

Written Assignment 5 - Graph Algorithms

1. Our goal is to find no. of trees in a forest.

We can find out with the help of below algorithm.

- ① Start from one node and initialize one counter variable to count no. of tree as c.
- ② We can implement DFS (Depth First Search) and traverse through all the nodes connected each other until we encounter a leaf node. A leaf node is a node with no children, so we backtrack as little as possible and find another undiscovered nodes.
- ③ We can use color notations for the above process of discovering nodes.
w = white = undiscovered node
g = grey = discovered but not complete
b = black = Completed
- ④ We will look for white nodes in every grey node discovered. Once every node turned black means we have traversed through the tree, we will increase the counter variable by 1.
- ⑤ We again repeat the ~~pro~~ above process and start from a different undiscovered white node.
- ⑥ When all the nodes are discovered.

and ~~if~~ complete, we will get the no. of trees as well.

Time complexity:-

Since we are using DFS, the time complexity will be $O(V+E)$. ~~and~~

~~Some~~ ~~constant~~ time where $V = \text{no. of vertices}$
 $E = \text{no. of edges}$.

2. We are given a tree with directed tree edges. We need to sort it topologically ~~search~~ without using DFS.

We can solve this with the following algorithm.

Topological sort means we need to arrange all the vertices such that there are no back edges.

- ① First we ~~can~~ need to find out incoming ~~no~~ edges or indegree to each node.
- ② Then append ^{all} the nodes and their indegree in some array.
- ③ Remove the node with indegree as 0 and append it to an array called B , simultaneously reduce the indegree of the nodes by 1, if their edges came from the recently removed node.
- ④ Repeat step 3 till we get all the nodes appended to array B .
- ⑤ Return array B .

→ The time complexity of this sorting will be $O(V+E)$ which is same as a DFS. Here V = vertices, E = edges.

3. We are provided a graph G . We have to write an algorithm that correctly determines if the graph is a DAG or not.

1st approach:

- ① First we need to find out the parent and child nodes.
- ② We can assign different colors to parents and its children.
- ③ We then append the colors along with the nodes into a dictionary.
- ④ If a node contains 2 different colors then we can say that it is not a DAG that means there exist a cycle.

2nd approach:-

- ① We can check each node and save the nodes in an array called visited.
- ② If there exist a node which we encounter in the process who has all the children that are ~~are~~ in visited array already, but then we say the graph is not a DAG.

→ Time complexity of this algorithm is $O(V+E)$ where V =vertices and E =edges in the worst case.

→ The 1st approach will fail if there is even no. of nodes in a cycle.

→ The 2nd approach will fail if one node has 2 incoming nodes from 2 different parent nodes.

3rd Approach:-

→ We can find the cycles by doing a topological sort on the graph (which does not have back edges).

→ We will find indegree as the incoming edges to a node.

→ Then we will append in an array called 'B', the node who has 0 indegree. Then decrease the neighbouring nodes indegree by 1 and remove the node.

→ Repeat the above steps until we get all the nodes.

→ This will give us the DAG, ^{as} we can't topologically sort a cycle). Running time: $O(V+E)$