, as shown:



Triangle *ABC* must have an area of $\frac{1}{2}$. So what must be the length of each of its legs, *AB* and *BC*? From the formula $\frac{1}{2}bh = \frac{1}{2}$, and noting that the base *BC* equals the height *AB*, the base *BC* must be 1 (the same as the height). Since the coordinates of point *C* are ($\sqrt{2}$, 0), the coordinates of point *B* must be ($\sqrt{2}$ – 1, 0). That is, the median is $\sqrt{2}$ – 1.

3. **(B).** One way to solve is to set up an implied equation or inequality, then make the same changes to both quantities, and finally compare after simplifying:

Quantity A		**Quantity B**	
$x^2 - 5x + 6$	?	$x^2 - 9x + 20$	
$-(x^2 - 5x + 6)$		$-(x^2 - 5x + 6)$	
0	?	$-9x - (-5x) + 20 - 6$	Notice that x^2 is common to both quantities, so it can be ignored (i.e., it cancels).
0	?	$-9x + 5x + 14$	
0	$<$	$-4x + 14$	

Because *x* is negative, –4*x* + 14 = –4(*neg*) + 14 = *pos* + 14, which is greater than 0.

Another way to solve is to factor and then compare based on number properties. Quantity A factors to (*x* – 2)(*x* – 3). Quantity B factors to (*x* – 4)(*x* – 5). Because *x* is negative, "*x* minus a positive number" is also negative. Each quantity is the product of two negative numbers, which is positive:

Quantity A: $(x - 2)(x - 3)$ = (*neg*)(*neg*) = *pos*
Quantity B: $(x - 4)(x - 5)$ = (more *neg*)(more *neg*) = more *pos*

Quantity B is greater.

4. **(A).** 24 to any power ends in the same units digit as 4 to the same power (if considering only the last digit of the product, consider only the last digits of the numbers being multiplied).

4 to any power ends in either 4 or 6 ($4^1 = 4$, $4^2 = 16$, $4^3 = 64$, etc.). If the power is odd, the answer ends in 4; if the power is even, the answer ends in 6. Since the exponent $5 + 2x$ is odd for any integer x, $24^{5 + 2x}$ ends in 4.

36 to any power ends in the same units digit as 6 to the same power. Powers of 6 always end in 6, so 36^6 ends in 6.

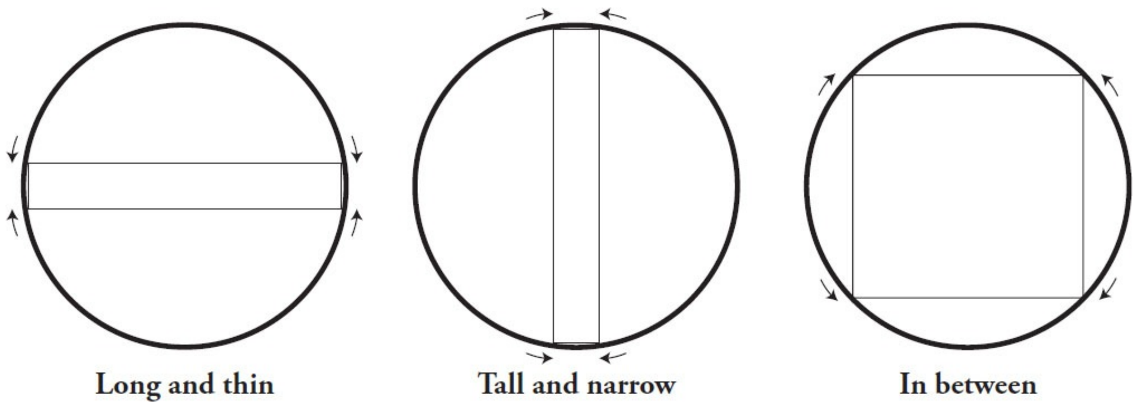
17 to any power ends in the same units digit as 7 to the same power. While the units digits of the powers of 7 do indeed create a pattern, 7^3 is just 343, which ends in 3. Thus:

$24^{5 + 2x}$ ends in 4
36^6 ends in 6
7^3 ends in 3

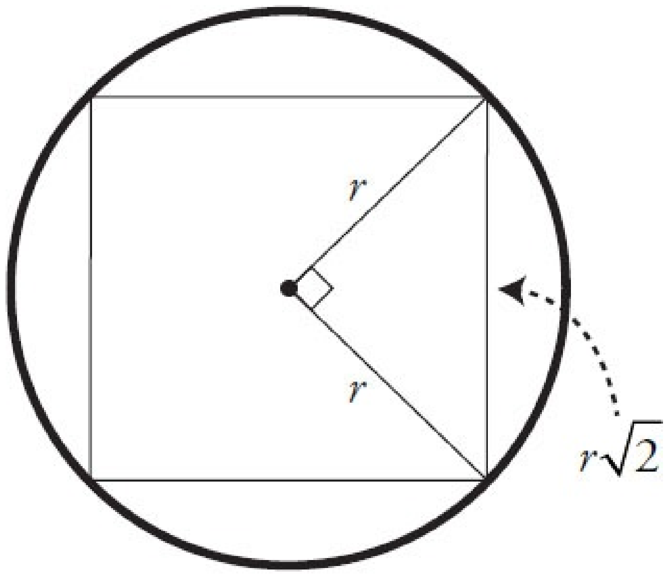
Multiplying three numbers that end in 4, 6, and 3 yields answer that ends in 2, because $(4)(6)(3) = 72$, which ends in 2.

5. **(B).** What are the possibilities for the inscribed rectangle?

The inscribed rectangle can be stretched and pulled to extremes: extremely long and thin, extremely tall and narrow, and somewhere in between:



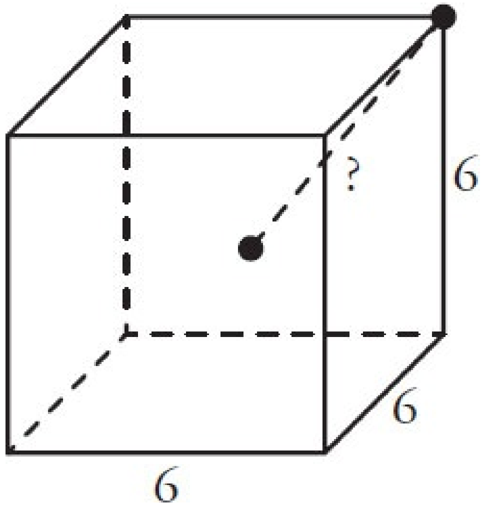
The "long and thin" and "tall and narrow" rectangles have a very small area, and the "in between" rectangle has the largest possible area. In fact, the largest possible rectangle inscribed inside a circle is a square:



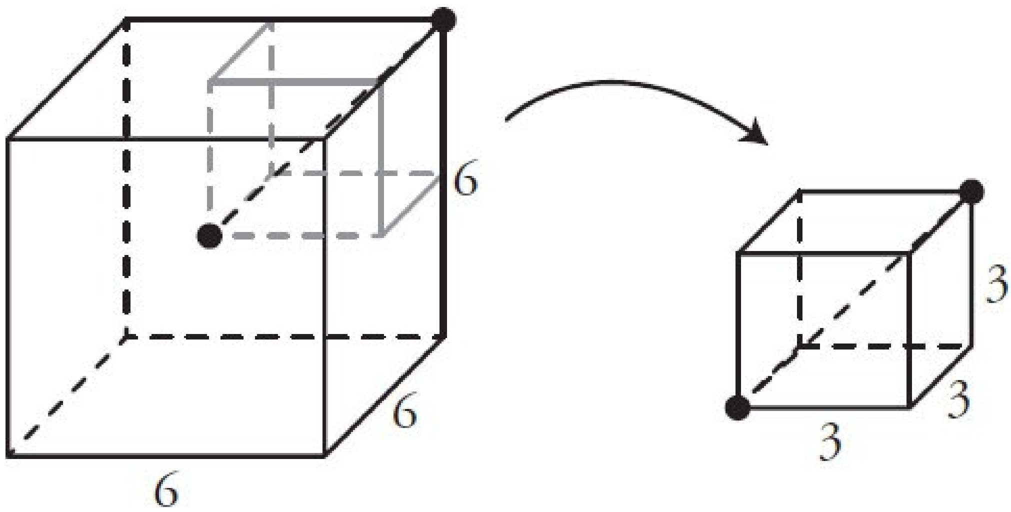
Square: maximal area

In this problem, the circumference is equal to $20\pi = 2\pi r$. Thus $r = 10$. The square then has a side length of $10\sqrt{2}$ and an area of $(10\sqrt{2})^2 = 200$.

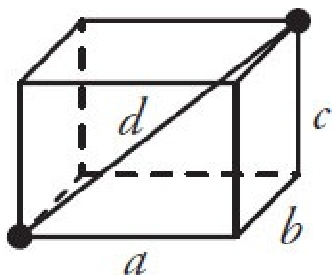
6. **(C).**



The length of any side of the cube is 6, and the question asks for the distance between the center of the cube and any of its vertices (corners). Chopping up the cube into 8 smaller cubes, the distance from the center of the 6 × 6 × 6 cube to any corner is the diagonal of a 3 × 3 × 3 cube.



There are several ways to find the diagonal of a cube. Probably the fastest is to use the Super Pythagorean theorem, which extends to three dimensions:



$a^2 + b^2 + c^2 = d^2$

In the special case when the three sides of the box are equal, as they are in a cube, then this is the equation, letting *s* represent any side of the cube:

$$
\begin{aligned}
s^2 + s^2 + s^2 &= d^2 \\
3s^2 &= d^2 \\
s\sqrt{3} &= d
\end{aligned}
$$

Since $s = 3$, $d = 3\sqrt{3}$.

7. $\frac{3}{4}$ **(or any equivalent fraction).** Probability is $\frac{(\text{favorable outcomes})}{(\text{total \# of possibilities})}$. There are 99 – 20 + 1 = 80 possible values for *c*, so the unknown is how many of these *c* values yield a $c^3 - c$ that is divisible by 12.

The prime factorization of 12 is 2 × 2 × 3. There are several ways of thinking about this: numbers are divisible by 12 if they are divisible by 3 and by 2 twice, or if they are multiples of both 4 and 3, or if half of the number is an even multiple of 3, etc.

The expression involving *c* can be factored:

$$c^3 - c = c(c^2 - 1) = c(c - 1)(c + 1)$$

These are consecutive integers. It may help to put them in increasing order: $(c - 1)c(c + 1)$. Thus, this question has a lot to do with *consecutive integers*, and not only because the integers 20 to 99 themselves are consecutive.

In any set of three consecutive integers, a multiple of 3 will be included. Thus, $(c - 1)c(c + 1)$ is always divisible by 3 for any integer *c*. This takes care of part of the 12. So the question becomes "How many of the possible $(c - 1)c(c + 1)$ values are divisible by 4?" Since the prime factors of 4 are 2's, it makes sense to think in terms of odds and evens.

$(c - 1)c(c + 1)$ could be (E)(O)(E), which is definitely divisible by 4, because the two evens would each provide at least one separate factor of 2. Thus, $c^3 - c$ is divisible by 12 whenever *c* is odd, which are the cases *c* = 21, 23, 25, …, 95, 97, 99. That's $\left(\frac{(99-21)}{2}\right)+1=\left(\frac{78}{2}\right)+1=40$ possibilities.

Alternatively, $(c - 1)c(c + 1)$ could be (O)(E)(O), which will only be divisible by 4 when the even term itself is a multiple of 4. Thus, $c^3 - c$ is also divisible by 12 whenever *c* is a multiple of 4, which are the cases *c* = 20, 24, 28, …, 92, 96. That's $\left(\frac{(96-20)}{4}\right)+1=\left(\frac{76}{4}\right)+1=20$ possibilities.

The probability is thus $\frac{(40+20)}{80}=\frac{60}{80}=\frac{3}{4}$.

8. **0.** Since the remainder is defined as what is left over after one number is divided by another, it makes sense that the leftover amount would be positive. So why is this information provided, if the remainder is "automatically"

positive? Because there is a third possibility: that the remainder is 0. If the remainder when 120 is divided by *m* is positive, the salient point is that $\frac{120}{m}$ does not have a remainder of 0. In other words, 120 is not divisible by *m*, or *m* is not a factor of 120. Similarly, *n* is not a factor of 120.

Another constraint on both *m* and *n* is that they are single-digit positive integers. So *m* and *n* are integers between 1 and 9, inclusive, that are not factors of 120. Only two such possibilities exist: 7 and 9.

Since $m > n$, $m = 9$ and $n = 7$. Thus, $m - n = 2$, and the remainder when 120 is divided by 2 is 0.

9. **50 centimeters.** Microchip radius = (2 cm)$\left(10\frac{mm}{cm}\right)$ = 20 mm.

Microchip diameter = 40 mm.

Blueprint diameter = 1 cm on blueprint per every 0.8 mm on the microchip:

$$= \left(\frac{1\text{ cm on blueprint}}{0.8\text{ mm on microchip}}\right)(40\text{ mm diameter on microchip})$$

$$= (1\text{ cm on blueprint})\left(\frac{40}{0.8}\right)$$

$$= (1\text{ cm on blueprint})\left(\frac{400}{8}\right)$$

$$= 50\text{ cm on blueprint}$$

10. **(D).** If $PV = kT$, then $P = \frac{kT}{V}$. Quantity A is $P = \frac{k(32)}{(20)} = \frac{8}{5}k$.

If $PV = kT$, then $T = \frac{PV}{k}$. Quantity B is $T = \frac{(78)(10)}{k} = \frac{780}{k}$.

Don't rush to judgment, thinking that $780 > \frac{8}{5}$ means that Quantity B is greater. Notice that the k term is in the numerator of one quantity (so Quantity A increases with k) and the denominator of the other (so the larger k is, the smaller Quantity B is).

If $k = 1$, then Quantity B is greater $\left(780 > \frac{8}{5}\right)$. But if $k = 100$, Quantity A is greater ($160 > 7.8$). The relationship cannot be determined from the information given.

11. **(C).** Since AC is a diameter of the circle, triangle ABC is a right triangle and angle ABC is a right angle. This means that angle CAB is 60°, and the ratios of a 30–60–90 triangle can be used to solve the problem.

The circumference of the circle, $\pi d = 12\pi\sqrt{3}$, so the diameter, which is also

AC, is $12\sqrt{3}$.

Now use ratios (specifically via the unknown multiplier *x* to find *AB*):

	AB	***BC***	***AC***
Basic Ratio	$1x$	$x\sqrt{3}$	$2x$
Known Side			$12\sqrt{3}$
Unknown Multiplier			$x = \dfrac{12\sqrt{3}}{2} = 6\sqrt{3}$
Compute Sides	$6\sqrt{3}$	$(6\sqrt{3})(\sqrt{3}) = 18$	$12\sqrt{3}$

Line segment *AB* has a length of $6\sqrt{3}$.

Alternatively, you could use estimation here if you forgot the ratios for 30–60–90 triangles. Since the longest side is always opposite the largest angle (and the shortest opposite the smallest), the sequence of sides must be $AC > BC > AB$. The diameter $AC = 12\sqrt{3} \approx 12(1.7) \approx 20.4$. From the looks of it, AB is about $\frac{1}{2}$ of AC's length (definitely not close to the length of AC), so answer choices (D) and (E) seem too long. Choices (A) and (B) are each less than $\frac{1}{3}$ of AC, which doesn't seem long enough.

12. **(B).** Profit equals revenue minus cost. The company's profit is:

$$
\begin{aligned}
p(9-p)-(p+15) &= 9p-p^2-p-15 \\
&= -p^2+8p-15 \\
&= -(p^2-8p+15) \\
&= -(p-5)(p-3)
\end{aligned}
$$

Profit will be zero if $p = 5$ or $p = 3$, which eliminates answers (A) and (C). For $p > 5$, both $(p-5)$ and $(p-3)$ are positive. In that case, the profit is negative (i.e., the company loses money). The profit is only positive if $(p-5)$ and $(p-3)$ have opposite signs, which occurs when $3 < p < 5$.

Alternatively, plug in the answer choices to see which value corresponds to a revenue that is higher than cost.

(A) Cost = 18, Revenue = 18, Profit = 0   Incorrect
(B) Cost = 19, Revenue = 20, Profit = 1   Correct
(C) Cost = 20, Revenue = 20, Profit = 0   Incorrect
(D) Cost = 21, Revenue = 18, Profit < 0   Incorrect
(E) Cost = 22, Revenue = 14, Profit < 0   Incorrect

13. $\frac{1}{4}$ **and** $\frac{1}{3}$ **only.** The sum

$\left(\frac{1}{41}+\frac{1}{42}+\frac{1}{43}+\frac{1}{44}+\cdots+\frac{1}{57}+\frac{1}{58}+\frac{1}{59}+\frac{1}{60}\right)$ has 20 fractional terms. It would be nearly impossible to compute if you had to find a common denominator and solve without a calculator and a lot of time. Instead, look at the maximum and minimum possible values for the sum.

Maximum: The largest fraction in the sum is $\frac{1}{41}$; k is definitely smaller than $20 \times \frac{1}{41}$, which is itself smaller than $20 \times \frac{1}{40} = \frac{1}{2}$.

Minimum: The smallest fraction in the sum is $\frac{1}{60}$; k is definitely larger than $20 \times \frac{1}{60} = \frac{1}{3}$.

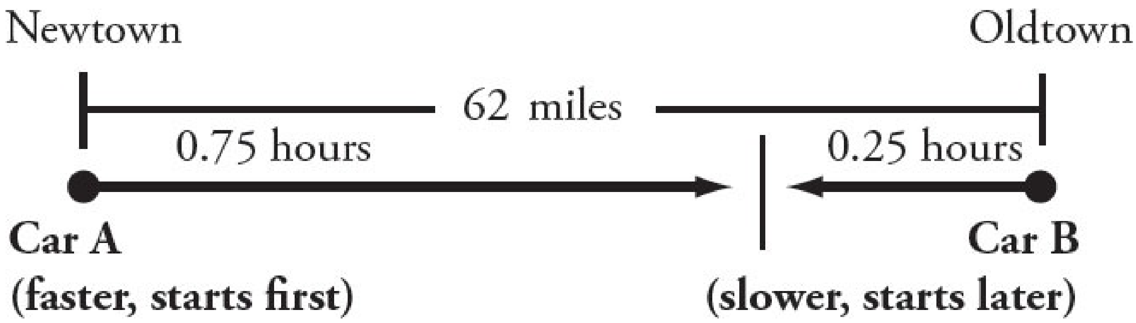
Therefore, $\frac{1}{3} < k < \frac{1}{2}$.

I. $\frac{1}{4} < \frac{1}{3} < k$ YES

II. $\frac{1}{3} < k$ YES

III. $\frac{1}{2} > k$ NO

14. **(A).** Draw a diagram to illustrate the moment at which car A and car B pass each other moving in opposite directions:



You could test the answer choices:

	B's distance (miles)	B's rate (mph) $= \frac{D}{T} = \frac{D}{0.25}$	A's rate (mph) = B's Rate + 8	A's distance (miles) $= R \times T$ $= R \times 0.75$	Total distance
(A)	**14**	**56**	**64**	**48**	**62**
(B)	12	48	56	42	54
(C)	10	40	48	36	46
(D)	9	36	44	33	42
(E)	8	32	40	30	38

Or solve algebraically, using an *RTD* chart. Convert 15 minutes to $\frac{1}{4}$ (or 0.25) hours:

	Rate	Time	Distance
Car A	$(r + 8)$ mph	0.75 hours	$(0.75)\ (r + 8)$ miles
Car B	r mph	0.25 hours	$0.25r$ miles
Total			62 miles

Set up and solve an equation for the total distance:

$$\begin{aligned} (0.75)(r + 8) + (0.25r) &= 62 \\ 0.75r + 6 + 0.25r &= 62 \\ r &= 56 \end{aligned}$$

Therefore, car B traveled a distance of $0.25r = (0.25)(56) = 14$ miles.

15. **(D).** Factor 10,125 to its prime factors: $10{,}125 = 3^4 5^3$.

So, $x^2 5^y = 3^4 5^3$.

In order to have 5^3 on the right side, there have to be three factors of 5 on the left side. All three could be in the 5^y term (i.e., y could equal 3). Or, one of the 5's could be in the 5^y term, and two of the 5's in the x^2 term (i.e., y could equal 1 and x could have a single factor of 5).

In order to have 3^4 on the right side, x^2 must have $3^4 = (3^2)^2$ as a factor. In other words, x must have 3^2 as a factor, because 3^2 is certainly not a factor of 5. Thus, x is a multiple of 9.

The possibilities:

Quantity A: x^2		Quantity B: 5^y	Check: The product must be 10,125	Check: Quantity A must be a perfect square	Check: Quantity B must be a power of 5
$x^2 = 9^2 = 81$	<	$5^y = 5^3 = 125$	(81)(125) = 10,125	Yes	Yes
$x^2 = (9 \times 5)^2 = 2{,}025$	>	$5^y = 5^1 = 5$	(2,025)(5) = 10,125	Yes	Yes

In one case, Quantity A is greater. In the other, Quantity B is greater. The relationship cannot be determined from the information given.

16. **(D).** Since there are variables in the answer choices, pick a number and test the choices. If $n = 2$, then

$$\frac{-2}{\sqrt{n-1}-\sqrt{n+1}} = \frac{-2}{\sqrt{2-1}-\sqrt{2+1}} = \frac{-2}{1-\sqrt{3}} \approx \frac{-2}{1-1.7} \approx \frac{-2}{-0.7},$$

which is greater than 2 (around 2.86). Now test the answer choices to see which one matches the target:

(A) –1 Too low

(B) 1 Too low

(C) $2\left(\sqrt{n-1}+\sqrt{n+1}\right) = 2\left(\sqrt{2-1}+\sqrt{2+1}\right) \approx 2(1+1.7)$ Too high

(D) $\sqrt{n-1}+\sqrt{n+1} = \sqrt{2-1}+\sqrt{2+1} \approx 1+1.7 \approx 2.7$ Correct

(E) $\dfrac{\sqrt{n-1}}{\sqrt{n+1}} = \dfrac{\sqrt{2-1}}{\sqrt{2+1}} \approx \dfrac{1}{1.7}$ Too low

Alternatively, solve this problem algebraically. The expression is in the form of $\dfrac{-2}{a-b}$, where $a = \sqrt{n-1}$ and $b = \sqrt{n+1}$.

Either simplify or cancel the denominator, as none of the answer choices have the same denominator as the original, and most of the choices have no denominator at all. To be able to manipulate a denominator with radical signs, first try to eliminate the radical signs entirely, leaving only a^2 and b^2 in the denominator. To do so, multiply by a fraction that is a convenient form of 1:

$$\frac{-2}{a-b}=\frac{-2}{a-b}\times\frac{(a+b)}{(a+b)}=\frac{-2(a+b)}{a^2-b^2}$$

Notice the “difference of two squares” special product created in the denominator with the choice of (*a* + *b*).

Substitute for *a* and *b*:

$$\frac{-2}{\sqrt{n-1}-\sqrt{n+1}}\times\frac{\sqrt{n-1}+\sqrt{n+1}}{\sqrt{n-1}+\sqrt{n+1}}=\frac{-2\left(\sqrt{n-1}+\sqrt{n+1}\right)}{(n-1)-(n+1)}=\frac{-2\left(\sqrt{n-1}+\sqrt{n+1}\right)}{-2}=\sqrt{n-1}+\sqrt{n+1}$$

The correct answer is (D).

17. **(A).** First, note the answer pairs (A) and (C), and (B) and (E), in which one ratio is the square of the other. This represents a likely trap in a problem that asks for the ratio of $\sqrt{x}$ to $\sqrt{y}$ rather than the more typical ratio of *x* to *y*. It is fairly safe to eliminate (D) as it is not paired with a trap answer and therefore probably not the correct answer. You should also suspect that the correct answer is (A) or (B), the "square root" answer choice in their respective pairs.

For problems involving successive changes in amounts—such as population growth problems, or compound interest problems—it is helpful to make a table:

	Account A	**Account B**
Now	x	y
After 1 month	$\left(\frac{9}{10}\right)x$	$\left(\frac{12}{10}\right)y$
After 2 months	$\left(\frac{9}{10}\right)\left(\frac{9}{10}\right)x=\left(\frac{81}{100}\right)x$	$\left(\frac{12}{10}\right)\left(\frac{12}{10}\right)y=\left(\frac{144}{100}\right)y$

If the accounts have the same amount of money after two months, then:

$$\left(\frac{81}{100}\right)x = \left(\frac{144}{100}\right)y$$

$$81x = 144y$$

This can be solved for $\frac{\sqrt{x}}{\sqrt{y}}$:

$$\frac{x}{y}=\frac{144}{81}$$

$$\frac{\sqrt{x}}{\sqrt{y}}=\frac{\sqrt{144}}{\sqrt{81}}=\frac{12}{9}=\frac{4}{3}$$

18. **(A).** First, note that the height of each person in question is fixed (no one grew taller or shorter); only weights changed. Second, note that BMI is always positive and is proportional to *w*; as weight increases, BMI increases, and vice versa. So the language of the quantities—“pounds gained … BMI increased” and “pounds lost … BMI decreased”—is aligned with this proportionality. Both quantities are a positive number of pounds.

Since BMI = $\frac{703w}{h^2}$, change in BMI =

$$\frac{703w_{before}}{h^2} - \frac{703w_{after}}{h^2} = \frac{703}{h^2}\left(w_{before} - w_{after}\right).$$

To simplify things, you can write this in terms of ΔBMI and Δw, the positive change in BMI and weight, respectively:

$$\Delta BMI = \frac{703}{h^2}\Delta w$$

(The triangle symbol indicating positive change in a quantity does not appear on the GRE—it is used here for convenience in notating an explanation.)

Since the quantities both refer to Δw, rewrite the relationship as $\Delta w = \frac{h^2}{703}\Delta BMI$. Both ΔBMI and h are given in each quantity, so Δw can be calculated and the relationship between the two quantities determined. (The answer is definitely not (D).)

Quantity A	**Quantity B**
$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{74^2}{703}(1.0) = \frac{74^2}{703}$	$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{65^2}{703}(1.2)$

Since the 703 in the denominator is common to both quantities, the comparison is really between $74^2 = 5{,}476$ and $65^2(1.2) = 4{,}225(1.2) = 5{,}070$. Quantity A is greater.

19. **(D).** There are four different cases that you must count: 5 7 _ _ _ ; _ 5 7 _ _ ; _ _ 5 7 _; and _ _ _ 5 7. In the case of 5 7 _ _ _, all three of the empty spaces can have any digit from 0–9, which is 10 possibilities, for a total of 10 × 10 × 10 = 1,000 possible numbers. In the case of _ 5 7 _ _, there are only 9 choices for the first digit since you cannot put a zero there if it is to be a five-digit positive integer. For the last two digits any number from 0–9 is still allowed, for a total of 9 × 10 × 10 = 900 possible numbers for the second case. The third and fourth cases are similar to the second; both include 900 possible numbers. Summing up the four cases, this adds up to 1,000 + 900(3) = 3,700 such integers. Note that this method will double count any integer that has two instances of the grouping "57" in it. For example, 57,357 will be counted both in the case of 57 _ _ _ and in the case of _ _ _ 57. In total, there are 10 ways a number could be counted both in the 57 _ _ _ and the _ _ _ 57 cases; there are 10 ways a number could be counted both in the 57 _ _ _ and the _ _ 57 _ cases; and there are 9 ways a number could be counted both in the _ 57 _ _ and _ _ _ 57 cases. This leaves 10 + 10 + 9 = 29 integers that are double-counted. However, it is fine that they are double-counted, because the question asks for the number of times the grouping "57" appears. These integers contain that grouping twice, so they should be counted twice, and the correct answer is 3,700.

20. **(A).** To find the average of any evenly spaced set, take the average of the first and last values. For the case of Quantity A, the first multiple of 5 is 200

and the last multiple of 5 is 705. The average then is $\frac{200+705}{2}=452.5$.

For Quantity B, the first multiple of 10 is also 200, however, the last multiple is 700, thus the average is $\frac{200+700}{2}=450$. Quantity A is greater.

21. **(C).** Set up an implied inequality and perform identical operations on each quantity, grouping variables:

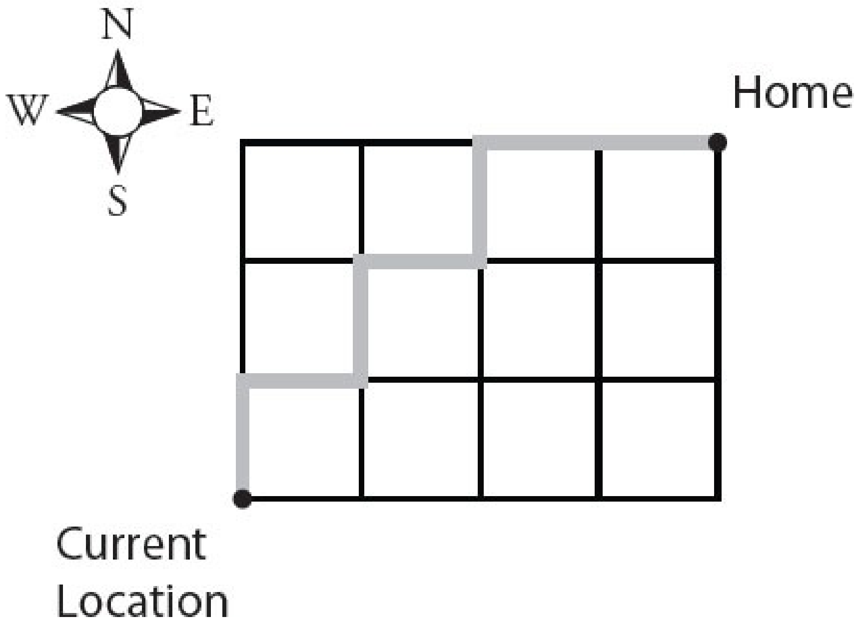
Quantity A		**Quantity B**
$a + d - c - 90$	?	$90 - e - b - f$
$a + d - c$	?	$180 - e - b - f$
$a + d - c + e + b + f$	?	180
$(a + d + e + f) + (b - c)$	?	180

In the last step above, only the order of the variables was changed and parentheses added to group certain terms. Notice that the angle at point *C* and point *D* is the same, as *AC* and *FD* are parallel lines intersected by the transversal *CE*. So, the first set of parentheses holds the sum of the interior angles of the biggest triangle *ACE*, which is 180. Also because *AC* and *FD* are parallel lines intersected by transversal *BE*, $b = c$, so $b - c = 0$ in the second set of parentheses.

Quantity A		Quantity B
$(a + d + e + f) + (b - c)$	?	180
$(180) + (0)$	=	180

The two quantities are equal.

22. **(D).** Given that the man can only move north and east, he must advance exactly 7 blocks from his current location to get home regardless of which path he takes. Of these 7 blocks, 4 must be moving east and 3 must be moving north. An example path is shown below:



The problem can then be rephrased as follows: "Of the 7 steps, when does the man choose to go east and when does he choose to go north?" Labeling each step as N for north and E for east, you can see the problem as the number of unique rearrangements of NNNEEEE (e.g., this arrangement corresponds to going north 3 times and then east 4 times straight to home). This is determined by $\frac{total!}{repeats!} = \frac{7!}{3!4!} = 35$.

23. **(D).** There are 2 red marbles and 7 not-red marbles. (It is irrelevant whether white or black is drawn; the question is about red or not.) There are two ways in which the second marble drawn is red: either not-red first, then red second OR red first, and then red again. Using $P(R)$ to indicate the probability of drawing red and $P(\text{not } R)$ to indicate the probability of drawing not-red, we have:

$$P(\text{not } R) \times P(R) = \left(\frac{7}{9}\right)\left(\frac{2}{8}\right) = \frac{14}{72}$$

$$P(R) \times P(R) = \left(\frac{2}{9}\right)\left(\frac{1}{8}\right) = \frac{2}{72}$$

Since a red on the second draw could happen one way or the other, sum these probabilities:

$$\frac{14}{72} + \frac{2}{72} = \frac{16}{72} = \frac{2}{9}\ ..$$

24. **(A).** Simplify both quantities, remembering that a power to a power means multiply the exponents. Also, 25 is 5 squared, so you can substitute, putting both quantities in terms of a base of 5:

Quantity A: $((25^x)^{-2})^3 = 25^{-6x} = (5^2)^{-6x} = 5^{-12x}$
Quantity B: $((5^{-3})^2)^{-x} = 5^{6x}$

Typically, when comparing exponents with the same base, the one with the larger exponent is greater. It might be tempting to conclude that $6x > -12x$, but be careful with negative variables.

If $x = -1$, Quantity A = 5^{12} and Quantity B = 5^{-6}, or $\frac{1}{5^6}$. In this case, Quantity A is much greater.

If $x = -\frac{1}{2}$, Quantity A = 5^6 and Quantity B = 5^{-3}, or $\frac{1}{5^3} = \frac{1}{125}$. Again, Quantity A is much greater.

If $x = -10$, Quantity A = 5^{120} and Quantity B = 5^{-60}. The more negative x gets, the greater the difference between Quantity A and Quantity B becomes. Quantity A is always greater.

Another way to look at it:

Quantity A: $5^{-12x} = 5^{-12 \times negative} = 5^{positive}$

Quantity B: $5^{6x} = 5^{6 \times negative} = 5^{negative}$

Even if $|x|$ is a tiny fraction, that is, the expression is some high order root of 5 such as $\sqrt[8]{5}$ or $\sqrt[100]{5}$, these quantities would approach 1 such that Quantity A is $5^{positive} > 1$ and Quantity B is $5^{negative} < 1$.

Since Quantity A is greater than 1 and Quantity B is less than 1, Quantity A is greater.

25. **(A).** First, note that some of the positive multiples of 6 in Quantity A are canceled out by the negative multiples of 6 in Quantity A (e.g., –126 + 126 = 0, –120 + 120 = 0, etc.).

To find the sum of the remaining multiples of 6 (i.e., 132 through 342, inclusive), find both the number of terms and the average of those terms: sum = (average) × (number of terms).

To find the number of terms, take the last multiple of 6 minus the first multiple of 6, divide by 6 and then add 1:

$$\text{Number of terms (multiples of } n\text{)} = \frac{\textit{last mult of } n - \textit{first mult of } n}{n} + 1$$

$$= \frac{342-(132)}{6}+1 = \frac{210}{6}+1 = 35+1 = 36$$

To find the average of any evenly spaced set, take the average of the first and last values:

$$\text{Average} = \frac{\textit{First} + \textit{Last}}{2} = \frac{132+342}{2} = 237$$

Therefore, the sum is equal to (Average) × (Number of terms) = (237)(36) = 8,532. Quantity A is greater.

26. **(B).** When a negative base is raised to an integer power, the question is about positives and negatives and odds and evens: $(-1)^{odd} = -1$ and $(-1)^{even} = +1$.

If x is even, all of the exponents in this question are even:

Quantity A: $(-1)^{even^2} + (-1)^{even^3} + (-1)^{even^4} = (-1)^{even} + (-1)^{even} + (-1)^{even} = 1 + 1 + 1 = 3$
Quantity B: $(-1)^{even} + (-1)^{2 \times even} + (-1)^{3 \times even} + (-1)^{4 \times even} = 1 + 1 + 1 + 1 = 4$

If x is odd, *some* of the exponents in this question are odd:

Quantity A: $(-1)^{odd^2} + (-1)^{odd^3} + (-1)^{odd^4} = (-1)^{odd} + (-1)^{odd} + (-1)^{odd} = (-1) + (-1) + (-1) = -3$
Quantity B: $(-1)^{odd} + (-1)^{2 \times odd} + (-1)^{3 \times odd} + (-1)^{4 \times odd} = (-1)^{odd} + (-1)^{even} + (-1)^{odd} + (-1)^{even}$

$= (-1) + 1 + (-1) + 1$
$= 0$

In both cases, Quantity B is greater than Quantity A.

27. **(B).** Because an exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle (in this case, the top small triangle), $c = a + b$.

Therefore, $d > c$ and $a + b = c$ taken together imply that $d > a + b$.

Subtract b from both sides: $d - b > a$.

Quantity B is greater.

28. **(A).** This problem introduces a square and a circle and states that the circumference of the circle is $\frac{7}{8}$ the perimeter of the square.

This is license to plug in. Both a square and a circle are regular figures—that is, all squares are in the same proportion as all other squares and all circles are in the same proportion as all other circles—plugging in only *one* set of values yields the same result as would plugging in *any* set of values. Because the figures are regular and related in a known way (circumference is equal to $\frac{7}{8} \times$ square perimeter), there is no need to repeatedly try different values as is often necessary on Quantitative Comparisons.

Because the circumference of a circle depends on $\pi (C = \pi d)$, it is best to pick values for the square. If the side of the square is 2, the perimeter is 4(2) = 8 and the area is (2)(2) = 4. Then, circumference of the circle is $\left(\frac{7}{8}\right)(8) = 7$.

Since circumference is $2\pi r = 7$, the radius of the circle is $r = \frac{7}{2\pi}$.

Using these numbers:

Quantity A: The area of the square = 4.
Quantity B: The area of the circle = $\pi r^2 =$

$$\pi \left(\frac{7}{2\pi} \right)^2 = \pi \left(\frac{49}{4\pi^2} \right) = \frac{49}{4\pi} \approx 3.9.$$

(Use the calculator and the approximation 3.14 for π to determine that Quantity A is greater.)

29. **(A).** One good way to work through this problem is to pick a number, ideally starting with the innermost shape, the small circle. Say this circle has radius 1 and diameter 2, which would also make the side of the smaller square equal to 2.

If the small square has side 2, its diagonal would be $2\sqrt{2}$ (based on the 45–45–90 triangle ratios, or you could do the Pythagorean theorem using the legs of 2 and 2). If the diagonal is $2\sqrt{2}$, then the diameter of the larger circle is also $2\sqrt{2}$ (and the radius of the larger circle is one-half of that, or $\sqrt{2}$), making the side of the larger square also equal to $2\sqrt{2}$. Therefore:

Small circle: radius = 1, area = π
Large circle: radius = $\sqrt{2}$, area = 2π

Small square: side = 2, area = 4
Large square: side = $2\sqrt{2}$, area = 8

Thus, the large circle has twice the area of the small circle, and the large square has twice the area of the small square. This will work for any numbers you choose. In fact, you may wish to memorize this as a shortcut: if a circle is inscribed in a square that is inscribed in a circle, the large circle has twice the area of the small circle; similarly, if a square is inscribed in a circle that is inscribed in a square, the large square has twice the area of the small square.

In Quantity A, the ratio of the area of the larger square to the smaller square is $\frac{2}{1} = 2$.

In Quantity B, twice the ratio of the area of the smaller circle to the area of the larger circle is equal to $2\left(\frac{1}{2}\right) = 1$.

30. **(A).** This problem is not as bad as it looks! Of course, the integers are much too large to fit in your calculator. However, all you need to know is that a pair consisting of one 2 and one 5 has a product of 10 and therefore adds a zero to the end of a number. For instance, a number with two 2's and two 5's in its prime factors ends with two zeros, because the number is a multiple of 100.

Quantity A has nineteen 5's and many more 2's (since 2^{16} and 4^{18} together is more than nineteen 2's—if you really want to know, it's 2^{16} and $(2^2)^{18}$, or 2^{16} and 2^{36}, or 2^{52}, or fifty-two 2's). Considering *pairs* made up of one 2 and one

5, exactly nineteen pairs can be made (the leftover 2's don't matter), and the number ends in nineteen 0's.

Quantity B has sixteen 5's and many more 2's (specifically, there are fifty-three 2's, but there's no need to calculate this). Considering *pairs* made up of one 2 and one 5, exactly sixteen pairs can be made (the leftover 2's don't matter), and the number ends in sixteen 0's. Thus, Quantity A is greater.

31. **(A).** Calculate several terms of the sequence defined by $a_n = 2^n - = \frac{1}{2^{n-33}}$ and look for a pattern:

$$a_1 = 2^1 - \frac{1}{2^{-32}} = 2^1 - 2^{32}$$

$$a_2 = 2^2 - \frac{1}{2^{-31}} = 2^2 - 2^{31}$$

…

$$a_{16} = 2^{16} - \frac{1}{2^{-17}} = 2^{16} - 2^{17}$$

$$a_{17} = 2^{17} - \frac{1}{2^{-16}} = 2^{17} - 2^{16}$$

…

$$a_{31} = 2^{31} - \frac{1}{2^{-2}} = 2^{31} - 2^2$$

$$a_{32} = 2^{32} - \frac{1}{2^{-1}} = 2^{32} - 2^1$$

Notice that the 16th and 17th terms (the two middle terms in a set of 32 terms) are arithmetic inverses, that is, their sum is zero. Likewise, the 1st and 32nd terms sum to zero, as do the 2nd and 31st terms. In the first 32 terms of the sequence, there are 16 pairs that each sum to zero. Thus, Quantity A is zero.

For the sum of the first 31 terms, you could either:

1. Subtract a^{32} from the sum of the first 32 terms: $0 - (2^{32} - 2^1) = 2^1 - 2^{32}$ = 2 – (a very large number) = negative, or
2. realize that in the first 31 terms, all terms except a_1 can be paired such that the pair sums to zero, so the sum of the first 31 terms = $a_1 = 2^1 - 2^{32}$ = 2 – (a very large number) = negative.

Thus, Quantity B is negative, which is less than zero. Quantity A is greater.

32. **(C).** The "interquartile range" of a group of 100 integers is found by splitting the 100 integers into two groups, a lower 50 and an upper 50. Then find the median of each of those groups. The median of the lower group is the first quartile (Q_1), while the median of the upper group is the third quartile (Q_3). Finally, $Q_3 - Q_1$ is the interquartile range.

The median of a group of 50 integers is the average (arithmetic mean) of the 25th and the 26th integers when ordered from smallest to largest. Order the list of 100 integers from smallest to largest, then, find #25 and #26 and average them to get the first quartile. Likewise, find #75 and #76 and average them to get the third quartile. Then perform the subtraction.

x = the integer label on the ball	$n_x = 18–(x – 4)^2$ = the number of balls with this label	Cumulative number of balls
1	$18 – (1 – 4)^2 = 18 – (– 3)^2 = 18 – 9 = 9$	9
2	$18 – (2 – 4)^2 = 18 – (– 2)^2 = 18 – 4 = 14$	9 + 14 = 23
3	$18 – (3 – 4)^2 = 18 – (– 1)^2 = 18 – 1 = 17$	23 + 17 = 40
4	$18 – (4 – 4)^2 = 18 – (0)^2 = 18$	40 + 18 = 58
5	$18 – (5 – 4)^2 = 18 – (1)^2 = 18 – 1 = 17$	58 + 17 = 75

Stop here. Ball #75 has a 5 on it (in fact, the last 5), while ball #76 must have a 6 on it (since 6 is the next integer in the list). Thus, the third quartile Q_3 is the average of 5 and 6, or 5.5. Count carefully—if you are off by even just one either way, you'll get a different number for the third quartile. Balls #25 and #26 both have a 3 on them. So the first quartile Q_1 is the average of 3 and 3, namely 3.

Finally, $Q_3 – Q_1 = 5.5 – 3 = 2.5$.

33. **(C).** Compute the expressions for each of the terms:

$x!4 = x^4 \times 4^{-x}$ and $4!x = 4^x \times x^{-4}$

Dividing the first by the second yields:

$$\frac{x^4 4^{-x}}{4^x x^{-4}} = \frac{x^4}{x^{-4}} \times \frac{4^{-x}}{4^x} = x^8 4^{-2x}$$

There are a number of ways to write $x^8 4^{-2x}$:

$$x^8 4^{-2x} = \frac{x^8}{4^{2x}} = \frac{x^8}{16^x}$$

The two quantities are equal.

34. **(C).** First find each of the sums. To find a sum of an evenly spaced set, use the formula:

$$\text{sum} = (\text{average}) \times (\text{number of terms})$$

For the odd positive integers between 1 and 100, inclusive (use “2” as the multiple; while odds are not multiples of two per se, they are evenly spaced every two numbers):

$$\text{Number of terms (odds)} = \frac{\textit{last mult of } n - \textit{first mult of } n}{n} + 1 = \frac{99-1}{2} + 1 = 50$$

To find the average of any evenly spaced set, take the average of the first and last values:

$$\text{Average (odds)} = \frac{\textit{First} + \textit{Last}}{2} = \frac{1+99}{2} = 50$$

Sum of the odd integers between 1 and 100, inclusive is equal to (average) × (number of terms) = 50 × 50 = 2,500.

For the even positive integers between 100 and 150, inclusive:

$$\text{Number of terms (evens)} = \frac{\textit{last mult of } n - \textit{first mult of } n}{n} + 1 = \frac{150-100}{2} + 1 = 26$$

$$\text{Average (evens)} = \frac{\textit{First} + \textit{Last}}{2} = \frac{100+150}{2} = 125$$

Therefore, the sum of the even integers between 100 and 150, inclusive is equal to (average) × (number of terms) = 125 × 26 = 3,250.

The ratio of the sum of the odd positive integers between 1 and 100, inclusive, to the sum of the even positive integers between 100 and 150, inclusive is equal to $\frac{2{,}500}{3{,}250} = \frac{250}{325} = \frac{(25)(10)}{(25)(13)} = \frac{10}{13}$, or 10 to 13

35. **(C).** The problem gives two ways to calculate the fourth term: (1) the

definition of the sequence tells you that $a_4 = a_3^2 - a_2^2$ and (2) the words indicate that $a_4 = a_1(a_2 + a_3) = 2(a_2 + a_3)$. Setting these two equal gives $a_3^2 - a_2^2 = 2(a_2 + a_3)$. Factor the left side: $(a_3 + a_2)(a_3 - a_2) = 2(a_2 + a_3)$. Since $a_n > 0$ for all possible n's, $(a_3 + a_2)$ does not equal 0 and you can divide both sides by it: $a_3 - a_2 = 2$ and $a_3 = a_2 + 2$. Using the definition of a_3, you know $a_3 = a_2^2 - a_1^2 = a_2^2 - 4$. Substituting for a_3 yields: $a_2 + 2 = a_2^2 - 4$ and $a_2^2 - a_2 - 6 = 0$. Factor and solve: $(a_2 - 3)(a_2 + 2) = 0$; $a_2 = 3$ or -2. a_n must be positive, so $a_2 = 3$ and $a_3 = a_2 + 2 = 3 + 2 = 5$.

36. **(D).** Since a_1 and a_3 are integers, a_2 must also be an integer: $a_3 = \frac{(a_1 + a_2)}{2}$ or $\text{INT} = \frac{(\text{INT} + a_2)}{2}$ so $2(\text{INT}) = \text{INT} + a_2$ and $a_2 = 2(\text{INT}) - \text{INT}$, which is itself an integer. a_4 is thus the average of two integers. If $a_2 + a_3$ is even, a_4 will be an integer. If $a_2 + a_3$ is odd, a_4 will be a decimal ending in 0.5. If a_4 is an integer, a_5 can be an integer or can be a decimal ending in 0.5. If a_4 is a decimal ending in 0.5, a_5 must be a decimal ending in 0.25 or 0.75. a_5 cannot be a decimal ending in 0.375 such as $\frac{75}{8} = 9.375$. Note that a_5 can be negative: even if a_1 and a_3 are positive, that does not rule out the possibility that a_2 (and subsequent terms) could be negative.

37. **(D).** Use the definition of @ to rewrite the equation:
$\left|\frac{x+1}{x}\right| - \frac{2+1}{2} = \frac{1}{2}\left(\left|\frac{x+1}{x}\right| - \frac{-1+1}{-1}\right)$. Simplifying yields:
$\left|\frac{x+1}{x}\right| - \frac{3}{2} = \frac{1}{2}\left|\frac{x+1}{x}\right|$. Let $z = \left|\frac{x+1}{x}\right|$. Substitute z into the equation: $z - \frac{3}{2} = (\frac{1}{2})z$ or $z = 3$. To solve $\left|\frac{x+1}{x}\right| = 3$, take two cases:

1. $\frac{x+1}{x} > 0$, so $\frac{x+1}{x} = 3$ or $x = 0.5$.
2. $\frac{x+1}{x} < 0$, so $\frac{x+1}{x} = -3$ or $x = -0.25$.

The sum of the solutions is 0.5 + (–0.25) = 0.25.

38. **(B).** Expressed algebraically, $\sqrt{10-3x} > x$. Because both sides of this inequality are non-negative, you can square both sides to result in the following:

$$10 - 3x > x^2$$
$$0 > x^2 + 3x - 10$$
$$0 > (x + 5)(x - 2)$$

Now, because the product of $(x + 5)$ and $(x - 2)$ is negative, you can deduce that the larger of the two expressions, $(x + 5)$, must be positive, and the smaller expression, $(x - 2)$, must be negative. Therefore, $x > -5$ and $x < 2$. Combining these yields $-5 < x < 2$.

However, because the question indicates that x is non-negative, x must be 0 or greater. Therefore, $0 \le x < 2$. The absolute value sign in Quantity A doesn't change anything—x is still greater than or equal to 0 and less than 2, and Quantity B is larger.

Alternatively, plug the value from Quantity B into $\sqrt{10-3x} > x$:

$$\sqrt{10-3(2)} > 2$$

$$\sqrt{4} > 2$$

$$2 > 2$$

This is FALSE, so *x* cannot be 2.

Now, plug in a smaller or larger value to determine whether *x* needs to be greater than or less than 2. If *x* = 1:

$$\sqrt{10-3(1)} > 1$$
$$\sqrt{7} > 1$$

$\sqrt{7}$ is between 2 and 3, so this is true.

Trying values will show that only values greater than or equal to 0 and less than 2 make the statement true, so Quantity A must be less than 2.

39. **(A).** The area of an equilateral triangle is $\frac{b^2\sqrt{3}}{4}$ where b is the length of one side. Since this area is between $25\sqrt{3}$ and $36\sqrt{3}$, substitute to get $25\sqrt{3} < \frac{b^2\sqrt{3}}{4} < 36\sqrt{3}$. Dividing all sides by $\sqrt{3}$ yields $25 < \frac{b^2}{4} < 36$. Multiplying all sides by 4 yields $100 < b^2 < 144$, and taking the square root of all sides yields $10 < b < 12$. Since every possibility for b is greater than 9, Quantity A is greater.

40. **(E).** When dealing with absolute values, consider two outcomes. First determine the outcome if the expression within the absolute value sign is positive. So, if $8 – 2x > 0$, then $|8 – 2x| = 8 – 2x$, and therefore $8 – 2x < 3y – 9$ or $2x > 17 – 3y$.

You also must determine the outcome if the expression within the absolute value sign is negative. So if $8 – 2x < 0$, then $|8 – 2x| = 2x – 8$, and therefore $2x – 8 < 3y – 9$ or $2x < 3y – 1$. Combining these two inequalities yields $3y – 1 > 2x > 17 – 3y$.

Now a quick sanity check to make sure the inequality makes sense: $3y – 9$ must be greater than 0 or the absolute value could not be less than $3y – 9$. So $y > 3$. This means $17 – 3y < 8$, and $3y – 1 > 8$, so there is definitely room for $2x$ to fit between those values. If the potential values of $17 – 3y$ and $3y – 1$ had overlapped, this would be an indication either that a mistake had been made or that the problem required further investigation to refine the result.

41. **(A).** This is an overlapping set problem. Matrix 1 shows an initial setup for a double-set matrix. The columns are headed “Skilled in BJJ” and “Not Skilled in BJJ.” The rows are headed “Skilled in Muy Thai” and “Not Skilled in Muy Thai.” There is also a total row and a total column.

When dealing with overlapping sets, consider whether the question is giving information regarding the population as a whole or regarding a subset of the population. While the first statement (“30% of all fighters”) refers to the whole population, the second statement (“20% of the fighters who are not skilled in Brazilian Jiu Jitsu”) refers to a subset of the population, in this case the 40% who are not skilled in Brazilian Jiu Jitsu. Thus, 8% are skilled in Muy Thai but not in Brazilian Jiu Jitsu, as seen in Matrix 1:

Matrix 1

	Skilled in BJJ	Not Skilled in BJJ	Total
Skilled in Muy Thai	> 30	8	
Not Skilled in Muy Thai			
Total	60	40	100

Matrix 2 shows how to fill out additional cells. Notably, there are some ranges of values that are possible for the cells in the first column. These ranges are limited by 0 on the low end and 60 on the high end:

Matrix 2

	Skilled in BJJ	Not Skilled in BJJ	Total
Skilled in Muy Thai	> 30 but ≤ 60	8	
Not Skilled in Muy Thai	≤ 0 but < 30	32	
Total	60	40	100

Matrix 3 sums across the rows to get subtotals. Particularly, the percent of fighters who are skilled in Muy Thai is greater than 38 but less than or equal to 68. Thus, Quantity A is greater:

Matrix 3

	Skilled in BJJ	Not Skilled in BJJ	Total
Skilled in Muy Thai	> 30 but ≤ 60	8	> 38 but ≤ 68
Not Skilled in Muy Thai	≤ 0 but < 30	32	≤ 32 but < 62
Total	60	40	100

42. **(B).** Express this situation with the equation $r = \dfrac{kb}{n^2}$, where *k* is a constant. Quadrupling *b* and more than tripling *n* yields the following equation: $r_1 = \dfrac{k \times 4b}{(\text{“greater than }3n\text{”})^2}$, where r_1 represents the new rate of data transfer.

Squaring a value that is greater than 3 produces a value that is greater than 9, allowing the equation to be rewritten as $r_1 = \dfrac{k \times 4b}{\text{“greater than }9n^2\text{”}}$.

Rearranging this equation yields $r_1 = \dfrac{4}{\text{“greater than 9”}} \times \dfrac{k \times b}{n^2}$ = less than $\dfrac{4}{9}r$. Thus, Quantity B is greater.