Page no:- 167, 168, 169, 170
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nth order derivative

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(i) measurement of angles
(ii) trigonometric functions
(iii) compound angles
(iv) transformation formulae
set 1 :- four formulae
set 2 :- four formulae
i.e., if $A+B=C$
and $A-B=D$
(v) multiple angles
(vi) sub-multiple angle
(vii) trigonometric ratios of some particular angles.
$\sin 18$, $\cos 18$, $\sin 36$, $\cos 36$
(viii) trigonometric equation (nine-formulae)

$\Rightarrow \frac{1}{2a}\left[\int \frac{(x+a)-(x-a)}{(x-a)(x+a)}\,dx\right] = \frac{1}{2a}\left[\int \frac{dx}{(x+a)} - \int \frac{dx}{x+a}\right]$

$\Rightarrow \frac{1}{2a}\left[\ln|x-a| - \ln|x+a|\right] = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

If the human brain were a computer, it could perform 38 thousand-trillion operations per second. The world's most powerful supercomputer, BlueGene, can manage only

The total weight of all the ants on Earth is greater than total weight of all the humans on the planet.

Amazing Facts

Starfish can re-grow their arms. In fact, a single arm can regenerate a whole body.

The average hummingbird's heart rate is more than 1,200 beats per minute.

S. No.	Date	Title	Page No.	Teacher's Sign. / Remarks

① $\frac{d}{dx}\left(\frac{ax^2+bx+c}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{ax^2}{\sqrt{x}} + \frac{bx}{\sqrt{x}} + \frac{c}{\sqrt{x}}\right)$

$= \frac{d}{dx}\left(a\cdot x^{3/2} + bx^{1/2} + c\cdot x^{-1/2}\right)$

$= a\,\frac{3}{2}x^{1/2} + b\frac{1}{2}x^{-1/2} + c\frac{-1}{2}x^{-3/2}$

$= \frac{3}{2}a\,x^{1/2} + b\frac{1}{2}x^{-1/2} - \frac{1}{2}cx^{-3/2}$

② $\frac{d}{dx}\left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right)^2 \Rightarrow \frac{d}{dx}\left(\frac{x}{a} + \frac{a}{x} + 2\right)$

$= \frac{1}{a} + a\cdot\frac{d}{dx}\left(\frac{1}{x}\right) + 0$

$= \frac{1}{a} + a\cdot\frac{d}{dx}\left(x^{-1}\right)$

$= \frac{1}{a} + a\left(-1\cdot x^{-2}\right)$

$= \frac{1}{a} - \frac{a}{x^2}$

③ $\frac{d}{dx}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$ let $f(x) = \frac{\sin x + \cos x}{\sin x \mp \cos x}$

$u = \sin x \mp \cos x$

$v = \sin x + \cos x$

$\therefore f(x) = \frac{u}{v}$

so, $f'(x) = \left(\frac{u}{v}\right)'$

$$f'(x) = \frac{u'v - v'u}{v^2}$$

$u = \sin x - \cos x$

$u' = (\sin x - \cos x)'$

$= (\sin x)' - (\cos x)'$

$= \cos x - (-\sin x)$

$= \cos x + \sin x$

$v = \sin x + \cos x$

$v' = (\sin x + \cos x)'$

$= (\sin x)' + (\cos x)'$

$= \cos x - \sin x$

$$f'(x) = \left(\frac{u}{v}\right)'$$

$$= \frac{u'v - v'u}{v^2}$$

$$= \frac{(\cos x + \sin x)(\sin x + \cos x) - (\cos x - \sin x)(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$\Rightarrow \frac{(\cos x - \sin x) \cdot (\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$\Rightarrow \frac{\cos x \cdot \sin x - \cos^2 x - \sin^2 x + \sin x \cos x}{(\sin x - \cos x)^2}$$

$$u = \frac{2\sin x \cos x - (\sin^2 x + \cos^2 x)}{(\sin x - \cos x)^2}$$

$$= \frac{2\sin x \cos x - 1}{(\sin x - \cos x)^2}$$

$(\sin x + \cos x)(\cos x + \sin x)$

$$v = \frac{\sin^2 x + 2\sin x \cos x + \cos^2 x}{(\sin x - \cos x)^2}$$

$$v = \frac{2\sin x \cos x + 1}{(\sin x - \cos x)^2}$$

Put the two fractions together to get:

$$y' = \frac{2\sin x \cos x - 1 - (2\sin x \cos x + 1)}{(\sin x - \cos x)^2}$$

$$y' = \frac{2\sin x \cos x + 1 - 2\sin x \cos x - 1}{(\sin x - \cos x)^2}$$

$$y' = \frac{-2}{(\sin x - \cos x)^2}$$

(4) $\frac{d}{dx}\left(\frac{\sec x - \tan x}{\sec x + \tan x}\right) \Rightarrow y = \frac{\sec x + \tan x}{\sec x - \tan x}$

$$= \frac{1 + \sin x}{1 - \sin x}$$

$$\frac{dy}{dx} = (1 - \sin x)\cos x$$

$$\frac{dy}{dx} = \frac{(1 - \sin x)\cos x + (1 + \sin x)\cos x}{(1 - \sin x)^2}$$

$$= \frac{\cos x - \sin x \cos x + \cos x + \sin x \cdot \cos x}{(1 - \sin x)^2}$$

$$= \frac{2\cos x}{(1-\sin x)^2}$$

(5) $\frac{d}{dx}\left(\frac{2^x \cot x}{\sqrt{x}}\right)$

$$\Rightarrow \frac{\sqrt{x}\,\frac{d}{dx}(2^x \cot x) - 2^x \cot x \,\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$$

$$= \frac{\sqrt{x}\left(-2^x \operatorname{cosec} x + 2^x \cot x \cdot \log 2\right) - \frac{2^x \cot x}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= \frac{2x\left(-2^x \operatorname{cosec}^2 x + 2^x \cot x \cdot \log 2\right) - 2^x \cot x}{2x\sqrt{x}}$$

$$= \frac{2^x\left(-2x \operatorname{cosec}^2 x + 2x \cot x \log 2 - \cot x\right)}{2x\sqrt{x}}$$

$$= \frac{2^x}{\sqrt{x}}\left(-\frac{2x \operatorname{cosec}^2 x}{2x} + \frac{2x \cot x \log 2}{2x} - \frac{\cot x}{2x}\right)$$

$$= \frac{2^x}{\sqrt{x}}\left(-\operatorname{cosec}^2 x + \log 2 \cot x - \frac{\cot x}{2x}\right)$$

$$= \frac{2^x}{\sqrt{x}}\left\{\log 2 \cot x - \operatorname{cosec}^2 x - \frac{\cot x}{2x}\right\}$$

Type (1)
Problem based upon direct results or related to substitution.

Q1. $I = \int \frac{dx}{1-4x^2} \Rightarrow$ 1st method :- $\Rightarrow I = \int \frac{dx}{(1)^2 - (2x)^2}$

$$\Rightarrow \frac{1}{2\times1\times2} \log\left|\frac{1+2x}{1-2x}\right| = \frac{1}{4}\log\left|\frac{1+2x}{1-2x}\right| + C \text{ Ans}$$

2nd method :-

$$I = \frac{1}{4}\int \frac{dx}{\left(\frac{1}{4} - x^2\right)} \Rightarrow \frac{1}{4}\int \frac{dx}{\left(\frac{1}{2}\right)^2 - x^2} \Rightarrow \frac{1}{4} \times \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{1/2+x}{1/2-x}\right|$$

$$\Rightarrow \frac{1}{4}\log\left|\frac{1+2x}{1-2x}\right| + C \text{ Ans}$$

Q2. $I = \int \frac{e^x}{3e^{2x}+2}\,dx \Rightarrow I = \int \frac{e^x\,dx}{3(e^x)^2+2}$, let $e^x = t$ $\therefore e^x dx = dt$

$$\Rightarrow I = \int \frac{dt}{3t^2+2} = \frac{1}{3}\int \frac{dt}{t^2+\frac{2}{3}} \Rightarrow \frac{1}{3}\int \frac{dt}{t^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{1}{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)} \tan^{-1}\left(\frac{t}{\sqrt{2}/\sqrt{3}}\right) = \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}\,e^x}{\sqrt{2}}\right) + C \text{ Ans}$$

Q3. $I = \int \frac{5x\,dx}{7x^4-2} \Rightarrow I = \int \frac{5x\,dx}{(\sqrt{7}x^2)^2 - (\sqrt{2})^2} = \frac{1}{\sqrt{7}}\int 5x\,dx$

$$= \frac{1}{\sqrt{7}}\, 5 \log\left|\frac{\sqrt{7}x+\sqrt{2}}{\sqrt{7}x-\sqrt{2}}\right| = \frac{1}{\sqrt{14}} \log\left|\frac{\sqrt{7}x+\sqrt{2}}{\sqrt{7}x-\sqrt{2}}\right| dx$$

$I = \int \frac{5x\,dx}{7(x^2)^2 - 2}$ $\Rightarrow$ let $x^2 = t$
$2x\,dx = dt$
$x\,dx = \frac{dt}{2}$

$I = \frac{5}{2} \int \frac{dt}{7t^2 - 2} \Rightarrow \frac{5}{2} \int \frac{dt}{(\sqrt{7}t)^2 - (\sqrt{2})^2}$

$\Rightarrow \frac{1}{\sqrt{7}} \cdot \frac{5}{2} \times \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{7}t - \sqrt{2}}{\sqrt{7}t + \sqrt{2}} \right|$

$= \frac{5}{4\sqrt{14}} \log \left| \frac{\sqrt{7}x^2 - \sqrt{2}}{\sqrt{7}x^2 + \sqrt{2}} \right| + c$

Type (II)
Integral of the form $\int \frac{dx}{(ax^2+bx+c)}$

Working Rule :-

(I) make the co-efficient of $x^2 = 1$
(II) In denominator, make perfect square

Q.1. $I = \int \frac{dx}{2x^2 - 3x + 1}$ Soln:- $I = \frac{1}{2} \int \frac{dx}{\left(x^2 - \frac{3}{2}x + \frac{1}{2}\right)}$

$\Rightarrow \frac{1}{2} \int \frac{dx}{\left(x^2 - 2 \cdot x \cdot \frac{3}{4} + \frac{9}{16}\right) - \frac{9}{16} + \frac{1}{2}}$

$\Rightarrow \frac{1}{2} \int \frac{dx}{\left(x - \frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2}$

$\Rightarrow \frac{1}{2} \times \frac{1}{2\left(\frac{1}{4}\right)} \log \left| \frac{x - 3/4 - 1/4}{x - 3/4 + 1/4} \right|$

$= \log \left| \frac{4x-4}{4x-2} \right| \Rightarrow \log \left| \frac{2x-2}{2x-1} \right|$

$\Rightarrow \log \left| \frac{x-1}{2x-1} \right| + c$

Q2 $I = \int \frac{dx}{1 + x - 2x^2}$ Soln = $\frac{1}{2} \int \frac{dx}{x^2 - \frac{x}{2} - \frac{1}{2}}$

$\Rightarrow \frac{1}{2} \int \frac{dx}{\left(x^2 - 2 \cdot x \cdot \frac{1}{4} + \frac{1}{16}\right) - \frac{1}{16} - \frac{1}{2}}$

$\Rightarrow \frac{1}{2} \int \frac{dx}{\left(x - \frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2}$ $\Rightarrow \frac{1}{2} \times \frac{1}{2 \times 3/4} \log \left| \frac{x - 1/4 - 3/4}{x - 1/4 + 3/4} \right|$

$= \frac{1}{3} \log \left| \frac{4x - 4}{4x + 2} \right| = \frac{1}{3} \log \left| \frac{2x-2}{2x+1} \right| \Rightarrow \frac{1}{3} \log \left| \frac{x-1}{2x+1} \right| + c$

Q3. $I = \int \frac{e^x\,dx}{3e^{2x} + e^x + 1}$ Q4. $I = \int \frac{\cos x\,dx}{(13\sin x - 5\sin^2 x + 2)}$

3. Soln:- $\int \frac{e^x\,dx}{3(e^x)^2 + e^x + 1}$ Let, $e^x = t$ 07.12.21

$\Rightarrow$ $\therefore e^x dx = dt$
$I = \int \frac{dt}{(3t^2 + t + 1)} \Rightarrow \frac{1}{3} \int \frac{dt}{\left(t^2 + \frac{t}{3} + \frac{1}{3}\right)}$

$\Rightarrow \frac{1}{3} \int \frac{dt}{\left(t^2 + 2 \cdot t \cdot \frac{1}{6} + \frac{1}{36}\right) - \frac{1}{36} + \frac{1}{3}} \Rightarrow \frac{1}{3} \int \frac{dt}{\left(t + \frac{1}{6}\right)^2 + \left(\frac{\sqrt{11}}{6}\right)^2}$

$\frac{1}{3} \cdot \frac{1}{\left(\frac{\sqrt{11}}{6}\right)} \tan^{-1}\left(\frac{t+\frac{1}{6}}{\frac{\sqrt{11}}{6}}\right)$

$\Rightarrow \frac{2}{\sqrt{11}} \tan^{-1}\left(\frac{6e^x+1}{\sqrt{11}}\right) + c$ Ans

Q4 sol^n:- Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$I = \int \frac{dt}{(3t - 5t^2 + 2)} = \frac{-1}{5}\int \frac{dt}{\left(t^2 - \frac{3t}{5} - \frac{2}{5}\right)}$

$I = \frac{1}{5}\int \frac{dt}{t^2 - 2t\cdot\frac{3}{10} + \frac{9}{100} - \frac{9}{100} - \frac{2}{5}}$

$\Rightarrow \frac{-1}{5}\int \frac{dt}{\left(t - \frac{3}{10}\right)^2 - \left(\frac{7}{10}\right)^2}$

$\Rightarrow \frac{-1}{5} \times \frac{1}{2\left(\frac{7}{10}\right)} \log\left|\frac{t - 3/10 - 7/10}{t - 3/10 + 7/10}\right|$

$\Rightarrow \frac{-1}{7} \log\left|\frac{10t - 10}{10t + 4}\right|$

$\Rightarrow \frac{-1}{7} \log\left|\frac{5t-5}{5t+2}\right|$

$I = \frac{-1}{7} \log\left|\frac{5\sin x - 5}{5\sin x + 2}\right| + c$ Ans.

$I = \frac{-1}{7} \log\left|\frac{\sin x - 1}{5\sin x + 2}\right| + c$ Ans

Type (III)

Integral of the form $\int \frac{px+q}{(ax^2+bx+c)}\,dx$

* Working Rule:-

(1) $(px+q) = A\frac{d}{dx}(ax^2+bx+c) + B$

Q. $I = \int \frac{(3x-1)}{(2x^2-3x+5)}\,dx$

sol^n:- Let,

$(3x-1) = A\frac{d}{dx}(2x^2-3x+5) + B$

$(3x-1) = A(4x-3) + B$ —(1)

$3x - 1 = 4Ax - 3A + B$

Equating like coefficient

$4A = 3 \Rightarrow \boxed{A = \frac{3}{4}}$

$-3A + B = -1$

$\Rightarrow -3\left(\frac{3}{4}\right) + B = -1$

$-\frac{9}{4} + B = -1$

$B = \frac{9-4}{4} \Rightarrow \boxed{B = \frac{5}{4}}$

Putting the value in eq^n (1)

$(3x-1) = \frac{3}{4}(4x-3) + \frac{5}{4}$

$\Rightarrow \frac{12x}{4} - \frac{9}{4} + \frac{5}{4}$

$\Rightarrow (3x-1)$

$\frac{(3x-1)}{(2x^2-3x+5)} = \frac{3(4x-3)}{4(2x^2-3x+5)} + \frac{5}{4(2x^2-3x+5)}$

$$= \int \frac{(3x-1)\,dx}{2x^2-3x+5} \Rightarrow \frac{3}{4}\int \frac{(4x-3)}{(2x^2-3x+5)}\,dx + \frac{5}{4}\int \frac{dx}{(2x^2-3x+5)}$$

$$I = \frac{3}{4} I_1 + \frac{5}{4} I_2 \quad \text{— (ii)}$$

Note: $I_1 \rightarrow$ substitution
$I_2 \rightarrow$ Type II

$$I_1 = \int \frac{(4x-3)\,dx}{(2x^2-3x+5)} \quad , \text{ let, } 2x^2-3x+5 = t$$
$$(4x-3)\,dx = dt$$

$$I_1 = \int \frac{dt}{t} \Rightarrow \log|t| = \log|2x^2-3x+5| + c$$

Next, $I_2 = \int \frac{dx}{(2x^2-3x+5)} \Rightarrow \frac{1}{2}\int \frac{dx}{x^2-\frac{3}{2}x+\frac{5}{2}}$

$$\Rightarrow \frac{1}{2}\int \frac{dx}{\left(x^2-2\cdot x\cdot\frac{3}{4}+\frac{9}{16}\right)-\frac{9}{16}+\frac{5}{2}} \Rightarrow \frac{1}{2}\int \frac{dx}{\left(x-\frac{3}{4}\right)^2+\left(\frac{\sqrt{31}}{4}\right)^2}$$

$$I_2 = \frac{1}{2}\times\frac{1}{\left(\frac{\sqrt{31}}{4}\right)}\tan^{-1}\left(\frac{x-\frac{3}{4}}{\frac{\sqrt{31}}{4}}\right) \Rightarrow \frac{2}{\sqrt{31}}\tan^{-1}\left(\frac{4x-3}{\sqrt{31}}\right)$$

from eqn (ii)

$$I = \frac{3}{4} I_1 + \frac{5}{4} I_2$$

$$I = \frac{3}{4}\log|2x^2-3x+5| + \frac{5}{4}\times\frac{2}{\sqrt{31}}\tan^{-1}\left(\frac{4x-3}{\sqrt{31}}\right) + c$$

Q. $I = \int \left(\frac{2x^2-3x+1}{5x^2+x-1}\right) dx$

Soln- $\left(\frac{2x^2-3x+1}{5x^2+x-1}\right) = \frac{2}{5} + \frac{\left(-\frac{17}{5}x+\frac{7}{5}\right)}{(5x^2+x-1)}$

$$5x^2+x-1 \,\big)\, 2x^2-3x+1 \,\big(\, 2/5$$
$$2x^2+\frac{2}{5}x-\frac{2}{5}$$
$$-\frac{17}{5}x+\frac{7}{5}$$

$$\Rightarrow \int \frac{(2x^2-3x+1)}{(5x^2+x-1)}\,dx = \frac{2}{5}\int dx + \frac{1}{5}\int \frac{(-17x+7)\,dx}{(5x^2+x-1)}$$

$$\therefore I = \frac{2}{5}x + \frac{1}{5} I_1 \quad \text{— (1)}$$

where, $I_1 = \int \frac{(-17x+7)}{(5x^2+x-1)}\,dx$, let, $(5x^2+x-1) = t$

$$(-17x+7) = A\frac{d}{dx}(5x^2+x-1) + B$$
$$(-17x+7) = A(10x+1) + B$$
$$-17x+7 = 10Ax + A + B$$
$$10Ax = -17x$$
$$A = \frac{-17}{10}$$

$\cancel{-17x+7 = 10x\frac{-17}{10} + 10 + B}$

$\cancel{-17x+7 = 10 + B}$

$\cancel{-17x-3 = B}$

$$A+B = 7$$
$$\frac{-17}{10} + B = 7$$
$$B = \frac{17}{10} + 7$$
$$B = \frac{87}{10}$$

$$\frac{-17x+7}{(5x^2+x-1)} = \frac{-17}{10}\frac{(-17x+7)}{(5x^2+x-1)} + \frac{87}{10(5x^2+x-1)}$$

~~Integrating both sides~~ :-

$$\int \frac{-17x+7}{(5x^2+x-1)} = \frac{-17}{10}\int$$

$$I = \frac{2}{5}x + \frac{1}{5}\left(\frac{-17}{10}\log|5x^2+x-1| + \frac{87}{10}\right)$$

$$= \int \frac{dx}{5x^2+x-1} \Rightarrow \frac{1}{5}\int \frac{dx}{x^2+\frac{x}{5}-\frac{1}{5}}$$

$$\Rightarrow \frac{1}{5}\int \frac{dx}{x^2+2\cdot x\cdot\frac{1}{10}+\frac{1}{100}-\frac{1}{100}-\frac{1}{5}}$$

$$\Rightarrow \frac{1}{5}\int \frac{dx}{\left(x+\frac{1}{10}\right)^2-\left(\frac{\sqrt{21}}{100}\right)^2}$$

$$\Rightarrow I = \frac{2}{5}x + \frac{1}{5}\left(\frac{-17}{10}\log|5x^2+x-1| + \frac{87}{10}\times\frac{1}{5}\times\frac{1}{2\left(\frac{\sqrt{21}}{10}\right)}\log\left|\frac{x+\frac{1}{10}-\frac{\sqrt{21}}{10}}{x+\frac{1}{10}+\frac{\sqrt{21}}{10}}\right|\right.$$
$+ c$ Ans

3Q. $I = \int \frac{(1-3x)}{1+5x-7x^2}\,dx$

4Q. $I = \int\left(\frac{3x^2+4x-2}{2x^2-x+1}\right)dx$

Sol^n 1)- $I = \int \frac{3x-1}{7x^2-5x-1}$ $\Rightarrow$ $3x-1 = A\frac{d}{dx}(7x^2-5x-1)+B$ — (1)

$3x-1 = A(14x-5)+B$

$3x-1 = 14Ax-5A+B$

$14A = 3 \Rightarrow \boxed{A = \frac{3}{14}}$

and, $-1 = -5A+B$

$-1 = -5\times 3 + B$

$-1 = \frac{-15}{14} + B$

$\frac{15}{14} - 1 = B$

$\boxed{B = \frac{+1}{14}}$

$I_1 = \int \frac{(14x-5)}{(7x^2-5x-1)}dx$

let, $7x^2-5x-1 = t$

$(14x-5)\,dx = dt$

$I_1 = \int \frac{dt}{t} \Rightarrow \log|7x^2-5x-1|$

Now, $I_2 = \int \frac{dx}{7x^2-5x-1} = \frac{1}{7}\int\frac{dx}{x^2-\frac{5}{7}x-\frac{1}{7}}$

$$\Rightarrow \frac{1}{7}\int\frac{dx}{x^2-2\cdot x\cdot\frac{5}{14}+\left(\frac{5}{14}\right)^2-\left(\frac{5}{14}\right)^2-\frac{1}{7}}$$

$$\Rightarrow \frac{1}{7}\int\frac{dx}{\left(x-\frac{5}{14}\right)^2-\left(\frac{\sqrt{53}}{14}\right)^2}$$

$$\Rightarrow \frac{1}{7}\times\frac{1}{2\times\frac{\sqrt{53}}{14}}\log\left|\frac{x-\frac{5}{14}-\frac{\sqrt{53}}{14}}{x-\frac{5}{14}+\frac{\sqrt{53}}{14}}\right|$$

$$\Rightarrow \frac{1}{\sqrt{53}}\log\left|\frac{14x-5-\sqrt{53}}{14x-5+\sqrt{53}}\right|$$

$$I = \frac{3}{14}\log|7x^2-5x-1| + \frac{1}{14\sqrt{53}}\log\left|\frac{14x-5-\sqrt{3}}{14x-5+\sqrt{3}}\right| + C$$

(2) $I = \int\left(\frac{3x^2+4x-2}{2x^2-x+1}\right)dx$

Sol^n:- $2x^2-x+1 \,\big)\, 3x^2+4x-2 \,\big(\, \frac{3}{2}$

$3x^2 - \frac{3x}{2} + \frac{3}{2}$

$\frac{11x}{2} - \frac{7}{2}$

$$\frac{3x^2+4x-2}{2x^2-x+1} \Rightarrow \frac{3}{2}\int dx + \frac{1}{2}\int\left(\frac{11x-7}{2x^2-x+1}\right)dx$$

and $I = \frac{3}{2}x + \frac{1}{2}\int\left(\frac{11x-7}{2x^2-x+1}\right)dx$

$I = \frac{3}{2}x + \frac{1}{2}\int\left(\frac{11x-7}{2x^2-x+1}\right)dx$

let, $I_1 = \int\frac{11x-7}{(2x^2-x+1)}dx$

$11x-7 = A\frac{d}{dx}\left(2x^2-x+1\right)+B$ — (ii)

$11x-7 = A(4x-1)+B \Rightarrow 11x-7 = 4Ax - A + B$

$4A = 11$

$\boxed{A = \frac{11}{4}}$

$-7 = -A+B$

$-7 = -\frac{11}{4}+B$

$\frac{11}{4}-7 = B \Rightarrow \frac{-14}{4} = B$

Put A and B in eqn (ii)

$11x-7 = \frac{11}{14}(4x-1)-\frac{17}{4}$

$I_1 = \frac{11}{4}\underbrace{\int\frac{(4x-1)}{2x^2-x+1}dx}_{I_2} - \frac{17}{4}\underbrace{\int\frac{dx}{2x^2-x+1}}_{I_3}$

$I_2 = \int\frac{4x-1}{2x^2-x+1}dx$

let, $2x^2-x+1 = t$

$(4x-1)dx = dt$

$I_2 = \int\frac{dt}{t} = \log|2x^2-x+1|$

$I_3 = \int\frac{dx}{2x^2-x+1} \Rightarrow \frac{1}{2}\int\frac{dx}{x^2-\frac{x}{2}+\frac{1}{2}}$

$= \frac{1}{2}\int\frac{dx}{x^2-2\cdot x\cdot\frac{1}{4}+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2+\frac{1}{2}}$

$\Rightarrow \frac{1}{2}\int\frac{dx}{\left(x-\frac{1}{4}\right)^2-\left(\frac{\sqrt{3}}{4}\right)^2}$

$\Rightarrow \frac{1}{\cancel{2}}\times\frac{1}{\sqrt{3}\times\frac{\cancel{2}}{4}}\log\left|\frac{x-\frac{1}{4}-\frac{\sqrt{3}}{4}}{x-\frac{1}{4}+\frac{\sqrt{3}}{4}}\right|$

$\Rightarrow \frac{1}{\sqrt{3}}\log\left|\frac{4x-1-\sqrt{3}}{4x-1+\sqrt{3}}\right|$

$I_1 = \frac{11}{4}\log|2x^2-x+1| - \frac{17}{4\sqrt{3}}\log\left|\frac{4x-1-\sqrt{3}}{4x-1+\sqrt{3}}\right| + c$

$\frac{3x}{2}+\frac{1}{2}\left(\frac{11}{4}\log|2x^2-x+1| - \frac{17}{2\sqrt{7}}\tan^{-1}\left(\frac{4x-1}{\sqrt{7}}\right)\right)+c$ 09|12|21

Type (iv)

Integral of the form $\int\left(\frac{x^2\pm1}{x^4+kx^2+1}\right)dx$

working ~~using~~ Rules

(i) dividing numerator and denominator by x^2

(ii) use suitable substitution.

Questions:-

(i) $\int \frac{x^2+1}{x^4+1}\,dx$ (ii) $\int \frac{x^2-1}{x^4+1}\,dx$

(iii) $\int \frac{dx}{x^4+1}\,dx$ (iv) $\int \frac{x^2}{x^4+3x^2+1} = x^4$

(v) $\int \sqrt{\tan x}\,dx$ (vi) $\int \sqrt{\cot\theta}\,d\theta$

(vii) $\int \sqrt{\tan x}+\sqrt{\cot x}\,dx$ (viii) $\int \frac{dx}{\sin^4 x+\cos^4 x}\,dx$

Sol^n- (1) $I = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx \Rightarrow \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$

let, $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$I = \int \frac{dt}{t^2+(\sqrt{2})^2} \Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$

$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\left(x-\frac{1}{x}\right)}{\sqrt{2}}\right) \Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}$

hence,

$$\boxed{\int \frac{x^2+1}{x^4+1} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C \quad \text{Proved}}$$

Sol^n (2):- $I = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx \Rightarrow \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2}\,dx$

let, $\left(x+\frac{1}{x}\right)=t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$

$I = \int \cancel{\frac{dt}{t^2-(\sqrt{2})^2}} \Rightarrow \cancel{\frac{1}{2}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)}$

$\cancel{\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{\left(x+\frac{1}{x}\right)}{\sqrt{2}}\right)}$

$\cancel{\Rightarrow \left(\frac{1}{2}\tan^{-1}\frac{x^2+1}{\sqrt{2}x}\right)+C}$ ~~Proved.~~

$\Rightarrow I = \frac{1}{2\sqrt{2}}\log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| \Rightarrow I = \frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right|$

$\Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$

hence,

$$\boxed{\int \frac{x^2-1}{x^4+1}\,dx = \frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C}$$

(3) $I = \int \frac{dx}{x^4+1} \Rightarrow I = \frac{1}{2}\int \frac{(x^2+1)-(x^2-1)}{(x^4+1)}\,dx$

$\Rightarrow \frac{1}{2}\int \frac{(x^2+1)}{(x^4+1)}\,dx - \frac{1}{2}\int \left(\frac{x^2-1}{x^4+1}\right)dx$

$\frac{1}{2}I_1 - \frac{1}{2}I_2$ ——— (1)

from Eq^n (1)

$I = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - \frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$

sol^n (4) $\int \frac{x^2 dx}{x^4+3x^2+1} \Rightarrow \frac{1}{2}\int \frac{(2x^2\, dx)}{(x^4+3x^2+1)}$

$= \frac{1}{2}\int \frac{(x^2+1)+(x^2-1)}{x^4+3x^2+1}dx \Rightarrow \frac{1}{2}\int \frac{x^2+1}{x^4+3x^2+1}\,dx + \frac{1}{2}\int\frac{(x^2-1)}{x^4+3x^2+1}dx$

$I = \frac{1}{2} I_1 + \frac{1}{2} I_2$ —(1)

$I_1 = \int \left(\frac{x^2+1}{x^4+3x^2+1}\right) dx \Rightarrow \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+3+\frac{1}{x^2}\right)} dx$

$\Rightarrow \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5}$ Let, $x-\frac{1}{x} = t$ $\therefore \left(1+\frac{1}{x^2}\right)dx = dt$

Next,

$I_2 = \int \frac{x^2-1}{x^4+3x^2+1}\,dx \Rightarrow \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x^2+3+\frac{1}{x^2}\right)}dx$

$\Rightarrow \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$ let, $x+\frac{1}{x} = t$

$\Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$

$I_2 = \int \frac{dt}{t^2+1} \Rightarrow \tan^{-1} t \Rightarrow \tan^{-1}\left(\frac{x+1}{x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{x^2+1}{x}\right)$

from eq^n (1)

$I = \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right) + \frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)$ Ans

(v) $I = \int \sqrt{\tan x}\, dx$

Sol^n:- Let, $\tan x = t^2$

$\therefore \sec^2 x\, dx = 2t\, dt$

$dx = \frac{2t\, dt}{\sec^2 x}$

$dx = \frac{2t\, dt}{1+\tan^2 x}$

$dx = \frac{2t\, dt}{1+t^4}$

$I = \int t\left(\frac{2t}{1+t^4}\right)dt$

$\Rightarrow \int \frac{2t^2}{t^4+1}\, dt$

$\Rightarrow \frac{(t^2+1)+(t^2-1)}{(t^4+1)}dt$

$\Rightarrow \int\left(\frac{t^2+1}{t^4+1}\right)dt + \int\left(\frac{t^2-1}{t^4+1}\right)dt$

$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right|$

hence,

$I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\left|\frac{\tan x - \sqrt{2\tan x}+1}{\tan^2 x + \sqrt{2\tan x}+1}\right| + c$ Ans

(vi) will be solve with same process

(viii) $I = \int \frac{dx}{\sin^4 x + \cos^4 x}$

sol^n:- dividing numerator and denominator by $\cos^4 x$

$I = \int \frac{\frac{dx}{\cos^4 x}}{\frac{\sin^4 x + \cos^4 x}{\cos^4 x}} dx \Rightarrow \int \frac{\sec^4 x}{\tan^4 x + 1} dx$

$\Rightarrow \int \frac{\sec^2 x \cdot \sec^2 x \, dx}{1 + \tan^4 x} \Rightarrow \int \frac{\sec^2 x (1 + \tan^2 x) \, dx}{1 + \tan^4 x}$

Let, $\tan x = t$

$I = \int \frac{\sec^2 x \, (1+t^2) \, dx}{t^4 + 1} \Rightarrow I = \int \frac{dt \, (1+t^2) \, dt}{(1+t^4)}$

$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x}\right) + c$ Ans

TYPE-(V)

INTEGRAL OF THE FORM $\int \frac{dx}{a + b\sin^2 x}$, $\int \frac{dx}{a + b\cos^2 x}$

and $\int \frac{dx}{a\sin^2 x + b\cos^2 x + c}$

Working Rule:-

(i) Dividing numerator and denomination by $\cos^2 x$

(ii) Let $\tan x = t$

Q1 $I = \int \frac{dx}{3 + 4\cos^2 x}$

sol^n $I = \int \frac{\sec^2 x \, dx}{3\sec^2 x + 4} \Rightarrow \int \frac{\sec^2 x \, dx}{3 + 3\tan^2 x + 4}$

$\Rightarrow \int \frac{\sec^2 x \, dx}{3\tan^2 x + 7}$, Let, $\tan x = t$ $\therefore \sec^2 x \, dx = dt$

$I = \int \frac{dt}{3t^2 + 7} \Rightarrow \int \frac{dt}{(\sqrt{3}t)^2 + (\sqrt{7})^2}$

$\Rightarrow \frac{1}{\sqrt{3}\sqrt{7}} \cdot \tan^{-1}\left(\frac{\sqrt{3}t}{\sqrt{7}}\right) \Rightarrow \frac{1}{\sqrt{21}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{7}}\right) + c$ Ans

Q2 $I = \int \frac{dx}{2\cos^2 x + 5\sin^2 x + 3}$

sol^n:- $I = \int \frac{\sec^2 x \, dx}{2 + 5\tan^2 x + 3\sec^2 x} \Rightarrow \int \frac{\sec^2 x \, dx}{5\tan^2 x + 3 + 3\tan^2 x + 2}$

$\Rightarrow \int \frac{\sec^2 x \, dx}{8\tan^2 x + 8}$ Put, $\tan x = t$ $\therefore \sec^2 x \, dx = dt$

$I = \int \frac{dt}{8t^2 + 5} \Rightarrow \int \frac{dt}{(2\sqrt{2}t)^2 + (\sqrt{5})^2}$

$\Rightarrow \frac{1}{2\sqrt{2}\sqrt{5}} \tan^{-1}\left(\frac{2\sqrt{2}t}{\sqrt{5}}\right)$

$$= \frac{1}{2\sqrt{10}} \tan^{-1}\left(\frac{2\sqrt{2}\tan x}{\sqrt{5}}\right) + c \quad \text{Ans}$$

Q8. $\int \frac{\sin x}{\sin 3x} dx$ Q. $\int \frac{\cos 5x}{\cos 3x} dx$

Soln:- $I = \int \frac{\sin x \, dx}{\sin 3x} \Rightarrow \int \frac{\sin x \, dx}{3\sin x - 4\sin^3 x}$

$$\Rightarrow \int \frac{dx}{3 - 4\sin^2 x} \Rightarrow \int \frac{\sec^2 x \, dx}{3\sec^2 x - 4\tan^2 x}$$

Let, $\tan x = t$

$\therefore \sec^2 x \, dx = dt$

$$I = \int \frac{dt}{3 - t^2} \Rightarrow \int \frac{dt}{(\sqrt{3})^2 - (t)^2}$$

$$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3}+t}{\sqrt{3}-t} \right|$$

$$= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x} \right| + c \quad \text{Ans}$$

Type (VI)

Integral of the form $\int \frac{dx}{a + b\sin x}$, $\int \frac{dx}{a + b\cos x}$ and

$$\int \frac{dx}{a\sin x + b\cos x} + c$$

Quitting Rule:-

(i) Put $\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$ and, $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

(ii) Let $\tan \frac{x}{2} = t$

Q1. $I = \int \frac{dx}{3 - 2\cos x}$

Soln:- $I = \int \frac{dx}{3 - 2\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)} \Rightarrow \int \frac{\sec^2 x/2 \, dx}{3 + 3\tan^2 x/2 - 2 + 2\tan^2 x/2}$

$$\Rightarrow \int \frac{\sec^2 x/2 \, dx}{5\tan^2 x/2 + 1}$$

Let, $\tan x/2 = t$

$\therefore \frac{1}{2}\sec^2 \frac{x}{2} \, dx = dt$

$\Rightarrow \sec^2 x/2 \, dx = 2dt$

$$I = 2\int \frac{dt}{5t^2+1} \Rightarrow 2\int \frac{dt}{(\sqrt{5}t)^2+1}$$

$$\Rightarrow \frac{2}{\sqrt{5}} \tan^{-1}\left(\frac{\sqrt{5}\,t}{1}\right)$$

$$\Rightarrow \frac{2}{\sqrt{5}} \tan^{-1}\left(\sqrt{5}\tan\frac{x}{2}\right) + c \quad \text{Ans}$$

Q2. $I = \int \frac{dx}{5\cos x - 2\sin x - 4}$

Solⁿ $I = \int \frac{dx}{5\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) - 2\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right) - 4}$

$I = \int \frac{\sec^2 x/2 \, dx}{5 - 5\tan^2\frac{x}{2} - 4\tan\frac{x}{2} - 4 - 4\tan^2\frac{x}{2}}$

$I = \int \frac{\sec^2 x/2 \, dx}{-9\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1}$

Let, $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2dt$

$I = 2\int \frac{dt}{-9t^2 - 4t + 1}$

$I = \frac{-2}{9}\int \frac{dt}{t^2 + \frac{4}{9}t - \frac{1}{9}}$

$I = \frac{-2}{9}\int \frac{dt}{\left(t^2 + 2\cdot t\cdot\frac{2}{9} + \frac{4}{81}\right) - \frac{4}{81} - \frac{1}{9}}$

$= \frac{-2}{9}\int \frac{dt}{\left(t+\frac{2}{9}\right)^2 - \left(\frac{\sqrt{13}}{9}\right)^2}$

$= \frac{2}{9}\int \frac{dt}{\left(\frac{\sqrt{13}}{9}\right)^2 - \left(t+\frac{2}{9}\right)^2}$

$\Rightarrow \frac{2}{9} \times \frac{1}{\frac{2\sqrt{13}}{9}} \log\left|\frac{\frac{\sqrt{13}}{9} + t + \frac{2}{9}}{\frac{\sqrt{13}}{9} - t - \frac{2}{9}}\right|$

$I = \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{13} + 2 + 9\tan x/2}{\sqrt{13} - 2 - 9\tan x/2}\right| + C$ Ans.

Type (VII)

Integral of the form $\int \left(\frac{a\sin x + b\cos x}{a\sin x + d\cos x}\right) dx$

working rule :

$(a\sin x + b\cos x) = A\frac{d}{dx}(c\sin x + d\cos x) + B(c\sin x + d\cos x)$

Q① $I = \int \left(\frac{2\sin x - \cos x}{-3\sin x + 4\cos x}\right)$ Q2. $I = \int \frac{dx}{1 - \tan x}$

Q3. $I = \int \frac{dx}{1 - \cot x}$

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Solⁿ ①:- let,

$(2\sin x - \cos x) = A\frac{d}{dx}(-3\sin x + 4\cos x) + B(-3\sin x + 4\cos x)$

$\Rightarrow A(-3\cos x - 4\sin x) + B(-3\sin x + 4\cos x)$ —①

$\Rightarrow (-4A - 3B)\sin x + (-3A + 4B)\cos x$

$\Rightarrow (-4A$

After Equating,

$(-4A - 3B = 2) \times 4$ $\quad = -16A - 12B = 8$

$(-3A + 4B = -1) \times 3$ $\quad -9A + 12B = -3$

$-25A = 5$

$A = \frac{-1}{5}$

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$-\frac{16 \times -1}{5} - 12B = 8$

$\frac{16}{5} - 12B = 8$

$\frac{16}{5} - 8 = 12B$

$\frac{16-40}{5} = 12B \Rightarrow \frac{-24}{5 \times 12} = B = \frac{-2}{5}$

from eqⁿ (1)

$$2\sin x - \cos x = \frac{-1}{5}\int \frac{(-3\cos x - 4\sin x)}{(-3\sin x + 4\cos x)}dx + \left(-\frac{2}{5}\right)\int \frac{(-3\sin x + 4\cos x)}{-(3\sin x + 4\cos x)}dx$$

$$\Rightarrow \frac{-1}{5}\log\left|-3\sin x + 4\cos x\right| - \frac{2}{5}x + C$$

Q. $I = \int \frac{dx}{1-\tan x} = \int \frac{dx}{1 - \frac{\sin x}{\cos x}} \Rightarrow \int \frac{dx}{\frac{\cos x - \sin x}{\cos x}}$

$$= \int \frac{\cos x \, dx}{\cos x - \sin x}$$

$$\cos x = A\frac{d}{dx}(\cos x - \sin x) + B(\cos x - \sin x) + C$$

$$= A(-\sin x - \cos x) + B(\cos x - \sin x) + C$$

$$\Rightarrow \sin x(-A - B) + \cos x(-A + B) + C$$

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Type VIII

Integral of the form $\int \left(\frac{a\sin x + b\cos x + c}{d\sin x + e\cos x + f}\right) dx$

Working rule:

$$(a\sin x + b\cos x + c) = A\frac{d}{dx}(d\sin x + e\cos x + f) + B(d\sin x + e\cos x + f) + C$$

Q. $I = \int \frac{\sin x + 3}{2\cos x + 4\sin x - 1}dx$

Solⁿ:- Let,

$$(\sin x + 3) = A\frac{d}{dx}(2\cos x + 4\sin x - 1) + B(2\cos x + 4\sin x - 1) + C$$

$$(\sin x + 3) = A(-2\sin x + 4\cos x) + B(2\cos x + 4\sin x - 1) + C$$

$$(\sin x + 3) = (-2A + 4B)\sin x + (4A + 2B)\cos x + (-B + C)$$

after equating,

$2 \times (-2A + 4B) = 1$ —(ii) $\quad -B + C = 3$ —(iv)

$2 \cdot 4A + 2B = 0$ —(iii)

$10B = \frac{1}{2}$ ... $B = \frac{2}{10} = \frac{1}{5}$

$\frac{-1}{5} + C = 3$

$C = 3 + \frac{1}{5}$

$= \frac{16}{5}$

$4A + 2 \times \frac{1}{5} = 0$

$4A + \frac{2}{5} = 0$

$4A = \frac{-2}{5} \quad A = \frac{-2}{20} = \frac{-1}{10}$

$\Rightarrow (\sin x + 3) = \frac{-1}{10}(-2\sin x + 4\cos x) + \frac{1}{5}(2\cos x + 4\sin x - 1) + \frac{16}{5}$

$\int (\sin x + 3)\,dx = \frac{-1}{10}\int \frac{(-2\sin x + 4\cos x)}{(2\cos x + 4\cos x - 1)}dx + \frac{1}{5}\int \frac{(2\cos x + 4\sin x - 1)}{(2\cos x + 4\sin x - 1)}dx + \frac{16}{5}\int \frac{dx}{2\cos x + 4\sin x - 1}$

$\Rightarrow \frac{1}{10}\log|2\cos x + 4\cos x - 1| + \frac{1}{5}x + \frac{16}{5}\int \frac{dx}{2\cos x + 4\sin x - 1}$ — (5)

Let, $I_1 = \int \frac{dx}{(2\cos x + 4\sin x - 1)}$

$\Rightarrow \int \frac{dx}{2\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right) + 4\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right) - 1}$

$I_1 = \int \frac{\sec^2 x/2 \, dx}{2 - 2\tan^2 x/2 + 8\tan x/2 - 1 - \tan^2 x/2}$

$I_1 = \int \frac{\sec^2 x/2 \, dx}{1 + 8\tan\frac{x}{2} - 3\tan^2\frac{x}{2}}$

Let, $\tan\frac{x}{2} = t \Rightarrow \sec^2 dx = dt$

$I_1 = 2\int \frac{dt}{1 + 8t + 0 - 3t^2}$

$I_1 = \frac{-2}{3}\int \frac{dt}{t^2 - \frac{8t}{3} - \frac{1}{3}}$

$I_1 = \frac{-2}{3}\int \frac{dt}{\left(t^2 - 2\cdot t\cdot\frac{4}{3} + \frac{16}{9}\right) - \frac{16}{9} - \frac{1}{3}}$

$\Rightarrow \frac{-2}{3}\int \frac{dt}{\left(t - \frac{4}{3}\right)^2 - \left(\frac{\sqrt{19}}{3}\right)^2}$

$\Rightarrow \frac{-2}{3}\int \frac{dt}{\left(\frac{\sqrt{19}}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2}$

$\Rightarrow \frac{-2}{3} \times \frac{1}{2\left(\frac{\sqrt{19}}{3}\right)} \log\left|\frac{\frac{\sqrt{19}}{3} + t - \frac{4}{3}}{\frac{\sqrt{19}}{3} - t + \frac{4}{3}}\right|$

$\Rightarrow \frac{-1}{\sqrt{19}} \log\left|\frac{\sqrt{19} - 4 + 3\tan x/2}{\sqrt{19} + 4 - 3\tan x/2}\right|$

from eq^n (5)

$I = \frac{-1}{10}\log|2\cos x + 4\sin x - 1| + \frac{1}{5}x + \frac{16}{5\sqrt{19}}\log\left|\frac{\sqrt{19} - 4 + 3\tan x/2}{\sqrt{19} + 4 - 3\tan x/2}\right| + C$

SET-(11) [REMEMBER]

(1) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$

(2) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$ $\quad x = a\tan\theta$

(3) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$ $\quad x = a\sin\theta$

Proof :-

(i) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| + C$

$I = \int \frac{dx}{\sqrt{x^2-a^2}}$, put $x = a\sec\theta$
$dx = a\sec\theta\tan\theta\, d\theta$

$I = \int \frac{a\sec\theta\tan\theta\, d\theta}{a\tan\theta} \Rightarrow \int \sec\theta\, d\theta$

$\Rightarrow \log|\sec\theta+\tan\theta| = \log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}-1}\right|$

$\log\left|\frac{x+\sqrt{x^2-a^2}}{a}\right|$

$I = \log|x+\sqrt{x^2-a^2}| - \log a$

hence,

$\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| - \log a$

(ii) $I = \int \frac{dx}{\sqrt{x^2+a^2}} \Rightarrow$ Let, $a = \tan\theta$
$dx = a\sec^2\theta\, d\theta$

Now,

$I = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a\tan^2\theta+a^2}} \Rightarrow I = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2(\tan^2\theta+1)}}$

$\Rightarrow \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2\sec^2\theta}} \Rightarrow \int \sec\theta\, d\theta$

$\Rightarrow \log|\sec\theta+\tan\theta| + c$

$\Rightarrow \log\left|\frac{\sqrt{x^2+a^2}}{a}+\frac{x}{a}\right| + c$

$\Rightarrow \log\left|\frac{\sqrt{x^2+a^2}+x}{a}\right|$

$\Rightarrow \log|\sqrt{x^2+a^2}+x| + C$ A.

(3) $I = \int \frac{dx}{\sqrt{a^2-x^2}}$ let, $x = a\sin\theta$
$dx = a\cos\theta\, d\theta$

$I = \int \frac{a\cos\theta}{\sqrt{a^2-a^2\sin^2\theta}}\, d\theta \Rightarrow \int \frac{a\cos\theta}{\sqrt{a^2(1-\sin^2\theta)}}\, d\theta$

$I = \int \frac{a\cos\theta}{\sqrt{a\cos^2\theta}}\, d\theta \Rightarrow \int \frac{a\cos\theta}{a\cos\theta}\, d\theta$

$\Rightarrow \int d\theta = \theta = \sin^{-1}\frac{x}{a} + c$

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TYPE-(I)

PROBLEMS RELATED TO DIRECT RESULT OR SUBSTITUTION

TYPE-(II)

INTEGRAL OF THE FORM $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

TYPE-(iii)
INTEGRAL OF THE FORM $\int \frac{(px+q)}{\sqrt{ax^2+bx+c}}$

Working Rule:

$$(px+q) = A\frac{d}{dx}(ax^2+bx+c) + B$$

Q. $I = \int \frac{(2x+3)}{\sqrt{1+x-3x^2}}\,dx$

Solⁿ $(2x+3) = A\frac{d}{dx}(1+x-3x^2) + B$

$(2x+3) \Rightarrow A(1-6x) + B$ — (1)

$(2x+3) \Rightarrow -6Ax + (A+B)$

Equating like coefficient, we get:

$-6A = 2 \Rightarrow A = \frac{-1}{3}$

$A + B = 3$

$\therefore B = \frac{3+1}{3} = \frac{10}{3}$

from Eqⁿ (1)

$(2x+3) = \frac{-1}{3}(1-6x) + \frac{10}{3}$

$$\Rightarrow \int \frac{2x+3}{\sqrt{1+x-3x^2}}\,dx = \frac{-1}{3}\int \frac{(1-6x)\,dx}{\sqrt{1+x-3x^2}} + \frac{10}{3}\int \frac{dx}{\sqrt{1+x-3x^2}}$$

Let, $I = \frac{-1}{3}I_1 + \frac{10}{3}I_2$ — (1)

where, $I_1 = \int \frac{(1-6x)\,dx}{\sqrt{1-x-3x^2}}$

Let, $1+x-3x^2 = t$

$\therefore (1-6x)\,dx = dt$

NOTE:-
I_1 = substitution
I_2 = Type II set (II)

$I_1 = \int \frac{dt}{\sqrt{t}} \Rightarrow 2\sqrt{t}$

$\Rightarrow 2\sqrt{1+x-3x^2}$

next, $I_2 = \int \frac{dx}{\sqrt{1+x-3x^2}} \Rightarrow \int \frac{dx}{\sqrt{-3\left(x^2-\frac{x}{3}-\frac{1}{3}\right)}}$

$$\Rightarrow \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{-\left\{\left(x^2-\frac{2x}{6}+\frac{1}{36}\right)-\frac{1}{36}-\frac{1}{3}\right\}}}$$

$$\Rightarrow \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{-\left\{\left(x-\frac{1}{6}\right)^2-\left(\frac{\sqrt{13}}{6}\right)^2\right\}}}$$

$$\Rightarrow \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{\left(\frac{\sqrt{13}}{6}\right)^2-\left(x-\frac{1}{6}\right)^2}} \Rightarrow \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{x-\frac{1}{6}}{\frac{\sqrt{13}}{6}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{6x-1}{\sqrt{13}}\right)$$

here, from Eqⁿ (1)

$$I = \frac{-1}{3}I_1 + \frac{10}{3}I_2$$

$$\Rightarrow \frac{-2}{3}\sqrt{1+x-3x^2} + \frac{10}{3\sqrt{3}}\sin^{-1}\left(\frac{6x-1}{\sqrt{13}}\right) + C \text{ A.}$$

SET-(iii)

TYPE-(1)

PROBLEMS RELATED TO DIRECT RESULT OR SUBSTITUTION

Type (ii)

INTEGRAL of the form $\int \sqrt{ax^2+bx+c}\,dx$

Type (iii)

Integral of the form $\int (px+q)\sqrt{ax^2+bx+c}\,dx$

Working Rule:-

$$(px+q) = A\frac{d}{dx}(ax^2+bx+c) + B$$

* REMEMBER:-

(1) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$

(2) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$

(3) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\,\cancel{\log}\,\sin^{-1}\frac{x}{a} + c$

TYPE-(iii)

Q. $I = \int (3x-1)\sqrt{5-4x-2x^2}\,dx$

Sol^n, let,

$$(3x-1) = A\frac{d}{dx}(5-4x-2x^2) + B$$

$$(3x-1) = A(-4x-4) + B \quad —(1)$$

$(3x-1) = -4Ax - 4A + B$

$-4A\cancel{x} = 3$ $\quad$ $\boxed{A = \frac{-3}{4}}$

$-1 = -4A + B$

$-1 = -4 \times \frac{-3}{4} + B$

$-1 = 3 + B$

$\boxed{B = -4}$

Now, from Eq^n (1)

$(3x-1) = \frac{-3}{4}(5-4x-2x^2$

$(3x-1) = \frac{-3}{4}(-4A-4) - 4$

$\frac{3x-1}{\sqrt{5-4x-2x}}$ $\quad (3x-1)\int\sqrt{5-4x-2x^2}\,dx = \frac{-3}{4}\int(-4-4x)\sqrt{5-4x-2x^2}\,dx$

$+ (-4)\int\sqrt{5-4x-2x^2}\,\cancel{dx}\,dx$

$I_1 = \frac{-3}{4}I_1 + (-4)I_2 \quad —(ii)$

Now, $I_1 = \int(-4-4x)\sqrt{5-4x-2x^2}\,dx$, ~~let~~.

let, $\quad 5-4x-2x^2\,dx = t$

$(-4-4x)\cdot dx = dt$

$I_1 = \int dt\,\sqrt{t}$

$\Rightarrow \frac{2t^{3/2}}{3} \Rightarrow \frac{2}{3}(5-4x-2x^2)^{3/2}$

$I_2 = \int \frac{\cancel{dx}}{\sqrt{5-4x-2x^2}}\,dx \Rightarrow \int \frac{\cancel{dx}}{\sqrt{-2\left(x^2-\frac{4x}{2}+\frac{5}{2}\right)}}\,dx$

$\Rightarrow \sqrt{2}\int \frac{\cancel{dx}}{\sqrt{-\left\{(x^2+2x\cdot1+1)-1-\frac{5}{2}\right\}}}\,dx \Rightarrow \sqrt{2}\int\sqrt{-\left\{(x+1)^2-\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2\right\}}$

$\Rightarrow \sqrt{2}\int\sqrt{\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2 - (x+1)^2}\,dx$

$$\Rightarrow \left(\frac{x+1}{2}\right)\sqrt{5-4x-2x^2} + \frac{7\sqrt{2}}{2\times 2}\sin^{-1}\left(\frac{x+1}{\sqrt{7}/2}\right)$$

$$\Rightarrow \left(\frac{x+1}{2}\right)\sqrt{5-4x-2x^2} + \frac{7\sqrt{2}}{2\sqrt{2}}\left(\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right)$$

from eqⁿ (ii)

$$I = -\frac{3}{4}I_1 - 4I_2$$

$$\Rightarrow -\frac{3}{4}\times\frac{2}{3}(5-4x-2x^2)^{3/2} - 2(x+1)\sqrt{5-4x-2x^2} - \frac{14}{\sqrt{2}}\sin^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right) + C \quad \text{Ans}$$

VB.NET

Note :-

① If you can't remember the name of a particular conversion function use c type function which let's you specify a type to convert into

Code :-

```
Option strict On
Module Type-example
Sub Main()
Dim db As Double
Dim intData As Integer
db = 3.1419
intData = CInt(db)
system.console.writeline ("Int data:" & str(
Int-Data))
End sub
End Module
```

Option Strict :-

It means that visual basic will not automatically convert data type when we assign the value of one type to a variable of another type

Str :- through this we can convert to string.

Syntax:-

CType (var-name, Data-type)

② The use of bright line to display text and the value by passing it the expression ("Int Data:" & Str (Int Data))

console.writeline

we can also ~~inbuilt~~ inbed ~~console.life~~ {0}{1}...) and so on into a text string which will then displaye replaced by successive values passed to right line

Example:

console.writeline ("The time is: {0} hours {1} minutes", 10, 2)

Output :- 10 hours 2 minutes.

* Variables & Constant

Variable :-

Variable is a simple name used to store the value of a specific data type in computers memory.

In VB.net each variable has a particular data type that determines the size, range and fixed space in computer memory.

With the help of variable we can perform

several operations and manipulate data values.

Syntax:-

for declaration: Dim varname As Data-Type

Dim keyword is used in class module structure, sub and a procedure.

It is used to declared and allocate the space for one or more variables in memory.

As keyword :-

It is a keyword that allows us to define the data type in the declaration statement.

* Getting values from user

In VB.net the console class provides the Read line() function in the System name space.

It is used to take input from the user and to assign a value to the variable.

Example:-

Dim name As String
name = Console.Readline()

code :-

```
Imports System
Module UserData
  Sub Main()
  Dim num As Integer
  console.writeline ("Enter any integer")
  num = console.Readline()
  console.writeline ("You entered : {0}", num)
  End sub
  End Module
```

applet :-

→ applet are small programs that are used in internet computing

→ It can be transported over the internet from one computer to another which can be run through applet viewer

→ It is an application program which is used to perform task like play sounds, also, graphics etc

you → It is like java program which is embedded in web page

User Name : [ ]
E-mail : [ ]
website : [ ]
comment : [ ]

Username:
last name: [ ]
first ~~front~~ name: [ ]
Gender : • male • female • other
[Submit] [Exit]

```
<html>
<head> form validation </head>
<body>
<form action="form.php" method="POST">
<?php
Username :<input type="text" name="username"
          value="priya03"><br>
first name: <input type="text" name="first name"
          value="priya"><br>
lastname : <input type="text" name="username"
          value="singh"><br>
E-mail: <input type="text" name="Email",
   value="priyangana03singh@gmail.com"><br>
website: <input type="text" name="website"><br>
comment : <text area input type="
   name="comment" rows="04" cols="10">
   </text area><br>
```

Gender :

```
<input type = "radio" name = "gender"
      value = "male" >Male <br>
<input type = "radio" name = "gender" value = "female">
Female <br>
<input type = "radio" name = "gender" value = "other">
Other <br>
<input type = "submit> <br>
<input type = "exit") <br>

</form>
</body>
</html>
```

* Definite integration

$$I = \int_a^b f(x)\,dx$$

Let, $\int f(x)\,dx = F(x) + C$

$I = [F(x) + C]_a^b$

$\Rightarrow F(b) + \cancel{C} - F(a) - \cancel{C}$

$= F(b) - F(a)$

Note:-

$$\int_a^b uv\,dx = u\int_a^b v\,dx - \int_a^b \left(\frac{du}{dx}\int v\,dx\right) dx$$

$$\Rightarrow [uw]_a^b - \int_a^b \left(\frac{du}{dx} \cdot \int v\,dx\right) dx$$

Let $\int v\,dx = w$

$$= [uw]_a^b - \int_a^b u'w\,dx$$

Ex:- $\displaystyle\int_0^{\pi/4} \frac{\tan^{-1}x}{1+x^2}\,dx$

Let, $\tan^{-1}x = t$

$\therefore \left(\dfrac{1}{1+x^2}\right) dx = dt$

when, $x = 0$, then $t = 0$

& when $x = \frac{\pi}{4}$ then $t = 1$

$$I = \int_0^1 t\,dt \Rightarrow \frac{[t^2]_0^1}{2} = \frac{1}{2} \text{ A.}$$

Imp. * Properties of Definite Integration:-

(P1) $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ Bnet

(P2) $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(P3) $\displaystyle\int_b^a f(x)\,dx = \int_b^a f(a-x)\,dx$

(P4) $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(P5) $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

where $a < c < b$

(P6) $\displaystyle\int_{-a}^{a} f(x)\,dx = \begin{cases} 0; & f(-x) = -f(x) \\ & \text{i.e., } f(x) \text{ is add function} \\ 2\displaystyle\int_0^a f(x)\,dx; & f(-x) = f(x) \\ & \text{i.e., } f(x) \text{ is an even function} \end{cases}$

TYPE-①

PROBLEM BASED UPON PROPERTY 3

Q1. Prove that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} = \frac{\pi}{4}$$

Sol^n: Let, $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ — ①

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\, dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx \quad \text{— (ii)}$$

Eq^n ① + Eq^n (ii)

$$2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right) dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4} \text{ A.}$$

Q. Prove $\int_0^{\pi/4} \log(1+\tan x)\, dx = \frac{\pi}{8}\log 2$

Proof: $I = \int_0^{\pi/4} \log$

$$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right) dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right) dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx$$

$$I = \int_0^{\pi/4} \{\log 2 - \log(1+\tan x)\}\, dx$$

$$I = \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log(1+\tan x)\, dx$$

$$I = (\log 2)\int_0^{\pi/4} dx - I$$

$$\Rightarrow 2I = (\log 2)\,[x]_0^{\pi/4}$$

$$\Rightarrow 2I = (\log 2)(\pi/4)$$

$$\therefore I = \pi/8 \log 2 \text{ A.}$$

Type (ii) Problems based upon property IV

Q. Prove that $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan x} = \frac{\pi}{12}$

Sol^n:- Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sin x}{\cos x}}$

$I = \int_{\pi/6}^{\pi/3} \frac{\cos x \, dx}{\cos x + \sin x} \, dx$ —— (i)

$I = \int_{\pi/6}^{\pi/3} \frac{\cos(\pi/3+\pi/6-x)\, dx}{\cos(\pi/3+\pi/6-x) + \sin(\pi/3+\pi/6-x)}$

$I = \int_{\pi/6}^{\pi/3} \frac{\cos(\pi/2-x)}{\cos(\pi/2-x)+\sin(\pi/2-x)}$

$I = \int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x} \, dx$ —— (ii)

Eq^n (i) & Eq^n (ii)

$2I = \int_{\pi/6}^{\pi/3} \left(\frac{\sin x + \cos x}{\sin x + \cos x}\right) dx$

$2I = \int_{\pi/6}^{\pi/3} dx$

$2I = [x]_{\pi/6}^{\pi/3}$

$\Rightarrow \left(\frac{\pi}{3} - \frac{\pi}{6}\right)$

$2I = \frac{\pi}{6}$

Hence, $I = \frac{\pi}{12}$ Pro

Type 2 :

Problems based upon modulus function and Property 5

Q1. $I = \int_0^{2\pi} |\sin x| \, dx$

Sol^n $\int_0^{\pi} |\sin x| \, dx + \int_{\pi}^{2\pi} |\sin x| \, dx$

$\Rightarrow \int_0^{\pi} \sin x \, dx - \int_{\pi}^{2\pi} \sin x \, dx$

$\Rightarrow -[\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$

$\Rightarrow -(\cos \pi - \cos 0) + (\cos 2\pi - \cos \pi)$

$\Rightarrow -(-1-1) + (1+1)$

$= 4$ Ans

Q2. $\int_0^4 \left(|x-1| + |x-2| + |x-3|\right) dx$

Sol^n:- $I = \int_0^4 |x-1| \, dx + \int_0^4 |x-2| \, dx + \int_0^4 |x-3| \, dx$

$\Rightarrow \int_0^1 |x-1| \, dx + \int_0^4 |x-1| \, dx + \int_0^2 |x-2| \, dx + \int_0^4 |x-2| \, dx + \int_0^3 |x-3| \, dx + \int_3^4 |x-3| dx$

$$= -\int_0^1 (x-1)\,dx + \int_1^4 (x-1)\,dx - \int_2^2 (x-2)\,dx + \int_2^4 (x-2)\,dx - \int_0^3 (x-3)\,dx + \int_3^4 (x-3)\,dx$$

$$\Rightarrow \int_0^1 (1-x)\,dx + \int_0^4 (x-1)\,dx + \int_0^2 (2-x)\,dx + \int_0^4 (x-2)\,dx + \int_0^3 (3-x)\,dx + \int_3^4 (x-3)\,dx$$

$$\Rightarrow \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4 + \left[2x - \frac{x^2}{2}\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^4 + \left[3x - \frac{x^2}{2}\right]_0^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$

A.F 19/2

$1 - \frac{1}{2}$

weight pre- ~~~~

CPU

in java.lang

lant executr

ead

th

wread

unboxing

integer i = new integer()

value.of(a); int i = a.intvalue();

int j = a;

+ i + " ");

SOP(a + " " + i + " ");

$= \int_0^1 (x-1)dx + \int_1^4 (x-1)dx = \int^1$

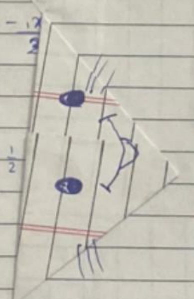
$\Rightarrow \int_0^1 (1-x)dx + \int_0^4 (x-1)dx$

$\Rightarrow \left[x \right.$

$\left[3x - \frac{x}{3} \right.$

$A_1 = 19/2$

$1 - \frac{1}{2}$



√B. NEP

Thread :- light weight pre- 11/08/22
processor
↳ basic unit of CPU
→ it is predefined class in java.lang
package
→ it is known for instant execute
new (start)
runnable read
running ~~run~~ execute
waiting blocked
dead - exit

```
import java.util.*;
public class abc extends Thread
{
  public void run()
  {
    s.o.p ("welcome");
  }
  p.s.v.m (String [] args)
  {
    abc c = new abc()   create th
    c.start();   // start thread
  }
}
```

autoboxing

```
int a=20;
integer i = integer.valueOf(a);
integer j = a
s.o.p (a+" "+i+" ");
```

unboxing

```
integer i = new integer(a)
int i = a.intValue();
int j = a;
s.o.p (a+" "+i+" ");
```

→ light weight process or it is a path of execution;
→ pre-defined class in java lang for
→ it is basic unit of CPU and also known for independent execution

(i) extend thread class
(ii) implements runnable interface

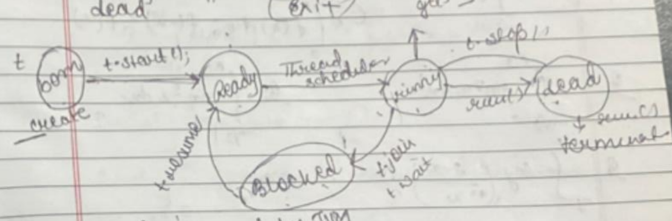
Thread class — runnable interface

extends — Thread — Implement

1. class A extends Thread
{
  overrride
  public void main ()
  { //code
  }
}

Thread life cycle

① new state (born)
runnable state (Ready)]
Running " (execution)] Active state
waiting " (Blocked)
dead " (Exit)



Thread is controlled by JVM

import java.util.*
public class MyThread Abc extends Thread
{
  public void run()
  {
    S.O.P ("welcome");
  }
  psvm (String[] args)
  {
    Abc c = new abc (); //Creating thre
    c.start (); // starting thread
  }
}

output
welcom

Type - IV

Problems based upon P(6)

i.e., $\int_{-a}^{a} f(x)\,dx = \begin{cases} 0; & f(-x) = -f(x) \\ 2\int_0^a f(x)\,dx; & f(-x) = f(x) \end{cases}$

Q. $\int_{-\pi/2}^{\pi/2} \sin^7 x\,dx$

Q. $\int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

Q. $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$

1 solⁿ:- Here,

$f(x) = \sin^7 x = (\sin x)^7$

$f(-x) = (\sin(-x))^7$

$= (-\sin x)^7$

$= -(\sin x)^7$

$\Rightarrow -\sin^7 x$

$f(-x) = -f(x)$

$\Rightarrow f(x) = \sin^7 x$ is an odd function

Hence by P(6)

$I = \int_{-\pi/2}^{\pi/2} \sin^7 x\,dx = 0$ Ans

2 solⁿ:- Here,

$f(x) = \sin^2 x = (\sin x)^2$

$\therefore f(-x) = (\sin(-x))^2$

$= (-\sin x)^2$

$= (\sin x)^2$

$= -\sin^2 x$

$\therefore f(-x) = f(x)$

$f(x) = \sin^2 x$ is an even function

Hence by P(6)

$I = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx = 2\int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

$I = 2\int_0^{\pi/2} \sin^2 x\,dx$ — (i)

$I = 2\int_0^{\pi/2} \sin^2\left(\frac{\pi}{2} - x\right) dx$ [by P(3)]

$I = 2\int_0^{\pi/2} \cos^2 x\,dx$ — (ii)

Eqⁿ (i) + Eqⁿ (ii)

$2I = 2\int_0^{\pi/2} (\sin^2 x + \cos^2 x)\,dx$

$I = \int_0^{\pi/2} dx$

$I = [x]_0^{\pi/2}$

$I = \pi/2$ Ans

3 solⁿ:- Here, $f(x) = \log\left(\frac{2-x}{2+x}\right)$

$\Rightarrow f(-x) = \log\left(\frac{2+x}{2-x}\right)$

$= -\log\left(\frac{2-x}{2+x}\right)$

$= -f(x)$

$$f(x) = \log\left(\frac{2-x}{2+x}\right)$$

is an odd function

$$\therefore I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$$

$$= 0 \quad \text{Ans. (by Property (6))}$$

Type (5)

Problem based upon property 7

i.e.,

$$\int_0^{2a} f(x)\,dx = \begin{cases} 0; & f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx; & f(2a-x) = f(x) \end{cases}$$

Q. Prove that

$$\int_0^{\pi/2} \log(\sin x)\,dx = \int_0^{\pi/2} \log(\cos x)\,dx = -\frac{\pi}{2}\log 2 = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$$

Sol^n:- $I = \int_0^{\pi/2} \log(\sin x)\,dx$

$$I = \int_0^{\pi/2} \log\left\{\sin\left(\frac{\pi}{2} - x\right)\right\} dx \quad \text{(by P(3))}$$

$$I = \int_0^{\pi/2} \log \cos x\,dx \quad \text{--- (ii)}$$

Eq^n (i) + Eq^n (ii)

$$2I = \int_0^{\pi/2} \log(\sin x) + \log(\cos x)\,dx$$

$$2I = \int_0^{\pi/2} \log(\sin x \cdot \cos x)\,dx$$

$$2I = \int_0^{\pi/2} \log\left(\frac{2\sin x \cos x}{2}\right) dx$$

$$2I = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$$

$$2I = \int_0^{\pi/2} (\log(\sin 2x) - \log 2)\,dx$$

$$2I = \int_0^{\pi/2} \log(\sin 2x)\,dx - \log 2 \int_0^{\pi/2} dx$$

$$2I = I_1 - (\log 2)\left[x\right]_0^{\pi/2}$$

$$2I = I_1 - \frac{\pi}{2}\log 2 \quad \text{--- (iii)}$$

Now,

$$I_1 = \int_0^{\pi/2} \log(\sin 2x)\,dx$$

let,

$2x = t$

$\therefore 2dx = dt$

$dx = \frac{dt}{2}$; when $x = 0$, then $t = 0$

when $x = \pi/2$ then $t = \pi$

$$I_1 = \frac{1}{2}\int_0^{\pi} \log(\sin t)\,dt$$

$$I_1 = \frac{1}{2}\int_0^{2\pi/2} \log \sin t \, dt$$

Here, $f(t) = \log(\sin t)$

$$f\left(\frac{2\pi}{2} - t\right) = \log f(\pi - t)$$
$$= \log(\sin(\pi - t))$$
$$= \log \sin t$$
$$= f(t)$$

By property (7)

$$I_1 = \frac{1}{2} \times 2 \int_0^{\pi/2} \log \sin t \, dt$$

$$I_1 = \int_0^{\pi/2} \log(\sin x)\, dx, \quad \text{by } P(1)$$

$$I_1 = I$$

from eq^n (iii)

$$2I = I_1 - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = -\frac{\pi}{2}\log 2$$

$$I = \frac{\pi}{2}\log\left(\frac{1}{2}\right) \quad \text{Proved}$$

H.W

$$\int_1^4 \left(|x-1| + |x-2| + |x-3|\right) dx = 19/2$$

(2) $\displaystyle\int_0^{\pi} \frac{x\,dx}{a^2\sin^2 x + b^2\cos^2 x}$ (3) $\displaystyle\int_{\pi/2}^{2\pi/2} |\sin x|\, dx$

(4) $\displaystyle\int_0^{\pi} |\cos x|\, dx$ (5) $\displaystyle\int_{-1}^{1} e^{|x|}\, dx$

(6) $\displaystyle\int_{-1}^{1} \sin|x| + \cos|x| \, dx$ (7) $\displaystyle\int_0^{\pi} |\cos x|\, dx$

08/01/22

Proof of properties of definite integration

(1) $\displaystyle\int_a^b f(x)\, dx = \int_a^b f(t)\, dt$

Proof:- Let, $f(x)\,dx = F(x) + c$

then,

$$L.H.S = \int_a^b f(x)\,dx = \left[F(x) + c\right]_a^b$$
$$= (F(b) + c) - (F(a) + c)$$
$$= F(b) + \cancel{c} - F(a) + \cancel{c}$$

$$L.H.S = F(b) - F(a)$$

$$R.H.S = \int_a^b f(t)\,dt = \left[F(t) + c\right]_a^b$$
$$= F(b) + \cancel{c} - F(a) - \cancel{c}$$
$$= F(b) - F(a)$$

hence,

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt \quad \text{Proved}$$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$

Proof:- Let, $\int f(x)dx = F(x) + c$

L.H.S when, $\int_a^b f(x)dx = \left[F(x)+c\right]_a^b$

$= F(b) + c - f(a) - c$

$= F(b) - F(a)$

R.H.S $= -\int_b^a f(x)\,dx$

$= -\left[F(x)+c\right]_b^a$

$= -\left(F(a) + c - F(b) - c\right)$

$= F(b) - F(a)$

Hence,

$\int_a^b f(x)\,dx = -\int_b^a f(x)dx$ Proved

Important 3 to 4 for exam

(iii) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Proof:- R.H.S $= I = \int_0^a f(a-x)\,dx$

Let, $a - x = t$

$-dx = dt$

$dx = -dt$

when $x = 0$, then $t = a$

and, when $x = a$ then $t = 0$

$I = -\int_a^0 f(t)\,dt$

$= \int_0^a f(t)\,dt$ (by Property (ii))

L.H.S $= \int_0^a f(x)\,dx$ (by property (i))

$=$ L.H.S

Hence, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Proved

(iv) $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

Proof:- R.H.S $= I = \int_a^b f(a+b-x)\,dx$

Let, $a+b-x = t$

$-dx = dt$

$dx = -dt$

when, $x = a$, then $t = b$

when $x = b$, then $t = a$

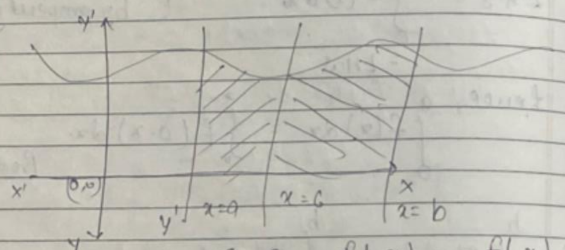
$$I = -\int_b^a -f(t)\,dt$$

$$= \int_a^b f(t)\,dt \quad (\text{by } p.(ii))$$

$$\int_a^b f(x)\,dx = L.H.S \quad (\text{by property (ii)})$$

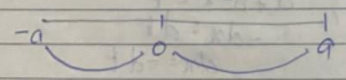
Proof

(v) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



(vi) $$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 \; ; & f(-x) = -f(x) \\ 2\int_0^a f(x)\,dx \; ; & f(-x) = f(x) \end{cases}$$

Proof:



$$I = \int_{-a}^{a} f(x)\,dx$$

$$I = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx \quad \text{— (1)}$$

Let, $I_1 = \int_{-a}^{0} f(x)\,dx$

Let, $x = -t$

$dx = -dt$

when,

$x = -a$, then $t = a$

and when $x = 0$ then $t = 0$

$$I_1 = -\int_a^0 f(-t)\,dt$$

$$= \int_0^a f(-t)\,dt \quad (\text{by property (ii)})$$

$$= \int_0^a f(-x)\,dx \quad (\text{By P(i)})$$

Case I:- when $f(-x) = -f(x)$

then,

$$I_1 = -\int_0^a f(x)\,dx$$

from eqn (1)

$$I = -\int_0^a f(x)\,dx + \int_0^a f(x)\,dx$$

$$I = 0$$

Case (ii) when $f(-x) = f(x)$

then, $I_1 = \int_0^a f(x)\,dx$

from Eqn (i)

$$I = \int_0^a f(x)\,dx + \int_0^a f(x)\,dx$$

$$I = 2\int_0^a f(x)\,dx$$

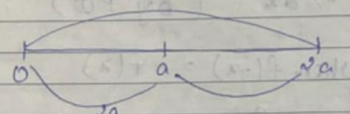
hence,

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0\ ; & f(-x) = -f(x) \\ 2\int_a^a f(x)\,dx\ ; & f(-x) = f(x) \end{cases}$$

Proved.

(viii) $$\int_a^{2a} f(x)\,dx = \begin{cases} 0\ ; & f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx\ ; & f(2a-x) = f(x) \end{cases}$$

Proof :



$$I = \int^{2a} f(x)\,dx$$

$$= \int_0^a f(x) + \int_a^{2a} f(x)\,dx \quad \text{—(i)}$$

Now, let, $I_1 = \int_a^{2a} f(x)\,dx$

$$I = \int_0^{2a} f(x)\,dx$$

$$= \int_0^a f(x) + \int_0^{2a} f(x)\,dx \quad \text{—(i)}$$

let, $2a - x = t$

$x = (2a - t)$

$\therefore -dx = dt$

$\Rightarrow dx = -dt$

where, $x = a$, then $t = a$

& when $x = 2a$, then $t = 0$

$$I_1 = -\int_a^0 f(2a-t)\,dt$$

$$= \int_0^a f(2a-t)\,dt \quad \text{(by P(ii))}$$

$$= \int_0^a f(2a-x)\,dx \quad \text{(by P(i))}$$

Case I : when $f(2a-x) = -f(x)$

then, $I_1 = -\int_0^a f(x)\,dx$

from Eqn (i)

$$I = \int_0^a f(x)\,dx - \int_0^a f(x)\,dx$$

$$I = 0$$

Case II : $f(2a-x) = f(x)$

then,

$$I = \int_0^a f(x)\,dx + \int_0^a f(x)\,dx$$

$$I = 2\int_0^a f(x)\,dx$$

hence, $$\int_0^{2a} f(x)\,dx = \begin{cases} 0\ ; & f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx\ ; & f(2a-x) = f(x) \end{cases}$$

Pr..

* Integration of Irrational function al or Integral of the form

$$\int \frac{dx}{x\sqrt{y}} \quad \text{or} \quad \int \frac{f(x)}{x\sqrt{y}}\,dx$$

Where, x = linear, y = linear

Working Rule :- put $y = t^2$

Q. $I = \int \frac{(2x-1)\,dx}{(3x-2)\sqrt{x+y}}$

Soln Let, $5x+4 = t^2$

$$x = \left(\frac{t^2-4}{5}\right)$$

$\therefore\ 5\,dx = 2t\,dt$

$$dx = \frac{2t\,dt}{5}$$

$$I = \int \frac{\left[2\left(\frac{t^2-4}{5}\right)-1\right]\frac{2t}{5}\,dt}{\left[3\left(\frac{t^2-4}{5}\right)-2\right]t}$$

$$I = \frac{2}{5}\int \frac{2t^2-8-5/\cancel{5}}{3t^2-12-10/\cancel{5}}\,dt$$

$$I = \frac{2}{5}\int \frac{2t^2-8-5}{3t^2-12-10}\,dt \Rightarrow \frac{2}{5}\int \frac{2t^2-13}{3t^2-22}\,dt$$

$$\Rightarrow \frac{2}{5}\int$$

* Integration by Partial fraction :-

Note :-

when degree of numerator is less than degree of denominator then partial fraction is applied.

when degree of numerator is greater

$f(x)$	Partial fraction
$\frac{1}{L_1L_2L_3}$	$\frac{A}{L_1}+\frac{B}{L_2}+\frac{C}{L_3}$
$\frac{1}{L_1L_2^3}$	$\frac{A}{L_1}+\frac{B}{L_2}+\frac{C}{L_2^2}+\frac{D}{L_2^2}$
$\frac{1}{LQ}$	$\frac{A}{L}+\frac{Bx+C}{Q}$
$\frac{1}{L^2Q}$	$\frac{A}{L}+\frac{B}{L^2}+\frac{Cx+D}{Q}$

$\frac{1}{Q_1 Q_2}$	$\frac{Ax+B}{Q_1} + \frac{Cx+D}{Q_2}$
$\frac{1}{LQ^2}$	$\frac{A}{L} + \frac{Bx+C}{Q} + \frac{Dx+E}{Q^2}$

where,
L, L_1, L_2, L_3 are linears
and Q, Q_1, Q_2 are quadratics

Type - ①

① $\int \frac{(2x-1)}{(x-1)(x+2)(x-3)}$

Sol^n:- let, $\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$ — ①

$(2x-1) = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$

Put, $x=1$
$1 = 3(-2)A \Rightarrow A = \frac{-1}{6}$

Put, $x=3$
$5 = (2)(5)C \Rightarrow C = \frac{1}{2}$

Put, $x=-2$
$-5 = (-3)(-5)B$
$B = \frac{-1}{3}$

from eq^n ①

$\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx = \int \frac{-1/6 \, dx}{(x-1)} + \int \frac{-1/3 \, dx}{x+2} + \int \frac{1/2 \, dx}{x-3}$

$I = \frac{-1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + c$

Type - 2

problem based upon substitution

Q $I = \int \frac{dx}{x(x^n+1)}$

let, $x^n = t$
$nx^{n-1} dx = dt$
$\frac{n x^n}{x} dx = dt$
$\frac{dx}{x} = \frac{dt}{nt} \Rightarrow \frac{1}{n} \int \frac{dt}{t(t+1)}$

$= \frac{1}{n} \int \frac{(t+1-t)}{t(t+1)} dt$

$= \frac{1}{n} \int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$

$= \frac{1}{n} \left(\log|t| - \log|t+1|\right)$

$= \frac{1}{n} \log \left|\frac{t}{t+1}\right|$

$= \frac{1}{n} \log \left|\frac{x^n}{x^n+1}\right|$

② $I = \int \frac{dx}{\sin x + \sin 2x}$

Sol^n, $I = \int \frac{dx}{\sin x + 2\sin x \cos x}$

$I = \frac{dx}{\sin x(1 + 2\cos x)}$

$$I = \int \frac{\sin x \, dx}{\sin^2 x (1+2\cos x)}$$

$$= \int \frac{\sin dx}{(1-\cos^2 x)(1+2\cos x)}$$

$$\Rightarrow \int \frac{\sin x \, dx}{(1+\cos x)(1-\cos x)(1+2\cos x)}$$

let $\cos x = t$

$= -\sin x \, dx = dt$

$\sin x \, dx = -dt$

$$I = \int \frac{-dt}{(1+t)(1-t)(1+2t)}$$

$$\Rightarrow \int \frac{dt}{(t+1)(t-1)(2t-1)} \quad \ldots \text{ solve}$$

Driver manager. getconnection (path, username, password)

oceat stat

Type :- 03

Problem based upon Replacement

Q. $\int \frac{x^2}{(x^2+2)(x^2+3)} dx$

$$\frac{x^2}{(x^2+2)(x^2+3)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+3} \times$$

Replace $x^2 = y$

i.e., $\frac{x^2}{(x^2+2)(x^2+3)} = \frac{y}{(y+2)(y+3)}$ — (1)

Now, $\frac{y}{(y+2)(y+3)} = \frac{A}{y+2} + \frac{B}{y+3}$ — (ii)

$y = A(y+3) + B(y+2)$

Put, $y = -2$

$-2 = A \Rightarrow \boxed{A = -2}$

Put, $y = -3$

$-3 = -B \Rightarrow \boxed{B = 3}$

from eqn (ii)

$$\frac{y}{(y+2)(y+3)} = \frac{-2}{(y+2)} + \frac{3}{(y+3)}$$

$$\frac{x^2}{(x^2+2)(x^2+3)} =$$

Register driver class
Creates connection object
Create statement object
Execute the query
close connection object

package pack
public class
{
// code
}

$$I = \frac{-2}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{3}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

$$I = -\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

(2) $I = \int \frac{(1-\cos x)\,dx}{\cos x(1+\cos x)}$

Replace $\cos x$ by y

then

$$I = \int \frac{(1-y)}{y(1+y)}\,dx$$

let, $\frac{1-y}{y(1+y)} = \frac{A}{y} + \frac{B}{(y+1)}$ — (1)

$(1-y) = A(y+1) + By$

Put, $y=0$

$1 = A \Rightarrow \boxed{A=1}$

Put, $y=-1$

$2 = -B \Rightarrow \boxed{B=-2}$

from Eq^n (1)

$$\frac{(1-y)}{y(1+y)} = \frac{1}{y} + \frac{(-2)}{(y+1)}$$

But, $y = \cos x$

$\therefore$

$$\int \frac{1-\cos x}{\cos x(1+\cos x)}\,dx = \int \frac{1}{\cos x}\,dx + \int \frac{(-2)}{(\cos x+1)}\,dx$$

$$\Rightarrow \int \sec x\,dx - 2\int \frac{dx}{2\cos^2\frac{x}{2}}$$

$$I = \int \sec x\,dx - \int \sec^2\frac{x}{2}\,dx$$

$\log|\sec x + \tan x| - 2\tan\frac{x}{2} + c$ Ans

Type-04

Note:- if degree of numerator is equal to degree of denominator and also co-efficient of highest degree term of numerator and denominator are equal then quotient must be 1

Q $I = \int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}\,dx$

let,

$$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \frac{A}{x-4} + \frac{B}{x-5} + \frac{C}{x-6} \quad — (1)$$

$(x-1)(x-2)(x-3) = (x-4)(x-5)(x-6) + A(x-5)(x-6)$
$+ B(x-4)(x-6)$
$+ C(x-4)(x-5)$

Put, $x=4$

$(3)(2)(1) = (-1)(-2)A$

$\boxed{A=3}$

Put, $x=5$

$(4)(3)(2) = B(1)(-1)$

$\boxed{B=-24}$

Put, $x=6$

$(5)(4)(3) = C(2)(1)$

$\boxed{C=30}$

from Eq^n (1)

$$\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = \int 1\,dx + \int \frac{3\,dx}{(x-4)} + \int \frac{-24\,dx}{(x-5)} + \int \frac{30\,dx}{x-6}$$

Separable/ele

hence,

$I = x + 3\log|x-4| - 24\log|x-5| + 30\log|x-6| + c$

$15\times2=30$

5×5

A^{-1}

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & -2 & 4 \\ 3 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & -2 & 4 \\ 0 & -1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + 3(R_2)$

$$\begin{bmatrix} 1 & 0 & -12 \\ 0 & -2 & 4 \\ 0 & -1 & -5 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 3 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

$R_2 \to \frac{-R_2}{2}$

$$\begin{bmatrix} 1 & 0 & -12 \\ 0 & 1 & -2 \\ 0 & -1 & -5 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 3 \\ 1 & -1/2 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

$A^{-1}x = B$

$3x + 4y + 2z = 2$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & -12 \\ 0 & 1 & -2 \\ 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 3 \\ 1 & -1/2 & 0 \\ -2 & -1/2 & 1 \end{bmatrix} A$$

$R_3 \to \frac{R_3}{-7}$

$$\begin{bmatrix} 1 & 0 & -12 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 3 \\ 1 & -1/2 & 0 \\ +2/7 & +1/14 & -1/7 \end{bmatrix} A$$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & -12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 3 \\ 11/7 & -5/14 & 2/7 \\ 2/7 & 1/14 & -1/7 \end{bmatrix}$$

$\frac{-56+12}{7}$

$3 - \frac{12}{7}$

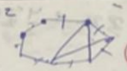
$R_1 \to R_1 + 12R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -44/7 & 12/14 & 9/7 \\ 11/7 & -5/14 & 2/7 \\ 2/7 & 1/14 & -1/7 \end{bmatrix}$$

(i) Register driver class forname() method
class.forname ("oracle.jdbc.driver.OracleDriver")

(ii) Create connection object
connection con = drivermanager.getconnection ("path", "username", "password")

(iii) create statement object
statement stat = con.createStatement();

$8\times(8-1)$ $\frac{7(7-1)}{2}$ 7×2  $\frac{n(n-1)}{2}$

$e^i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 1 0 0 1

$\frac{9!}{4!\times3!}$

$\frac{9\times8\times7\times6\times5\times4!}{4!\times3!}$

(3)

(ϕ) (1)

36 × 240

1440

$+ 8 \times$

2160

$\{a,b,c\}$

$\{(\phi),(a),(b),(c),(a,b),(a,c)(b,c),(a,b,c)\}$

$((p\to q)\wedge(q\to r)) = (p\to r)$

p	q	p→q	q
T	T	T	
T	F	T	
F	T	F	
F	P	~~T~~	

datatype arrayname [ ]

int a[ ] ;

a = new int [5];

int a [ ] = new int [5];

It is a collection of similar data elements or different

JDBC API

Java → JDBC driver → database → Oracle, my sql, MS Access

JDBC → Java DataBase Connectivity

It is a java API to create and execute query with database.

API → set of rule

JDBC API → java driver → database (my sql, oracle, ms access)

Java application

(1) Register driver class

for name () method

class.forname ("oracle.jdbc.driver.OracleDriver").

(i) init () method → initialization One tym

(ii) start() method → after many time

(iii) stop () → invoked when applet is stop

destroy →

Syntax :

```
import java.awt.*;
"       .applet.*;
public class abc extends applet
{   // code
public void paint
{
    // code
}
}
```

<html>
<head></head>
<body>
<applet code>
</applet>
</body>
</html>

new – start
runnable → execution
runn → blocked
wait → destroye
dead

Born → initial → init
running → start
Idol → stop
dead → destr
display → pau.

primitive
Autoboxing	Unboxing
public class Wrapper	→
psvm (string args[])	→
{	{
int a = 20;	Integer a = new integer(3)
integer i = integer.valueOf(a);	int i = a.intValue();
integer j = a;	int j = a;
S.O.P (a+" "+i+" "+j);	S.O.P (a+" "+i+" "+j);
} }	} }
20 20 20	3 3 3

int → Integer   Integer → Int

```
int a = 20;
integer i = integer.valueOf(a)
integer j = a;
S.O.P (a+" "+i+" "+j);

integer a = new integer(3)
int i = a.intValue();
integer j = a;
```

The mechanism of constructing a new class from existing class. This is basically done by creating a new class reusing the properties of existing class.

syntax :-

```
class A // super class
{ //code
}
class B extends A // sub class
{ //code
}
```

single Inheritance — Sup: Parent → Child

multilevel → Parent → Child 1 → ~~Parent~~ → Child 2

Multiple → doesn't support

Parent 1, Parent 2 → Child

ambiguity will occur

through Integra

int a * [ ]
a = new int [5];

Hierarchical Inheritance

Parent
child child child

Polymorphism

many — form

Same object having different behaviour

compile syntax returntype methodname(par

```
package simple; mypack;
public class simple {
psvm (s[] args)
{
s.o.p ("welcome");
}
}
```

package mypack; public class

```
package mypack;
public class simple;
{
psvm (s[] args)
{
S.O.P ("welcom");
}
}
```

compile
javac -d . simple.java

java mypack.simple

public
10
15
s.o.p

```
class main {
psvm (string[] args)
{
int [] age = {12, 4, 5};
SOP ("using for loop:");
for (int i=0; i< age.length; i++) {
s.o.p (age [i]);
}
}
}
```

age = {12, 4, 5}

```
package mypack {
public class abc {
p svm
{
s.o.p ("welcome")
}
}
```

```
class abc {
psvm ( ) {
int [] age = {12, 4, 5}
}
s.o.p ("using for loop:")
for (i=0; i< age.length; i++) {
s.o.p (age [i]);
}
}
}
```

```
package mypack
public class abc,
{
psvm ( ) {
S.O.P
```

2019 PYQ	2021 PYQ	2022 PYQ
(1) (c)	(1) (b)	(1) (c)
(2) (b)	(2) (b)	(2) (d)
(3) (a)	(3) (d)	(3) (a)
(4) (b)	(4) (a) switch, if	(4) (b)
(5) (a)	(5) (d)	(5) (d)
(6) (b)	(6) (a)	(6) (c)
(7) (a)	(7) (c)	(7) (a)
(8) (c)	(8) (c)	(8) (a)
(9) (d)	(9) (a)	(9) thrown
(10) (a)	(10) (b)	(11) (b)

→ String is a sequence of character
→ double quote is used to represent the string.
→ Example

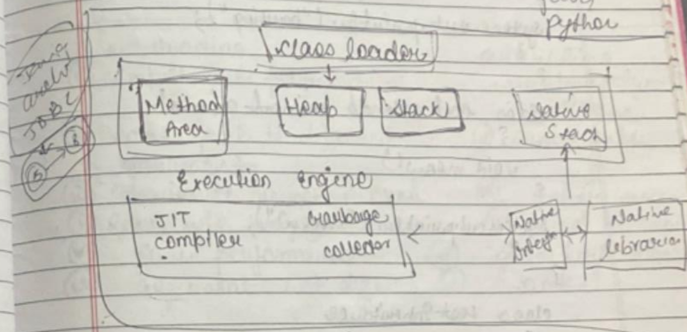
string type = "java"
↓ data type

String in java is not primitive type

```
class abc {
    psvm (str [] args) {
        // create a string
        string first = "java";
        string second = "python";
        // print st
        system.out.println(first); //,
                           (second);
    }
}
Sys.
```

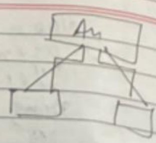
```
class abc {
. psvm () {
    // create str
    string first = "java";
    "      second = "python";
    // print string
    system.out.println(first);
                      (second);
```

output
java
python




```
class abc
{
    psvm (string [] args)
    {                               // create string
        string first = "java";
        string second = "python";
        // print string
        system.out.println(first);
                          (second);
    }
}
```

```
class animal
{
void eat ()
{
system.out.println ("eating");
}
}
class dog extends animal
{
void bark ()
{
system.out.println ("barking");
}
}
class cat extends ~~animal~~ animal
{
void meow()
{
system.out.println ("meow");
}
}
class test Inheritance
{
public static void main (String args [])
{
cat c= new cat ();
c.meow ();
c.bark ();
c. eat ();
}
}
```




```
class A
{
psvm
{
int a=10;
integer j = a;
s.o.p( a); //autoboxing
}
}
a = 10.
```

```
class B
{
psvm
{
integer a = new integer(3);
int j = a;
s.o.p (a); //unbox
}
}
b = 3.
```

overloading	overriding
(i) compile time polymorphism	runtime polymorphism
(ii) same name but different parament	(ii) same both
(iii) Inheritance is not involved	(iii) Inheritance
(iv) Return type may be same	(iv) must be same
(v) Poor performance	(v) good performance
(vi) argument list different	(vi) same

compile	run
(i) static binding	(ii) dynamic binding
(ii) method call is resolved at compile time	(ii) at run time
(iii) flexible	(iii) not flexible
(iv) method overloading	(iv) method overriding
(v) error, will be visible at compile time	(v) error, will be caught at runtime
(vi) fast execution	(vi) slow execution

HLL → sourcecode → bytecode → binary code → (gsd) asm
CIL/MIL, compile

```
public class abc extends Thread
{
    public void run()
    {
        s.o.p("welcome")
    }
    psvm
    {
        abc c = new abc
        c.start();
    }
}
```

mach → LLL → HLL

source cod ↓ (IL/MIL) → Bytecod ↓ JIT → Machin → binary → relocatable address

Array

HLL ↓ source code ↓ bytecode ↓ binary compile

assemble ↓ LLL

```
class abc
{
    psvm
    {
        int [] age = {12,4,5}
        s.o.p("using for loops");
        for (i=0; i<age.length; i++)
        {
            s.o.p(age[i]);
        }
    }
}
```

CLR, CTS, CLS, BCL, FCL, .Net Fw, window services

Tautology ↓

$\wedge$ and, $\vee$ or, $\sim$ not

if all values in truth then only tautology

P	~P	P∨~P
T	F	T
F	T	T

Tautology

P	~P	P∧~P
T	F	F
F	T	F

contradiction

logical equivalence
$P \equiv Q$ (if P & Q are proposition)

condition statement
$P \rightarrow Q$
if Pq → true & false → False

Biconditional statement
$P \leftrightarrow Q$

Principle of duality
→ every valid boolean expression remains valid if the operators and identity elements are interchanged

$+ \leftrightarrow \cdot$
$1 \leftrightarrow 0$

$A+B\cdot C = (A+B)\cdot(A+C)$
$A\cdot(B+C) = (A\cdot B) + (A\cdot C)$
$A+1=1 \Rightarrow A\cdot 0=0$

Eg:- $\overline{A}\overline{B}(C+\overline{D})(\overline{C}+D)$ → Boolean expression
$\Rightarrow (\overline{A}+\overline{B}) + (C\cdot\overline{D}) + (\overline{C}\cdot D)$ ← this is duality

Eg:- $F = AB + \overline{C}D + E + 0$
$\Rightarrow (A+B)\cdot(\overline{C}+D)\cdot E\cdot 1$

Hand Shaking lemma

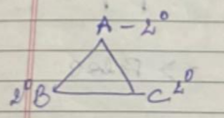
graph = sum of degree of vertex is twice number of edge

$\boxed{\text{sum of degree} = 2 \times \text{no. of edge}}$

(1) A — B

$D = 2 \times 6$

$= 2$



$6 = 2 \times 3$

$\boxed{D = 2 \times 6}$

walk in graph theory
→ total no. of edges covered in a walk = length of the walk
→ vertex, edge can be repeated.
if length of walk = 0, then it is called as a trivial walk.

Trail in graph theory
→ open walk
→ no edge repeated
→ vertices repeated

Path in graph theory
→ open walk
→ neither ~~edge~~ edges & vertices repeated

Circuit walk

1 → rank-10 (5) — (15)
→ 15 ✗

→ circuit walk in which no edge is repeated
→ vertex may be repeated.

SIMPLE GRAPH
multiple edges

TREE

root node / top level node
level 0
n: nodes
edge = n-1
fig: - tree structure

degree → total no. of children of a node.

node A
left subtree {B, D, E}
right subtree {C, F}
height → bottom to top
depth → top to bottom

Binary tree → each node has almost 2 child

full binary tree → each internal node has exactly two children.

complete :- each node will have to be at same level.

complete / almost Binary trees → all level are completely filled
→ from L → R

Tree Traversal Method

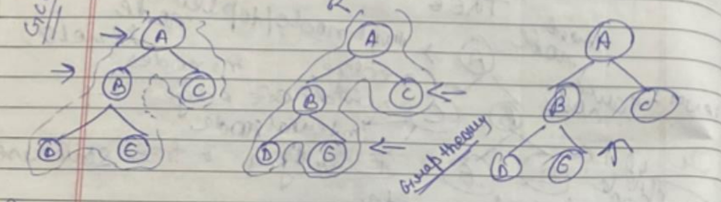
→ systematic way to so that each vertex is visited exactly once

(1) Pre-order :- visit the node
L → R

Post order → L → R
→ visit root

Inorder → L
visit node
R

GCD



Preorder
ABDEC

DEBCA
Postorder

ABEAC
Inorder

Edges

vertices [edges]

Q. 272 & 1479.

Gcd (272, 1479) = 17

Q. Gcd (256, 1166) ⇒ $256x + 1166y$

$1166 = 4 \times 256 + 142$ (4)

$256 = 1 \times 142 + 114$ (3)

$142 = 1 \times 114 + 28$ (2)

$114 = 4 \times 28 + 2$ — (1)

$28 = 14 \times 2 + 0$

Eqn (1)

$2 = 114 - 4 \times 28$

$2 = 114 - (142 - 1 \times 114) \times 4$ (value of 28)

$2 ⇒ 114 - 142 \times 4 + 4 \times 114$

$2 ⇒ 114(1+4) - 142 \times 4$

$2 ⇒ 114(5) - 142 \times 4$

$2 ⇒ (256 - 1 \times 142) \times 5 - 142 \times 4$

$2 ⇒ 256 \times 5 - 142 \times 5 - 142 \times 4$

$2 ⇒ 256 \times 5 - (5+4) 142$

$2 ⇒ 256 \times 5 - 9 \times 142$

$2 ⇒ 256 \times 5 - 9(1166 - 4 \times 256)$

$⇒ 256 \times 5 - 9 \times 1166 + 9(4 \times 256)$

$⇒ 256 \times 5 - 9 \times 1166 + 36 \times 256$

$⇒ 256(5+36) - 9 \times 1166$

$2 ⇒ 256(41) + (-9) \times 1166$ ⇒ $256x + 1166y$

Say $x_0 = 41$

$y_0 = -9$

$\boxed{2 = 256x_0 + 1166y_0}$

g.cd.

$2x + y + 2z = 0$
$2x - y + z = 10$
$x + 3y - z = 5$

$\Rightarrow \begin{bmatrix} 2 & 01 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 5 \end{bmatrix}$

Q

$AX = D$

$A^{-1}(AX) = A^{-1}D$

$(A^{-1}A)X = A^{-1}D$

$IX = A^{-1}D$

$X = A^{-1}D$ ---- (1)

$A^{-1} = \frac{\text{adj } A}{|A|} \Rightarrow$

det A

$|A| = 2(0-(-4)) - 3(10-(-1)) + 9(8-0)$

$\Rightarrow 2(4) + (-3)(11) + 9(8)$

$\Rightarrow 8 + 33 + 72$

$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

A^{-1}
$AX = B$

$AX = B \Rightarrow A^{-1}AX = A^{-1}B$

$X = A^{-1}B \quad | \quad IAX = A^{-1}B$

$\begin{array}{r} 72 \\ 8 \\ \hline 80 \end{array} \qquad \begin{array}{r} 80 \\ 33 \\ \hline 113 \end{array}$

$A^{-1} = \frac{\text{adj } A}{|A|}$

$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = 3|2-3| + 2|4+4| + 3|-6-4|$

$\Rightarrow 3(-1) + 2(8) + 3(-10)$

$\Rightarrow -3 + 16 - 30$

$= -17 \neq 0$

$|A| = 2(0-(-4)) + 3(10-(-1)) + 1(8-0)$

$\Rightarrow 2\times4 + 3(11) + 8$

$\Rightarrow 8 + 33 + 8$

$\Rightarrow 49$

$\begin{array}{r} 16 \\ 33 \\ \hline 49 \end{array}$

$A_{11} \Rightarrow (-1)^{1+1}(2-3) \Rightarrow (-1)^2(-1) = -1$

$A_{12} \Rightarrow (-1)^{1+2}(4+4) \Rightarrow (-1)^3(8) = -8$

$A_{13} \Rightarrow (-1)^{1+3}(-6-4) \Rightarrow (-1)^4(-10) = -10$

$A_{21} \Rightarrow (-1)^{2+1}(-4+9) \Rightarrow (-1)^3(5) = -5$

$A_{22} \Rightarrow (-1)^4(6-12) \Rightarrow (-1)^4(6) = -6$

$A_{23} \Rightarrow (-1)^5(9-8) \Rightarrow (-1)^5 \cdot 1 = 9$

$A_{31} \Rightarrow (-1)^4(2-3) \Rightarrow -1$

$A_{32} \Rightarrow (-1)^5(-3-6) \Rightarrow 9$

$A_{33} \Rightarrow (-1)^6(3+4) \Rightarrow 7$

cofactor

$\text{Adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & -18 & 7 \end{bmatrix}$

$A^{-1} = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & +18 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -18 & -7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{17}\begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

$\Rightarrow \frac{1}{17}\begin{bmatrix} 8+5+4 \\ 64+6-36 \\ 80-1-28 \end{bmatrix} \Rightarrow \frac{1}{17}\begin{bmatrix} 17 \\ 34 \\ 51 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \therefore x = 1, y = 2, z = 3$

Q $f(x) \in [-1, 1]$

(i) f is one-one,

Let, $x_1, x_2 \in [-1, 1]$ & $f(x_1) = f(x_2)$

$\frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$

$x_1 \cdot x_2 + 2x_1 = x_1 \cdot x_2 + 2x_2$

$[x_1 = x_2]$

for onto

(ii) let $y = \frac{x}{x+2}$

$xy + 2y = x$

$2y = x - xy \Rightarrow 2y = x(1-y)$

$$\left[x = \frac{2y}{1-y}\right] \quad y \neq 1$$

$$\left[f^{-1}(x) = \frac{2x}{1-x}\right]$$

3\. $f(x) = x^2$

y = range (f)

co-domain = range

∴ f is onto

∴ f is one-one as

let, $x_1, x_2 \in N$

$\Rightarrow f(x_1) = f(x_2)$

$(x_1)^2 = (x_2)^2$

$[x_1 = x_2]$ ∴ f is one-one

f is one & onto both.

so, f is invertible

Let, $y = x^2$

$\left[f^{-1}(x) = \sqrt{x}\right]$

$\cancel{2x - x - 8 = 0}$

$\cancel{2x = 8}$

$2x^2 - 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$P(n,4) = 20 \times P(n,2)$

$\Rightarrow \frac{n!}{(n-4)!} \Rightarrow 20 \times \frac{n!}{(n-2)!}$

$\Rightarrow (n-2)! = 20\,(n-4)!$ $\quad n(n-1)!$

$(n-2)(n-3) = 20$

$n^2 - 3n - 2n + 6 = 20$

$n^2 - 5n = 14$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7) + 2(n-7) = 0$

$(n-7)(n+2) = 0$

$n = -2 \;/\; n = 7$

n = natural no

$P(n,4) = 42\,P(n,2)$

$\frac{n!}{(n-4)!} = 42 \times \frac{n!}{(n-2)!}$

$(n-2)! = 42 \times (n-4)!$

$(n-2)(n-3)(n-4)! = 42 \times (n-4)!$

$(n-2)(n-3) = 42$

$n^2 - 2n - 3n + 6 = 42$

$n^2 - 5n = 36$

$n^2 - 5n - 36 = 0$

$n^2 - 9n + 4n - 36 = 0$

$n(n-9) + 4(n-9) = 0$

$(n-9)(n+4) = 0$

$n = 9 \;/\; \cancel{n = -4}$

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \Rightarrow ad - bc$$

$2x^2 - 8 = 0$

$2(x^2 - 4) = 0$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = 2 \;/\; -2$

4\. If $P(5, r) = 2\,P(6, r-1)$

$\Rightarrow \frac{5!}{(5-r)!} \Rightarrow 2 \times \frac{6!}{(6-(r-1))!}$

$\Rightarrow \frac{6!}{(6-r+1)!}$

$\Rightarrow \frac{6!}{(7-r)!}$

Given, $P\; {}^5P_r = 2\;{}^6P_{r-1}$

$\frac{(7-r)!}{(5-r)!} \Rightarrow 2 \times \frac{6!}{5!}$

$\Rightarrow \frac{(7-r)(6-r)(5-r)!}{(5-r)!} = \frac{2 \times 6 \times 5!}{5!}$

$(7-r)(6-r) = 12$

$(7-x)(6-x) = 12$

$42 - 7x - 6x + x^2 = 12$

$(42 - 13x + x^2) = 12$

$+x^2 - 13x = 12 - 42$

$+x^2 - 13x + 30 = 0$

$+x^2 - 13x + 30 = 0$

$+x^2 - 10x - 3x + 30 = 0$

$x(x-10) - 3(x-10) = 0$

$(x-10)(x-3) = 0$

$x = 3, 10$

$x < 5$ and $x < 6$

$\therefore x = 3$

sequence

$a_{n+1} = a_n^m$, $a_n = 3$

$f(x_1) = f(x_2)$

$3 - 4x_1 = 3 - 4x_2$

$-4x_1 = -4x_2$

$\boxed{x_1 = x_2}$

one-one

onto, $y = f(x)$

$y = 3 - 4x$

$y - 3 = -4x$

$\frac{-1}{4}(y-3) = x$

$\frac{-y+3}{4} = x$

$\boxed{\frac{3-y}{4} = x}$ $\boxed{R \to R}$

onto function

$3 - y$ invertible

$f^{-1}(x)$:

$y = 3 - 4x$

$f^{-1}(x) =$

$(3-4x)$

^5P_x & $^6P_{x-1}$

$\frac{5!}{(5-x)!} \Rightarrow \frac{6!}{6-(x-1)!}$

$\Rightarrow \frac{5!}{(6-x+1)!} \Rightarrow \frac{6!}{(7-x)!}$

$\frac{(7-x)!}{(5-x)!} = \frac{6!}{5!}$

$\frac{(7-x)!}{(5-x)!} \Rightarrow \frac{6 \times 5!}{5!}$

$(7-x)(6-x) = 6$

$7(6-x) - x(6-x) = 6$

$42 - 7x - 6x + x^2 = 6$

$42 - 13x + x^2 = 6$

$x^2 - 13x + 36 = 0$

$x^2 - 9x - 4x + 36 = 0$

$x(x-9) - 4(x-9) = 0$

$(x-9)(x-4) = 0$

$x = 9$ / $x = 4$

$\therefore x > 5$ and $x > 6$, $x = 4$

2, 4, 10, 28, ...

$2, 2_0, 2+$

$a_1 = 40$, $u_{n+1} = 3 \cdot 2 + 1 = 8$

u_{n+1}

$u_{n+1} = u_{n-1}$

$= 2 - 1 \Rightarrow 2$

$u_{n+1} = 3 \times 2 - 2 = 6 - 2 = 4$

$u_{n+1} = 3 \times 2 - 2 = 6 - 2 = 4$

$u_{n+1} = 3 \times 4 - 2 \Rightarrow 12 - 2 = 10$

$\frac{4^2}{36}$, 13, 2×18

Q $f(x) = 3 - 4x$, $f^{-1}(x)$

for one-one,

$3 - 4x_1 = 3 - 4x_2$

$x_1 = x_2$

$\therefore$ it is one-one.

for onto, $y = 3 - 4x$

$y - 3 = 4x$

$x = \frac{y-3}{4} = \frac{-1}{4}(y-3)$

$x \Rightarrow \frac{-y+3}{4} \Rightarrow \frac{3-y}{4}$

for $y \in R$, $\exists$ $x = \frac{3-y}{4}$

$f(x) = f\left(\frac{3-y}{4}\right)$

$\Rightarrow 3 - 4\left(\frac{3-y}{4}\right)$

$\Rightarrow 3 - 3 + y = y$

$f(x) = y$

$\left(f^{-1}(x) = \frac{3-x}{4}\right)$

calculate:-

(i) $\phi(1001)$

$1001 = 7^1 \times 11^1 \times 13^1$

$1001 \times \left(\frac{6}{7}\right)\left(\frac{10}{11}\right)\left(\frac{12}{13}\right)$

$\Rightarrow 720$ ans

7	1001
11	143
13	13
	1

Q $\phi(5040)$

$5040 \Rightarrow 2^4 \times 3^2 \times 5^1 \times 7^1$

$5040 \times \left(\frac{1}{2}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{6}{7}\right)$

1008, 144

$\Rightarrow$ 1152 ans

2	5040
2	2520
2	1260
2	630
3	315
5	105
3	21
7	7
	1

5	315
3	63
3	21
7	7
	1

1444×3, 162

2) $5040(2520$, 18%

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}, \quad B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$\theta - \phi = \pm \frac{n\lambda}{2}$ $n \Rightarrow$ odd no.

$\theta - \phi$

$AB = \begin{bmatrix} \cos^2\theta\cos^2\phi + \cancel{\sin\theta\sin\theta \cos\phi\sin\phi} \\ \cos^2\theta \end{bmatrix}$

$$AB = \begin{bmatrix} \cos^2\theta\cdot\cos^2\phi + \cos\theta\sin\theta\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cdot\cos^2\phi + \sin^2\theta\cdot\cos\phi\sin\phi & \cos\theta\sin\theta\cdot\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$\cos\theta\cos\phi$

$$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi(\cos\theta\cos\phi + \sin\theta\cdot\sin\phi) & \cos\theta\sin\phi(\cos\theta\cdot\cos\phi + \sin\theta\cdot\sin\phi) \\ \cos\phi\cdot\sin\theta(\cos\phi\cos\theta + \sin\theta\sin\phi) & \sin\theta\sin\phi(\cos\theta\cos\phi + \sin\theta\sin\phi) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi(\cos(\theta-\phi)) & \sin\phi\cos\theta\cos(\theta-\phi) \\ \cos\phi\cdot\sin\theta(\cos(\theta-\phi)) & \cos\theta\sin\phi(\cos(\theta-\phi)) \end{bmatrix}$$

$A\cdot B = 0$

$\theta - \phi = n\lambda/2$

$\cos(\theta-\phi) = \cos(\pm n\lambda/2) = 0$ (bcoz n is odd)

$$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi(\cos n\lambda/2) & \sin\phi\cos\theta(\cos n\lambda/2) \\ \cos\phi\cdot\sin\theta(\cos(n\lambda/2)) & \cos\theta\cdot\sin\phi(\cos n\lambda/2) \end{bmatrix}$$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ null matrix.

$x^2 - y^2$, x^3, y^2, 3

given, $f(x) = x^2+1$

verify $fog = f(g(x))$

$\Rightarrow f(x^2)$ $f(\sin x)$

edge

$\Rightarrow (\sin x)^2 + 1$

$\Rightarrow \sin^2 x + 1$

$gof = g(f(x)) \Rightarrow g(x^2+1)$

$\Rightarrow \sin(x^2+1)$

rank of matrix $A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 4 & 10 & 18 \end{vmatrix}$

$|A| = 1(5\cdot18 - 10\cdot8) - 2(2\cdot18 - 8\cdot4) + 3(2\cdot10 - 5\cdot4)$

$\Rightarrow (90-80) - 2(36-32) + 3(20-20)$

$= 10 - 8 + 0$

$= 2$

13

$\therefore |A| \neq 0$

Rank of $A = 3$

if 0, $\begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix} = 16 - 15$

$= 1$

$$A = \begin{array}{c|cccc|} R_2 & 2 & -1 & 1 & 4 \\ R_1 & 1 & 0 & 1 & 5 \\ R_3 & 4 & -1 & 3 & 14 \\ R_4 & 3 & -1 & 2 & 9 \end{array}$$

$R_4 \to R_4 - 3R_1$

$R_3 \to R_3 - 4R_1$

$R_2 \to R_2 - 2R_1$

$$|A| \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & -1 & -1 & -6 \\ 0 & -1 & -1 & -6 \\ 0 & -1 & -1 & -6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - R_2$

14

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & -1 & -1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

minor: $\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} = -1$

$2 \times 2 \neq 0$

$\rho(A) = 2$

Q

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad R_2 \to R_2 + 2R_1$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad R_3 \to R_3 - R_1$$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \to 1/3 R_2$

$R_3 \to 1/2 R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank(A) = 2

	2	-2	1	2
-1	3	0	-1	3
0	-2	1	0	-2
1	2	-2		

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}_{4\times4}$$

Inverse of a matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$|A| = 1(3-0) - 2(-1-0) - 2(2-0)$

$\Rightarrow 1(3) - 2(-1) - 2(2)$

$\Rightarrow 3 + 2 - 4$

$\Rightarrow 1 \neq 0 \Rightarrow A^{-1}$ exists

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$A = \begin{bmatrix} 2 & 7 \\ -1 & 4 \end{bmatrix}_{2\times2} \qquad \begin{array}{l} 8-(-7) \\ 8+7 \\ \Rightarrow 15 \end{array}$$

$$A = \begin{bmatrix} -1 & 6 & -2 \\ 2 & 1 & 1 \\ 4 & 1 & -3 \end{bmatrix} \Rightarrow +(-1)(1\times-3-1\times1) - 6(2\times-3-4\times1) + (-2)(2\times1-4\times1)$$

$\Rightarrow -1(-3-1) - 6(-6-4) - 2(2-4)$

$\Rightarrow -1(-4) - 6(-10) - 2(-2)$

$\Rightarrow +4 + 60 + 4$

$\Rightarrow +68$

1	2	-2	1	2	
-1	3	0	-1	3	
0	-2	1	0	-2	
1	2	-2	1	2	
-1	3	0	-1	3	

$$\begin{bmatrix} 3 & 2 & 6 \\ -1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} - \operatorname{adj} A$$

Elementary transformation

$A = IA$

$I \quad B\ A$

$B = A^{-1}$

$$A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} \Rightarrow |A| \Rightarrow -4-(-6)$$

$\Rightarrow -4+6 = 2$

$|A| \neq 0$

$A = IA$

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \qquad 1-0 \Rightarrow 0-1$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0-1 \\ 0 & 1 \end{bmatrix} A \qquad 1-(-1)$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

$R_2 \to 1/2 R_2$

$$\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1/2 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + 4R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}_{4\times 4}$$

$$|A| = 2\begin{vmatrix} 2 & 0 & 2 \\ -1 & 0 & 3 \\ -2 & 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 & 2 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 4 & 2 & 2 \\ 1 & -1 & 3 \\ 1 & -2 & 1 \end{vmatrix} - 6\begin{vmatrix} 4 & 2 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{vmatrix}$$

$$|A| = 2[2(0-3) - 0(-2+6) + 2(-1-2)]$$
$$+ 2[4(0-3) - 0(2-3) + 2(1-0)] + 0[4(-2-3) - 2(2-3) + 2(-1+\dots)]$$
$$- 6[4(-1+0) - 2(1+1) + 0(-2+1)]$$

$$|A| = 2[2(-3) - 0(+4) + 2(+1)] + 2[4(-3) - 0(-1) + 2(1)] + 0[4(-5) - 2(-1) \dots$$
$$+ 2(-1)] - 6[4(-1) - 2(2) + 0(-1)]$$

$$\Rightarrow 2[-6-0+2] + 2[-12-0+2] + 0 - 6[-4-4+0]$$
$$\Rightarrow 2[-4 -20+0+48]$$
$$\Rightarrow 2[-24+48]$$
$$\Rightarrow 2[24] = 48$$

$$2[2(-3) + 2(-1+2)]$$
$$2[-6+2]$$
$$2[-4] = -8$$
$$2[-12+2] \Rightarrow -20$$
$$-28 - 48$$
$$76$$

$$6(-4-4)$$
$$-48$$

$$2\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & 3 \\ -2 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 4 & 0 & 2 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix} + 0\begin{bmatrix} 0 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$2[2(0-3) - 0(0) + 2(-1-0)] \Rightarrow 2[2(-3) - 2]$$
$$\Rightarrow 2[-6-2] = 2\times -8 = -16$$
$$-16 \neq 0$$

$$2[4(0-3) - 0(0) + 2(1-0)]$$
$$\Rightarrow 2[4(-3) + 2] \Rightarrow 20$$

$$6[4(-1+2) - 2(1+1) + 0(0)]$$
$$\Rightarrow 6[4(1) - 2(2)]$$
$$6(4-4)$$

$$6[4(-1+0) - 2(1+1) + 0(1-0)]$$
$$\Rightarrow 6[4(-1) - 2(1)]$$
$$\Rightarrow 6(-4-8) \quad -6$$
$$\Rightarrow 48 \quad 36$$

$$-16 + 20 - 48$$
$$-36 \quad 36$$
$$-16 + 20 + 36$$
$$= 0$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$|A| \Rightarrow 1(0-(-25)) - 3(0-(-10)) + 2(-15-0)$

$\Rightarrow 1(25) - 3(10) + 2(-15)$

$\Rightarrow 25 - 30 + 30$

$\Rightarrow 25 \neq 0 \quad A^{-1}$ exists

$R_2 \to R_2 + 3R_1$

$R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 8R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix}$$

$R_3 \to 1/25\, R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -15/25 & 1/25 & 9/25 \end{bmatrix} A$$

$R_2 \to R_2 - 21R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/5 & 4/25 & 11/25 \\ -3/5 & 1/25 & 9/25 \end{bmatrix}$$

$R_1 \to R_1 + 2R_3$

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/5 & 2/25 & 18/25 \\ -2/5 & 4/25 & 11/25 \\ -3/5 & 1/25 & 9/25 \end{bmatrix}$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2/5 & -3/5 \\ -2/5 & 4/25 & 11/25 \\ 3/5 & 1/25 & 9/25 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$R_2 \to 1/4\, R_2$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ [illegible] & [illegible] \end{bmatrix} A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} [illegible] & [illegible] \\ -2/5 & -1/5 \end{bmatrix} A$$

Q

$$\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_1 \to 1/10\, R_1$

$$\begin{bmatrix} 1 & -1/5 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 5R_1$

$$\begin{bmatrix} 1 & -1/5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ -1/2 & -1 \end{bmatrix}$$

Rank of the Word

3 4 6 2 1 5
M O T H E R
2 2 3 1 0 0

$\boxed{E\ H\ M\ O\ R\ T}$

5! 4! 3! 2! 1! 0!
240 48 18 2 0 0 + 1
240 + 48 + 18 + 2
⇒ 308 + 1
⇒ 309

4 5 2 3 1
P R I M E
3 3 1 1 0
4! 3! 2! 1! 0!
72 18 2 1 0 + 1
72 + 18 + 2 + 1
94th

4 2 1 3 2 5
S E C R E T
4 1 0 1 0 0
5! 2! 2! 1! 1! 1! 0!
5! 4! 3! 2! 1! 0!
240 12 0 2 0 0 + 1
255

2 4 4 2 3 1
G O O G L E
1 3 3 1 1 0
2! 2! 2! 2! 1! 1!
5! 4! 3! 2! 1! 0
30 36 18 2 1 0 + 1

$^4P_2 \times {}^5P_3$

$\frac{4!}{2!} \times \frac{5!}{2!}$

$\frac{n!}{(n-r)!}$

$^4P_3$

atmost

at least

$^4P_3$

3! 24

4 (3) (2) (5)
P A R I S
2 0 1 0 0
4! 3! 2! 1! 0
48 + 0 + 2 + 0 + 0 + 1
51th rank

P A R I S
→ A I P R S
A _ _ _ _ ⇒ 4! ⇒ 4×3×2×1 = 24
I _ _ _ _ ⇒ 4! ⇒ 4×3×2×1 = 24
P A I R S ⇒ 49th
$\boxed{P\ A\ I\ S\ R}$ 50th

A G A I N
→ A A G I N
A _ _ _ _ 4!/2! ⇒ 24
G _ _ _ _ 4!/2! ⇒ 12
I _ _ _ _ 4!/2! ⇒ 12
N A A G I 49th 4!/2! ⇒ 12
N A A I G 50th

2022			2021
1		(11) (c)	1. (a)
2 (a)		(12) (d)	
3 (c)		(13) (c)	
4 (a)		(14) (a)	
5		(15)	
6 (d)			
7 (d)			
8			
9			
10 (b)			

$f(x_1) = f(x_2)$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$f(x) = \frac{x-2}{x-3} = y$$

$xy - 3y = x-2$

$x(y-3y) = x-2$

$$\boxed{x = \frac{x-2}{1-3y}}$$

$x-2 = xy-3y$

$x - xy = 2 - 3y$

$x(1-y) = 2-3y$

$$\boxed{x = \frac{2-3y}{1-y}}$$

⇒ conditional

p	q	p → q
T	F	F
T	F	F
F	T	T
F	T	T
F	F	T

biconditional
$\Leftrightarrow$

p	q	p ⟺ q
T	F	F
T	F	F
T	T	T
F	F	T
F	T	F

272 and 1479
$\boxed{272x + 1479y}$

3. adjacency matrix

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 3 & 0 & 0 \\ b & 3 & 0 & 1 & 1 \\ c & 0 & 1 & 2 & 2 \\ d & 0 & 1 & 2 & 0 \end{array}$$

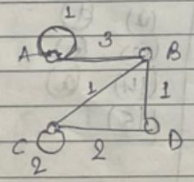


fig.

$A_{12} = 3$

$A_{ij} = A_{11} = 1$
so, there will be self loop

$2^{24} \Rightarrow \left((2)^4\right)^6$

$\Rightarrow (16)^6 \Rightarrow (17-1)^6 \quad (-1)^6 \Rightarrow 1$

(b) one-one

$3 - 4x = y$

$-4x = y - 3$

$$\boxed{x = \frac{3-y}{4}} = f^{-1}(y)$$

one-one & onto

$$f^{-1}(y) \Rightarrow f^{-1}(x) = \frac{3-x}{4}$$

$\Rightarrow$ to $\Rightarrow$

binary → voltage pulse
1 → +ve value, 0 → zero value
high voltage, zero voltage

NRZ-L :- 0 → +ve, 1 → -ve

NRZ-I 1-change in voltage or no voltage

INDEPENDENCE

P — — — — — — — — — — I

$$\frac{10!}{2 \times 3! \times 2! \times 4!}$$

$$\Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2 \times 3 \times 2 \times 2 \times 1 \times 1}$$

$\Rightarrow 10 \times 9 \times 2 \times 7 \times 5$

180 5
126 3
63 0

Case 1 :- selecting together
$10-3=7$
$25-3=22$
${}^{22}C_7$

$\frac{5!}{(5-4)!} = \frac{6!}{(6-4-1)!}$

Case 2 :- left out together
10 students
$25-3=22$

Total no. of selection $= {}^{22}C_7 \times {}^{22}C_{10}$

Q. 4g 7b = 5 C
(a) ${}^{4}C_1$ ${}^{7}C_5$
(b) 1g + 4 b
2g + 3 b
3g + 2b
4g + 1 b

(c) 3g + 2 b
4g + 1 b

Q. B G B G B G B G B G B $\Rightarrow {}^6P_3 \times 5!$

A G A I N
A A G I N
A _ _ _ _ $\quad {}^4P_4 \Rightarrow 4! \Rightarrow 4\times3\times2\times1 \Rightarrow 24$
G A _ _ _ $\quad {}^4P_4 \Rightarrow 4!/2! \Rightarrow 12$
I A _ _ _ $\quad 4!/2! \Rightarrow 12$
N A A G I $\Rightarrow 49^{th}$ world
N A A I G $= 50^{th}$ word

42

$36 + 12 = 48$, $+1$

${}^6P_1 \times {}^6P_4$

6 6 5 4 3

(a) $7\times6\times5\times4 \Rightarrow$
${}^6P_1 \times {}^6P_3$
$\Rightarrow \frac{6!}{5!} \times \frac{6!}{3!} \Rightarrow 6\times6\times5\times4$

6 6 5 4

$\frac{n!}{(n-4)!}$ $\quad \frac{}{(n-2)!}$

$(n-2)! \Rightarrow 42\times(n-4)!$
$(n-2)(n-3)(n-4)! \Rightarrow 42\times(n-4)!$
$(n-2)(n-3)=42$
$n^2-3n-2n+6=42$
$n^2-5n-36=0$
$n^2-9n+4n-36=0$
$n(n-9)+4(n-9)=0$
$(n-9)(n+4)=0$
so, $n=9$, $n \neq -4$
$n > 4$

Q. GCD (256, 1166) $\Rightarrow 256x + 1166y$
$1166 = 256\times4+142$ — (IV)
$256 = 142\times1+114$ — (III)
$142 = 114\times1+28$ — (II)
$114 = 28\times4+2$ — (I)
$28 = 2\times14+0$

$114 = 28\times4+2$
$2 = 114-28\times4$
$2 = 114-(142-114\times1)\times4$
$= 114 - 142\times4 + 114\times4$
$\Rightarrow 114(1+4) - 142\times4$
$\Rightarrow 114(5) - 142\times4$
$\Rightarrow (256-142\times1)5 - 142\times4$
$\Rightarrow 256\times5 - 142\times5 - 142\times4$
$\Rightarrow 256\times5 - 142(5+4)$
$\Rightarrow 256\times5 - 142(9)$

$256)\,1166\,(4$
1024
$142)\,256\,(1$
142
$114)\,142\,(1$
114
$28)\,114\,(4$
112
$2)\,28\,(14$

chromatic no: 2

$2 \Rightarrow 256 \times 5 - (1166 - 256 \times 4)(9)$

$2 \Rightarrow 256 \times 5 - 1166 \times 9 + 256 \times 36$

$\Rightarrow 256(5+36) - 1166 \times 9$

$\Rightarrow 256(41) + 1166 \times (-9)$

$\boxed{2 = 256\, x_0 + 1166\, y_0}$

G.C.D

2022	2021
1	(b)
2	(a)
3	(c)
4	(a)
5	
6	
7	
8	
9	
10	
11	

12, 13, 14, 15

$P(n,4)$

$\Rightarrow \frac{n!}{(n-4)!} = 42 \times \frac{n!}{(n-2)!}$

$(n-2)! = 42 \times (n-4)!$

$(n-2)(n-3)(n-4)! = 42 \times (n-4)!$

$n^2 - 5n + 6 = 42$

$n^2 - 5n - 36 = 0$

$n^2 - 9n + 4n - 36 = 0$

$y = \frac{3}{4}$

$\boxed{\frac{3-y}{4} = x}$

Bijective

$^4P_3$

$\frac{4 \times 3}{2}$

$^4P_2 \times {^5P_3}$

$^7P_3 \times {^5P_2}$

$7 \times 6 \times 5$

210

$\Rightarrow \frac{7!}{4!} \times \frac{5!}{1}$

$5 \times 4 \times 3 \times 2$

210×60

$2(x^2 - 4)$

3	4	2	3	4
0	2	-3	0	2
1	-2	6	1	-2
3	4	2	3	4
0	2	-3	0	2

6	-28	-16
-9	16	9
-2	10	6

$A^{-1} = \frac{adj A}{|A|}$

$X = A^{-1}B$

$-4 - 24$

$-12 - 4$

1888 − 112 = 1776 ; 188

2000 → 1888 ; 2000 − 1888 = 112

Math pmp

Prove & state Hand shaking lemma //15 marks

fundamental theorem of Arith/Ar

fermits little theorem /15 marks

Graph theory : adjacency & incidence matrix

Elementary matrix, ~~Gotti~~ reduction

rank of matrix

one to, onto

Injective, surjective, bijective

Inverse / Adjacent matrix

Principle of duality

Recurrence

~~Pure vertices~~ n with (n-1)

Prove that n trees have (n-1) edges. /15 marks

Bipartite graph

~~Ter~~ seven bridge problem

Tautology

Truth table

$A^{-1} = \frac{adj(A)}{|A|}$

one-one onto

Overloading	Overriding
different parameter	→ same
compile time polymorphism	→ runtime
Occours betw subclass & superclass	→ occours betw the method of same class
An error, it will be visible on compiletime	→
method call is determined at compile time	→ runtime

compiler — java compiler — jscript compiler

language specific compiler

Metadata | C.IL

JIT compiler

Native code

Execution

Run

$x + 4y + 2x = 8$

$2y + 6z = -2$

$x^2 + 4y + 12z = -4$

$3x + 4y + 17z = 8$

$2y - 3z = 3$

$5x + 17 + \frac{(2y-3)}{3}$

$5x + \frac{14y - 21}{3}$

$15x + 14y = 24 + 21 = 45$

$\int \sec^3 x \, dx$ $\qquad 1 + \tan^2 x = t$

$2\tan x \sec^2 x$

$\int \sec x (\sec^2 x \, dx)$ $\qquad \sec$

$\int \sec x (1 + \tan^2 x) \, dx$

$\int \sec x\,dx + \sec x \tan^2 x \, dx$

$\sec x$.

$\tan x = t$

$\sec^2 x = \frac{dt}{dx}$

$\frac{1}{\tan x}$ $\qquad \sec^2 x \, dx = dt$

$\int \frac{4}{3\,x + \cos 3x} \, dx$

$\sec x = t$

$\sec x \tan x = \frac{dt}{dx}$

$\sec x \int \sec^2 x - \int \frac{d\sec x}{dx} \int \sec^2 x \, dx$

$\sec x \tan x - \int (\sec x \tan x \tan x) \, dx$

$\sec x \tan - \int \sec x \tan^2 x \, dx$

$\sec x \tan x - \int \sec x (1 + \sec^2 x) \, dx$

$\sec \tan - \int \sec x\,dx + \int \sec^3 x \, dx$

$I = \sec \tan x - \int \sec x\,dx - I$

$2I = \sec x \tan - \log \tan(\frac{\pi}{4} + x)$

$u \int v \, dx - \int \left(\frac{du}{dx} \cdot \int v \, dx \right)$

Autoboxing $\qquad$ Unboxing

```
int a = 20;
integer i = integer.forvalue();
integer j = a;
```

```
integer a = new integer(3);
integer i = a.int value();
integer j = a;
s.o.p (a" + j + "");
```

overloading	overriding
(i) it has differ parameter	same different parameter
(ii) compile time polymorphism	run time polymorphism
(iii) method call determined at runtime compile time	method call determine at runtime
(iv) occurs bet^n superclass & subclass	methods in the same class
(v) error will be caught at compile time	caught at runtime

$\tan^{-1}x = z$ $\qquad$ $\tan^{-1}x = z$

$x = \tan z$

$\frac{1}{(1+x^2)} = \frac{dz}{dx}$

$\frac{1}{1+x^2}\,dx = dz$

$\frac{e^{mz}}{(1+x^2)(1+x^2)}$

$\frac{e^{mz}}{(1+x^2)}\,dz$

$\int \frac{e^{mz}}{(1+\tan^2 z)}\,dz$

$\int \frac{1}{1+\tan^2 z} \times e^{mz}\,dz$

$\frac{1}{1+\tan^2 z}$

$\frac{1}{1+\tan^2 z}\,\frac{e^{mz}}{m} - \int 1 \cdot \frac{e^{mz}}{m}$

$\frac{1}{1+\tan^2 z}\,\frac{e^{mz}}{m} - \frac{1}{m}\int = - I,$

$2z \quad 2$

{2 seat → 24001/- geyser}

(2BK) → Nitu galaria mall. | 8340285289

2 room 5000/- | 9412706662

Fayshswani

I LATE
2/11

Leela Krishna Apart

(Li)

Boolean algebra
Boolean function

42000
12000
12

- Rare species

- Abendence level
- Describe 5 raw species which have medical value
- Document characteristic features
- Role of plant in ecosystem
- how it is use by whom
- is the plant being collected sustainably or over harvested

- what is the human economy with current problem

JAVA → 1 hour
PHP → 1 hour
VB.net → 1 hour

HTML → 1 hour
CSS

→ 02 hour

MHTCET

$1+x^2 = \tan x$

$2x \frac{dy}{dx} = \sec^2 x$

$\frac{dy}{dx} = \frac{\sec^2 x}{2x}$

want to burst into tears can't hold more
→ no you can't do it, control, keep calm, just calm

If the human brain were a computer, it could perform 38 thousand-trillion operations per second. The world's most powerful supercomputer, BlueGene, can manage only

The total weight of all the ants on Earth is greater than total weight of all the humans on the planet.

Amazing Facts

Starfish can re-grow their arms. In fact, a single arm can regenerate a whole body.

The average hummingbird's heart rate is more than 1,200 beats per minute.