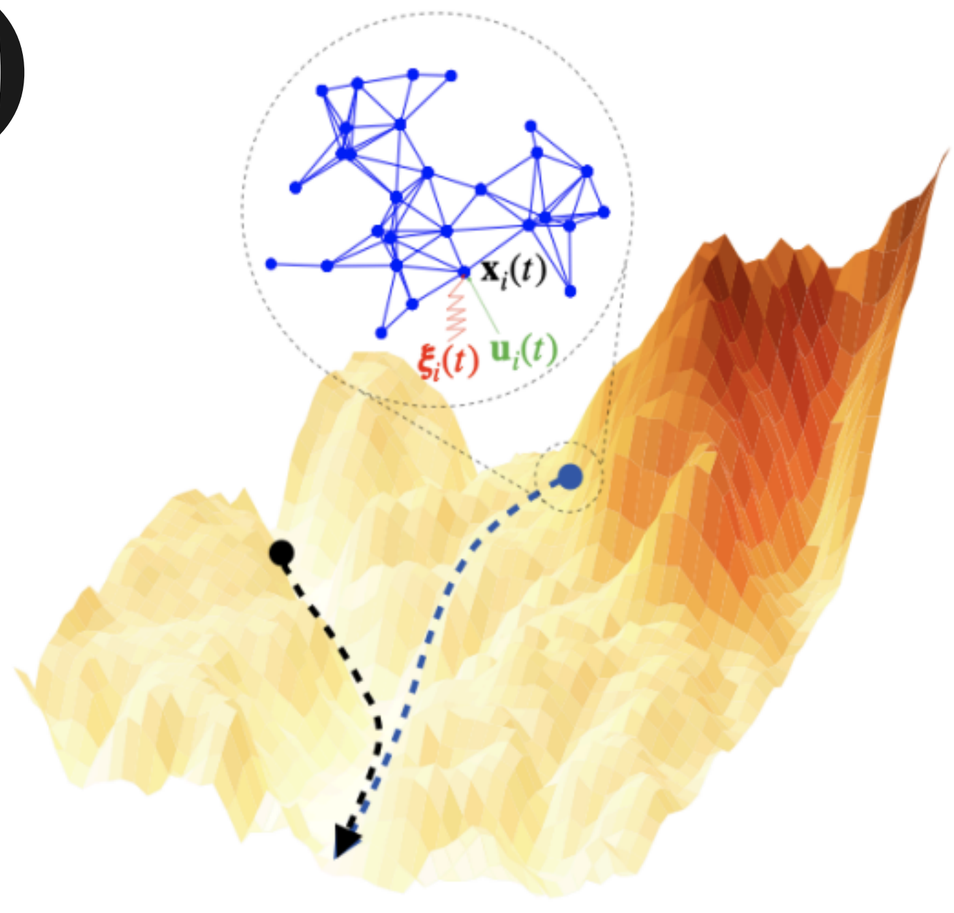
a)



b)

Variables

$$\underline{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \vdots \\ \mathbf{x}_i(t) \\ \vdots \\ \mathbf{x}_N(t) \end{bmatrix} \quad \underline{\mathbf{u}}(t) = \begin{bmatrix} \mathbf{u}_1(t) \\ \vdots \\ \mathbf{u}_i(t) \\ \vdots \\ \mathbf{u}_N(t) \end{bmatrix} \quad \underline{\boldsymbol{\xi}}(t) = \begin{bmatrix} \boldsymbol{\xi}_1(t) \\ \vdots \\ \boldsymbol{\xi}_i(t) \\ \vdots \\ \boldsymbol{\xi}_N(t) \end{bmatrix}$$

Dynamics

$$\begin{cases} \dot{\mathbf{x}}_i(t) = -\frac{1}{\eta} \nabla_i V(\underline{\mathbf{x}}(t)) + \mathbf{u}_i(t) + \sqrt{2D}\, \boldsymbol{\xi}_i(t) \\ \mathbf{x}_i(0) = \mathbf{x}_{i_0} \end{cases} \quad (1)$$

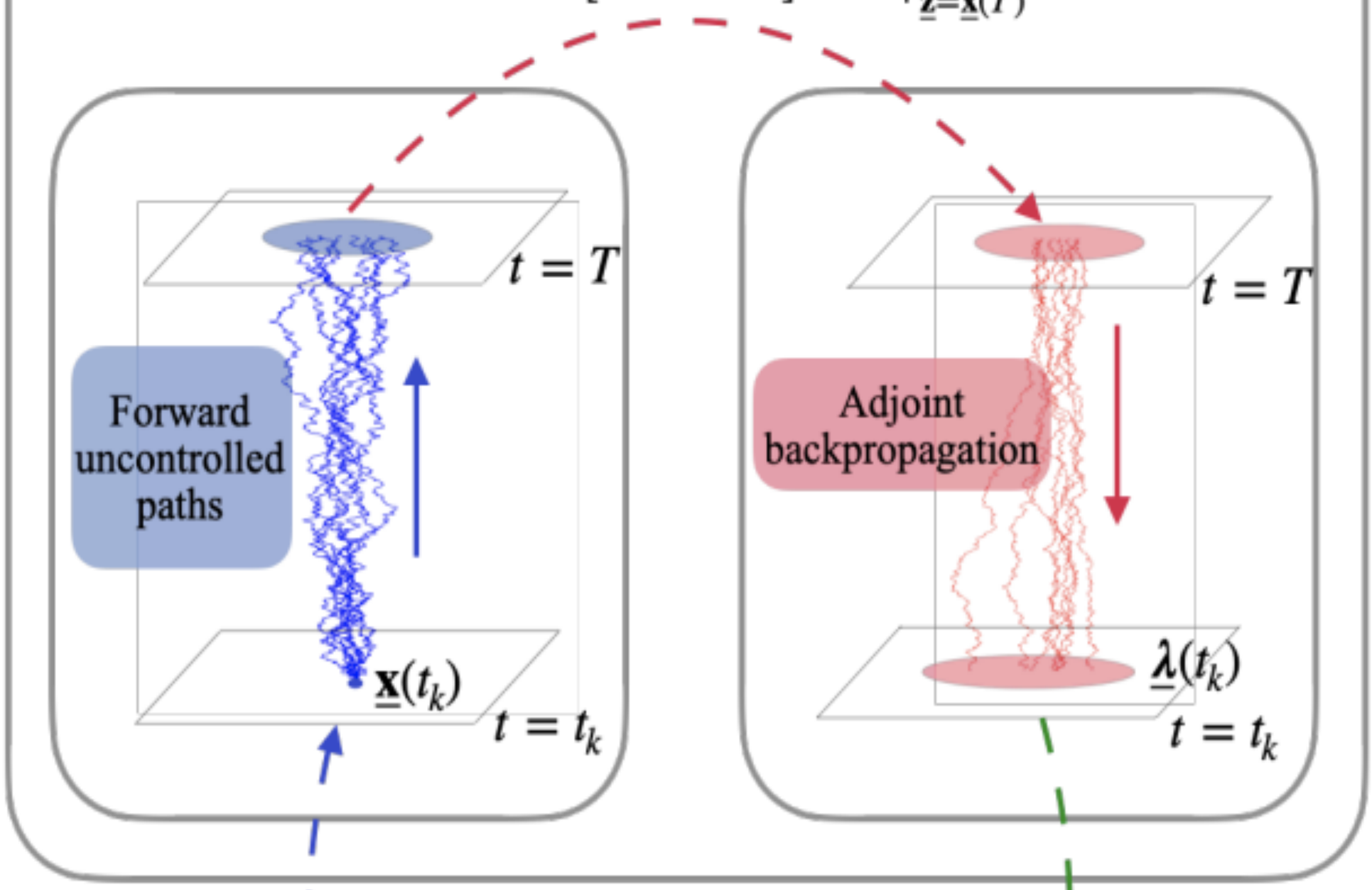
Control

$$\min_{\underline{\mathbf{u}}} \mathbb{E}\left[\frac{\gamma}{2}\int_0^T \sum_{i=1}^N \mathbf{u}_i^2(t)\, dt + \Psi(\underline{\mathbf{x}}(T))\right] \text{ given (1) holds}$$

c)

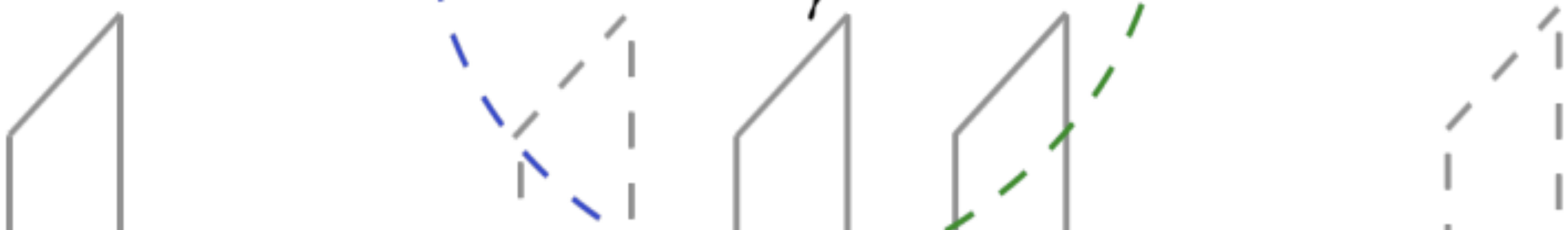
Adjoint terminal condition

$$\underline{\boldsymbol{\lambda}}(T) = \frac{e^{-\beta\Psi(\underline{\mathbf{x}}(T))}}{\mathbb{E}\left[e^{-\beta\Psi(\underline{\mathbf{x}}(T))}\right]} \frac{\partial\Psi}{\partial\underline{\mathbf{z}}}\bigg|_{\underline{\mathbf{z}}=\underline{\mathbf{x}}(T)}$$



Optimal control at $t = t_k$

$$\underline{\mathbf{u}}^*(t_k) = -\frac{1}{\gamma}\mathbb{E}\left[\underline{\boldsymbol{\lambda}}(t_k)\right]$$



Displacement at $t = t_k$

$$\underline{\mathbf{x}}(t_{k+1}) = \underline{\mathbf{x}}(t_k) - \nabla V(\underline{\mathbf{x}}(t_k))\Delta t + \underline{\mathbf{u}}^*(t_k)\Delta t + \sqrt{2D}\underline{\boldsymbol{\xi}}(t_k)\Delta t$$