# Basic Probability Homework

## Problem 1: Rule of Complements

Suppose the probability of an event $A$ is given by $P(A) = 0.3$. Calculate the probability of the complement of event $A$, $P(A^c)$.

**Solution:**
Using the rule of complements, we have $P(A^c) = 1 - P(A)$. Thus, $P(A^c) = 1 - 0.3 = 0.7$.

## Problem 2: Definition of Conditional Probability

The probabilities of events $A$ and $B$ are given by $P(A) = 0.4$ and $P(B) = 0.5$, respectively. The probability of both events occurring is given by $P(A \text{ and } B) = 0.2$. Calculate the conditional probability $P(A|B)$.

**Solution:**
Using the definition of conditional probability, we have $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$. Thus, $P(A|B) = \frac{0.2}{0.5} = 0.4$.

## Problem 3: Law of Total Probability

A box contains three bags: Bag 1, Bag 2, and Bag 3. Bag 1 contains 2 red and 3 green balls, Bag 2 contains 4 red and 1 green balls, and Bag 3 contains 1 red and 4 green balls. You randomly choose a bag and then randomly choose a ball from that bag. Calculate the probability of drawing a red ball.

**Solution:**
Let $R$ be the event of drawing a red ball, and let $B_i$ be the event of choosing Bag $i$. Using the law of total probability, we have:

$$\begin{aligned}
P(R) &= P(R|B_1)P(B_1) + P(R|B_2)P(B_2) + P(R|B_3)P(B_3) \\
&= \frac{2}{5} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{1}{3} + \frac{1}{5} \cdot \frac{1}{3} \\
&= \frac{2}{15} + \frac{4}{15} + \frac{1}{15} \\
&= \frac{7}{15}.
\end{aligned}$$

## Problem 4: Independent Events

Given events $A$ and $B$, with $P(A) = 0.6$, $P(B) = 0.7$, and $P(A \text{ and } B) = 0.42$. Determine if events $A$ and $B$ are independent.

**Solution:**
Events $A$ and $B$ are independent if $P(A \text{ and } B) = P(A)P(B)$. We check this condition:
$P(A) \times P(B) = 0.6 \times 0.7 = 0.42$, which is the same as $P(A \text{ and } B)$, so A and B are independent.

## Problem 5: Conditional Probability

Let $P(A) = 0.3$, $P(B) = 0.4$, and $P(A \text{ and } B) = 0.1$. Calculate the conditional probability $P(A^c|B)$.

**Solution:**

We first find $P(A^c \text{ and } B)$:
$P(A^c \text{ and } B) = P(B) - P(A \text{ and } B) = 0.4 - 0.1 = 0.3$
Now, we can find $P(A^c|B)$:
$P(A^c|B) = \frac{P(A^c \text{ and } B)}{P(B)} = \frac{0.3}{0.4} = 0.75$

## Problem 6: Law of Total Probability

A company has two warehouses (Warehouse 1 and Warehouse 2). Warehouse 1 has 60% of the total stock, while Warehouse 2 has 40% of the total stock. The probability of a damaged item in Warehouse 1 is 2%, and in Warehouse 2 it is 5%. Calculate the probability of choosing a damaged item from the company's stock.

**Solution:**

Let $D$ be the event of choosing a damaged item, and let $W_i$ be the event of choosing from Warehouse $i$. Using the law of total probability, we have:

$$\begin{aligned} P(D) &= P(D|W_1)P(W_1) + P(D|W_2)P(W_2) \\ &= 0.02 \times 0.6 + 0.05 \times 0.4 \\ &= 0.012 + 0.02 \\ &= 0.032. \end{aligned}$$

## Problem 7: Conditional Probability and Independence

Given events $A$ and $B$, with $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \text{ and } B) = 0.3$. Calculate the conditional probability $P(B|A)$ and determine if events $A$ and $B$ are independent.

**Solution:**

First, we find $P(B|A)$:
$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.3}{0.5} = 0.6$
To check for independence, we compare $P(B)$ and $P(B|A)$:
$P(B) = P(B|A) \implies 0.6 = 0.6$
Since the equality holds, events $A$ and $B$ are independent.