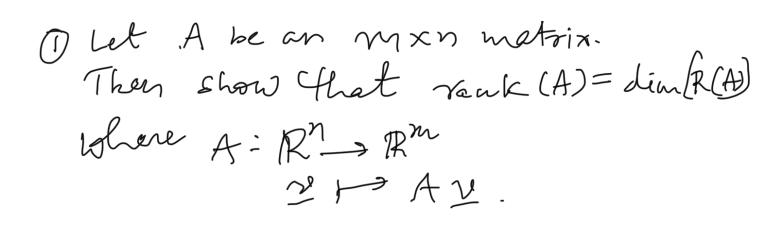
## **Exercises 2**



Let  $V=\mathbb{R}^{2}$  Define  $= [x_{1}x_{1}]_{-1}^{2} - [y_{1}]_{-1}^{2}$   $\angle y_{2} = 2x_{1}y_{1} - x_{1}y_{2} - x_{2}y_{1}^{2}, y_{1}^{2}$   $+5x_{1}y_{2}$ Check whether  $\langle y \rangle$  is an inner product or not.

Borow that the following!

(i) The eigenvalues of a symmetric matrix over R (or a Hermitian matrix over C) and real.

(ii) The eigenvolues of a skursynm or skur-Hermitian matrix on Zero or purely imaginary.

(ii) The eigenvalus of an orthogonal matrix or cenitary matrix have absolute value 1.

Proof Let  $A_{n \times n}$  matrix.

Let row-rack (A) = k  $W \perp 0 G$  alme the first k rowvertors of A forms a borns of row space (A).

Say  $T_1, -, T_k$ Let  $B = \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix}$  submatrix of A.

rowspace (B) = rowspace (A)

· rowrk(B) = rowrk(A) = k.

Column (B)

WLOG Single be the first & columns & B which forms a basis for the Columns & (B).

Let M be the Submetrix of B where rows one Minner x & Column are 51 2 - 2 CK of Cours Mis also a brokmatrix of A. Now det [M] \$0. Thus we find a kxt hubmatrix whole det is non-zero. Also if we take ony Internation of A of KH vow of A, Then they are lid. Thin implies that all memetises of B' of hige (k+1) have det = 0. => all (K+1) x(E+1) sumedinces of A (K+1) xn.
here del=0. By for on s > 16. : k = rank(A).rowrk (A) = dim(R(At)) = dim (R(A)), Observatione R(A) = CohnegelA)  $R(A^{t}) = Rams_{t}(A)$ din [L(AU) = lin (R (At))

$$2,2 \in \mathbb{R}^{n}$$

$$2,2 = (21.12) \quad (2n0.) = y^{2}2$$

$$2.12 \Leftrightarrow 2.2 = 6$$

Let A be a Hermitian matrix.

(7)  $A = A^{\dagger} = A^{\dagger}$ .

Let A be an eigender of A & 20 be its corresponding exquestor.

Toshow: A is a red number.

$$A = \lambda 2$$

$$(A \times )^{t} = (A \times )^{t}$$

$$\Rightarrow x^{t} A^{t} = x^{t}$$

$$\Rightarrow (x^{t} A \times )^{t} = (A \times )^{t}$$

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$$\Rightarrow (x^{t} A \times$$

Theorem! Let Anxi be a matrix

& di,,-, by be the distinct edgerables

of A. Suppose 121,-, by be the corresponding

eigenvectors of di,-, by respectively. Then

\[
\{\frac{1}{2}\],-,\frac{1}{2}\] is \lambda.i.

Proof: We have \[
A\frac{1}{2}\] = \frac{1}{2}\]

\[
A\frac{1}{2}\]

( 1/4 --- + Cx 2/2 = 6 for mue TO Show C=====0 prove by industion on r. { 24. } { 24, 22} (e, 24+c2 22 = 0) -> () A (42) + 62 22 = 0 C, A14+ C2 A12=0 (c, d, 20) + c2 y2 1/2 = 0 C, 2, 21 + C2 2, 22 = 2 c, d, 121 + c2d2 12= 0  $+ c_2(\lambda_1 - \lambda_2) \underline{v}_2 = \underline{o}$ But d, # 22, 22 # 6. =  $|e_z=0\rangle$  =  $|c=0\rangle$ 

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By andution hypothers we get 
$$C_1(d_r-d_r)=0, \cdots, C_{m_1}(d_r-d_{m_1})=0$$

$$rac{1}{2}$$
  $rac{1}{2}$