CSCI B505 – Fall 2018
Written Assignment 1(SOlUTIONS):

Use the formal definitions of big-Oh, etc for the asymptotic complexity questions.

1. Suppose $f(x) = 3x^2+5x+3$ and $g(x) = 2x^3+x-100$. Recall the formal definitions of big-Oh. Write down one combination of constants c, n_0 such that $f(x) = O(g(x))$ and explain why you chose those constants.

   Solutions. By definition, Set c $= 1$, $n_0 = 5$, then for $\forall x \geq n_0$, $c \cdot g(x) \geq f(x)$ because $cg(x) - f(x) = 2x^3 - 3x^2 - 4x - 103$ takes the value $250 - 75 - 20 - 103 = 52$ and its derivative $d(x) = 6x^2 - 6x - 4 > 0$ for $\forall x \geq n_0$. □

2. What is the run time for the following function? Justify your answer.

```
int foo(int n){
    int i,j,k=0;
    for(i = n/2; i <= n; i++){
        for(j = 2; j<= n; j = j * 2){
            k += n/2;
        }
    }
    return k;
}
```

   Solutions. There are two loops in the function. The inner loop contains logarithmically many increments($O(\log n)$), while the outer loop contains linearly many increments ($O(n)$). Hence the total runtime is $O(n \log n)$. □

3. Show that $f(n) = 1/n \in O(1)$ using the formula.

   Proof. Set $c = 1, n_0 = 1$. Then for $\forall n \geq n_0$, $f(n) = \frac{1}{n} \leq 1 = c \cdot 1$. □

4. You are given $f(n) = O(g(n))$ and $f(n) = O(h(n))$. Give an example where $g(n) = O(h(n))$ and where $g(n) \neq O(h(n))$

   Solutions.

$g(n) = O(h(n))$: Let $f(n) = 1$, $g(n) = n, h(n) = n^2$.

$g(n) \neq O(h(n))$: Let $f(n) = 1$, $g(n) = n^2, h(n) = n$. $\square$

5. Compare the following pairs of functions, and show which one is big-Oh of the other one (prove using the definition): $(n^2, 2^n), (2^n, 3^n), (\log n, \log^2 n)$, $(n^{\sqrt{n}}, n^n)$.

   Solutions. $n^2 = O(2^n)$: Set $c = 1, n_0 = 5$. The rest follows by taking the difference between the two functions and finding the derivative of the difference(measuring increment).
   $2^n = O(3^n)$: Set $c = 1, n_0 = 1$. Trivial comparison.
   $\log n = O(\log^2 n)$: Set $c = 1, n_0 = 2$. Trivial comparison.
   $n^{\sqrt{n}} = O(n^n)$: Set $c = 1, n_0 = 2$. Trivial comparison. $\square$