

Questions

Geometry

1. Let $ABCD$ be a convex quadrilateral with perpendicular diagonals. If $AB = 20$, $BC = 70$, and $CD = 90$, then what is the value of DA ?
2. In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 17. What is the greatest possible perimeter of the triangle?
3. In a triangle ABC , X and Y are points on the segments AB and AC , respectively, such that $AX : XB = 1 : 2$ and $AY : YC = 2 : 1$. If the area of triangle AXY is 10, then what is the area of triangle ABC ?
4. Let $ABCD$ be a convex quadrilateral with $\angle DAB = \angle BDC = 90^\circ$. Let the incircles of triangles ABD and BCD touch BD at P and Q , respectively, with P lying in between B and Q . If $AD = 999$ and $PQ = 200$, then what is the sum of the radii of the incircles of triangles ABD and BDC ?
5. Let XOY be a triangle with $\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY , respectively. Suppose that $XN = 19$ and $YM = 22$. What is XY ?

Number System

1. A natural number k is such that $k^2 < 2014 < (k+1)^2$. What is the largest prime factor of k ?
2. The first term of a sequence is 2014. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2014th term of the sequence?
3. What is the smallest possible natural number n for which the equation $x^2 - nx + 2014 = 0$ has integer roots?
4. If $x^{(x^4)} = 4$, what is the value of $x^{(x^2)} + x^{(x^8)}$?
5. Let S be a set of real numbers with mean M . If the means of the sets $S \cup \{15\}$ and $S \cup \{15, 1\}$ are $M + 2$ and $M + 1$, respectively, then how many elements does S have?
6. Natural numbers k, l, p , and q are such that a and b are roots of the equation $x^2 - kx + l = 0$ such that $a + \frac{1}{b}$ and $b + \frac{1}{a}$. What is the sum of all possible values of q ?
7. For natural numbers x and y , let (x, y) denote the greatest common divisor of x and y . How many pairs of natural numbers x and y with $x \leq y$ satisfy the equation $xy = x + y + (x, y)$?
8. For how many natural numbers n between 1 and 2014 (both inclusive) is $\frac{8n}{9999-n}$ an integer?
9. For a natural number b , let $N(b)$ denote the number of natural numbers a for which the equation $x^2 + ax + b = 0$ has integer roots. What is the smallest value of b for which $N(b) = 20$?
10. One morning, each member of Manjul's family drank an 8-ounce mixture of coffee and milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Manjul drank $\frac{1}{7}$ -th of the total amount of milk and $\frac{2}{17}$ -th of the total amount of coffee. How many people are there in Manjul's family?

Algebraic Equations

1. If real numbers a, b, c, d, e satisfy

$$a + 1 = b + 2 = c + 3 = d + 4 = e + 5 = a + b + c + d + e + 3,$$

what is the value of $a^2 + b^2 + c^2 + d^2 + e^2$?

2. Let $x_1, x_2, \dots, x_{2014}$ be real numbers different from 1, such that $x_1 + x_2 + \dots + x_{2014} = 1$ and

$$\frac{x_1}{1 - x_1} + \frac{x_2}{1 - x_2} + \dots + \frac{x_{2014}}{1 - x_{2014}} = 1.$$

What is the value of

$$\frac{x_1^2}{1 - x_1} + \frac{x_2^2}{1 - x_2} + \frac{x_3^2}{1 - x_3} + \dots + \frac{x_{2014}^2}{1 - x_{2014}}?$$

Discrete

1. What is the number of ordered pairs (A, B) where A and B are subsets of $\{1, 2, \dots, 5\}$ such that neither $A \subseteq B$ nor $B \subseteq A$?

Functions

1. Let f be a one-to-one function from the set of natural numbers to itself such that $f(mn) = f(m)f(n)$ for all natural numbers m and n . What is the least possible value of $f(999)$?

Trigonometry

1. In a triangle ABC , let I denote the incenter. Let the lines AI , BI , and CI intersect the incircle at P , Q , and R , respectively. If $\angle BAC = 40^\circ$, what is the value of $\angle QPR$ in degrees?