Assignment

sumeeth kumar
AI21BTECH11008

CONTENTS

***Abstract*—This manual provides solutions to the Assignment of Random Numbers**

I. Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

I.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/sumeethkumar12/
    random−variables/blob/main/1/unidat.c
wget https://github.com/sumeethkumar12/
    random−variables/blob/main/1/coeffs.h
```

Download the above files and execute the following commands

a) $ gcc unidat.c -lm
b) $ ./a.out

I.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1}$$

Solution: The following code plots Fig. I.2

```
wget https://github.com/sumeethkumar12/
    random−variables/blob/main/1/1_2.py
```

Download the above files and execute the following commands to produce Fig.I.2

a) $ python3 1_2.py

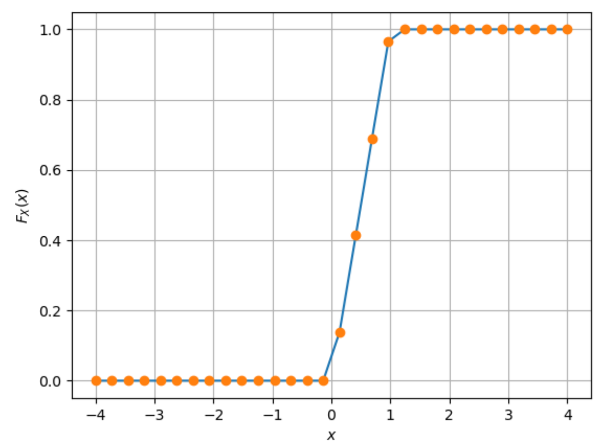

Fig. I.2. The CDF of U

I.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for} \tag{2}$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx \tag{3}$$

$$\implies F_U(x) = x \tag{4}$$

I.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{6}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program.

```
wget ttps://github.com/sumeethkumar12/
    random−variables/blob/main/1/1_4.c
wget https://github.com/sumeethkumar12/
    random−variables/blob/main/1/coeffs.h
```

Download the above files and execute the following commands

a) $ gcc 1_4.c
b) $ ./a.out

I.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{7}$$

Solution:

$$\text{var}\left[U\right] = E\left[U - E\left[U\right]\right]^2 \tag{8}$$

$$\implies \text{var}\left[U\right] = E\left[U^2\right] - E\left[U\right]^2 \tag{9}$$

$$E\left[U\right] = \int_{-\infty}^{\infty} x dF_U(x) \tag{10}$$

$$E\left[U\right] = \int_{0}^{1} x \tag{11}$$

$$\implies \boxed{E\left[U\right] = \frac{1}{2}} \tag{12}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{13}$$

$$E\left[U^2\right] = \int_{0}^{1} x^2 dF_U(x) \tag{14}$$

$$\implies E\left[U^2\right] = \frac{1}{3} \tag{15}$$

$$\implies \boxed{\text{var}\left[U\right] = \frac{1}{12} = 0.0833} \tag{16}$$

II. Central Limit Theorem

II.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{17}$$

using a C program, where $U_i, i = 1, 2, \dots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program.

```
wget https://github.com/sumeethkumar12/
    random−variables/blob/main/2/2_1.c
wget ttps://github.com/sumeethkumar12/
    random−variables/blob/main/1/coeffs.h
```

Download the above files and execute the following commands

a) $ gcc 2_1.c
b) $ ./a.out

II.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution: The CDF of X is plotted in Fig. II.2 using the code below

```
wget https://github.com/sumeethkumar12/
    random−variables/blob/main/2/2_2.py
```

Download the above files and execute the following commands to produce Fig.II.2

a) $ python3 2.2.py

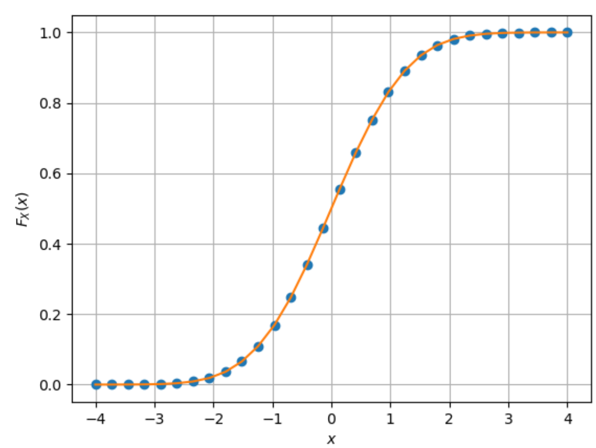

Fig. II.2. The CDF of X

Some of the properties of CDF

a) $F_X(x)$ is non decreasing function.
b) Symmetric about one point.
c) The Q function is defined as, $Q(X) = Pr(X > x)$
d) Hence we can use it to calculate $F_x(x)$, $F_x(x) = 1 - Q(x)$

II.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{18}$$

What properties does the PDF have?

Solution: The PDF of X is plotted in Fig. II.3 using the code below

```
wget https://github.com/sumeethkumar12/
    random-variables/blob/main/2/2_3.py
```

Download the above files and execute the following commands to produce Fig.II.3

a) $ python3 2_3.py

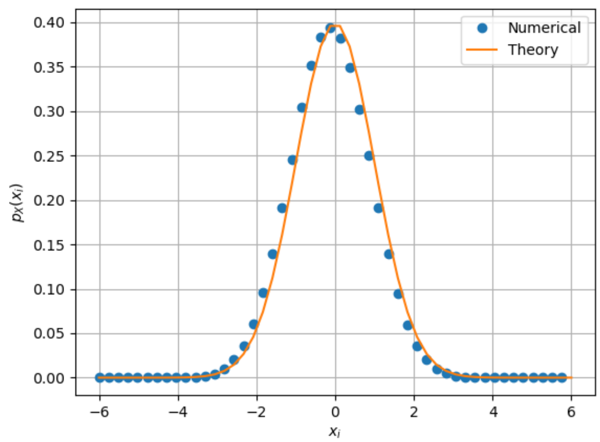

Fig. II.3. The PDF of X

Some of the properties of the PDF:

a) Symmetric about $x = \mu$
b) decreasing function for $x < \mu$ and increasing for $x > \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

II.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program.

```
wget https://github.com/sumeethkumar12/
    random-variables/blob/main/2/2_4.c
wget https://github.com/sumeethkumar12/
    random-variables/blob/main/2/coeffs.h
```

Download the above files and execute the following commands

a) $ gcc 2_4.c
b) $ ./a.out

II.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{19}$$

repeat the above exercise theoretically.

Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \tag{20}$$

$$\boxed{F_X(x) = 1} \tag{21}$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} x p_X(x)dx \tag{22}$$

$$\implies \boxed{E(x) = 0} \tag{23}$$

3) Variance is given by

$$\text{var}[U] = E(U^2) - (E(U))^2 \tag{24}$$

$$\implies \boxed{\text{var}[U] = 1} \tag{25}$$

III. From Uniform to Other

III.1 Generate samples of

$$V = -2\ln(1-U) \tag{26}$$

and plot its CDF.
Solution: Download the following files and execute the C program.

```
wget https://github.com/sumeethkumar12/
    random-variables/blob/main/3/3_1.c
wget https://github.com/sumeethkumar12/
    random-variables/blob/main/3/coeffs.h
```

Download the above files and execute the following commands

a) $ gcc 3_1.c -lm
b) $ ./a.out

The CDF of V is plotted in Fig. III.1 using the code below

```
wget https://github.com/sumeethkumar12/
    random-variables/blob/main/3/3_1.py
```

Download the above files and execute the following commands to produce Fig.III.1

a) $ python3 3_1.py

III.2 Find a theoretical expression for $F_V(x)$.
Solution: If Y = g(X), we know that $F_Y(y) = F_X(g^{-1}(y))$, here

$$V = -2\ln(1-U) \tag{27}$$

$$1 - U = e^{\frac{-V}{2}} \tag{28}$$

$$U = 1 - e^{\frac{-V}{2}} \tag{29}$$

$$F_V(X) = F_U(1 - e^{\frac{-X}{2}}) \tag{30}$$

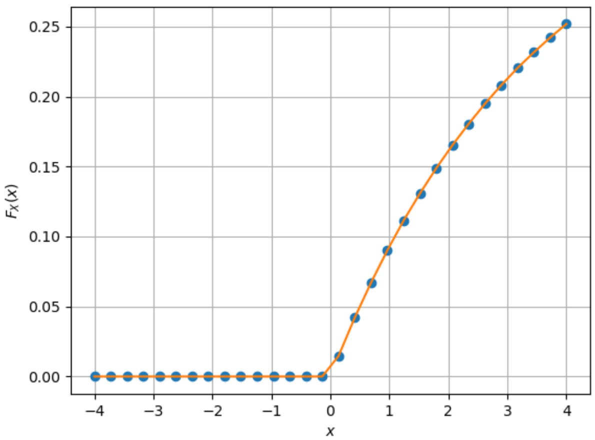

Fig. III.1. The PDF of X

$$\text{when },\ 0 \le 1 - e^{\frac{-X}{2}} \le 1$$

$$0 \le e^{\frac{-X}{2}} \le 1 \tag{31}$$

$$X \ge 0, \text{So,} \tag{32}$$

$$F_V(X) = 1 - e^{\frac{-X}{2}}, X \ge 0 \tag{33}$$

$$F_V(X) = 0, X < 0 \tag{34}$$