Presentation EE1390

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Question

Verify if the circles $C_1 : x^2 + y^2 = 1$ and $C_2 : (x + 7)^2 + (y - 1)^2 = 49$ are orthogonal.

Solution

$C_1 : x^2 + y^2 = 1 \qquad C_2 : (x+7)^2 + (y-1)^2 = 49$

General form of a circle $= x^T Vx + 2u^T x + F = 0$

For circle C_1 :$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $u_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $F_1 = -1$

For circle C_2 :$V_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $F_2 = 1$

Matrix transformation of the question

Verify if the circles C_1: $x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + 2 \begin{bmatrix} 0 & 0 \end{bmatrix} x - 1 = 0$ and C_2 : $x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + 2 \begin{bmatrix} 1 & -1 \end{bmatrix} x + 1 = 0$ are orthogonal.

Tangent to any circle at Point P is $(P^T V + u^T)x + P^T u + F = 0$

Let P be the point of intersection of C_1 and C_2

Tangent to C_1 at P is $(P^T V_1 + u_1^T)x + P^T u + F_1 = 0$

$T1 : (P^T + u_1^T)x + P^T u_1 - 1 = 0$

Tangent to C_2 at P is $(P^T V_2 + u_2^T)x + P^T u_2 + F_2 = 0$

$T2 : (P^T + u_2^T)x + P^T u_2 + 1 = 0$

For orthogonality these tangents have to be perpendicular.

Direction vector T_1 is $D_1 : P^T + u_1^T$

Direction vector T_2 is $D_2 : P^T + u_2^T$

Orthogonality condition : $D_1 D_2^T = 0$

$(P^T + u_1^T)(P + u_2) = 0$

$P^T P + P^T u_2 + u_1^T P + u_1^T u_2 = 0$

Result

One point of intersection of C_1 and C_2 is $\text{P} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

On substituting in previous equation we get

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0$$

As we proved for one point of intersection the the tangents will also be perpendicular at the other point of intersection.

Therefore the circles are orthogonal.

Tangents

$$T1 : (P^T + u_1^T)x + P^T u_1 - 1 = 0$$

$$(\begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix})\mathrm{x} + \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ - } 1 = 0$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix}\mathrm{x} = 1$$

Which implies T_1 : Line y=1

$$T2 : (P^T + u_2^T)x + P^T u_2 + 1 = 0$$

$$(\begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \end{bmatrix})\mathrm{x} + \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 1 = 0$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix}\mathrm{x} = 0$$

Which implies T_2 : y-axis

Figure