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Chaotic flows with a single nonquadratic term

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ABSTRACT

This paper addresses a previously unexplored regime of three-dimensional dissipative chaotic flows in which all but one of the nonlinearities are quadratic. The simplest such systems are determined, and their equilibria and stability are described. These systems often have one or more infinite lines of equilibrium points and sometimes have stable equilibria that coexist with the strange attractors, which are sometimes hidden. Furthermore, the coefficient of the single nonquadratic term provides a simple means for scaling the amplitude and frequency of the system.

1. Introduction

Many of the common examples of dissipative chaotic flows involve three-dimensional autonomous systems with quadratic nonlinearities, by which we mean to include cross-product terms like xy in addition to squared terms like x^2. For example, the classic Lorenz system [1] has two quadratic terms and five linear terms, while the Rössler system [2] has one quadratic term and six linear terms. The simplest such system has one quadratic term and four linear terms [3], and it has been proved that chaotic systems with fewer than five terms and with linear damping and quadratic nonlinearities cannot exist [4]. A general class of system, with both general linear and quadratic terms, was recently considered in [5] since it represents one of the simplest systems capable of producing chaos. An unexplored regime involves chaotic systems in which most of the terms are quadratic. We will identify the simplest such systems and show that they require at least one nonquadratic term to give chaos, but that one is a sufficient number to produce many examples of strange attractors with unusual properties including multiple line equilibria, hidden attractors, and a convenient amplitude–frequency control parameter.

Such systems are of practical use because the amplitude/frequency control knob provides a good secure key for secret communication or replaces the extra amplifier or attenuator in radar or other communication systems when the chaotic systems are used as the signal sources. Systems with many nonlinearities are common in single-mode lasers [6], closed-loop convection [7], waterwheels [8], plasma, and propagation of the dipole domains [9].

2. Chaotic flows with mostly quadratic terms

We consider the most general parametric 3-D form containing all possible quadratic nonlinearities except for a single nonquadratic term $f(x, y)$ that without loss of generality is placed in the first ($\dot{x}$) equation,

$$\dot{x} = f(x, y) + a_1x^2 + a_2y^2 + a_3z^2 + a_4xy + a_5xz + a_6yz \tag{1.1}$$

$$\dot{y} = a_7x^2 + a_8y^2 + a_9z^2 + a_{10}xy + a_{11}xz + a_{12}yz \tag{1.2}$$

$$\dot{z} = a_{13}x^2 + a_{14}y^2 + a_{15}z^2 + a_{16}xy + a_{17}xz + a_{18}yz \tag{1.3}$$

We further consider the simplest forms for $f(x, y)$ given by

$$f(x, y) = \begin{cases} 0 & \text{case A} \\ 1 & \text{case B} \\ \pm x & \text{case C} \\ y & \text{case D} \end{cases} \tag{2}$$

As shown later, any multiplicative coefficient of $f(x, y)$ can be set to unity without loss of generality.

2.1. Case A: $f(x, y) = 0$

The simplest case contains only quadratic terms. Such a system has nullclines that are planes passing through the origin so that they intersect only at the origin or along one or more lines passing through the origin. Thus there is either a single equilibrium point at the origin or one or more lines of equilibrium points

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stretching to infinity in both directions. In either case, there cannot be an attractor of finite size (a limit cycle or strange attractor) since there is no characteristic length to determine its scale. Indeed, a linear rescaling of the three variables leaves the system unchanged except for a change in the time scale. The origin can be a weak attractor, but only by virtue of the nonlinearities since the trace of the Jacobian matrix vanishes at the origin. Any periodic solutions must be invariant cycles whose size and shape depend on the initial conditions.

A simple example that illustrates the dynamics of the more general case is one in which each variable depends only on the other two,

$$\dot{x} = yz \tag{3.1}$$

$$\dot{y} = xz \tag{3.2}$$

$$\dot{z} = -xy \tag{3.3}$$

This system has three perpendicular line equilibria, $(x, 0, 0)$, $(0, y, 0)$, and $(0, 0, z)$, whose eigenvalues are $\lambda_1 = 0$, $\lambda_{2,3} = \pm xi$, $\pm yi$ and $\pm z$, respectively. Trajectories follow the curves $\mathrm{d}y/\mathrm{d}x = x/y$ and $\mathrm{d}z/\mathrm{d}x = -x/z$, which can be integrated to give two constants of the motion: $y^2 - x^2 = y_0^2 - x_0^2$ and $z^2 + x^2 = z_0^2 + x_0^2$. Thus the dynamics are constrained to a one-dimensional manifold that depends only on the initial conditions. The manifold is a hyperbola when projected onto the xy-plane and a circle when projected onto the xz- or yz-plane. Initial conditions in the planes $x_0 = \pm y_0$ at $x_0^2 + z_0^2 = z^2$ remain in that plane and attract to a point $(0, 0, \mp|z|)$ on the z-axis. Other initial conditions lie on periodic cycles whose frequency and amplitude depend on the initial conditions and that are invariant since the trace of the Jacobian matrix is zero. Thus limit cycles and strange attractors are not possible for these systems.

2.2. Case B: $f(x, y) = 1$

An extensive search for the simplest chaotic system that contains a single constant term found the example

$$\dot{x} = 1 + yz \tag{4.1}$$

$$\dot{y} = -xz \tag{4.2}$$

$$\dot{z} = y^2 + ayz \tag{4.3}$$

Since this system has five terms and the variables x, y, z, and t can be linearly rescaled without altering the dynamics, it has a single parameter a that is arbitrarily put into the last term. The system has two equilibrium points $(0, \pm\sqrt{a}, \frac{\mp 1}{\sqrt{a}})$ with complex eigenvalues. When $a = 2$, the eigenvalues are 2.9793, $-0.0754 \pm 0.9714i$ and -2.9793, $0.0754 \pm 0.9714i$, respectively, and the system is chaotic with Lyapunov exponents (0.1519, 0, -2.3871) and a Kaplan–Yorke dimension of $D_{\mathrm{KY}} = 2 - \lambda_1/\lambda_3 = 2.0636$. The corresponding attractor is shown in Fig. 1, and the corresponding signals and their power spectra are shown in Figs. 2 and 3, respectively. The time series shows the rather disparate time scales that lead to the broadband power spectrum. Since the system has only even powers of the variables, it has the unusual property that it is inversion invariant under the transformation $(x \to -x, y \to -y, z \to -z, t \to -t)$, and thus it has a symmetric attractor/repellor pair. This is probably the simplest example of a system of the type considered in this paper.

The dynamics of this system are otherwise unremarkable, with an elongated limit cycle born at $a = 0$ and undergoing a series of period-doubling bifurcations culminating in chaos at $a \approx 1.0354$. The chaos persists to large values of a with occasional periodic windows. For $a \approx 2.0573$ and $a \approx 2.3236$, there are homoclinic orbits on the attractor, but they do not appear to be associated with

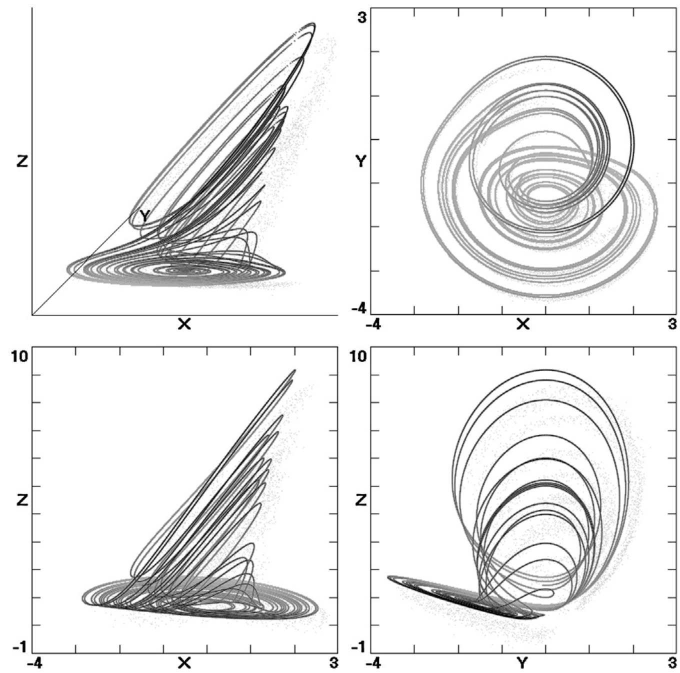

Fig. 1. Chaotic attractor from Eq. (4) with $a = 2$ and initial condition $(x_0, y_0, z_0) = (0, 0, 2)$.

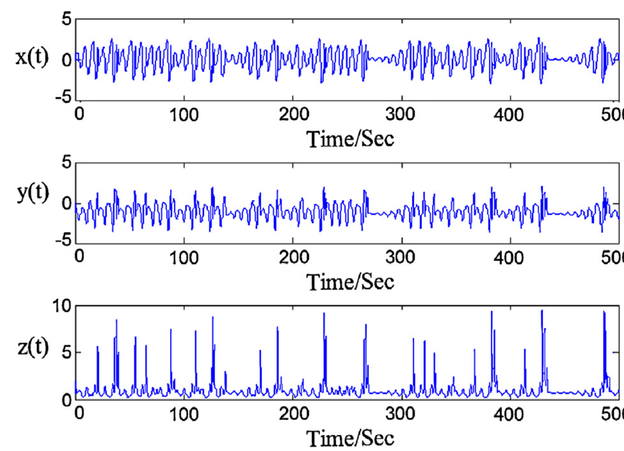

Fig. 2. Chaotic signals from Eq. (4) with $a = 2$ and initial conditions $(x_0, y_0, z_0) = (0, 0, 2)$.

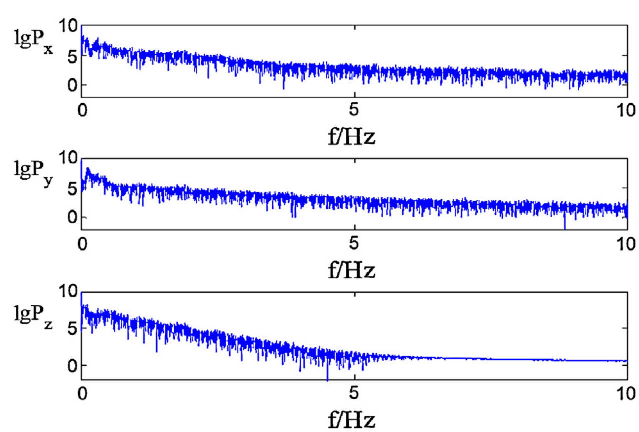

Fig. 3. Power spectrum of signals from Eq. (4) with $a = 2$.

Table 1
Chaotic flows with a single linearity in x.

Model	Equations	Parameter values admitting chaos	Equilibria	Eigenvalues	x_0, y_0, z_0	LEs	D_{KY}
SL_1	$\dot{x} = -x + ay^2 - xy$ $\dot{y} = xz$ $\dot{z} = z^2 - bxy$	$a = 2$ $b = 1$	$(0, 0, 0)$	$(0, 0, -1)$	3.4 3 0	0.1889 0 −1.6864	2.1120
SL_2	$\dot{x} = -ax + xy$ $\dot{y} = z^2 + xz$ $\dot{z} = y^2 - byz$	$a = 2$ $b = 1$	$(0, 0, 0)$ $(-a/b, a, a)$	$(0, 0, -2)$ $(-2.9311, 0.4656 \pm 1.5851i)$	−2 0 3	0.2191 0 −2.2191	2.0987
SL_3	$\dot{x} = x + ay^2 - z^2$ $\dot{y} = x^2 - by^2$ $\dot{z} = xz$	$a = 2.4$ $b = 1$	$(0, 0, 0)$ $(-\frac{b}{a}, \pm\frac{\sqrt{b}}{a}, 0)$	$(1, 0, 0)$ $(-0.4167, 0.0833 \pm 0.9091i)$ $(2.2103, -0.3770 - 0.4167)$	0 0.9 ±0.6	0.0734 0 −1.5866	2.0463
SL_4	$\dot{x} = -x + by^2 + xz$ $\dot{y} = xz$ $\dot{z} = -axy + yz$	$a = 0.1$ $b = 1$	$(0, 0, z)$	$(0, 0, z - 1)$	8 0 0.7	0.2390 0 −1.3956	2.1713
SL_5	$\dot{x} = -x + az^2$ $\dot{y} = z^2 - bxz$ $\dot{z} = xy - yz$	$a = 1$ $b = 2$	$(0, y, 0)$ $(\frac{1}{ab^2}, 0, \frac{1}{ab})$	$(0, -1, -y)$ $(0.2685, -0.6342 \pm 0.2516i)$	2 3 0	0.0748 0 −0.8856	2.0845

any bifurcation. A variant of the system in which the yz term in the first equation is replaced with $-y^2$ has similar dynamics.

2.3. Case C: $f(x, y) = \pm x$

When the nonquadratic term is a single linear (SL) function of x, the simplest chaotic examples that were found have six terms. Table 1 lists five such cases, chosen from a much larger list because they have different numbers and types of equilibria, and Fig. 4 shows the corresponding attractors projected onto the xy-plane for the given parameters that produce chaos. With six terms, there are two parameters, chosen to be ± 1 where possible, or otherwise to be a small integer or a decimal fraction with the fewest possible digits. The simplest system, SL_1, has a single equilibrium with two eigenvalues zero, while the most complicated system, SL_5, has an infinite line of equilibrium points plus a single isolated equilibrium. Many of the equilibria have a largest eigenvalue that is zero for the chosen parameters, and thus the stability is often determined by the nonlinearities. All the cases for which the largest real part of the eigenvalue is zero appear to be unstable when nonlinearities are considered except for Model SL_5 where the line of equilibrium points in nonlinearly stable in the sense that initial conditions that start in its vicinity forever remain near their starting point. Four of the five cases have negative x in the $\dot{x}$ equation as expected for a dissipative system, but Model SL_3 has a positive x, which would normally imply anti-damping, but the $-by^2$ nonlinear damping term in the $\dot{y}$ equation overwhelms the linear anti-damping. Dissipative systems with linear anti-damping are not widely known and have been relatively little studied. This case has three equilibrium points, all unstable, and a reflection invariance about the $z = 0$ plane in which all three of the equilibria lie, as well as a symmetric pair of strange attractors. All other cases are asymmetric.

Another common feature in systems with mostly quadratic nonlinearities is that the dynamics often have two disparate time scales, making the equations stiff and hard to integrate. In addition to the case in Fig. 2, another more extreme example is Model SL_5 where there is a fast growing oscillation in the yz-plane followed by a slow relaxation oscillation in the x-direction.

2.4. Case D: $f(x, y) = y$

When the nonquadratic term is a linear function of y, the simplest chaotic examples that were found have five terms, one of which with a single equilibrium at the origin and all eigenvalues

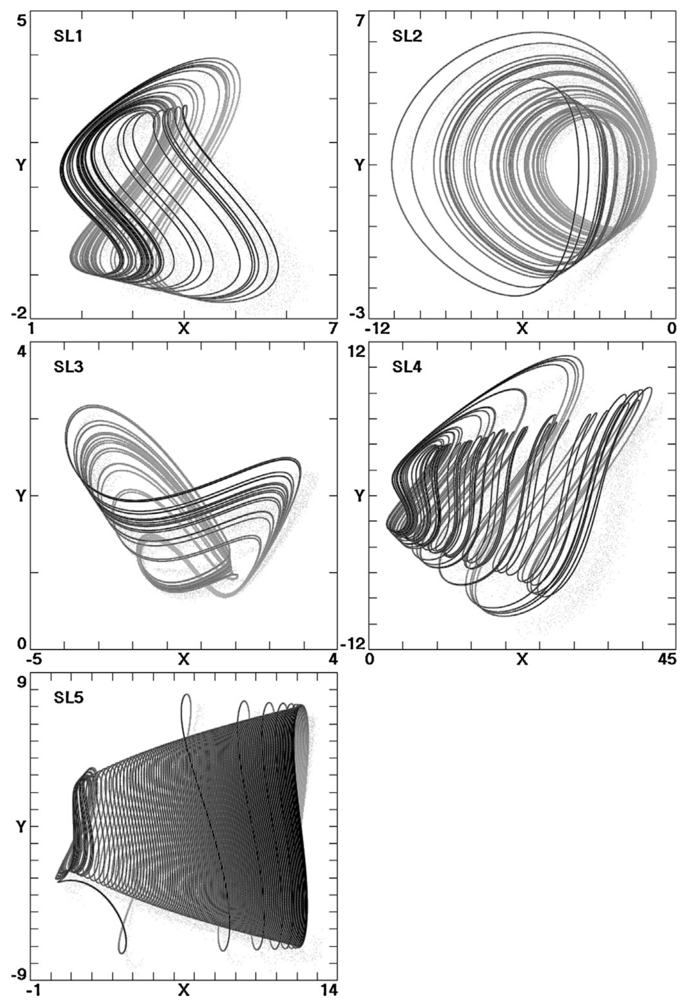

Fig. 4. State space diagram for the cases in Table 1 projected onto the xy-plane.

zero is shown as Model SL_6 in Table 2. The other six systems in the table have six terms and were chosen from a much larger list because they have different numbers and types of equilibria. Fig. 5 shows the corresponding attractors projected onto the xy-plane. The simplest of these systems, SL_7, has three equilibrium points, while the most complicated system, SL_{12}, has two infinite parallel lines of equilibrium points plus a single isolated equilibrium.

Table 2
Chaotic flows with a single linearity in y.

Model	Equations	Parameter values admitting chaos	Equilibria	Eigenvalues	x_0, y_0, z_0	LEs	D_{KY}
SL_6	$\dot{x}=y-z^2$ $\dot{y}=-axz$ $\dot{z}=x^2-yz$	$a=0.9$	$(0,0,0)$	$(0,0,0)$	0 0 1.4	0.1304 0 −3.9246	2.0332
SL_7	$\dot{x}=-y-yz$ $\dot{y}=x^2+axz$ $\dot{z}=z^2+byz$	$a=14$ $b=1$	$(0,0,0)$ $(0,1/b,-1)$ $(a,1/b,-1)$	$(0,0,0)$ $(-2.7937, 0.8969\pm 2.0511i)$ $(0.0714, -0.5357\pm 13.9925i)$	0 1 −0.7	0.1340 0 −0.5489	2.2441
SL_8	$\dot{x}=y-y^2$ $\dot{y}=az^2+xy$ $\dot{z}=-x^2-bxy$	$a=0.3$ $b=1$	$(0,0,0)$ $(0,1,0)$ $(-1,1,1/\sqrt{a})$ $(-1,1,-1/\sqrt{a})$	$(0,0,0)$ $(0,\pm i)$ $(-1.5302, 0.2651\pm 0.8035)$ $(0.4098, -0.7049\pm 1.4752i)$	0 1.3 −1	0.0337 0 −0.2544	2.1324
SL_9	$\dot{x}=y$ $\dot{y}=ay^2-xz$ $\dot{z}=x^2+xy-bxz$	$a=0.4$ $b=1$	$(0,0,z)$	$(0,\pm\sqrt{-z})$	0 4 5	0.0749 0 −0.7391	2.1014
SL_{10}	$\dot{x}=y+axz$ $\dot{y}=xy-xz$ $\dot{z}=x^2+bxy$	$a=0.2$ $b=3$	$(0,0,z)$ $(-\frac{1}{a},\frac{1}{ab},\frac{1}{ab})$	$(0,\frac{z\pm\sqrt{z^2-100z}}{10})$ $(0.3565, -2.5116\pm 7.9885i)$	−0.2 0 0	0.0280 0 −0.2397	2.1167
SL_{11}	$\dot{x}=y+y^2-ayz$ $\dot{y}=-z^2+byz$ $\dot{z}=xy$	$a=0.9$ $b=1$	$(x,0,0)$ $(0,-1,0)$ $(0,\frac{1}{ab-1},\frac{b}{ab-1})$	$(0,0,0)$ $(-0.7113, 0.3556\pm 1.1311i)$ $(0.9911, -5.4956\pm 8.4079i)$	0.8 −2 0	0.1401 0 −0.8573	2.1634
SL_{12}	$\dot{x}=-y+x^2-y^2$ $\dot{y}=-xz$ $\dot{z}=ax^2+bxy$	$a=0.3$ $b=1$	$(0,0,z)$ $(0,-1,z)$ $(\frac{ab}{a^2-b^2},\frac{a^2}{b^2-a^2},0)$	$(0,\pm\sqrt{z})$ $(0,\pm\sqrt{-z})$ $(-0.5690, -0.0452\pm 0.2351i)$	0 −0.3 2	0.0096 0 −0.2660	2.0362

Most of the equilibria have a largest eigenvalue that is either zero or negative for the chosen parameters, and thus the stability is often determined by the nonlinearities. All the isolated equilibrium points (points that are not part of a line of equilibria) that are neutrally stable according to their eigenvalues appear to be nonlinearly unstable. Some of the lines of equilibrium points with a zero eigenvalue appear to be nonlinearly stable over a portion of their length. All of these systems listed are asymmetric, but symmetric cases also exist.

3. Hidden attractors

Systems with many quadratic terms tend to have multiple equilibria, sometimes even stable ones coexisting with the strange attractor, an example of which is SL_{12}. Usually limit cycles and strange attractors are associated with an equilibrium point that has lost its stability but that remains in its basin of attraction. Such attractors are called "self-excited", and they can be found by starting with an initial condition in the neighborhood of the unstable equilibrium. Nearly all strange attractors that have been studied are of this type, typical examples of which are in [1,10–12]. Limit cycles and strange attractors whose basins of attraction do not intersect with small neighborhoods of any equilibrium points are called "hidden" since there is no way to choose an initial condition that guarantees that they will be found [13,14]. Chaotic systems without any equilibria [15] or with only stable equilibria [16] are obvious examples of hidden attractors, but some systems have only unstable equilibrium points that are far from the basin of the hidden attractor. Furthermore, any chaotic system that has an infinite line of equilibrium points is likely to have a hidden strange attractor since most points in the neighborhood of the line will usually lie outside the basin of the strange attractor [17]. Such hidden attractors are important in engineering applications because they can lead to unexpected and potentially disastrous behavior.

An example of a system with not one, but two hidden strange attractors is Model SL_3. This system has symmetry about the plane $z=0$ and a symmetric pair of strange attractors that lie above and below the plane, respectively. It has three unstable equilibrium points that lie in the $z=0$ plane. Since that plane does not intersect either of the strange attractors or their basins of attraction, it is not possible to show a two-dimensional plane in which all three equilibrium points and the two basins appear. However, Fig. 6 shows a cross section of the basins in the $x=y$ plane where two of the equilibrium points are shown as small open circles. Fig. 7 shows a similar plot in the $x=-y$ plane where the other equilibrium point appears. None of the equilibria lie near the basins of the strange attractors, and all orbits that start in their vicinity are unbounded. Thus both strange attractors are hidden from all three equilibria.

4. Controllability of amplitude and frequency

Usually, a dynamical system has amplitude parameters and bifurcation parameters, which influence the size of the attractor and its topology, respectively. For the above systems, the coefficient of the single nonquadratic term $f(x,y)$ controls both the amplitude and frequency of the signals generated by the systems since it is the only term whose dimensions are different from the remaining quadratic terms. By contrast, in order to have only amplitude control, all of the quadratic terms as well as any constant terms must be controlled.

As an illustration, consider a simultaneous amplitude and frequency control of Eq. (4) by the transformation $x\to x/c$, $y\to y/c$, $z\to z/c$, $t\to ct$. Then Eq. (4) becomes

$$\dot{x}=c^2+yz \tag{5.1}$$

$$\dot{y}=-xz \tag{5.2}$$

$$\dot{z}=y^2+ayz \tag{5.3}$$

Thus, if the constant term in the $\dot{x}$ equation changes according to c^2, it will scale the amplitude and frequency according to c.

For those systems with a single linear term such as Model SL_7, a transformation $x\to x/c$, $y\to y/c$, $z\to z/c$, $t\to ct$ gives simultaneous amplitude and frequency control. The system becomes

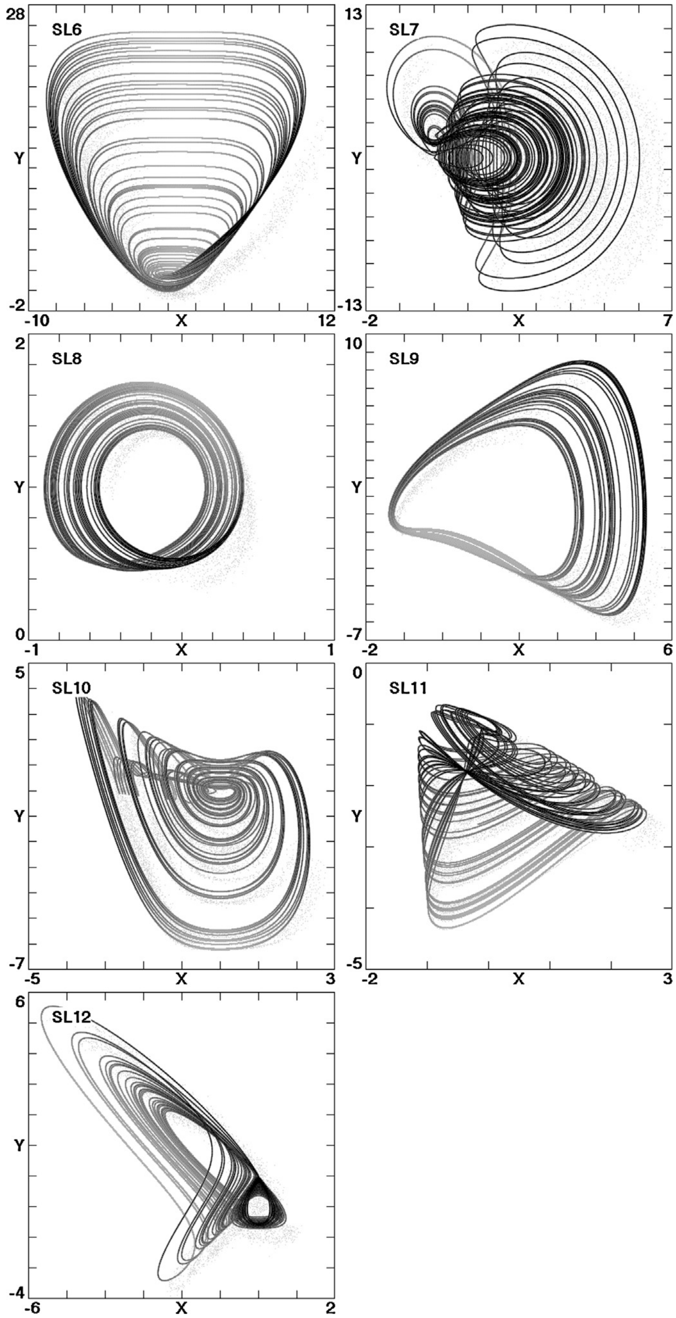

Fig. 5. State space diagram for the cases in Table 2 projected onto the xy-plane.

$$\dot{x} = -cy - yz \tag{6.1}$$

$$\dot{y} = x^2 + axz \tag{6.2}$$

$$\dot{z} = z^2 + byz \tag{6.3}$$

Therefore, the coefficient c of y in the $\dot{x}$ equation scales the amplitude and frequency according to c.

For comparison, we show partial pure amplitude control by changing the coefficient of a nonlinear term [18]. For Model SL_2, there is a one-dimensional amplitude control parameter in the second equation,

$$\dot{x} = -ax + xy \tag{7.1}$$

$$\dot{y} = z^2 + cxz \tag{7.2}$$

$$\dot{z} = y^2 - byz \tag{7.3}$$

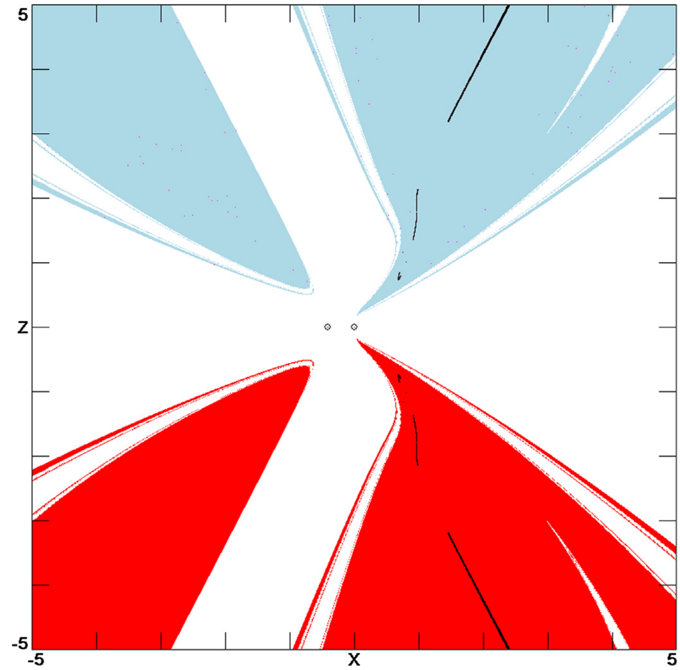

Fig. 6. Cross sections in the plane $x = y$ of the basins of attraction (red and blue, respectively, in the web version) for the two hidden strange attractors whose cross sections are shown in black for Model SL_3. Two of the three unstable equilibria (shown as small open circles) lie outside both basins as does the third equilibrium (shown in Fig. 5), and all orbits that start in the vicinity of these equilibria are unbounded.

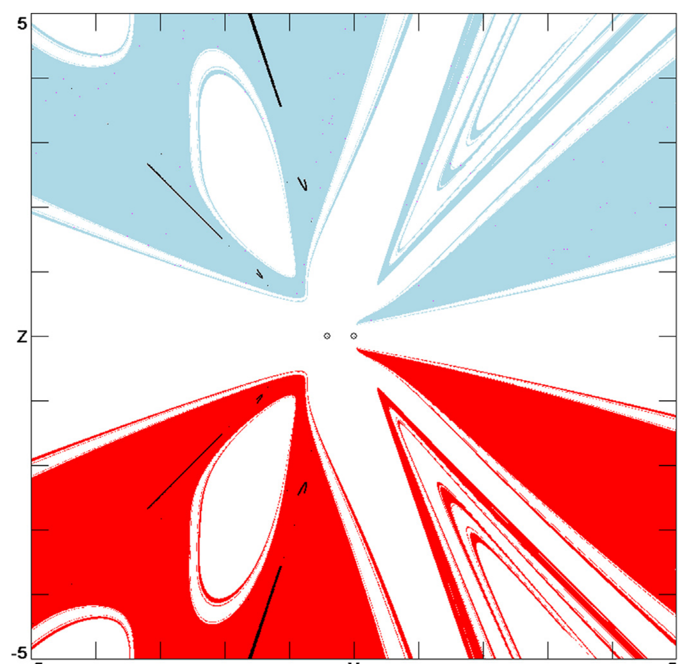

Fig. 7. Cross sections in the plane $x = -y$ of the basins of attraction (red and blue, respectively, in the web version) for the two hidden strange attractors whose cross sections are shown in black for Model SL_3. Two of the three unstable equilibria (shown as small open circles) lie outside both basins as does the third equilibrium (shown in Fig. 4), and all orbits that start in the vicinity of these equilibria are unbounded.

The coefficient c in the xz term controls the amplitude of x according to $\frac{1}{c}$, while the amplitude of y and z remain unchanged, and thus it is not a good bifurcation parameter. This can be proved by the transformation $x \to \frac{1}{c}x$, $y \to y$, $z \to z$. The coefficient of a quadratic term can also give two-dimensional amplitude control when the system has rotational symmetry [18].

5. Discussion and conclusions

When considering systems with mostly quadratic terms, new regimes of chaotic flows appear. A variety of simple dynamical systems with four or five quadratic terms and a single constant or linear term have been found, which have different numbers and types of equilibria. Many systems have a line of equilibrium points, and some of them have a line and additional isolated equilibrium points. One of the systems described even has two lines of equilibrium points coexisting with another isolated equilibrium point. The variety of equilibria makes the origin of the chaotic attractor ambiguous and sometimes hidden. In addition, the coefficient of the nonquadratic term provides a good control knob for amplitude and frequency adjustment. Meanwhile, the coefficients of the nonlinear terms are useful bifurcation parameters, although some combinations of them can provide partial or total amplitude control. Finally, we note that most of the systems described have a two-dimensional parameter space, and only a single combination of those parameters has been examined for each system. Thus these systems likely have additional dynamic regions that would be worth further detailed study.

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References

[1] E.N. Lorenz, J. Atmos. Sci. 20 (1963) 130.
[2] O.E. Rössler, Phys. Lett. A 57 (1976) 397.
[3] J.C. Sprott, Phys. Lett. A 228 (1997) 271.
[4] Z. Fu, J. Heidel, Nonlinearity 10 (1997) 1289.
[5] S.R. Choudhury, R.A. Van Gorder, Nonlinear Dyn. 69 (2012) 2255.
[6] S.D. Gupta, G.S. Agarwal, J. Opt. Soc. Am. B 8 (1991) 1712.
[7] D.M. Bošković, M. Krstić, Automatica 37 (2001) 2033.
[8] L.E. Matson, Am. J. Phys. 75 (2007) 1114.
[9] B.R. Andrievskii, A.L. Fradkov, Autom. Remote Control 65 (2004) 505.
[10] W. Zhou, Y. Xu, H. Lu, L. Pan, Phys. Lett. A 372 (2008) 5773.
[11] S. Dadras, H.R. Momeni, Phys. Lett. A 373 (2009) 3637.
[12] S.T. Kingni, L. Keuninckx, P. Woafo, G. Van der Sande, J. Danckaert, Nonlinear Dyn. 73 (2013) 1111.
[13] G.A. Leonov, V.I. Vagaitsev, N.V. Kuznetsov, Phys. Lett. A 375 (2011) 2230.
[14] G.A. Leonov, V.I. Vagaitsev, N.V. Kuznetsov, Physica D 241 (2012) 1482.
[15] S. Jafari, J.C. Sprott, S.M.R.H. Golpayegani, Phys. Lett. A 377 (2011) 699.
[16] M. Molaei, S. Jafari, J.C. Sprott, S.M.R.H. Golpayegani, Int. J. Bifurc. Chaos 23 (2013) 1350188.
[17] S. Jafari, J.C. Sprott, Chaos Solitons Fractals 57 (2013) 79.
[18] C. Li, J.C. Sprott, Nonlinear Dyn. 73 (2013) 1335.