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Assignment-2

① (a) $(\lambda z.z)(\lambda y.yy)(\lambda x.xa)$

$= (\lambda y.yy)(\lambda x.xa)$ (Reduction)

$= (\lambda x.xa)(\lambda x.xa)$ β red.^{n} (replace z with λy yy)

$= (\lambda x.xa)\,a$ β red.^{n} (replace y with λx xa)

$= (aa)$ β red.^{n} (replace x with λx xa)

β red.^{n} (replace x with a)

(b) $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$

$= (\lambda z.zz)(\lambda z.zy)$ β red.^{n} (replace z with λz.zz)

$= (\lambda z.zy)(\lambda z.zy)$ β red.^{n} (replace z with λz.zy)

$= (\lambda z.zy)\,y$ β red.^{n} (replace z with λz.zy)

$= (yy)$ β red.^{n} (replace z with y)

(c) $(\lambda x.\lambda y\,x\,yy)(\lambda a.a)b$

$= (\lambda y.((\lambda a.a)yy))b$ β red.^{n} (replace x with λa.a)

$= (\lambda y.yy)b$ β red.^{n} (replace y with b)

$= (bb)$ β red.^{n} (replace a with b)

(d) $(\lambda x.\lambda y\,x\,y\,y)(\lambda y.y)\,y$

$= ((\lambda x.\lambda z\,x\,zz)(\lambda a.\lambda a))\,y$ α conv.^{n} (rename y to z)

$= (\lambda z((\lambda a.a)\,z\,z)\,y$ β red.^{n} (replace z with λy y)

$= (\lambda z.z\,z)\,y$ β red.^{n} (replace z with y)

$= (yy)$ β red.^{n} (replace y with y)

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(e) $(\lambda x.xx)(\lambda y.yx)z$

$= ((\lambda y.yx)(\lambda y.yx))z$ — β red^n (replace x with $\lambda y\,yx$)

$= ((\lambda y.yx)x)z$ — β red^n (replace y with $\lambda y\,yx$)

$= ((xx)z)$ — β red^n (replace y with x)

(f) $(\lambda x.(\lambda y(xy))y)z$

$= (\lambda x.(\lambda a.(xa))y)z$ — α conv^n (rename y to a)

$= (\lambda x.ya)z\ (\lambda x(xy))z$ — β red^n (replace a with y)

$= zy$ — β red^n (replace x with z)

(g) $(((\lambda x.(\lambda y.(xy))(\lambda y.y))\omega)$

$= [((\lambda x.(\lambda y.(xy))(\lambda a.a))\omega)$ — α conv^n (rename y to a)

$= ((\lambda y.((\lambda a.a)y))\omega)$ — β red^n (replace x with $\lambda y\,y$)

$= ((\lambda y.y)\omega)$ — β red^n (replace a with ω)

$= \omega$ — β red^n (replace y with $\lambda y\,y$)

(2) (a) ∴

$F = \lambda f.\lambda n\,(if(=n1)1\,(if(=n2)2\,(if(=n3)5\,(*\,(n\,(+\,F(n-1)\,(+\,F(n-2)\,F(n-3))))))))$

∴ Tri Product$(n) = (YF)n$

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(b) Now, Triproduct 4 = (YF)4

$YF = F(YF)$

$(YF)1 = F(YF)1$

$= \lambda F.\lambda n\ (if(=n1)\ 1\ if(=n2)2\ (if(=n3)5\ (*(n\ (+$
$(F(n-1)\ (+F(n-2)\ F(n-3)))))))))(YF)1$

$= \lambda n\ [if(=n1)1\ (if(=n2)2\ (if(=n3)5\ (*(n\ (+\ (YF(n-1)$
$(+YF(n-2)\ YF(n-3)))))))\ 1$

{replace F with YF} [β reduction]

$= if(=11)1\ (if(=12)2\ (if(=13)5\ (*(1(+(YF10)\ (+YF(-1)\ YF(-2)))))))$

{replace n with 1} [β reduction]

$= 1$ (Since $1=1$ is true)

$(YF)2 = F(YF)2$

$= \lambda F.\lambda n\ (if(=n1)1\ (if(=n2)2\ (if(=n3)5\ (*(n(+(F(n-1)(+$
$F(n-2)\ F(n-2)))))))))\ (YF)2$

{replace F with YF} [β-red.^{n}]

$= \lambda n\ (if(=n1)1\ (if(=n2)2\ (if(=n3)5\ (*\ (n\ (+\ (YF(n-1)$
$(+YF(n-2)\ YF(n-3)))))))))\ 2$

$= if(=21)1\ (if(=22)2\ (if(=23)5\ (*2(+(YF(1)\ (+YF(0)\ YF(-1)))))))$

$= 2$ (Since $2=2$ is true)

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YF(3) = F(YF)3

= λF.λn (if(=n1)1 (if(=n2)2 (if(=n3)5 (*n(+F(n-1)(+
F(n-2)F(n-3)))))))(YF)(3)

= λn(if(=n1)1 (if(=n2)2(if(=n3)5 (*n(+(YF(n-1)(+
YF(n-2)YF(n-3)))))))3

{replace F with YF} [β-red^n]

= if(=31)1(if(=32)2(if(=33)5 (*3(+(YF(2)(+YF(1)
YF(0))))))

= 5 (since (3=3) is true)

YF(4) = F(YF)4

= λF.λn (if(=n1)1(if(=n2)2(if(=n3)5(*n(+(F(n-1)
(+F(n-2)F(n-3)))))))(YF)(4)

{replace F with YF} [β-red^n]

= if(=41)1(if(=42)2(if(=43)5(*4(+(YF(3)
(+YF(2)(YF(1)))))))

= *4(+(YF(3)(+YF(2)YF(1))))

= *4(+(5(+2 1))) [δ red^n 2+1=3]

= *4(+(5 3)) [δ red^n 5+3=8]

= *4 (8) [δ red^n 4*8=32]

= 32