

Lecture 17 – Query Processing, Continued

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The next thing we would like to do is expand to more advanced queries, specifically, join queries.

Join queries will be cripplingly inefficient if they operate on unsorted data.

Remember that in a join, we need to match a tuple of the left hand relation with a tuple of the right hand relation.

If for every tuple we had to linearly search the right hand relation, that would be painful.



One way or another, we have to sort one of the relations to make this work.

Sorting relations that do not fit into memory is called external sorting, and we will learn the external sort-merge algorithm.

Divide the data into smaller units, sort the smaller units, then merge the smaller sorted units into a larger sorted unit.

The sorting of each smaller unit will take place in memory and then we need to do an N -way merge where N is the number of smaller units to be merged.

Step one of the algorithm is to divide the file into N chunks of size M where M is the number of blocks that can fit into the area of memory available for sorting.

Each chunk is as big as it can be for the constraints of the system, but no bigger.

This chunk is called a **run**.

Each run i is then sorted and it is written to a temporary file called R_i .

Step two of the algorithm now merges it.

If N is less than $M - 1$ we have the simpler case and we can complete the merge in one pass.

In that case we load the first block of R_i for each i from 0 to $N - 1$ into memory and we allocate an output block.

Then we choose the first tuple from the all of the blocks, and move it into the output block.

If a block R_i becomes empty, replace it with the next block in that run (if any).

If the output block becomes full, write it out and allocate a new output block.

Continue this algorithm until all runs are empty.

We're gonna need a bigger boat...

If N is large enough that we can't do it all in one pass, we need multiple passes.



We will combine the first $M - 1$ runs into a temporary file, and then the next (up to) $M - 1$ runs, and so on, until the last run has been processed.

Repeat the process with the larger runs as input until we produce the sorted file.

Suppose the file consists of 10 000 blocks and we can fit 50 blocks in memory to do the sort.

We can therefore create runs of $10\,000/50 = 200$ blocks each.

Each run is sorted.

Then we come to merge them.

External Sort-Merge Example

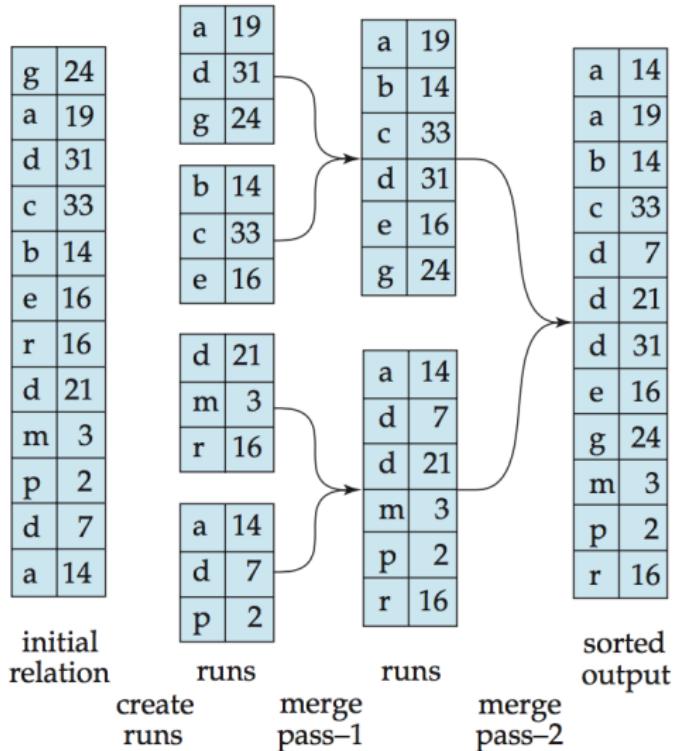
We cannot fit a block from each of the 200 runs inside the 49 available ($50 - 1$ block for output) so we must do multiple passes.

The first pass sorts runs 1 through 49 into a new run (let's call it R'_1), then runs 50 - 98 into R'_2 , et cetera, until the last one which is then 197-200 in R'_5 .

These larger R' runs are then combined using the same merge procedure as before.

Since there are only 5 we are sure the merge will complete in this second pass and we have the output file we wanted.

External Sort-Merge



Suppose the number of blocks in the relation r is b_r .

The first step of the algorithm requires us to read in each block of the relation and write it out again for a cost of $2b_r$.

The number of runs decreases by $M - 1$ in each merge pass, so the number of merge passes is $\lceil \log_{M-1}(b_r/M) \rceil$.

Each pass reads every block of the relation and writes it once, except the last pass doesn't need to write the last output to disk.

So the total number of block transfers is $b_r(2\lceil \log_{M-1}(b_r/M) \rceil + 1)$.

If each run has b_b blocks, each merge pass requires $\lceil b_r/b_b \rceil$ seeks to read data.

And we need to then write the output which costs $2\lceil b_r/b_b \rceil$ for every merge pass, except the last, since it gets sent on.

So the total number of seeks is $2\lceil b_r/M \rceil + \lceil b_r/b_b \rceil(2\lceil \log_{M-1}(b_r/M) \rceil - 1)$.

We get a total of 44 seeks: computed as $8 + 12 \times (2 \times 2 - 1)$ seeks if $b_b = 1$.

There are several ways to compute a join, and we will analyze the costs of a few of the different approaches.



Whatever route is actually taken, it's important to note that joins can be very expensive depending on how this is to be computed.

The join we are looking to examine is the one introduced in relational algebra as \bowtie_θ , specifically $r_1 \bowtie_{r_1.a=r_2.b} r_2$.

Or in SQL, an `INNER JOIN` with an explicit `ON` predicate.

Join Strategy 1: Nested-Loop

The first option we are going to look at is basically a linear search:

The algorithm described in pseudocode is:

1. for each tuple i in r1
2. for each tuple j in r2
3. if i.a equals j.b
4. add (i join j) to the result
5. end if
6. end for
7. end for

Join Strategy 1: Nested-Loop

We expect that the number of tuples that needs to be examined is the number of tuples in r_1 (let's call it n_1) multiplied by the number of tuples in r_2 (n_2).

In the best case scenario, both relations fit in memory at the same time.

The cost is just two seeks plus a transfer for each of the blocks of r_1 (which we will call b_1) and a transfer for each of the blocks of r_2 (b_2).

In the worst case we get only one block of each relation at a time.

The inner loop then is one seek plus b_2 transfers to read it into memory.

The outer loop requires one seek and one transfer for each block of r_1 , plus one run of the inner loop for each tuple in b_1 .

So the total amount of seeks is $b_1 + n_1$ and the number of blocks to be transferred is $b_1 + n_1 \times b_2$.

To illustrate with an example the difference between the best and worst case scenario, let us assign some numbers to n_1 , b_1 and b_2 .

Suppose n_1 is 1600, b_1 is 100 blocks, n_2 is 10000, and b_2 is 500.

Sticking with values used earlier, a seek takes 4 ms and a transfer 0.1 ms.

In the best case scenario, the cost is 2 seeks plus a transfer for each block, or:

$$2 \times t_s + (b_1 + b_2) \times t_T = 2 \times 4 + (100 + 500) \times 0.1 = 8 + 60 = 68 \text{ ms.}$$

In the worst case scenario, the number of seeks is $2b_1$ plus the number of transfers as $b_1 + n_1 \times b_2$, or:

$$b_1 \times t_s + (b_1 + n_1 \times b_2) \times t_T = 100 \times 4 + (100 + 1600 \times 500) \times 0.1 = 100 \times 4 + 800100 \times 0.1 = 400 + 80010 = 80410 \text{ ms.}$$

Yikes. That is a really huge difference. Conclusion: buy more RAM.

Assuming that b_1 is not equal to b_2 we can actually observe something interesting: it could be worse!

The so-called worst case scenario is actually the second-worst case scenario.

Imagine if the order of the relations is switched in the join loop).

Let's do the math again:

$$2 \times b_2 \times t_s + (b_2 + n_2 \times b_1 \times t_T) = 2 \times 500 \times 4 + (500 + 10000 \times 100) \times 0.1 = \\ 1000 \times 4 + 1000500 \times 0.1 = 4000 + 100050 = 104050 \text{ ms.}$$

That's about 1.29 times more just based on the choice of which relation is b_1 and which is b_2 .

Under this strategy it matters quite a lot which relation is the outer loop and which one is the inner loop.

It would be vastly preferable to make the outer loop the smaller of the two relations!

Join Strategy 2: Block Nested-Loop

The previous strategy could be improved by thinking in terms of block operations rather than in terms of individual tuples.

Instead of running the inner loop once for each tuple in the outer loop, we could run it once for each block in the other relation.

That might be a lot less painful in terms of count of disk reads.

Join Strategy 2: Block Nested-Loop

If everything fits in memory, then the block nested-loop strategy is really no improvement over the regular nested loop algorithm.

It gains an advantage if there will be more memory reads.

Instead of loading the inner blocks once for every tuple of the outer relation, they are loaded once per block of the outer relation.

Thus there are $b_1 \times b_2 + b_1$ transfers and $2 \times b_1$ seeks that take place.

Join Strategy 2: Block Nested-Loop

Now: $2 \times b_1 = 2 \times 100$ seeks (4 ms each) = $200 \times 4 = 800$ ms seek time, plus
 $b_1 \times b_2 + b_1 = 100 \times 500 + 100 = 50100$ (0.1 ms each) = 5010 ms.

Summed up, a total of 5810 ms to complete this.

Join Strategy 2: Block Nested-Loop

As before it could be worse if we swapped the order of the relations.

$2 \times b_2 = 2 \times 500$ seeks which at a cost of 4 ms = $1000 \times 4 = 4000$ ms seek time, plus $b_2 \times b_1 + b_2 = 500 \times 100 + 500 = 50500$ at a cost of 0.1 ms = 5050 ms.

The total is then 9050 ms which is 1.56 times the cost of the other ordering.

Join Strategy 3: Index Nested-Loop

Suppose an index is available on one of the two relations.

If so, then we don't have to do this in the most painful way; we can use the index instead.

Since we will iterate over the inner relation more times, we want the index to be on the inner relation, if we have only one index to work with.

Join Strategy 3: Index Nested-Loop

To find which tuples in the inner relation match the outer relation tuple $t_1.a$ then we do an index lookup on r_2 to find the tuple(s) that match.

The worst case scenario will once again be that we can only get one block of the relation into memory.

We will need to do one seek for each block of the outer relation b_1 and then one transfer for it.

We will also need to, once for each tuple in b_1 look into the relation r_2 .

The formula is $b_1(t_s + t_T) + n_1 \times c$ where c is the cost of doing a single selection in that table using an index (if available).

The merge join might be more properly called the *sort merge join* approach.

Suppose relations r_1 and r_2 have some set of common attributes $r_1 \cap r_2$ and we wish to compute the natural join.

If both relations are sorted on $r_1 \cap r_2$ then we can do a merge-sort-like algorithm to compute the join.

Join Strategy 4: Merge-Join

The algorithm has one pointer for each relation, initialized to the first tuple in that relation.

Then advance those pointers.

Then each tuple of r_1 with the same value of the join attributes is read into a temporary block.

Then the tuples of r_2 are read and processed as they go.

Join Strategy 4: Merge-Join

The temporary block does need to be big enough to contain all the tuples of r_1 for each value of the join attributes.

That is usually not a problem since we hope not too many tuples match that particular condition.

The fallback plan is then the block nested-loop strategy.

Once again, the best case scenario is the two seeks plus reading in each block of each relation.

In the worst case, we one seek for each block of the relation as well as a transfer for each block of each relation: $(b_1 + b_2)(t_s + t_T)$.

There is also an assumption that the relations in question are sorted on the attributes of the join.

Join Strategy 5: Hybrid Merge-Join

We can execute a variant of the merge-join approach on unsorted tuples if a secondary index exists on the join attribute(s).

Normally if we are accessing the second relation through an index of some sort, we get the tuples in sorted order (from the view of that index)...

But they could be anywhere in the disk blocks meaning that each time we go to the next tuple is potentially another disk seek and transfer.

Join Strategy 5: Hybrid Merge-Join

To save ourselves some work we can use the hybrid merge-join algorithm.

The basic idea here is to merge the sorted relation r_1 with the leaf entries of r_2 's secondary index.

The output is then made up of tuples of r_1 and pointers to the tuples of r_2 .

This file can then be sorted on the pointers to r_2 tuples so that accesses to r_2 go in physical storage order rather than jumping everywhere back and forth.

Another important strategy can be used for joining relations and it relies on hashing as we have already discussed.

However, the complexity of the hash join is high and we will consider it beyond the scope of this course.

Other Considerations in Query Processing

There are a few more situations beyond the select and join scenarios that we will care about when we have to actually execute a query.

- Distinct
- Projection
- Set Operations
- Outer Join
- Aggregation
- Order of Operations

Often times in performing a query we are asked to remove duplicates.

This is easy enough if we are sorting the data because two identical values will be next to each other for merging.

Alternatively, we could simply avoid writing them to the output at all if the sort-inserted routine finds something already in the space for that value.

This can be expensive, so this is generally done only if we really ask for it.

Projection could happen in several steps.

If employee ID is used in a where clause, for example, but not asked for in the output, first project down to the output fields plus those in the where clause.

Then later throw away some more when presenting the final results.

The same is true of an order-by clause: if that field is not needed in the output we will toss it out once the ordering is complete.

Set operations (union, intersection, difference) are fairly self explanatory.

Sorting the data is necessary, generally, to make this efficient.

If there is no index, we might need to build one.

Each relation r_x is partitioned into different parts which we will label $r_{x_0}, r_{x_1} \dots$

- 1 Build an in memory hash index on the $r_{1,i}$
- 2 Add the tuples $r_{2,i}$ to the index if they are not present
- 3 Add the tuples referenced in the index to the result.

Set Operations: Intersection

- 1 Build an in memory hash index on the $r_{1,i}$
- 2 For each of the tuples in $r_{2,i}$, see if it is already in the index, and if so, add it to the output.

- 1 Build an in memory hash index on the r_{1_i}
- 2 For each of the tuples in r_{2_i} , see if it is already in the index, delete it from the index.
- 3 Add the tuples referenced in the index to the result.

The outer join operation requires us to add some tuples that don't have a match in the other relation.

A nested loop algorithm isn't too bad: if there is no match then pad the tuples with nulls on the appropriate side(s) and add it to the output.

Or: compute the (regular) join, then do a select on all tuples, subtracting those without a match, and pad those tuples and add them to the result.

One approach to aggregation is like the idea of duplicate removal.

We select all the ones we need to aggregate, then take a second pass through the data to sort it, and then merge the values as are necessary.

Count might be done slightly more efficiently since we can skip the sort and merge entries part by simply keeping a running count.

That might also be applied to min, max, sum if we are clever enough to maintain those values and update them accordingly.

Average would require maintaining two values and then performing a division... but somehow this seems rather possible.

$r_1 \bowtie r_2 \bowtie r_3$ could be executed with two different groupings and with different orders of operands:

- 1** $(r_1 \bowtie r_2) \bowtie r_3,$
- 2** $(r_2 \bowtie r_1) \bowtie r_3,$
- 3** $r_3 \bowtie (r_1 \bowtie r_2),$
- 4** $r_3 \bowtie (r_2 \bowtie r_1),$
- 5** $r_1 \bowtie (r_2 \bowtie r_3).$
- 6** $r_1 \bowtie (r_3 \bowtie r_2).$
- 7** $(r_2 \bowtie r_3) \bowtie r_1$
- 8** $(r_3 \bowtie r_2) \bowtie r_1$

But which of these is likely to be the best?