Numerical analysis Experiment Report

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1 Introduction

1.1 Task 1

Construct Lagrange interpolation polynomial pL(x) for the function

$$f(x) = \frac{1}{1+25x^2}$$

and the interpolation node is:

$$x_i = 1 - \frac{2i}{N}$$

$$x_i = -cos\left(\frac{i+1}{N+2}\pi\right)$$

Then calculate the integral of Lagange interpolation polynomial and function $f(x)$. After that,calcualte the error of them.

1.2 Task 2

Write general programs for calculating integrals using the complex Simpson integral formula and the complex trapezoidal integral formula respectively.Calculate the integral using the above procedure

$$\int_0^4 sin(x)\,dx$$

Take nodes with

$$x_i = \frac{i}{N}$$

2 Method

Lagrange interpolation.

3 Results

3.1 Task 1

In this image, integral_1 and integral_2 represent

$$\int_{-1}^{1} p_L(x)\,dx$$

with different interpolation nodes.

```
>> HW6_1
Exact integral result:
    0.5494


N = 5
integral_1 = 0.46153846
integral_2 = 0.48114044
error_1 = 0.08782185
error_2 = 0.06821986
N = 10
integral_1 = 0.93466011
integral_2 = 0.55408569
error_1 = 0.38529980
error_2 = 0.00472538
N = 15
integral_1 = 0.83111180
integral_2 = 0.54758613
error_1 = 0.28175149
error_2 = 0.00177418
N = 20
integral_1 = -5.36991042
integral_2 = 0.55001079
error_1 = 5.91927072
error_2 = 0.00065048
N = 25
integral_1 = -5.39986108
integral_2 = 0.54935991
error_1 = 5.94922139
error_2 = 0.00000039
N = 30
integral_1 = 153.79793084
integral_2 = 0.54932164
error_1 = 153.24857053
error_2 = 0.00003867
N = 35
integral_1 = 173.88036818
integral_2 = 0.54935707
error_1 = 173.33100788
error_2 = 0.00000323
N = 40
integral_1 = -4912.41691205
integral_2 = 0.54936481
error_1 = 4912.96627236
error_2 = 0.00000451
```

3.2 Task 2

In following image, the result represent N from left to right with

$$N = 2^i$$

```
>> HW6_2
The error of the integral of sinx from 0 to 4 using Simpson method is:
    0.0104    0.0006    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000


The error of the integral of sinx from 0 to 4 using trapezoid method is:
    0.5919    0.1402    0.0346    0.0086    0.0022    0.0005    0.0001    0.0000    0.0000    0.0000    0.0000    0.0000


The convergence rate of the integral of sinx from 0 to 4 using Simpson method is:
    4.1367    4.0327    4.0081    4.0020    4.0005    4.0001    4.0000    3.9995    4.0176    4.3099    0.1520


The convergence rate of the integral of sinx from 0 to 4 using trapezoid method is:
    2.0782    2.0184    2.0045    2.0011    2.0003    2.0001    2.0000    2.0000    2.0000    2.0000    2.0000
```

4 Discussion

A Computer Code

See attached files