Adapting to Changing Signal Statistics Using Compressed Measurements

B.Tech Project

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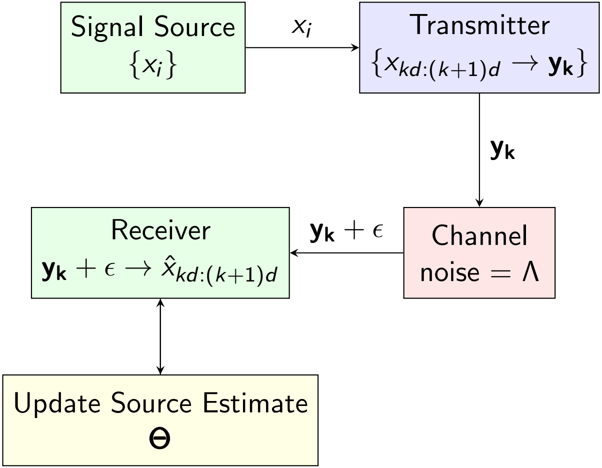
Introduction



Introduction

1. Our Source generates a continuous stream of data and forwards it to the transmitter
2. The transmitter considers a sequence of d data points, compresses it using $0-1$ random matrices and sends it over the channel
3. The receiver receives the noisy signal and reconstructs it using Statistical Compressed Sensing. It has an estimate of the source trained on uncompressed samples from the source
4. On detecting drift in the source distribution, the receiver proceeds to update its source estimate

Gaussian Mixture Model

Definition 1

$$\boldsymbol{\Theta} = \bigcup_{k=1}^{K} \{\pi_k, \mu_k, D_k\} \quad \text{where } k \in 1, 2, \ldots, K$$

$$p_{\boldsymbol{\Theta}}(x) = \sum_{k=1}^{K} \pi_k \mathcal{N}(x|\mu_k, D_k)$$

We assume that a sequence patch-size d data points of the input can be modelled by a GMM and hence, use uncompressed data from the source to train our source distribution. The GMM is trained using the well-known Expectation Maximisation (EM) algorithm

Statistical Compressed Sensing

$$\text{Suppose} \quad \mathbf{x} \in \mathbb{R}^d \sim GMM(\Theta)$$
$$\boldsymbol{\phi} \in \mathbb{R}^{m \times d} \quad \text{where } m < n$$
$$\mathbf{y} = \boldsymbol{\phi}\mathbf{x} + \boldsymbol{\epsilon}$$

In previous work [6], a Piecewise Linear Estimate [5] was used to reconstruct the patch. However, it has been shown in [4] that a linear combination of the estimates using the weights of the components minimises the MSE of the reconstructions. We assume that the noise is zero-mean with a diagonal covariance matrix Λ

Reconstruction

As per [4], the MMSE estimate $\hat{x}$ is

$$\hat{\mathbf{x}}_{\text{MMSE}}(\mathbf{y},\boldsymbol{\Theta}) = \sum_{k=1}^{K} \rho_k(\mathbf{y},\boldsymbol{\Theta})\boldsymbol{\eta}_k(\mathbf{y},\boldsymbol{\Theta})$$

$$\boldsymbol{\eta}_z(\mathbf{y},\boldsymbol{\Theta}) = \boldsymbol{\mu}_z + \mathbf{D}_z\phi'(\boldsymbol{\Lambda} + \phi\mathbf{D}_z\phi')^{-1}(\mathbf{y} - \phi\boldsymbol{\mu}_{\boldsymbol{z}})$$

$$\mathbf{C}_z(\mathbf{y},\boldsymbol{\Theta}) = \mathbf{D}_z - \mathbf{D}_z\phi'(\boldsymbol{\Lambda} + \phi\mathbf{D}_z\phi')^{-1}\phi\mathbf{D}_z$$

$$\mathbf{R}_z(\boldsymbol{\Theta}) = \boldsymbol{\Lambda} + \phi\mathbf{D}_z\phi'$$

$$\rho_z(\mathbf{y},\boldsymbol{\Theta}) = \frac{\pi_z\mathcal{N}(\mathbf{y};\phi\boldsymbol{\mu}_{\boldsymbol{z}},\mathbf{R}_{\mathbf{z}}\boldsymbol{\Theta})}{\sum_{l=1}^{K}\pi_l\mathcal{N}(\mathbf{y};\phi\boldsymbol{\mu}_{\boldsymbol{l}},\mathbf{R}_{\mathbf{l}}\boldsymbol{\Theta})}$$

Drift Detection

We use cross-validation as a measure to detect drift in the source distribution. Suppose we take m measurements of the signal x, then we only use 80% of the measurements for reconstruction, and the remaining measurements are used to calculate cross-validation error. Suppose the mean cross-validation error was $\boldsymbol{\epsilon}_{cv}$ on the test set of the data set the GMM was trained on, and σ_{cv} was the standard deviation. Then we set threshold as $t = \boldsymbol{\epsilon}_{cv} + 0.5\sigma_{cv}$. If for a continuous sequence of samples, the cross-validation error exceeds this threshold, then we flag a drift in the distribution.

Updating the Source Estimate

Suppose our original estimate of the source GMM was Θ; however, over time, its parameters changed to Θ'. We wish to update our estimate using the compressed measurements $\mathbf{y}$. In [4], a method based on the EM algorithm was presented to solve this problem. The updates are given by

$$\pi_k^{(t)} = \frac{\sum_{i=1}^{N} \rho_{ik}^{(t-1)}}{\sum_{l=1}^{K} \sum_{i=1}^{N} \rho_{il}^{(t-1)}} \quad \mu_k^{(t)} = \frac{\sum_{i=1}^{N} \rho_{ik}^{(t-1)} \boldsymbol{\eta}_{ik}^{(t-1)}}{\sum_{i=1}^{N} \rho_{ik}^{(t-1)}}$$

$$\mathbf{D}_k^{(t)} = \frac{\sum_{i=1}^{N} \rho_{ik}^{(t-1)} [(\boldsymbol{\eta}_{ik}^{(t-1)} - \boldsymbol{\mu}_k^{(t)})(\boldsymbol{\eta}_{ik}^{(t-1)} - \boldsymbol{\mu}_k^{(t)})' + \mathbf{C}_{ik}^{(t-1)}]}{\sum_{i=1}^{N} \rho_{ik}^{(t-1)}}$$

Synthetic Data

We started with a random GMM and sampled 10000 samples from it to train our source estimate. Following this, we compressed the data using 10 random $0-1$ matrices and performed reconstruction using SCS. The following are some plots of the reconstruction and the RMSE using different methods and at different compression ratios

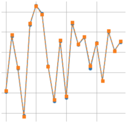


Figure: CR=1.25

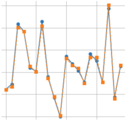


Figure: CR=1.67

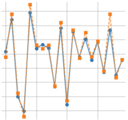


Figure: CR=2.5

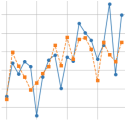


Figure: CR=5

Synthetic Data

	Mean RMSE with method		
Compression	Min MSE	Max Max	DCT
5	5.265	7.770	7.189
2.5	2.278	2.677	6.203
2	1.527	1.579	5.798
1.67	1.124	1.128	4.939
1.25	0.549	0.549	3.48

Table: Comparison of reconstruction methods

	Mean RMSE (min MSE method)	
Compression	Updated Estimate	Old Estimate
5	5.59	5.60
2.5	2.58	2.61
2	1.89	1.92
1.67	1.48	1.53
1.25	0.74	1.07

Table: RMSE pre & post update

Following this, we introduce drift synthetically in the source GMM by adding drift to the means of the components whose norm was at most 20% of the norm of the mean. The covariance matrices of the components remained the same. We then updated our source estimate using 2000 compressed samples from this source. Then, we compared the reconstructions using our updated estimate and the old estimate.

Synthetic Data - Post Update

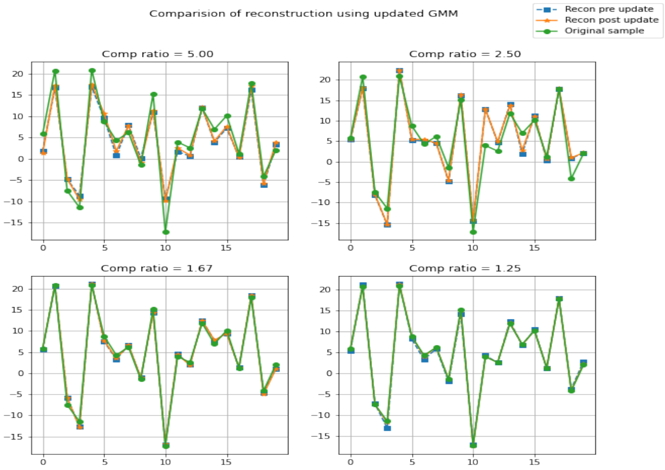


Figure: Comparison of reconstructions pre & post update

Aluminium Panel Dataset

We consider an aluminium panel with four symmetric points on it. An input was given on one of the ends, and the output response was measured on the other 3. Following this, a damage was introduced in the form of a rivet hole in the centre. The experiment was conducted at various temperatures.

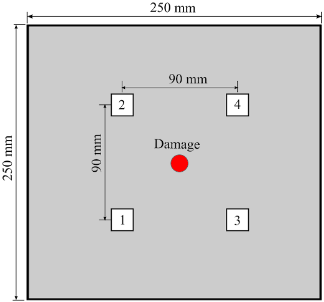


Figure: Aluminium Panel

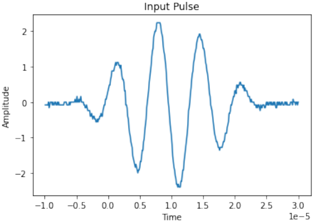


Figure: Input Signal

Aluminium Panel Dataset

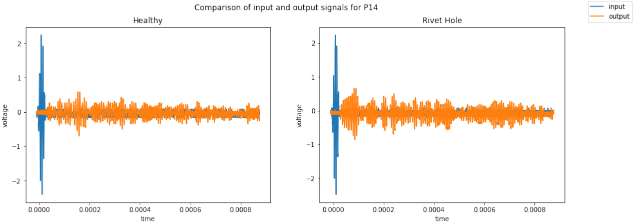


Figure: Healty and Rivet Hole Panel response

Now we considered patches of different sizes $d = 50, 100, 150$ and train our source estimate of the healthy panel response. We then reconstruct compressed samples and observe the PSNR. Following this, we used the source estimate to reconstruct compressed samples of the damaged panel and observed the PSNR.

Aluminium Panel Dataset - PSNR

	PSNR with Percent measurements - healthy signal									
Temp	10	20	30	40	50	60	70	80	90	100
20	29.37	31.204	32.40	33.13	33.717	34.425	35.193	36.083	37.291	38.755
30	29.823	33.52	34.49	34.969	35.58	36.40	37.10	38.07	39.195	40.65
40	29.28	31.86	32.504	33.185	33.82	34.544	35.25	36.07	37.13	38.689
50	30.044	32.14	33.23	33.829	34.460	35.143	35.83	36.70	37.78	39.25
60	29.15	32.243	33.25	33.973	34.609	35.15	35.940	36.89	38.08	39.735

Table: PSNR on the reconstruction of healthy test set

	PSNR with Percent measurements - damaged signal									
Temp	10	20	30	40	50	60	70	80	90	100
20	32.89	35.481	36.727	37.385	38.002	38.682	39.436	40.283	41.401	42.844
30	30.621	34.685	35.574	36.134	36.765	37.428	38.139	39.009	40.146	41.784
40	22.47	32.567	33.60	34.403	35.341	36.140	36.897	37.874	39.076	40.719
50	30.323	32.508	33.880	34.471	35.055	35.750	36.50	37.365	38.43	39.96
60	26.63	31.515	32.628	33.289	33.880	34.48	35.37	36.343	37.420	38.85

Table: PSNR on the reconstruction of damaged test set

Aluminium Panel Dataset - PSNR

We observe that the PSNR for reconstructing the damaged signals is comparable to that of the healthy signals. This means that our model is accurately representing the distribution of the damaged signal patches as well. Hence, we weren't able to detect drift in this case.

To detect drift in larger time scales, we tried to increase the patch size to $d = 300$. However, this resulted in some of the covariance matrices of the components of the GMM being singular, which causes issues in reconstruction. $|\Sigma_z| = 0$ would make the likelihood of any sample ∞

Here are some of the reconstructions at different compression ratios

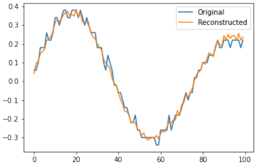


Figure: Healthy signal at CR=2

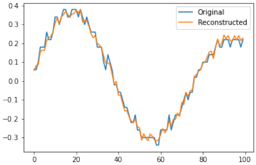


Figure: Healthy signal at CR=1.25

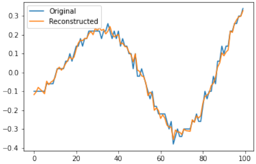


Figure: Damaged signal at CR=2

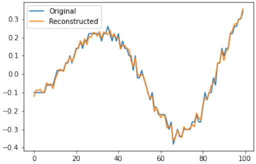


Figure: Damaged signal at CR=1.25

Conclusion

We presented a model of a transmitter-receiver system which uses compressed sensing to reduce the bandwidth requirements. By modelling the consecutive samples of the signal as being drawn from a GMM, we could use Statistical Compressed Sensing to get accurate reconstructions of the signals. We also present the experiments done to validate the theoretical results.

In previous work [6], the method for GMM update after detecting drift relied on the transmitter sending uncompressed data. It has been updated to use compressed measurements only. The setting of the threshold was also quite ad-hoc, and we tried to set it empirically by using the mean and variance of the cross-validation error.

There need to be improvements in the method to detect drift since the choice of threshold in our experiments is still data dependent. Also, we need to prove that cross-validation is a good measure to detect drift.

Thank You

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