HOME WORK-4

1 (a) $N = 30$
$m = 10$
$k = 15$

The probability that 6 marked tigers will be observed is -

dhyper(6, 10, 20, 15)
$= \boxed{0.2273}$

(b) Since all the tigers have the same probability of being captured.

$\frac{6}{15} = \frac{10}{N}$ $\left(\frac{\text{observed}}{\text{selected}} = \frac{\text{marked}}{\text{total}}\right)$

$\Rightarrow \boxed{N = 25}$

2 (a) Jim's waiting time is uniformly distributed between (5, 30) minutes.

By formula -

$E(x) = \int_a^b \frac{x}{b-a}\,dx$

$= \frac{b^2 - a^2}{2(b-a)}$

$= \frac{a+b}{2}$

$= \frac{5+30}{2} = \frac{35}{2}$

$\boxed{E(x) = 17.5 \text{ minutes}}$

$SD = \sqrt{\frac{(b-a)^2}{12}}$

$= \frac{b-a}{\sqrt{12}} = \frac{25}{2\sqrt{3}}$

$\boxed{SD = 7.2168}$

(b) $P(X > 10)$

$= 1 - P(X \leq 10)$

$= 1 - \left(\frac{10-5}{30-5}\right)$ $\left(\text{By formula } \frac{x-a}{b-a}\right)$

$= 1 - \frac{1}{5}$

$= \boxed{\frac{4}{5} = 0.80}$

(c) $\frac{P(X < 20)}{P(X > 15)}$ (Since 8:30 Am means 15 minutes waiting time)

$= \frac{F(20) - F(15)}{1 - F(15)}$

$= \frac{\left(\frac{20-0}{25-0}\right) - \left(\frac{15-0}{25-0}\right)}{1 - \left(\frac{15-0}{25-0}\right)}$

$= \frac{\frac{4}{5} - \frac{3}{5}}{1 - \left(\frac{3}{5}\right)}$

$= \frac{\frac{1}{5}}{\frac{2}{5}}$

$= \boxed{\frac{1}{2}}$

(c)

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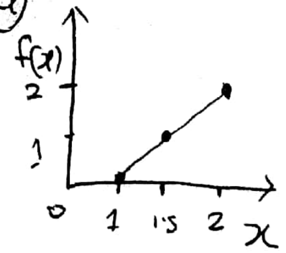
4.

(a)

(5.6.2)

3. (a)



(b) $f(x) = \begin{cases} 0, & \text{if } x<1 \\ 2(x-1), & \text{if } 1<x\leq 2 \\ 0, & \text{if } x>2 \end{cases}$

$0 + 0 + \int_1^2 2(x-1)dx$

$= 2\int_1^2 (x-1)\,dx$

$= 2\left[\frac{x^2}{2}\right]_1^2 - 2[x]_1^2$

$= 3 - 2$

$= 1$

Since sum of all the probabilities is 1, therefore the pdf is valid and $f(x) \geq 0$.

(c) $P(1.50 < x < 1.75)$

$= \int_{1.5}^{1.75} 2(x-1)dx$

$= 2\left[\frac{x^2}{2} - x\right]_{1.5}^{1.75}$

$= [x^2]_{1.5}^{1.75} - 2[x]_{1.5}^{1.75}$

$= \boxed{0.3125}$

4. (5.6.3)

(a) $f(x) = \begin{cases} 0, & x<0 \\ cx, & 0<x<1.5 \\ c(3-x), & 1.5<x<3 \\ 0, & x>3 \end{cases}$

$c\int_0^{1.5} x\,dx + c\int_{1.5}^{3} (3-x)dx = 1$

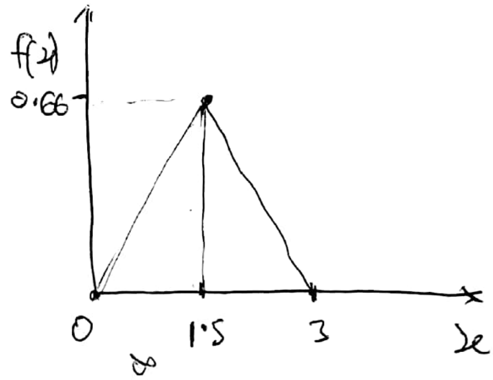
$\Rightarrow c \times \frac{1.5^2}{2} + c\left[3x - \frac{x^2}{2}\right]_{1.5}^{3} = 1$

$\Rightarrow c \times \frac{1.5^2}{2} + c\left[3\times\frac{3}{2} - \left(\frac{3^2 - 1.5^2}{2}\right)\right] = 1$

$\Rightarrow c\,[1.5^2 + 9 - 9 + 1.5^2] = 2$

$\Rightarrow c = \frac{1}{1.5^2} = \frac{1}{2.25} = \boxed{0.44}$

(b)



$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^{1.5} x \times cx\,dx + \int_{1.5}^{3} x \times c(3-x)dx$

$= c\int_0^{1.5} x^2 dx + c\int_{1.5}^{3} (3x - x^2)dx$

$= c\left[\left[\frac{x^3}{3}\right]_0^{1.5} + 3\left[\frac{x^2}{2}\right]_{1.5}^{3} - \left[\frac{x^3}{3}\right]_{1.5}^{3}\right]$

$= 0.44\left[\frac{1.5^3}{3} + 3\left(\frac{3^2 - 1.5^2}{2}\right) - \left(\frac{3^3 - 1.5^3}{3}\right)\right]$

$\boxed{E(X) = 1.485}$

(c) $P(X>2)$

$= 1 - P(X \le 2)$

$= 1 - \left\{ \left[\int_{1.5}^{2} c(3-x) \right] dx + \int_{0}^{1.5} cx\,dx \right\}$

$= 1 - c\left[\left(3x - \frac{x^2}{2}\right)_{1.5}^{2} + \left(\frac{x^2}{2}\right)_{0}^{1.5} \right] \quad \left(\frac{1}{b-a}\right)$

$= 1 - 0.77$

$= \boxed{0.23}$

(d) $Y \sim \text{Uniform}(0,3)$

$\text{Var}(Y) = \sigma^2 = \frac{(b-a)^2}{12}$

$= \frac{9}{12} = \frac{3}{4}$

$\boxed{\text{Var}(Y) = 0.75}$

$\text{Var}(X) = E(X^2) - (E(X))^2$

$E(x^2) = \int_{0}^{1.5} x^2 f(x)\,dx + \int_{1.5}^{3} x^2 f(x)\,dx$

$= \int_{0}^{1.5} c x^3\,dx + \int_{1.5}^{3} x^2 c(3-x)\,dx$

$= c\left[\frac{1.5^4}{4}\right] + c\left[\frac{3x^3}{3} - \frac{x^4}{4}\right]_{1.5}^{3}$

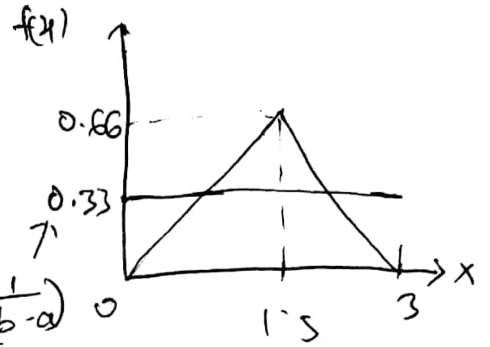
$= c\left[\frac{1.5^4}{4}\right] + c\left[(3^3 - 1.5^3) - \frac{1}{4}(3^4 - 1.5^4)\right]$

$= 2.59875$

$\therefore \text{Var}(x) = 2.59875 - (1.485)^2$

$= \boxed{0.393525}$

$\therefore \boxed{\text{Var}(Y) > \text{Var}(X)}$



(e) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$F(x) = \int_{-\infty}^{x} 0\,dx = 0 \left(\text{for } x<0\right)$

for $0 \le x < 1.5$:-

$F(x) = c\int_{0}^{x} x\,dx = 0.44 \times \frac{x^2}{2}\Big]_{0}^{x}$

$= 0.22x^2$

for $1.5 \le x < 3$:-

$F(x) = c\int_{0}^{1.5} x\,dx + c\int_{1.5}^{x} (3-x)\,dx$

$= 0.44\left[\left(\frac{x^2}{2}\right)_{0}^{1.5} + \left(3x - \frac{x^2}{2}\right)_{1.5}^{x}\right]$

$= 0.44\left[\frac{1.5^2}{2} + 3(x-1.5) - \frac{1}{2}(x^2 - 1.5^2)\right]$

$= 0.44\left[1.125 + 3x - 4.5 - \frac{x^2}{2} + 1.125\right]$

$= 0.44\left[3x - \frac{x^2}{2} - 2.25\right]$

$= (1.32x - 0.22x^2 - 0.99)$

$= -0.22x^2 + 1.32x - 0.99$

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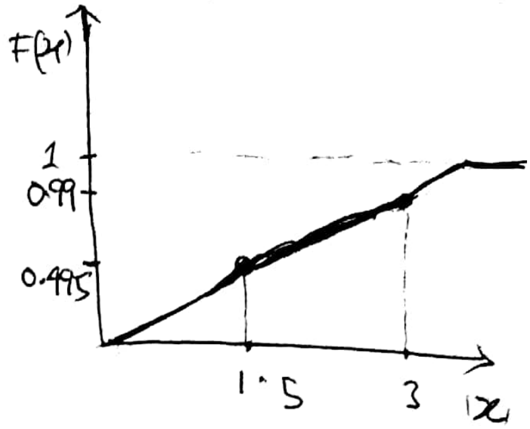
$$\therefore F(x) = \boxed{\begin{cases} 0 & , x < 0 \\ 0.22x^2 & , 0 < x < 1.5 \\ -0.22x^2 + 1.32x - 0.99 & , 1.5 < x < 3 \\ 1 & , x > 3 \end{cases}}$$



5. (6.4.4)

(a) X can take values from 0 to 1. (1 being the radius)

(b) $P(X \leq 0.5) = ?$

$$P(A) = \frac{\text{area}(A)}{\text{area}(B)} = \frac{\text{area}(A)}{\pi}$$

$$= \frac{\pi x^2}{\pi} = x^2$$

$$\boxed{\therefore P(X \leq 0.5) = (0.5)^2 = 0.25}$$

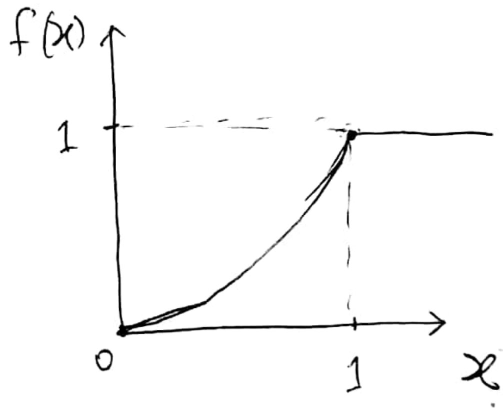
(c) $P(0.5 \leq X \leq 0.7)$

$= P(0.7) - P(0.5)$

$= 0.7^2 - 0.5^2$

$= \boxed{0.24}$

(d) $F(x) = \begin{cases} 0 & , x < 0 \\ x^2 & , 0 < x < 1 \\ 1 & , x > 1 \end{cases}$



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6(a) $f(x) = \begin{cases} 2k, & 0 \le x < 3 \\ 3k, & 3 \le x < 5 \\ 0, & \text{otherwise} \end{cases}$

By formula -

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_0^3 2k\,dx + \int_3^5 3k\,dx = 1$

$\Rightarrow 2kx\Big]_0^3 + 3kx\Big]_3^5 = 1$

$\Rightarrow 6k + 6k = 1$

$\Rightarrow \boxed{k = \frac{1}{12}}$

(b) $F(4)$

$= \int_0^3 2k\,dx + \int_3^4 3k\,dx$

$= 2kx\Big]_0^3 + 3kx\Big]_3^4$

$= 6k + 3k$

$= 9k$

$\Rightarrow \boxed{F(4) = \frac{9}{12} = \frac{3}{4}}$

(c) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^3 x \times 2k\,dx + \int_3^5 x \times 3k\,dx$

$= 2k\frac{x^2}{2}\Big]_0^3 + 3k\frac{x^2}{2}\Big]_3^5$

$= 9k + \frac{3k}{2}(5^2 - 3^2)$

$= 9k + \frac{3k}{2}(25-9)$

$= 9k + \frac{3k}{2}(16)$

$= 9k + 24k$

$\Rightarrow \boxed{E(X) = 33 \times \frac{1}{12} = 2.75}$

7. $f(x) = \begin{cases} \frac{1}{30}, & 0 \le x < 20 \\ \frac{1}{60}, & 20 \le x < 40 \\ 0 & \text{otherwise} \end{cases}$

(a) for $x < 0$:

$F(x) = 0$

for $0 \le x < 20$:

$F(y) = \int_0^y \frac{1}{30}\,dy = \frac{y}{30}$

for $20 \le x < 40$:

$F(y) = \int_0^{20} \frac{1}{30}\,dy + \int_{20}^{y} \frac{1}{60}\,dy$

$= \frac{y}{30}\Big]_0^{20} + \frac{y}{60}\Big]_{20}^{y}$

$= \frac{2}{3} + \left(\frac{y-20}{60}\right)$

$= \frac{2}{3} + \frac{y}{60} - \frac{1}{3}$

$= \frac{1}{3} + \frac{y}{60}$

for $x > 40$:

$F(y) = 1$

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$$\therefore \boxed{F(y) = \begin{cases} 0, & y < 0 \\ \dfrac{y}{30}, & 0 \le y < 20 \\ \left(\dfrac{1}{3} + \dfrac{y}{60}\right), & 20 < y \le 40 \\ 1, & y > 40 \end{cases}}$$

(b)

Since $F(y) = \frac{y}{30}$ (for $0 \le y < 20$)

$$= \frac{20}{30} = 0.66$$

so, $(F(y) = 0.5)$ lies between

$0 \le y < 20$ interval.

$$\therefore F(y) = \frac{y}{30}$$

$$\Rightarrow 0.5 = \frac{y}{30}$$

$$\Rightarrow \boxed{y = \frac{1}{2} \times 30 = 15} \text{ (for } F(y) = 0.5\text{)}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^{20} x \times \frac{1}{30}\, dx + \int_{20}^{40} x \times \frac{1}{60}\, dx$$

$$= \frac{1}{30} \times \frac{20^2}{2} + \frac{1}{60} \cdot \frac{40^2 - 20^2}{2}$$

$$= 16.66$$

$\therefore \boxed{E(X) > y} \to$ for $F(y) = 0.5$.

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