Digital Signal Processing Assignment 1

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***Abstract*—This submission is part of the assignments from the Oppenhiem Textbook of the course EE-3900 DIgital Signal Processing**

Contents

1 Oppenhiem 3.3-a

1) Determine the z-transform if each of the following sequences, Included with your answer with region of convergence in the z-plane. Express all sums in closed form ; α can be complex.

a) $x_a[n] = \alpha^{|n|}$,$1 > |\alpha| > 0$

Solution: The z transform of $x_b[n]$ is given by

$$x_a[z] = \sum_{n=-\infty}^{\infty} x_a[n] z^{-n} \tag{1.1}$$

$$= \sum_{n=-\infty}^{\infty} \alpha^{|n|} z^{-n} \tag{1.2}$$

$$= \sum_{n=-\infty}^{-1} \alpha^{-n} z^{-n} + \sum_{n=0}^{\infty} \alpha^{n} z^{-n} \tag{1.3}$$

$$= \frac{\alpha z}{1-\alpha z} + \frac{1}{1-\frac{\alpha}{z}} \tag{1.4}$$

$$= \frac{z(1-\alpha^2)}{(1-\alpha z)(z-\alpha)} \tag{1.5}$$

The given sequence $x_a[n]$ is two sided exponential sequence
from above equation it is clear that the z transform $x_a[z]$ converges for

$$|\alpha| < |z| < |\frac{1}{|\alpha|} \tag{1.6}$$

so **the ROC is annular region,** $|\alpha| < |z| < \frac{1}{\alpha}$ where annular region is meant by region between two conc circles.