

数值代数实验报告

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一、问题描述

用C++编写计算矩阵奇异值的通用程序，参考课本7.6.2 节 (P234-240) SVD 迭代完成SVD 算法7.6.3，并对附件svddata.txt 中的矩阵作SVD 分解 $A = P\Sigma Q$ 。并计算 $P^T I, QQT I, P\Sigma Q$ ， A 的绝对值最大的元素，依次用 ep , eq , et 表示。要求输出迭代次数，从小到大排序的所有奇异值以及上面要求的三个值。

二、程序介绍

实现书上的算法有：课本7.6.2 节 (P234-240) SVD 迭代，完成SVD 算法7.6.3

额外的算法：矩阵 A 二对角化的算法，householder变换，生成Jacobi矩阵的算法。

其余函数代码内均有注释。

三、实验结果

第一题实验结果:

```
A =      214.31      0 -1.72841e-15 -1.47981e-14 -2.24376e-15 -2.69359e-15 -5.48173e-15 -8.87745e-16 -5.60454e-14 -5.51213e-16 4.80302e-17 -2.22045e-16
-6.93889e-17      32.2979      0 -1.31015e-15 5.85469e-17 -9.2581e-16 -2.99153e-15 -1.34658e-15 3.69062e-16 -8.19874e-16 -2.88398e-17      0
-6.93889e-17 -1.14839e-15      8.66648 4.52209e-11 -6.245e-17 -4.33681e-17 4.16334e-16 3.46945e-17 -7.49401e-16 9.02056e-17 3.33067e-16      0
-6.93889e-17 -4.996e-15 -1.38778e-17 5.94702      0 2.1684e-18 4.33681e-17 -1.73472e-17 -2.22045e-16 -8.23994e-16 6.93889e-18 1.11022e-16
-2.91434e-16 -1.35308e-15 6.45317e-16 -2.77556e-17 4.98101      0 1.85182e-16 3.32633e-16 -6.10406e-16 3.26995e-16 -3.54751e-16 -2.77556e-17
-2.91434e-16 4.4964e-15 -2.32453e-16 1.94289e-16 -2.0383e-16 3.1445      0 5.20417e-17 1.83881e-16 2.63678e-16 -2.39392e-16 5.55112e-17
-2.91434e-16 -1.49888e-15 -3.81639e-17 8.32667e-17 -3.46945e-18 -2.94903e-17 2.60205      0 -1.11022e-16 -5.55112e-17 3.60822e-16      0
-1.80411e-16 -2.25514e-16 1.32706e-16 -8.32667e-17 -9.54098e-18 2.60209e-17 6.59195e-17 1.89063      0 1.82146e-17 -1.04083e-16      0
-1.80411e-16 2.23432e-15 2.14412e-15 -1.94289e-16 6.50521e-18 -2.5377e-16 -3.43475e-16 -2.01933e-18 0.375993      0 2.77556e-16 8.32667e-17
-1.80411e-16 8.79852e-15 -1.92207e-15 4.16334e-17 3.17021e-16 -6.93889e-18 2.4981e-16 8.45678e-17 1.38778e-17 1.14018      0 2.77556e-17
-1.80411e-16 4.57967e-16 1.38778e-16 -6.93889e-18 4.66207e-17 7.63278e-17 -5.55112e-17 9.3648e-18 5.55112e-17 5.3668e-17 0.703989      0
-1.80411e-16 4.64906e-16 1.97758e-16 2.22045e-16 -1.92988e-16 -1.21431e-17 -6.93889e-17 4.00477e-18 5.55112e-17 -8.22638e-18 -3.46945e-17 0.880006
-1.80411e-16 1.13104e-15 -3.88578e-16 -1.66533e-16 -1.83772e-17 -2.81025e-16 1.56125e-16 -2.09793e-17 2.77556e-17 7.69784e-17 -6.93889e-18 4.16334e-17
-1.80411e-16 -9.02056e-17 7.11237e-17 3.46945e-17 -8.78204e-18 2.67147e-16 -1.11022e-16 -3.95734e-18      0 2.81893e-18 1.73472e-17 -4.68375e-17
-1.80411e-16 4.996e-16 3.1225e-17 -1.38778e-16 2.15756e-17 5.68122e-17 2.91434e-16 1.13164e-17 -5.55112e-17 4.06576e-17 1.56125e-17 1.38778e-17
-1.80411e-16 2.18575e-16 1.40513e-16 2.77556e-17 1.20563e-16 2.90566e-17 2.25514e-17 1.50162e-17 -2.77556e-17 -1.34441e-17 -1.04083e-16 2.94903e-17
-1.80411e-16 -9.29812e-16 3.81639e-17 2.22045e-16 3.11383e-16 8.1532e-17 -4.16334e-17 6.61363e-18 -1.38778e-17 4.66207e-18 8.67362e-18 -4.77049e-18
-1.80411e-16 -6.245e-16 -2.72352e-16 -6.93889e-18 3.75134e-17 4.85723e-17 -1.17961e-16 -1.19262e-17 1.38778e-17 2.9924e-17 1.73472e-18 -6.50521e-18
-1.80411e-16 6.66134e-16 7.1991e-17 2.77556e-17 1.42247e-16 1.71738e-16 3.05311e-16 2.64545e-17 1.38778e-17 9.84565e-17 8.67362e-18 1.84314e-18
-1.80411e-16 -1.30451e-15 6.41848e-17 1.11022e-16 1.04083e-17 -2.08167e-16 4.996e-16 -5.38848e-17 -4.16334e-17 1.63715e-17 2.08167e-17 3.55618e-17
-1.80411e-16 -4.92661e-16 9.93889e-17 -1.38778e-17 -1.73472e-18 4.16334e-16 1.21431e-17 -4.09828e-17 -4.85723e-17 2.47198e-17 1.95156e-18 -9.1073e-18
-1.80411e-16 1.27676e-15 -1.26288e-15 -5.55112e-17 5.26922e-17 -9.97466e-18 -7.97973e-17 7.86047e-17 -2.77556e-17 3.25261e-18 8.67362e-19      0
-1.80411e-16 -1.36002e-15 -5.68989e-16 1.38778e-16 2.34188e-16 -1.40513e-16 -1.73472e-17 2.49366e-17 4.16334e-17 8.13152e-18 5.20417e-18 1.21431e-16
-1.80411e-16 3.46945e-16 2.01228e-16 1.11022e-16 -1.61438e-16 1.21431e-16 1.38778e-16 4.7217e-17 -3.46945e-18 -2.1684e-17 -3.46945e-17 -1.04083e-17
-1.80411e-16 -9.1034e-16 7.9073e-17 -2.77556e-17 2.38524e-17 7.97073e-17 -2.91434e-16 -1.17094e-17 2.08167e-17 -2.27682e-18 -6.07153e-18 1.76942e-16
-1.94289e-16 -4.44809e-16 -1.56125e-17 2.22045e-16 -1.50921e-16 1.38778e-17 1.31839e-16 -1.28071e-18 -2.77556e-17 -2.86229e-17 4.33681e-19 -3.46945e-18
-2.08167e-16 3.1225e-16 -1.68268e-16 1.66533e-16 3.02709e-16 3.33067e-16      0 3.3561e-17 6.93889e-18 1.68051e-18 -4.85723e-17 -1.04083e-17
-2.22045e-16      0 -2.22045e-16 -4.44809e-16      0      0 2.22045e-16      0 1.11022e-16 1.38778e-17 5.55112e-17 5.55112e-17
```

```
P =      0.201521 0.0911402      0 -1.3986e-07 0.0772605 0.322807 0.256887 0.123667 -0.0866028 -0.240969 0.348862 -0.197716 -0.404687 0.017673 0.217549
0.109832 0.319241 0.0283557 0.204913 0.00458787 -0.0809262 0.114699 0.258142 0.0145209 0.0209009 0.00908854 0.0584687 0.183148 -0.146504
0.29061 0.264505 -0.0512033 0.342982 0.113254 0.0561503 -0.0906387 0.231329 0.0919456 -0.171139 -0.124253 -0.267726 -0.129713 -0.104737 0
-1.99909 -0.106910 0.0794768 -0.0317489 -0.268255 -0.121089 0.021411 -0.290670 0.188472 0.0724663 0.176073 -0.343131 -0.104515 -0.193299
0.190181 0.119826 -0.173717 0.0785137 0.223697 0.089275 0.164854 -0.0819218 0.165363 0.00102446 -0.0595711 0.0671138 0.250287 0.329738 -0
0.136077 -0.160351 -0.140856 0.0490834 0.0636028 0.115358 0.231153 -0.344275 0.151327 -0.23119 0.13246 0.222884 -0.213133 0.433599
0.254386 0.207301 -0.0813732 -0.245351 0.0130674 -0.0217869 0.0358542 0.0978748 0.0428182 -0.150386 0.0624607 0.0199971 -0.029447 -0.0393121 -0
-3.19297 0.031931 0.104175 0.056245 0.0558364 0.338733 -0.253273 -0.213282 0.109336 0.00550046 0.125366 0.435922 0.151119 -0.44472
0.201216 0.0880326 -0.0512625 -0.133406 0.0860741 0.0273551 0.0542833 0.0443752 0.0629634 0.0780019 0.0110657 0.0359494 0.191129 -0.179997 -0
0.0462278 0.235666 -0.410109 -0.0780643 0.414204 -0.194218 -0.471498 -0.00513573 0.0735477 0.21939 0.0181743 -0.209465 -0.261031 0.0916836
0.262533 0.237432 0.077748 0.0303058 0.0132068 0.0444983 0.0386113 0.0887603 -0.316229 0.0352413 -0.0808656 0.0910414 -0.0729861 -0.274096 -0.0
0.0419546 -0.0975325 -0.351879 -0.0428883 -0.297513 0.0762631 -0.0363362 0.173699 -0.296973 -0.400814 -0.292688 0.160523 -0.193329 0.0320098
0.243492 0.116939 0.0629639 0.0582823 -0.0623611 -0.0322901 0.00267559 -0.420532 0.0668712 0.123296 0.299432 -0.00659083 -0.196687 0.225251 -
0.52179 -0.0646878 0.0411685 -0.193775 -0.118273 0.364739 0.0276899 0.09987 0.0652384 -0.131602 0.0622103 -0.162586 -0.0749841 -0.0967181
0.159834 -0.0893072 0.442065 -0.055982 -0.0886118 0.047855 0.282982 0.0764946 0.225137 0.169305 0.0169856 0.0909175 0.0557834 -0.163936 0
0.0216529 -0.391865 0.141508 0.54024 0.0784058 -0.133658 -0.108029 0.160214 0.0217606 -0.0629571 0.124682 0.0464528 -0.0637602 0.0221223
0.216831 -0.278939 -0.235113 0.349371 -0.317596 0.112187 0.253236 0.0171741 0.0826097 -0.340402 0.337239 -0.0542116 -0.0237766 0.0810098 0
-1.55807 0.229727 -0.190813 0.196726 0.131412 0.0871175 0.111339 0.0423341 -0.142878 -0.0697631 0.0937626 0.111044 0.0138085 -0.190623
0.0928883 -0.547772 -0.0245587 -0.23847 -0.158042 0.321641 -0.0880091 0.038254 -0.148838 -0.18902 -0.277632 0.212802 0.184725 0.0883405 -0
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0.158165 -0.0463303 0.1225787 0.120003 0.0878498 0.00909474 -0.275837 0.22448 0.296071 0.0730955 0.0015338 0.21431 -0.182303 0.00268426 0
0.031961 0.398798 0.102057 0.044012 0.0593127 -0.0156348 -0.168542 0.0623894 0.0739371 -0.490599 0.0773139 -0.0546809 0.346084 0.228771
0.147892 -0.0300931 0.0489356 0.119416 0.254437 -0.123956 0.177335 -0.0450998 0.09215 -0.315841 -0.277662 -0.163854 -0.112729 0.121324 -0
-1.71715 -0.19823 0.120147 -0.085721 0.230549 -0.0529103 -0.157578 -0.081221 -0.354912 0.188287 -0.39123 -0.0805662 0.323582 0.158621
0.147795 -0.0396272 0.0496149 0.105018 -0.0284197 -0.461003 0.111084 0.0907401 -0.183368 0.029631 -0.142796 0.254262 0.638237 0.0381783 -0
0.0673855 0.103311 0.0917778 0.02351 -0.0698319 -0.202744 0.129175 -0.0438198 0.0169323 -0.0694598 0.0400717 -0.136299 0.19522 -0.22037
0.158657 -0.0550523 0.0999677 -0.040313 0.0760899 0.00576923 -0.322409 0.235791 0.217393 0.131325 0.0293666 0.197126 0.0269945 0.599482 0
-1.87933 -0.167699 -0.109431 0.0222309 -0.118297 0.0599771 -0.119294 0.0947295 -0.200511 0.180555 -0.159921 -0.0352724 -0.193161 -0.277192
0.218459 0.130461 0.0143121 -0.34007 -0.250533 -0.192067 0.17721 -0.409195 0.0835638 -0.0582173 -0.0891446 0.0451626 -0.0915488 0.0842198 0
-6.12805 -0.0194187 0.115887 -0.206599 -0.0266816 -0.0285801 -0.16263 -0.0230479 0.0505677 -0.118497 0.0303232 -0.00966164 0.0830718 -0.00538069
0.159834 -0.0893072 0.442065 -0.055982 -0.0886118 0.047855 0.282982 0.0764946 0.225137 0.169305 0.0169856 0.0909175 0.0240103 -0.0606404 -0
0.0674816 0.435496 0.102038 0.271746 0.213332 0.199587 0.213747 0.215199 -0.134006 0.232812 0.140117 0.0308402 -0.154488 0.0051095
0.242183 0.0464139 -0.249362 -0.0912947 0.0822891 0.099419 -0.0619317 0.179179 -0.210653 0.035524 0.353346 0.118232 0.0974041 -0.108985 0
0.0726473 -0.0253388 0.060923 -0.0533792 0.130421 -0.0510381 -0.0136692 0.0608072 -0.144889 0.0340365 -0.148838 -0.0120323 -0.244382 0.192548
0.114773 -0.158967 0.0266431 0.140529 0.198157 0.197554 -0.0982307 -0.524557 0.103017 -0.00235237 -0.0446769 0.048229 -0.0145618 -0.154016 0
0.0125502 0.101905 0.0427995 0.385468 -0.171806 0.224319 -0.11891 -0.215646 0.0325636 0.155124 -0.104843 -0.187668 0.00583749 -0.0460658
0.0933567 -0.230244 0.0202884 -0.0679714 0.0431394 0.0784963 0.108533 0.10475 0.136759 0.241306 -0.217479 0.276888 -0.348509 -0.0754392 0
0.0268042 -0.131768 -0.0213873 -0.157435 0.465802 -0.0531484 0.339082 -0.231688 0.0120718 -0.161575 0.00893414 -0.102 -0.0802051 -0.315397
0.12281 -0.233625 -0.0284907 -0.161752 0.187375 -0.494987 0.0526245 0.0364021 -0.127682 -0.0474879 0.242878 -0.246161 -0.158028 0.0758759 -0
0.0352377 0.0145888 -0.0893203 0.141246 0.0329373 0.466794 0.0592561 0.0892004 0.0825633 -0.100716 0.100567 -0.380861 -0.116152 0.069423
0.194173 0.0273062 -0.0760688 -0.267384 -0.010747 -0.00408776 -0.435188 -0.19226 0.389877 -0.195929 -0.152068 -0.118635 0.19595 -0.284525 -0
0.0658806 0.0148488 -0.0340441 0.0648447 0.142297 0.0187205 0.454612 0.212791 -0.10155 -0.00934464 -0.0840452 -0.0160227 -0.0731663 -0.0726484
0.113186 -0.127864 -0.109075 0.218761 0.115968 0.0845831 0.299806 0.0206603 0.200705 -0.0419144 -0.220654 0.268292 -0.00394195 -0.0824178 -0
0.0552526 -0.102355 0.0718321 -0.292095 -0.101005 0.25261 -0.0317328 0.585665 0.202158 0.135156 0.152462 0.00112569 -0.0365073 -0.0183303
0.245103 -0.0379005 0.249796 0.223658 0.212712 0.00142139 -0.250621 -0.0438574 -0.236907 0.0374695 0.0317127 -0.10255 0.0526566 0.0365469 0
0.0433022 0.113337 -0.0299223 -0.0845067 0.191159 0.140691 0.0330611 0.063840 0.583883 0.130682 -0.397614 0.140045 0.107705 0.100606
0.212867 0.0408414 0.0432211 0.3223 0.0098765 -0.0012797 0.0282962 0.217003 -0.228191 -0.33744 -0.133818 0.275680 -0.287306 0.133114 -
0.09523 0.103352 -0.0949957 0.216425 -0.15649 -0.244084 0.208054 0.0754158 0.11225 0.32407 0.140605 0.0131654 0.0701753 0.279555
0.235342 -0.0342118 0.208583 0.22741 -0.12882 0.0526029 -0.242403 -0.113701 -0.260678 0.0223432 -0.00639523 -0.0795438 0.0585996 0.0160812 0
0.0442937 -0.0739062 -0.0375475 -0.169321 0.186961 0.159976 -0.00779487 0.0409998 -0.407153 0.145878 0.594529 0.0992192 0.110496 0.128561
0.0325802 -0.309093 0.0897124 0.0442409 0.477681 -0.292797 -0.119177 -0.0925929 0.00905826 -0.0647401 0.0899299 -0.052934 -0.104726 -0.155627 0
-1.54788 0.0328707 -0.0166423 -0.110141 -0.113296 -0.338967 -0.0450548 -0.02668 0.0864267 0.0813027 0.075012 0.489293 -0.143249 -0.0385487
0.159496 -0.174221 -0.129762 -0.224787 0.123585 0.210925 0.0472228 0.102244 0.0738503 0.179292 0.391638 0.0307355 0.12817 -0.182757 0
-0.070913 -0.290496 -0.267671 -0.174687 -0.19764 -0.0208991 0.0884289 -0.108163 -0.013922 0.102836 -0.029847 -0.104377 0.534584 0.0609172
```

```
Q =      0.0234413 -0.0422221 0.00963128 -0.028024 0.123837 -0.0889256 0.059089 -0.0510038 0.701523 0.195669 -0.543076 0.376767
0.161979 -0.044267 -0.521551 0.724297 -0.384544 -0.21098 0.117628 0.10304 0.0294532 0.12333 0.074329 -0.0451588
0.0292717 -0.0036797 -0.0630708 0.0106822 -0.0430269 0.105246 0.102769 -0.015717 -0.506422 -0.230952 -0.470916 0.555693
0.144091 0.543618 0.759407 0.726692 -0.188343 -0.0280853 -0.116490 0.0746045 -0.0156241 0.013904 0.00256561 0.00491023
0.0349575 -0.116109 0.0186828 0.0570462 -0.0056745 0.21143 0.202736 0.0606621 0.355333 -0.866511 0.116359 0.0283752
0.0312827 -0.0873538 -0.135289 0.156338 0.191302 -0.0528119 -0.891951 0.252501 0.0351058 0.167604 0.0119204 0.143043
0.157212 -0.363467 -0.0490554 0.08802 0.642166 0.257032 0.0950931 -0.365931 -0.0162775 0.152785 0.369728 0.22571
0.0779178 -0.175794 -0.0480063 0.060128 0.296522 0.432264 0.0125808 0.212771 -0.0653433 0.0801522 -0.493572 -0.618539
0.956374 0.283782 0.0112589 0.0227738 -0.0556006 -0.0284664 0.00250434 0.0149853 -0.00382375 -0.00448851 0.00146235 0.00152511
0.0544552 -0.0039516 -0.348422 -0.924485 0.0347159 0.0458657 -0.0085951 -0.00296719 0.00722707 -0.0429476 0.0259679 -0.0132883
0.0262712 -0.0036574 0.0351311 -0.0290843 0.436249 -0.77887 0.119498 -0.0067562 -0.172791 -0.252212 -0.194836 -0.002625
0.00800058 -0.0294925 -0.0302003 0.0421216 -0.260624 
```

四、结果分析

第一题：用SVD隐式QR迭代的方法计算矩阵奇异值，速度还是很快的，耗时很短，精度也很高。