

Linear Algebra

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SLI

Vector Spa

Sub-Spa

macpenaene

Column Cons

Quadratic form

Regression Analysis Linear Algebra

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Simple linear regression with Vector notation

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- Consider a data set $D = \{(x_i, y_i) | x_i \in \mathbb{R}, y_i \in \mathbb{R}, \forall i = 1, 2, \cdots, n\}$
- \blacksquare x_i s are non stochastic
- y_i s are stochastic and realized values of random variable Y_i s
- **y** = $(y_1, y_2, ..., y_n)^T$, **x** = $(x_1, x_2, ..., x_n)^T$, $\beta = (\beta_0, \beta_1)^T$ and **1** = $(1, 1, ..., 1)^T$

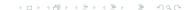
Problem statement (Redefined)

We are interested to have a prediction vector

$$\hat{\mathbf{y}} = g(\mathbf{x}, \boldsymbol{\beta}) = [\mathbf{1} \ \mathbf{x}] \boldsymbol{\beta}$$

which will approximate well the observed vector \mathbf{v} for known vector \mathbf{x} .

It is a problem in \mathbb{R}^n now !!





Other uses of vector representation

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Column Space

Column Space

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- Weighted sum / Averaging
- Expectation of discrete random variable
- Combing audio signals for music composition
- Image representation in pic-cell.
- Principal component Analysis
- \mathbb{P}_n = Polynomial up to degree n



Vector Space $(V, +, \cdot)$

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Definition

A vector space V over real numbers $\mathbb R$ is a collection of vectors such that

- $1 + : V \times V \rightarrow V$ [closed under vector addition]
- (x + y) + z = x + (y + z), for all $x, y, z \in V$ [associative]
- There exists $0 \in V$ such that 0 + x = x + 0 = x for all $x \in V$ [identity element exists]
- 1 There exists $-\mathbf{x} \in \mathbf{V}$ for each \mathbf{x} such that $(-\mathbf{x}) + \mathbf{x} = \mathbf{x} + (-\mathbf{x}) = \mathbf{0}$ [inverse exists]
- $\mathbf{S} \mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$ [commutative]
- **6** $a \cdot (b \cdot \mathbf{x}) = (ab) \cdot \mathbf{x}$ for all $a, b \in \mathbb{R}$ and $\mathbf{x} \in \mathbf{V}$
- 7 $1 \cdot \mathbf{x} = \mathbf{x}$ for all $\mathbf{x} \in \mathbf{V}$
- **8** $(a+b) \cdot \mathbf{x} = (a \cdot \mathbf{x}) + (b \cdot \mathbf{x})$ for all $a, b \in \mathbb{R}$ and $\mathbf{x} \in \mathbf{V}$
- $\mathbf{9} \ a \cdot (\mathbf{x} + \mathbf{y}) = a \cdot (\mathbf{x}) + a \cdot (\mathbf{y})$



Sub-Space $(S, +, \cdot)$

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Definition

If a subset S of V is a vector space itself then S is celled subspace of V.

How to check S is a subspace of V?

- (1) Whether $\mathbf{0} \in \mathbf{S}$?
- (2) Whether $\mathbf{x} + a \cdot \mathbf{y} \in \mathbf{S}$? for all $\mathbf{x}, \mathbf{y} \in \mathbf{S}$ and $a \in \mathbb{R}$.

Example:

- (1) All lines passing through (0,0) in \mathbb{R}^2 .
- (2) All planes passing through origin in \mathbb{R}^n .
- (3) \mathbb{P}_5 in \mathbb{P}_7



Span

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Definition

The span of a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \cdots \mathbf{v}_k\} \in \mathbf{V}$ is the collection

$$Sp\{\mathbf{v}_1,\mathbf{v}_2,\cdots\mathbf{v}_k\} = \left\{\sum_{i=1}^k c_i\mathbf{v}_i|c_i\in\mathbb{R}\right\}$$

which is the collection of all possible linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2, \cdots \mathbf{v}_k\}$.

Note: A span is always a subspace.

Example:

(a)
$$Sp\{(0,1),(1,1)\} = Sp\{(0,1),(1,0)\} = \mathbb{R}^2$$

(b)
$$Sp\{(0,1,0),(1,1,0)\} = \mathbb{R} \times \mathbb{R} \times \{0\} = xy$$
 - pane in \mathbb{R}^3

In regression $\hat{\mathbf{y}} \in Sp\{1,\mathbf{x}\}$ which is closest to $\mathbf{y} \in \mathbb{R}^n$



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