

CSCI B505 – Fall 2018

Written Assignment 4 - Greedy Algorithms:

Due online Nov 8, 2018, 11:59pm EST.

You can use LaTeX, Word, or even pen and paper to write down your answers. But **please try to submit a PDF file.**

1. Suppose you are going to drive from Bloomington to New York. Your gas tank, when full, holds enough gas to travel m miles, and you have a map that gives distances between gas stations along the route. Let $d_1 < d_2 < \dots < d_n$ be the locations of all the gas stations along the route, where d_i is the distance from Bloomington to the i -th gas station. You can assume that the distance between neighboring gas stations is at most m miles.
Your goal is to make as few gas stops as possible along the way. Describe an algorithm that determines which gas stations you should stop and prove the correctness and optimality of your algorithm.
2. You are given n events, each of which takes one minute. Event i will provide a profit of g_i dollars ($g_i > 0$) if it starts at or before time $t_i \in \mathbf{R}^+$. (Note: If an event doesn't start by t_i then there is no benefit in scheduling it at all. All events can start as early as time 0.)
Describe and prove the most efficient algorithm to find a schedule that maximizes the profit.
3. Given a set $x_1 \leq x_2 \leq \dots \leq x_n$ of points on the real line, describe and prove an algorithm that yields the smallest set of unit-length closed intervals that covers all of the points. (A unit-length closed interval can be represented in the form $[n, n + 1]$.)
For example, the interval $[1, 2]$ covers all points x_i such that $1 \leq x_i \leq 2$.
4. Describe and prove an efficient algorithm that makes change for n cents, using the least number of quarters, dimes, nickels and pennies.