数值代数实验报告

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一、问题描述

将p150 算法5.3.1 实用共轭梯度法编写为通用的程序。并分别用该程序完成：1.用共轭梯度法求解差分方程，要求4 位有效数字。观察迭代次数和求解所用时间。2.用Hilbert 矩阵测试你所编写的共轭梯度法程序，对n = 20, 40, 60, 80 分别求解，观察解是否准确，迭代停止条件自定，给出迭代次数和求解所用时间。3. 分别用Jacobi 迭代法，G-S 迭代法和共轭梯度法求解方程，观察迭代次数和求解所用时间，并对结果给出解释。

二、程序介绍

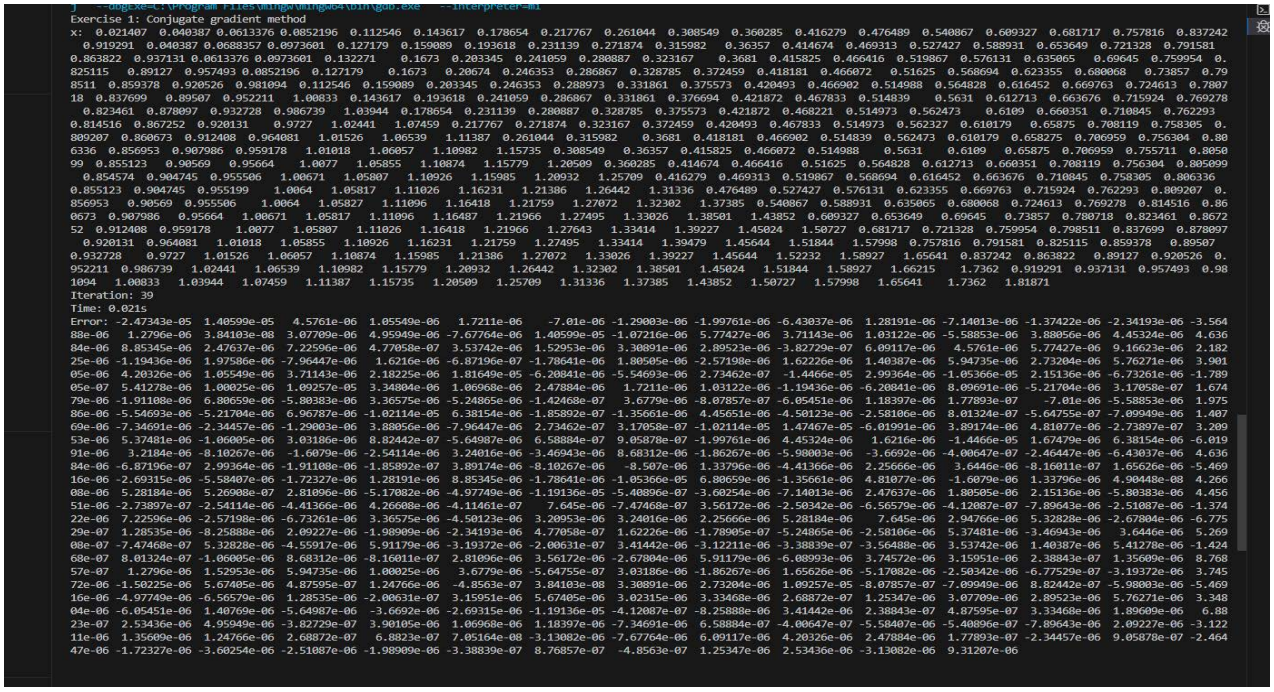
涉及的算法有矩阵方程的Jacobi 迭代、G-S 迭代和实用共轭梯度法。

主代码写在了homework.cpp里并由主函数输出相应结果.引用的函数均在function.h，function.cpp， exercise.h，C++库文件eigen-3.4.0里。函数名字均相当程度上反映了函数作用。

第二题选用1e-10为收敛条件。第三题采用1e-7为收敛条件。

三、实验结果

第一题实验结果：



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Exercise 1: Conjugate gradient method
x:  0.021407  0.040387 0.0613376 0.0852196  0.112546  0.143617  0.178654  0.217767  0.261044  0.308549  0.360285  0.416279  0.476489  0.540867  0.609327  0.681717  0.757816  0.837242
  0.919291  0.040387 0.0688357 0.0973601  0.127179  0.159089 0.193618  0.231139  0.271874  0.315982   0.36357  0.414674  0.469313  0.527427  0.588931  0.653649  0.721328  0.791581
0.863822  0.937131 0.0613376 0.0973601  0.132271    0.1673  0.203345  0.241059  0.280887  0.323167    0.3681  0.415825  0.466416  0.519867  0.576131  0.635065   0.69645  0.759954  0.
825115   0.89127  0.957493 0.0852196  0.127179    0.1673   0.20674  0.246353  0.286867  0.328785  0.372459  0.418181  0.466072   0.51625  0.568694  0.623355  0.680068   0.73857  0.79
8511  0.859378  0.920526  0.981094  0.112546  0.159089  0.203345  0.246353  0.288973  0.331861  0.375573  0.420493  0.466902  0.514988  0.564828  0.616452  0.669763  0.724613  0.7807
18  0.837699   0.89507  0.952211   1.00833  0.143617  0.193618  0.241059  0.286867  0.331861  0.376694  0.421872  0.467833  0.514839    0.5631  0.612713  0.663676  0.715924  0.769278
  0.823461  0.878097  0.932728  0.986739   1.03944  0.178654  0.231139  0.280887  0.328785  0.375573  0.421872  0.468221  0.514973  0.562473    0.6109  0.660351  0.710845  0.762293
0.814516  0.867252  0.920131    0.9727   1.02441   1.07459  0.217767  0.271874  0.323167  0.372459  0.420493  0.467833  0.514973  0.562327  0.610179   0.65875  0.708119  0.758305  0.
809207  0.860673  0.912408  0.964081   1.01526   1.06539   1.11387  0.261044  0.315982    0.3681  0.418181  0.466902  0.514839  0.562473  0.610179  0.658275  0.706959  0.756304  0.80
6336  0.856953  0.907986  0.959178   1.01018   1.06057   1.10982   1.15735  0.308549   0.36357  0.415825  0.466072  0.514988    0.5631    0.6109   0.65875  0.706959  0.755711  0.8050
99  0.855123   0.90569   0.95664    1.0077   1.05855   1.10874   1.15779   1.20509  0.360285  0.414674  0.466416   0.51625  0.564828  0.612713  0.660351  0.708119  0.756304  0.805099
  0.854574  0.904745  0.955586   1.00671   1.05807   1.10926   1.15985   1.20932   1.25709  0.416279  0.469313  0.519867  0.568694  0.616452  0.663676  0.710845  0.758305  0.806136
0.855123  0.904745  0.955199    1.0064   1.05817   1.11026   1.16231   1.21386   1.26442   1.31336  0.476489  0.527427  0.576131  0.623355  0.669763  0.715924  0.762293  0.809207  0.
856953   0.90569  0.955586    1.0064   1.05827   1.11096   1.16418   1.21759   1.27072   1.32302   1.37385  0.540867  0.588931  0.635065  0.680068  0.724613  0.769278  0.814516  0.86
0673  0.907986   0.95664   1.00671   1.05817   1.11096   1.16487   1.21966   1.27495   1.33026   1.38501   1.43852  0.609327  0.653649   0.69645   0.73857  0.780718  0.823461  0.8672
52  0.912408  0.959178    1.0077   1.05807   1.11026   1.16418   1.21966   1.27643   1.33414   1.39227   1.45024   1.50727  0.681717  0.721328  0.759954  0.798511  0.837699  0.878097
  0.920131  0.964081   1.01018   1.05855   1.10926   1.16231   1.21759   1.27495   1.33414   1.39479   1.45644   1.51844   1.57998  0.757816  0.791581  0.825115  0.859378   0.89507
0.932728    0.9727   1.01526   1.06057   1.10874   1.15985   1.21386   1.27072   1.33026   1.39227   1.45644   1.52232   1.58927   1.65641  0.837242  0.863822   0.89127  0.920526  0.
952211  0.986739   1.02441   1.06539   1.10982   1.15779   1.20932   1.26442   1.32302   1.38501   1.45024   1.51844   1.58927   1.66215   1.7362  0.919291  0.937131  0.957493  0.98
1094   1.00833   1.03944   1.07459   1.11387   1.15735   1.20509   1.25709   1.31336   1.37385   1.43852   1.50727   1.57998   1.65641    1.7362   1.81871
Iteration: 39
Time: 0.021s
Error: -2.47343e-05  1.40599e-05   4.5761e-06  1.05549e-06   1.7211e-06    -7.01e-06 -1.29003e-06 -1.99761e-06 -6.43037e-06  1.28191e-06 -7.14013e-06 -1.37422e-06 -2.34193e-06 -3.564
88e-06   1.2796e-06  3.84103e-08  3.07709e-06  4.95949e-06 -7.67764e-06  1.40599e-05 -1.07216e-06  5.77427e-06  3.71143e-06  1.03122e-06 -5.58853e-06  3.88056e-06  4.45324e-06  4.636
84e-06  8.85345e-06  2.47637e-06  7.22596e-06  4.77058e-07  3.53742e-06  1.52953e-06  3.30891e-06  2.89523e-06 -3.82729e-07  6.09117e-06   4.5761e-06  5.77427e-06  9.16623e-06  2.182
25e-06 -1.19436e-06  1.97586e-06 -7.96447e-06   1.6216e-06 -6.87196e-07 -1.78641e-06  1.80505e-06 -2.57198e-06  1.62226e-06  1.40387e-06  5.94735e-06  2.73204e-06  5.76271e-06  3.901
05e-06  4.20326e-06  1.05549e-06  3.71143e-06  2.18225e-06  1.81649e-05 -6.20841e-06 -5.54693e-06  2.73462e-07  -1.4466e-05  2.99364e-06 -1.05366e-05  2.15136e-06 -6.73261e-06 -1.789
05e-07  5.41278e-06  1.00025e-06  1.09257e-05  3.34804e-06  1.06968e-06  2.47884e-06   1.7211e-06  1.03122e-06 -1.19436e-06 -6.20841e-06  8.09691e-06 -5.21704e-06  3.17058e-07  1.674
79e-06 -1.91108e-06  6.80659e-06 -5.80383e-06  3.36575e-06 -5.24865e-06 -1.42468e-07   3.6779e-06 -8.07857e-07 -6.05451e-06  1.18397e-06  1.77893e-07    -7.01e-06 -5.58853e-06  1.975
86e-06 -5.54693e-06 -5.21704e-06  6.96787e-06 -1.02114e-05  6.38154e-06 -1.85892e-07 -1.35661e-06  4.45651e-06 -4.50123e-06 -2.58106e-06  8.01324e-07 -5.64755e-07 -7.09949e-06  1.407
69e-06 -7.34691e-06 -2.34457e-06 -1.29003e-06  3.88056e-06 -7.96447e-06  2.73462e-07  3.17058e-07 -1.02114e-05  1.47467e-05 -6.01991e-06  3.89174e-06  4.81077e-06 -2.73897e-07  3.209
53e-06  5.37481e-06 -1.06005e-06  3.03186e-06  8.82442e-07 -5.64987e-06  6.58884e-07  9.05878e-07 -1.99761e-06  4.45324e-06   1.6216e-06  -1.4466e-05  1.67479e-06  6.38154e-06 -6.019
91e-06   3.2184e-06 -8.10267e-06  -1.6079e-06 -2.54114e-06  3.24016e-06 -3.46943e-06  8.68312e-06 -1.86267e-06 -5.98003e-06  -3.6692e-06 -4.00647e-07 -2.46447e-06 -6.43037e-06  4.636
84e-06 -6.87196e-07  2.99364e-06 -1.91108e-06 -1.85892e-07  3.89174e-06 -8.10267e-06   -8.507e-06  1.33796e-06 -4.41366e-06  2.25666e-06   3.6446e-06 -8.16011e-07  1.65626e-06 -5.469
16e-06 -2.69315e-06 -5.58407e-06 -1.72327e-06  1.28191e-06  8.85345e-06 -1.78641e-06 -1.05366e-05  6.80659e-06 -1.35661e-06  4.81077e-06  -1.6079e-06  1.33796e-06  4.90448e-08  4.266
08e-06  5.28184e-06  5.26908e-07  2.81096e-06 -5.17082e-06 -4.97749e-06 -1.19136e-05 -5.40896e-07 -3.60254e-06 -7.14013e-06  2.47637e-06  1.80505e-06  2.15136e-06 -5.80383e-06  4.456
51e-06 -2.73897e-07 -2.54114e-06 -4.41366e-06  4.26608e-06 -4.11461e-07    7.645e-06 -7.47468e-07  3.56172e-06 -2.50342e-06 -6.56579e-06 -4.12087e-07 -7.89643e-06 -2.51087e-06 -1.374
22e-06  7.22596e-06 -2.57198e-06 -6.73261e-06  3.36575e-06 -4.50123e-06  3.20953e-06  3.24016e-06  2.25666e-06  5.28184e-06    7.645e-06  2.94766e-06  5.32828e-06 -2.67804e-06 -6.775
29e-07  1.28535e-06 -8.25888e-06  2.09227e-06 -1.98909e-06 -2.34193e-06  4.77058e-07  1.62226e-06 -1.78905e-07 -5.24865e-06 -2.58106e-06  5.37481e-06 -3.46943e-06   3.6446e-06  5.269
08e-07 -7.47468e-07  5.32828e-06 -4.55917e-06  5.91179e-06 -3.19372e-06 -2.00631e-07  3.41442e-06 -3.12211e-06 -3.38839e-07 -3.56488e-06  3.53742e-06  1.40387e-06  5.41278e-06 -1.424
68e-07  8.01324e-07 -1.06005e-06  8.68312e-06 -8.16011e-07  2.81096e-06  3.56172e-06 -2.67804e-06  5.91179e-06 -6.08993e-06  3.74572e-06  3.15951e-06  2.38843e-07  1.35609e-06  8.768
57e-07   1.2796e-06  1.52953e-06  5.94735e-06  1.00025e-06   3.6779e-06 -5.64755e-07  3.03186e-06 -1.86267e-06  1.65626e-06 -5.17082e-06 -2.50342e-06 -6.77529e-07 -3.19372e-06  3.745
72e-06 -1.50225e-06  5.67405e-06  4.87595e-07  1.24766e-06  -4.8563e-07  3.84103e-08  3.30891e-06  2.73204e-06  1.09257e-05 -8.07857e-07 -7.09949e-06  8.82442e-07 -5.98003e-06 -5.469
16e-06 -4.97749e-06 -6.56579e-06  1.28535e-06 -2.00631e-07  3.15951e-06  5.67405e-06  3.02315e-06  3.33468e-06  2.68872e-07  1.25347e-06  3.07709e-06  2.89523e-06  5.76271e-06  3.348
04e-06 -6.05451e-06  1.40769e-06 -5.64987e-06  -3.6692e-06 -2.69315e-06 -1.19136e-05 -4.12087e-07 -8.25888e-06  3.41442e-06  2.38843e-07  4.87595e-07  3.33468e-06  1.89609e-06   6.88
23e-07  2.53436e-06  4.95949e-06 -3.82729e-07  3.90105e-06  1.06968e-06  1.18397e-06 -7.34691e-06  6.58884e-07 -4.00647e-07 -5.58407e-06 -5.40896e-07 -7.89643e-06  2.09227e-06 -3.122
11e-06  1.35609e-06  1.24766e-06  2.68872e-07   6.8823e-07  7.05164e-08 -3.13082e-06 -7.67764e-06  6.09117e-06  4.20326e-06  2.47884e-06  1.77893e-07 -2.34457e-06  9.05878e-07 -2.464
47e-06 -1.72327e-06 -3.60254e-06 -2.51087e-06 -1.98909e-06 -3.38839e-07  8.76857e-07  -4.8563e-07  1.25347e-06  2.53436e-06 -3.13082e-06  9.31207e-06
```

第二题实验结果：

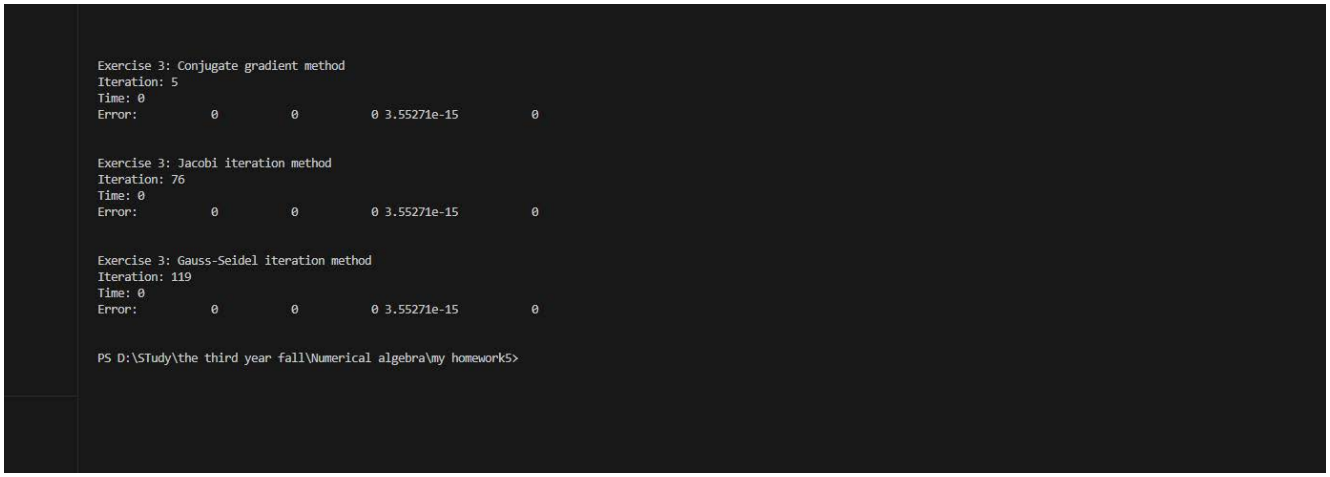
```
Exercise 2: Conjugate gradient method
N: 20
Iteration: 5
Time: 0s
Error:  6.58026e-08  -5.6482e-07  1.17561e-06  2.16464e-07 -5.13234e-07 -6.99536e-07  -5.4908e-07 -2.64697e-07  2.86217e-08  2.67849e-07  4.28094e-07  5.05377e-07   5.0608e-07  4.409
52e-07  3.21894e-07  1.60338e-07 -3.34788e-08 -2.50765e-07 -4.84172e-07 -7.27682e-07


Exercise 2: Conjugate gradient method
N: 40
Iteration: 6
Time: 0s
Error: -7.02667e-07 -1.27387e-07 -1.04557e-06 -3.40505e-07  1.11662e-07    1.981e-07  9.07939e-08  -7.4931e-08 -2.27463e-07  -3.3793e-07 -4.00537e-07 -4.20233e-07 -4.05968e-07 -3.673
06e-07 -3.12904e-07 -2.49959e-07 -1.84127e-07 -1.19663e-07 -5.96305e-08 -6.12484e-09   3.9518e-08  7.65462e-08  1.04647e-07  1.23827e-07  1.34326e-07  1.36538e-07  1.30964e-07  1.181
66e-07  9.87382e-08  7.32853e-08  4.24045e-08  6.67539e-09  -3.3348e-08  -7.7142e-08 -1.24216e-07 -1.74115e-07 -2.26418e-07 -2.80738e-07 -3.36721e-07 -3.94046e-07


Exercise 2: Conjugate gradient method
N: 60
Iteration: 6
Time: 0s
Error: -2.49487e-06  3.76803e-07 -4.32117e-06 -2.27088e-06 -1.49228e-07  8.78339e-07  1.04606e-06  7.39361e-07  2.40752e-07  -2.8144e-07 -7.40075e-07  -1.0982e-06 -1.34747e-06 -1.494
52e-06 -1.55296e-06 -1.53874e-06 -1.46771e-06 -1.35436e-06 -1.21133e-06 -1.04925e-06 -8.76877e-07 -7.01221e-07 -5.27807e-07 -3.60881e-07 -2.03626e-07  -5.8347e-08  7.33655e-08  1.904
95e-07  2.92482e-07  3.79127e-07  4.50511e-07  5.06929e-07  5.48835e-07  5.76801e-07   5.9148e-07   5.9358e-07  5.83843e-07  5.63023e-07  5.31879e-07  4.91159e-07  4.41595e-07  3.838
96e-07  3.18746e-07  2.46799e-07  1.68676e-07  8.49694e-08 -3.76456e-09 -9.70004e-08 -1.94245e-07 -2.95037e-07 -3.98944e-07 -5.05564e-07 -6.14522e-07 -7.25471e-07 -8.38087e-07 -9.520
71e-07 -1.06715e-06 -1.18306e-06 -1.29958e-06 -1.41648e-06


Exercise 2: Conjugate gradient method
N: 80
Iteration: 8
Time: 0s
Error:  1.00502e-06  9.96324e-08  5.90323e-07 -1.88855e-08 -3.60882e-07 -4.09089e-07 -3.17162e-07 -1.91623e-07 -8.37641e-08 -1.12058e-08  2.51719e-08  3.14243e-08  1.55392e-08 -1.485
52e-08 -5.33753e-08 -9.50828e-08 -1.36384e-07 -1.74818e-07 -2.08819e-07  -2.3751e-07 -2.60516e-07 -2.77818e-07 -2.89647e-07 -2.96391e-07 -2.98532e-07 -2.96604e-07 -2.91155e-07 -2.827
26e-07 -2.71835e-07 -2.58967e-07  -2.4457e-07  -2.2905e-07 -2.12771e-07 -1.96055e-07 -1.79189e-07 -1.62418e-07 -1.45955e-07 -1.29982e-07 -1.14653e-07 -1.00093e-07 -8.64083e-08 -7.368
13e-08 -6.19777e-08 -5.13471e-08 -4.18248e-08 -3.34339e-08  -2.6187e-08 -2.00876e-08 -1.51312e-08 -1.13068e-08 -8.59754e-09 -6.98216e-09 -6.43528e-09 -6.92843e-09 -8.43054e-09 -1.090
85e-08 -1.43278e-08 -1.86527e-08 -2.38467e-08 -2.98729e-08 -3.66942e-08 -4.42735e-08 -5.25741e-08 -6.15595e-08 -7.11939e-08 -8.14421e-08 -9.22696e-08 -1.03643e-07 -1.15529e-07 -1.278
97e-07 -1.40714e-07 -1.53953e-07 -1.67583e-07 -1.81577e-07 -1.95908e-07  -2.1055e-07 -2.25479e-07 -2.40671e-07 -2.56103e-07 -2.71753e-07
```

第三题实验结果：



四、结果分析

第一题：对19*19的大矩阵，仅迭代39次，耗时0.02s，效率相当高。
第二题：对相当病态的Hilbert矩阵，共轭梯度法表现优异，不仅结果准确，迭代次数还很少，相应地，耗时非常少。
第三题：因为方程组太小，三个迭代方法时间是比较不了。但是从迭代次数可以推测出，共轭梯度法的速度远快于Jacobi迭代法和G-S迭代法！！