Assignment 4: Word Embeddings

Welcome to the fourth (and last) programming assignment of Course 2!

In this assignment, you will practice how to compute word embeddings and use them for sentiment analysis.

- To implement sentiment analysis, you can go beyond counting the number of positive words and negative words.
- You can find a way to represent each word numerically, by a vector.
- The vector could then represent syntactic (i.e. parts of speech) and semantic (i.e. meaning) structures.

In this assignment, you will explore a classic way of generating word embeddings or representations.

- You will implement a famous model called the continuous bag of words (CBOW) model.

By completing this assignment you will:

- Train word vectors from scratch.
- Learn how to create batches of data.
- Understand how backpropagation works.
- Plot and visualize your learned word vectors.

Knowing how to train these models will give you a better understanding of word vectors, which are building blocks to many applications in natural language processing.

Outline

1. The Continuous bag of words model

Let's take a look at the following sentence:

> **'I am happy because I am learning'**.

- In continuous bag of words (CBOW) modeling, we try to predict the center word given a few context words (the words around the center word).
- For example, if you were to choose a context half-size of say $C = 2$, then you would try to predict the word **happy** given the context that includes 2 words before and 2 words after the center word:

> C words before: [I, am]
>
> C words after: [because, I]

- In other words:

$$context = [I, am, because, I]$$
$$target = happy$$

The structure of your model will look like this:

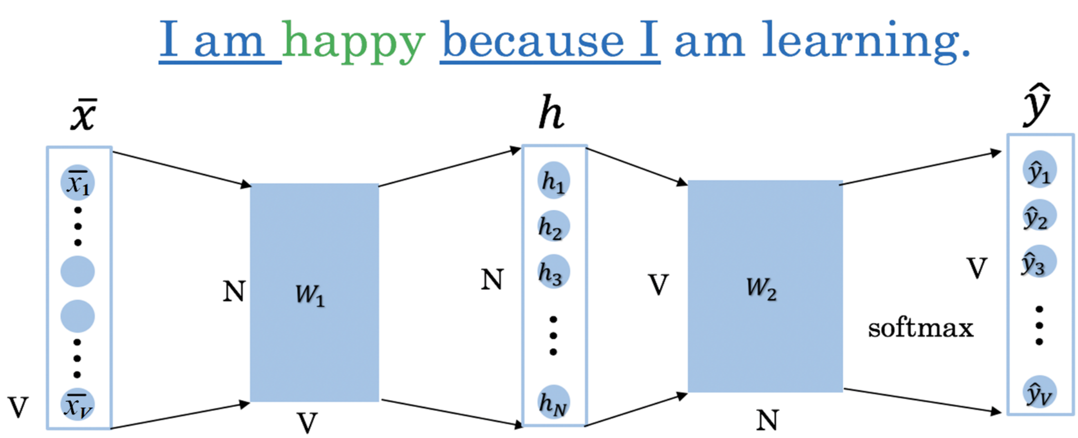
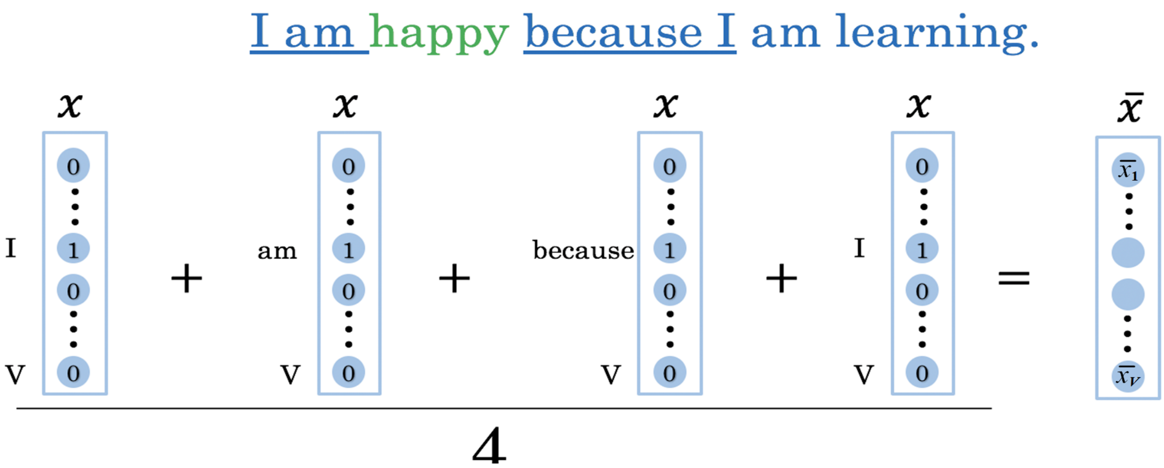


Figure 1

Where $\bar{x}$ is the average of all the one hot vectors of the context words.



In [18]:
```
# Import Python libraries and helper functions (in utils2)
import nltk
from nltk.tokenize import word_tokenize
import numpy as np
from collections import Counter
from utils2 import sigmoid, get_batches, compute_pca, get_dict
```

In [19]:
```
# Download sentence tokenizer
nltk.data.path.append('.')
```

In [20]:
```
# Load, tokenize and process the data
import re                                                              #  Load th
e Regex-modul
with open('shakespeare.txt') as f:
    data = f.read()                                                    #  Read in
the data
data = re.sub(r'[,!?;-]', '.',data)                                    #  Punktua
tions are replaced by .
data = nltk.word_tokenize(data)                                        #  Tokeniz
e string to words
data = [ ch.lower() for ch in data if ch.isalpha() or ch == '.']       #  Lower c
ase and drop non-alphabetical tokens
print("Number of tokens:", len(data),'\n', data[:15])                  #  print d
ata sample
```

```
Number of tokens: 60996
 ['o', 'for', 'a', 'muse', 'of', 'fire', '.', 'that', 'would', 'ascend', 'th
e', 'brightest', 'heaven', 'of', 'invention']
```

In [21]:
```
# Compute the frequency distribution of the words in the dataset (vocabulary)
fdist = nltk.FreqDist(word for word in data)
print("Size of vocabulary: ",len(fdist) )
print("Most frequent tokens: ",fdist.most_common(20) ) # print the 20 most fre
quent words and their freq.
```

```
Size of vocabulary:  5778
Most frequent tokens:  [('.', 9630), ('the', 1521), ('and', 1394), ('i', 125
7), ('to', 1159), ('of', 1093), ('my', 857), ('that', 781), ('in', 770),
('a', 752), ('you', 748), ('is', 630), ('not', 559), ('for', 467), ('it', 46
0), ('with', 441), ('his', 434), ('but', 417), ('me', 417), ('your', 397)]
```

Mapping words to indices and indices to words

We provide a helper function to create a dictionary that maps words to indices and indices to words.

In [22]:
```
# get_dict creates two dictionaries, converting words to indices and viceversa.
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
print("Size of vocabulary: ", V)
```

```
Size of vocabulary:  5778
```

In [23]:
```
# example of word to index mapping
print("Index of the word 'king' :  ",word2Ind['king'] )
print("Word which has index 2743:  ",Ind2word[2743] )
```

```
Index of the word 'king' :   2745
Word which has index 2743:   kindness
```

2 Training the Model

Initializing the model

You will now initialize two matrices and two vectors.

- The first matrix (W_1) is of dimension $N \times V$, where V is the number of words in your vocabulary and N is the dimension of your word vector.
- The second matrix (W_2) is of dimension $V \times N$.
- Vector b_1 has dimensions $N \times 1$
- Vector b_2 has dimensions $V \times 1$.
- b_1 and b_2 are the bias vectors of the linear layers from matrices W_1 and W_2.

The overall structure of the model will look as in Figure 1, but at this stage we are just initializing the parameters.

Exercise 01

Please use [numpy.random.rand (https://numpy.org/doc/stable/reference/random/generated/numpy.random.rand.html)](https://numpy.org/doc/stable/reference/random/generated/numpy.random.rand.html) to generate matrices that are initialized with random values from a uniform distribution, ranging between 0 and 1.

Note: In the next cell you will encounter a random seed. Please **DO NOT** modify this seed so your solution can be tested correctly.

In [24]:
```
# UNQ_C1 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
# GRADED FUNCTION: initialize_model
def initialize_model(N,V, random_seed=1):
    '''
    Inputs:
        N:  dimension of hidden vector
        V:  dimension of vocabulary
        random_seed: random seed for consistent results in the unit tests
     Outputs:
        W1, W2, b1, b2: initialized weights and biases
    '''

    np.random.seed(random_seed)

    ### START CODE HERE (Replace instances of 'None' with your code) ###
    # W1 has shape (N,V)
    W1 = np.random.rand(N,V)
    # W2 has shape (V,N)
    W2 = np.random.rand(V,N)
    # b1 has shape (N,1)
    b1 = np.random.rand(N,1)
    # b2 has shape (V,1)
    b2 = np.random.rand(V,1)
    ### END CODE HERE ###

    return W1, W2, b1, b2
```

In [25]:
```
# Test your function example.
tmp_N = 4
tmp_V = 10
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)
assert tmp_W1.shape == ((tmp_N,tmp_V))
assert tmp_W2.shape == ((tmp_V,tmp_N))
print(f"tmp_W1.shape: {tmp_W1.shape}")
print(f"tmp_W2.shape: {tmp_W2.shape}")
print(f"tmp_b1.shape: {tmp_b1.shape}")
print(f"tmp_b2.shape: {tmp_b2.shape}")
```

```
tmp_W1.shape: (4, 10)
tmp_W2.shape: (10, 4)
tmp_b1.shape: (4, 1)
tmp_b2.shape: (10, 1)
```

Expected Output

```
tmp_W1.shape: (4, 10)
tmp_W2.shape: (10, 4)
tmp_b1.shape: (4, 1)
tmp_b2.shape: (10, 1)
```

2.1 Softmax

Before we can start training the model, we need to implement the softmax function as defined in equation 5:

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{i=0}^{V-1} e^{z_i}} \tag{5}$$

- Array indexing in code starts at 0.
- V is the number of words in the vocabulary (which is also the number of rows of z).
- i goes from 0 to |V| - 1.

Exercise 02

Instructions: Implement the softmax function below.

- Assume that the input z to `softmax` is a 2D array
- Each training example is represented by a column of shape (V, 1) in this 2D array.
- There may be more than one column, in the 2D array, because you can put in a batch of examples to increase efficiency. Let's call the batch size lowercase m, so the z array has shape (V, m)
- When taking the sum from $i = 1 \cdots V - 1$, take the sum for each column (each example) separately.

Please use

- numpy.exp
- numpy.sum (set the axis so that you take the sum of each column in z)

In [26]:
```
# UNQ_C2 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
# GRADED FUNCTION: softmax
def softmax(z):
    '''
    Inputs:
        z: output scores from the hidden layer
    Outputs:
        yhat: prediction (estimate of y)
    '''

    ### START CODE HERE (Replace instances of 'None' with your own code) ###

    # Calculate yhat (softmax)
    e_z = np.exp(z)
    yhat = e_z/np.sum(e_z,axis=0)
    ### END CODE HERE ###

    return yhat
```

In [27]:
```
# Test the function
tmp = np.array([[1,2,3],
                [1,1,1]
               ])
tmp_sm = softmax(tmp)
display(tmp_sm)
```

```
array([[0.5       , 0.73105858, 0.88079708],
       [0.5       , 0.26894142, 0.11920292]])
```

Expected Ouput

```
array([[0.5       , 0.73105858, 0.88079708],
       [0.5       , 0.26894142, 0.11920292]])
```

2.2 Forward propagation

Exercise 03

Implement the forward propagation z according to equations (1) to (3).

$$h = W_1 \; X + b_1 \tag{1}$$
$$a = ReLU(h) \tag{2}$$
$$z = W_2 \; a + b_2 \tag{3}$$

For that, you will use as activation the Rectified Linear Unit (ReLU) given by:

$$f(h) = \max(0, h) \tag{6}$$

Hints

In [28]:
```
# UNQ_C3 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
# GRADED FUNCTION: forward_prop
def forward_prop(x, W1, W2, b1, b2):
    '''
    Inputs:
        x:  average one hot vector for the context
        W1, W2, b1, b2:  matrices and biases to be learned
     Outputs:
        z:  output score vector
    '''

    ### START CODE HERE (Replace instances of 'None' with your own code) ###

    # Calculate h
    h = np.dot(W1,x)+b1

    # Apply the relu on h (store result in h)
    h = np.maximum(0,h)

    # Calculate z
    z = np.dot(W2,h)+b2

    ### END CODE HERE ###

    return z, h
```

In [29]:
```
# Test the function

# Create some inputs
tmp_N = 2
tmp_V = 3
tmp_x = np.array([[0,1,0]]).T
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(N=tmp_N,V=tmp_V, random_seed
=1)

print(f"x has shape {tmp_x.shape}")
print(f"N is {tmp_N} and vocabulary size V is {tmp_V}")

# call function
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print("call forward_prop")
print()
# Look at output
print(f"z has shape {tmp_z.shape}")
print("z has values:")
print(tmp_z)

print()

print(f"h has shape {tmp_h.shape}")
print("h has values:")
print(tmp_h)
```

```
x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

z has shape (3, 1)
z has values:
[[0.55379268]
 [1.58960774]
 [1.50722933]]

h has shape (2, 1)
h has values:
[[0.92477674]
 [1.02487333]]
```

Expected output

```
x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

z has shape (3, 1)
z has values:
[[0.55379268]
 [1.58960774]
 [1.50722933]]

h has shape (2, 1)
h has values:
[[0.92477674]
 [1.02487333]]
```

2.3 Cost function

- We have implemented the *cross-entropy* cost function for you.

In [30]:
```
# compute_cost: cross-entropy cost functioN
def compute_cost(y, yhat, batch_size):
    # cost function
    logprobs = np.multiply(np.log(yhat),y) + np.multiply(np.log(1 - yhat), 1 - y)
    cost = - 1/batch_size * np.sum(logprobs)
    cost = np.squeeze(cost)
    return cost
```

In [31]:
```
# Test the function
tmp_C = 2
tmp_N = 50
tmp_batch_size = 4
tmp_word2Ind, tmp_Ind2word = get_dict(data)
tmp_V = len(word2Ind)

tmp_x, tmp_y = next(get_batches(data, tmp_word2Ind, tmp_V,tmp_C, tmp_batch_siz
e))

print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")

tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)

print(f"tmp_W1.shape {tmp_W1.shape}")
print(f"tmp_W2.shape {tmp_W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")
print(f"tmp_b2.shape {tmp_b2.shape}")

tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp_h.shape: {tmp_h.shape}")

tmp_yhat = softmax(tmp_z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")

tmp_cost = compute_cost(tmp_y, tmp_yhat, tmp_batch_size)
print("call compute_cost")
print(f"tmp_cost {tmp_cost:.4f}")
```

```
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)
tmp_yhat.shape: (5778, 4)
call compute_cost
tmp_cost 9.9560
```

Expected output

```
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)
tmp_yhat.shape: (5778, 4)
call compute_cost
tmp_cost 8.9542
```

2.4 Training the Model - Backpropagation

Exercise 04

Now that you have understood how the CBOW model works, you will train it.
You created a function for the forward propagation. Now you will implement a function that computes the gradients to backpropagate the errors.

In [32]:

```
# UNQ_C4 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
# GRADED FUNCTION: back_prop
def back_prop(x, yhat, y, h, W1, W2, b1, b2, batch_size):
    '''
    Inputs:
        x:  average one hot vector for the context
        yhat: prediction (estimate of y)
        y:  target vector
        h:  hidden vector (see eq. 1)
        W1, W2, b1, b2:  matrices and biases
        batch_size: batch size
     Outputs:
        grad_W1, grad_W2, grad_b1, grad_b2:  gradients of matrices and biases
    '''
    ### START CODE HERE (Replace instanes of 'None' with your code) ###

    # Compute l1 as W2^T (Yhat - Y)
    # Re-use it whenever you see W2^T (Yhat - Y) used to compute a gradient
    l1 = np.dot(W2.T,(yhat-y))
    # Apply relu to l1
    l1 = np.maximum(0,l1)
    # Compute the gradient of W1
    grad_W1 = (1/batch_size)*np.dot(l1,x.T)    #1/m * relu(w2.T(yhat-y)) . xT
    # Compute the gradient of W2
    grad_W2 = (1/batch_size)*np.dot(yhat-y,h.T)
    # Compute the gradient of b1
    grad_b1 = np.sum((1/batch_size)*np.dot(l1,x.T),axis=1,keepdims=True)
    # Compute the gradient of b2
    grad_b2 = np.sum((1/batch_size)*np.dot(yhat-y,h.T),axis=1,keepdims=True)
    ### END CODE HERE ###

    return grad_W1, grad_W2, grad_b1, grad_b2
```

In [33]:

```
# Test the function
tmp_C = 2
tmp_N = 50
tmp_batch_size = 4
tmp_word2Ind, tmp_Ind2word = get_dict(data)
tmp_V = len(word2Ind)

# get a batch of data
tmp_x, tmp_y = next(get_batches(data, tmp_word2Ind, tmp_V,tmp_C, tmp_batch_siz
e))

print("get a batch of data")
print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")

print()
print("Initialize weights and biases")
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)

print(f"tmp_W1.shape {tmp_W1.shape}")
print(f"tmp_W2.shape {tmp_W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")
print(f"tmp_b2.shape {tmp_b2.shape}")

print()
print("Forwad prop to get z and h")
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp_h.shape: {tmp_h.shape}")

print()
print("Get yhat by calling softmax")
tmp_yhat = softmax(tmp_z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")

tmp_m = (2*tmp_C)
tmp_grad_W1, tmp_grad_W2, tmp_grad_b1, tmp_grad_b2 = back_prop(tmp_x, tmp_yhat
, tmp_y, tmp_h, tmp_W1, tmp_W2, tmp_b1, tmp_b2, tmp_batch_size)

print()
print("call back_prop")
print(f"tmp_grad_W1.shape {tmp_grad_W1.shape}")
print(f"tmp_grad_W2.shape {tmp_grad_W2.shape}")
print(f"tmp_grad_b1.shape {tmp_grad_b1.shape}")
print(f"tmp_grad_b2.shape {tmp_grad_b2.shape}")
```

```
get a batch of data
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)

Initialize weights and biases
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)

Forwad prop to get z and h
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)

Get yhat by calling softmax
tmp_yhat.shape: (5778, 4)

call back_prop
tmp_grad_W1.shape (50, 5778)
tmp_grad_W2.shape (5778, 50)
tmp_grad_b1.shape (50, 1)
tmp_grad_b2.shape (5778, 1)
```

Expected output

```
get a batch of data
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)

Initialize weights and biases
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)

Forwad prop to get z and h
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)

Get yhat by calling softmax
tmp_yhat.shape: (5778, 4)

call back_prop
tmp_grad_W1.shape (50, 5778)
tmp_grad_W2.shape (5778, 50)
tmp_grad_b1.shape (50, 1)
tmp_grad_b2.shape (5778, 1)
```

Gradient Descent

Exercise 05

Now that you have implemented a function to compute the gradients, you will implement batch gradient descent over your training set.

Hint: For that, you will use `initialize_model` and the `back_prop` functions which you just created (and the `compute_cost` function). You can also use the provided `get_batches` helper function:

`for x, y in get_batches(data, word2Ind, V, C, batch_size):`

`...`

Also: print the cost after each batch is processed (use batch size = 128)

In [34]:
```
# UNQ_C5 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
# GRADED FUNCTION: gradient_descent
def gradient_descent(data, word2Ind, N, V, num_iters, alpha=0.03):

    '''
    This is the gradient_descent function

      Inputs:
        data:      text
        word2Ind:  words to Indices
        N:         dimension of hidden vector
        V:         dimension of vocabulary
        num_iters: number of iterations
     Outputs:
        W1, W2, b1, b2:  updated matrices and biases

    '''
    W1, W2, b1, b2 = initialize_model(N,V, random_seed=282)
    batch_size = 128
    iters = 0
    C = 2
    for x, y in get_batches(data, word2Ind, V, C, batch_size):
        ### START CODE HERE (Replace instances of 'None' with your own code) #
##
        # Get z and h
        z, h = forward_prop(x, W1, W2, b1, b2)
        # Get yhat
        yhat = softmax(z)
        # Get cost
        cost = compute_cost(y, yhat, batch_size)
        if ( (iters+1) % 10 == 0):
            print(f"iters: {iters + 1} cost: {cost:.6f}")
        # Get gradients
        grad_W1, grad_W2, grad_b1, grad_b2 = back_prop(x, yhat, y, h, W1, W2,
b1, b2, batch_size)

        # Update weights and biases
        W1 -= alpha*grad_W1
        W2 -= alpha*grad_W2
        b1 -= alpha*grad_b1
        b2 -= alpha*grad_b2

        ### END CODE HERE ###

        iters += 1
        if iters == num_iters:
            break
        if iters % 100 == 0:
            alpha *= 0.66

    return W1, W2, b1, b2
```

In [35]:
```
# test your function
C = 2
N = 50
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
num_iters = 150
print("Call gradient_descent")
W1, W2, b1, b2 = gradient_descent(data, word2Ind, N, V, num_iters)
```

```
Call gradient_descent
iters: 10 cost: 0.107177
iters: 20 cost: 0.038644
iters: 30 cost: 0.023685
iters: 40 cost: 0.017095
iters: 50 cost: 0.013380
iters: 60 cost: 0.010995
iters: 70 cost: 0.009333
iters: 80 cost: 0.008108
iters: 90 cost: 0.007168
iters: 100 cost: 0.006423
iters: 110 cost: 0.005992
iters: 120 cost: 0.005633
iters: 130 cost: 0.005314
iters: 140 cost: 0.005030
iters: 150 cost: 0.004774
```

Expected Output

```
iters: 10 cost: 0.435713
iters: 20 cost: 0.053471
iters: 30 cost: 0.028200
iters: 40 cost: 0.019141
iters: 50 cost: 0.014486
iters: 60 cost: 0.011652
iters: 70 cost: 0.009746
iters: 80 cost: 0.008376
iters: 90 cost: 0.007343
iters: 100 cost: 0.006537
iters: 110 cost: 0.006076
iters: 120 cost: 0.005693
iters: 130 cost: 0.005356
iters: 140 cost: 0.005057
iters: 150 cost: 0.004789
```

Your numbers may differ a bit depending on which version of Python you're using.

3.0 Visualizing the word vectors

In this part you will visualize the word vectors trained using the function you just coded above.

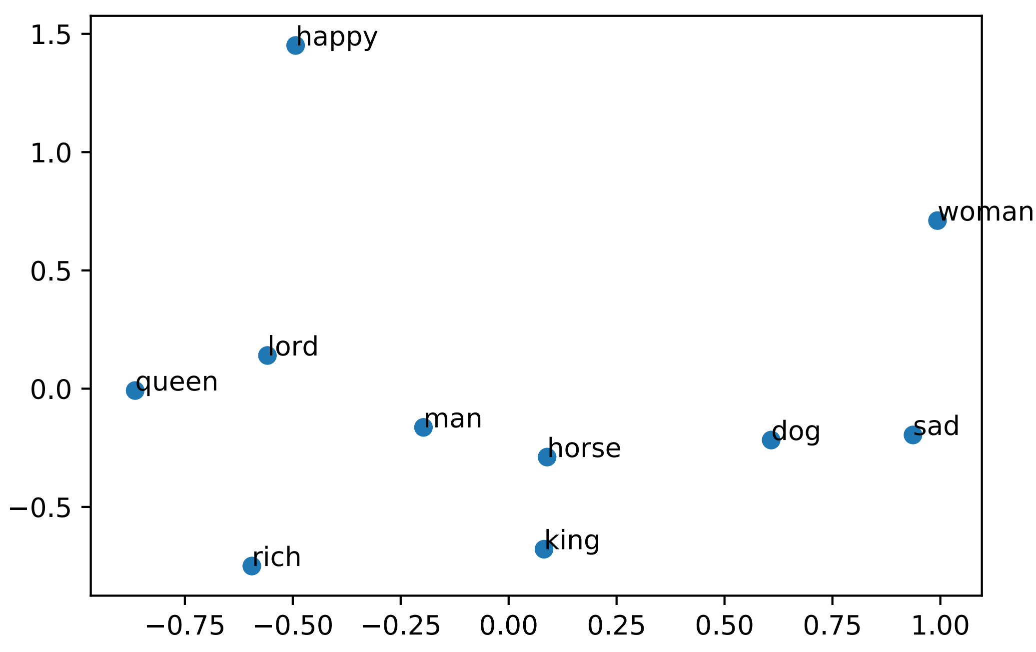
In [36]:
```
# visualizing the word vectors here
from matplotlib import pyplot
%config InlineBackend.figure_format = 'svg'
words = ['king', 'queen','lord','man', 'woman','dog','horse',
         'rich','happy','sad']

embs = (W1.T + W2)/2.0

# given a list of words and the embeddings, it returns a matrix with all the e
mbeddings
idx = [word2Ind[word] for word in words]
X = embs[idx, :]
print(X.shape, idx)  # X.shape:  Number of words of dimension N each
```

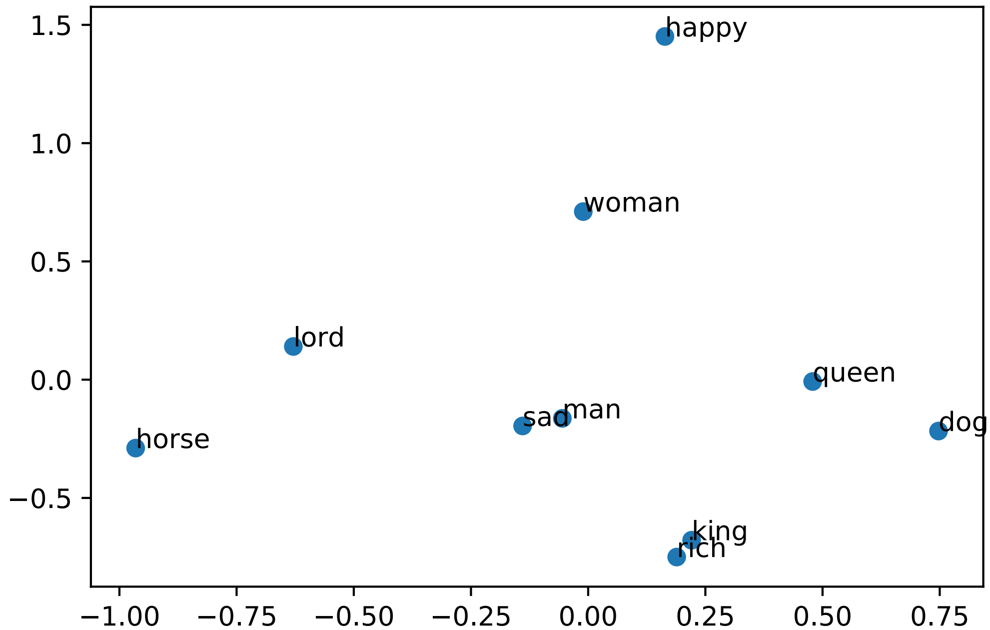
```
(10, 50) [2745, 3951, 2961, 3023, 5675, 1452, 2472, 4191, 2316, 4278]
```

In [37]:
```
result= compute_pca(X, 2)
pyplot.scatter(result[:, 0], result[:, 1])
for i, word in enumerate(words):
    pyplot.annotate(word, xy=(result[i, 0], result[i, 1]))
pyplot.show()
```



You can see that man and king are next to each other. However, we have to be careful with the interpretation of this projected word vectors, since the PCA depends on the projection -- as shown in the following illustration.

In [38]:
```
result= compute_pca(X, 4)
pyplot.scatter(result[:, 3], result[:, 1])
for i, word in enumerate(words):
    pyplot.annotate(word, xy=(result[i, 3], result[i, 1]))
pyplot.show()
```



In [ ]: