数值代数实验报告

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一、问题描述

考虑两个偏微分方程问题离散化得到的大型线性方程组的求解问题。

1.分别用Jacobi 迭代法,G-S 迭代法和SOR 迭代法求线性方程组的解，要求4 位有效数字，然后比较迭代次数，运行时间与精确解的误差。对epsilon= 0.1, 0.01, 0.0001, 考虑同样的问题。要求输出计算结果，收敛所需要的迭代次数和运行时间。

2.要求仿照下面写的Jacobi 迭代格式的推导过程推导处G-S 迭代和SOR 迭代的格式（在报告中写出推导过程），在用SOR 迭代法求解的过程中，请对不同的N 使用合适的松弛因子ω，并在程序输出中打印松弛因子的值。观察运行结果后选取合适的。（代码中不需要体现选取过程，只需给出即可）。

二、程序介绍

涉及的算法有矩阵方程的Jacobi 迭代、G-S 迭代和SOR 迭代方法。第二题中的算法单独推导实现。第二题的算法推导放在同文件另外一个pdf中。

主代码写在了homework.cpp里并由主函数输出相应结果.引用的函数均在function.h，function.cpp， exercise.h，C++库文件eigen-3.4.0里。函数名字均相当程度上反映了函数作用。

三、实验结果

第一题实验结果：

```
------------------------------------------------------------aipu =1
Jacobi_iteration_solve: 0.0128168 0.0255564 0.0382192 0.0508065 0.0633186 0.0757567 0.0881212  0.100413  0.112633  0.124782   0.13686  0.148869  0.160809  0.172681  0.184484  0.19622
2  0.207892  0.219498  0.231038  0.242515  0.253928  0.265278  0.276566  0.287793  0.298958  0.310064  0.321109  0.332096  0.343023  0.353894  0.364706  0.375463  0.386162  0.396807
 0.407397  0.417932  0.428413  0.438841  0.449216  0.459539  0.469809   0.48003  0.490198  0.500317  0.510386  0.520406  0.530377    0.5403  0.550175  0.560003  0.569783  0.579518  0
.589206   0.59885  0.608447  0.618001   0.62751  0.636975  0.646397  0.655777  0.665113  0.674408   0.68366  0.692872  0.702042  0.711173  0.720262  0.729313  0.738323  0.747296  0.7
56229  0.765124  0.773981    0.7828  0.791582  0.800328  0.809037   0.81771  0.826346  0.834948  0.843514  0.852045  0.860542  0.869005  0.877433  0.885828   0.89419  0.902519  0.910
815  0.919079  0.927311  0.935511  0.943679  0.951816  0.959922  0.967998  0.976043  0.984059  0.992044
jacobi:                 time:0.618              iter_num:13138          error:-5.36282e-05  -0.00010626 -0.000157993 -0.000208602 -0.000258347 -0.000306843 -0.000354509 -0.000400809
 -0.000446314  -0.00049034 -0.000533607 -0.000575287 -0.000616244 -0.000655515 -0.000694098 -0.000730902 -0.000767055 -0.000801341 -0.000835014 -0.000866741  -0.00089789 -0.000927022
 -0.000955613 -0.000982123  -0.00100813    -0.001032   -0.0010554  -0.00107661   -0.0010974  -0.00111596  -0.00113412  -0.00115002  -0.00116557  -0.00117883  -0.00119177  -0.00120241
  -0.00121276  -0.00122081   -0.0012286  -0.00123409  -0.00123935  -0.00124232  -0.00124509  -0.00124559  -0.00124591    -0.001244  -0.00124194  -0.00123767  -0.00123329  -0.00122673
  -0.00122009  -0.00121132  -0.00120249  -0.00119159  -0.00118065  -0.00116769  -0.00115473  -0.00113981  -0.00112491  -0.00110812  -0.00109137  -0.00107282  -0.00105431  -0.00103409
  -0.00101393 -0.000992146 -0.000970439 -0.000947193 -0.000924042 -0.000899447  -0.00087496 -0.000849126 -0.000823414 -0.000796454 -0.000769627 -0.000741654 -0.000713825 -0.000684955
 -0.000656236 -0.000626583 -0.000597088 -0.000566766 -0.000536609 -0.000505731 -0.000475024 -0.000443704 -0.000412558 -0.000380908 -0.000349434 -0.000317563  -0.00028587 -0.000253885
 -0.000222079 -0.000190086 -0.000158271 -0.000126373   -9.465e-05 -6.29463e-05 -3.14139e-05
GS:             time:1.533              iter_num:6557           error:-5.61086e-05 -0.000111194 -0.000165231 -0.000218195 -0.000270062 -0.000320808 -0.000370412 -0.000418852 -0.0004
66107 -0.000512158 -0.000556987 -0.000600575 -0.000642905 -0.000683963 -0.000723732 -0.000762199 -0.000799352 -0.000835178 -0.000869667 -0.000902809 -0.000934595 -0.000965017  -0.000
99407  -0.00102175  -0.00104804  -0.00107296  -0.00109648  -0.00111862  -0.00113937  -0.00115874  -0.00117672  -0.00119331  -0.00120853  -0.00122237  -0.00123485  -0.00124596  -0.001
25573  -0.00126414  -0.00127122  -0.00127698  -0.00128143  -0.00128457  -0.00128643  -0.00128701  -0.00128634  -0.00128443  -0.00128129  -0.00127695  -0.00127141  -0.00126471  -0.001
25686  -0.00124788  -0.00123779  -0.00122661  -0.00121437  -0.00120109  -0.00118679   -0.0011715  -0.00115525  -0.00113805  -0.00111993  -0.00110092  -0.00108105  -0.00106035  -0.001
03883  -0.00101653  -0.00099348 -0.000969703 -0.000945229 -0.000920088 -0.000894307 -0.000867917 -0.000840947 -0.000813426 -0.000785384  -0.00075685 -0.000727854 -0.000698426 -0.0006
68595 -0.000638391 -0.000607843 -0.000576981 -0.000545834 -0.000514431 -0.000482801 -0.000450973 -0.000418975 -0.000386836 -0.000354583 -0.000322245 -0.000289849 -0.000257421  -0.000
22499 -0.000192579 -0.000160217 -0.000127928 -9.57377e-05 -6.36701e-05 -3.17497e-05
SOR:            time:1.328              iter_num:5614           error:-4.94366e-05 -9.79296e-05 -0.000145459 -0.000192007 -0.000237554 -0.000282082 -0.000325575 -0.000368016 -0.0004
09389 -0.000449679 -0.000488873 -0.000526956 -0.000563917 -0.000599743 -0.000634423 -0.000667946 -0.000700305 -0.000731489 -0.000761492 -0.000790305 -0.000817924 -0.000844343 -0.0008
69557 -0.000893563 -0.000916359 -0.000937942 -0.000958313 -0.000977469 -0.000995414  -0.00101215  -0.00102767  -0.00104199  -0.00105511  -0.00106704  -0.00107777  -0.00108732  -0.001
09569   -0.0011029  -0.00110895  -0.00111385  -0.00111761  -0.00112024  -0.00112176  -0.00112218   -0.0011215  -0.00111976  -0.00111695   -0.0011131  -0.00110822  -0.00110233  -0.001
09544  -0.00108758  -0.00107876    -0.001069  -0.00105832  -0.00104675  -0.00103429  -0.00102097  -0.00100682 -0.000991851 -0.000976089 -0.000959557 -0.000942277 -0.000924273 -0.0009
05567 -0.000886183 -0.000866147  -0.00084548 -0.000824209 -0.000802358 -0.000779951 -0.000757013 -0.00073357 -0.000709646 -0.000685266 -0.000660456 -0.000635241 -0.000609646 -0.0005
83697 -0.000557417 -0.000530833 -0.00050397 -0.000476852 -0.000449503 -0.000421949 -0.000394215 -0.000366323 -0.000338298 -0.000310165 -0.000281945 -0.000253664 -0.000225343 -0.0001
97085 -0.000168673 -0.000140368 -0.000112112 -8.39268e-05 -5.58328e-05 -2.78505e-05
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```
------------------------------------------------------------aipu =0.1
Jacobi_iteration_solve: 0.0504207 0.0967123  0.139251  0.178376    0.2144  0.247604  0.278245  0.306555  0.332747  0.357012  0.379528  0.400451  0.419929   0.43809  0.455056  0.47093
5  0.485826  0.499818  0.512995  0.525428  0.537187  0.548331  0.558919  0.568999  0.578619  0.587818  0.596638   0.60511  0.613268  0.621139  0.628751  0.636125  0.643285  0.650248
 0.657035  0.663658  0.670136  0.676479  0.682701  0.688812  0.694824  0.700743   0.70658  0.71234  0.718033  0.723662  0.729235  0.734756  0.740231  0.745662  0.751055  0.756412  0
.761737  0.767033  0.772302  0.777547   0.78277  0.787973  0.793158  0.798326   0.80348  0.808619  0.813746  0.818861  0.823967  0.829062   0.83415  0.839229  0.844302  0.849367  0.8
54427  0.859482  0.864532  0.869577  0.874619  0.879656   0.88469  0.889722   0.89475  0.899776    0.9048  0.909821  0.914841  0.919859  0.924875   0.92989  0.934903  0.939916  0.944
927  0.949937  0.954946  0.959955  0.964962  0.969969  0.974976  0.979982  0.984987  0.989992  0.994996
jacobi:                 time:0.262              iter_num:5919           error: -0.00216271  -0.00392644  -0.00534622  -0.00647127  -0.00734324  -0.00800035  -0.00847379  -0.00879321
  -0.00898167  -0.00906217  -0.00905146  -0.0089674  -0.00882187  -0.00862876  -0.00839621  -0.00813504  -0.00785057  -0.00755125  -0.00724026  -0.00692424  -0.00660478  -0.00628713
  -0.00597169  -0.00566265  -0.00535959  -0.00506586  -0.00478046  -0.0045061  -0.00424139  -0.00398856  -0.00374596  -0.00351547  -0.00329527  -0.00308696  -0.00288867  -0.00270176
  -0.00252437  -0.00235769  -0.00219987  -0.00205198  -0.00191225  -0.00178161  -0.00165839  -0.00154342  -0.00143515  -0.00133431  -0.00123946  -0.00115128  -0.00106842 -0.000991488
 -0.000919272 -0.000852317 -0.000789513 -0.000731356 -0.000676841 -0.000626416 -0.000579177 -0.000535527 -0.000494657 -0.000456928 -0.000421618  -0.00038905 -0.000358584 -0.000330506
 -0.000304251 -0.000280073 -0.000257472 -0.000236675 -0.000217242 -0.000199371 -0.000182679 -0.000167338 -0.000153016  -0.00013986 -0.000127584 -0.000116313 -0.000105802 -9.61562e-05
 -8.71658e-05 -7.89193e-05 -7.12385e-05 -6.41961e-05 -5.76422e-05 -5.16351e-05   -4.605e-05 -4.09325e-05 -3.61797e-05  -3.1826e-05 -2.77878e-05 -2.40894e-05 -2.06641e-05 -1.75277e-05
 -1.46278e-05 -1.19728e-05 -9.52283e-06    -7.28e-06 -5.21508e-06 -3.32479e-06 -1.58902e-06
GS:             time:0.704              iter_num:2977           error: -0.00216713  -0.00393459  -0.00535787  -0.00648559  -0.00736029  -0.00801918  -0.00849471  -0.00881517  -0.009
00518  -0.00908612  -0.00907652  -0.00899241  -0.00884763  -0.00865409    -0.008422  -0.0081601  -0.00787586  -0.00757559  -0.00726465  -0.00694753  -0.00662798  -0.00630911  -0.005
99348  -0.00568316  -0.00537983  -0.0050848  -0.00479908  -0.00452342  -0.00425836  -0.00400426  -0.00376131  -0.00352957  -0.00330902  -0.00309952  -0.00290089  -0.00271286  -0.002
53515  -0.00236741  -0.00220929  -0.00206042  -0.00192041  -0.00178887   -0.0016654  -0.00154961  -0.00144111  -0.00133952  -0.00124448  -0.00115562  -0.00107259 -0.000995059 -0.0009
22701 -0.000855209 -0.000792287 -0.000733655 -0.000679045 -0.000628203  -0.00058089 -0.000536878 -0.000495952 -0.000457909  -0.00042256 -0.000389724 -0.000359233 -0.000330928  -0.000
30466  -0.00028029 -0.000257688 -0.000236731 -0.000217305 -0.000199303 -0.000182626 -0.000167179 -0.000152877 -0.000139637 -0.000127385  -0.00011605 -0.000105565 -9.58711e-05 -8.6909
8e-05 -7.86286e-05 -7.09781e-05 -6.39125e-05 -5.73892e-05 -5.13685e-05 -4.58136e-05 -4.06904e-05  -3.5967e-05 -3.16139e-05 -2.76037e-05 -2.39109e-05  -2.0512e-05  -1.7385e-05 -1.4509
5e-05 -1.18668e-05 -9.43918e-06 -7.21053e-06 -5.16575e-06 -3.29088e-06 -1.57296e-06
SOR:            time:0.637              iter_num:2527           error: -0.00216016  -0.00392134  -0.00533896  -0.00646162  -0.00733181   -0.0079867  -0.00845872  -0.00877611  -0.008
96347  -0.00904215  -0.00903064  -0.00894495  -0.00879889  -0.00860434  -0.0083715  -0.0081091  -0.00782456  -0.00752419  -0.00721332  -0.00689643  -0.00657725  -0.00625889  -0.005
94388  -0.0056343  -0.00533178  -0.00503765  -0.00475289  -0.00447826  -0.00421427  -0.00396129  -0.00371948  -0.00348893  -0.00326958  -0.0030613  -0.00286389  -0.0026771  -0.002
50062  -0.00233411  -0.00217721  -0.00202955  -0.00189074  -0.00176038  -0.00163807  -0.00152343  -0.00141605  -0.00131556  -0.00122159  -0.00113378  -0.00105177 -0.000975228 -0.0009
03833 -0.000837274 -0.000775257 -0.000717499 -0.000663734 -0.000613709 -0.000567182 -0.000523926 -0.000483729 -0.000446386 -0.000411708 -0.000379515  -0.00034964 -0.000321925 -0.0002
96221 -0.000272389  -0.0002503 -0.000229833 -0.000210873 -0.000193314 -0.000177057  -0.00016201 -0.000148086 -0.000135206 -0.000123293 -0.000112279 -0.000102098 -9.26893e-05 -8.3997
5e-05 -7.59699e-05 -6.85579e-05 -6.17163e-05 -5.54031e-05 -4.95792e-05 -4.42086e-05 -3.92574e-05 -3.46947e-05 -3.04912e-05 -2.66203e-05  -2.3057e-05 -1.97781e-05 -1.67623e-05 -1.3989
6e-05 -1.14416e-05  -9.1013e-06 -6.95284e-06 -4.98157e-06  -3.1739e-06 -1.51726e-06
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```

```
------------------------------------------------------------aipu =0.01
Jacobi_iteration_solve: 0.254993 0.384991  0.45249 0.488741 0.509366  0.52218 0.531087 0.538041 0.544019 0.549508 0.554753 0.559875 0.564937 0.569968 0.574983 0.579991 0.584995 0.589
997 0.594998 0.599999 0.604999     0.61    0.615     0.62    0.625     0.63    0.635     0.64    0.645     0.65    0.655     0.66    0.665     0.67    0.675     0.68    0.685     0.6
9    0.695     0.7    0.705     0.71    0.715     0.72    0.725     0.73    0.735     0.74    0.745     0.75    0.755     0.76    0.765     0.77    0.775     0.78    0.785     0.79
   0.795     0.8    0.805     0.81    0.815     0.82    0.825     0.83    0.835     0.84    0.845     0.85    0.855     0.86    0.865     0.87    0.875     0.88    0.885     0.89
 0.895     0.9    0.905     0.91    0.915     0.92    0.925     0.93    0.935     0.94    0.945     0.95    0.955     0.96    0.965     0.97    0.975     0.98    0.985     0.99    0
.995
jacobi:                 time:0.032              iter_num:568            error:   -0.066067   -0.0573415   -0.0376168   -0.0221017   -0.0122651   -0.00658072  -0.00345718  -0.00179092
 -0.000919721 -0.000469414 -0.000239117  -0.00012159 -6.21353e-05 -3.18249e-05 -1.65798e-05 -8.70483e-06 -4.75753e-06 -2.63567e-06 -1.57493e-06 -9.50201e-07 -6.38841e-07 -4.22382e-07
 -3.14913e-07 -2.22189e-07 -1.76325e-07 -1.28558e-07  -1.0499e-07 -7.75109e-08 -6.39686e-08  -4.7356e-08  -3.9172e-08  -2.8958e-08 -2.39262e-08 -1.76323e-08 -1.45313e-08 -1.06679e-08
 -8.76406e-09 -6.40758e-09 -5.24605e-09 -3.81924e-09  -3.1158e-09  -2.2586e-09  -1.8359e-09 -1.32503e-09 -1.07305e-09 -7.71056e-10 -6.22069e-10 -4.45015e-10 -3.57648e-10 -2.54709e-10
 -2.03902e-10 -1.44556e-10  -1.1526e-10 -8.13374e-11 -6.45887e-11 -4.53662e-11 -3.58737e-11 -2.50768e-11 -1.97442e-11 -1.37343e-11 -1.07654e-11 -7.45093e-12 -5.81268e-12 -4.00191e-12
 -3.10685e-12 -2.12674e-12 -1.64246e-12 -1.11733e-12  -8.5798e-13 -5.79536e-13 -4.42313e-13 -2.95874e-13 -2.24265e-13 -1.48215e-13 -1.11466e-13 -7.24976e-14 -5.37348e-14 -3.43059e-14
  -2.4869e-14 -1.54321e-14 -1.08802e-14 -6.66134e-15 -4.66294e-15 -2.77556e-15 -2.10942e-15 -1.44329e-15 -1.33227e-15 -1.11022e-15 -1.11022e-15 -8.88178e-16 -8.88178e-16 -6.66134e-16
 -6.66134e-16 -4.44089e-16 -4.44089e-16 -2.22045e-16 -1.11022e-16           0  1.11022e-16
GS:             time:0.08               iter_num:333            error:   -0.066067   -0.0573416   -0.0376159   -0.0221009   -0.0122636   -0.00657942  -0.00345546  -0.00178953 -0.0009
18148 -0.000468174 -0.000237819 -0.000120577  -6.1128e-05 -3.10454e-05 -1.58313e-05 -8.12906e-06 -4.21879e-06 -2.22342e-06 -1.19687e-06 -6.62283e-07  -3.7905e-07 -2.25443e-07 -1.3960
7e-07 -8.98891e-08 -5.99182e-08 -4.10973e-08 -2.88154e-08 -2.05293e-08 -1.47877e-08 -1.07281e-08 -7.81653e-09 -5.70806e-09 -4.17188e-09 -3.04874e-09 -2.22619e-09 -1.62354e-09 -1.1821
8e-09 -8.59292e-10 -6.23403e-10 -4.51363e-10 -3.26126e-10 -2.35139e-10 -1.69174e-10  -1.2145e-10 -8.69985e-11 -6.21825e-11 -4.43464e-11 -3.15556e-11 -2.24035e-11 -1.58696e-11 -1.1215
5e-11 -7.90767e-12 -5.56233e-12 -3.90299e-12  -2.7317e-12 -1.90681e-12 -1.32727e-12 -9.21263e-13 -6.37268e-13 -4.39426e-13 -3.01537e-13 -2.06168e-13  -1.4011e-13  -9.4702e-14 -6.3504
8e-14 -4.24105e-14 -2.79776e-14 -1.84297e-14 -1.18794e-14 -7.88258e-15 -5.21805e-15 -3.66374e-15 -3.10862e-15 -2.77556e-15 -2.66454e-15 -2.55351e-15 -2.44249e-15 -2.33147e-15 -2.2204
5e-15 -2.10942e-15  -1.9984e-15 -1.88738e-15 -1.77636e-15 -1.55431e-15 -1.55431e-15 -1.33227e-15 -1.33227e-15 -1.11022e-15 -1.11022e-15 -8.88178e-16 -8.88178e-16 -6.66134e-16 -6.6613
4e-16 -4.44089e-16 -4.44089e-16 -2.22045e-16 -1.11022e-16           0  1.11022e-16
SOR:            time:0.071              iter_num:277            error:   -0.0660655   -0.0573394   -0.0376137   -0.0220988   -0.0122617   -0.0065779   -0.0034542  -0.00178851 -0.0009
17331 -0.000467524 -0.000237306 -0.000120174 -6.08126e-05 -3.07997e-05 -1.56405e-05 -7.98147e-06 -4.10493e-06 -2.13585e-06  -1.1297e-06 -6.10906e-07 -3.39854e-07 -1.95618e-07  -1.169
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6e-10 -5.89932e-10 -4.26743e-10 -3.08121e-10 -2.22037e-10 -1.59679e-10 -1.14596e-10 -8.20691e-11 -5.86492e-11 -4.18222e-11 -2.97584e-11 -2.11279e-11  -1.4967e-11 -1.05792e-11 -7.4602
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1e-14 -2.67564e-14 -1.75415e-14 -1.16573e-14 -7.32747e-15 -5.21805e-15 -3.66374e-15 -3.21965e-15 -2.88658e-15 -2.77556e-15 -2.66454e-15 -2.55351e-15 -2.44249e-15 -2.33147e-15 -2.2204
5e-15 -2.10942e-15  -1.9984e-15 -1.88738e-15 -1.77636e-15 -1.55431e-15 -1.55431e-15 -1.33227e-15 -1.33227e-15 -1.11022e-15 -1.11022e-15 -8.88178e-16 -8.88178e-16 -6.66134e-16 -6.6613
4e-16 -4.44089e-16 -4.44089e-16 -2.22045e-16 -1.11022e-16           0  1.11022e-16
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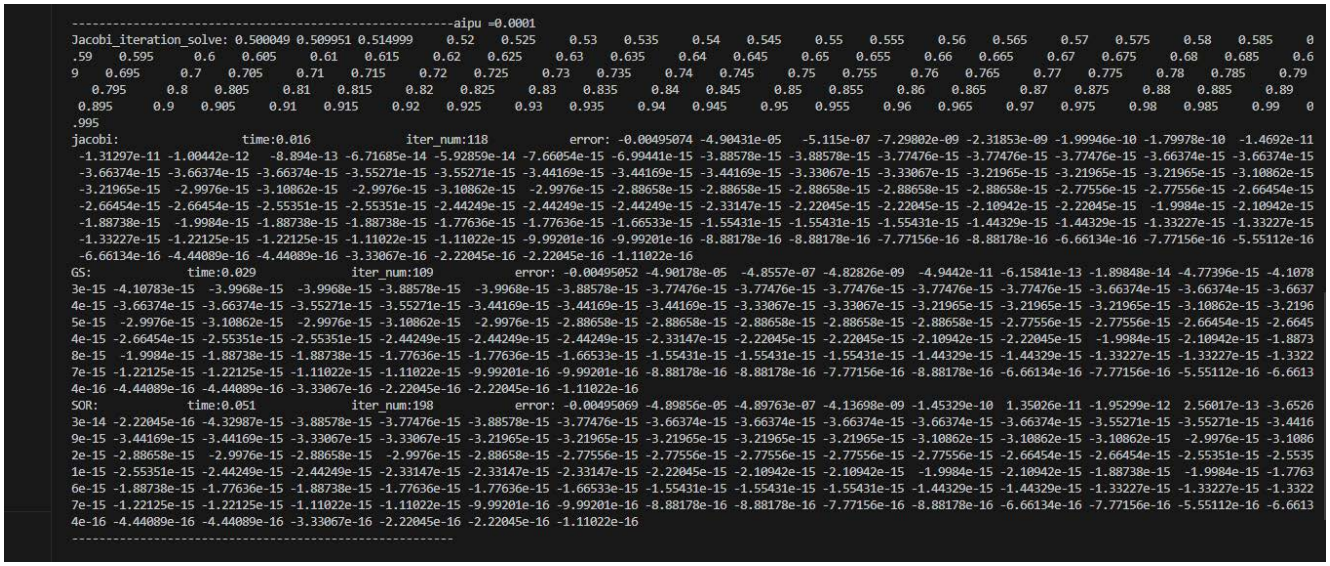
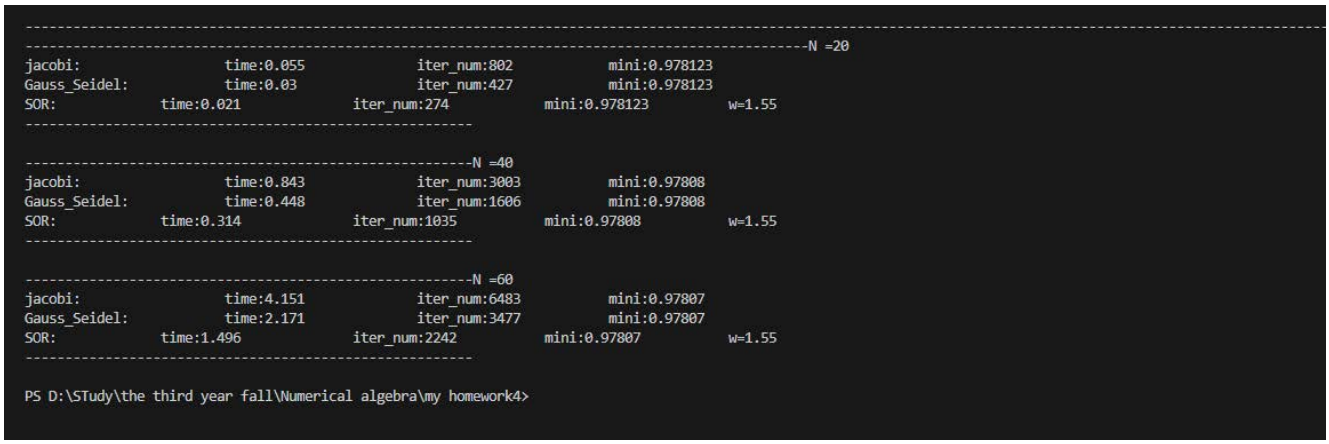


第二题实验结果：



四、结果分析

第一题：一般来说收敛速度上SOR>G-S>Jacobi，精确度上相差不大.

第二题：经过测验，w=1.55的时候，收敛速度相对比较快，可以看出速度上比G-S迭代要快出不少。而迭代次数貌似呈现N平方量级的增长。