ASSIGNMENT-0

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1.1 Problem A.1: $G_1(s) = \dfrac{10}{s+10}$

1. Find the pole and DC gain:

$$\text{Pole: } s = \_\_\_\_\_\_\_\_\_\_, \quad G_1(0) = \_\_\_\_\_\_\_\_\_\_$$

Pole:-10 $G_1(0)$=1

2. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.

2) Magnitude plot:

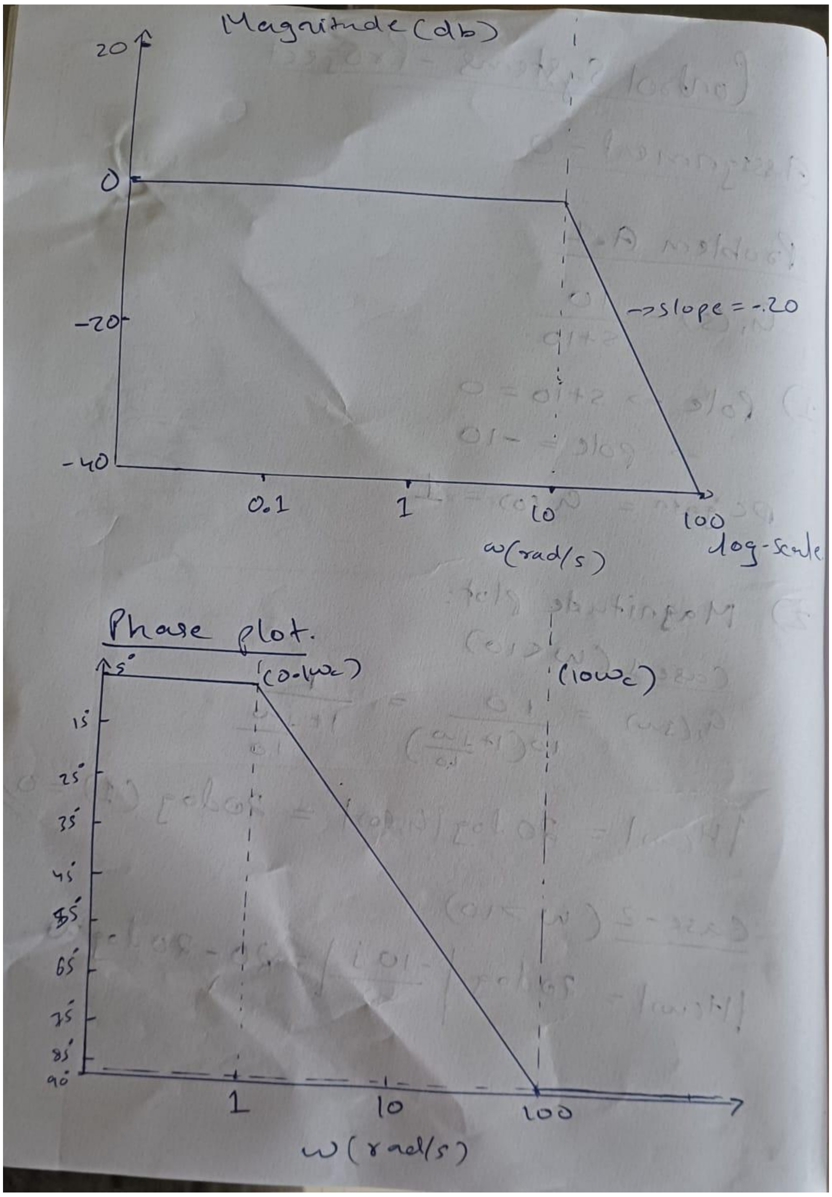
<u>Case 1</u> ($\omega < 10$)

$$G_1(j\omega) = \frac{10}{10\left(1+\frac{j\omega}{10}\right)} = \frac{1}{1+\frac{j\omega}{10}}$$

$$|H(\omega)| = 20\log|G_1(0)| = 20\log(1) = 0$$

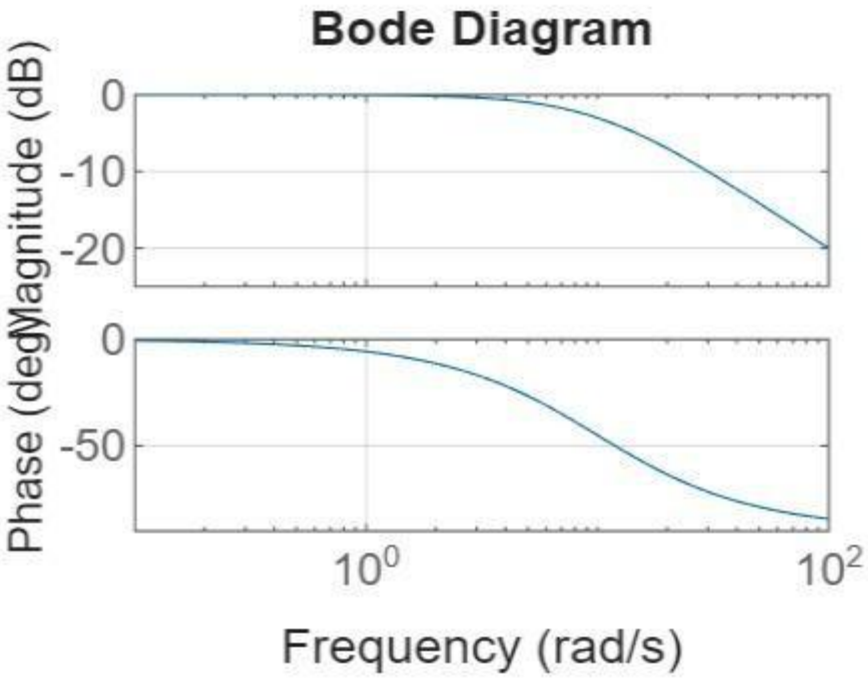
<u>Case-2</u> ($\omega > 10$)

$$|H(\omega)| = 20\log\left|\frac{-10j}{\omega}\right| = 20 - 20\log\omega.$$



3. Attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for $G_1(s)$.



1.2 Problem A.2: $G_2(s) = \dfrac{s-2}{s+10}$

1. Find the zero, pole, and DC gain:

   Zero: $s =$ __________, Pole: $s =$ __________, $G_2(0) =$ __________

Zero:2 Pole:-10 $G_2(0)$=-0.2

2. Sketch the **asymptotic Bode magnitude** and **phase** plots for $\omega \in [0.1, 100]$ rad/s.

1) $G_2(s) = \frac{s-2}{s+10}$

Zero: $\Rightarrow s-2=0$

$\Rightarrow s=2$ zero $= 2$

Pole: $\Rightarrow s+10=0$

$\Rightarrow s=-10$ pole $= -10$

$G_2(0)$: $G_2(0) = \frac{0-2}{0+10} = \frac{-1}{5}$ //

2) $G_2(s) = (-0.2)\left[\frac{1-\frac{s}{2}}{1+\frac{s}{10}}\right]$

$G_2(j\omega) = -0.2\left[\frac{1-\frac{j\omega}{2}}{1+\frac{j\omega}{10}}\right]$

$\omega_z = 2,\ \omega_p = 10$

Case-1 $(\omega < 2)$

$20\log(|-0.2|) = -14\,dB$

Case-2 $(2 < \omega < 10)$

$\cancel{G_2(s) = -0.2\left(\frac{-j\omega}{2}\right)} = G_2(j\omega) = 0.2\left(\frac{j\omega}{2}\right)$

Slope: $+20dB$ as it is a zero.

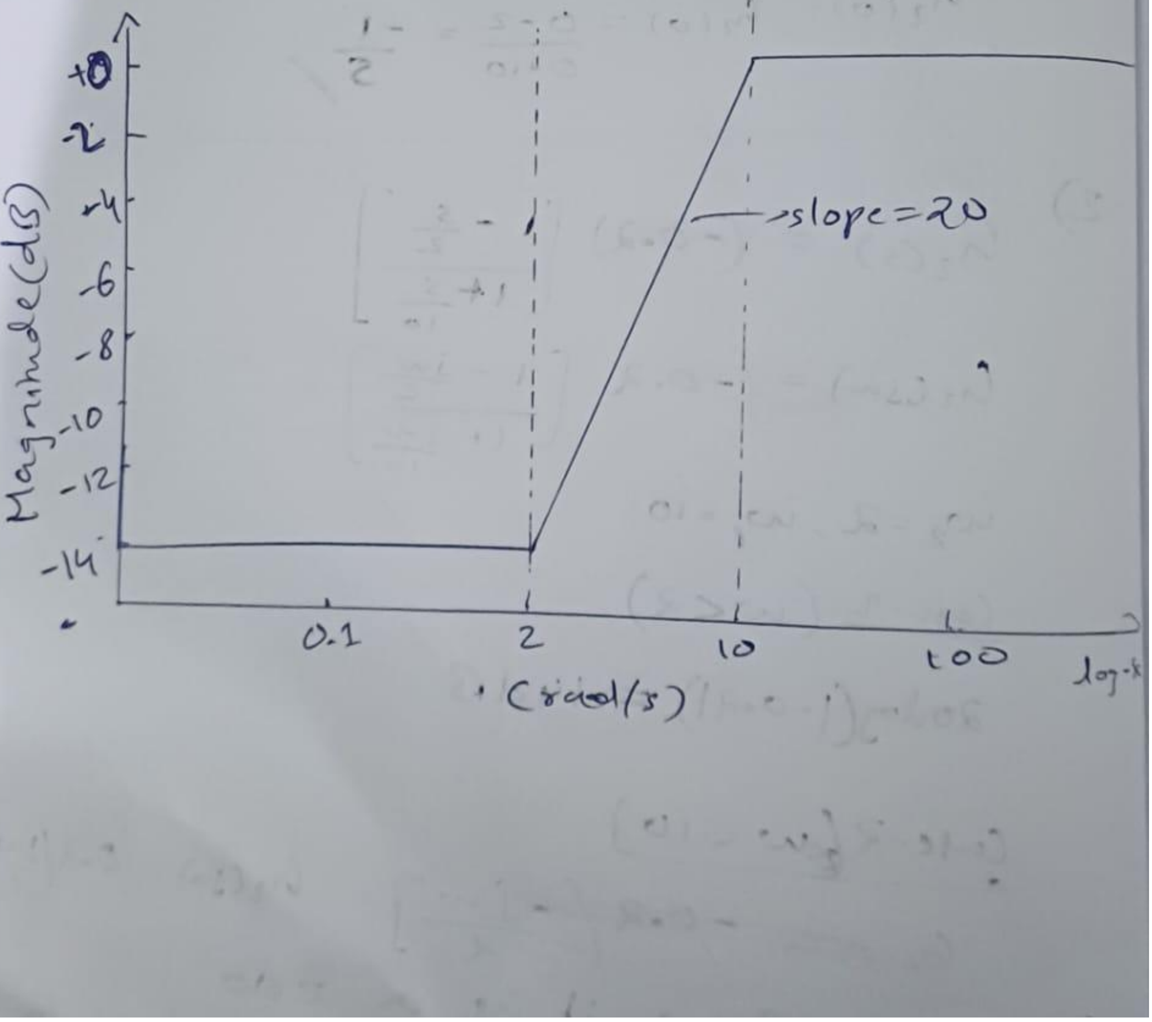
$|H(\omega)| = -14 + 20\log(\omega) - 20\log 2$

Case: 3 $(\omega > 10)$

Slope: 0 dB as reduced by −20 dB at pole of order 1.

$$G_2(j\omega) = (0.2)\left[\frac{\frac{-j\omega}{2}}{\frac{j\omega}{10}}\right] = 1$$

$$|H(\omega)| = 20\log(1) = 0$$



Phase plot

$$G_1(j\omega) = \frac{-0.2\left[1 - \frac{j\omega}{2}\right]}{1 + \frac{j\omega}{10}}$$

$$\arg(G_1(j\omega)) = 180 + \arg\left(1 - \frac{j\omega}{2}\right] - \arg\left(1 + j\frac{\omega}{10}\right)$$

$\arg\left(1 - j\frac{\omega}{2}\right]$:

→ $0°$ for $\omega << 0.2$

→ decreases linearly from 0 to $-90°$ $(0.2 < \omega < 20)$

→ $-90°$ $\omega >> 20$

$\arg\left(1 + j\frac{\omega}{10}\right)$:

→ $0°$ for $\omega << 1$

→ goes from 0 to $90°$ $(1 < \omega < 100)$

→ $90°$ $\omega >> 100$

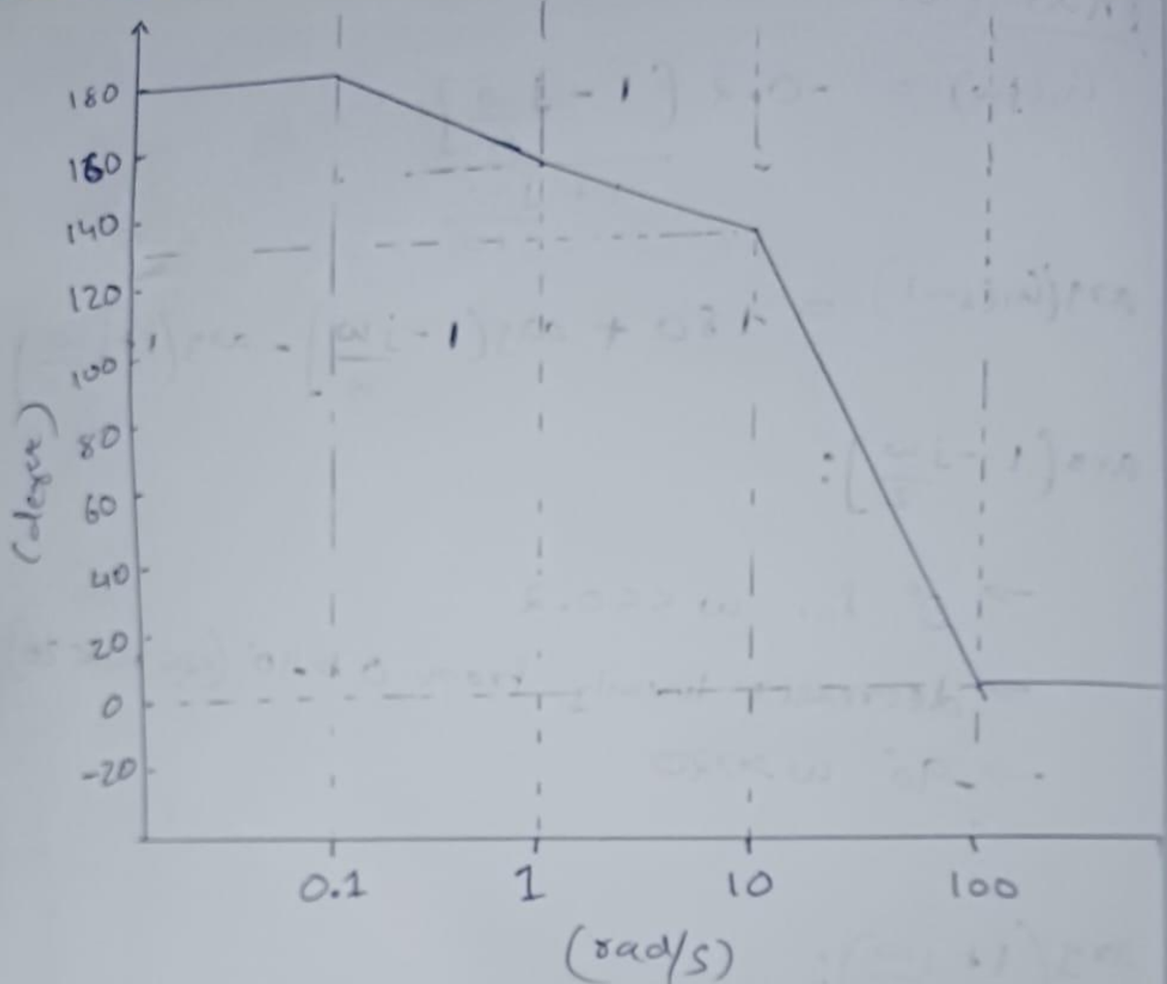
Approximately:

(i) $\omega = 0.1 : 180°$

(ii) $\omega = 1 : 150°$

(iii) $\omega = 10 : 135°$

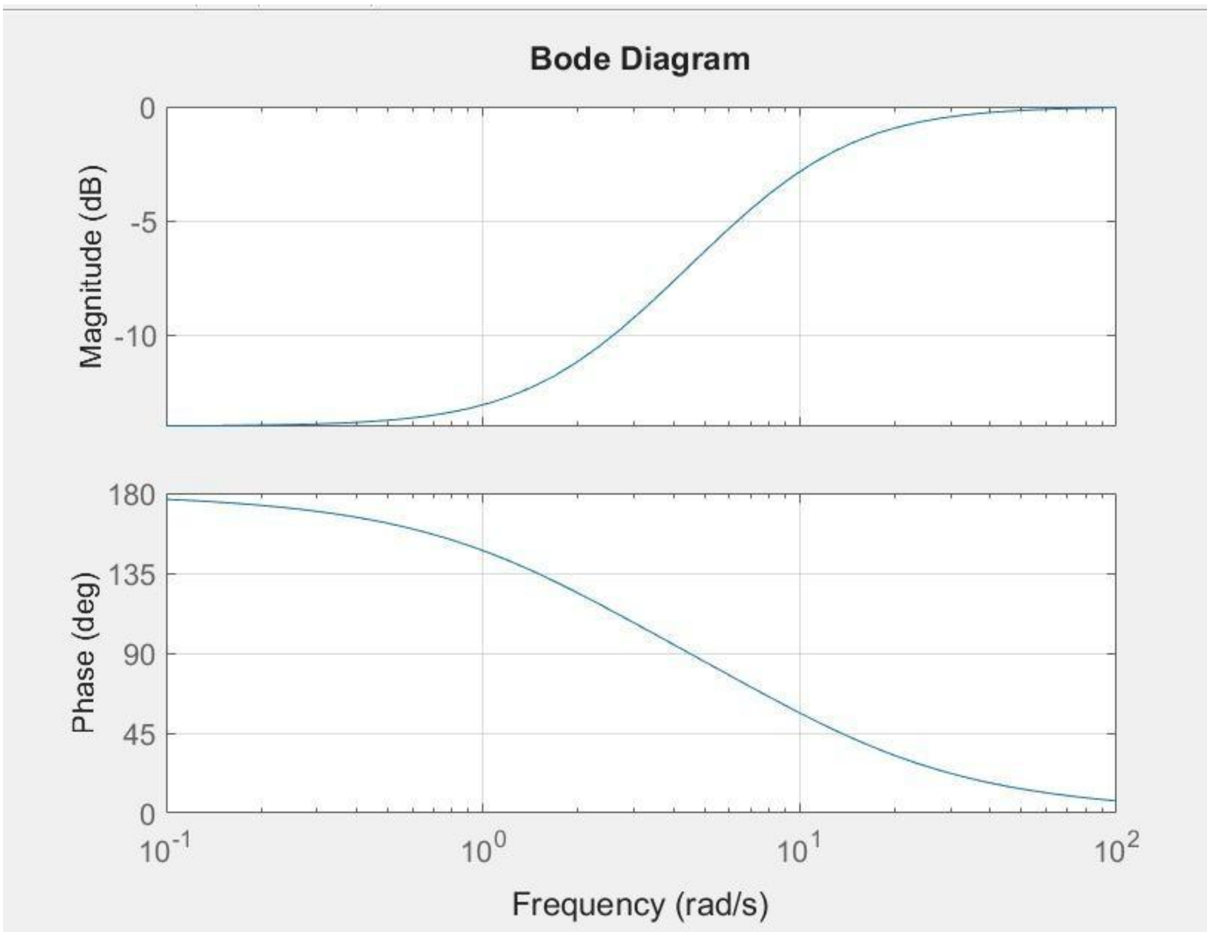
(iv) $\omega = 100 : 0$



4) Compared to a left-half plane zero a right half-plane zero produces phase lag. Its contribution is about 180° less phase than an equivalent left-half plane-zero.

3. Attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for $G_2(s)$.



4)

4. **Very short question:** This system has a **right-half-plane zero**. One sentence: how does that usually affect the phase (compared to a left-half-plane zero)?

Compared to a left-half plane zero a right-half plane zero produces phase lag,its contribution is about 180^0 less than an equivalent lefthalf plane zero.

1.3 Problem A.3: $G_3(s) = \dfrac{100}{s^2 + 10s + 100}$

1. Find the poles:

$s_{1,2} =$ ______________________________

Problem 4.3

1) for poles:

$$s^2 + 10s + 100 = 0$$

$$\Rightarrow (s+5)^2 = -75 \Rightarrow s = 5 \pm j5\sqrt{3}$$

$$s_{1,2} = 5 \pm j5\sqrt{3}$$

2) Magnitude plot:

$$G_3(j\omega) = \frac{100}{(j\omega)^2 + 10j\omega + 100}$$

$$= \frac{1}{\left(\frac{j\omega}{10}\right)^2 + \left(\frac{j\omega}{10}\right) + 1} = \frac{1}{\left(1 - \left(\frac{\omega}{10}\right)^2\right) + j\frac{\omega}{10}}$$

$$= \frac{\left[1 - \left(\frac{\omega}{10}\right)^2\right] - j\frac{\omega}{10}}{\left[\left(1 - \left(\frac{\omega}{10}\right)^2\right]^2 + \left(\frac{\omega}{10}\right)^2\right]}$$

taking standard 2^{nd} order form:

$$s^2+10s+100 = s^2+2\xi\omega_n s+\omega_n^2$$

$$\Rightarrow \omega_n = 10 \text{ rad/s}$$

$$\xi = 0.5$$

$$G_3(s) = \frac{100}{s^2+10s+100} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$G_3(0) = \frac{100}{100} = 1$$

for $\underline{\omega << \omega_n}$

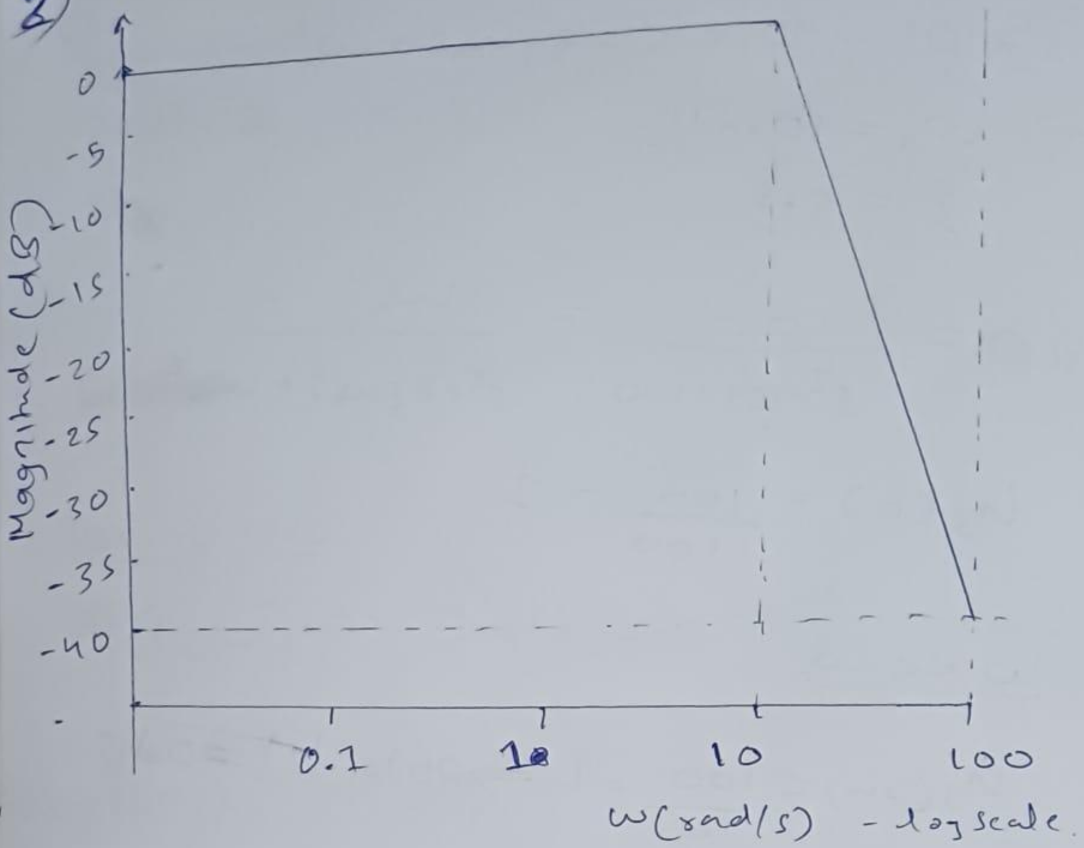
$$G_3(j\omega) \simeq \frac{100}{100} = 1 \Rightarrow 20\log_{10}|1| = 0\,dB$$

for $\underline{\omega >> \omega_n}$

$$G_3(j\omega) \simeq \frac{100}{-\omega^2}$$

$$M(\omega) = 20\log\left|\frac{100}{-\omega^2}\right| = 40-40\log_{10}\omega$$

2)



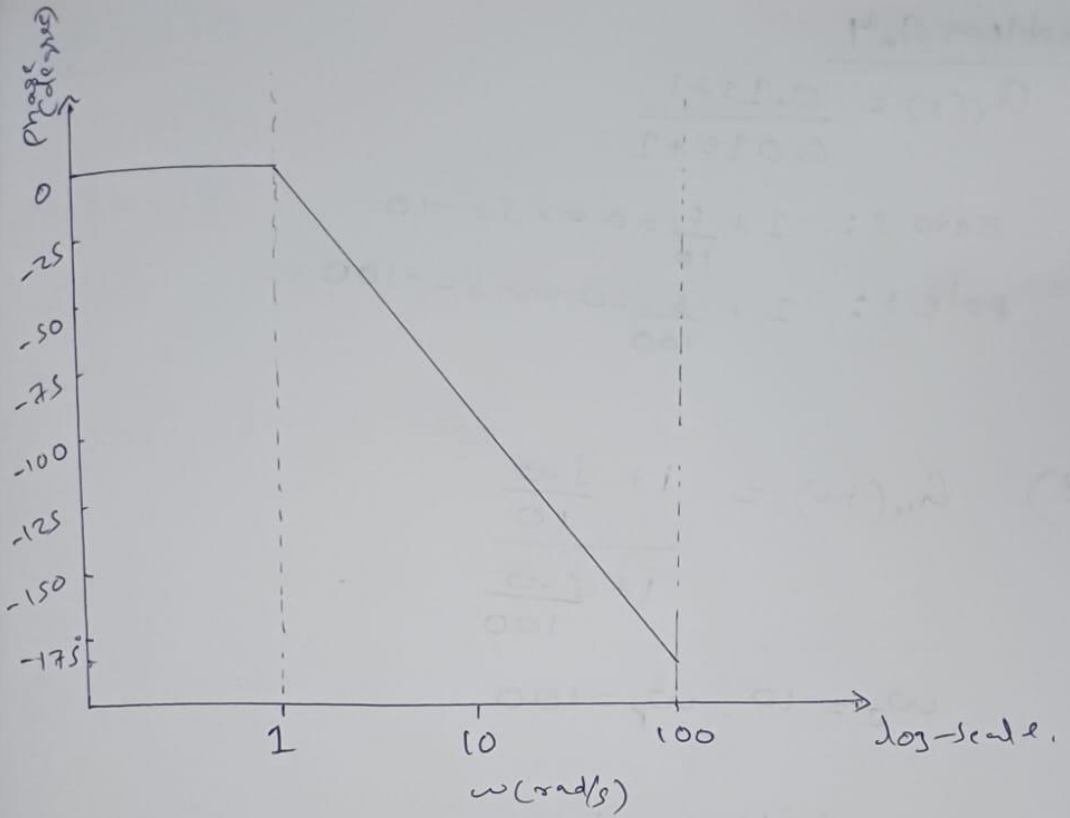
Phase plot

~~for a straight~~

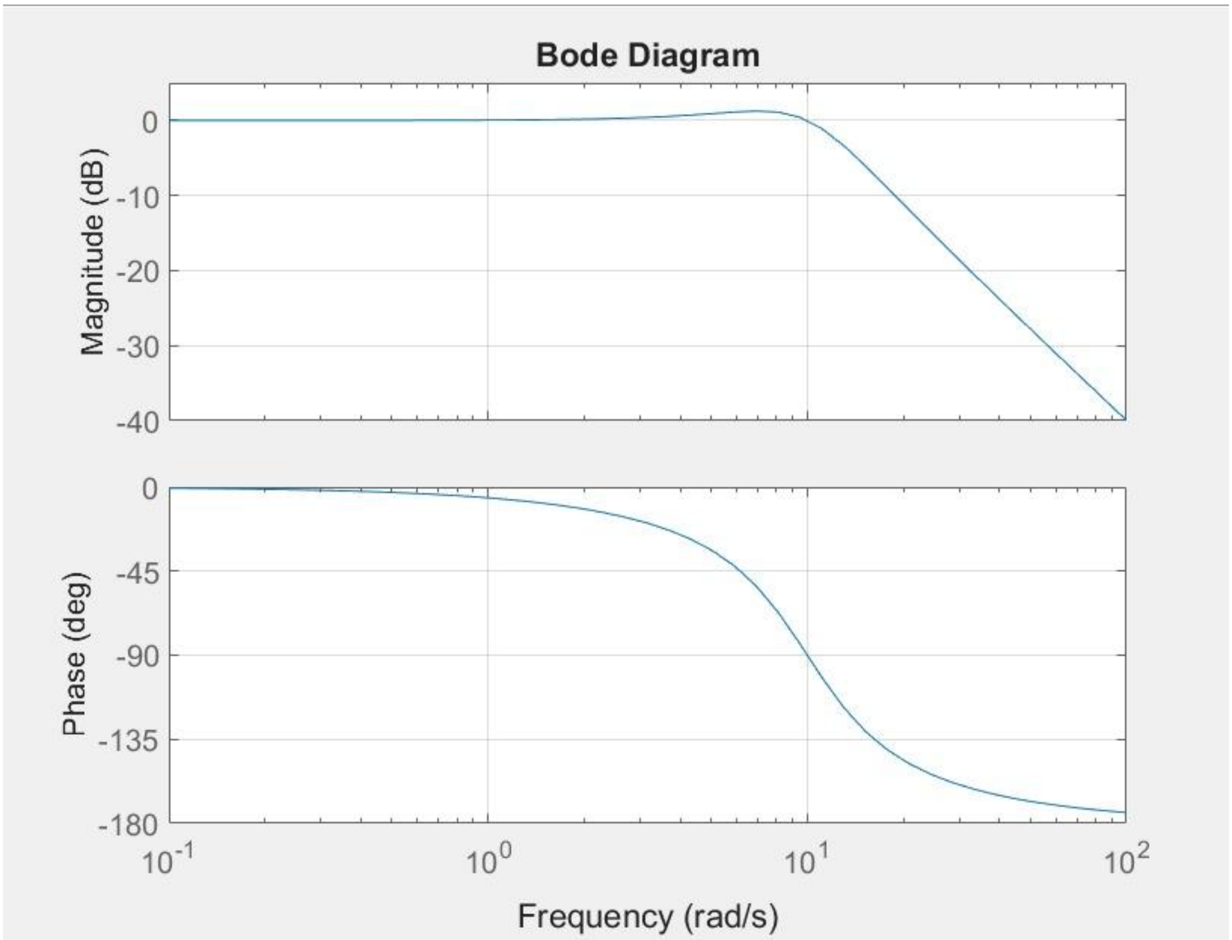
- at low frequency $Im(h_3) = 0$
  so phase = 0°.

* at $\omega = \omega_n/10$ phase starts dropping linearly

* at $\omega = 10\omega_n$ phase approaches 180°.



Phase (degrees)
0
-25
-50
-75
-100
-125
-150
-175°
1
10
100
log-scale.
ω(rad/s)



1.4 Problem A.4: $G_4(s) = \dfrac{0.1s + 1}{0.01s + 1}$

1. Find the zero and pole:

$$\text{Zero at } s = \underline{\hspace{3cm}}, \quad \text{Pole at } s = \underline{\hspace{3cm}}$$

Problem A.4

1) $G_4(s) = \dfrac{0.1s+1}{0.01s+1}$

zeros: $1 + \dfrac{s}{10} = 0 \Rightarrow s = -10$

poles: $1 + \dfrac{s}{100} = 0 \Rightarrow s = -100$

2) $G_4(j\omega) = \dfrac{1+\dfrac{j\omega}{10}}{1+\dfrac{j\omega}{100}}$

$\omega_z = 10, \ \omega_p = 100$

$M(\omega) = 20\log|G_4(j\omega)|$

$= 20\log_{10}|1+j0.1\omega| - 20\log_{10}|1+j0.01\omega|$

Slopes contributed by pole & zero:

- by ω_z:
  - 0 dB/decade for $\omega << \omega_z$
  - +20 dB/decade for $\omega >> \omega_z$

- by ω_p
  - 0 dB/decade for $\omega << \omega_p$
  - −20 dB/decade for $\omega >> \omega_p$

$\underline{\omega < 10}$

$\rightarrow$ slope $= 0$

$\underline{<\omega < 100}$

slope $= +20$

$\underline{\omega > 100}$

slope $= 20 - 20 = 0$

Phase plot

$$G_4(j\omega) = \frac{1 + j0.1\omega}{1 + j0.01\omega}$$

$$\arg(G_4(j\omega)) = \arg(1 + j0.1\omega) - \arg(1 + j0.001\omega)$$

Consider zero (ω_z)

$\omega << \omega_z$

$\Rightarrow \simeq \arg(1) \cancel{- \arg(1)} = 0$

$\omega >> \omega_z$

$\simeq \arg(j0.1\omega) = 90^\circ$

* in b/w use linear approximation.

Consider pole

$\omega << \omega_p$

$\Rightarrow \simeq -\arg(1) = 0$

$\omega >> \omega_p$

$\simeq -\arg(j0.01\omega) = -90^\circ$

* use linear approximation in b/w

taking sum of both contributions.

<u>$0.1 < \omega < 1$</u>

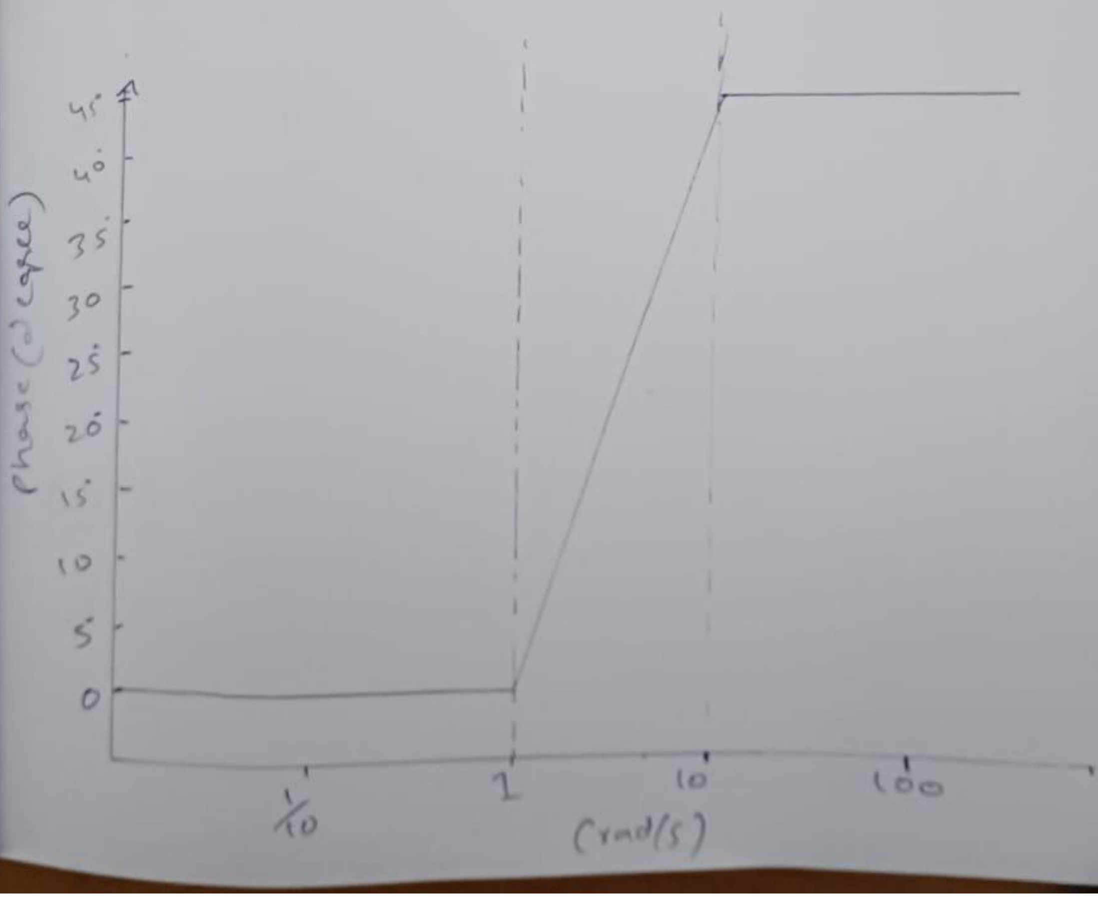
$\varphi = 0°$

w) <u>$1 < \omega < 10$</u>

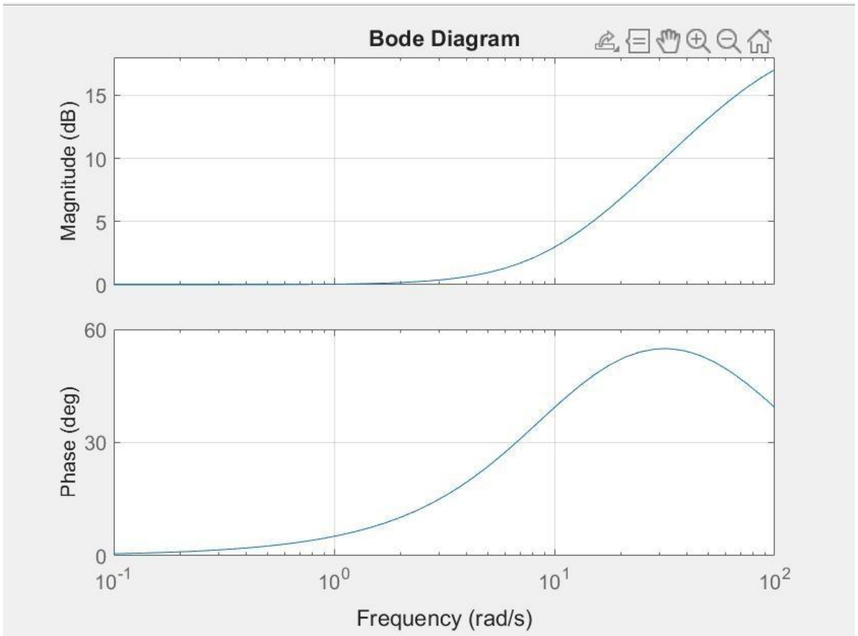
$\varphi \Rightarrow$ rises from $0°$ to $45°$ linearly
as negligible contribution of pole.

<u>$10 < \omega < 100$</u>

$\varphi \rightarrow$ pole & zero cancel out each others
effect.



4) because zero is at lower frequency than the pole the zero's contribution gets active before the poles, resulting in a positive face.



4. **Very short question:** Around the frequency between the zero and pole, does $G_4(s)$ tend to add **positive** phase (phase lead) or **negative** phase (phase lag)?

Because Zero is at lower frequency than the pole the zeros contribution gets active before the poles, resulting in a positive phase

2 Part B: Mass–Spring–Damper Transfer Function

Consider the mechanical system shown below. A mass m is connected to a fixed wall via a spring (stiffness c) and a damper (damping coefficient d). An external force $F(t)$ is applied to the mass, producing displacement $x(t)$.

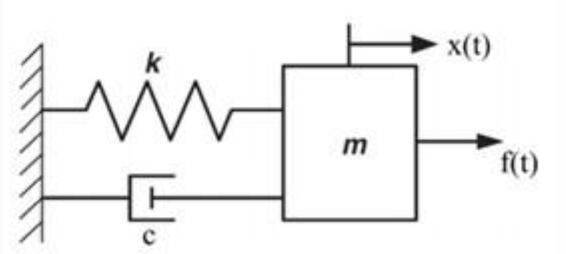


Figure 1: Mass–spring–damper system with input force $F(t)$ and output displacement $x(t)$.

B.1 Derive the Differential Equation and Transfer Function

1. Using Newton's second law and the diagram, write the differential equation relating $x(t)$ and $F(t)$:

   ____

2. Assume zero initial conditions and apply the Laplace transform to obtain an equation in $X(s)$ and $F(s)$:

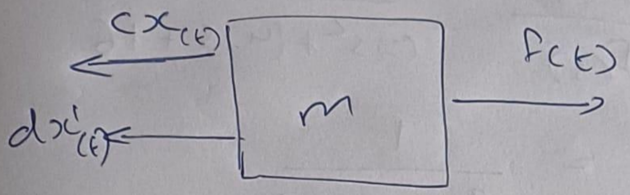
   ____

3. Derive the transfer function:

$$G(s) = \frac{X(s)}{F(s)} = \underline{\hspace{8cm}}$$

Part B.1

FBD



$$\Sigma F_x = m x''_{(t)}$$

1) $m x''(t) = f(t) - c x_{(t)} - d x'(t)$

2) $\xrightarrow{\mathcal{L}} m s^2 X_{(s)} = F(s) - c X_{(s)} - d s X(s)$

$\Rightarrow X(s) \left[m s^2 + d s + c \right] = F(s)$

3) $G(s) = \dfrac{X(s)}{F(s)} = \dfrac{1}{m s^2 + d s + c}$

Part B.2

$m = 1\,kg \quad d = 4\,N.s/m \quad c = 16\,N/m$

1) $G(s) = \frac{X(s)}{F(s)} = \frac{1}{(1)s^2 + (4)s + 16}$

$= \frac{1}{s^2 + 4s + 16}$

2) poles:

$\Rightarrow s^2 + 4s + 16 = 0$

$\Rightarrow (s+2)^2 = -12$

$s + 2 = \pm j 2\sqrt{3}$

$s = -2 \pm j 2\sqrt{3}$

$$\boxed{s_{1,2} = -2 \pm j 2\sqrt{3}}$$

3) the standard form of a 2^{nd} order transfer function:

$\Rightarrow \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

~~for easy~~

Convert $G(s)$ to that form.

$$G(s) = \frac{1}{16}\left[\frac{16}{s^2+4s+16}\right], \quad \omega_n = 4$$

$$G(0) = 20\log_{10}\left|\frac{1}{16}\right| \approx -24\,dB$$

when $\underline{\omega << \omega_n}$

* we can take the DC gain as $\omega \to 0$

$$M(\omega) = -24\,dB$$

when $\underline{\omega >> \omega_n}$

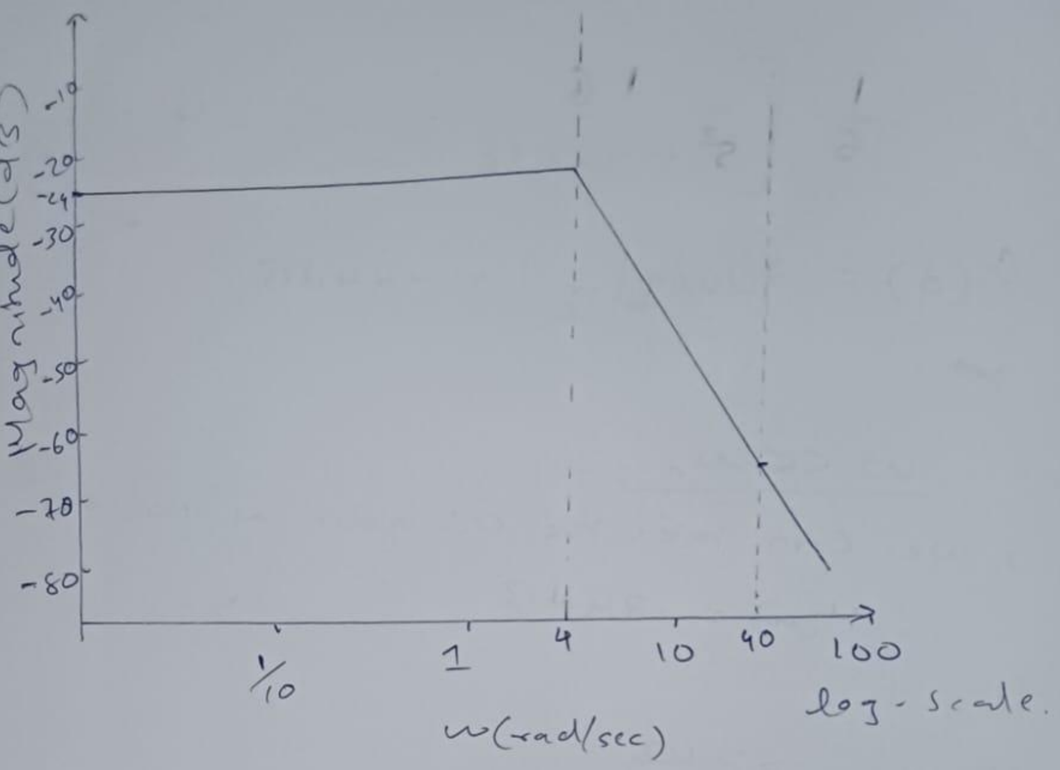
* $$G(j\omega) \approx \frac{1}{16}\left[-\frac{16}{\omega^2}\right]$$

$$\Rightarrow \approx -24.1 - 40\log\left(\frac{\omega}{4}\right) dB$$

: $\omega = 0.1$, $-24\,dB$

$\omega = 4$, $-24\,dB$. (starts changing linearly)

$\omega = 40$: $-24 - 40 \approx -64\,dB$



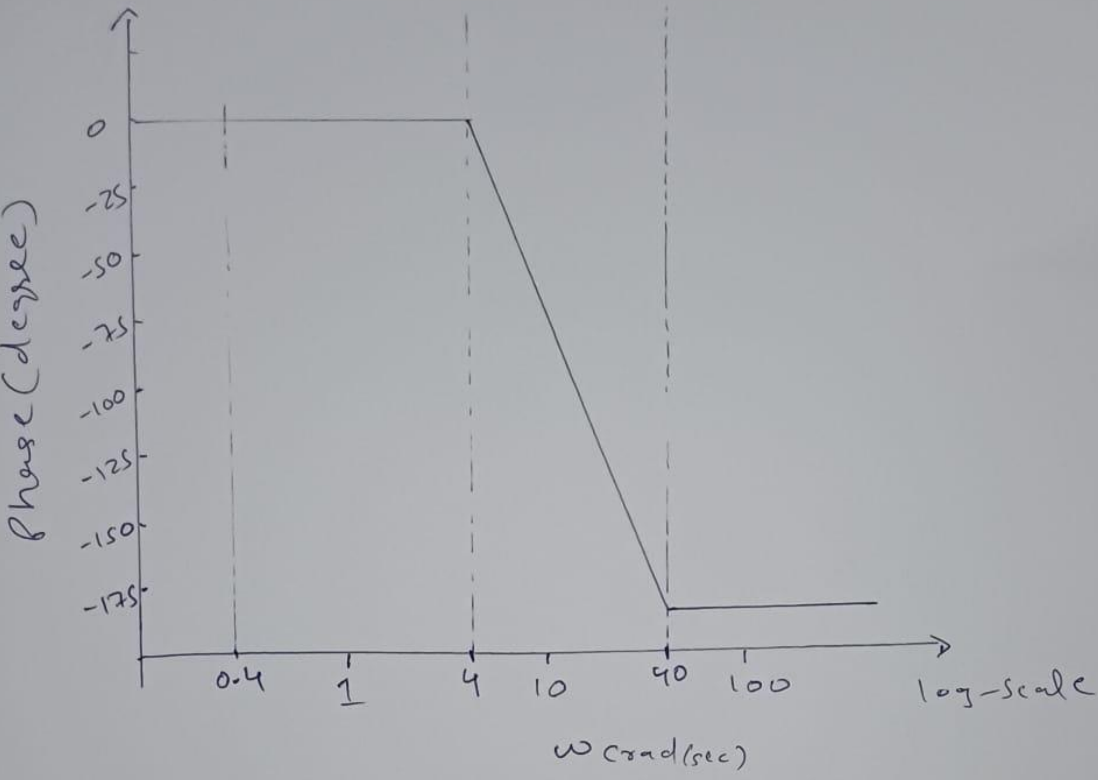
Phase plot

$\omega = 0.1\omega_n$: $0°$

$\omega = \omega_n$: $-90°$

$\omega > 10\omega_n$: $180°$

we use linear approximation b/w ω_n & $10\omega_n$.



Phase (degree)
0
-25
-50
-75
-100
-125
-150
-175
0.4
1
4
10
40
100
log-scale
ω (rad/sec)

4. Generate Bode plots in MATLAB/Octave/Python and attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for this numerical example.