Announcements / Reminders

- Please follow the Week 08 To Do List instructions

Plan for Today

- Predicate / First-Order Logic

Proof by Resolution

The process of proving by resolution is as follows:

A. Formalize the problem: "English to Predicate Logic"
B. Derive $KB \wedge \neg Q$ [in general: NOT(Some Sentence)]
C. Convert $KB \wedge \neg Q$ into CNF ("standardized") form
D. Apply resolution rule to resulting clauses. New clauses will be generated (add them to the set if not already present)
E. Repeat (D) until:
   a. no new clause can be added (KB does NOT entail Q)
   b. last two clauses resolve to yield the empty clause (KB entails Q)

Proof by Resolution

The process of proving by resolution is as follows:

A. Formalize the problem: "English to Predicate Logic"
B. Negate the input statement/claim C to obtain $\neg C$
C. Convert $\neg C$ into CNF ("standardized") form
D. Apply resolution rule to resulting clauses. New clauses will be generated (add them to the set if not already present)
E. Repeat (D) until:
   a. no new clause can be added (C is false)
   b. last two clauses resolve to yield the empty clause (C is true)

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

1. Eliminate all equivalences $\Leftrightarrow$ and implications $\Rightarrow$
2. Reduce the scope of all $\neg$ to single term (De Morgan)
3. Make all variable names unique (standardize apart)
4. Move quantifiers left (convert to prenex normal form)
5. Eliminate Existential quantifiers (skolemization)
6. Eliminate Universal quantifiers
7. Convert to conjunction of disjuncts
8. Create separate clause for each conjunct

Converting FOL to CNF: Example 1

Original sentence S:

$$\forall w\, ([P_1(w) \vee P_2(w) \Rightarrow P_3(w)] \vee [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

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By Implication Law ($p \Rightarrow q \equiv \neg p \vee q$):

$$\forall w\, ([\underline{P_1(w) \vee P_2(w) \Rightarrow P_3(w)}] \vee [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

becomes:

$$\forall w\, ([\underline{\neg(P_1(w) \vee P_2(w)) \vee P_3(w)}] \vee [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

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becomes:

$$\forall w\, ([\neg(P_1(w) \vee P_2(w)) \vee P_3(w)] \vee [\exists x\, (\exists y\, (\underline{\neg P_6(x, y) \vee P_4(w, x)})))]) \wedge [\forall w\, (P_5(w))]$$

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By De Morgan's Law ($\neg(p \vee q) \equiv \neg p \wedge \neg q$):

$$\forall w\, ([\underline{\neg(P_1(w) \vee P_2(w))} \vee P_3(w)] \vee [\exists x\, (\exists y\, (\neg P_6(x, y) \vee P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

becomes:

$$\forall w\, ([\underline{(\neg P_1(w) \wedge \neg P_2(w))} \vee P_3(w)] \vee [\exists x\, (\exists y\, (\neg P_6(x, y) \vee P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

Predicate (First-Order) Logic to CNF

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Converting FOL to CNF: Example 1

Original sentence S:

$$\forall w\, ([P_1(w) \vee P_2(w) \Rightarrow P_3(w)] \vee [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

Variable w (w and w) is bound to two different quantifiers:

$$\forall w\, ([\neg(P_1(w) \vee P_2(w)) \vee P_3(w)] \vee [\exists x\, (\exists y\, (\neg P_6(x, y) \vee P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

Replace w with z and the sentence S becomes:

$$\forall w\, ([(\neg P_1(w) \wedge \neg P_2(w)) \vee P_3(w)] \vee [\exists x\, (\exists y\, (\neg P_6(x, y) \vee P_4(w, x)))]) \wedge [\forall z\, (P_5(z))]$$

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

1. Eliminate all equivalences $\Leftrightarrow$ and implications $\Rightarrow$
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Original sentence S:

$$\forall w\, ([P_1(w) \vee P_2(w) \Rightarrow P_3(w)] \vee [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \wedge [\forall z\, (P_5(z))]$$

Quantified variables unique, move quantifiers left (order!):

$$\forall w\, ([\neg(P_1(w) \vee P_2(w)) \vee P_3(w)] \vee [\underline{\exists x}\, (\underline{\exists y}\, (\neg P_6(x, y) \vee P_4(w, x)))]) \wedge [\underline{\forall z}\, (P_5(z))]$$

becomes:

$$\forall w\, \underline{\exists x}\, \underline{\exists y}\, \underline{\forall z}\, ([(\neg P_1(w) \wedge \neg P_2(w)) \vee P_3(w)] \vee [(\neg P_6(x, y) \vee P_4(w, x))]) \wedge [(P_5(z))]$$

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

1. Eliminate all equivalences $\Leftrightarrow$ and implications $\Rightarrow$
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Converting FOL to CNF: Example 1

Original sentence S:

$$\forall w\, ([P_1(w) \vee P_2(w) \Rightarrow P_3(w)] \vee [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \wedge [\forall w\, (P_5(w))]$$

We have two existential quantifiers to remove here:

$$\forall w\, \underline{\exists x}\, \exists y\, \forall z\, ([(\neg P_1(w) \wedge \neg P_2(w)) \vee P_3(w)] \vee [(\underline{\neg P_6(x, y)} \vee \underline{P_4(w, x)})]) \wedge [(P_5(z))]$$

and:

$$\forall w\, \exists x\, \underline{\exists y}\, \forall z\, ([(\neg P_1(w) \wedge \neg P_2(w)) \vee P_3(w)] \vee [(\underline{\neg P_6(x, y)} \vee P_4(w, x))]) \wedge [(P_5(z))]$$

Both $\exists x$ and $\exists y$ are inside the scope of the universal quantifier $\forall w$. We need to use Skolem function substitution (Skolemization).

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall z\, (P_5(z))]$

Let's start with $\exists x$ and replace x with a Skolem function:

$\forall w\, \underline{\exists x}\, \exists y\, \forall z\, ([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\underline{\neg P_6(x, y)} \lor \underline{P_4(w, x)})]) \land [(P_5(z))]$

becomes:

$\forall w \exists y \forall z\, ([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\underline{\neg P_6(f(w), y)} \lor \underline{P_4(w, f(w))})]) \land [(P_5(z))]$

Quantified variable x was replaced with Skolem function $f(w)$. Existential quantifier $\exists x$ was removed.

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall z\, (P_5(z))]$

Now: remove $\exists y$ and replace y with a Skolem function:

$\forall w \exists y \forall z\, ([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\underline{\neg P_6(f(w), y)} \lor P_4(w, f(w)))]) \land [(P_5(z))]$

becomes:

$\forall w \forall z\, ([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\underline{\neg P_6(f(w), g(w))} \lor P_4(w, f(w)))]) \land [(P_5(z))]$

Quantified variable y was replaced with Skolem function $g(w)$. Existential quantifier $\exists y$ was removed.

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6. **Eliminate Universal quantifiers**
7. **Convert to conjunction of disjuncts**
8. **Create separate clause for each conjunct**

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall z\, (P_5(z))]$

Remaining quantified variables are universally quantified:

$\forall w \forall z\, ([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall z\, (P_5(z))]$

We can simply "drop" universal quantifiers:

$\forall w \forall z\, ([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

becomes:

$([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

We are "dropping" universal quantifiers for inferential purposes only. Equivalence is lost, but we can still use the remaining sentence to infer.

Predicate (First-Order) Logic to CNF

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Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall z\, (P_5(z))]$

By Associative Law $((p \lor q) \lor r \Leftrightarrow p \lor (q \lor r))$:

$([(\neg P_1(w) \land \neg P_2(w)) \lor P_3(w)] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

becomes:

$([P_3(w) \lor (\neg P_1(w) \land \neg P_2(w))] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

By Associative Law $((p \lor q) \lor r \Leftrightarrow p \lor (q \lor r))$:

$([\underline{P_3(w) \lor (\neg P_1(w) \land \neg P_2(w))}] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

becomes:

$([\underline{P_3(w) \lor (\neg P_1(w) \land \neg P_2(w))}] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

By Distributive Law $(p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r))$:

$([\underline{P_3(w) \lor (\neg P_1(w) \land \neg P_2(w))}] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

becomes:

$([\underline{(P_3(w) \lor \neg P_1(w)) \land (P_3(w) \lor \neg P_2(w))}] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

Let's make some substitutions:

$([(P_3(w) \lor \neg P_1(w)) \land (P_3(w) \lor \neg P_2(w))] \lor [(\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))]) \land [(P_5(z))]$

$A \equiv (P_3(w) \lor \neg P_1(w))$
$B \equiv (P_3(w) \lor \neg P_2(w))$
$C \equiv (\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$

so the sentence becomes:

$([A \land B] \lor [C]) \land [(P_5(z))]$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

By Distributive Law $(p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r))$:

$([A \land B] \lor [C]) \land [(P_5(z))]$

becomes:

$((A \lor C) \land (B \lor C)) \land [(P_5(z))]$

where:

$A \equiv (P_3(w) \lor \neg P_1(w))$
$B \equiv (P_3(w) \lor \neg P_2(w))$
$C \equiv (\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

Remove substitutions:

$((A \lor C) \land (B \lor C)) \land [(P_5(z))]$

becomes:

$(((P_3(w) \lor \neg P_1(w)) \lor (\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))) \land ((P_3(w) \lor \neg P_2(w)) \lor (\neg P_6(f(w), g(w)) \lor P_4(w, f(w))))) \land [(P_5(z))]$

where:

$A \equiv (P_3(w) \lor \neg P_1(w))$
$B \equiv (P_3(w) \lor \neg P_2(w))$
$C \equiv (\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

We can remove some parentheses:

$(((P_3(w) \lor \neg P_1(w)) \lor (\neg P_6(f(w), g(w)) \lor P_4(w, f(w)))) \land ((P_3(w) \lor \neg P_2(w)) \lor (\neg P_6(f(w), g(w)) \lor P_4(w, f(w))))) \land [(P_5(z))]$

becomes:

$$(P_3(w) \lor \neg P_1(w) \lor \neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$$
$$\land (P_3(w) \lor \neg P_2(w) \lor \neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$$
$$\land (P_5(z))$$

Converting FOL to CNF: Example 1

Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

We obtained sentence S in CNF form:

$$(P_3(w) \lor \neg P_1(w) \lor \neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$$
$$\land (P_3(w) \lor \neg P_2(w) \lor \neg P_6(f(w), g(w)) \lor P_4(w, f(w)))$$
$$\land (P_5(z))$$

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

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2. **Reduce the scope of all $\neg$ to single term (De Morgan)**
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8. **Create separate clause for each conjunct**

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Original sentence S:

$\forall w\, ([P_1(w) \lor P_2(w) \Rightarrow P_3(w)] \lor [\exists x\, (\exists y\, (P_6(x, y) \Rightarrow P_4(w, x)))]) \land [\forall w\, (P_5(w))]$

Let's number all clauses:

$$(P_3(w) \lor \neg P_1(w) \lor \neg P_6(f(w), g(w)) \lor P_4(w, f(w)))_1$$
$$\land (P_3(w) \lor \neg P_2(w) \lor \neg P_6(f(w), g(w)) \lor P_4(w, f(w)))_2$$
$$\land (P_5(z))_3$$

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

1. Eliminate all equivalences ⇔ and implications ⇒
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3. Make all variable names unique (standardize apart)
4. Move quantifiers left (convert to prenex normal form)
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6. Eliminate Universal quantifiers
7. Convert to conjunction of disjuncts
8. Create separate clause for each conjunct

Converting FOL to CNF: Example 2

Original sentence S:

$$\forall x\ [\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \Rightarrow [\exists y\ \text{Loves}(y, x)]$$

"Everyone who loves all animals is loved by someone"

Converting FOL to CNF: Example 2

Original sentence S:

$$\forall x\ [\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \Rightarrow [\exists y\ \text{Loves}(y, x)]$$

By Implication Law ($p \Rightarrow q \equiv \neg p \vee q$):

$$\forall x\ \underline{[\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \Rightarrow [\exists y\ \text{Loves}(y, x)]}$$

becomes:

$$\forall x\ \underline{\neg[\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \vee [\exists y\ \text{Loves}(y, x)]}$$

Converting FOL to CNF: Example 2

Original sentence S:

$$\forall x\ [\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \Rightarrow [\exists y\ \text{Loves}(y, x)]$$

By Implication Law ($p \Rightarrow q \equiv \neg p \vee q$):

$$\forall x\ \neg[\forall y\ \underline{(\text{Animal}(y) \Rightarrow \text{Loves}(x, y))}] \vee [\exists y\ \text{Loves}(y, x)]$$

becomes:

$$\forall x\ \neg[(\forall y\ (\neg\text{Animal}(y) \vee \text{Loves}(x, y))] \vee [\exists y\ \text{Loves}(y, x)]$$

Predicate (First-Order) Logic to CNF

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1. Eliminate all equivalences ⇔ and implications ⇒
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Converting FOL to CNF: Example 2

Original sentence S:

$$\forall x\ [\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \Rightarrow [\exists y\ \text{Loves}(y, x)]$$

By the equivalence ($\neg\forall x\ (p) \equiv \exists x\ (\neg p)$):

$$\forall x\ \underline{\neg[(\forall y\ (\neg\text{Animal}(y) \vee \text{Loves}(x, y))]} \vee [\exists y\ \text{Loves}(y, x)]$$

becomes:

$$\forall x\ \underline{[(\exists y\ \neg(\neg\text{Animal}(y) \vee \text{Loves}(x, y))]} \vee [\exists y\ \text{Loves}(y, x)]$$

Converting FOL to CNF: Example 2

Original sentence S:

$$\forall x\ [\forall y\ (\text{Animal}(y) \Rightarrow \text{Loves}(x, y))] \Rightarrow [\exists y\ \text{Loves}(y, x)]$$

By De Morgan's Law ($\neg(p \vee q) \equiv \neg p \wedge \neg q$):

$$\forall x\ [(\exists y\ \underline{\neg(\neg\text{Animal}(y) \vee \text{Loves}(x, y))}] \vee [\exists y\ \text{Loves}(y, x)]$$

becomes:

$$\forall x\ [(\exists y\ \underline{(\neg\neg\text{Animal}(y) \wedge \neg\text{Loves}(x, y))}] \vee [\exists y\ \text{Loves}(y, x)]$$