

The foundations of the rigorous study of *analysis* were laid in the nineteenth century, notably by the mathematicians Cauchy and Weierstrass. Central to the study of this subject are the formal definitions of *limits* and *continuity*.

Let  $D$  be a subset of  $\mathbf{R}$  and let  $f: D \rightarrow \mathbf{R}$  be a real-valued function on  $D$ . The function  $f$  is said to be *continuous* on  $D$  if, for all  $\epsilon > 0$  and for all  $x \in D$ , there exists some  $\delta > 0$  (which may depend on  $x$ ) such that if  $y \in D$  satisfies

$$|y - x| < \delta$$

then

$$|f(y) - f(x)| < \epsilon.$$

One may readily verify that if  $f$  and  $g$  are continuous functions on  $D$  then the functions  $f + g$ ,  $f - g$  and  $f \cdot g$  are continuous. If in addition  $g$  is everywhere non-zero then  $f/g$  is continuous.