# Questions

## Geometry

1. Let $ABCD$ be a convex quadrilateral with perpendicular diagonals. If $AB = 20$, $BC = 70$, and $CD = 90$, then what is the value of $DA$?

2. In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 17. What is the greatest possible perimeter of the triangle?

3. In a triangle $ABC$, $X$ and $Y$ are points on the segments $AB$ and $AC$, respectively, such that $AX : XB = 1 : 2$ and $AY : YC = 2 : 1$.If the area of triangle $AXY$ is 10, then what is the area of triangle $ABC$?

4. Let $ABCD$ be a convex quadrilateral with $\angle DAB = \angle BDC = 90^\circ$. Let the incircles of triangles $ABD$ and $BCD$ touch $BD$ at $P$ and $Q$, respectively, with $P$ lying in between $B$ and $Q$. If $AD = 999$ and $PQ = 200$, then what is the sum of the radii of the incircles of triangles $ABD$ and $BDC$?

5. Let $XOY$ be a triangle with $\angle XOY = 90^\circ$. Let $M$ and $N$ be the midpoints of legs $OX$ and $OY$, respectively. Suppose that $XN = 19$ and $YM = 22$. What is $XY$?

# Number System

1. A natural number $k$ is such that $k^2 < 2014 < (k+1)^2$. What is the largest prime factor of $k$?

2. The first term of a sequence is 2014. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the $2014^{th}$ term of the sequence?

3. What is the smallest possible natural number $n$ for which the equation $x^2 - nx + 2014 = 0$ has integer roots?

4. If $x^{\left(x^4\right)} = 4$, what is the value of $x^{\left(x^2\right)} + x^{\left(x^8\right)}$?

5. Let $S$ be a set of real numbers with mean $M$. If the means of the sets $S \cup \{15\}$ and $S \cup \{15, 1\}$ are $M + 2$ and $M + 1$, respectively, then how many elements does $S$ have?

6. Natural numbers $k, l, p$, and $q$ are such that $a$ and $b$ are roots of the equation $x^2 - kx + l = 0$ such that $a + \frac{1}{b}$ and $b + \frac{1}{a}$.What is the sum of all possible values of $q$?

7. For natural numbers $x$ and $y$, let $(x, y)$ denote the greatest common divisor of $x$ and $y$. How many pairs of natural numbers $x$ and $y$ with $x \leq y$ satisfy the equation $xy = x + y + (x, y)$?

8. For how many natural numbers $n$ between 1 and 2014 (*bothinclusive*) is $\frac{8n}{9999-n}$ an integer?

9. For a natural number $b$, let $N(b)$ denote the number of natural numbers $a$ for which the equation $x^2 + ax + b = 0$ has integer roots. What is the smallest value of $b$ for which $N(b) = 20$?

10. One morning, each member of Manjul's family drank an 8-ounce mixture of coffee and milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Manjul drank $\frac{1}{7}$-th of the total amount of milk and $\frac{2}{17}$-th of the total amount of coffee. How many people are there in Manjul's family?

# Algebraic Equations

1. If real numbers $a, b, c, d, e$ satisfy

$$a + 1 = b + 2 = c + 3 = d + 4 = e + 5 = a + b + c + d + e + 3,$$

   what is the value of $a^2 + b^2 + c^2 + d^2 + e^2$?

2. Let $x_1, x_2, \cdots, x_{2014}$ be real numbers different from 1, such that$x_1 + x_2 + \cdots + x_{2014} = 1$ and

$$\frac{x_1}{1 - x_1} + \frac{x_2}{1 - x_2} + \cdots + \frac{x_{2014}}{1 - x_{2014}} = 1.$$

   What is the value of

$$\frac{x_1^2}{1 - x_1} + \frac{x_2^2}{1 - x_2} + \frac{x_3^2}{1 - x_3} + \cdots + \frac{x_{2014}^2}{1 - x_{2014}}?$$

# Discrete

1. What is the number of ordered pairs $(A, B)$ where $A$ and $B$ are subsets of $\{1, 2, \ldots, 5\}$ such that neither $A \subseteq B$ nor $B \subseteq A$?

# Functions

1. Let $f$ be a one-to-one function from the set of natural numbers to itself such that $f(mn) = f(m) f(n)$ for all natural numbers $m$ and $n$. What is the least possible value of $f(999)$?

# Trignometry

1. In a triangle $ABC$, let $I$ denote the incenter. Let the lines $AI$, $BI$, and $CI$ intersect the incircle at $P$, $Q$, and $R$, respectively. If $\angle BAC = 40^\circ$, what is the value of $\angle QPR$ in degrees?