

附加作业9

Problem (2022-1). Look for a *traveling wave solution* of the PDE

$$u_{tt} + (u^2)_{xx} = -u_{xxxx}$$

of the form $u(x, t) = v(x - ct)$. In particular, you should find an ODE for v . Under the assumption that v goes to a constant as $|x| \rightarrow \infty$, describe the form of the solution.

Problem (2022-2). Let $\Omega \in \mathbb{R}^n$. Let $u(x, t)$ be a smooth solution of the following initial boundary value problem:

$$\begin{aligned} u_{tt} - \Delta u + u^3 &= 0 & \text{for } (x, t) \in \Omega \times [0, T] \\ u(x, t) &= 0 & \text{for } (x, t) \in \partial\Omega \times [0, T]. \end{aligned}$$

a) Derive an *energy equality* for u . (Hint: Multiply by u_t and integrate over $\Omega \times [0, T]$.)

b) Show that if $u|_{t=0} = u_t|_{t=0} = 0$ for $x \in \Omega$, then $u \equiv 0$.