Problem 1 [5 pts]:

Convert English sentences to FOL. Write each of the following English sentences using First Order Logic. Use the following predicates and constants only.

- Nationality(**x**, **y**): Predicate. Person **x** nationality is **y**.
- Cousin(**p1**, **p2**): Predicate. Person **p1** is a cousin of person **p2**.
- Drives(**z1**, **z2**): Predicate. Person **z1** drives car brand **z2**.
- American, Dutch, Australian, Swedish, Brazilian: constants denoting some nationalities. This list is not comprehensive. **There are also other nationalities not mentioned here**.
- Chrysler, Ford, BMW: constants denoting some car brands. This list is not comprehensive. **There are also other car brands not mentioned here**.
- Amy, George, Mia: constants denoting some people. This list is not comprehensive. **There are also other people not mentioned in this list**.

OTHER ANSWERS POSSIBLE

a) Mia is either Dutch or Swedish **[1 pts]**.

Your solution:
Nationality(Mia, Dutch) $\vee$ Nationality(Mia, Swedish)

b) George is not Brazilian and he drives the same car brand as all his cousins **[1 pts]**.

Your solution:
$\neg$Nationality(George, Brazilian) $\wedge$ $\forall$y [Cousin(George, y) $\Rightarrow$ $\exists$x [Drives(George, x) $\wedge$ Drives(y, x)]]

c) Not all people who drive BMWs are Swedish **[1 pts]**.

Your solution:
$\neg\forall$x [Drives(x, BMW) $\Rightarrow$ Nationality(x, Swedish)]

d) Every Brazilian has a cousin in Australia **[1 pts]**.

Your solution:
$\forall$x [Nationality(x, Brazilian) $\Rightarrow$ $\exists$y [Cousin(x, y) $\wedge$ Nationality(y, Australian)]]

e) Driving a Chrysler means that you are either an American or have no cousins **[1 pts]**.

Your solution:
$\forall x\ [\text{Drives}(x, \text{Chrysler}) \Rightarrow [\text{Nationality}(x, \text{American}) \vee \neg\exists y\ [\text{Cousin}(x, y)]]]$

Problem 2 [7 pts]:

Consider the following predicate / First-Order logic sentence:

$$\forall a\ (\exists b\ (X(a, b) \wedge Y(b)) \Rightarrow [\forall c\ (\neg Z(c, a))])$$

Assume that X, Y, and Z are predicates. Domains for variables are:

$D_a = \{A1, A2, A3\}$
$D_b = \{B1, B2\}$
$D_c = \{C1, C2, C3, C4\}$

Now convert the sentence above to its **CNF form** (**show and explain all steps**):

Your answer:

Step 1) Remove implications and equivalences.

$$\forall a\ (\exists b\ (X(a, b) \wedge Y(b)) \Rightarrow [\forall c\ (\neg Z(c, a))])$$

becomes (using Implication Law):

$$\forall a\ (\neg\exists b\ (X(a, b) \wedge Y(b)) \vee [\forall c\ (\neg Z(c, a))])$$

Step 2) Reduce the scope of all negations to single term.

$$\forall a\ (\neg\exists b\ (X(a, b) \wedge Y(b)) \vee [\forall c\ (\neg Z(c, a))])$$

becomes (using quantifier equivalence laws):

$$\forall a\ (\forall b\ \neg(X(a, b) \wedge Y(b)) \vee [\forall c\ (\neg Z(c, a))])$$

and later (using De Morgan's rule):

$$\forall a\ (\forall b\ (\neg X(a, b) \vee \neg Y(b)) \vee [\forall c\ (\neg Z(c, a))])$$

Step 3) Make all variables unique. All variables already unique, there is no need for this step.

Step 4) Move all quantifiers to the left.

$$\forall a\ (\forall b\ (\neg X(a, b) \vee \neg Y(b)) \vee [\forall c\ (\neg Z(c, a))])$$

becomes:

$$\forall a\ \forall b\ \forall c\ [(\neg X(a, b) \vee \neg Y(b)) \vee (\neg Z(c, a)))]$$

Step 5) Eliminate all existential quantifiers. There are no existential quantifiers in this sentence, there is no need for this step

Step 6) Eliminate universal quantifiers.

$$\forall a\ \forall b\ \forall c\ [(\neg X(a, b) \vee \neg Y(b)) \vee (\neg Z(c, a)))]$$

becomes [**one could replace variables with actual values here as well**]:

$$[(\neg X(a, b) \vee \neg Y(b)) \vee (\neg Z(c, a)))]$$

and we can remove some parentheses

$$[\neg X(a, b) \vee \neg Y(b) \vee \neg Z(c, a)]$$

Step 7) Convert to conjunction of disjuncts (CNF form). Above sentence is already in CNF form. No need to do anything.

Step 8) Create separate clause for each conjunct. Above sentence is already a single clause in CNF form. No need to do anything.

$$(\neg X(a, b) \vee \neg Y(b) \vee \neg Z(c, a))_1$$

Problem 3 [8 pts]:

We are given the following joint distribution for variables A, B, and C. Please compute the requested probabilities. **Show each <u>probability distribution</u> P() as a table/vector**.

A	B	C	P(A, B, C)
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

a) Probability distribution $\mathbf{P}(B, \neg C)$ **[2 pts]**

Your solution:

B	$\neg$C	P(B, $\neg$C)
true	true	= 0.126 + 0.168 = 0.294
true	false	= 0.014 + 0.392 = 0.406
false	true	= 0.048 + 0.096 = 0.144
false	false	= 0.012 + 0.144 = 0.156

b) Probability distribution $\mathbf{P}(B)$ – you can use your answer to part a to compute the answer to this question. **[2 pts]**

Your solution:

B	P(B)
true	= 0.014 + 0.126 + 0.392 + 0.168 = 0.7
false	= 0.012 + 0.048 + 0.144 + 0.096 = 0.3

c) Probability distribution $\mathbf{P}(B \mid \neg C)$ – you can use your answers to parts a and b to compute the answer to this question. **[2 pts]**

Your solution:

From conditional probability formula:

$P(B \mid \neg C) = P(B, \neg C) / P(\neg C)$

First, let's obtain $\mathbf{P}(\neg C)$:

$\neg C$	$P(\neg C)$
true	= 0.126 + 0.048 + 0.168 + 0.096 = 0.438
false	= 0.014 + 0.012 + 0.392 + 0.144 = 0.562

Now, we can obtain $\mathbf{P}(B \mid \neg C)$:

B	$\neg C$	$P(B \mid \neg C)$
true	true	$= P(B=true, \neg C=true) / P(\neg C=true) = 0.294 / 0.438 = 0.671$
true	false	$= P(B=true, \neg C=false) / P(\neg C=false) = 0.406 / 0.562 = 0.722$
false	true	$= P(B=false, \neg C=true) / P(\neg C=true) = 0.144 / 0.438 = 0.329$
false	false	$= P(B=false, \neg C=false) / P(\neg C=false) = 0.156 / 0.562 = 0.278$

d) Probability distribution $\mathbf{P}(A, B \mid \neg C)$ – you can use your answers from previous parts if they are relevant. **[2 pts]**

Your solution:

From conditional probability formula:

$P(A, B \mid \neg C) = P(A, B, \neg C) / P(\neg C)$

Now, let's obtain $\mathbf{P}(A, B \mid \neg C)$:

A	B	$\neg C$	$P(A, B \mid \neg C)$
true	true	true	$= P(A=true,B=true,\neg C=true)/P(\neg C=true) = 0.126 / 0.438 = 0.288$
true	true	false	$= P(A=true,B=true,\neg C=false)/P(\neg C=false) = 0.014 / 0.562 = 0.025$
true	false	true	$= P(A=true,B=false,\neg C=true)/P(\neg C=true) = 0.048 / 0.438 = 0.110$
true	false	false	$= P(A=true,B=false,\neg C=false)/P(\neg C=false) = 0.012 / 0.562 = 0.021$
false	true	true	$= P(A=false,B=true,\neg C=true)/P(\neg C=true) = 0.168 / 0.438 = 0.383$
false	true	false	$= P(A=false,B=true,\neg C=false)/P(\neg C=false) = 0.392 / 0.562 = 0.698$
false	false	true	$= P(A=false,B=false,\neg C=true)/P(\neg C=true) = 0.096 / 0.438 = 0.220$
false	false	false	$= P(A=false,B=false,\neg C=false)/P(\neg C=false) = 0.144 / 0.562 = 0.256$