Hadamard transformation :-

$$H_m = \frac{1}{\sqrt{2}} \begin{bmatrix} H_{m-1} & H_{m-1} \\ H_{m-1} & -H_{m-1} \end{bmatrix}$$

$$H_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_2 = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & +1 \end{bmatrix}$$

$$H_3 = \frac{1}{\sqrt{8}} \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}$$

$$V(K) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} u(m) (-1)^{b(K m)}$$

$$u(m) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} V(K) (-1)^{b(K,m)} \quad 0 \le m \le N-1$$

Unsharp masking & High boost filter

$$u(m,n) = \frac{1}{N^2} \sum_{k=0} \sum_{l=0} v(k,l)\, e^{\frac{j2\pi}{N} km}\, e^{\frac{j2\pi}{N} ln}$$

$0 \le k, l < N$

forward transformation

$$a(k,n) = \cos\left[\frac{\pi(2n+1)k}{2N}\right]$$

$$v(k) = \alpha(k) \sum_{n=0}^{N-1} u(n) \cos\left[\frac{\pi(2n+1)k}{2N}\right] \quad 0 \le k \le N-$$

$$\alpha(k) = \frac{1}{\sqrt{N}} \quad k=0$$

$$\alpha(k) = \sqrt{\frac{2}{N}} \quad k \neq 0$$

$$u(n) = \sum_{k=0}^{N-1} a(k) v(k) \cos\left[\frac{\pi(2n+1)k}{2N}\right] \quad 0 \le n \le N-1$$

2DCT

$$v(k,l) = \alpha(k)\alpha(l) \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} u(m,n)$$

$$= \cos\left[\frac{\pi(2m+1)k}{2N}\right] \cos\left[\frac{\pi(2n+1)l}{2N}\right]$$

$$\alpha(k), \alpha(l) = \frac{1}{\sqrt{N}} \quad k, l = 0$$

$$u(n) = \sum_{k=0}^{N-1} a(k)\, v(k) \cos\left[\frac{\pi(2n+1)k}{2N}\right] \qquad 0 \le n \le N-1$$

2D DCT

$$v(k,l) = \alpha(k)\,\alpha(l) \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} u(m,n)$$

$$= \cos\left[\frac{\pi(2m+1)k}{2N}\right] \cos\left[\frac{\pi(2n+1)l}{2N}\right]$$

$$\alpha(k),\ \alpha(l) = \frac{1}{\sqrt{N}} \qquad k, l = 0$$

$$\alpha(k),\ \alpha(l) = \sqrt{\frac{2}{N}} \qquad k, l \neq 0$$

$$u(n) = \sum_{k=0}^{N-1} v(k)\, a^*(k,n) \qquad 0 \le n \le N-1$$

Two dimensional

forward

$$v(k,l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} a(m,n)\, u(m,n) \qquad 0 \le k \le N-1$$

Inverse

$$u(m,n) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a^*(m,n)\, v(k,l) \qquad 0 \le m,n \le N-1$$

1DFT

$$a(k,n) = e^{-j \frac{2\pi}{N} kn}$$

12/9

One dimensional

$$v(k) = \sum_{n=0}^{N-1} a(k,n)\, u(n) \qquad 0 \le k \le N-1$$

$$u(n) = \sum_{k=0}^{N-1} v(k)\, a^*(k,n) \qquad 0 \le n \le n-1$$

Two dimensional

forward

$k \geq 0 \quad l \geq 0$

No derivation is required

The elements $V(k,l)$ are called transform coefficients

2D - Separable Transformation

$$a_{k,l}(m,n) = a_k(m)\ b_l(n)$$

$$a_{k,l}(m,n) = a(k,m) \cdot \frac{b}{a}(l,n)$$

$$V_{k,l} = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} a(k,m) . a(l,n) . u(m,n) \qquad 0 < k,l < N-1$$

$$\frac{\partial f}{\partial x} = f(x+1) - f(\ldots$$

$$\frac{\partial^2 f}{\partial x^2} =$$

Gradient filters

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}$$

Roberts operator

Prewirtt

$$G_x = (z_9 - z_5) \qquad G_y = (z_8 - z_6)$$

Sobel operator

$$G_x = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$G_y = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

Prewitt

$$G_x = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)$$

$$G_y = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$$

2nd Derivative

$A^{*T} V A = U$ $\quad A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix}$

$= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

The term spatial frequency generally refers to the continuous fourier transform frequency and is not same as the discreate fourier frequency.

→ <u>Optimum transform</u>

Another imp. consideration in selecting a transform is its performance in filtering and data compression

eg:- karhunen Loeve transform.

<u>properties of unitary transform</u>

1) energy conservation and rotation

- it preserves signal energy or equivalently the length of vector u in the N- dimensional vector space.

2) energy compaction and variances of transform coeffients

- Most unitary transform have a tendency to pack a large fraction of the average energy of the image into relatively few components of the transform coeffients.

3) decorrelation

- when the i/p vector elements are highly co-related. the transform coeffients tend to be uncorrelated.

1DFT

The discrete fourier ~~series~~ transform (DFT) of a sequence is defined as

<u>forward</u> $$v(k) = \sum_{n=0}^{N-1} u(n)\, e^{\frac{-j2\pi}{N}}$$

<u>inverse</u> $$u(n) = \frac{1}{N} \sum_{k=0}^{N-1} v(k)\, e^{\frac{j2\pi}{N}}$$

Properties of DFT and Unitary DFT

DFT and unitary DFT
- matrices are symmetric
- The extensions are periodic
- The DFT is the sampled spectrum of the finite sequence $u(n)$ extended by zeroes outside the interval $[0, N-1]$
- The DFT and unitary DFT of dimension N can be implemented by a fast algorithm in $O(N \log_2 N)$ operations
- The DFT and unitary DFT of a real sequence is conjugate symmetric about $N/2$

2D - DFT

- The 2D DFT of $N \times N$ image is a separable transform defined as

$$v(k,l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} u(m,n)\, e^{-\frac{j2\pi}{N} km}\, e^{-\frac{j2\pi}{N} ln} \qquad 0 \le k,l \le N-1$$

<u>properties</u>
- symmetric, unitary
- periodic extension
- sampled fourier spectrum
- fast transform
- conjugate symmetry
- Basis images.

cosine

<u>properties</u>
- it is real and orthogonal.
- it is not the real part of the unitary DFT.
- it is a fast transform.
- has excellent energy compaction for highly correlated data
- the basis vectors are the eigen vectors of symmetric tridiagonal matrix.

Scanned by CamScanner

Sine transform

$$V(k) = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{N-1} u(n) \sin \frac{\pi (k+1)(n+1)}{N+1} \qquad 0 \le k \le N-1$$

$$u(n) = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{N-1} V(k) \sin \frac{\pi (k+1)(n+1)}{N+1} \qquad 0 \le n \le N-1$$

properties

- real, symmetric and orthogonal.
- not imaginary part of unitary DFT.
- it is a fast transform.
- The basis vectors are the eigen vectors of symmetric tridiagonal matrix

Hadamard transform

- is real, symmetric and orthogonal.
- it is a fast transform
- has good energy compaction for highly correlated images
- The natural order of Hadamard transform coefficients turns out to be equal to the bit reversed grey code representation of its sequence s.

Scanned by CamScanner