Digital Signal Processing

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Abstract—**This manual provides a simple introduction to digital signal processing.**

1 SOFTWARE INSTALLATION

Run the following commands (commands may change depending on Linux distro)

```
$ sudo apt update && sudo apt upgrade
$ sudo apt install libffi−dev libsndfile1 python3−
    scipy python3−numpy python3−matplotlib
$ pip3 install cffi pysoundfile
```

2 DIGITAL FILTER

2.1 Download the sound file using

```
$https://github.com/sumeethkumar12/signal−
    processing/blob/main/
    Sound_With_ReducedNoise.wav
```

2.2 You will find a spectrogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in Problem **??** in the spectrogram and play. Observe the spectrogram. What do you find?
Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band noise and execute the code.
Solution: Download the source code using

```
$ https://github.com/sumeethkumar12/signal−
    processing/blob/main/codes/cancel_noise.
    py
```

and execute it using

```
$ python3 cancel_noise.py
```

2.4 The output of the python script in Problem **??** is the audio file `Sound_With_ReducedNoise.wav`. Play the file in the spectrogram in Problem **??**. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz.

3 DIFFERENCE EQUATION

3.1 Let
$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \quad (3.1)$$

Sketch $x(n)$.

3.2 Let
$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2),$$
$$y(n) = 0, n < 0 \quad (3.2)$$

Sketch $y(n)$.
Solution: The following code yields Fig. (**??**).

```
$ https://github.com/sumeethkumar12/signal−
    processing/blob/main/codes/xnyn.py
```

and execute it using

```
$ python3 xnyn.py
```

3.3 Repeat the above exercise using a C code.

```
$ https://github.com/sumeethkumar12/signal−
    processing/blob/main/codes/xnyn.c
```

4 Z-TRANSFORM

4.1 The *Z*-transform of $x(n)$ is defined as

$$X(z) = Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (4.1)$$ Show that
$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \quad (4.2)$$

and find
$$\mathcal{Z}\{x(n-k)\} \quad (4.3)$$

Fig. 3.2: Plot of $x(n)$ and $y(n)$

4.2 Now we will find Z transform of the signal

$$Z\{x(n)\} = \sum_{n=0}^{5} x(n) z^{-n} \tag{4.12}$$

$$= 1z^0 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + 1z^{-5} \tag{4.13}$$

$$= 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5} \tag{4.14}$$

4.3 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.15}$$

from (??) assuming that the Z-transform is a linear operation.

Solution:

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.16}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{4.17}$$

Solution: Now we will rewrite ,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{4.18}$$

Now since Z-transform is a linear operator we can write that,

$$Y(n) + \frac{1}{2}Y(n-1) = X(n) + X(n-2) \tag{4.19}$$

$$Y(n) + \frac{z^{-1}}{2}Y(n) = X(n) + z^{-2}X(n) \tag{4.20}$$

$$\implies \frac{Y(n)}{X(n)} = \frac{1 + z^{-2}}{1 + \frac{z^{-1}}{2}} \tag{4.21}$$

4.4 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.22}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.23}$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.24}$$

Solution: Given that,

$$X(z) = Z\{x(n)\} \tag{4.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4.5}$$

So,

$$Z\{x(n-1)\} = \sum_{n=-\infty}^{\infty} x(n-1) z^{-n} \tag{4.6}$$

Take $k = n - 1$,

$$= \sum_{k=-\infty}^{\infty} x(k) z^{-(k+1)} \tag{4.7}$$

$$= z^{-1} \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{4.8}$$

$$= z^{-1} \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4.9}$$

$$= z^{-1} X(z) \tag{4.10}$$

resulting in and similarly following the above steps you will get,

$$Z\{x(n-k)\} = z^{-k} X(n) \tag{4.11}$$

Solution: The Z-transform of δn is,

$$\mathcal{Z}\{\delta n\} = \sum_{n=-\infty}^{\infty} \delta(n)\, z^{-n} \qquad (4.25)$$

$$= \delta(0)\, z^0 + 0 \quad (\text{Using } (\textbf{??})) \qquad (4.26)$$

$$= 1 \qquad (4.27)$$

and the Z-transform of unit-step function $u(n)$ is,

$$U(n) = \sum_{n=-\infty}^{\infty} u(n)\, z^{-n} \qquad (4.28)$$

$$= 0 + \sum_{n=0}^{\infty} 1.z^{-n} \qquad (4.29)$$

$$= 1 + z^{-1} + z^{-2} + \ldots \qquad (4.30)$$

Above is a infinite geometric series with z^{-1} as common ratio , so we can write it as

$$U(n) = \frac{1}{1 - z^{-1}} \quad \because |z| > 1 \qquad (4.31)$$

4.5 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \qquad |z| > |a| \qquad (4.32)$$

Solution: The Z- transform will be

$$\mathcal{Z}\{a^n u(n)\} = \sum_{n=0}^{\infty} a^n z^{-n} \qquad (4.33)$$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \ldots. \qquad (4.34)$$

Above is a infinite geometric series with first term 1 and common ratio as $\frac{a}{z}$ and it can be written as,

$$\mathcal{Z}\{a^n u(n)\} = \frac{1}{1 - \frac{a}{z}} \quad \because |a| < |z| \qquad (4.35)$$

Therefore,

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \qquad |z| > |a| \qquad (4.36)$$

4.6 1et

$$H(e^{\jmath\omega}) = H(z = e^{\jmath\omega}). \qquad (4.37)$$

Plot $|H(e^{\jmath\omega})|$. Comment. $H(e^{\jmath\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $x(n)$. **Solution:** The following code plots Fig. .

$ https://github.com/sumeethkumar12/signal−
    processing/blob/main/codes/dtft.py

The figure can be generated using

$ python3 dtft.py

We observe that $\left|H\left(e^{\jmath\omega}\right)\right|$ is periodic with fundamental period 2π. Now using we will find

Fig. 4.6: Plot of $\left|H\left(e^{\jmath\omega}\right)\right|$ against ω

$\left|H\left(e^{\jmath\omega}\right)\right|,$

$$H\left(e^{\jmath\omega}\right) = \frac{1 + e^{-2\jmath\omega}}{1 + \frac{e^{-\jmath\omega}}{2}} \qquad (4.38)$$

$$\implies \left|H\left(e^{\jmath\omega}\right)\right| = \frac{\left|1 + e^{-2\jmath\omega}\right|}{\left|1 + \frac{e^{-\jmath\omega}}{2}\right|} \qquad (4.39)$$

$$= \frac{\left|1 + e^{2\jmath\omega}\right|}{\left|e^{2\jmath\omega} + \frac{e^{\jmath\omega}}{2}\right|} \qquad (4.40)$$

$$= \frac{|1 + \cos 2\omega + j\sin 2\omega|}{\left|e^{\jmath\omega} + \frac{1}{2}\right|} \qquad (4.41)$$

$$= \frac{\left|4\cos^2(\omega) + 4j\sin(\omega)\cos(\omega)\right|}{|2e^{\jmath\omega} + 1|} \qquad (4.42)$$

$$= \frac{|4\cos(\omega)|\,|\cos(\omega) + j\sin(\omega)|}{|2\cos(\omega) + 1 + 2j\sin(\omega)|} \qquad (4.43)$$

$$\therefore \left|H\left(e^{\jmath\omega}\right)\right| = \frac{|4\cos(\omega)|}{\sqrt{5 + 4\cos(\omega)}} \qquad (4.44)$$

Using the above expression we can say that graph is symmetric about origin and has a period of 2π.

4.7 Express $h(n)$ in terms of $H\left(e^{\jmath\omega}\right)$.

Solution:

$$\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}\omega \tag{4.45}$$

$$= \int_{-\pi}^{\pi} \sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}e^{j\omega n}\omega \tag{4.46}$$

$$= \sum_{k=-\infty}^{\infty} h(k) \int_{-\pi}^{\pi} e^{j\omega(n-k)}\omega \tag{4.47}$$

Now,

$$\int_{-\pi}^{\pi} e^{j\omega(n-k)}\omega = \begin{cases} \int_{-\pi}^{\pi}\omega & n-k=0 \\ \frac{\exp(j\omega(n-k))}{j(n-k)}\Big|_{-\pi}^{\pi} & n-k \neq 0 \end{cases} \tag{4.48}$$

$$= \begin{cases} 2\pi & n-k=0 \\ 0 & n-k \neq 0 \end{cases} \tag{4.49}$$

$$= 2\pi\delta(n-k) \tag{4.50}$$

Thus,

$$\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}\omega = 2\pi \sum_{k=-\infty}^{\infty} h(k)\delta(n-k) \tag{4.51}$$

$$= 2\pi h(n) * \delta(n) \tag{4.52}$$

$$= 2\pi h(n) \tag{4.53}$$

Therefore, $h(n)$ is given by the inverse DTFT (IDTFT) of $H(e^{j\omega})$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}\omega \tag{4.54}$$

5 IMPULSE RESPONSE

5.1 Using long division, find

$$h(n), \quad n < 5 \tag{5.1}$$

for H(z) in .
Solution: we can write

$$H(z) = \frac{1+z^{-2}}{1+\frac{z^{-1}}{2}} \tag{5.2}$$

$$\begin{array}{r} 2z^{-1} \quad -4 \\ \hline 1+z^{-1}/2| \quad 1 \quad +z^{-2} \\ 2z^{-1} \quad +z^{-2} \\ \hline 1 \quad -2z^{-1} \\ -4 \quad -2z^{-1} \\ \hline 5 \end{array}$$

So we can replace as,

$$\frac{1+z^{-2}}{1+\frac{z^{-1}}{2}} = 2z^{-1} - 4 + \frac{5}{1+z^{-1}/2} \tag{5.3}$$

Now we can expand the second term of above expression as an infinite geometric series,

$$\frac{5}{1+z^{-1}/2} = 5\left(1 + \left(\frac{-1}{2z}\right) + \left(\frac{-1}{2z}\right)^2 + \ldots\right) \tag{5.4}$$

where we assume $\left|\frac{1}{2z}\right| < 1$. So will become,

$$= 2z^{-1} - 4 + 5 + \frac{-5}{2}z^{-1} + \frac{5}{4}z^{-2} + \frac{-5}{8}z^{-3} + \frac{5}{16}z^{-4} + \ldots \tag{5.5}$$

$$= 1z^0 + \frac{-1}{2}z^{-1} + \frac{5}{4}z^{-2} + \frac{-5}{8}z^{-3} + \frac{5}{16}z^{-4} + \ldots \tag{5.6}$$

Now to get $h(n)$ for $n < 5$ we will compare with the below equation,

$$H(z) = \sum_{n=-\infty}^{n=\infty} h(n)z^{-n} \tag{5.7}$$

$h(n)$ will be the coefficient of z^{-n}.
Using this, we can write,

$$h(0) = 1 \tag{5.8}$$

$$h(1) = \frac{-1}{2} \tag{5.9}$$

$$h(2) = \frac{5}{4} \tag{5.10}$$

$$h(3) = \frac{-5}{8} \tag{5.11}$$

$$h(4) = \frac{5}{16} \tag{5.12}$$

And for $n < 0$ $h(n) = 0$.
For $n > 5$, we can get h(n) from the geometric series,

$$h(n) = 5\left(\frac{-1}{2}\right)^n \tag{5.13}$$

5.2 Find an expression for $h(n)$ using $H(z)$, given that

h(n) $\overset{\mathcal{Z}}{\rightleftharpoons}$ $H(z)$(5.14)and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse response* of the system defined .

Solution: The $H(z)$ can be written as,

$$H(z) = \frac{1}{1 + \frac{z^{-1}}{2}} + \frac{z^{-2}}{1 + \frac{z^{-1}}{2}} \qquad (5.15)$$

we can write it as,

$$h(n) = \left(\frac{-1}{2}\right)^n u(n) + \left(\frac{-1}{2}\right)^{n-2} u(n-2) \quad (5.16)$$

5.3 Sketch $h(n)$. Is it bounded? Justify Theoritically.
   Solution: Download the code for the plot from the below link,

> wget https://github.com/
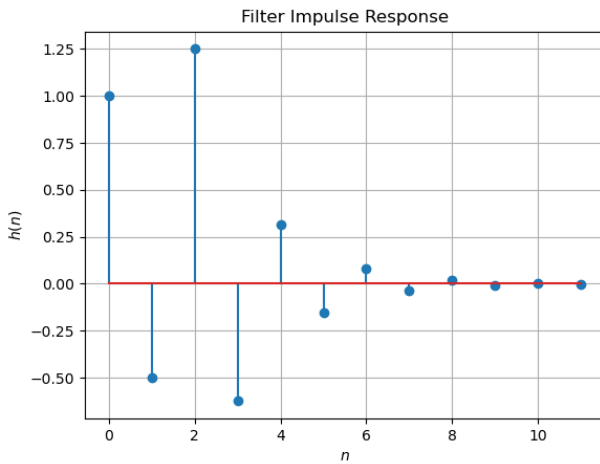>   sumeethkumar12Signal_Processing/blob/
>   master/Sound%201/Codes/hn.py

Fig. 5.3: $h(n)$ as inverse of $H(n)$

From the plot it seems like $h(n)$ is bounded and becomes smaller in magnitude as n increases. Using , we can get theoritical expression as,

$$h(n) = \begin{cases} 0 & , n < 0 \\ \left(\frac{-1}{2}\right)^n & , 0 \le n < 2 \\ 5\left(\frac{-1}{2}\right)^n & , n \ge 2 \end{cases} \qquad (5.17)$$

A sequence $\{x_n\}$ is said to be bounded if and only if there exist a positive real number M such that,

$$|x_n| \le M, \forall n \in \mathcal{N} \qquad (5.18)$$

So to say $h(n)$ is bounded we should able to find the M which satisfies .

For $n < 0$,

$$|h(n)| \le 0 \qquad (5.19)$$

For $0 \le n < 2$,

$$|h(n)| = \left|\frac{-1}{2}\right|^n \qquad (5.20)$$

$$= \left(\frac{1}{2}\right)^n \le 1 \qquad (5.21)$$

And for $n \ge 2$,

$$|h(n)| = \left|5\left(\frac{-1}{2}\right)\right|^n \qquad (5.22)$$

$$= \left(\frac{5}{2}\right)^n \le \frac{5}{4} \qquad (5.23)$$

From above three cases, we can get M as,

$$M = max\left\{0, 1, \frac{5}{4}\right\} \qquad (5.24)$$

$$= \frac{5}{4} \qquad (5.25)$$

Therefore, $h(n)$ is bounded with $M = \frac{5}{4}$ i.e.,

$$|h(n)| \le \frac{5}{4} \forall n \in \mathcal{N} \qquad (5.26)$$

5.4 Convergent? Justify using the ratio test.
   Solution: We can say a given real sequence $\{x_n\}$ is convergent if

$$\lim_{n \to \infty} \left|\frac{x_{n+1}}{x_n}\right| < 1 \qquad (5.27)$$

This is known as Ratio test.
In this case the limit will become,

$$\lim_{n \to \infty} \left|\frac{h(n+1)}{h(n)}\right| = \lim_{n \to \infty} \left|\frac{5\left(\frac{-1}{2}\right)^{n+1}}{5\left(\frac{-1}{2}\right)^n}\right| \qquad (5.28)$$

$$= \lim_{n \to \infty} \left|\frac{-1}{2}\right| \qquad (5.29)$$

$$= \frac{1}{2} \qquad (5.30)$$

As $\frac{1}{2} < 1$, from root test we can say that $h(n)$ is convergent.

5.5 The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \qquad (5.31)$$

Is the system defined stable for the impulse

response ?
Solution: ,

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty} \left(\left(\frac{-1}{2} \right)^n u(n) + \left(\frac{-1}{2} \right)^{n-2} u(n-2) \right) \tag{5.32}$$

$$= 2 \left(\frac{1}{1 + \frac{1}{2}} \right) \tag{5.33}$$

$$= \frac{4}{3} \tag{5.34}$$

∴ the system is stable.

5.6 Verify the above result using a python code.
Solution: Download the python code from the below link

> wget https://github.com/sumeethkumar12/
> Signal_Processing/blob/master/Sound
> %201/Codes/hnstable.py

Then run the following command,

> python3 hn_stable.py

5.7 Compute and sketch $h(n)$ using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \tag{5.35}$$

This is the definition of $h(n)$.
Solution: Download the code for the plot **??** from the below link,

> wget https://github.com/sumeethkumar12/
> Signal_Processing Codes/hndef.py

Note that this is same as **??**.
For $n < 0$, $h(n) = 0$ and,

$$h(0) = \delta(0) \tag{5.36}$$

$$= 1 \tag{5.37}$$

For $n = 1$,

$$h(1) + \frac{1}{2}h(0) = \delta(1) + \delta(-1) \tag{5.38}$$

$$\implies h(1) = -\frac{1}{2}h(0) \tag{5.39}$$

$$= -\frac{1}{2} \tag{5.40}$$
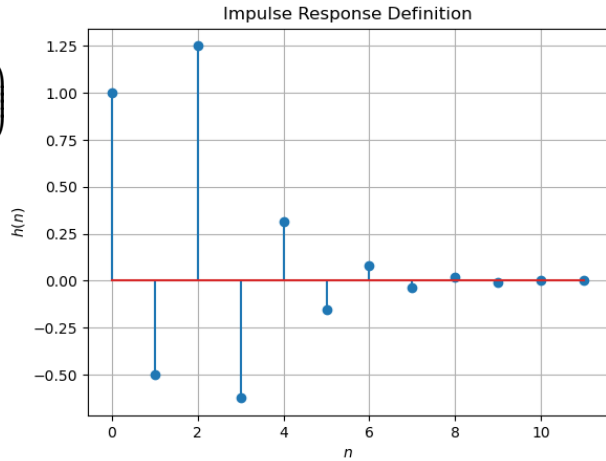
Fig. 5.7: From the definition of $h(n)$

$n = 2$,

$$h(2) + \frac{1}{2}h(1) = 0 + \delta(0) \tag{5.41}$$

$$h(2) = 1 + \frac{1}{4} \tag{5.42}$$

$$= \frac{5}{4} \tag{5.43}$$

And for $n > 2$ RHS will be 0 so,

$$h(n) = -\frac{1}{2}h(n-1) \tag{5.44}$$

Overall

$$h(n) = \begin{cases} 0 & , n < 0 \\ 1 & , n = 0 \\ -\frac{1}{2} & , n = 1 \\ \frac{5}{4} & , n = 2 \\ -\frac{1}{2}h(n-1) & , n > 2 \end{cases} \tag{5.45}$$

5.8 Compute

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{5.46}$$

Comment. The operation is known as *convolution*.
Solution: Download the code for plot from the below link

> wget https://github.com/
> sumeethkumar12Signal_Processing/blob/
> master/Sound%201/Codes/ynconv.py

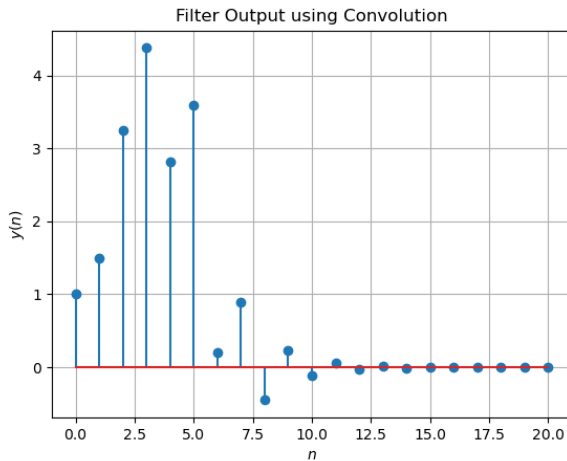Note that the plot is same that as in .

Fig. 5.8: $y(n)$ using the convolution definition

5.9 Express the above convolution using a Toeplitz matrix.
Solution: Download the python code from the below link for the plot,

```
wget https://github.com/
    sumeethkumar12Signal_Processing/blob/
    master/Sound%201/Codes/
    ynconv_toeplitz.py
```

Then run the following command,

```
python3 ynconv_toeplitz.py
```

we express $y(n)$ as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) \tag{5.47}$$

To understand how we can use a Toeplitz matrix, we will see what we are doing in

$$y(0) = x(0) h(0) \tag{5.48}$$
$$y(1) = x(0) h(1) + x(1) h(0) \tag{5.49}$$
$$y(2) = x(0) h(2) + x(1) h(1) + x(2) h(0) \tag{5.50}$$

.

.

The same thing can be written as,

$$y(0) = \begin{pmatrix} h(0) & 0 & 0 & . & . & .0 \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.51}$$

$$y(1) = \begin{pmatrix} h(1) & h(0) & 0 & 0 & . & . & .0 \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.52}$$

$$y(2) = \begin{pmatrix} h(2) & h(1) & h(0) & 0 & . & .0 \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.53}$$

.

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Using Toeplitz matrix of $h(n)$ we can simplify it as,

$$y(n) = \begin{pmatrix} h(0) & 0 & 0 & . & . & .0 \\ h(1) & h(0) & 0 & . & . & .0 \\ h(2) & h(1) & h(0) & . & . & .0 \\ & & .. & & & \\ & & .. & & & \\ 0 & 0 & 0 & . & . & .h(m-1) \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.54}$$

Now we will take n

$$x(n) = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \end{pmatrix} \tag{5.55}$$

And ,

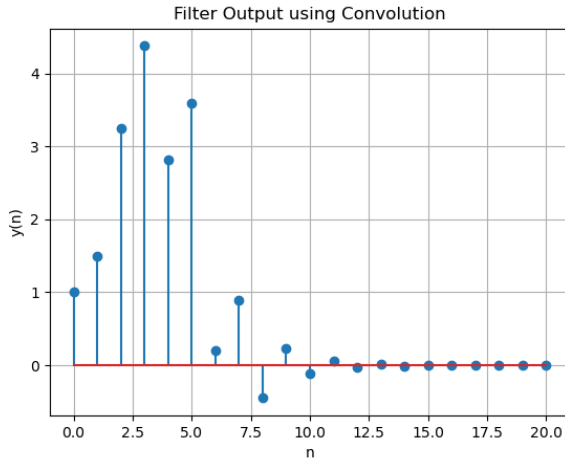$$h(n) = \begin{pmatrix} 1 \\ -0.5 \\ 1.25 \\ . \\ . \end{pmatrix} \tag{5.56}$$

Fig. 5.9: Convolution of $x(n)$ and $h(n)$ using toeplitz matrix

6 DFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{6.1}$$

and $H(k)$ using $h(n)$.

Solution: Run the following codes to compute $X(k)$ which is plotted in fig:6.1.

wget https://github.com/sumeethkumar12/
  signal−processing/blob/main/codes/6.1.py

Run the following codes to compute $H(k)$.

wget https://github.com/sumeethkumar12/
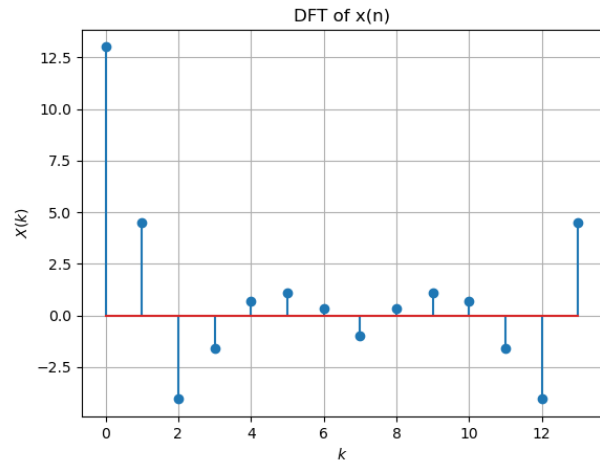 signal−processing/blob/main/codes/6.12.
  py

Now ,

$$y(n) = x(n) * h(n) \tag{5.57}$$

$$= \begin{pmatrix} 1 & 0 & 0 & . & . & .0 \\ -0.5 & 1 & 0 & . & . & .0 \\ 1.25 & -0.5 & 1 & . & . & .0 \\ & & & .. & & \\ & & & .. & & \\ 0 & 0 & 0 & . & . & . \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ . \\ . \\ x(5) \end{pmatrix} \tag{5.58}$$

$$= \begin{pmatrix} 1 \\ 1.5 \\ 3.25 \\ . \\ . \\ . \end{pmatrix} \tag{5.59}$$

Fig. 6.1: DFT of $x(k)$
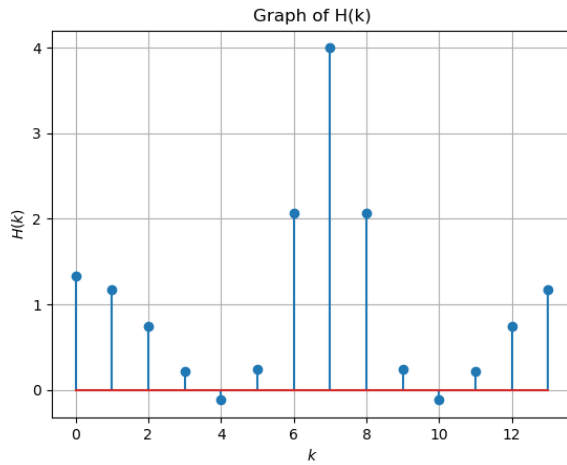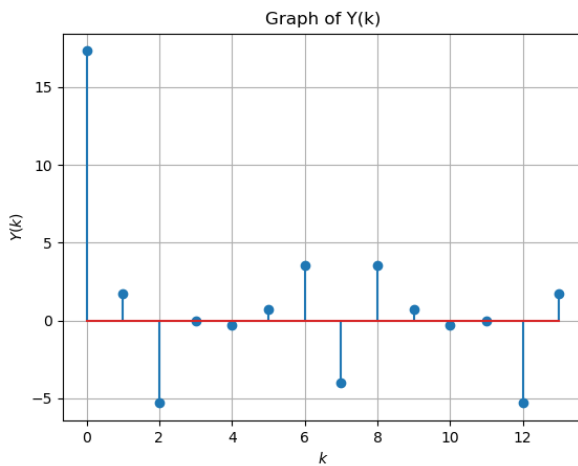
6.2 Compute

$$Y(k) = X(k)H(k) \tag{6.2}$$

Solution: Run the following codes to compute $Y(k)$ respectively.

wget https://github.com/sumeethkumar12/
  signal−processing/blob/main/codes/6.2.py

5.10 Show that

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k) \tag{5.60}$$

Solution: Substitute $k := n - k$, we will get

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{5.61}$$

$$= \sum_{n-k=-\infty}^{\infty} x(n-k)h(k) \tag{5.62}$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)h(k) \tag{5.63}$$

6.3 Compute

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) \cdot e^{j2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \tag{6.3}$$

Solution: The following code plots Fig. fig:6.3. Note that this is the same as $y(n)$ in Fig. fig:3.1

Fig. 6.1: $H(n)$

Fig. 6.3: $y(n)$

Fig. 6.2: DFT of xyn)

wget https://github.com/sumeethkumar12/
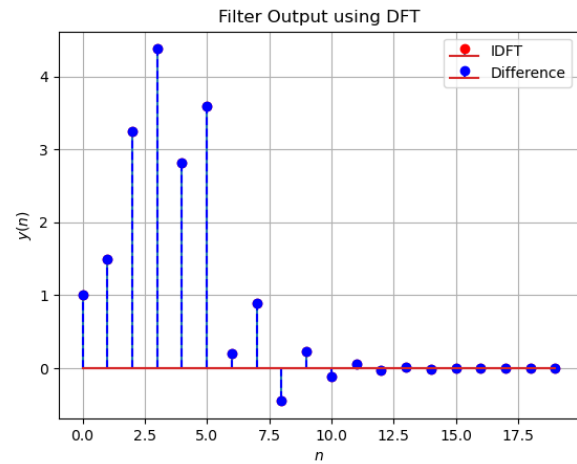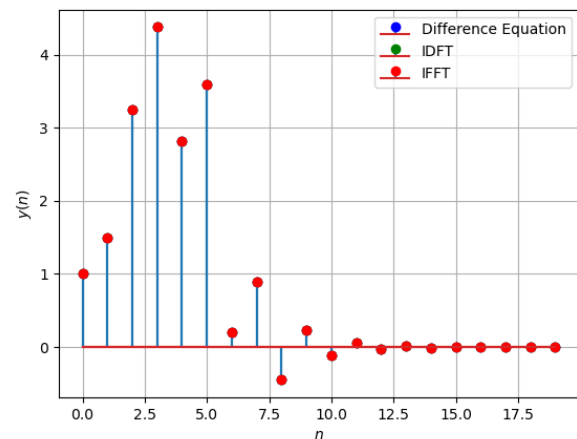signal−processing/blob/main/codes/6.3.py

6.4 Repeat the previous exercise by computing $X(k), H(k)$ and $y(n)$ through FFT and IFFT.
Solution: Download the below python code for the plot **??**,

wget https://github.com/sumeethkumar12/
signal−processing/blob/main/codes/6.4.py

Then run the following command,

python3 yn_ifft.py

Fig. 6.4: The plot of $y(n)$ using IFFT