

# 数值代数实验报告

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## 一、问题描述

将算法2.5.1，即估计矩阵的1范数，优化法编写成通用的子程序。估计5到20阶Hilbert矩阵的 $\infty$ 范数条件数并求解若干方程组A的逆的无穷范数，计算解x的精度，并与真实相对误差作比较。

## 二、程序介绍

涉及的算法有，列主元guass消去法，前代法，回代法，对角元为1的前代法，对角元为1的前代法，回代法，矩阵无穷范数，封装函数列主元消去解线性方程组。矩阵运算，向量运算均引用Eigen库实现。平台是VScode，语言为C++。

主要代码写在了homework.cpp里并由主函数输出相应结果，引用的函数均在function.h和文件eigen-3.4.0里。函数名字均相当程度上反映了函数作用。

## 三、实验结果

展示实验产生的结果

作业1结果：

```
PS D:\Study(the third year fall\Numerical algebra\my homework2> g++ -std=c++11 -I c:\Users\26957\vscode\extensions\ms-vscode.cpptools-1.17.5-win32-x64\debugadapters\tdin=Microsoft-MIEngine-In-xrtze5ow.5nn' '--stdout=Microsoft-MIEngine-Out-ok3qv3xq.dde' '--stderr=Microsoft-MIEngine-Error-q0ghwn2r.fk2' '--pid=Microsoft-Exe=C:\Program Files\mingw\mingw64\bin\gdb.exe' '--interpreter=mi'
size of Matrix=5 condition number by infinity norm of Hilbert matrix A = 943656
size of Matrix=6 condition number by infinity norm of Hilbert matrix A = 2.98703e+07
size of Matrix=7 condition number by infinity norm of Hilbert matrix A = 9.85195e+08
size of Matrix=8 condition number by infinity norm of Hilbert matrix A = 3.38728e+10
size of Matrix=9 condition number by infinity norm of Hilbert matrix A = 1.09965e+12
size of Matrix=10 condition number by infinity norm of Hilbert matrix A = 3.53538e+13
size of Matrix=11 condition number by infinity norm of Hilbert matrix A = 1.23862e+15
size of Matrix=12 condition number by infinity norm of Hilbert matrix A = 3.83167e+16
size of Matrix=13 condition number by infinity norm of Hilbert matrix A = 4.6293e+17
size of Matrix=14 condition number by infinity norm of Hilbert matrix A = 1.37122e+19
size of Matrix=15 condition number by infinity norm of Hilbert matrix A = 1.12484e+18
size of Matrix=16 condition number by infinity norm of Hilbert matrix A = 1.34428e+18
size of Matrix=17 condition number by infinity norm of Hilbert matrix A = 1.97137e+18
size of Matrix=18 condition number by infinity norm of Hilbert matrix A = 9.12824e+19
size of Matrix=19 condition number by infinity norm of Hilbert matrix A = 3.3957e+19
size of Matrix=20 condition number by infinity norm of Hilbert matrix A = 3.1628e+18
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size of Matrix=5 Relative error of estimate = 4.99973e-16
```

```
size of Matrix-5      Relative error of estimate = 4.88823e-16
real relative error = 2.22682e-16
size of Matrix-6      Relative error of estimate = 5.93986e-16
real relative error = 2.80386e-16
size of Matrix-7      Relative error of estimate = 3.85671e-16
real relative error = 1.14452e-16
size of Matrix-8      Relative error of estimate = 1.48248e-15
real relative error = 5.60344e-16
size of Matrix-9      Relative error of estimate = 2.38669e-14
real relative error = 6.51656e-15
size of Matrix-10     Relative error of estimate = 7.36239e-15
real relative error = 3.1658e-15
size of Matrix-11     Relative error of estimate = 2.83227e-14
real relative error = 6.44323e-15
size of Matrix-12     Relative error of estimate = 1.20342e-13
real relative error = 4.19263e-14
size of Matrix-13     Relative error of estimate = 5.55139e-14
real relative error = 1.60667e-14
size of Matrix-14     Relative error of estimate = 1.83046e-12
real relative error = 2.69971e-13
size of Matrix-15     Relative error of estimate = 7.64401e-13
real relative error = 1.1968e-13
size of Matrix-16     Relative error of estimate = 9.13492e-12
real relative error = 1.48293e-12
size of Matrix-17     Relative error of estimate = 2.95597e-12
real relative error = 5.71346e-13
size of Matrix-18     Relative error of estimate = 3.72097e-12
real relative error = 5.14228e-13
size of Matrix-19     Relative error of estimate = 4.06228e-11
real relative error = 5.45158e-12
size of Matrix-20     Relative error of estimate = 8.13301e-12
real relative error = 2.2125e-12
size of Matrix-21     Relative error of estimate = 5.55767e-11
real relative error = 7.02516e-12
size of Matrix-22     Relative error of estimate = 9.17764e-11
real relative error = 8.8281e-12
size of Matrix-23     Relative error of estimate = 6.33312e-10
real relative error = 8.35996e-11
size of Matrix-24     Relative error of estimate = 1.72122e-10
real relative error = 3.17178e-11
size of Matrix-25     Relative error of estimate = 2.89612e-11
real relative error = 5.37911e-12
size of Matrix-26     Relative error of estimate = 5.4171e-09
real relative error = 7.05402e-10
size of Matrix-27     Relative error of estimate = 1.97876e-08
real relative error = 2.29819e-09
size of Matrix-28     Relative error of estimate = 2.01213e-08
real relative error = 3.46742e-09
size of Matrix-29     Relative error of estimate = 1.33747e-08
real relative error = 3.53852e-09
size of Matrix-30     Relative error of estimate = 2.22395e-08
real relative error = 2.51357e-09
PS D:\Study\the third year fall\Numerical algebra\my homework2>
```

## 四、结果分析

作业一：Hilbert矩阵作为相当病态的矩阵，其无穷范数条件数相当大。

算法2.5.1在估计相对误差上的表现相当不错。因为我选取的是双精度的变量，由此可见在双精度下，列主元消去的计算精度相当高，误差很小。并且优化法估计矩阵1范数也表现得很好，由其计算出的估计相对误差与真实相对误差非常接近。

