

附加作业7

1. 令常数 $a > 0$,用Fourier展开法求解以下定解问题

$$\begin{cases} u_t = a^2 u_{xx} + f(x, t), 0 < x < l, t > 0 \\ u|_{t=0} = \varphi(x) \\ u|_{x=0} = u|_{x=l} = 0 \end{cases}$$

2. 找出所有 $\omega \in \mathbb{R}$ 使

$$\begin{cases} u_{tt} = 4u_{xx} + \sin(\pi x)\sin(\omega t), 0 < x < 1, t > 0 \\ u|_{t=0} = u_t|_{t=0} = 0 \\ u|_{x=0} = u|_{x=1} = 0 \end{cases}$$

的解 $u(x, t)$ 在 $[0, 1] \times [0, +\infty)$ 上有界

3. 求解以下一维波动方程初边值问题

$$\begin{cases} u_{tt} = u_{xx} - 2u_x + e^x \sin(\pi x), 0 < x < 1, t > 0 \\ u|_{t=0} = u_t|_{t=0} = 0 \\ u|_{x=0} = u|_{x=1} = 0 \end{cases}$$

4. 求解以下一维热方程初边值问题

$$\begin{cases} u_t = u_{xx} + \sin(\pi x), 0 < x < 1, t > 0 \\ u|_{t=0} = x \\ u|_{x=0} = t, u|_{x=1} = 1 - t \end{cases}$$