Department of Electronics & Communication Engineering
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<u>ECN 614 – Adaptive Signal Processing Techniques</u>

TUTORIAL #1

1. Show that the product $p^*[-k]w_k$ is an analytic function of the complex tap weight w_k but $w_k^* p[-k]$ is not an analytic function.

2. A second-order autoregressive process is defined by the difference equation

$$x[n] = x[n-1] - 0.6x[n-2] + w[n]$$

   where $\{w[n]\}$ is a zero-mean white-noise process with variance σ_w^2. Determine the autocorrelation function values $r_{xx}(0)$, $r_{xx}(1)$ and $r_{xx}(2)$.

3. Consider a Wiener filtering problem characterized as follows: The correlation matrix of the tap-input vector $\mathbf{x}[n]$ is

$$\mathbf{R}_{\mathrm{xx}} = \begin{bmatrix} 1 & 0.5 & 0.25 \\ 0.5 & 1 & 0.5 \\ 0.25 & 0.5 & 1 \end{bmatrix}$$

   and the cross-correlation vector between the tap-input vector $\mathbf{x}[n]$ and the desired response $d[n]$ is

$$\mathbf{p} = [0.5 \quad 0.25 \quad 0.125]^T.$$

   (a) Evaluate the tap weights of the Wiener filter.

   (b) What is the minimum mean-square error produced by this Wiener filter?

   (c) Formulate a representation of the Wiener filter in terms of the eigenvalues of the matrix $\mathbf{R}_{\mathrm{xx}}$ and associated eigenvalues.

4. The tap-input vector of a transversal filter is defined by

$$\mathbf{x}[n] = \alpha[n]\mathbf{s}[\omega] + \boldsymbol{\nu}[n],$$

   where

$$\mathbf{s}[\omega] = [\,1 \quad e^{-j\omega} \quad \ldots \quad e^{-j\omega(M-1)}\,]^T$$

   and

$$\boldsymbol{\nu}[n] = [\,\nu[n] \quad \nu[n-1] \quad \ldots \quad \nu[n-1+M]\,]^T$$

   The complex amplitude $\alpha[n]$ of the sinusoidal vector $\mathbf{s}[\omega]$ is a random variable with zero mean and variance $\sigma_\alpha^2 = E\{\,|\alpha[n]|^2\,\}$.

(a) Determine the correlation matrix of the tap-input vector $\mathbf{x}[n]$.

(b) Suppose that the desired response $d[n]$ is uncorrelated with $\mathbf{x}[n]$. What is the value of the tap-weight vector of the corresponding Wiener filter?

(c) Suppose that the variance σ_α^2 is zero and the desired response is defined by $d[n] = v[n-k]$, where $0 \le k \le M-1$. What is the new value of the tap-weight vector of the Wiener filter?

(d) Determine the tap-weight vector of the Wiener filter for a desired response defined by $d[n] = \alpha[n]e^{-j\omega\tau}$, where τ is a prescribed delay.

5. Consider a signal $x[n] = s[n] + w[n]$, where $s[n]$ is an autoregressive process that satisfies the difference equation

$$s[n] = 0.8s[n-1] + v[n]$$

where $\{v[n]\}$ is a zero-mean white-noise sequence with variance $\sigma_v^2 = 0.49$ and $\{w[n]\}$ is another zero-mean white-noise sequence with variance $\sigma_w^2 = 1$. The processes $\{v[n]\}$ and $\{w[n]\}$ are uncorrelated.

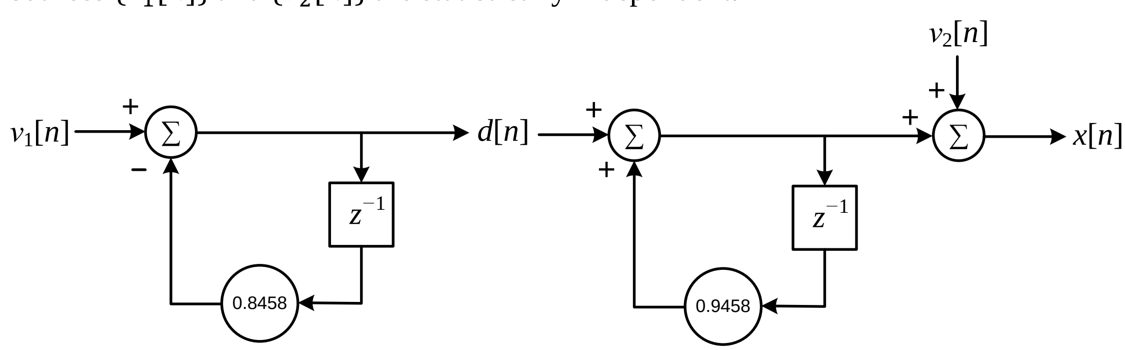
(a) Determine the autocorrelation sequences $\{ r_{ss}(m) \}$ and $\{ r_{xx}(m) \}$.

(b) Design a Wiener filter of length $M = 2$ to estimate $\{s[n]\}$.

(c) Repeat Part (b) above for $M = 3$.

(d) Compare the minimum mean-square errors produced by the filters in Part (b) and Part (c) above and comment on the difference.

6. The figure below shows the autoregressive model of the transmitted signal $d[n]$ and the model of a noisy communication channel over which the signal is transmitted. $\{v_1[n]\}$ is a white-noise process of zero-mean with variance $\sigma_1^2 = 0.27$. $\{v_2[n]\}$ is another white-noise source of zero-mean with variance $\sigma_2^2 = 0.1$. The two noise sources $\{v_1[n]\}$ and $\{v_2[n]\}$ are statistically independent.



A Wiener filter of length two is employed at the receiver to estimate the transmitted signal $d[n]$. Determine the optimum weight vector of the Wiener filter and the minimum mean-square error produced by the filter.