Homework 5

S520, Spring 2019

Due at the beginning of class, Tuesday February 12th. Please upload your file to Canvas no later than 1pm on the due date. Late submission will be accepted (but penalized) before the solutions are posted.

Trosset question numbers refer to the hardcover textbook. Show all work. **Use R to find the answers in questions 1, 3, 4 and 5(b). Provide your code and round your final answers to 3 decimal places.**

1. Trosset exercise 5.6.7
2. Trosset exercise 5.6.8
3. Let X_1, X_2, X_3, and X_4 be independent standard normal random variables.
   (a) Find the probability that at least two of the random variables X_1, X_2, X_3, X_4 are greater than 1.96.
   (b) Let $\bar{X}$ be the mean of X_1 to X_4:
   $$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4}{4}.$$
   Find $P(\bar{X} > 1.96)$.
4. Let X_1, X_2 and X_3 be independent and Normally distributed random variables. The mean and standard deviation of X_k are $4k$ and $2k$, respectively (for k = 1, 2, 3).
   (a) What is the distribution of $X_1 + X_2 + X_3$?
   (b) What is the probability that $X_1 + X_2 + X_3 > 32$?
   (c) What is the distribution of $X_1 + X_2 - X_3$?
   (d) What is the probability that $X_1 + X_2 - X_3 > 5$?
5. Understanding the nice symmetry of the Normal distribution can be helpful when dealing with percentiles and probabilities. Suppose $Z \sim N(0, 1)$ and assume $z > 0$.
   (a) Let $p = P(Z \leq -z)$. Write an expression for $P(-z \leq Z \leq z)$ in terms of p.
   (b) Use (a) to find an interval $[-z, z]$ such that $P(-z \leq Z \leq z) = 0.95$.