

ME308 PROJECT – TEAM 36

Bus Route Optimization

Members

Sumeet Ranjan	: 190040120
Aditya Chopra	: 190020034
Rezeena Chinthamalla	: 190100099
Thurkapally Manogna	: 190100130



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MODEL

- As the first approach we try to model the bus routing problem for the campus, to transport the students from their respective residence to lecture hall.
- This model incorporates bus capacity and time constrained vehicle problem.
- The model optimises for the least operating cost for both operating cost and amortized servicing cost of the busses, giving us the least number of bus required to cover all the bus stops/

Parameters, Variables and Objective Function

```
param N;                                # Number of Intermediate Nodes + 1;
set N1 := {0..N};                        # 0 U {I} U d
set N2 := {1..N-1} ;                    # I
set N3 := {0..N-1};                     # 0 U {I}
set N4 := {1..N};                        # {I} U d
set S := 1 .. N;
set SS := 1 .. 2**N-1;
set POW {k in SS} := {i in S: (k div 2**(i-1)) mod 2 = 1};

# Part1: Decision Variables
var x{N1,N1} binary;                     #Select the path_{ij}
var k integer;                           #Number of buses
var u{N2};
var v{N2};
var c{N1,N1};                            #Cost_{i,j}

param d{N1,N1};                          #Distance_{i,j}
param f;                                 #Fixed Cost
param Q;
param T;
param a;

param b{N2};                             # q_i

# Part2: Objective Function
minimize z: f*k + sum{ i in N1 , j in N1 }c[i,j]*x[i,j];
```

VARIABLES

- $x_{ij} = \begin{cases} 1 & \text{if any bus traverses through } i, j \\ 0 & \text{otherwise} \end{cases}$
- $k = \text{number of buses}$
- $u_i = \text{Total number of students picked just after leaving node } i.$
- $v_i = \text{distance travelled by bus upto node } i.$

PARAMETERS

- N = Number of intermediate nodes + 1.
- $\{0\}$: origin ; $\{d\}$: dummy node.
- I = set of intermediate nodes.
- Sets : $N_1 = I \cup \{0\} \cup \{d\}$; $N_2 = I$; $N_3 = \{0\} \cup I$; $N_4 = I \cup \{d\}$
- d_{ij} = distance between nodes i and j
- f = amortized cost for each bus
- Q = Upper bound on total capacity of each bus.
- T = Upper bound on distance travlled by each bus.
- b_i = number of students to be picked up at each node i

OBJECTIVE FUNCTION

$$\text{minimise : } f * k + \sum_{i \in N_1} \sum_{j \in N_1} c_{ij} * x_{ij}$$

*The objective function is the total cost of travel and fixed amortized cost for k buses used.
The aim of this formulation is to minimise the objective function.*

Constraints

```
# Part 3: Constraints
s.t. M0 {i in N1, j in N1}: c[i,j] =a*d[i,j];

s.t. M1: sum{i in N2}x[0,i] = k;
s.t. M2: sum{i in N2}x[i,N] = k;

s.t. M3 { i in N2}: sum{ j in N4 }x[i,j] = 1;
s.t. M4 { j in N2 }: sum{ i in N3 }x[i,j] = 1;

## alternate capacity constraint ##
#s.t. MA1 {i in N2 , j in N2: i<>j}: u[i]-u[j]+Q*x[i,j]+(Q-b[i]-b[j])*x[j,i] <= Q- b[j];

##### capacity constraints #####

s.t. M5 {i in N2}: b[i]<=u[i];
s.t. M6 {i in N2, j in N2: i<>j}: u[i]+b[j]<=1000000*(1-x[i,j])+u[j];
s.t. M7 {i in N2, j in N2: i<>j}: u[i]+b[j]+1000000*(1-x[i,j])>=u[j];

s.t. M8 {i in N2}: u[i]<=Q;
s.t. M9 {i in N2}: u[i]-b[i]*x[0,i]+Q*x[0,i] <= Q;

##### travelling distance cap #####

s.t. M10 {i in N2, j in N2: i<>j}: v[i]+d[i,j]<=1000000*(1-x[i,j])+v[j];
s.t. M11 {i in N2, j in N2: i<>j}: v[i]+d[i,j]+1000000*(1-x[i,j])>=v[j];

s.t. M12 {i in N2}: v[i] - d[0,i]*x[0,i] >= 0;
s.t. M13 {i in N2}: v[i]-d[0,i]*x[0,i]+T*x[0,i]<=T;
#s.t. MA2 {i in N2}: v[i]<=T;
#s.t. MA3 {i in N2}: x[i,i] = 0;

## alternate distance cap constraint ##
#s.t. MA4 {i in N2 , j in N2 : i!=j}: v[i] - v[j] + (T - d[i,N] - d[0,j] + d[i,j])*x[i,j]+ (T-d[i,N]-d[0,j]-d[j,i])*x[j,i] <= T-d[i,N]-d[0,j];
#s.t. MA5 {i in N2}: v[i] - d[0,i]*x[0,i] >= 0;
#s.t. MA6 {i in N2}: v[i]-d[0,i]*x[0,i]+T*x[0,i]<=T;

##### subtour elimination Dantzig-Fulkerson-Johnson formulation #####
s.t. M14 {s in SS: card(POW[s])>=2}: sum{i in POW[s], j in POW[s]: i<>j} x[i,j]<=card(POW[s])-1;
```


CONSTRAINTS

$$M0 : c_{ij} = a * d_{ij} \quad \forall i, j \in N_1$$

Setting the cost matrix proportional to distance matrix.

$$M1 : \sum_{i \in N_2} x_{0i} = k$$

Sum of buses leaving the origin (o) is equal to number of buses.

$$M2 : \sum_{i \in N_2} x_{id} = k$$

Sum of buses entering the dummy node (d) is equal to number of buses.

$$M3 : \sum_{j \in N_4} x_{ij} = 1 ; \quad \forall i \in N_2$$

Sum of buses leaving an intermediate node = 1.

$$M4 : \sum_{i \in N_3} x_{ij} = 1 ; \quad \forall j \in N_2$$

Sum of buses entering an intermediate node = 1.

$$M5 : b_i \leq u_i ; \quad \forall i \in N_2$$

Total number of students picked up just after leaving node i is greater than or equal to number of students picked up at node i.

$$M6 : u_i + b_j \leq 1000000 * (1 - x_{ij}) + u_j \quad ; \forall i, j \in N_2, i \neq j$$

$$M7 : u_i + b_j + 1000000 * (1 - x_{ij}) \geq u_j \quad ; \forall i, j \in N_2, i \neq j$$

Trivial for $x_{ij} = 0$. For $x_{ij} = 1$, M6 and M7 combine to give $u_i + b_j = u_j$

i.e. total number of students picked up till node i + number of students at node j = total number of students picked up till node j , as bus moves from i to j

$$M8 : u_i \leq Q \quad ; \forall i \in N_2$$

Total number of students picked up just after leaving node i is less than or equal to maximum capacity possible

$$M9 : u_i - b_i * x_{0i} + Q * x_{0i} \leq Q \quad \forall i \in N_2$$

Trivial for nodes for which $x_{0i} = 0$. For $x_{0i} = 1$; M9 and M5 combine to give $b_i = u_i$.

$$M10 : v_i + d_{ij} \leq 1000000 * (1 - x_{ij}) + v_j \quad ; \forall i, j \in N_2, i \neq j$$

$$M11 : v_i + d_{ij} + 1000000 * (1 - x_{ij}) \geq v_j \quad ; \forall i, j \in N_2, i \neq j$$

*Trivial for $x_{ij} = 0$. For $x_{ij} = 1$, M10 and M11 combine to give $v_i + d_{ij} = v_j$.
i.e. distance travelled by bus upto node i + distance between nodes i and j =
distance travelled by bus upto node j, as bus moves from i to j.*

$$M12 : v_i - d_{oi} * x_{oi} \geq 0; \forall i \in N_2$$

Trivial for $x_{oi} = 0$. For $x_{oi} = 1$; M12 and M13 combine to give $v_i = d_{oi}$.

$$M13 : v_i - d_{oi} * x_{oi} + T * x_{oi} \leq T ; \forall i \in N_2$$

For $x_{oi} = 0$; v_i is limited by maximum distance that the bus can travel. For $x_{oi} = 1$, M12 and M13 combine to give $v_i = d_{oi}$

$$M14 : \sum_i \sum_j x_{ij} \leq |POW(s)| - 1 ; \forall s \in \{1, 2, \dots, (2^n - 1)\}, i, j \in POW(s), i \neq j, |POW(s)| \geq 2$$

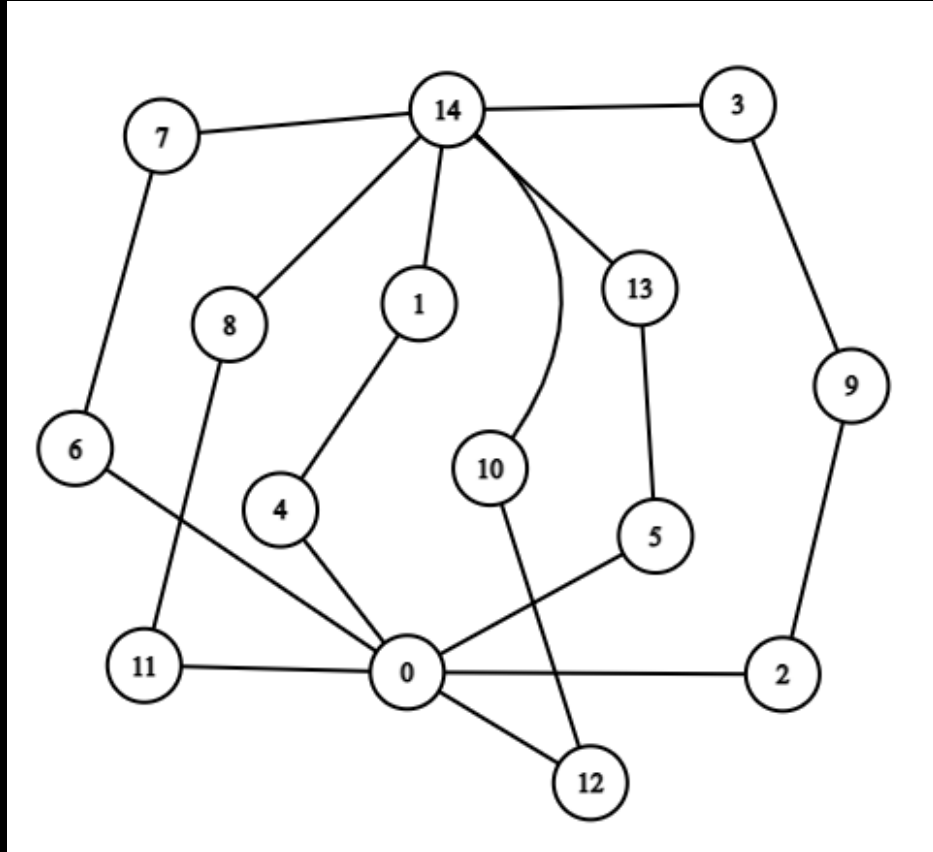
To ensure that cycles do not occur.

DATA SET

```
param N = 14;
param a = 1;
param f = 20000;
param Q = 30;
param T = 10000;
param d :
    0  1  2  3  4  5  6  7  8  9  10 11 12 13 14 :=
0  50000  401 102 245 34  22 15 27 338 12 397 78 37 95 50000
1  401 50000  501 179 369 411 414 374 85 391 5  363 426 494 0
2  102 501 50000  346 135 100 92 127 439 112 497 176 104 10 0
3  245 179 346 50000  214 256 259 219 95 236 175 205 271 339 0
4  34 369 135 214 50000  45 48 10 303 25 365 57 59 128 0
5  22 411 100 256 45 50000  13 37 348 23 406 98 52 93 0
6  15 414 92 259 48 13 50000  40 352 26 410 102 39 85 0
7  27 374 127 219 10 37 40 50000  309 17 370 62 54 120 0
8  338 85 439 95 303 348 352 309 50000  329 78 298 364 432 0
9  12 391 112 236 25 23 26 17 329 50000  387 78 47 106 0
10 397 5  497 175 365 406 410 370 78 387 50000  359 422 490 0
11 78 363 176 205 57 98 102 62 298 78 359 50000  75 170 0
12 37 426 104 271 59 52 39 54 364 47 422 75 50000  99 0
13 95 494 10 339 128 93 85 120 432 106 490 170 99 50000 0
14 50000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 50000;

param b :=
1  5
2  6
3  9
4  25
5  11
6  13
7  17
8  14
9  15
10 11
11 16
12 18
13 19;
```


RESULTS



7764 branch-and-bound nodes
 absmipgap = 11.7178, relmipgap = 9.61591e-05
 k = 6

```
x[i,j] [*,*]
:   0   1   2   3   4   5   6   7   8   9  10  11  12  13  14   :=
0   0   0   1   0   1   1   1   0   0   0   0   1   1   0   0
1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
2   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
3   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
4   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0
5   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0
6   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
7   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
8   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
9   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
10  0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
11  0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
12  0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
13  0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
14  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
;
```

```
: u[i] v[i]   :=
1   30  403
2    6  102
3   30  450
4   25   34
5   11   22
6   13   15
7   30   55
8   30  376
9   21  214
10  29  459
11  16   78
12  18   37
13  30  115
;
```

z = 121858

MODEL 2

- This model deals with the case of variable initial and final stations.
- For each of the $\{R\}$ routes, the initial and final stations are fixed.
- Number of Passengers demanding to travel from node o to destination d are denoted as D^{od} .
- Demand D^{od} can make transfer at any node i , from one route to the other.
- For such system, we need to specify the distance between nodes, initial and final stations for each route and demand od .

Parameters, Variables and Objective Function

#Part1 Parameters

```
param r0;  
set R = {1..r0};  
  
param n;  
set N = {1..n};  
set S := 1 .. n;  
set SS := 1 .. 2**n;  
set POW {k in SS} := {i in S: (k div 2**(i-1)) mod 2 = 1};
```

```
param cap;  
param D{N,N};  
param e{R};  
param f{R};  
param l{N,N};          #C_bus  
param trmax;  
param M;  
param Ca{N,N};
```

Part2: Decision Variables

```
var x{N,N,R} binary;      #[I][J][R]  
var p{N,N,N,N,R} binary;  #[O][D][I][J][R]  
var t{N,N,N} binary;      #[O][D][I]  
var m{N,N} binary;  
var Cb{N,N};  
var u{N,N,R};
```

Part3: Objective Function

```
maximize z: sum {o in N, d in N} (D[o,d]*m[o,d]) -sum{o in N, d in N}Cb[o,d] -sum{i in N, j in N, r in R} (l[i,j]*x[i,j,r])- 0.001*sum{o in N, d in N, i in N} (m[o,d]*D[o,d]*t[o,d,i]);
```

VARIABLES

- $x_{ijr} = \begin{cases} 1 & \text{if route } r \text{ passes through segment } ij \\ 0 & \text{otherwise} \end{cases}$
- $p_{ijr}^{od} = \begin{cases} 1 & \text{if demand } od \text{ uses segment } ij \text{ on route } r \\ 0 & \text{otherwise} \end{cases}$
- $t_i^{od} = \begin{cases} 1 & \text{if demand } od \text{ stops at node } i \text{ for transfer} \\ 0 & \text{otherwise} \end{cases}$
- $m^{od} = \begin{cases} 1 & \text{if demand } od \text{ uses bus for travelling} \\ 0 & \text{otherwise} \end{cases}$
- $C_b^{od} = \text{Cost for demand } od, \text{ for using bus system.}$
- $u_{ijr} = \text{Number of passengers travelling from node } i \text{ to node } j, \text{ using route } r.$

PARAMETERS

- r_0 : *Number of routes*
- n : *Number of nodes*
- D^{od} : *Demand for travelling from node o to d .*
- $e(r)$: *initial station for route r .*
- $f(r)$: *final station for route r .*
- l_{ij} : *length between node i and j .*
- tr_{max} : *maximum number of transfers desired.*
- M : *constant with high value*
- cap : *maximum capacity the bus can fill.*

SETS

- $R = \{1, 2, 3 \dots r_0\}$
- $N = \{1, 2, 3 \dots n\}$
- $S = \{1, 2, \dots n\}$
- $SS = \{1, 2, \dots 2^n\}$
- $POW = \text{power set of } S$

OBJECTIVE FUNCTION

$$\text{maximize : } \sum_o \sum_d D^{od} m^{od} - \sum_o \sum_d C_b^{od} - \sum_i \sum_j \sum_r l_{ij} * x_{ijr} - 0.001 \sum_o \sum_d \sum_i m^{od} * D^{od} * t_i^{od}$$

*The objective function is combination of smaller objective functions : $z = w_1 * z_1 - w_2 * z_2 - w_3 * z_3 - w_4 * z_4$;
Here w_i is the weight of i^{th} (smaller) objective function*

z_1 : maximizing demand served by bus system.

z_2 : minimizing cost of bus system for demand od .

z_3 : minimizing total bus route length.

z_4 : minimizing the total number of transfers made by all users.

Constraints

Part 4: Constraints

s.t. M01 {r in R}: sum{j in N} x[e[r],j,r]=1;

s.t. M02 {r in R}: sum{j in N} x[j,f[r],r]=1;

s.t. M03 {i in N, r in R: i!=e[r] and i!=f[r]}: sum{j in N}x[j,i,r]=sum{k in N}x[i,k,r];

s.t. M04{o in N, d in N}: Cb[o,d]=sum{i in N, j in N, r in R}l[i,j]*p[o,d,i,j,r];

#s.t. MA1 {i in N, r in R}: x[i,i,r] = 0;

s.t. M05 {s in SS, r in R: card(POW[s])>=2 }:sum{i in POW[s], j in POW[s]: i<>j}x[i,j,r]<=card(POW[s])-1;

#s.t. MA2 {r in R, i in N , j in N : i<j}: x[j,i,r] = x[i,j,r];

s.t. M06 {o in N, d in N}: sum{j in N, r in R}p[o,d,o,j,r] = 1;

s.t. M07 {o in N, d in N}: sum{j in N, r in R}p[o,d,j,d,r] = 1;

#s.t. MA3 {o in N, d in N, i in N , j in N, r in R}: p[o,d,i,j,r]*p[o,d,j,i,r]<=0;

s.t. M08 {o in N, d in N, i in N: i!=o and i!=d}: sum{j in N, r in R:i<>j}p[o,d,j,i,r] = sum{k in N, r in R:i<>k}p[o,d,i,k,r] ;

#s.t. MA4 {o in N, d in N}: Cb[o,d]<=Ca[o,d]+M*(1-m[o,d]);

s.t. M09 {o in N, d in N, r in R, i in N: i!=o and i!=d}: sum{j in N}p[o,d,j,i,r] - sum{k in N}p[o,d,i,k,r]<=t[o,d,i];

#s.t. MA5 {o in N, d in N}: sum{i in N} t[o,d,i] <= trmax;

s.t. M10 {r in R , i in N, j in N}: sum{o in N, d in N} (p[o,d,i,j,r]+p[o,d,j,i,r]) <= M*(x[i,j,r]+x[j,i,r]);

s.t. M11 {i in N,j in N, r in R}: (sum{o in N,d in N}D[o,d]*p[o,d,i,j,r]*m[o,d])<=cap;

s.t. M12 {i in N,j in N, r in R}: u[i,j,r] <= (sum{o in N,d in N}D[o,d]*p[o,d,i,j,r]*m[o,d]);

s.t. M13 {i in N,j in N, r in R}: u[i,j,r] >= (sum{o in N,d in N}D[o,d]*p[o,d,i,j,r]*m[o,d]);

#s.t. MA6 {i in N, r in R}:u[i,r]<=cap;

CONSTRAINTS

$$M1 : \sum_j x_{e(r)jr} = 1 ; \forall r$$

For each route 'r', number of buses leaving e(r) is 1.

$$M2 : \sum_j x_{jfr} = 1 ; \forall r$$

For each route 'r', number of buses entering f(r) is 1.

$$M3 : \sum_j x_{jir} = \sum_k x_{ikr} ; \forall r, i \neq e(r) \neq f(r)$$

For each node i (other than e(r) and f(r)), : a segment of route r arrives at node i, then a segment must also leave from it.

$$M4 : C_b^{od} = \sum_i \sum_j \sum_r l_{ij} * p_{ijr}^{od} ; \forall o, d$$

Cost of bus for demand od is proportional to total length travelled by the bus, via all routes possible.

$$M5 : \sum_{i \in POW(s)} \sum_{j \in POW(s)} x_{ijr} \leq |POW(s)| - 1 ; \forall r, s \in SS ; |POW(s)| \geq 2$$

To ensure that no cycles will occur, we use this constraint.

$$M6 : \sum_j \sum_r p_{ojr}^{od} = 1 ; \forall o, d$$

From the trip origin o of each demand od, the demand should move from o to j via 1 route.

$$M7 : \sum_j \sum_r p_{jdr}^{od} = 1 ; \forall o, d$$

Towards the final destination trip d of each demand od, the demand enters d via 1 route.

$$M8 : \sum_j \sum_{\substack{r \\ i \neq j}} p_{jir}^{od} = \sum_k \sum_{\substack{r \\ i \neq k}} p_{ikr}^{od} \quad \forall o, d, i \neq o \neq d$$

In nodes i, different from the trip origin or destination of each demand od, if a traveler arrives at that node, he must continue his journey to another node.

$$M9: \sum_j p_{jir}^{od} - \sum_k p_{ikr}^{od} \leq t_i^{od} \quad \forall o, d, r, i \neq o \neq d$$

Constraint to track transfer at each node i, for demand od.

$$M10 : \sum_o \sum_d p_{ijr}^{od} + p_{jir}^{od} \leq M * (x_{ijr} + x_{jir}) ; \forall r, i, j$$

Trivial for either or both x_{ijr} or $x_{jir} = 1$. For $x_{ijr} = x_{jir} = 0$, for every demand od , $p_{ijr}^{od} = p_{jir}^{od} = 0$

$$M11 : u_{ijr} \leq cap ; \forall r, i, j$$

The number of passengers in the bus is bounded by the maximum capacity of bus.

$$M12 : u_{ijr} = \sum_o \sum_d D^{od} * p_{ijr}^{od} * m^{od} ; \forall r, i, j$$

Number of passengers travelling from node i to node j , using route r .

Data set

```
param r0 = 3;  
param n = 5;  
param cap = 50;
```

```
param D:  
      1      2      3      4      5      :=  
1      0      50      20      11      24  
2      17      0      3      21      27  
3      1      8      0      31      32  
4      6      8      3      0      9  
5      15      18      19      5      0;
```

```
param e:=  
1 1  
2 2  
3 3;
```

```
param f:=  
1 3  
2 4  
3 5;
```

```
param l:  
      1      2      3      4      5      :=  
1      99999      1.6      3.8      5.6      4.9  
2      1.6      99999      99      9.1      6.3  
3      3.8      4.8      99999      2.1      9.0  
4      5.6      9.1      2.1      99999      12.3  
5      4.9      6.3      9.0      12.3      99999;
```

```
param trmax = 10;  
param M = 1000;
```

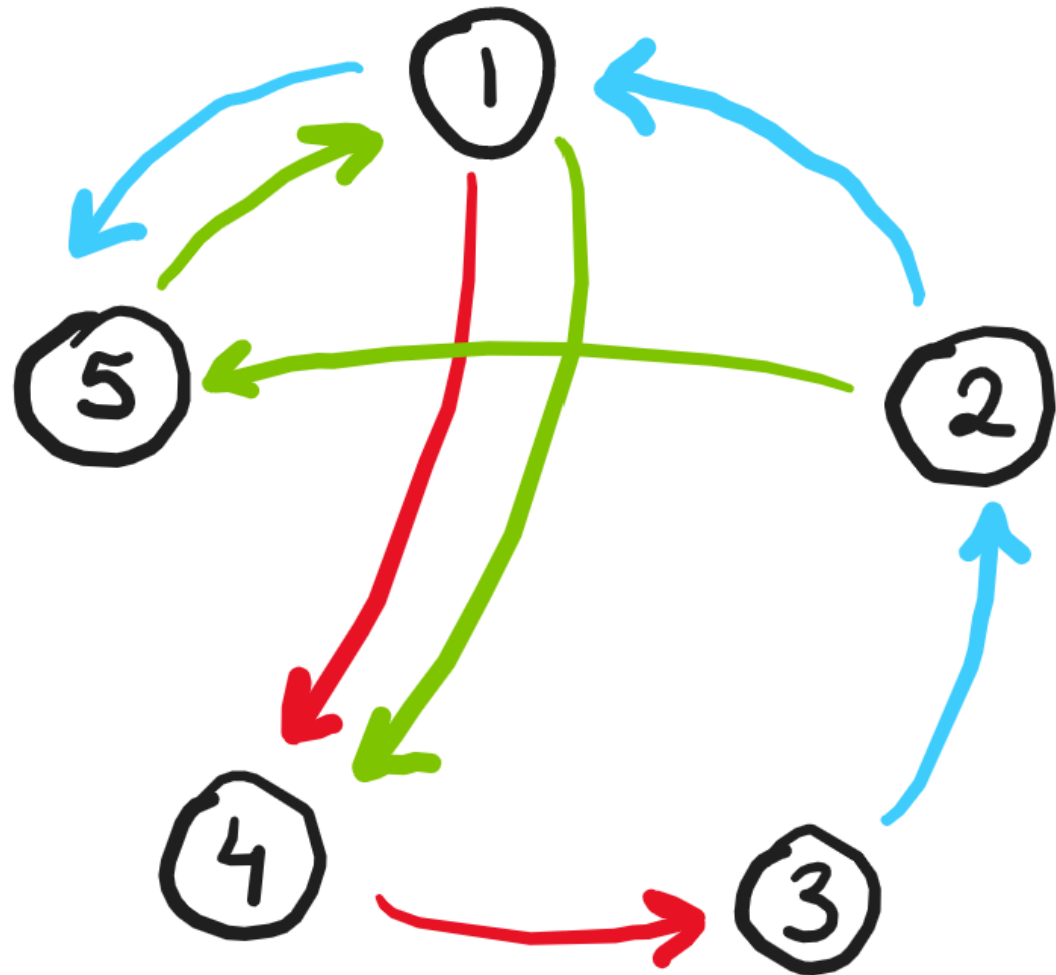
RESULTS

```
x[i,j,r] [*,*,1]
:   1   2   3   4   5   :=
1   0   0   0   1   0
2   0   0   0   0   0
3   0   0   0   0   0
4   0   0   1   0   0
5   0   0   0   0   0
```

```
[*,*,2]
:   1   2   3   4   5   :=
1   0   0   1   0   0
2   0   0   0   0   1
3   0   0   0   1   0
4   0   0   0   0   0
5   1   0   0   0   0
```

```
[*,*,3]
:   1   2   3   4   5   :=
1   0   0   0   0   1
2   1   0   0   0   0
3   0   1   0   0   0
4   0   0   0   0   0
5   0   0   0   0   0
;
```

z = 155.954



```

p[2,4,i,j,r] [*,* ,1]
:   1   2   3   4   5   :=
1   0   0   0   1   0
2   0   0   0   0   0
3   0   0   0   0   0
4   0   0   0   0   0
5   0   0   0   0   0

```

```

[*,* ,2]
:   1   2   3   4   5   :=
1   0   0   0   0   0
2   0   0   0   0   0
3   0   0   0   0   0
4   0   0   0   0   0
5   0   0   0   0   0

```

```

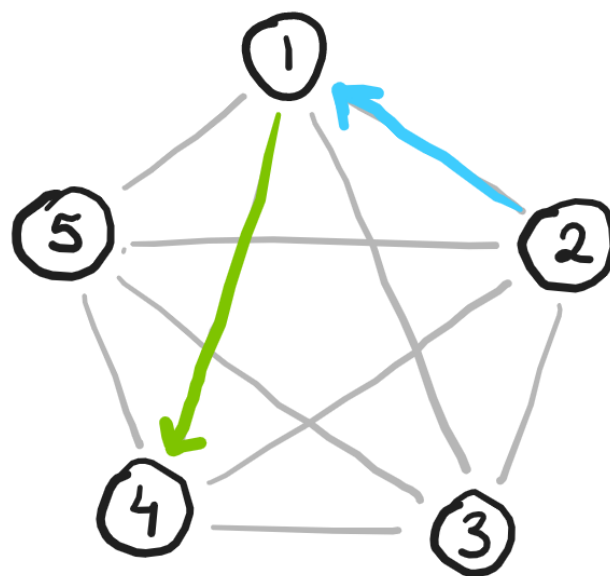
[*,* ,3]
:   1   2   3   4   5   :=
1   0   0   0   0   0
2   1   0   0   0   0
3   0   0   0   0   0
4   0   0   0   0   0
5   0   0   0   0   0
;

```

```

t[2,4,i] [*] :=
1   1
2   0
3   0
4   0
5   0
;

```



$2 \rightarrow 1 \quad (r=3)$

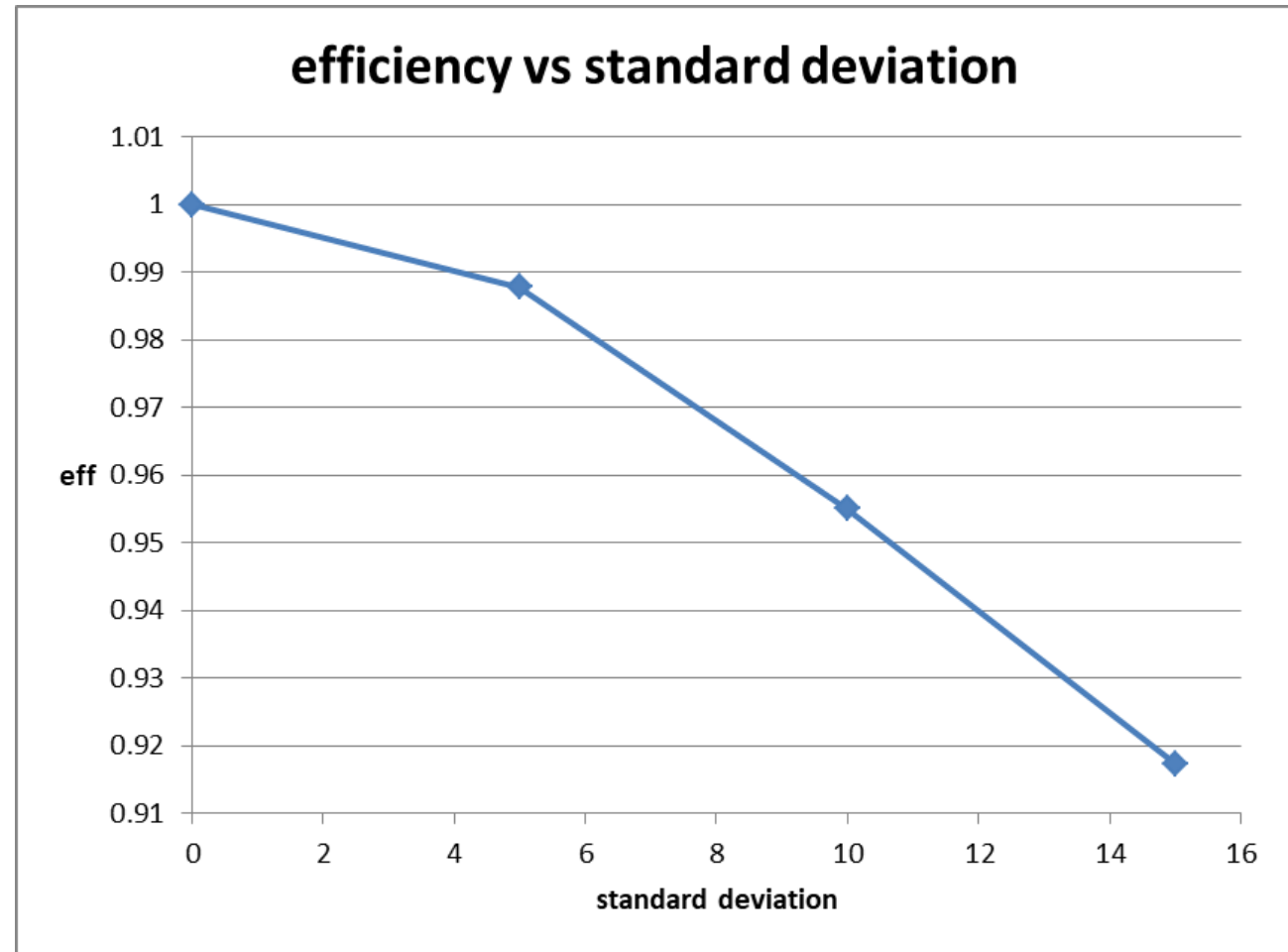
$1 \rightarrow 4 \quad (r=2)$

MODEL ROBUSTNESS

- In order to access the robustness of the model, we use greedy method to simulate the model in real world.
- The demand is sampled from gaussian distribution with mean as the demand used during optimization and fixed standard deviation.
- We get the fraction of total passengers served over the entire demand.
- The average of the score gives a good idea about the robustness of the model.

RESULTS

- Using greedy method, the fraction of passengers that can get on the bus depends on number of passenger already on the bus and its capacity.
- We see pretty high fraction of passengers served.
- The fraction of demand served reduces as the uncertainty in demand increase



RESULTS

- We observe that the optimised model depends on the weights assigned to each term in the objective function.
- Changing weights can aggressively
 - Minimise the number of bus changes passengers have to make.
 - Minimise the average cost of travel for passengers.
 - Maximise the number of passengers served
 - Minimise the operating cost.

MODEL

- <https://github.com/sumeet-ranjan/bus-route-optimization>