Department of Electronics & Communication Engineering
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ECN 614 – Adaptive Signal Processing Techniques

TUTORIAL #2

1. The sampled form of the transmitted radar signal is $A_0 e^{-j\Omega_0 n}$ where Ω_0 is the angular frequency and A_0 is the transmitted complex amplitude. The signal received is given as

$$x[n] = A_1 e^{-j\Omega_1 n} + \nu[n]$$

   where $|A_1| < |A_0|$ and Ω_1 differs from Ω_0 by virtue of Doppler shift produced by the motion of the target of interest, and $\nu[n]$ is a sample of white noise.

   The time-series $x[n]$ is applied to an M-tap Wiener filter with cross-correlation vector between $x[n]$ and the desired response preset to $\mathbf{p} = \sigma_0^2\, \mathbf{a}(\Omega_0)$, where $\sigma_0^2 = E\{|A_0|^2\}$ and $\mathbf{a}(\Omega_0) = [1 \quad e^{-j\Omega_0} \quad \ldots \quad e^{-j\Omega_0(M-1)}]^T$. Derive expression for the tap-weight vector of the Wiener filter.

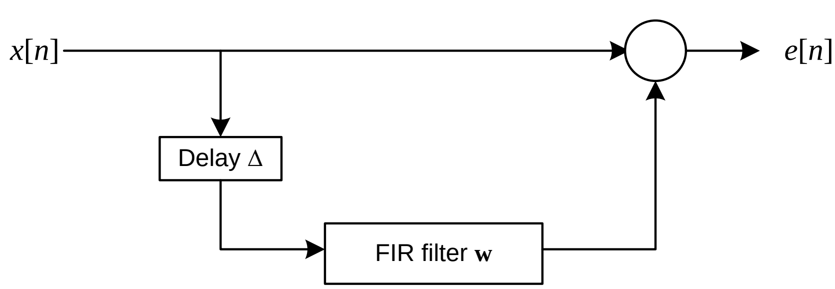
2. Derive the augmented Wiener-Hopf equations of a forward prediction-error filter by
   (a) Formulate the expression for the mean-square value of the forward prediction error in terms of the tap-weight vector of the forward prediction-error filter.
   (b) Minimize this mean-square prediction error, subject to the constraint that the first tap-weight (fed with the current sample $x[n]$) is equal to unity.

3. Consider the linear prediction of a stationary autoregressive process generated from the difference equation

$$x[n] = 0.9x[n-1] + \nu[n]$$

   where $\{\nu[n]\}$ is a zero-mean white-noise process with unit variance. Determine the tap weights of the forward prediction-error filter.

4. Consider the filtering structure described in the figure below, where the delay Δ is an integer greater than unity.



   Find the optimum value of the weight-vector $\mathbf{w}[n]$ so as to minimize the estimation error $e[n]$.

5. A process $x[n]$ consists of a single sinusoidal process of complex amplitude α and angular frequency Ω in additive white noise of zero mean and variance σ_v^2, as given by

$$x[n] = \alpha\, e^{j\Omega n} + v[n]$$

where $E\{|\alpha|^2\} = \sigma_\alpha^2$ and $\{|v[n]|^2\} = \sigma_v^2$. Determine the tap weights of the forward prediction-error filter of order M and the final value of the prediction-error power.

6. Consider a wide-sense stationary process $x[n]$ whose auto-correlation function has the following values for different lags:

$$r(0) = 1; \quad r(1) = 0.8; \quad r(2) = 0.6; \quad r(3) = 0.4$$

(a) Use the Levinson-Durbin recursion to evaluate the tap weights of the forward prediction-error filter of order three and the final value of the prediction-error power.

(b) Based on the Levinson-Durbin recursion, determine the transfer function of the forward prediction-error filter and check your result with that in Part (a) above.

7. A second-order autoregressive process is defined by the difference equation

$$x[n] = x[n-1] - 0.5x[n-2] + v[n]$$

where $\{v[n]\}$ is a zero-mean white-noise process with variance $\sigma_v^2 = 0.5$.

(a) Find the average power of $x[n]$.

(b) Find the reflection coefficients κ_1 and κ_2.

(c) Using the results obtained in Part (a) and Part (b) above, compute the autocorrelation function values $r(1)$ and $r(2)$, and the average prediction-error powers P_1 and $P2$.