IEOR E4650 Business Analytics

Session 21: Multiple Objectives and Portfolio Analytics

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Outline

- Decision making with multiple goals
- Risk and reward tradeoff in financial portfolios
  - Benefits of diversification
  - Using optimization to find efficient solutions
- Revisiting financial analytics
  - Using optimization to solve large-scale portfolio problems
- Portfolio approach to search engine marketing

Multiple Goals

- General tradeoffs when making decisions
  - Profit versus market share
  - Profit versus service level
  - Cost versus quality
  - Short term versus long term
- Portfolio construction
  - Investors would like portfolios with high return and low risk

How can we find the best tradeoffs between conflicting goals?

Computing Risk and Return

Harry Markowitz won the Nobel Prize
for his portfolio optimization ideas

Computing Risk and Return of Individual Assets

Table of annual returns

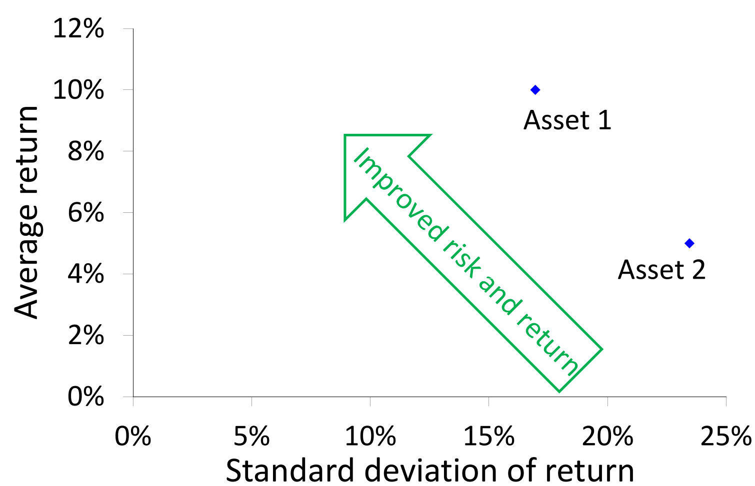
Scenario	Asset 1 return, R_1	Asset 2 return, R_2
1	30%	−20%
2	20%	−20%
3	15%	15%
4	−5%	20%
5	−10%	30%

- Reward: which asset has a higher average return?
- Risk: which asset has a smaller standard deviation of return?

Computing Risk and Return of Individual Assets

	Asset 1 return	Asset 2 return
Average	10.0%	5.0%
Std dev	17.0%	23.5%

Scenario	R_1	R_2
1	30%	−20%
2	20%	−20%
3	15%	15%
4	−5%	20%
5	−10%	30%

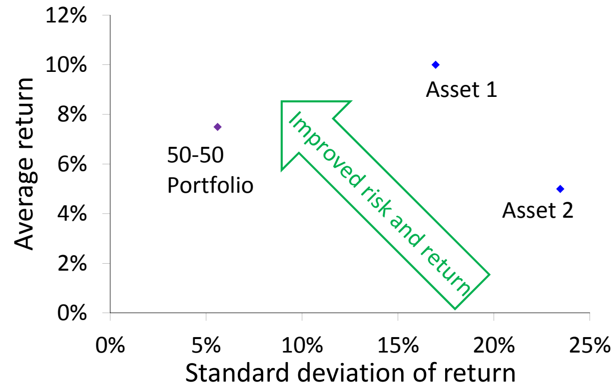


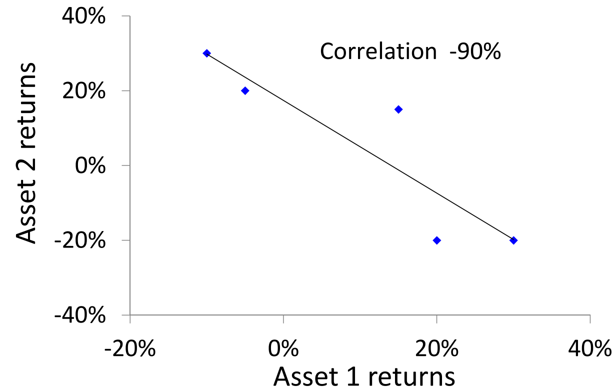
Is there any reason to include asset 2 in your portfolio?

Computing Risk and Return of a Portfolio

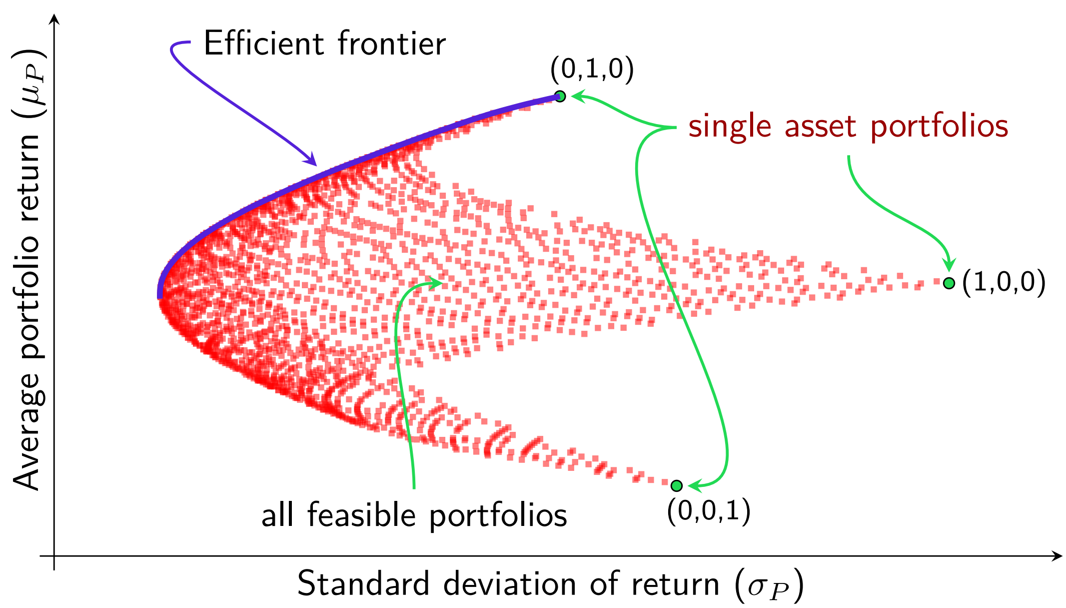
	Asset 1 return R_1	Asset 2 return R_2	50-50 portfolio
Average	10.0%	5.0%	7.5%
Std dev	17.0%	23.5%	5.6%
Scenario	R_1	R_2	$0.5R_1 + 0.5R_2$
1	30%	−20%	5.0%
2	20%	−20%	0.0%
3	15%	15%	15.0%
4	−5%	20%	7.5%
5	−10%	30%	10.0%

- 50-50 portfolio has a smaller risk than either asset 1 or asset 2
- Returns are negatively correlated
- Diversification benefit: diversified portfolios can have less risk





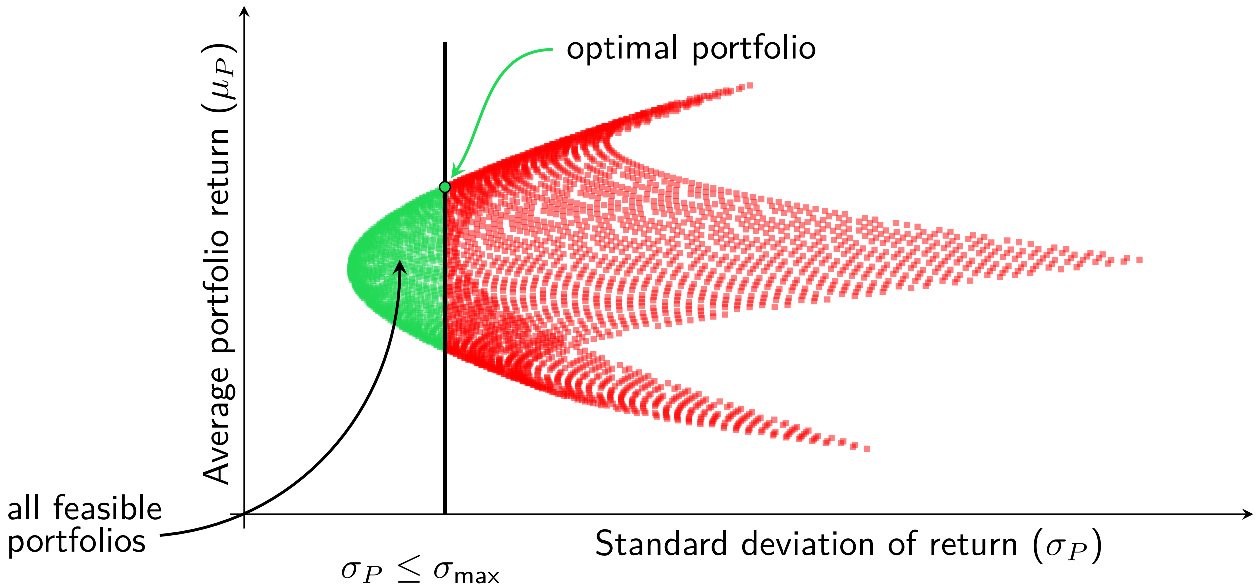
Efficient Frontier: 3 Asset Example



Two conflicting goals

- Maximize expected return
- Minimize standard deviation of return

Using Optimization to Find an Efficient Portfolio

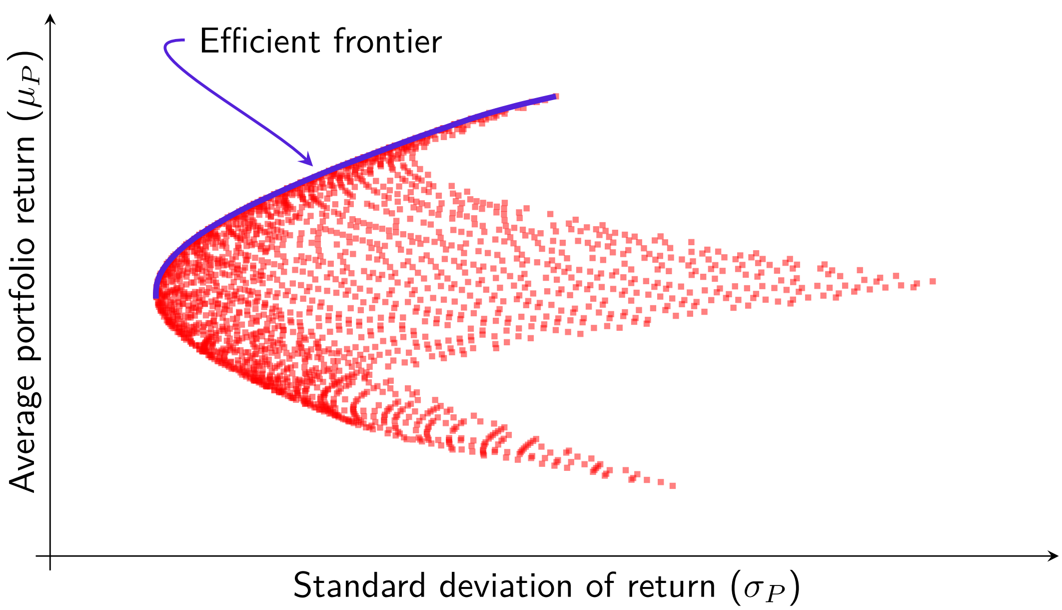


Optimization to find one efficient portfolio

- Optimize one goal (e.g., max expected return)
- Constrain the other goal (e.g., risk limit)
- Decision variables are the portfolio weights

maximize	μ_P
subject to	$\sigma_P \leq \sigma_{\text{max}}$

Efficient Frontier



By varying the risk limit, σ_{max}, and solving for optimal (σ_P, μ_P), we can construct the efficient frontier

(Decision variables are the portfolio weights)

maximize	μ_P
subject to	$\sigma_P \leq \sigma_{\text{max}}$

Portfolio Optimization in More Detail

Portfolio optimization ingredients:

- Random returns of n assets: $R_1, R_2, \ldots, R_n$
- Decision variables: portfolio weights $(w_1, w_2, \ldots, w_n)$
- Objective: maximize expected portfolio return
- Constraints: risk limit, sum of portfolio weights, bounds on portfolio weights

Portfolio optimization model:

$$\begin{array}{lll}
\underset{w_1, w_2, \ldots, w_n}{\text{maximize}} & \mathsf{E}[w_1 R_1 + \cdots + w_n R_n] & \text{[Max expected portfolio return]} \\
\text{subject to} & (1) \;\; \sqrt{\mathsf{Var}[w_1 R_1 + \cdots + w_n R_n]} \leq \sigma_{\mathsf{max}} & \text{[Risk limit]} \\
 & (2) \;\; w_1 + w_2 + \cdots + w_n = 1 & \text{[Portfolio weights sum to one]} \\
 & (3) \;\; l_i \leq w_i \leq u_i, \;\; i = 1, \ldots, n & \text{[Bounds on portfolio weights]}
\end{array}$$

Computing Expected Return for Several Assets

Portfolio weights: $(w_1, w_2, \ldots, w_n)$

Portfolio expected return

- Use stats formula: $\mathsf{E}[aX + bY] = a\mathsf{E}[X] + b\mathsf{E}[Y]$
- 2 assets: $\mathsf{E}[w_1 R_1 + w_2 R_2] = w_1 \mathsf{E}[R_1] + w_2 \mathsf{E}[R_2]$
- n assets: $\mathsf{E}[w_1 R_1 + \cdots + w_n R_n] = w_1 \mathsf{E}[R_1] + \cdots + w_n \mathsf{E}[R_n]$

Interpretation: the expected return of a portfolio is a weighted average of the expected returns of individual assets

Computing Risk for Several Assets

c	ac	bc	c^2
b	ab	b^2	bc
a	a^2	ab	ac
	a	b	c

Portfolio weights: $(w_1, w_2, \dots, w_n)$

Variance of portfolio return

- Use stats formula: $\text{Var}[aX + bY] = a^2\,\text{Var}[X] + b^2\,\text{Var}[Y] + 2ab\,\text{Cov}[X, Y]$
- 2 assets: $\text{Var}[w_1R_1 + w_2R_2] = w_1^2\,\text{Var}[R_1] + w_2^2\,\text{Var}[R_2] + 2w_1w_2\,\text{Cov}[R_1, R_2]$
- n assets: $\text{Var}[w_1R_1 + \cdots + w_nR_n] = w_1^2\,\text{Var}[R_1] + \cdots + w_n^2\,\text{Var}[R_n] +$
  $2w_1w_2\,\text{Cov}[R_1, R_2] + 2w_1w_3\,\text{Cov}[R_1, R_3] + \cdots + 2w_{n-1}w_n\,\text{Cov}[R_{n-1}, R_n]$
- Standard deviation: $\text{Stdev}[X] = \sqrt{\text{Var}[X]}$

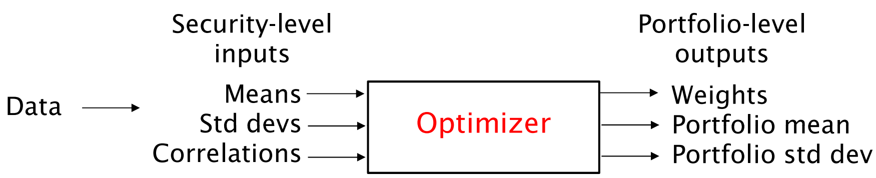
Interpretation: the variance of portfolio return depends on the variances of individual assets and the correlation of returns between all pairs of assets

Computing an Efficient Portfolio in Excel

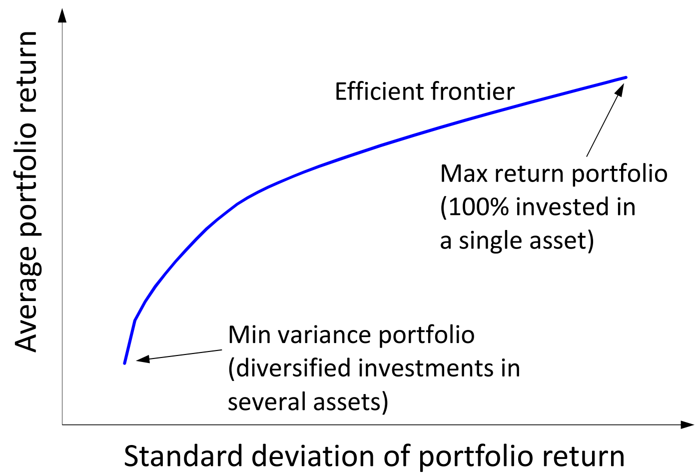
	A	B	C	D	E	F	G
3		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6
4	Expected return	12%	14%	13%	11%	10%	16%
5	Std dev return	18%	15%	17%	12%	11%	14%
6							
7		Correlation matrix of returns					
8		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6
9	Asset 1	100.0%	5.0%	18.8%	48.9%	31.6%	48.7%
10	Asset 2	5.0%	100.0%	34.5%	-7.2%	40.4%	20.3%
11	Asset 3	18.8%	34.5%	100.0%	17.4%	47.1%	23.7%
12	Asset 4	48.9%	-7.2%	17.4%	100.0%	51.4%	49.5%
13	Asset 5	31.6%	40.4%	47.1%	51.4%	100.0%	57.4%
14	Asset 6	48.7%	20.3%	23.7%	49.5%	57.4%	100.0%
15							
16	User defined parameters					Mean variance output	
17	Max std dev	11.0%				Portfolio exp return	15.10%
18	Min weight	0.0%				Portfolio std dev return	11.00%
19	Max weight	100.0%			Portfolio weights:	Asset 1	0.00%
20						Asset 2	30.80%
21						Asset 3	8.07%
22						Asset 4	0.84%
23						Asset 5	0.00%
24						Asset 6	60.29%

- Solve portfolio optimization problems up to 50 assets
- Optional lower and upper bounds on portfolio weights
- Lower bound less than zero allows short selling
- Efficient frontiers from multiple calls to Solver

Steps to Construct an Efficient Frontier



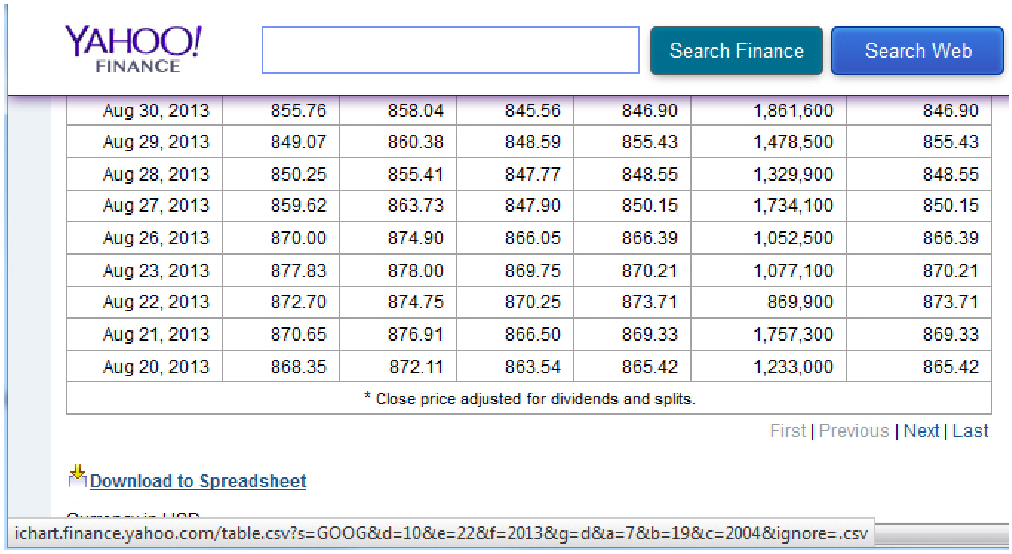
1. Find the max return portfolio:
2. Find the min risk portfolio:
3. Reoptimize by varying $\sigma_{\max}$ between the two extreme standard deviations



Steps to Construct an Efficient Frontier in Excel

1. Find stock price data at: `http://finance.yahoo.com`
2. Import the data into Excel (using Excel's web query)
3. Convert prices to returns
4. Compute average returns, standard deviations and correlations
5. Compute optimal portfolios using the Excel Solver

Getting Data from Yahoo Finance Into Excel



Aug 30, 2013	855.76	858.04	845.56	846.90	1,861,600	846.90
Aug 29, 2013	849.07	860.38	848.59	855.43	1,478,500	855.43
Aug 28, 2013	850.25	855.41	847.77	848.55	1,329,900	848.55
Aug 27, 2013	859.62	863.73	847.90	850.15	1,734,100	850.15
Aug 26, 2013	870.00	874.90	866.05	866.39	1,052,500	866.39
Aug 23, 2013	877.83	878.00	869.75	870.21	1,077,100	870.21
Aug 22, 2013	872.70	874.75	870.25	873.71	869,900	873.71
Aug 21, 2013	870.65	876.91	866.50	869.33	1,757,300	869.33
Aug 20, 2013	868.35	872.11	863.54	865.42	1,233,000	865.42

* Close price adjusted for dividends and splits.

First | Previous | Next | Last

Download to Spreadsheet

ichart.finance.yahoo.com/table.csv?s=GOOG&d=10&e=22&f=2013&g=d&a=7&b=19&c=2004&ignore=.csv

- Yahoo finance | quotes | historical prices: has a "download to spreadsheet" button
- Process can be automated using the "web query" feature of Visual Basic (VBA)

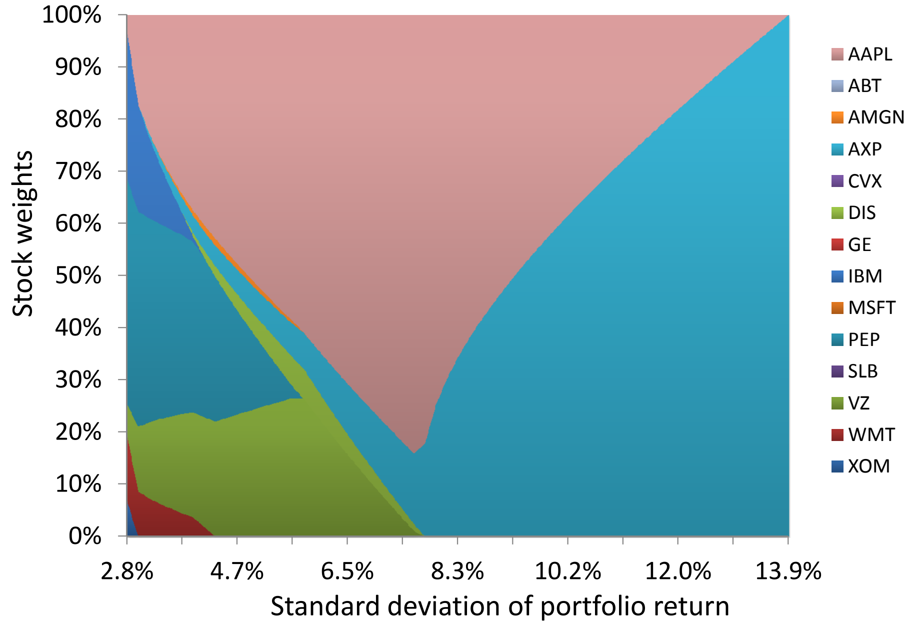
Getting Data from Yahoo Finance Into Excel

The key is to look at the URL used by the "download to spreadsheet" button:

```
http://real-chart.finance.yahoo.com/table.csv
                        ?s=AAPL&a=07&b=19&c=2004&d=10&e=22&f=2013&g=d
```

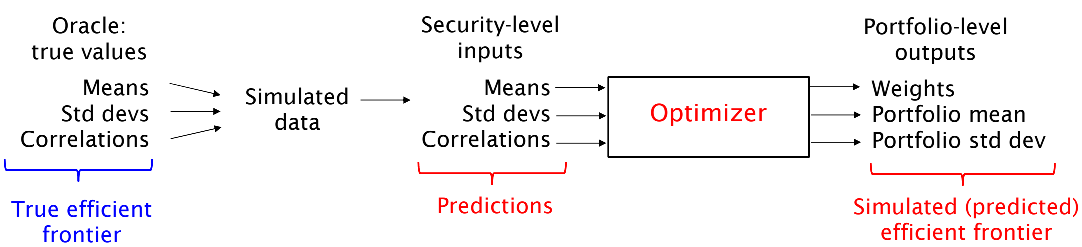
s=AAPL	ticker
a=07	start month, '00'=January
b=19	start day
c=2004	start year
d=10	end month
e=22	end day
f=2013	end year
g=d	frequency, 'd'=daily data

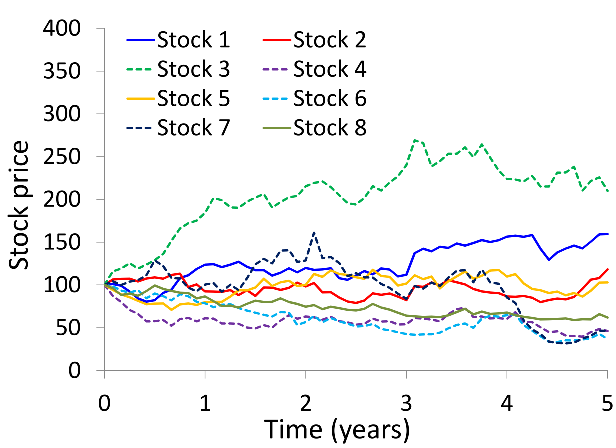
14 Asset Example: Visualizing the Efficient Portfolios



- Minimum risk: diversified portfolio invested in 6 stocks (AAPL, IBM, PEP, VZ, WMT and XOM)
- Maximum return: specialized portfolio 100% invested in 1 stock (AXP)

Testing the Portfolio Optimization Process with Simulation

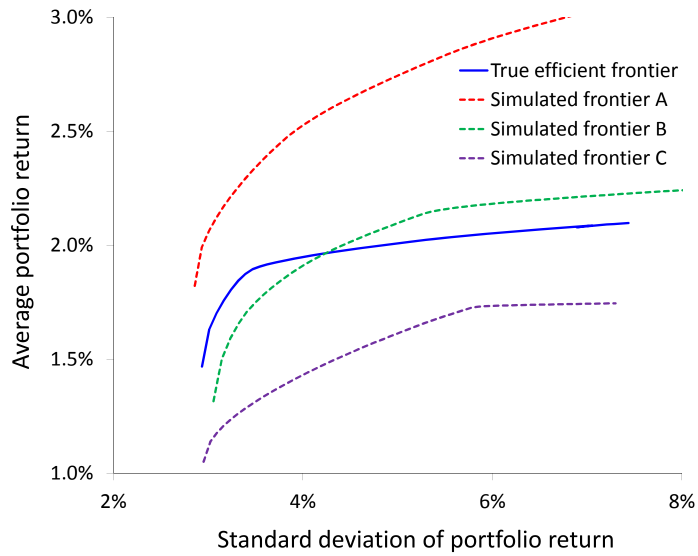




Monte Carlo Simulation can be used to test the portfolio optimization process

- Oracle provide a "true" efficient frontier that is never known in practice
- Simulation creates data that mimics the historical data we use in practice
- Simulation experiment can be repeated to see possible patterns or biases

Testing the Portfolio Optimization Process with Simulation



Question: Which simulated efficient frontier is more likely?

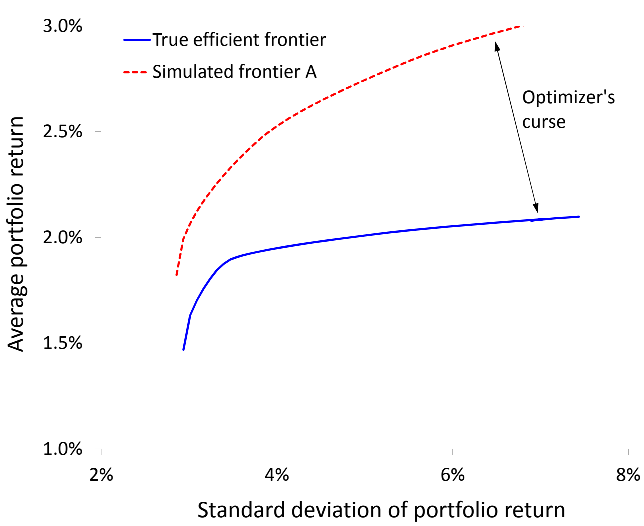
(A) The red frontier: above the true frontier

(B) The green frontier: nearly equal to the true frontier

(C) The purple frontier: below the true frontier

(D) The three simulated frontiers are equally likely

Why are the Frontiers with Simulated Data Biased?



- Suppose we have eight asset classes, with equal mean returns

$$\mu_1 = \mu_2 = \cdots = \mu_8 = \mu$$

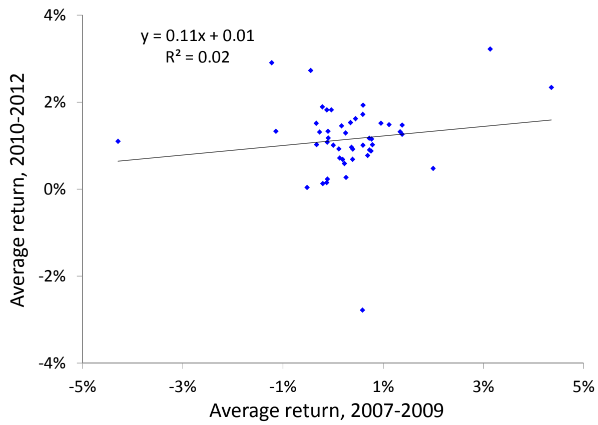
- Simulating returns gives predicted average returns

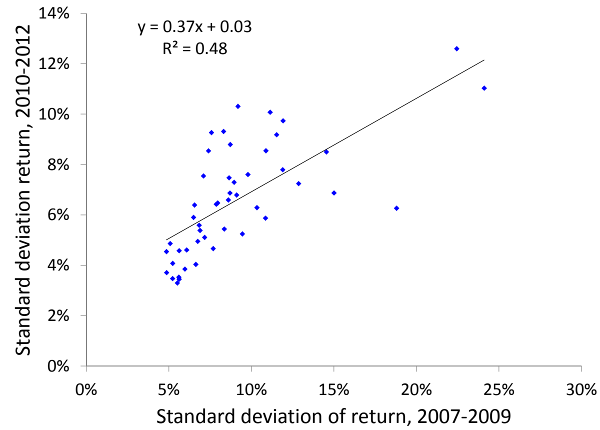
$$\bar{R}_1, \bar{R}_2, \ldots, \bar{R}_8$$

- It is very likely that one asset will have $\bar{R}_i \gg \mu$

- The maximum return portfolio is 100% invested in the single security with the largest predicted average return (i.e., the largest $\bar{R}_i$)
- The maximum return portfolio on the simulated (predicted) efficient frontier is very likely to like above the true efficient frontier
- Optimizer's curse: extra weight is put on securities with large estimation errors

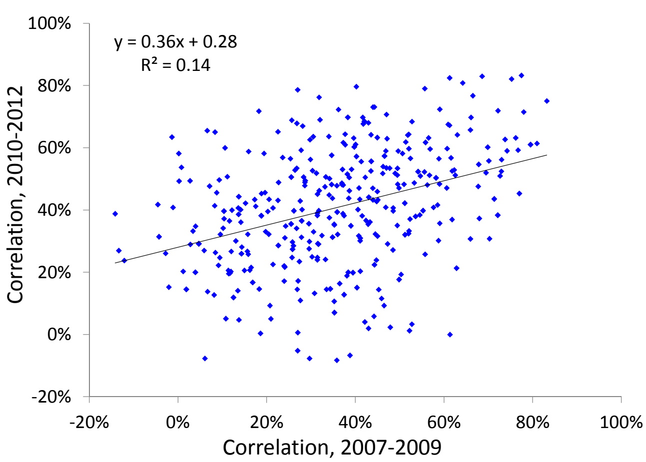
Luck and Skill: Predicting Means and Standard Deviations

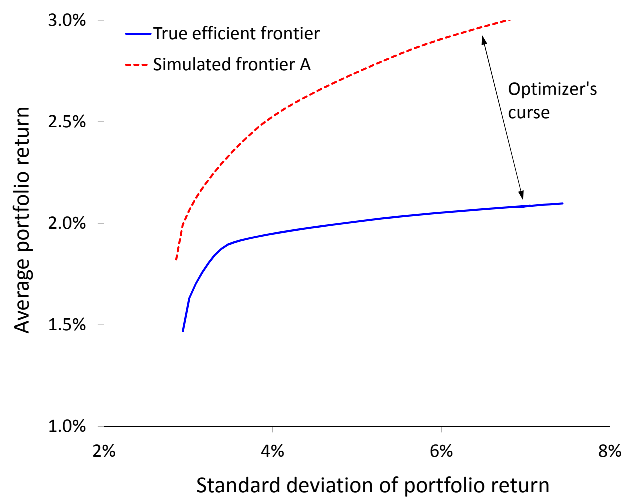




- Predicting mean returns is hard!
- Predicting standard deviations of returns (volatility) is relatively easy

Luck and Skill: Predicting Correlations



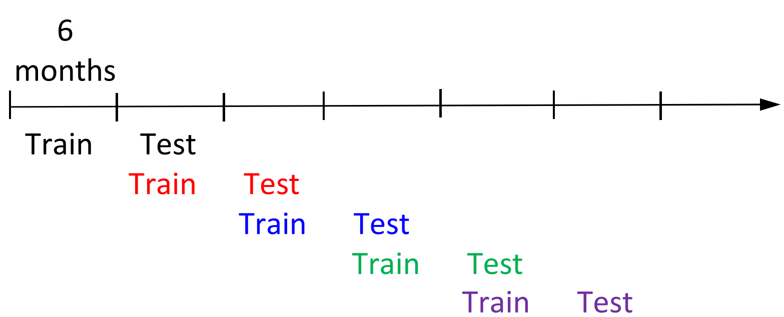


- Predicting correlations is relatively easy (comparable to predicting standard deviations)
- Simulated (predicted) frontiers are close to the true frontier at the minimum risk end of the frontiers
    - Because the minimum risk end of the frontier only depends on standard deviations and correlations, not on mean returns

Mitigating the Optimizer's Curse

- Robust optimization (Axioma)
  - Include prediction errors in the optimization
- Use better predictors of average returns
  - Blind use of hist. average returns is dangerous
  - It doesn't make sense to predict negative expected returns in the future for all stocks that went down in the past
  - It's not likely that the stock that went up the most in the past will go up the most in the future
- Bounds on portfolio weights
- Split data into training and test sets (session 5)

Financial Analytics Revisited

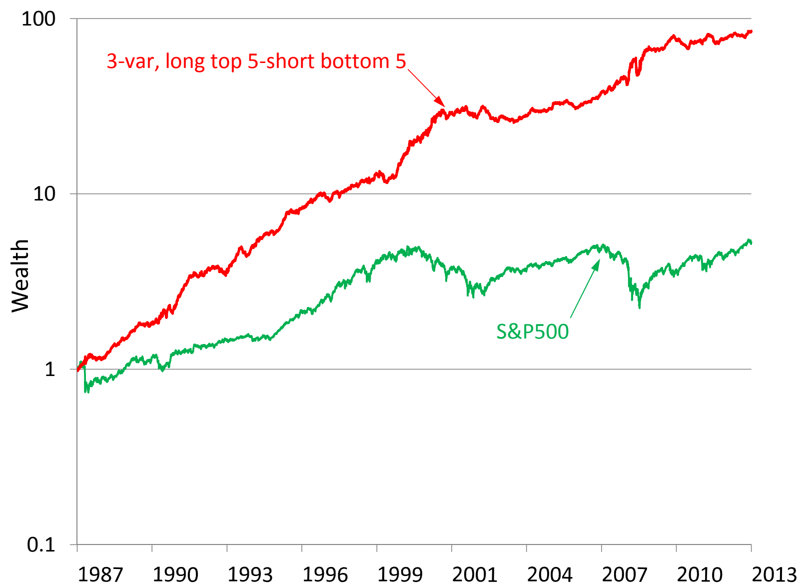


- Three prediction variables: 1D (k =1), 1W (k =5), 2W (k =10)
- Estimate prediction equations (one for each stock) using 6 months of data

$$Y = \beta_0 + \beta_1(\text{1D}) + \beta_2(\text{1W}) + \beta_3(\text{2W}) + \text{error},$$

  where Y is the next day's predicted return
- Each day in the test set, use predictions to rank stocks
- Form portfolio: long top 5 stocks, short bottom 5 stocks
- Re-estimate the regression equations (train) every 6 months

Financial Analytics Revisited



- S&P500: avg return: 9%, std dev: 19%
- Three variables: 1D (k =1), 1W (k =5), 2W (k =10)
- Long top 5 - short bottom 5 stocks: avg return: 18%, std dev: 13%
- How could we do better? Portfolio optimization!

Long-Short Portfolio Optimization

$$\begin{array}{lll} \underset{w_1, w_2, \ldots, w_n}{\text{maximize}} & \text{Expected portfolio return} & \\ \text{subject to} & (1) & \text{Portfolio std dev of return} \leq \sigma_{\text{max}} \\ & (2) & \text{Dollar long position} = \text{Dollar short position} \\ & (3) & l_i \leq w_i \leq u_i \end{array}$$

- Decision variables: portfolio weights $(w_1, w_2, \ldots, w_n)$
- Expected return

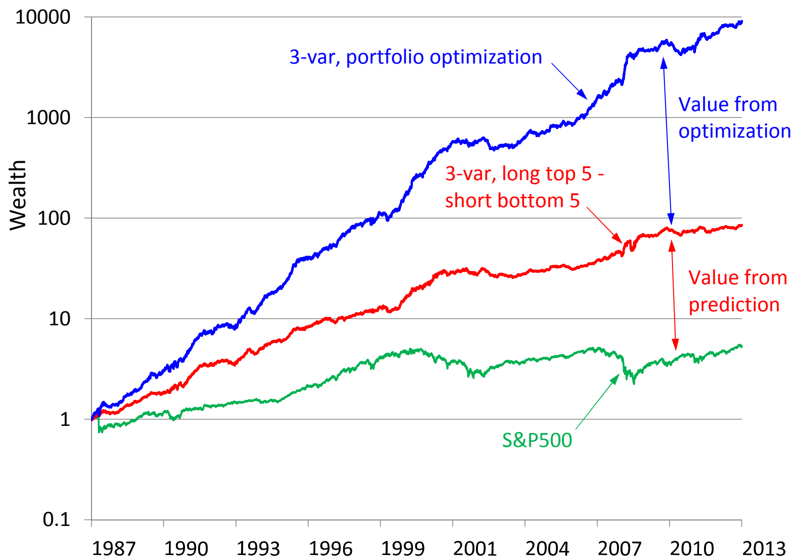
$$\sum_{i=1}^{50} w_i Y_i,$$

where Y_i is next day's predicted return of stock i

- Constraints
  (1) Risk (standard deviation of portfolio return) is bounded
    - Previous 6 months of data are used to estimate standard deviations and correlations

  (2) Portfolio is "market neutral" (longs equal shorts)
  (3) Lower and upper bounds on portfolio weights

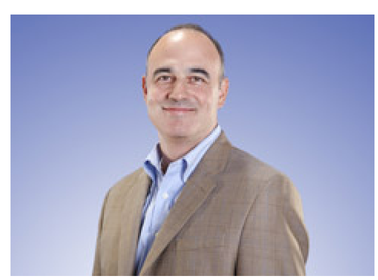
Long-Short Portfolio Optimization



- S&P500: avg return: 9%, std dev: 19%
- Long top 5 - short bottom 5 stocks: avg return: 18%, std dev: 13%
- Long-short optimization: avg return: 37%, std dev: 19%
  (Long-short optimization uses the 3-variable regression for predicted returns)

Axioma

Sebastian Ceria, CEO
Former CBS professor



Rob Bender, CFO
CBS 1996

Clients include: hedge funds, pension funds, investment banks, asset management firms

Axioma website: www.axioma.com

Porfolio Approach to Search Engine Marketing

Decision Variables: marketing “portfolio”

- which search keywords
- how much to bid per click by keyword
- which position to place ads

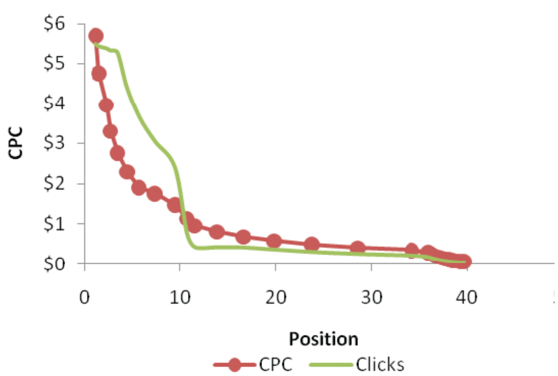
Predictive Models: (by keyword)

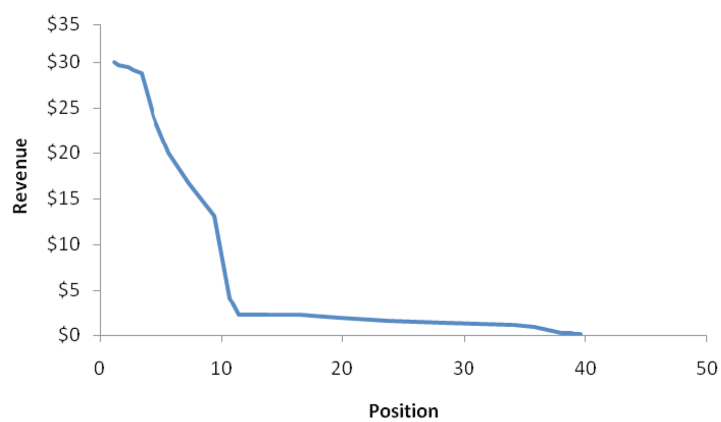
- number of impressions
- required bids, click probablity
- conversion probability, revenue

Optimization Objectives:

- maximize revenue/orders/registrations
- minimize cost spent on SEM

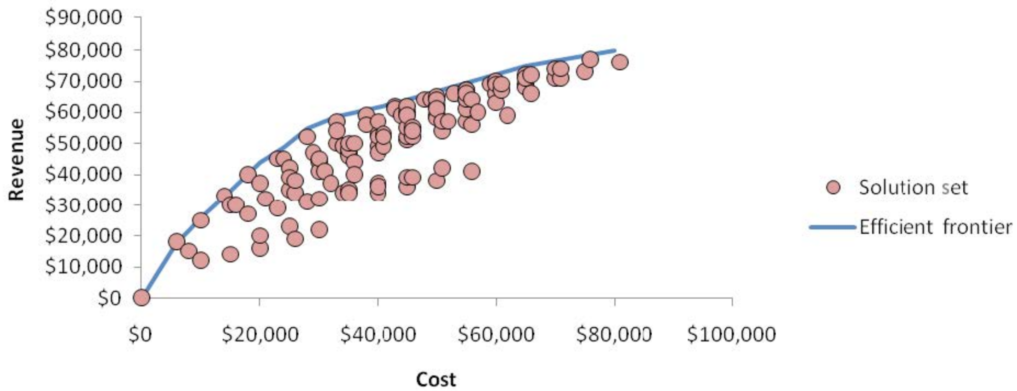
Costs and Revenues based on Position





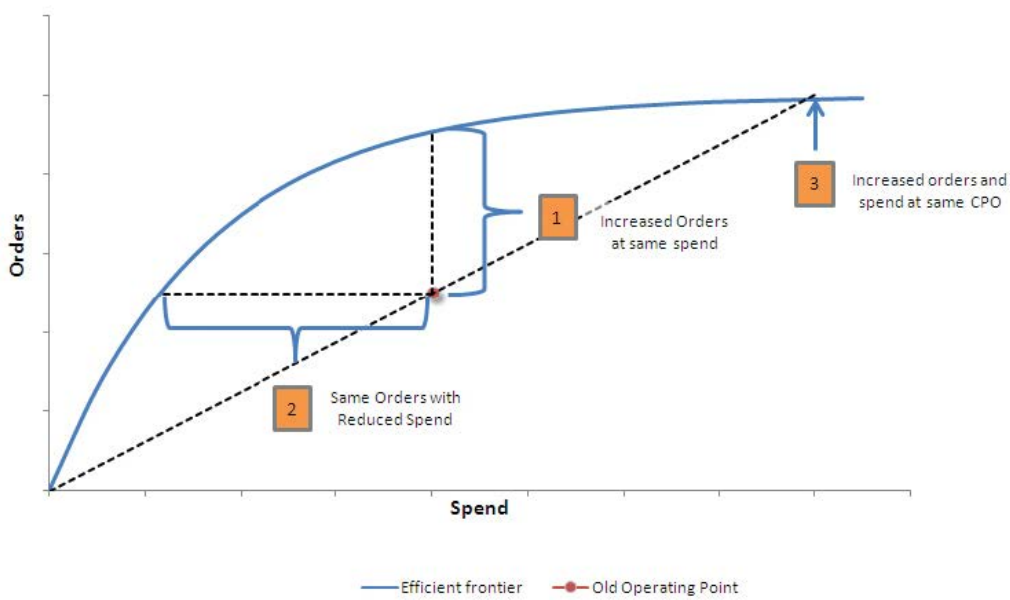
(See “Algorithms and optimization for SEM”, Adobe Corp.)

Porfolio Approach to Search Engine Marketing



(See "Algorithms and optimization for SEM", Adobe Corp.)

Efficient Frontier Lift



(See "Algorithms and optimization for SEM", Adobe Corp.)

Wrap Up

Decision making with multiple goals

- Many applications
  - Risk and reward; short term and long term; cost and quality
- Find efficient strategies: optimize one goal and constrain the other
  - construct efficient frontier

Optimizer's curse

- Optimization results can be overly optimistic
  - Optimization will tend to put higher weight on decisions with large prediction errors
- Testing can be used to develop more realistic expectations of performance

Financial investing

- Estimate expected returns
  - Fundamental analysis (corporate finance course)
  - Predictive analytics
- Portfolio construction and re-balancing
  - Systematic approach using optimization