Assignment No 4

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1 Overview

This assignment is to fit two functions e^x and cos(cos(x)) over the interval $[0,2\pi)$ using the fourier series and find the coefficients of the fourier series.

We then have to find the best estimates of the coefficients using the **Least Square** method and also using **Matrix Product**. We then find the absolute difference and also the deviation of the coefficients and finally, plot the matrix product of the coefficients found from the least square method along with the true graph to note the differences.

2 Fourier Series

The fourier series expression is given by :

$$a_o + \sum_{n=1}^{\infty} a_n cos(nx) + b_n sin(nx)$$

where the coefficients satisfy the equations :

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx$$
$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)cos(nx)dx$$
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sin(nx)dx$$

3 Plots and Output

3.1 Functions

The two functions are defined in python using the `def f(x) :` code and the input is sent to this function either as a vector or a scalar to compute the values. They are then plotted as follows :

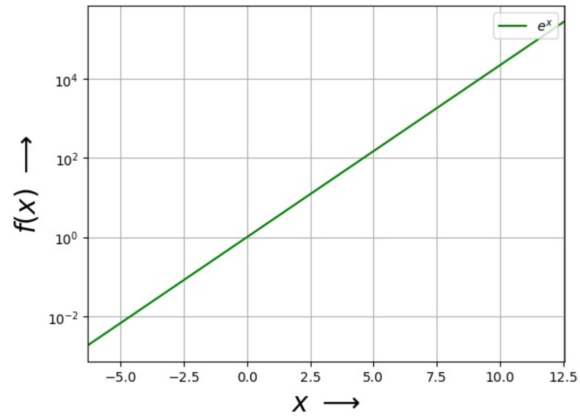

Figure 1: Functions plotted using semilogy

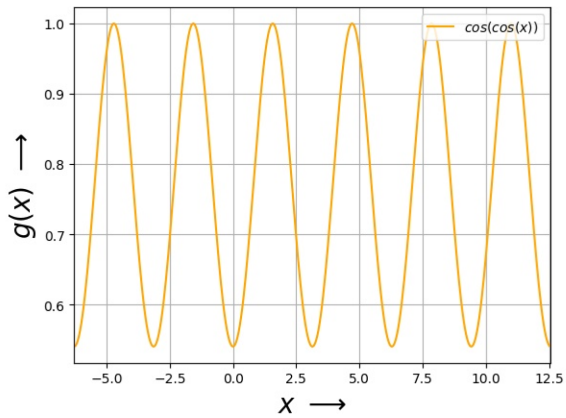

Figure 2: Functions plotted using plot

It turns out that the first function is non-periodic whereas the second one is.

3.2 Coefficients of the series using *quad* and their plots

We now find the coefficients of the fourier series for the two functions by using the quad function, which integrates a function of double variables, given by the equation :

$$\text{rtnval} = \text{quad(u,0,2*pi,args=(k))}$$

This integrates the product of two functions defined in the above equations for coefficients and returns the coefficients.

The coefficients are then listed into a vector which is then plotted against n, n being the subscript value. Plots for the first 25 coefficients for both functions are as follows :

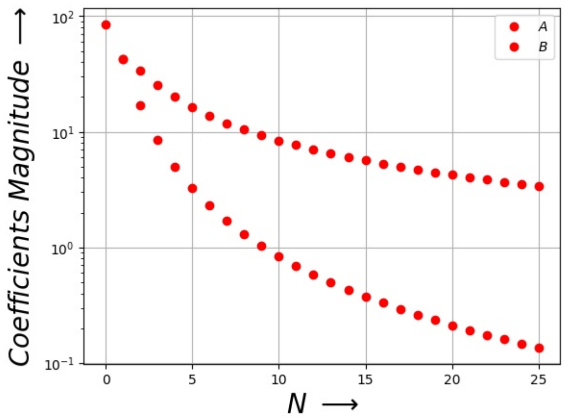

Figure 3: Coefficients of e^x using semilogy

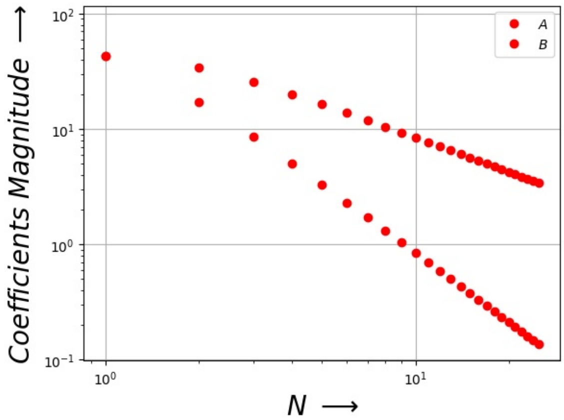

Figure 4: Coefficients of e^x using loglog

- The magnitude for b_n is nearly zero since the function is even function and the integral vanishes in theory.
- For the first equation, higher order fourier terms become irrelevant but the lower order terms determine the equation whereas for the second one, only some terms hugely determine the shape of the function for n close to 10 and other such values and that too being a_n, b_n being zero.

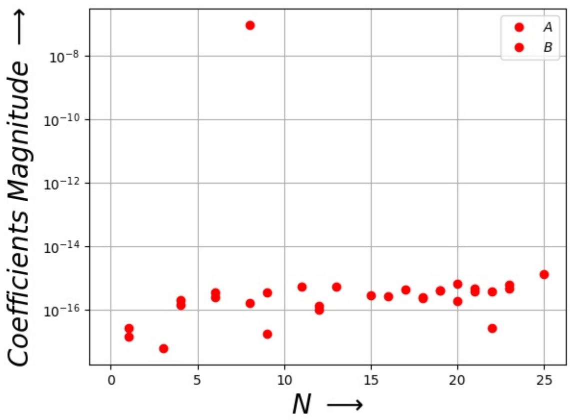

Figure 5: Coefficients of cos(cos(x)) using semilogy

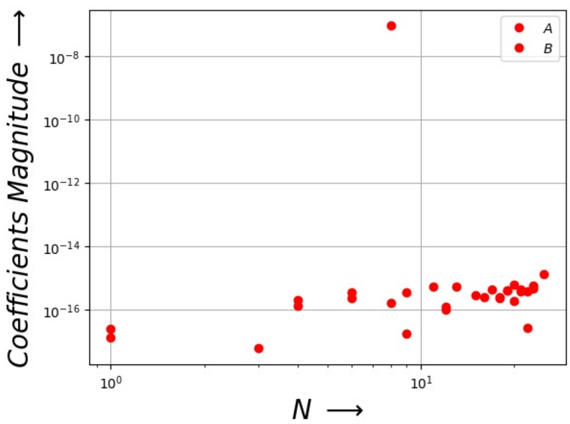

Figure 6: Coefficients of cos(cos(x)) using loglog

- For computing the a_n values, we convert the cos(nx) term to e^{jnx} term, hence having an exponential relation with x. And so, the semilog plot for the second function is linear. But for the first function, since both the terms have a $\frac{1}{n^2}$ relation, loglog plot for the first function is linear.

3.3 Finding estimates of coefficients using *lstsq*

We find the best estimates for the coefficients by converting it to a matrix form and using the *Least Squares* method to find their values.

We define x using `linspace` with 400 steps from 0 to 2π. Using that, we compute the various entries for the matrix and the column vector f(x), and find the values of the coefficients.

The matrix equation is given as :

$$\begin{pmatrix} 1 & cosx_1 & sinx_1 & \cdots & cos25x_1 & sin25x_1 \\ 1 & cosx_2 & sinx_2 & \cdots & cos25x_2 & sin25x_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & cosx_{400} & sinx_{400} & \cdots & cos25x_{400} & sin25x_{400} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ b_1 \\ \cdots \\ a_{25} \\ b_{25} \end{pmatrix} = \begin{pmatrix} f(x_1) \\ f(x_2) \\ \cdots \\ f(x_{400}) \end{pmatrix}$$

We then use the **lstsq** function to find the best fit numbers that will satisfy the matrix equation. We then convert these estimated coefficients to a vector and plot them against N.

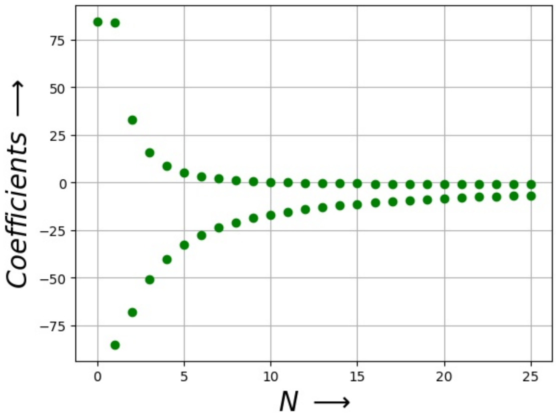

Figure 7: Coefficients of e^x using lstsq

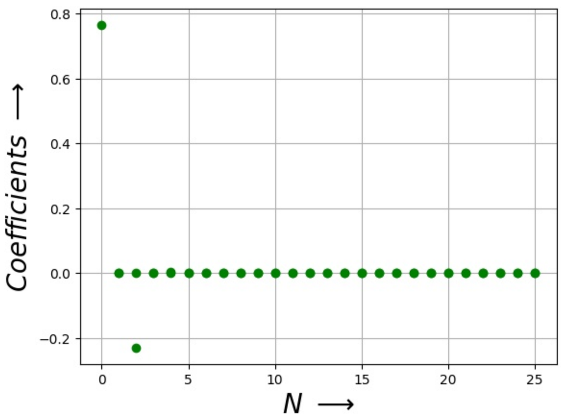

Figure 8: Coefficients of cos(cos(x)) using lstsq

3.4 Comparing the calculated coefficients based on two different operations

The coefficients for the fourier series are obtained by direct integration and lstsq methods. These are then compared by taking the absolute difference and the largest deviation in the difference vectors as well as their standard deviations are calculated and printed.

The output is given as :
`The largest deviations are 85.063491 and 0.114903`
`The standard deviations are given by 25.956963 and 0.015928`

The above values are such based on the current computation of **lstsq**. They may change if the calculation is performed differently again.

3.5 Comparing plots of the functions with the estimated coefficients

The coefficients calculated with lstsq method is then matrix multiplied with the square matrix to get the estimated values of the two functions. These are then plotted on the same plots respectively.

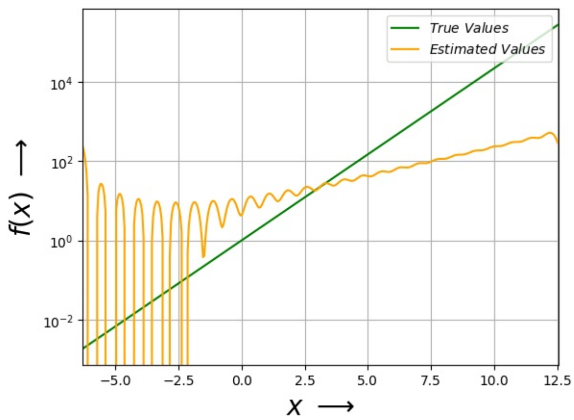

Figure 9: Coefficients of cos(cos(x)) using lstsq

The large deviation in the first function is due to the fact that e^x is non-periodic whereas the second function cos(cos(x)) is periodic. This implies that the first function estimated using fourier series is incorrect since this estimated function is supposed to be periodic but the actual function isn't.

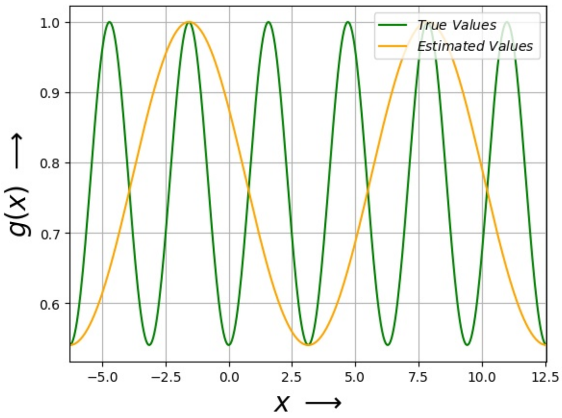

Figure 10: Coefficients of cos(cos(x)) using lstsq

4 Conclusion

The python code is too large for it to be present here and could be referred to in the folder.