HOME WORK 5

1. (5.6.7)

(a) $\mu = -5, \sigma = 10$

$P(X<0)$

$= \text{pnorm}(0, -5, 10)$

$= \boxed{0.691}$

(b) $P(X>5)$

$= 1 - \text{pnorm}(5, -5, 10)$

$= \boxed{0.159}$

(c) $P(-3<X<7)$

$= \text{pnorm}(7, -5, 10)$
$- \text{pnorm}(-3, -5, 10)$

$= \boxed{0.306}$

(d) $P(|X+5| < 10)$

$P(-10 < X+5 < 10)$

$= P(-15 < X < 5)$

$= \text{pnorm}(5, -5, 10)$
$- \text{pnorm}(-15, -5, 10)$

$= \boxed{0.683}$

(e) $P(|X-3|>2)$

$= P(-2 > (X-3) > 2)$

$= P(1 > X > 5)$

$= 1 - \text{pnorm}(5, -5, 10)$
$+ \text{pnorm}(1)$

$= \boxed{0.884}$

2. (5.6.8)

(a) $X_1 \sim \text{Normal}(1, 9)$

$X_2 \sim \text{Normal}(3, 16)$

$Y = X_1 + X_2$

$\boxed{\mu_Y = 1 + 3 = 4}$

$\sigma_Y^2 = 3^2 + 4^2$

$\boxed{\sigma_Y^2 = 25}$

(b) $Y = -X_2$

$\boxed{\mu_Y = -3}$

$\sigma_Y^2 = (-4)^2$

$\boxed{\sigma_Y^2 = 16.}$

(c) $Y = X_1 - X_2$

$\boxed{\mu_Y = 1 - 3 = -2}$

$\sigma_Y^2 = 3^2 + (-4)^2$

$\boxed{\sigma_Y^2 = 25}$

(d) $Y = 2X_1$

$\boxed{\mu_Y = 2 \times 1 = 2}$

$\sigma_Y^2 = (2 \times 3)^2$

$\boxed{\sigma_Y^2 = 36}$

(e) $Y = 2X_1 - 2X_2$

$\mu_Y = 2\times 1 - 2 \times 3$

$\boxed{\mu_Y = 2 - 6 = -4}$

$\sigma_Y^2 = (2\times 3)^2 + (2 \times 4)^2$

$= 36 + 64$

$\boxed{\sigma_Y^2 = 100}$

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3. (a) $\mu = 0, \sigma^2 = 1$ (since standard normal distribution)

P(2 random variable $\geq$ 1.96)

$= 1 - [$P(no random variable > 1.96)

+ P(1 random variable > 1.96)$]$

X $\rightarrow$ can take x_1, x_2, x_3, x_4

P(~~random variable~~ X > 1.96)

= 1 − Pnorm(1.96, 0, 1)

= 0.024

P(no random variable > 1.96)

= Pbinom (0, 4, 0.024)

= 0.907

P(1 random variable > 1.96)

= dbinom(1, 4, 0.024)

= 0.089

$\therefore$ P(2 random variable > 1.96)

$= 1 - [0.907 + 0.089]$

$= \boxed{0.004}$

(b) $\bar{x} = \dfrac{x_1 + x_2 + x_3 + x_4}{4}$

$\mu_{\bar{x}} = 0$

$\sigma_{\bar{x}}^2 = \left(\frac{1}{2} \times 1\right)^2 = \frac{1}{4}$

$P(\bar{x} > 1.96)$

$= 1 - \text{Pnorm}(1.96, 0, \frac{1}{2})$

$= \boxed{\cancel{4.427 \times 10^{-05}}} = \boxed{4.427 \times 10^{-5}}$

4.

(a) $Y = x_1 + x_2 + x_3$

$\mu_y = 4 + 8 + 12 = 24$

$\sigma_y^2 = 2^2 + 4^2 + 6^2 = 4 + 16 + 36$

$= 56$

distribution of Y −

~~rnorm(10,000, 24, $\sqrt{56}$)~~

$\boxed{N(\mu_y = 24, \sigma_y^2 = 56)}$ ~~Can take any value~~

(b) $P(Y > 32)$ $\quad y = x_1 + x_2 + x_3$

~~= 1 − Pnorm(32, 24, $\sqrt{56}$)~~

= 1 − Pnorm (32, 24, sqrt(56))

$= \boxed{0.143}$

(c) $y = x_1 + x_2 - x_3$

Distribution of y −

~~rnorm(10,000, 0, $\sqrt{56}$)~~ ~~Can take any value~~

$\boxed{N(\mu_y = 0, \sigma_y^2 = 56)}$

(d) $y = x_1 + x_2 - x_3$

$P(Y > 5)$

~~= 1 − Pnorm(5, 0, $\sqrt{56}$)~~

= 1 − Pnorm (5, 0, sqrt(56))

= ~~0.143~~ $\boxed{0.252}$

5. $p = P(Z \leq -z)$

(a)

$P(-z \leq Z \leq z)$

$= P(Z \leq z) - P(Z \leq -z)$

$= (1-p) - p$

$= \boxed{1-2p}$

(b) From (a)

$P(-z \leq Z \leq z) = 1-2p$ — (1)

Given

$P(-z \leq Z \leq z) = 0.95$ — (2)

From (1) & (2) -

$1-2p = 0.95$

$\Rightarrow 2p = 0.05$

$\Rightarrow p = 0.025$

$\Rightarrow P(Z \leq -z) = 0.025$

qnorm(0.025, 0, 1) = $-z$

(As standard normal $\sim N(0,1)$)

$\Rightarrow \boxed{-1.960 = -z}$

$\Rightarrow \boxed{z = 1.960}$

$\therefore$ The interval is -

$\boxed{[-z, z] = [-1.960, 1.960]}$