Assignment :-3

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Ans 1 :-

(a) let $\mathcal{E}_1 = \mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash y : Ref\ Bool$ [Identifier rule) — (1)

$\because$ $y := true$ {given}

$\mathcal{E}_1 \vdash true : Bool$ (constant rule) [$true \in \Sigma c$] — (2)

$$\dfrac{\overline{\mathcal{E}_1 \vdash true : Bool}^{\,(2)}, \quad \overline{\mathcal{E}_1 \vdash y : Ref\ Bool}^{\,(1)}}{\mathcal{E}_1 \vdash y := true : command} \quad \text{(assignment rule)}$$

$\therefore$ given expression is of command type.

(b) Given the types of func1 and func2, let

$\mathcal{E}_0 = \{func1 : A \to B,\ func2 : C \to B\}$

$\mathcal{E}_1 = \mathcal{E}_0 \cup \{x : A\} \cup \{q : C\}$

therefore, $\mathcal{E}_1 \vdash func1 : A \to B$ (constant rule) — (1)

$\mathcal{E}_1 \vdash func2 : C \to B$ (constant rule) — (2)

$\mathcal{E}_1 \vdash x : A$ (Identifier rule) — (3)

$\mathcal{E}_1 \vdash q : C$ (Identifier rule) — (4)

$$\dfrac{\dfrac{\overline{\mathcal{E}_1 \vdash func1 : A \to B}^{\,(1)}, \quad \overline{\mathcal{E}_1 \vdash x : A}^{\,(3)}}{\mathcal{E}_1 \vdash (func1\ x) : B}^{\,(5)} \quad \& \quad \overline{\mathcal{E}_1 \vdash x : A}^{\,(3)}}{\mathcal{E}_1 \vdash \lambda(x : A).(func1\ x) : A \to B}^{\,(6)} \quad \begin{matrix}\text{(Application rule)} \\ \text{(Function rule)}\end{matrix}$$

Since domain of func2 and type of q are same, we get

$$\dfrac{\dfrac{\overline{\mathcal{E}_1 \vdash func2 : C \to B}^{\,(2)}, \quad \overline{\mathcal{E}_1 \vdash q : C}^{\,(4)}}{\mathcal{E}_1 \vdash (func2\ q) : B}^{\,(7)} \quad \overline{\mathcal{E}_1 \vdash q : C}^{\,(4)}}{\mathcal{E}_1 \vdash \lambda(q : C).(func2\ q) : C \to B}^{\,(8)} \quad \begin{matrix}\text{(Application rule)} \\ \text{(function rule)}\end{matrix}$$

from (6) and (8) we have (Sumit, 18CS30042)

$$\dfrac{}{\mathcal{E}_1 \vdash \lambda(x:A).(\text{func}\,1\,x) : A \to B}(6) \quad \dfrac{}{\mathcal{E}_1 \vdash \lambda(q:C).(\text{fun}2\,q) : C \to B}(8)$$

$$\dfrac{}{\mathcal{E}_1 \vdash \lambda(x:A).(\text{func}1\,x);\ \lambda(q:C).(\text{fun}2\,q) : C \to B}(9)$$

(Sequencing)

Hence, type of given λ-expression is $C \to B$.

(c) given the type of 1,

let $\mathcal{E}_0 = \{1 : \text{Bool} \to \text{Bool} \to \text{Bool}\}$

$\mathcal{E}_1 = \mathcal{E}_0 \cup \{\omega : \text{Bool} \to \Pi\} \cup \{x : \text{Bool}\}$

$\therefore \quad \mathcal{E}_1 \vdash 1 : \text{Bool} \to \text{Bool} \to \text{Bool}$ (constant rule) —(1)

$\mathcal{E}_1 \vdash \omega : \text{Bool} \to \Pi$ (Identifier rule) —(2)

$\mathcal{E}_1 \vdash x : \text{Bool}$ (Identifier rule) —(3)

Since, $\text{true} \in \mathcal{E}C$, $\mathcal{E}_1 \vdash \text{true} : \text{Bool}$ (constant rule) —(4)

~~Hence, the type of given λ exp is Bool → Π → Bool → Π.~~

$(x \mid \text{true}) = ((1\,x)\,\text{true})$ (in prefix notation)

Now, $\because$ type of x and domain of 1 are same we have

$$\dfrac{\dfrac{}{\mathcal{E}_1 \vdash 1 : \text{Bool} \to \text{Bool} \to \text{Bool}}(1) \quad , \quad \dfrac{}{\mathcal{E}_1 \vdash x : \text{Bool}}(3)}{\mathcal{E}_1 \vdash (1x) : \text{Bool} \to \text{Bool}}(5)$$

(Application rule)

$$\dfrac{\mathcal{E}_1 \vdash (1x) : \text{Bool} \to \text{Bool} \qquad \dfrac{\cancel{\mathcal{E}_1 \vdash \text{tr}}}{\mathcal{E}_1 \vdash \text{true} : \text{Bool}}(4)}{\mathcal{E}_1 \vdash ((1x)\,\text{true}) \vdash \text{Bool}}(6)$$

(Application rule)

$\because$ domain of $(1x)$ and type of true are same

~~Hence, the type of given λ exp is (Bool → Π) → (Bool → Π)~~

(Sumit , 18CS30042)

$$\dfrac{\overline{\mathcal{E}_1 \vdash \omega : Bool \to \pi}\,(2) \quad \overline{\mathcal{E}_1 \vdash ((1x)\ true) : Bool}\,(6)}{\mathcal{E}_1 \vdash \omega\,(1x\ true) : \pi}\,(7) \quad \overline{\mathcal{E}_1 \vdash x : Bool}\,(3)$$

(application rule)

$$\overline{\mathcal{E}_1 \vdash \lambda(x : Bool).(\omega(1x\ true)) : Bool \to \pi}\,(8)$$

(function rule)

Now, using (2) & (8)

$$\dfrac{\overline{\mathcal{E}_1 \vdash \lambda(x : Bool).(\omega(1x\ true)) : Bool \to \pi}\,(8) \quad \overline{\mathcal{E}_1 \vdash \omega : Bool \to \pi}\,(2)}{\mathcal{E}_1 \vdash \lambda(\omega : Bool \to \pi).\lambda(x : Bool).(\omega(1x\ true)) : Bool \to \pi \to Bool \to \pi}$$

(function rule)

Hence, the type of given λexp is $Bool \to \pi \to Bool \to \pi$.

(Sumit, 18CS30042)

(d) $\because$ given the type of $+$,

let $\mathcal{E}_0 = \{ + : S \rightarrow S \}$

$\mathcal{E}_1 = \mathcal{E}_0 \cup \{ x : S \} \cup \{ f : S \rightarrow C \}$

$\mathcal{E}_1 \vdash + : S \rightarrow S$ (const rule) — (1)

$\mathcal{E}_1 \vdash x : S$ (Identifier rule) — (2)

$\mathcal{E}_1 \vdash f : S \rightarrow C$ (Identifier rule) — (3)

$$\dfrac{\dfrac{\dfrac{}{\mathcal{E}_1 \vdash f : S \rightarrow C}(3) \quad \dfrac{\dfrac{}{\mathcal{E}_1 \vdash + : S \rightarrow S}(1) \quad \dfrac{}{\mathcal{E}_1 \vdash x : S}(2)}{\mathcal{E}_1 \vdash (+x) : S}(4)\ \text{(application rule)}}{\mathcal{E}_1 \vdash f(+x) : C}(5) \quad \dfrac{}{\mathcal{E}_1 \vdash x : S}(2)\ \text{(application rule)}}{\mathcal{E}_1 \vdash \lambda(x:S).\, f(+x) : S \rightarrow C}(6)\ \text{(function rule)}$$

$$\dfrac{\mathcal{E}_1 \vdash \lambda(x:S).\, f(+x) : S \rightarrow C \quad \dfrac{}{\mathcal{E}_1 \vdash f : S \rightarrow C}(3)}{\mathcal{E}_1 \vdash \lambda(f : S \rightarrow C).\, \lambda(x:S).\, f(+x) : S \rightarrow C \rightarrow S \rightarrow C}(7)$$

Hence, type of given λexp is $(S \rightarrow C) \rightarrow (S \rightarrow C)$

(e) $\mathcal{E}_0 : \{ x : \text{Ref Bool},\ y : \text{Bool} \}$

let $\mathcal{E}_1 = \mathcal{E}_0 \cup \{ \text{succ} : \text{Int} \rightarrow \text{Int},\ \text{true} : \text{Bool},\ 4 : \text{Int} \}$

So, $\mathcal{E}_1 \vdash x : \text{Ref Bool}$ (Identifier rule) — (1)

$\mathcal{E}_1 \vdash \text{succ} : \text{Int} \rightarrow \text{Int}$ (const. rule) — (2)

$\mathcal{E}_1 \vdash \text{true} : \text{Bool}$ (const. rule) — (3)

$\mathcal{E}_1 \vdash 4 : \text{Int}$ (const. rule) — (4)

(Sumit, 18CS30042)

$$\frac{\overline{E_1 \vdash succ : Int \to Int}^{\,(2)} \quad , \quad \overline{E_1 \vdash 4 : Int}^{\,(4)}}{E_1 \vdash (succ\ 4) : Int} \; (5)$$

(Application rule)

Now,

$$\frac{\overline{E_1 \vdash x : Ref\ Bool}^{\,(1)} \quad , \quad \overline{E_1 \vdash true : Bool}^{\,(3)}}{E_1 \vdash x := true : command} \; (6)$$

(Assignment rule)

using (5) & (6)

$$\frac{\overline{E_1 \vdash succ\ 4 : Int}^{\,(5)} \quad \overline{E_1 \vdash x := true : command}^{\,(6)}}{E_1 \vdash succ 4\ ;\ x := true : command} \; (7)$$

(Sequencing rule)

Hence, type of given λexp is command.

Ans :-2

(a) ~~ϕ : float → integer~~

~~let $E_0 = \{p : float \to integer, f : float \to float, y : float\}$~~

~~$E_0 \vdash$~~

Let F denote float, I denote integer, B denote Bool, C denote char

∴ given the type of ϕ

let $E_0 = \{\phi : F \to I\}$

$E_1 = E_0 \cup \{p : F \to I, f : F \to F, y : F\}$

∴ $E_1 \vdash \phi : F \to I$ (Const. rule) — (1)

$E_1 \vdash p : F \to I$ (Const. rule) — (2)

$E_1 \vdash f : F \to F$ (Const. rule) — (3)

$E_1 \vdash y : F$ (Const. rule) — (4)

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$$\dfrac{\dfrac{}{\varepsilon_1 \vdash f : F \to F}(3) \quad \dfrac{\dfrac{}{\varepsilon_1 \vdash f : F \to F}(3), \quad \dfrac{}{\varepsilon_1 \vdash y : F}(4)}{\varepsilon_1 \vdash (fy) : F}(5)}{\varepsilon_1 \vdash f(fy) : F}(7)$$

(app. rule)

(application rule)

$$\dfrac{\dfrac{}{\varepsilon_1 \vdash p : F \to I}(2) \quad \varepsilon_1 \vdash f(fy) : F}{\varepsilon_1 \vdash p(f(fy)) : I}(8)$$

(application rule)

$$\dfrac{\dfrac{}{\varepsilon_1 \vdash y : F}(4) \quad \dfrac{}{\varepsilon_1 \vdash p(f(fy)) : I}(8)}{\varepsilon_1 \vdash \lambda(y : F).\, p(f(fy)) : F \to I}(9)$$

(function rule)

$$\dfrac{\dfrac{}{\varepsilon_1 \vdash f : F \to F}(3) \quad \varepsilon_1 \vdash \lambda(y : F).\, p(f(fy)) : F \to I}{\varepsilon_1 \vdash \lambda(f : F \to F).\, \lambda(y : F).\, p(\cancel{\ }f(fy)) : F \to F \to F \to I}(10)$$

(function rule)

using (2) & (10)

$$\dfrac{\dfrac{}{\varepsilon_1 \vdash p : F \to I}(2) \quad \dfrac{}{\varepsilon_1 \vdash \lambda(f : F \to F).\, \lambda(y : F).\, p(f(fy)) : F \to F \to F \to I}(10)}{\varepsilon_1 \vdash \lambda(p : F \to I).\, \lambda(f : F \to F).\, \lambda(y F).\, p(f(fy)) : F \to I \to F \to F \to F \to I}(11)$$

function rule)

$$\dfrac{\dfrac{}{\varepsilon_1 \vdash \phi : F \to I}(1) \quad \varepsilon_1 \vdash \lambda(p : F \to I).\, \lambda(f : F \to F).\, \lambda(y F).\, p(f(fy)) : F \to I \to F \to F \to F \to I}{\varepsilon_1 \vdash \lambda(p : F \to I).\, \lambda(f : F \to F).\, \lambda(y : F).\, p(f(fy))\phi : F \to F \to F \to I}(12)$$

application rule)

Hence, the type of given λexp is $F \to F \to F \to I$.

(Sumit, 18CS30042)

(b) $\mathcal{E}_0 : \{\phi : B \to B \to B\} \cup \{true : Bool\}$ where, B : Bool, C : char } let's define this

$\mathcal{E}_1 = \mathcal{E}_0 \cup \{func1 : B \to C\} \cup \{\tau : B\}$

$\therefore$ $\mathcal{E}_1 \vdash \phi : B \to B \to B$ (Const rule) — (1)

$\mathcal{E}_1 \vdash true : B$ (const rule) — (2)

$\mathcal{E}_1 \vdash func1 : B \to C$ (Identifier rule) — (3)

$\mathcal{E}_1 \vdash \tau : B$ (Identifier rule) — (4)

Assuming that $\tau \phi$ true can be reordered as $((\phi\, \tau)\, true)$ { $\because$ for the given expression to be type correct }

$$\dfrac{\dfrac{\overline{\mathcal{E}_1 \vdash \phi : B \to B \to B}^{(1)} \quad \overline{\mathcal{E}_1 \vdash \tau : B}^{(4)}}{\mathcal{E}_1 \vdash (\phi\, \tau) : B \to B}(5) \quad \overline{\mathcal{E}_1 \vdash true : B}^{(2)}}{\mathcal{E}_1 \vdash ((\phi\, \tau)\, true) : B}(6)$$

(application rule)

$$\dfrac{\overline{\mathcal{E}_1 \vdash func1 : B \to C}^{(3)} \quad \mathcal{E}_1 \vdash ((\phi\, \tau)\, true) : B}{\mathcal{E}_1 \vdash func1\, ((\phi\, \tau)\, true) : C}(7)$$

(application rule)

$$\dfrac{\overline{\mathcal{E}_1 \vdash \tau : B}^{(4)} \quad \mathcal{E}_1 \vdash func1\, ((\phi\, \tau)\, true) : C}{\mathcal{E}_1 \vdash \lambda(\tau : B).\, func1\, ((\phi\, \tau)\, true) : B \to C}(8)$$

(function rule)

$$\dfrac{\overline{\mathcal{E}_1 \vdash func1 : B \to C}^{(3)} \quad \mathcal{E}_1 \vdash \lambda(\tau : B).\, func1\, ((\phi\, \tau)\, true) : B \to C}{\mathcal{E}_1 \vdash \lambda(func1 : B \to C).\, \lambda(\tau : B).\, func1\, ((\phi\, \tau)\, true) : B \to C \to B \to C}(9)$$

(function rule)

Hence, type of given λexp is $B \to C \to B \to C$.