

Question 1: A die is rolled. What is the probability of getting:

- (a) An even number
- (b) A number greater than 4

Answer:

A fair die is rolled once.

Sample Space:

$S = \{1, 2, 3, 4, 5, 6\}$

Total outcomes = 6

(a) Probability of getting an even number

Even numbers on a die are: $\{2, 4, 6\}$

Number of favourable outcomes = 3

$$P(\text{Even number}) = \frac{3}{6} = \frac{1}{2}$$

Answer:

The probability of getting an even number is $\frac{1}{2}$.

(b) Probability of getting a number greater than 4

Numbers greater than 4 are: $\{5, 6\}$

Number of favourable outcomes = 2

$$P(\text{Number greater than 4}) = \frac{2}{6} = \frac{1}{3}$$

Answer:

The probability of getting a number greater than 4 is $\frac{1}{3}$.

Question 2: In a class of 50 students:

20 like Mathematics (M)

15 like Science (S)

5 like both subjects

What is the probability that a student chosen at random likes Mathematics or Science?

Answer:

Total number of students = 50

Students who like Mathematics (M) = 20

Students who like Science (S) = 15

Students who like both subjects = 5

To find:

Probability that a student likes Mathematics or Science.

Solution:

Since some students like both Mathematics and Science, we use the formula:

$$\begin{aligned}n(M \cup S) &= n(M) + n(S) - n(M \cap S) \\ &= 20 + 15 - 5 = 30\end{aligned}$$

So, 30 students like Mathematics or Science.

$$P(M \cup S) = \frac{30}{50} = \frac{3}{5}$$

Answer:

The probability that a randomly chosen student likes Mathematics or Science is **3/5**.

Question 3: A bag has 3 red and 2 blue balls. If one ball is drawn randomly and is red, what is the probability that the next ball is also red (without replacement)?

Answer:

Given:

A bag contains 3 red balls and 2 blue balls.

One ball is drawn and it is red.

The ball is not replaced.

To find:

Probability that the next ball is also red.

Solution:

After drawing one red ball, the remaining balls are:

- Red balls = 2
- Blue balls = 2

So, total remaining balls = 4

Probability that the next ball is red:

$$P(\text{Red}) = \frac{2}{4} = \frac{1}{2}$$

Answer:

The probability that the next ball is also red is $\frac{1}{2}$.

Question 4: The population of a school is divided into 60% boys and 40% girls. If you want equal representation of both genders in the sample, which method should you use: Simple Random Sampling or Stratified Sampling? Why?

Answer:

Given:

The school population consists of:

- Boys = 60%
- Girls = 40%

To find:

Which sampling method should be used to get equal representation of boys and girls.

Solution:

To ensure equal representation of both boys and girls in the sample, Stratified Sampling should be used.

In stratified sampling, the population is divided into groups (strata) such as boys and girls, and samples are taken from each group separately. This ensures that both genders are properly represented in the sample.

Simple random sampling may not guarantee equal representation, as one group may be selected more than the other by chance.

Answer:

Stratified Sampling should be used because it ensures equal representation of both boys and girls in the sample.

Question 5: The average height of 1000 students = 160 cm. A sample of 100 students shows an average height = 158 cm. Find the sampling error.

Answer:

Given:

Average height of population (1000 students) = 160 cm

Average height of sample (100 students) = 158 cm

To find:

Sampling error

Solution:

Sampling error is the difference between the sample mean and the population mean.

$$\begin{aligned}\text{Sampling Error} &= \text{Sample Mean} - \text{Population Mean} \\ &= 158 - 160 = -2 \text{ cm}\end{aligned}$$

Answer:

The sampling error is **-2 cm**.

(This shows that the sample average is 2 cm less than the population average.)

Question 6: The population mean salary is ₹50,000 with $\sigma = ₹5,000$. If we take a sample of 100 employees, what is the standard error of the mean (SEM)?

Answer:

Given:

Population mean salary = ₹50,000

Population standard deviation (σ) = ₹5,000

Sample size (n) = 100

To find:

Standard Error of the Mean (SEM)

Solution:

The formula for Standard Error of the Mean is:

$$SEM = \sigma / \sqrt{n}$$

Substituting the given values:

$$SEM = \frac{5000}{\sqrt{100}} = \frac{5000}{10} = 500$$

Answer:

The standard error of the mean (SEM) is ₹500.

Question 7: In a group of 100 students:

40 like Cricket (C)

30 like Football (F)

10 like both Cricket and Football

Find the probability that a student likes at least one sport.

Answer:

Given:

Total number of students = 100

Students who like Cricket (C) = 40

Students who like Football (F) = 30

Students who like both Cricket and Football = 10

To find:

Probability that a student likes at least one sport.

Solution:

“At least one sport” means Cricket or Football, that is $C \cup F$.

We use the formula:

$$\begin{aligned}n(C \cup F) &= n(C) + n(F) - n(C \cap F) \\ &= 40 + 30 - 10 = 60\end{aligned}$$

So, 60 students like at least one sport.

$$P(\text{At least one sport}) = \frac{60}{100} = \frac{3}{5}$$

Answer:

The probability that a randomly chosen student likes at least one sport is $\frac{3}{5}$.

Question 8: From a deck of 52 cards, two cards are drawn without replacement. What is the probability that both are Aces?

Answer:

Given:

A standard deck has 52 cards.

Number of Aces = 4.

Two cards are drawn without replacement.

To find:

Probability that both cards are Aces.

Solution:

Probability that the first card is an Ace:

$$\frac{4}{52}$$

After drawing one Ace, remaining cards = 51

Remaining Aces = 3

Probability that the second card is an Ace:

$$\frac{3}{51}$$

So, required probability:

$$\begin{aligned} p &= \frac{4}{52} \times \frac{3}{51} \\ &= \frac{12}{2652} = \frac{1}{221} \end{aligned}$$

Answer:

The probability that both cards drawn are Aces is 1/221.

Question 9: A factory produces bulbs with 2% defective rate. If 5 bulbs are chosen at random, what is the probability that all are non-defective?

Answer:

Given:

Defective rate = 2%

So, probability that a bulb is non-defective = $1 - 0.02 = 0.98$

Number of bulbs chosen = 5

To find:

Probability that all 5 bulbs are non-defective.

Solution:

Each bulb being non-defective is an independent event.

$$\begin{aligned} P(\text{All non-defective}) &= (0.98)^5 \\ &= 0.9039 \text{ (approx)} \end{aligned}$$

Answer:

The probability that all 5 bulbs are non-defective is approximately 0.904.

Question 10: Differentiate between discrete and continuous random variables with examples.

Answer:

Discrete Random Variable vs Continuous Random Variable

Discrete Random Variable

A discrete random variable takes countable values.
These values are usually whole numbers.

Examples:

- Number of heads when a coin is tossed
- Number of defective items in a batch

Continuous Random Variable

A continuous random variable takes uncountable values and can take any value within a given range.

Examples:

- Height of students
- Weight of a person
- Time taken to complete a task

Difference Between Discrete and Continuous Random Variables

Discrete Random Variable	Continuous Random Variable
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Takes countable values	Takes uncountable values
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Usually whole numbers	Can be decimals
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Based on counting

Based on measurement

Answer:

Discrete random variables have countable values, while continuous random variables can take any value within a range and are measured rather than counted.