assignment 8

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June 16, 2022

Outline

EX:9.5,question

Q)The random variables a and b are independent $N(0;\sigma)$ and p is the probability that the process $x(t) = a - bt$ crosses the t axis in the intervel(0,T) . show that $\pi p = \arctan T$.

solution:

the process crosses the t axis at $x(t) = 0$, $a - bt = 0$,iff $t = \frac{a}{b}$
setting $\sigma_1 = \sigma_2 = \sigma$ and $r = 0$, in theorm we get,

THEORM:

$$F_z(z) = \frac{1}{2} + \frac{1}{\pi} arc \tan \frac{\sigma_2 z - r\sigma_1}{\sigma_1\sqrt{1 - r^2}} \tag{1}$$

substituting the values we get,

$$p[0 < t < T] = \frac{1}{2} + \frac{1}{\pi} arc \tan \frac{\sigma(T)}{\sigma} - \left(\frac{1}{2} + \frac{1}{\pi} arc \tan 0\right) \tag{2}$$

$$= \frac{1}{2} + \frac{1}{\pi} arc \tan(T) - \left(\frac{1}{2} + \frac{1}{\pi} arc \tan 0\right) \tag{3}$$

solution

$$p = \frac{1}{\pi} arc \tan T \tag{4}$$

$$p\pi = arc \tan T \tag{5}$$

so, hence the given statement i.e $\pi p = \arctan T$ is proved