Module 09: CS31003: Compilers: Fundamentals of Data Flow Analysis

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Module Objectives

- Understanding Data Flow Analysis (DFA) to estimate various data propagation entities in programs statically
- Understanding DFA formulation with forward / backward flow and inclusive / exclusive confluence
- Understanding formulation for various DFA solutions for Reaching Definitions, Available Expressions, Live Variables, Def-Use Chains, Copy Propagation etc.
- Understanding use of DFA in global optimization

Module Outline

Data-flow analysis

- These are techniques that derive information about the flow of data along program execution paths
- An *execution path* (or *path*) from point p_1 to point p_n is a sequence of points $p_1, p_2, ..., p_n$ such that for each $i = 1, 2, ..., n-1$, either
  1. p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that same statement, or
  2. p_i is the end of some block and p_{i+1} is the beginning of a successor block
- In general, there is an infinite number of paths through a program and there is no bound on the length of a path
- Program analyses summarize all possible program states that can occur at a point in the program with a finite set of facts
- No analysis is necessarily a perfect representation of the state

Path Examples


```
    p0
100: n = 5
    p1
101: i = 0
    p2
102: if i < n goto 106
    p3
103: goto 124
    p4
104: i = i + 1
    p5
105: goto 102
    p6
106: t4 = i << 2
    p7
107: t5 = a[t4]
    p8
```

```
    p8
108: t6 = i << 2
    p9
109: t7 = b[t6]
    p10
110: if t5 >= t7 goto 120
    p11
111: t8 = i << 2
    p12
112: t9 = c + t8
    p13
113: t10 = i << 2
    p14
114: t11 = a[t10]
    p15
115: t12 = i << 2
    p16
```

```
    p16
116: t13 = b[t12]
    p17
117: t14 = t11 * t13
    p18
118: *t9 = t14
    p19
119: goto 104
    p20
120: t15 = i << 2
    p21
121: t16 = c + t15
    p22
122: *t16 = 0
    p23
123: goto 104
    p24
124: return
```

Path-1: p0-p1-p2-p3-p24

Path-2: p0-p1-p2-p6-p7-p8-p9-p10-p20-p21-p22-p23-p4-p5-p2

Path-3: p0-p1-p2-p6-p7-p8-p9-p10-p20-p21-p22-p23-p4-p5-p2-p6-p7-p8-p9-p10-p20-p21-p22-p23-p4-p5-p2

Path-4: p0-p1-p2-p6-p7-p8-p9-p10-p20-p21-p22-p23-p4-p5-p2-p3-p24

Path Examples: Basic Block

```
p0: // Block B1
  0: n = 5
  1: i = 0
p1: // goto B2

p2: // Block B2
  0: if i < n goto B4
p3: // goto B7

p12: // Block B7
  0: return
p13:

p4: // Block B4
  0: t4 = 4 * i
  1: t5 = a[t4]
  2: t6 = 4 * i
  3: t7 = b[t6]
  4: if t5 >= t7 goto B6
p5: // goto B5

p6: // Block B5
  0: t8 = 4 * i
  1: t9 = c + t8
  2: t10 = 4 * i
  3: t11 = a[t10]
  4: t12 = 4 * i
  5: t13 = b[t12]
  6: t14 = t11 * t13
  7: *t9 = t14
p7: // goto B3

p8: // Block B6
  0: t15 = 4 * i
  1: t16 = c + t15
  2: *t16 = 0
p9: // goto B3

p10: // Block B3
  0: i = i + 1
p11: // goto B2
```

Path: p0-p1-p2-p4-p5-p8-p9-p10-p11-p2-p3-p12-p13

Uses of Data-flow Analysis

- Program debugging
  - Which are the definitions (of variables) that *may* reach a program point? These are the *reaching definitions*
  - Can a variable may potentially be used without being initialized?
- Program optimization
  - Constant folding
  - Copy propagation
  - Common sub-expression elimination etc.

Data-Flow Analysis Schema

- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point
- The set of all possible data-flow values is the *domain* for the application under consideration
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of definitions in the program
  - A particular data-flow value is a set of definitions
- $IN[s]$ and $OUT[s]$: data-flow values *before* and *after* each statement s
- The *data-flow problem* is to find a solution to a set of constraints on $IN[s]$ and $OUT[s]$, for all statements s

Data-Flow Analysis Schema (2)

- Two kinds of constraints
  - Those based on the semantics of statements (*transfer functions*)
  - Those based on flow of control
- A DFA schema consists of
  - A control-flow graph
  - A direction of data-flow (forward or backward)
  - A set of data-flow values
  - A confluence operator (usually set union or intersection)
  - Transfer functions for each block
- We always compute *safe* estimates of data-flow values
- A decision or estimate is *safe* or *conservative*, if it never leads to a change in what the program computes (after the change)
- These safe values may be either subsets or supersets of actual values, based on the application

Reaching Definitions

Reaching Definitions

- We *kill* a definition of a variable *a*, if between two points along the path, there is an assignment to *a*
- A definition *d* reaches a point *p*, if there is a path from the point immediately following *d* to *p*, such that *d* is not *killed* along that path
- Unambiguous and ambiguous definitions of a variable

  a := b+c

  (unambiguous definition of 'a')

  ...

  *p := d

  (ambiguous definition of 'a', if 'p' may point to variables other than 'a' as well; hence does not kill the above definition of 'a')

  ...

  a := k-m

  (unambiguous defn. of 'a'; kills the above defn. of 'a')

Reaching Definitions

- We compute super-sets of definitions as *safe* values
- It is safe to assume that a definition reaches a point, even if it does not.
- In the following example, we assume that both a=2 and a=4 reach the point after the complete if-then-else statement, even though the statement a=4 is not reached by control flow

```
if (a==b) a=2; else if (a==b) a=4;
```

Reaching Definitions: How to use them?

- Build use / def Chains
- Constant Propagation: For a use like

  ```
  n: x = ... v ...
  ```

  if all definitions that reach `n` are of the form

  ```
  d: v = c // c is a constant
  ```

  we can replace `v` in `n` by `c`.
- Un-initialized Variables: How to detect?
- Loop-invariant Code Motion: For

  ```
  d1: a = . . .;
  d2: b = . . .;
  for (. . .) {
      . . .
      n: x = a + b;
      . . .
  }
  ```

  if all definitions of variables on RHS of `n` and that reach `n` are outside the loop like `d1` and `d2`, `n` can also be moved outside the loop.

Reaching Definitions Problem: DFA Formulation

- The data-flow equations (constraints)

$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]) \\
IN[B] &= \phi, \textit{for all } B \; (\textit{initialization only})
\end{aligned}
$$

- If some definitions reach B_1 (entry), then $IN[B_1]$ is initialized to that set
- Forward flow DFA problem (since $OUT[B]$ is expressed in terms of $IN[B]$), confluence operator is $\cup$
  - Direction of flow does not imply traversing the basic blocks in a particular order
  - The final result does not depend on the order of traversal of the basic blocks

Reaching Definitions Problem: DFA Formulation

- $GEN[B]$ = set of all definitions inside B that are "visible" immediately after the block - *downwards exposed* definitions
  - If a variable x has two or more defintions in a basic block, then only the last definition of x is downwards exposed; all others are not visible outside the block
- $KILL[B]$ = union of the definitions in all the basic blocks of the flow graph, that are killed by individual statements in B
  - If a variable x has a definition d_i in a basic block, then d_i kills all the definitions of the variable x in the program, except d_i

Reading Definitions Analysis: GEN and KILL

In other blocks:

d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

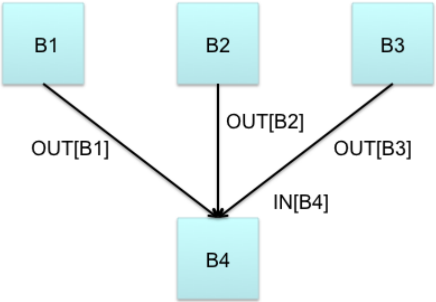
d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

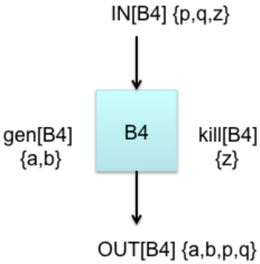
B

Set of all definitions = {d1,d2,d3,d4,d5,d6,d7,d8,d9,10}

GEN[B] = {d2,d3,d4}
KILL[B] = {d4,d9,d5,d10,d1}

Reaching Definitions Analysis: DF Equations





IN[B4] = OUT[B1] U OUT[B2] U OUT[B3]

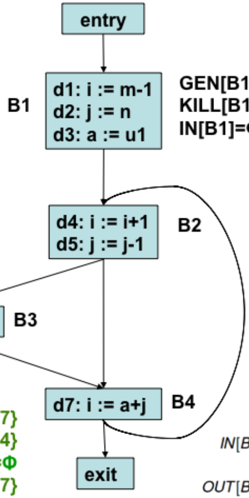
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

OUT[B4] = gen[B4] U (IN[B4] – kill[B4])

Reaching Definitions Analysis: An Example - Pass 1

Pass 1



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]=Φ
OUT[B2]={d4,d5}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]=Φ
OUT[B4]={d7}

Adapted from the "Dragon Book", A-W, 1986

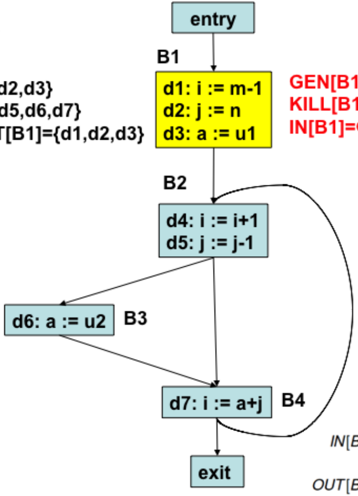
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.1

Pass 2

GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

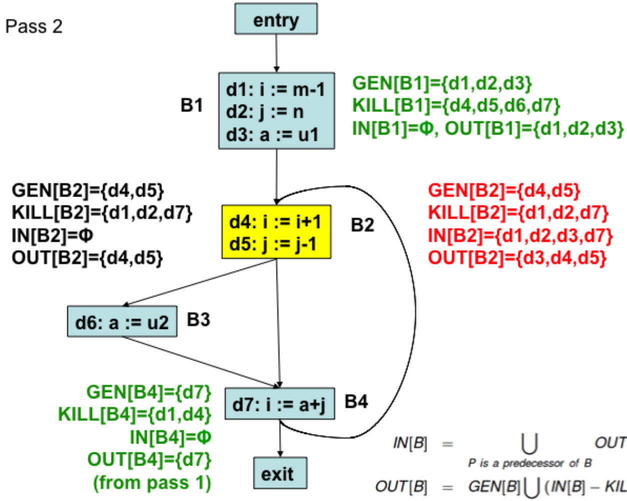


GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

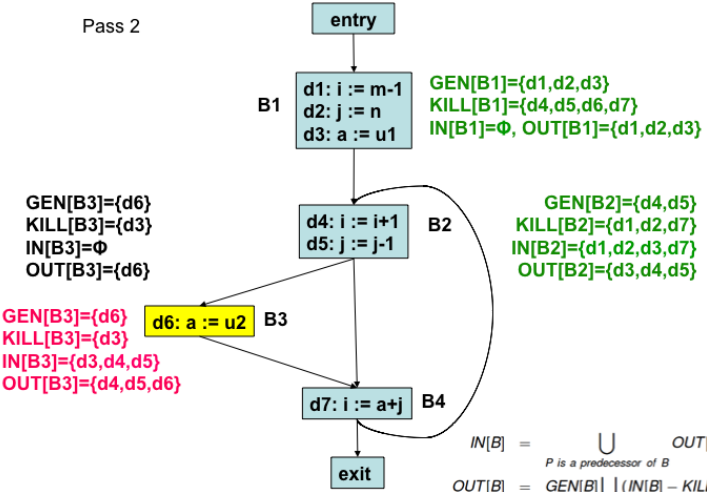
Reaching Definitions Analysis: An Example - Pass 2.2



$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

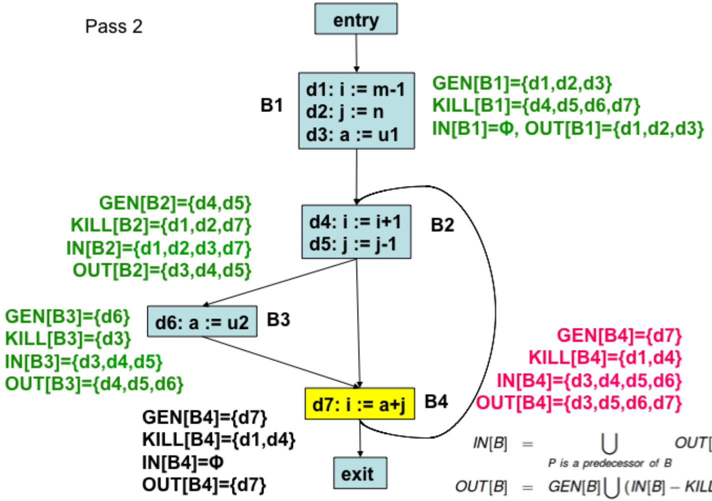
Reaching Definitions Analysis: An Example - Pass 2.3



$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

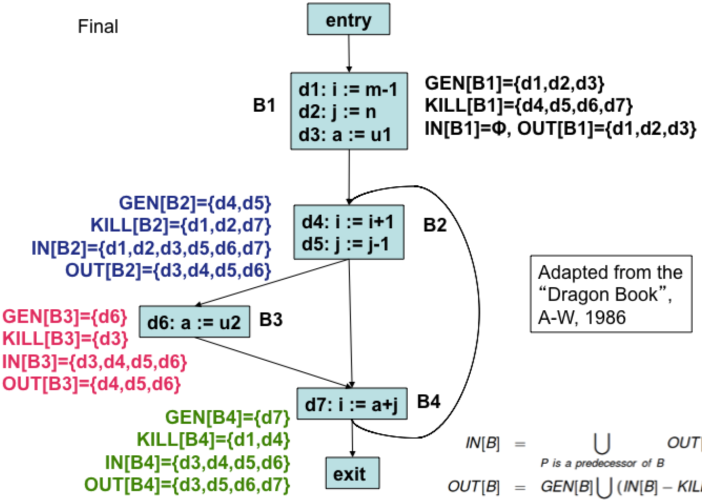
Reaching Definitions Analysis: An Example - Pass 2.4



$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Final



Adapted from the "Dragon Book", A-W, 1986

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

An Iterative Algo. for Computing Reaching Def.

for each block B do $\{\ IN[B] = \phi;\ OUT[B] = GEN[B];\ \}$
$change = true;$
while $change$ do $\{\ change = false;$
  for each block B do $\{$

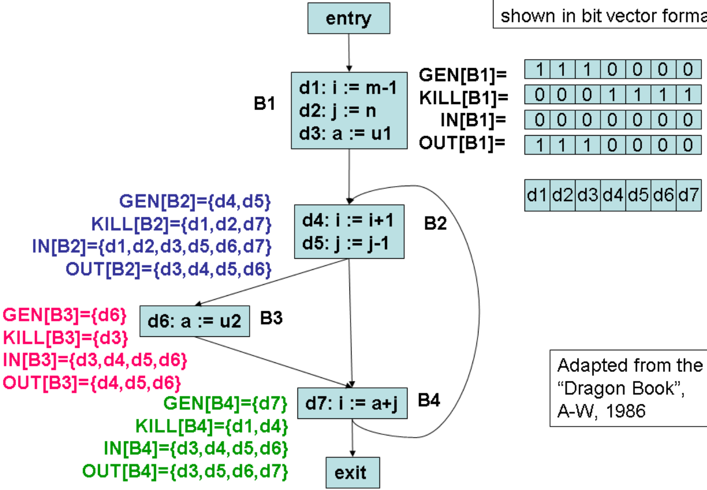
$$\begin{aligned} IN[B] &= \bigcup_{P \text{ a predecessor of } B} OUT[P]; \\ oldout &= OUT[B]; \\ OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]); \end{aligned}$$

    if $(OUT[B] \neq oldout)$ $change = true;$
  $\}$
$\}$

- GEN, $KILL$, IN, and OUT are all represented as bit vectors with one bit for each definition in the flow graph

Reaching Definitions: Bit Vector Representation

Final dataflow value sets shown in bit vector format



Adapted from the "Dragon Book", A-W, 1986

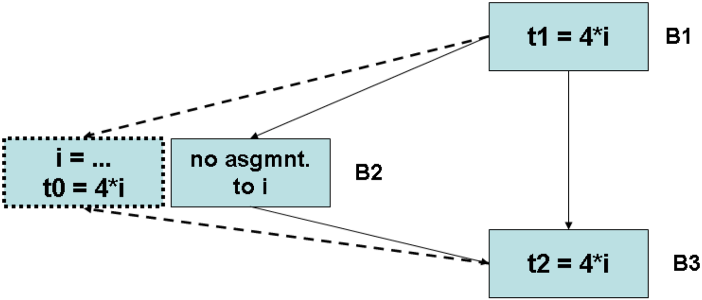
Available Expressions

Available Expression Computation

- Sets of expressions constitute the domain of data-flow values
- Forward flow problem
- Confluence operator is $\cap$
- An expression $x + y$ is *available* at a point p, if every path (not necessarily cycle-free) from the initial node to p evaluates $x + y$, and after the last such evaluation, prior to reaching p, there are no subsequent assignments to x or y
- A block *kills* $x + y$, if it assigns (or may assign) to x or y and does not subsequently recompute $x + y$.
- A block *generates* $x + y$, if it definitely evaluates $x + y$, and does not subsequently redefine x or y

Available Expression Computation(2)

- Useful for global common sub-expression elimination
- $4 * i$ is a CSE in $B3$, if it is available at the entry point of $B3$ *i.e.*, if i is not assigned a new value in $B2$ or $4 * i$ is recomputed after i is assigned a new value in $B2$ (as shown in the dotted box)



Computing e_gen and e_kill

Module 09

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Objectives & Outline

Data-flow Analysis
Points & Paths
Debugging & Optimization
DFA Schema

DFA Problems
Reaching Definitions
Available Expressions
Live Variable
DU Chains
Copy Propagation

- For statements of the form $x = a$, step 1 below does not apply
- The set of all expressions appearing as the RHS of assignments in the flow graph is assumed to be available and is represented using a hash table and a bit vector

e_gen[q] = A   **q**
x = y + z
p

Computing e_gen[p]

1. A = A **U** {y+z}
2. A = A – {all expressions involving x}
3. e_gen[p] = A

e_kill[q] = A   **q**
x = y + z
p

Computing e_kill[p]

1. A = A – {y+z}
2. A = A **U** {all expressions involving x}
3. e_kill[p] = A

Available Expression Computation - EGEN and EKILL

In other blocks:

d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

B

Set of all expressions = {f+1,a+7,b+d,d+c,a+4,e+c,a+b,c+f,e+a}

EGEN[B] = {f+1,b+d,d+c}
EKILL[B] = {a+4,a+b,e+a,e+c,c+f,a+7}

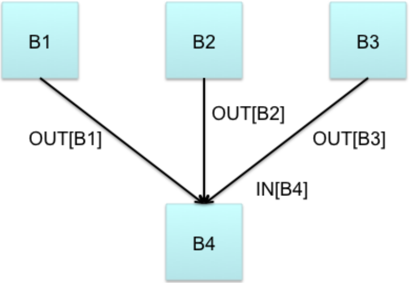
Available Expression Computation - DF Equations (1)

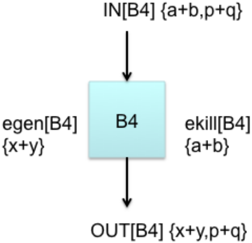
- The data-flow equations

$$
\begin{aligned}
IN[B] &= \bigcap_{P \text{ is a predecessor of } B} OUT[P], \quad B \text{ not initial} \\
OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B]) \\
IN[B1] &= \phi \\
IN[B] &= U, \text{ for all } B \neq B1 \text{ (initialization only)}
\end{aligned}
$$

- $B1$ is the intial or entry block and is special because nothing is available when the program begins execution
- $IN[B1]$ is always ϕ
- U is the universal set of all expressions
- Initializing $IN[B]$ to ϕ for all $B \neq B1$, is restrictive

Available Expression Computation - DF Equations (2)



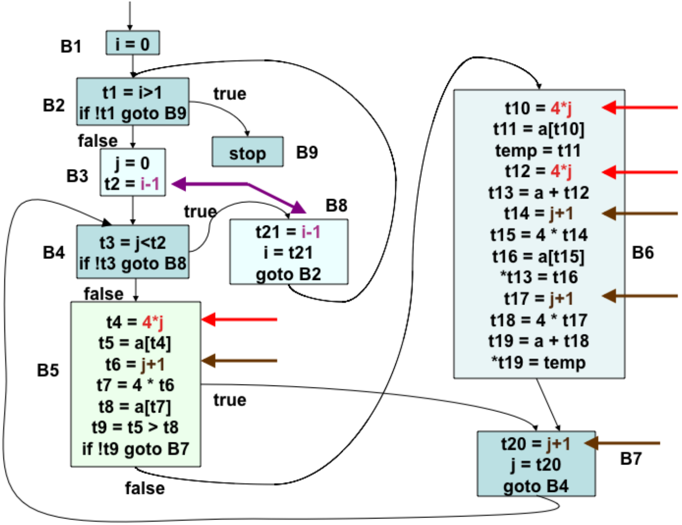


IN[B4] = OUT[B1] $\bigcap$ OUT[B2] $\bigcap$ OUT[B3]

OUT[B4] = egen[B4] $\bigcup$ (IN[B4] – ekill[B4])

$$
\begin{aligned}
IN[B] &= \bigcap_{P \text{ is a predecessor of } B} OUT[P], \; B \text{ not initial} \\
OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B])
\end{aligned}
$$

Available Expression Computation - An Example



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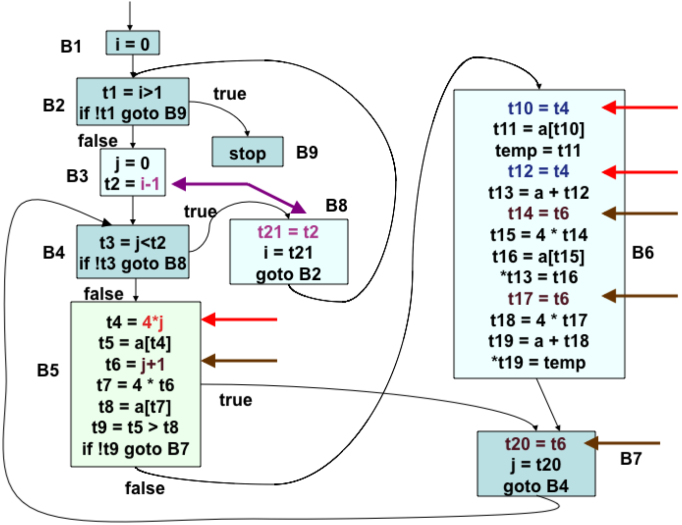
Copy Propagation

Available Expression Computation - An Example (2)



An Iterative Algorithm for Computing Available Expressions

for each block $B \neq B1$ do $\{OUT[B] = U - e\_kill[B]; \}$
/* You could also do $IN[B] = U$;*/
/* In such a case, you must also interchange the order of */
/* $IN[B]$ and $OUT[B]$ equations below */
$change = true$;
while $change$ do { $change = false$;
for each block $B \neq B1$ do {

$$
\begin{aligned}
IN[B] &= \bigcap_{P \;\; a \;\; predecessor \;\; of \;\; B} OUT[P]; \\
oldout &= OUT[B]; \\
OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B]);
\end{aligned}
$$

if $(OUT[B] \neq oldout)$ $change = true$;
}
}

DFA: Live Variables

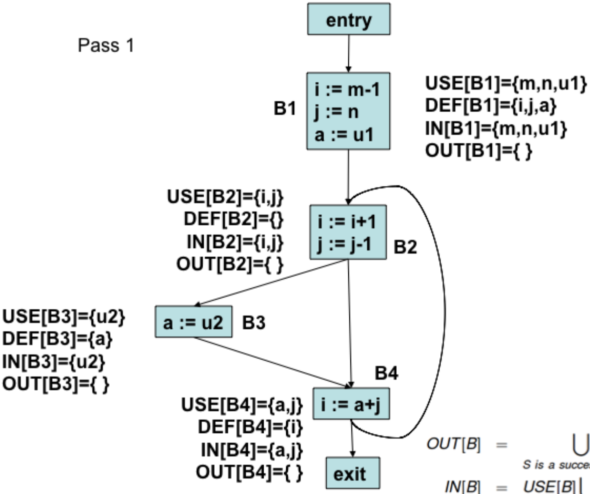
Live Variables

Live Variable Analysis

- The variable x is *live* at the point p, if the value of x at p could be used along some path in the flow graph, starting at p; otherwise, x is *dead* at p
- Sets of variables constitute the domain of data-flow values
- Backward flow problem, with confluence operator $\bigcup$
- $IN[B]$ is the set of variables live at the beginning of B
- $OUT[B]$ is the set of variables live just after B
- $DEF[B]$ is the set of variables definitely assigned values in B, prior to any use of that variable in B
- $USE[B]$ is the set of variables whose values may be used in B prior to any definition of the variable

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
OUT[B] &= \phi, \textit{ for all } B \textit{ (initialization only)}
\end{aligned}
$$

Live Variable Analysis: An Example - Pass 1

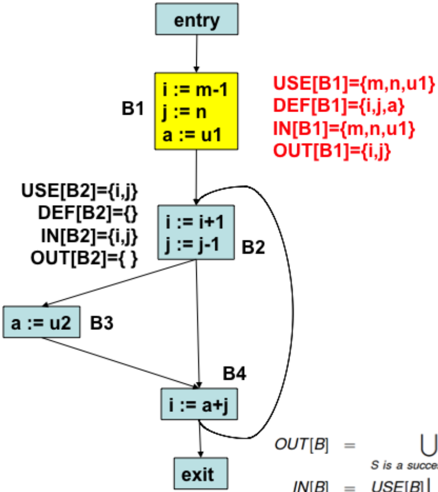


$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Pass 2.1

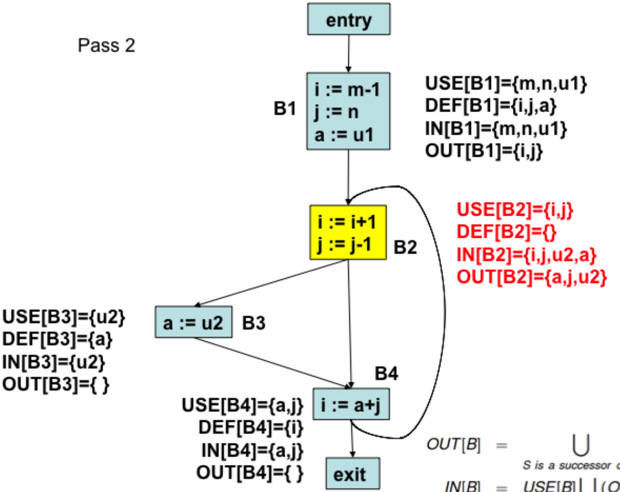
Pass 2



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

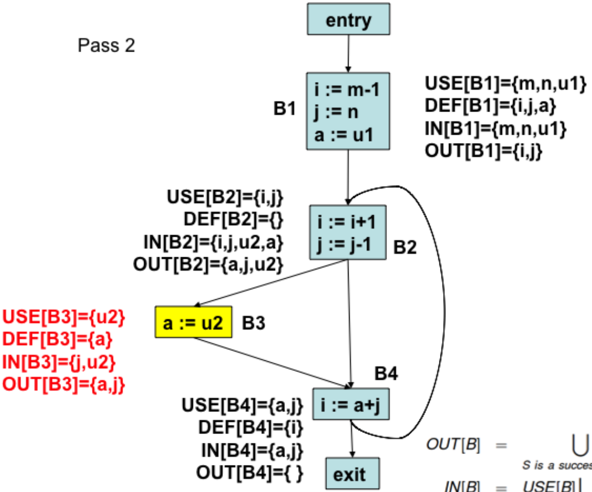
Live Variable Analysis: An Example - Pass 2.2



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

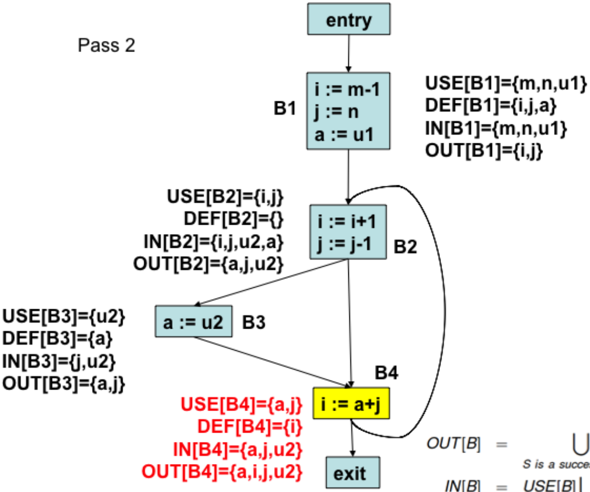
Live Variable Analysis: An Example - Pass 2.3



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

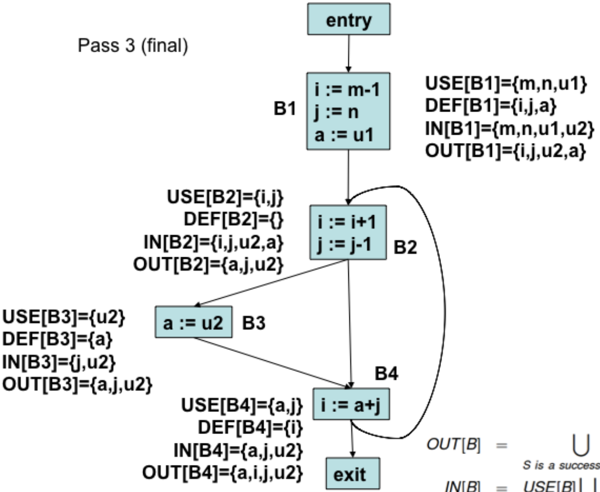
Live Variable Analysis: An Example - Pass 2.4



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Final pass



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Definition-Use Chains

DFA: Definition-Use Chains

- For each definition, we wish to attach the statement numbers of the uses of that definition
- Such information is very useful in implementing register allocation, loop invariant code motion, etc.
- This problem can be transformed to the data-flow analysis problem of computing for a point p, the set of uses of a variable (say x), such that there is a path from p to the use of x, that does not redefine x.
- This information is represented as sets of (x; s) pairs, where x is the variable used in statement s
- In live variable analysis, we need information on whether a variable is used later, but in (x; s) computation, we also need the statement numbers of the uses
- The data-flow equations are similar to that of LV analysis
- Once IN[B] and OUT[B] are computed, d-u chains can be computed using a method similar to that of u-d chains

Data Flow Analysis for (x, s) Pairs

- Sets of pairs (x, s) constitute the domain of data-flow values
- Backward flow problem, with confluence operator $\bigcup$
- $IN[B]$ is the set of pairs (x, s), such that statement s uses variable x and the value of x at $IN[B]$ has not been modified along the path from $IN[B]$ to s
- $OUT[B]$ is the set of pairs (x, s), such that statement s uses variable x and the value of x at $OUT[B]$ has not been modified along the path from $OUT[B]$ to s
- $DEF[B]$ is the set of pairs (x, s), such that s is a statement which uses x, s is not in B, and B contains a definition of x
- $USE[B]$ is the set of pairs (x, s), such that s is a statement in B which uses variable x and such that no prior definition of x occurs in B

Data Flow Analysis for (x, s) Pairs

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
OUT[B] &= \phi, \text{ for all } B \text{ (initialization only)}
\end{aligned}
$$

DFA: Copy Propagation

Copy Propagation

Copy Propagation

- Eliminate copy statements of the form $s: \ x := y$, by substituting y for x in all uses of x reached by this copy
- Conditions to be checked
  1. u-d chain of use u of x must consist of s only. Then, s is the only definition of x reaching u
  2. On every path from s to u, including paths that go through u several times (but do not go through s a second time), there are no assignments to y. This ensures that the copy is valid
- The second condition above is checked by using information obtained by a new data-flow analysis problem
  - $c\_gen[B]$ is the set of all copy statements, $s : x := y$ in B, such that there are no subsequent assignments to either x or y within B, after s
  - $c\_kill[B]$ is the set of all copy statements, $s : x := y$, s not in B, such that either x or y is assigned a value in B
  - Let U be the universal set of all copy statements in the program

Copy Propagation - The Data-flow Equations

- $c\_in[B]$ is the set of all copy statements, $x := y$ reaching the beginning of B along every path such that there are no assignments to either x or y following the last occurrence of $x := y$ on the path
- $c\_out[B]$ is the set of all copy statements, $x := y$ reaching the end of B along every path such that there are no assignments to either x or y following the last occurrence of $x := y$ on the path

$$
\begin{aligned}
c\_in[B] &= \bigcap_{P \text{ is a predecessor of } B} c\_out[P],\ B \text{ not initial} \\
c\_out[B] &= c\_gen[B] \bigcup (c\_in[B] - c\_kill[B]) \\
c\_in[B1] &= \phi,\ \text{where } B1 \text{ is the initial block} \\
c\_out[B] &= U - c\_kill[B],\ \text{for all } B \neq B1 \text{ (initialization only)}
\end{aligned}
$$

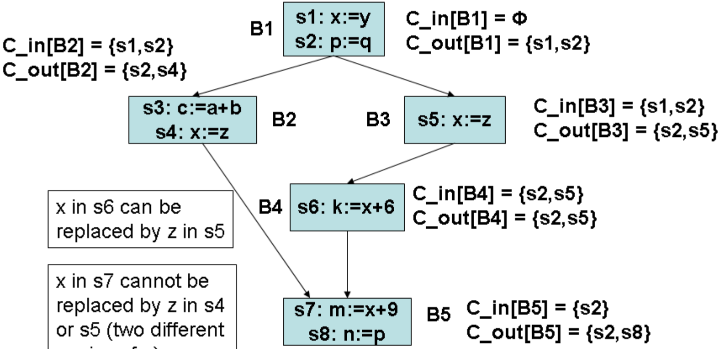
Algorithm for Copy Propagation

For each copy, $s : x := y$, do the following

1. Using the $du - chain$, determine those uses of x that are reached by s
2. For each use u of x found in (1) above, check that
   (i) u-d chain of u consists of s only
   (ii) s is in $c\_in[B]$, where B is the block to which u belongs. This ensures that
      - s is the only definition of x that reaches this block
      - No definitions of x or y appear on this path from s to B

   (iii) no definitions x or y occur within B prior to u found in (1) above
3. If s meets the conditions above, then remove s and replace all uses of x found in (1) above by y

Copy Propagation Example 1



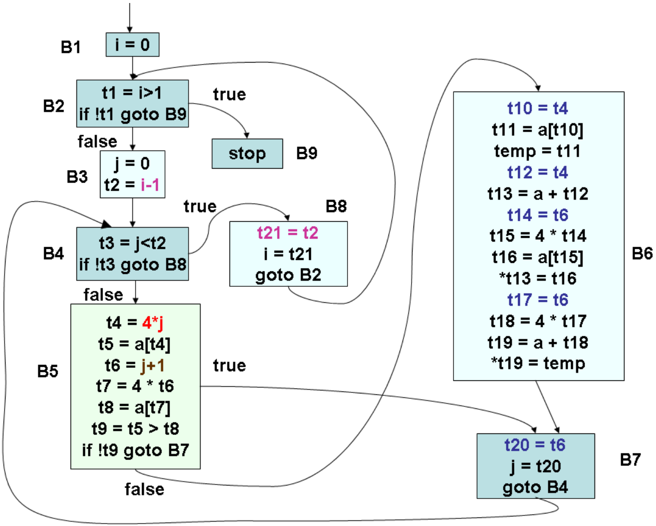
x in s6 can be replaced by z in s5

x in s7 cannot be replaced by z in s4 or s5 (two different copies of z)

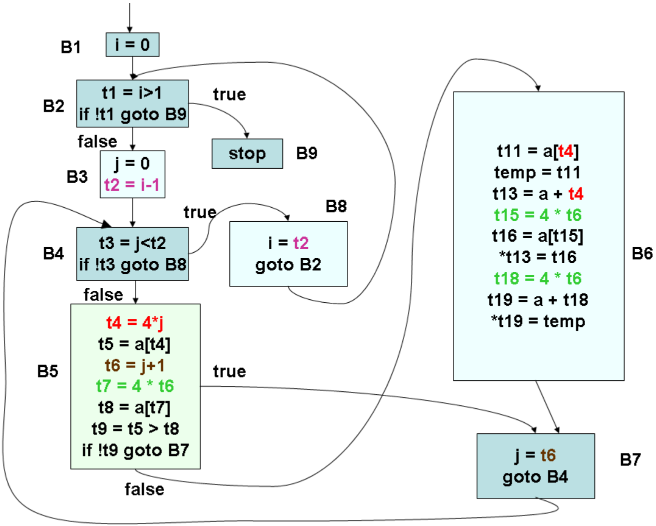
p in s8 can be replaced by q in s2 (s2 reaches B5 thro' both the paths)

Adapted from "The Dragon Book" A-W 1986

Copy Propagation on Running Example 1.1



Copy Propagation on Running Example 1.2



GCSE and Copy Propagation on Running Example 1.1

Module 09

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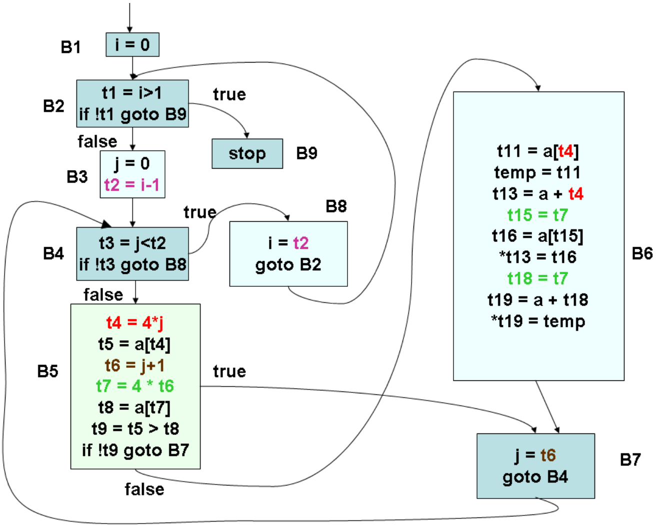

GCSE and Copy Propagation on Running Example 1.2