

5) a) Given,

$$x_{11} = x_{12} = x_1 \quad \& \quad x_{21} = x_{22} = x_2,$$

the ridge regression problem seeks to minimize

$$= (y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1)^2 + (y_2 - \hat{\beta}_1 x_2 - \hat{\beta}_2 x_2)^2 + \lambda (\hat{\beta}_1^2 + \hat{\beta}_2^2)$$

where  $\hat{\beta}_1$  &  $\hat{\beta}_2$  are estimates of coefficients

b) Deriving the expression in (a) with respect to  $\hat{\beta}_1$  &  $\hat{\beta}_2$  & setting them to zero yields.

$$\frac{\partial}{\partial \hat{\beta}_1} = 2 \hat{\beta}_1 x_{11}^2 - 2 x_{11} y_1 + 2 \hat{\beta}_2 x_{11} x_{12} + 2 \hat{\beta}_1 x_{21}^2 - 2 x_{21} y_2 + 2 \hat{\beta}_2 x_{21} x_{22} + 2 \lambda \hat{\beta}_1 = 0$$

$$\text{and } \frac{\partial}{\partial \hat{\beta}_2} =$$

$$(2 \hat{\beta}_1 x_{11}^2 - 2 x_{11} y_1 + 2 \hat{\beta}_2 x_{11} x_{12}) +$$

$$(2 \hat{\beta}_1 x_{21}^2 - 2 x_{21} y_2 + 2 \hat{\beta}_2 x_{21} x_{22}) + 2 \lambda \hat{\beta}_2 = 0$$

$$= 0$$

Rearranging above equations,

$$\textcircled{I} \quad \lambda \hat{\beta}_1 = x_1 y_1 + x_2 y_2 + 2 \hat{\beta}_1 x_1 x_2 + 2 \hat{\beta}_2 x_1 x_2$$

$$\textcircled{II} \quad \text{and } \lambda \hat{\beta}_2 = x_1 y_1 + x_2 y_2 + 2 \hat{\beta}_1 x_1 x_2 + 2 \hat{\beta}_2 x_1 x_2$$

This implies that  $\hat{\beta}_1 = \hat{\beta}_2$ .

c) Least optimization problem with the given information can be stated as

$$(y_1 - \beta_1 x_1 - \beta_2 x_2)^2 + (y_2 - \beta_1 x_2 - \beta_2 x_1)^2 + \lambda (\beta_1 + |\beta_2|)$$

d) Replacing the variables as mentioned in (b) we get

$$= (b - \beta_1 a - \beta_2 a)^2 + (b + \beta_1 a + \beta_2 a)^2 + \lambda (|\beta_1| + |\beta_2|)$$

$$= 2 [b - a(\beta_1 + \beta_2)]^2 + \lambda (|\beta_1| + |\beta_2|)$$

Taking partial derivatives w.r.t  $\beta_1$  &  $\beta_2$  and setting them to 0, we get

$$= 4a [b - a(\beta_1 + \beta_2)] = \pm \lambda$$

This eqn indicates the boundary of Lasso constraint. If  $\beta_1$  &  $\beta_2$  are negative the sign will be - and if they are positive, it will be positive. So there are many possible values of coefficients.