

23.5

for $j=1:n$

~~Insertion Sort (Arr, n)~~

void Insertion Sort (Arr, n):

{ if $n \leq 1$, return;

Insertion Sort (Arr, $n-1$);

int $j = n-2$, int $last = arr[n-1]$

while ($j \geq 0$ && $arr[j] > last$)

{ $arr[j+1] = arr[j]$;

$j--$;

}

$arr[j+1] = last$

}

running time:

$$T(n) = \begin{cases} T(n-1) + O(n) & n \neq 1 \\ 1 & n = 1 \end{cases}$$

2-1

a. Using insertion sort to sort one length k sublist will cost $\Theta(k^2)$ time, then,

there are n/k sublists. so will cost

$$\Theta(k^2 \cdot \frac{n}{k}) = \Theta(nk) \text{ in total}$$

b. Let Merge Sort cost $T(n)$ time.

then

~~$T(n) = \Theta(nk) + \Theta(n)$~~
~~Our Modified Sort~~ $T(n)$

$$T\left(\frac{n}{k}\right) = 2T\left(\frac{n}{2k}\right) + O(n)$$

$$\Rightarrow T(n) = \Theta\left(n \lg\left(\frac{n}{k}\right)\right)$$

e

c. Standard merge sort : $\Theta(n \log n)$

when $k=1$. It has the same running time as standard merge sort

d. let $f(k) = n \log(\frac{n}{k})$

$$f'(k) = n + n \cdot \frac{k}{k^2} \cdot (-\frac{1}{k^2}) = n - \frac{n}{k}$$

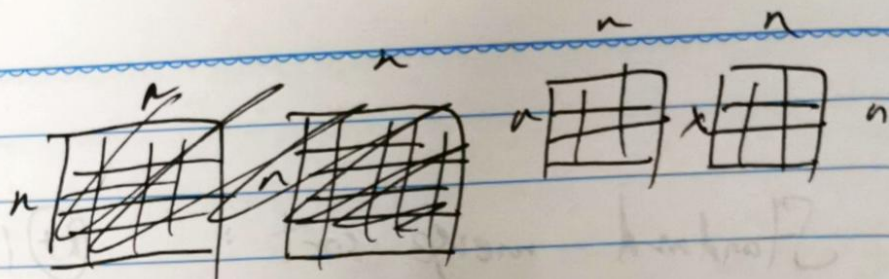
so f $\begin{cases} \searrow & k \in (0, 1) \\ \nearrow & k \in (1, \infty) \end{cases}$

so f has minimum with $k=1$

$f_{\min} = 1$. However, I

will choose k with experiment's result.

4.2-3



$$T(n) = kT\left(\frac{n}{3}\right) + O(n^2) \quad [12]$$

$$\Rightarrow T(n) = k\left(kT\left(\frac{n}{9}\right) + O\left(\frac{n^2}{9}\right)\right) + O(n^2)$$

$$\dots \Rightarrow T(n) = k^m T\left(\frac{n}{3^m}\right) + \left(1 + \frac{k}{3} + \frac{k^2}{3^2} + \dots + \frac{k^{m-1}}{3^{m-1}}\right) O(n^2)$$

Let $m = \log_3 n + 1$

$$\Rightarrow T(n) = k^{\log_3 n + 1} \cdot T(1) + \frac{\left(\frac{k}{3}\right)^{\log_3 n + 1} - 1}{\frac{k}{3} - 1} O(n^2)$$

$$= k^{\log_3 n + 2} + \frac{\left(\frac{k}{3}\right)^{\log_3 n + 1} - 1}{\frac{k}{3} - 1} O(n^2)$$

$$= O(n^{\log_3 k})$$

$$\Rightarrow \log_3 k < \log 7$$

$$\Rightarrow k < 3^{\log 7}$$