Introduction to Algorithms

Binary Search Trees

Dynamic Sets

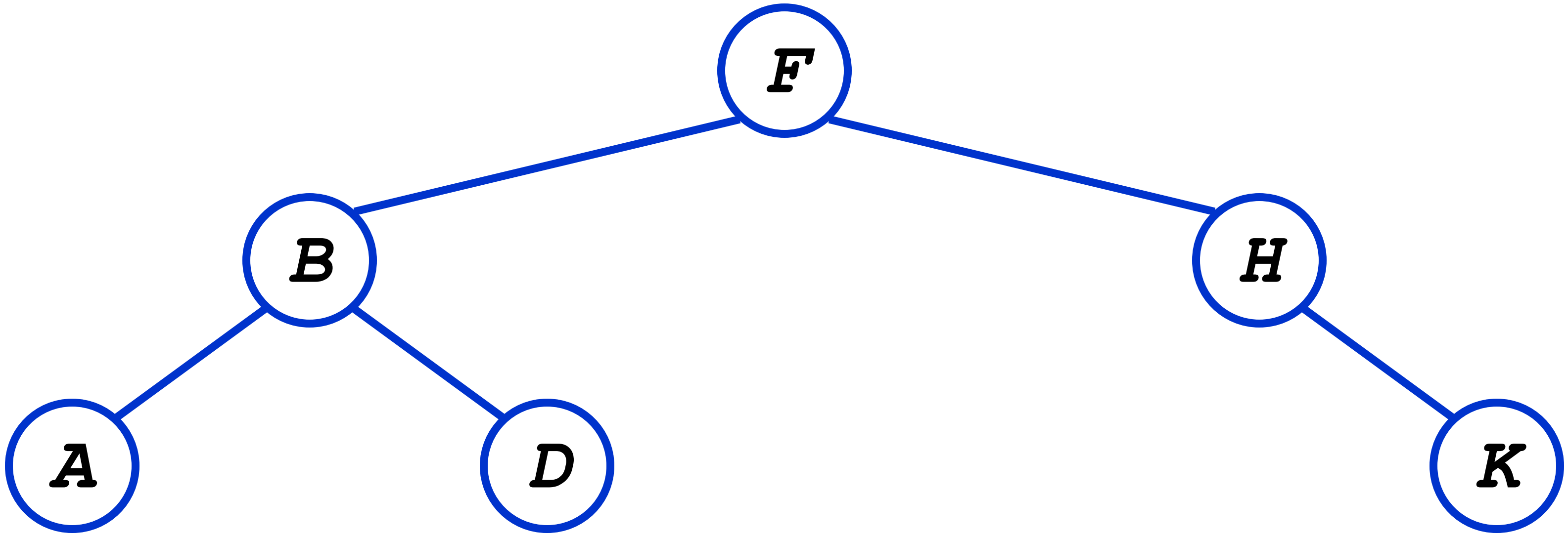
- Next few lectures will focus on data structures rather than straight algorithms
- In particular, structures for *dynamic sets*
  - Elements have a *key* and *satellite data*
  - Dynamic sets support *queries* such as:
    - ***Search(S, k), Minimum(S), Maximum(S), Successor(S, x), Predecessor(S, x)***
  - They may also support *modifying operations* like:
    - ***Insert(S, x), Delete(S, x)***

Binary Search Trees

- *Binary Search Trees* (BSTs) are an important data structure for dynamic sets
- In addition to satellite data, elements have:
  - *key*: an identifying field inducing a total ordering
  - *left*: pointer to a left child (may be NULL)
  - *right*: pointer to a right child (may be NULL)
  - *p*: pointer to a parent node (NULL for root)

Binary Search Trees

- BST property:
  key[leftSubtree(x)] $\leq$ *key[x]* $\leq$ *key[rightSubtree(x)]*
- Example:



Inorder Tree Walk

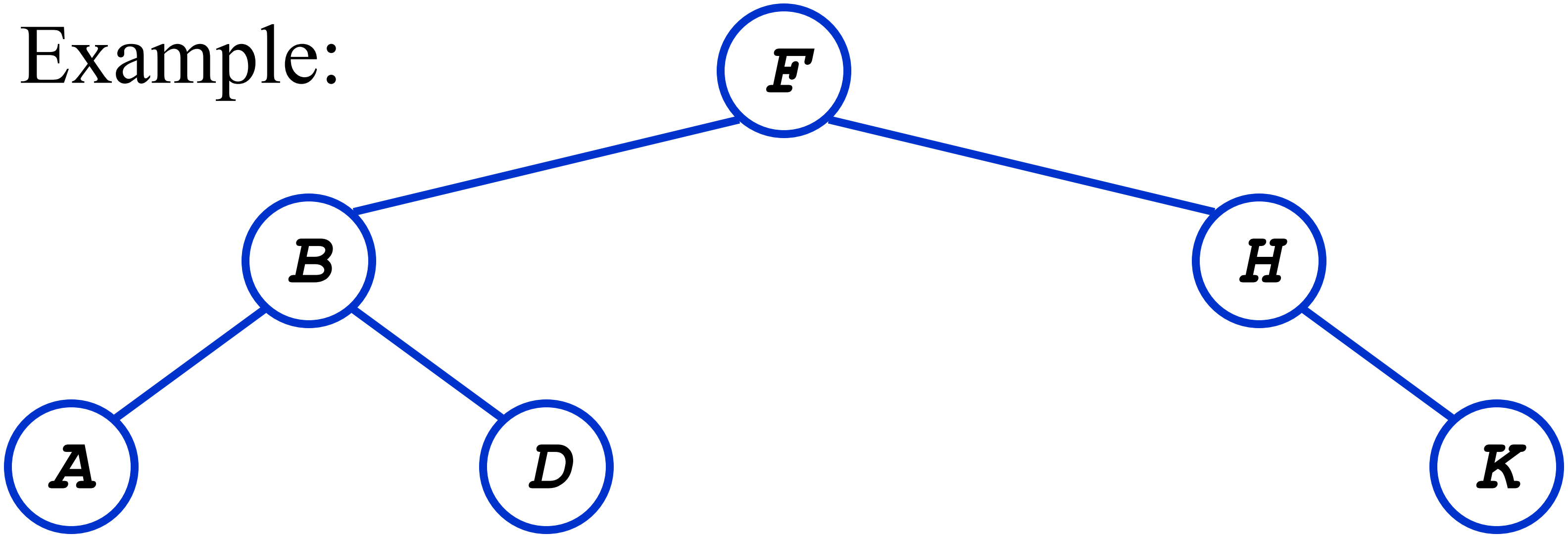
- *What does the following code do?*

```
TreeWalk(x)
    TreeWalk(left[x]);
    print(x);
    TreeWalk(right[x]);
```

- A: prints elements in sorted (increasing) order
- This is called an *inorder tree walk*
  - *Preorder tree walk*: print root, then left, then right
  - *Postorder tree walk*: print left, then right, then root

Inorder Tree Walk

- Example:



- *How long will a tree walk take?*
- *Prove that inorder walk prints in monotonically increasing order*

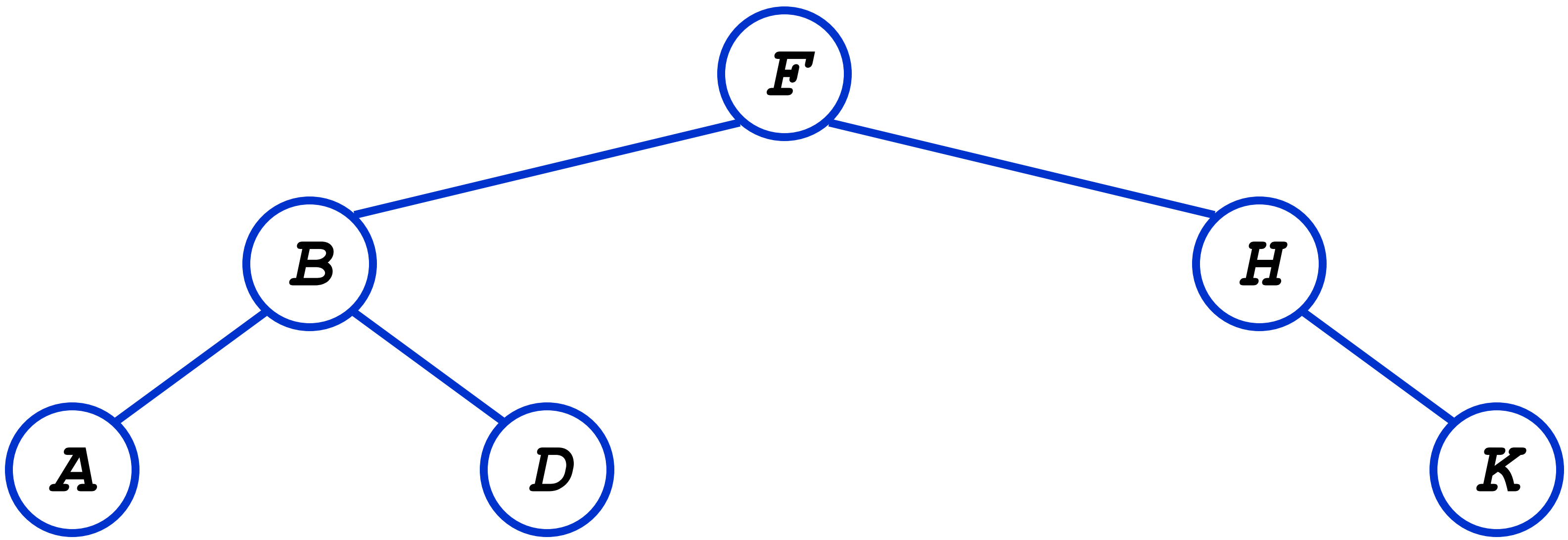
Operations on BSTs: Search

- Given a key and a pointer to a node, returns an element with that key or NULL:

```
TreeSearch(x, k)
     if (x = NULL  or  k = key[x])
         return x;
     if (k < key[x])
         return TreeSearch(left[x], k);
     else
         return TreeSearch(right[x], k);
```

BST Search: Example

- Search for *D* and *C*:



Operations on BSTs: Search

- Here's another function that does the same:

```
TreeSearch(x, k)
    while (x != NULL  and  k != key[x])
        if (k < key[x])
            x = left[x];
        else
            x = right[x];
    return x;
```

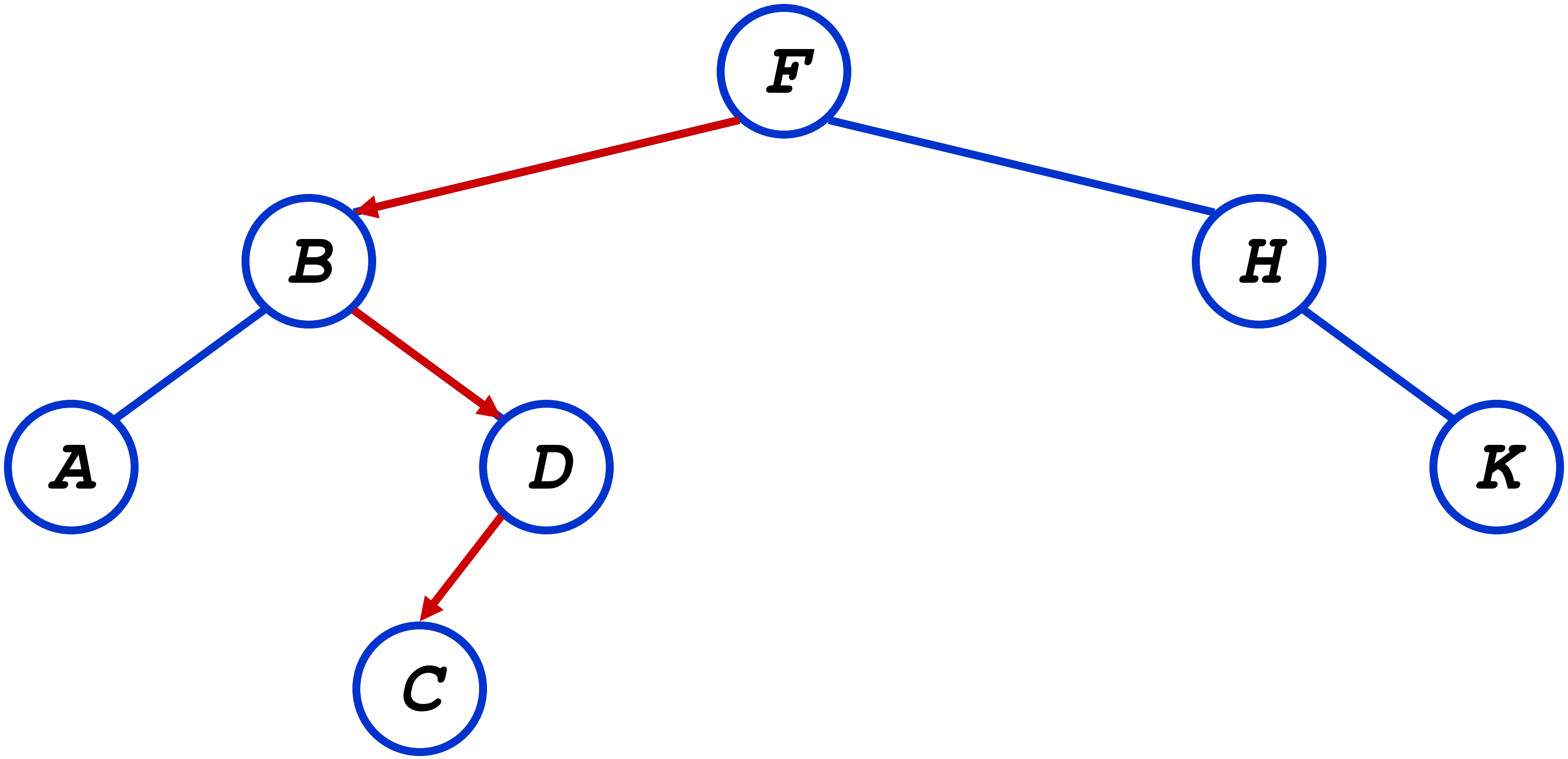
- *Which of these two functions is more efficient?*

Operations of BSTs: Insert

- Adds an element x to the tree so that the binary search tree property continues to hold
- The basic algorithm
  - Like the search procedure above
  - Insert x in place of NULL
  - Use a “trailing pointer” to keep track of where you came from (like inserting into singly linked list)

BST Insert: Example

- Example: Insert C



BST Search/Insert: Running Time

- *What is the running time of TreeSearch() or TreeInsert()?*
- A: O(h), where h = height of tree
- *What is the height of a binary search tree?*
- A: worst case: h = O(n) when tree is just a linear string of left or right children
  - We'll keep all analysis in terms of h for now
  - Later we'll see how to maintain h = O(lg n)

Sorting With Binary Search Trees

- Informal code for sorting array A of length *n*:

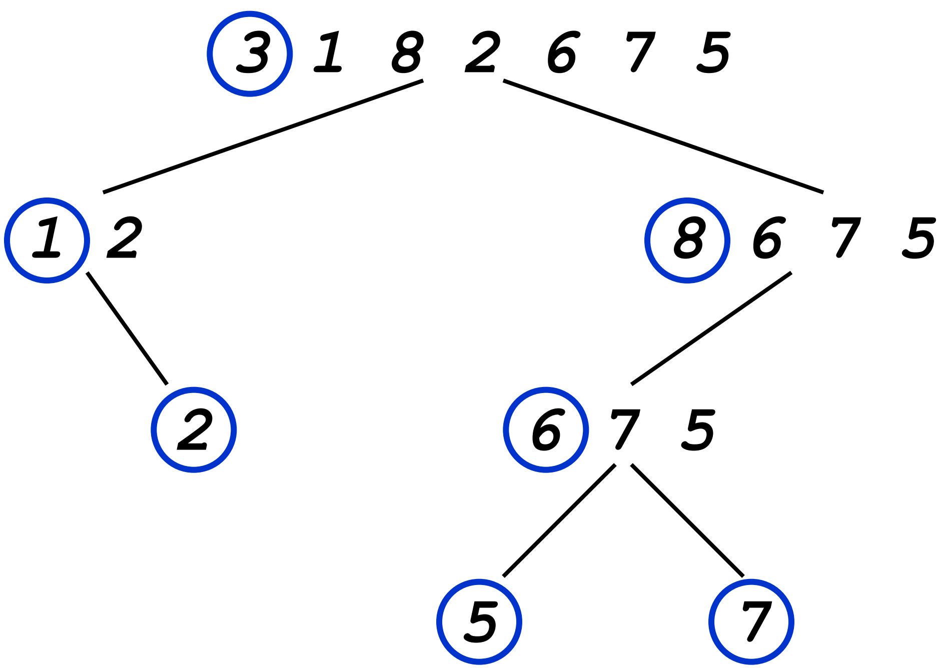
```
BSTSort(A)
    for i=1 to n
        TreeInsert(A[i]);
    InorderTreeWalk(root);
```

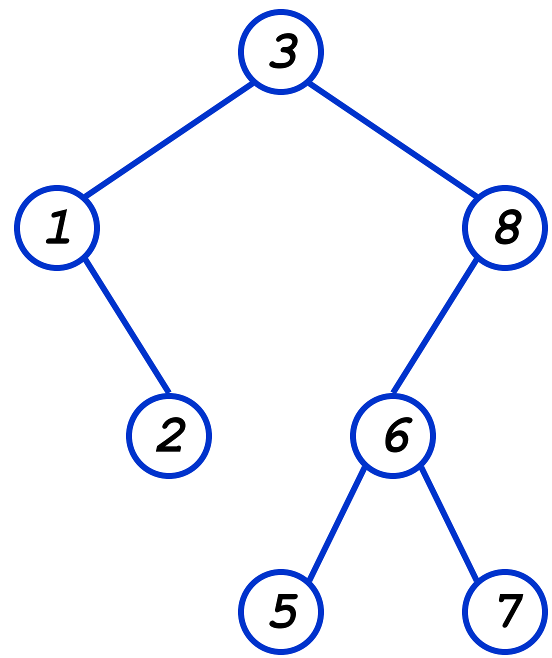
- *Argue that this is $\Omega(n \lg n)$*
- *What will be the running time in the*
  - *Worst case?*
  - *Average case? (hint: remind you of anything?)*

Sorting With BSTs

- Average case analysis
  - It's a form of quicksort!

```
for i=1 to n
    TreeInsert(A[i]);
InorderTreeWalk(root);
```





Sorting With BSTs

- Same partitions are done as with quicksort, but in a different order
  - In previous example
    - Everything was compared to 3 once
    - Then those items < 3 were compared to 1 once
    - Etc.
  - Same comparisons as quicksort, different order!
    - Example: consider inserting 5

Sorting With BSTs

- Since run time is proportional to the number of comparisons, same time as quicksort: $\Omega(n \lg n)$
- *Which do you think is better, quicksort or BSTsort? Why?*

Sorting With BSTs

- Since run time is proportional to the number of comparisons, same time as quicksort: O(n lg n)
- *Which do you think is better, quicksort or BSTSort? Why?*
- A: quicksort
  - Better constants
  - Sorts in place
  - Doesn't need to build data structure

More BST Operations

- BSTs are good for more than sorting. For example, can implement a priority queue
- *What operations must a priority queue have?*
  - Insert
  - Minimum
  - Extract-Min

BST Operations: Minimum

- *How can we implement a Minimum() query?*
- *What is the running time?*

BST Operations: Successor

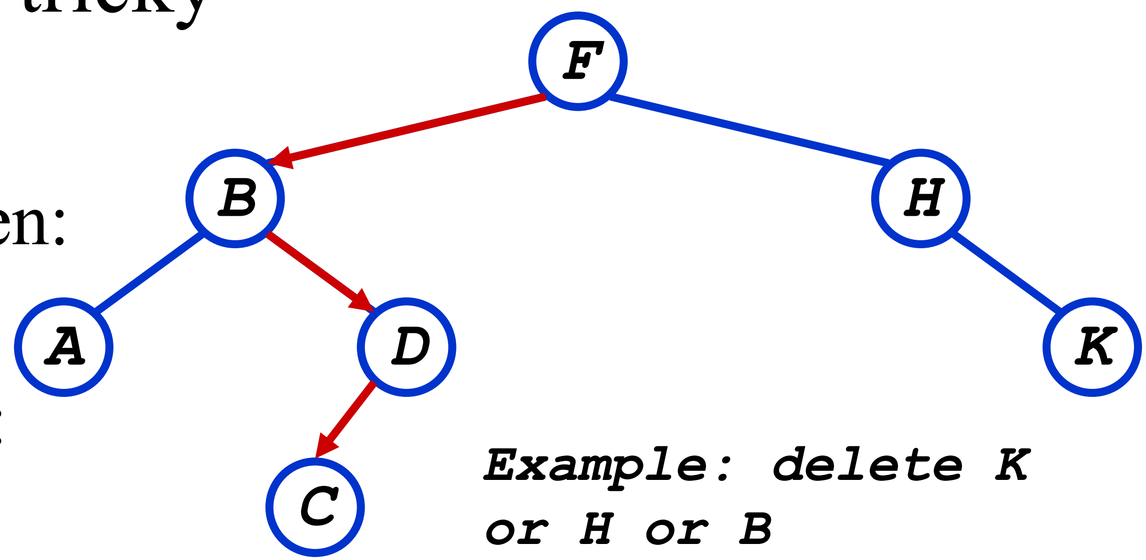
- For deletion, we will need a Successor() operation
- Draw Fig 12.2
- *What is the successor of node 3? Node 15? Node 13?*
- *What are the general rules for finding the successor of node x? (hint: two cases)*

BST Operations: Successor

- Two cases:
  - x has a right subtree: successor is minimum node in right subtree
  - x has no right subtree: successor is first ancestor of x whose left child is also ancestor of x
    - Intuition: As long as you move to the left up the tree, you're visiting smaller nodes.
- Predecessor: similar algorithm

BST Operations: Delete

- Deletion is a bit tricky
- 3 cases:
  - x has no children:
    - Remove x
  - x has one child:
    - Splice out x
  - x has two children:
    - Swap x with successor
    - Perform case 1 or 2 to delete it



Example: delete K or H or B

BST Operations: Delete

- *Why will case 2 always go to case 0 or case 1?*
- A: because when x has 2 children, its successor is the minimum in its right subtree
- *Could we swap x with predecessor instead of successor?*
- A: yes. *Would it be a good idea?*
- A: might be good to alternate

The End

- Up next: guaranteeing a O(lg n) height tree