Applied Algorithms
Written Assignment-1

1. $f(x)=3x^2+5x+3$
   $g(x)=2x^3+x-100$

   We need to prove f(x)=O(g(x))

   By law of Big O-
   0<=f(x)<=c*g(x) for positive c,n0 and n>=n0

   Let c=20, n0=5

   f(5)=75+25+3=103
   g(5)=250+5-100=155

   f(5)>0 and f(5)<g(5)
   so f(5)<=20*g(5)
   Therefore 0<=f(x)<=c*g(x) holds true for n>5
   I choose n0=5 so as to negate the -100 in the g(x) and a sufficient large value of c.

2.

```
int foo(int n){
    int i,j,k=0;
       for(i = n/2; i
          <=n;i++){
        for(j = 2; j<= n; j = j * 2){
            k +=
            n/2;
        }
    }
    return k;
}
```

   The inner for loop will run for logn times as we are incrementing the value of j as 2*j.
   The outer for loop will run for n/2 times as we have initialized the value as n/2 and then incrementing it by +1.
   The initialization of variables and return the values steps are executed once each.

   So the run time for this algorithm will be (logn)*(n/2)+2 which is equals to O(nlogn).

3.

$f(n)=1/n$

we need to prove $f(n)=O(1)$

To prove this we need to prove-

$0<=f(n)<=c*g(n)$

For positive c,n0 and n>=n0

Here $c*g(n)=1$

Let c=1 and n0=1

$f(1)=1$

so $0<=f(1)<=1$ holds true for n0=1.

4.

$f(n)=O(g(n))$ and $f(n)=O(h(n))$

Example1 (to prove g(n)=O(h(n)))

Let $f(n)=n$

$g(n)=n^2$

$h(n)=2^n$

It is clear that $n=O(n^2)$ and $n=O(2^n)$ for n>=1.

We need to prove $g(n)=O(h(n))$

Let c=4,n0=5

$g(5)=25$

$h(5)=32$

So $g(n)<=h(n)$ for n>5

That means $0<=g(n)<=c*h(n)$ holds true for n>=5.

Therefore $g(n)=O(h(n))$ for n>=5

Example2 (to prove g(n)!=O(h(n)))

Let $f(n)=n$

$g(n)=n^3$

$h(n)=n\log n$

It is clear that $n=O(n^3)$ and $n=O(n\log n)$ for n>=10.

We need to prove $g(n)!=O(h(n))$

Let c=1 and n0=10

$g(10)=1000$
$h(10)=10$

So $g(n)>h(n)$ for $n=10$
That means $0<=g(n)<=c*h(n)$ holds false.
Therefore $g(n)!=O(h(n))$ holds true.

5.

Pair1(n^2,2^n)

Let $f(n)=n^2$
$g(n)=2^n$

Let $c=1,n0=5$

$f(5)=25$
$g(5)=32$

So $f(n)<=g(n)$ for $n>=5$
That means $0<=f(n)<=c*g(n)$ holds true for $n>=5$.
Therefore $(n^2)=O(2^n)$ for $n>=5$.

Pair2(2^n,3^n)

Let $f(n)=2^n$
$g(n)=3^n$

let $c=1,n0=1$

$f(1)=2$
$g(1)=3$

so $f(n)<=g(n)$ for $n>=1$
That means $0<=f(n)<=c*g(n)$ holds true for $n>=1$.
Therefore $(2^n)=O(3^n)$ for $n>=1$.

Pair3(logn,$\log^2$n)

Let $f(n)=\log n$
$g(n)=\log^2 n$

let $c=1,n0=11$

$f(11)=1.041$
$g(11)=1.083$

so $f(n)<=g(n)$ for $n>=11$
That means $0<=f(n)<=c*g(n)$ holds true for $n>=11$.
Therefore $(\log n)=O(\log^2 n)$ for $n>=11$.

Pair4($n^{sqrt(n)}$,n^n)

Let f(n)= $n^{sqrt(n)}$

g(n)=n^n

let c=1,n0=4

f(4)=4^2=16

g(4)=4^4=256

so f(n)<=g(n) for n>=4

That means 0<=f(n)<=c*g(n) holds true for n>=4.

Therefore ($n^{sqrt(n)}$)=O(n^n) for n>=4.