Problem Sheet 1 (Matrix Algebras–MA20107)

(1) Using Gram-Schmidt orthogonalization process, find the orthonormal sets for the following linearly independent vectors

(i) $(1, 2, 1), (1, 0, 1), (1, 0, 2)$ in $\mathbb{R}^3$

(ii) $(2, 1, 0), (0, 1, 1), (2, 0, 2)$ in $\mathbb{R}^3$

(iii)$(0, 3, 4), (3, 5, 0), (2, 5, 5)$ in $\mathbb{R}^3$

(iv) $(1, 1, 0, 1), (1, 2, 1, 0), (0, 1, 2, 1), (1, 0, 1, 1)$ in $\mathbb{R}^4$.

(2) Find x, y so that $(x, y, 1)$ is orthogonal to both $(1, 2, 3), (1, 1, 1)$.

(3) Find A^{39} for $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.

(4) Find $A^{202} - 3A^{147} + 2I$, for $A = \begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}$.

(5) Find $A^{24} - 3A^{15}$, for $A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 1 & 0 \\ -1 & -3 & -1 \end{bmatrix}$.

(6) Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -7 & 3 \end{bmatrix}$. Find a non-singular matrix P such that $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -2 & 1 \end{bmatrix}$.

(7) Find e^A, where

(i) $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(8) Find $\cos(A)$, where $A = \begin{bmatrix} \pi & 3\pi \\ 2\pi & 2\pi \end{bmatrix}$

(9) Find a unitary matrix (or orthogonal matrix) S such that S^*AS is an upper triangular matrix, where

(i) $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & i & 1 \\ -i & 2 & i \\ 1 & -i & 1 \end{bmatrix}$.

(10) Find a unitary matrix (or orthogonal matrix) S such that S^*AS is a diagonal matrix, where

(i) $A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$.