ANLP Assignment 1

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1. (exercise 2.3, page 59) Compute the probability of the event "A period occurs after a three-letter word and this period indicates an abbreviation," assuming the following probabilities:
a. P(is-abbreviation/ three-letter-word) = 0.8
b. P(three-letter-word) = 0.0003

Ans) The conditional probability of is-abbreviation given that it's a three letter word is 0.8 and probability and the probability of a three letter word is 0.0003.

So, the probability of (A period occurs after a three-letter word and this period indicates an abbreviation) is -
P(is-abbreviation/ three-letter-word) x P(three-letter-word) = 0.8 x 0.0003=0.00024

2. (exercise 2.4, page 59) Are X and Y as defined in the following table independently distributed?
x 0 0 1 1
y 0 1 0 1
p(X=x,Y=y) 0.32 0.08 0.48 0.12

Ans)
We know that X and Y are independent if P(X$\cap Y) = P(X) * P(Y)$
From the above table –
$P(X = 0) = 0.32 + 0.08 = 0.4$
$P(X = 1) = 0.48 + 0.12 = 0.6$
$P(Y = 0) = 0.32 + 0.48 = 0.8$
$P(Y = 1) = 0.08 + 0.12 = 0.2$

$P(X=0)*P(Y=0) = 0.4*0.8 = 0.32$
$P(X=0)*P(Y=1) = 0.4*0.2 = 0.08$
$P(X=1)*P(Y=0) = 0.6*0.8 = 0.48$
$P(X=1)*P(Y=1) = 0.6*0.2 = 0.12$

Since P(X)*P(Y) is same as P(X$\cap Y$)in every case, we can say that X and Y are independent.

3. Conditional Probability:

a. If two fair dice are rolled, what is the conditional probability that at least one lands on 1 given that the dice land on different numbers?.

Ans) Total outcomes = 6* 6=36 (6 for each die)
Total outcomes with both numbers being different= 36 - 6=30 (removing (1,1),(2,2)...(6,6))

P ((1st die =1 or 2nd die=1) and both dies are different numbers)=10 (5 for each die)

Therefore conditional probability of at least one lands on 1 given that the dice land on different numbers=P((1st die =1 or 2nd die=1) and both dies are different numbers)/P(Total outcomes with both numbers being different)=10/30=1/3

b. A bin contains 25 light bulbs, 5 of which are in good condition and will function for at least 30 days, 10 of which are partially defective and will fail in their second day of use, and 10 of which are totally defective and will not light up. Given that a randomly chosen bulb initially lights what is the probability that it will still be working after one week?

Ans)
P(the bulb still works after one week / it initially lights)=P(the bulb works after one week and the bulb initially lights)/P(the bulb initially lights)

P(the bulb initially lights)=15/25

P(the bulb works after one week and the bulb initially lights)=5/25

Therefore -
P(the bulb still works after one week / it initially lights)=(5/25)/(15/25)=1/3

4. Bayes Theorem: British and American spelling are 'rigour' and 'rigor', respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 40% of the English-speaking men at the hotel are British and 60% are Americans, what is the probability that the writer is British?
Ans)
P(American)=0.6
P(British)=0.4
P(vowel/American)=2/5=0.4
P(vowel/British)=3/6=0.5

P(vowel)=P(vowel/American)*P(American)+P(vowel/British)*P(British)
=(0.4*0.6)+(0.5*0.4)=0.44
From above,we can calculate the probability of vowel and British as-
$P(British \cap vowel)$=P(British)*P(vowel/British)=0.4*0.5=0.2

Therefore-
P(British/vowel)=$P(British \cap vowel)$/P(vowel)
=0.2/0.44=0.454

5. In the slides, we calculated the entropy for a fair 8-sided die. Let's assume that this 8-sided die is not actually fair, but instead has this distribution:

1 2 3 4 5 6 7 8
(1/8) (1/16) (1/4) (1/8) (1/16) (1/16) (1/4) (1/16)
What is the entropy of this distribution?

Ans)
Formula for entropy-

- $\sum_{x=1}^{8} f(x) * log(f(x))$
for x as the number appear on the die and f(x) as the probability associated with that number

Entropy=-($(1/8) * log(1/8) + (1/16) * log(1/16) + (1/4) * log(1/4) + (1/8) * log(1/8) + (1/16) * log(1/16) + (1/16) * log(1/16) + (1/4) * log(1/4) + (1/8) * log(1/8)$)

=$-((1/8) * (-3) + (1/16) * (-4) + (1/4) * (-2) + (1/8) * (-3) + (1/16) * (-4) + (1/16) * (-4) + (1/4) * (-2) + (1/8) * (-3))$

=-((-0.75)+(-1)+(-1))=2.75