1. Simple linear Regression.

$$\underset{\sim}{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \underset{\sim}{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad \underset{\sim}{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \underset{\sim}{\hat{y}} = g(\underset{\sim}{x}, \underset{\sim}{\beta}) = \underset{n\times 2}{[\underset{\sim}{1}\ \underset{\sim}{x}]}\ \underset{\sim}{\beta} = \begin{bmatrix} \beta_0 + \beta_1 x_1 \\ \beta_0 + \beta_1 x_2 \\ \vdots \\ \beta_0 + \beta_1 x_n \end{bmatrix}$$

more equations than unknown parameters.

$\underset{\sim}{y} = [\underset{\sim}{1}\ \underset{\sim}{x}]\ \underset{\sim}{\beta} \Rightarrow$ more equations than unknown parameters.

2. <u>Weighted Sum</u>

$$\underset{\sim}{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} \quad \underset{\sim}{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \quad \underset{\sim}{w} \cdot \underset{\sim}{v} = \underset{\sim}{w}^T \underset{\sim}{v} = \sum_{i=1}^{n} w_i v_i$$

w_i = cradit of ith subject. $\quad \underset{\sim}{w} \cdot \underset{\sim}{v}$ = SGPA.
v_i = score/Grade in ith sub.

3. <u>Expection.</u>

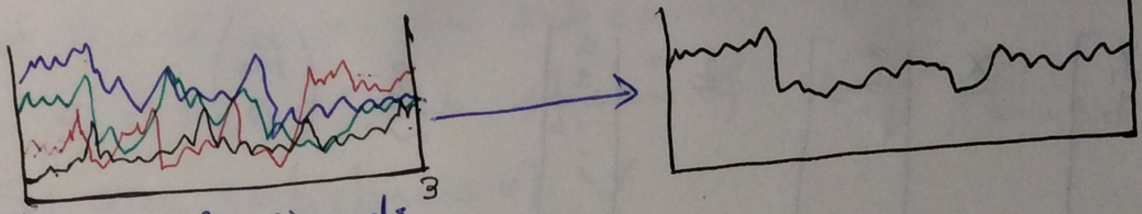
Random variable X takes values a_i with ~~prob~~. p_i $\quad \sum_i p_i = 1$

i.e. $P(X = a_i) = p_i$

then $E(X) = \sum_i a_i p_i = \underset{\sim}{a}^T \underset{\sim}{p}$. <u>if exists.</u>

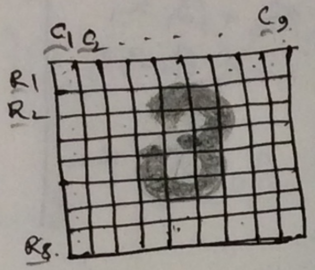
$$\boxed{\sum_i |a_i|\, p_i < \infty}$$

4



audio signals

5



$$\rightarrow \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \\ \tilde{c}_3 \\ \vdots \\ \tilde{c}_9 \end{pmatrix}$$

Is it 3 or 8 ??

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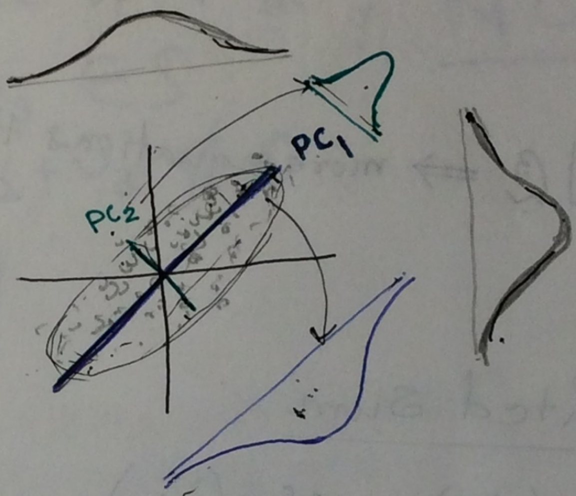
PCA

SVD



7

$$P_n = \sum_{k=0}^{n} a_k x^k$$

$$= \tilde{a}^T \tilde{X}$$

$$\tilde{a} \in \mathbb{R}^{n+1}$$

$a_i \in \mathbb{R}$

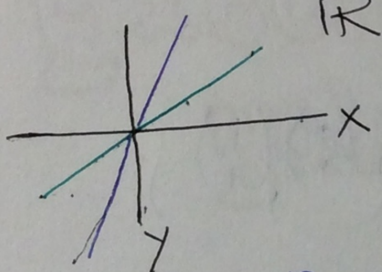
$$\tilde{a} = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{pmatrix} \quad \tilde{X} = \begin{pmatrix} x^0 \\ x \\ x^2 \\ \vdots \\ x^n \end{pmatrix}$$

Examples of vector space.

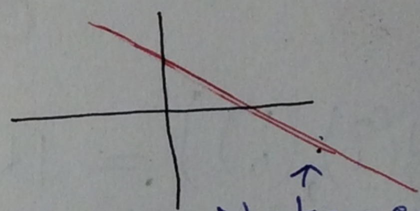
① $\mathbb{R}$.
② $\mathbb{R}^2$. (x y - plane)
③ $\mathbb{R}^3$ (x, y, z - 3D space)
④ $\mathbb{R}^n$. In general.
⑤ $\mathbb{P}_n \rightarrow$ Polynomials degree upto n.
⑥ All continuous functions on $[0, 1]$

Example of subspace.

① $\mathbb{R}^2 = V$.

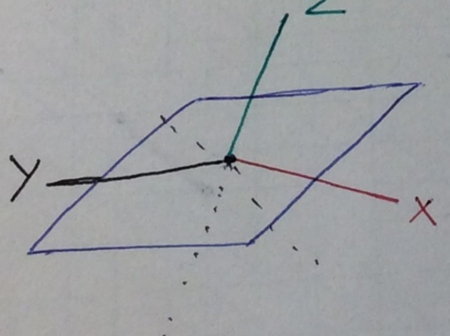


Lines passing through (0,0)



Not an example of a subspace.

② $\mathbb{R}^3$.



③ $\mathbb{P}_5 \subset \mathbb{P}_7$.

$$\mathbb{P}_7 = \left\{ \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_7 \end{pmatrix} \middle| \; a_i \in \mathbb{R} \quad i = 0, 1, \cdots, 7 \right\}$$

$$\mathbb{P}_5 = \left\{ \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_5 \\ 0 \\ 0 \end{pmatrix} \; a_i \in \mathbb{R} \quad i = 0, 1, 2, \cdots 5 \right\}$$

$$\mathrm{sp}\left\{\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}1\\0\end{pmatrix}\right\} = \mathbb{R}^2 = \mathrm{sp}\left\{\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix}\right\}$$

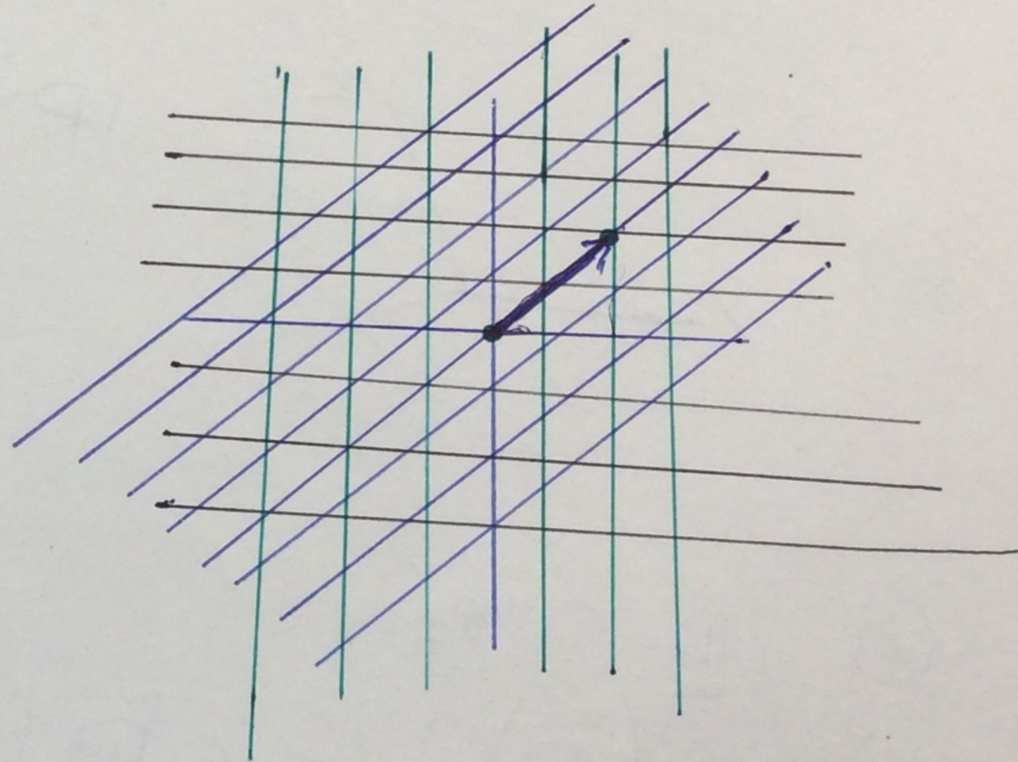
example of <u>span</u>:

$$\begin{pmatrix}2\\2\end{pmatrix} = 2\begin{pmatrix}0\\1\end{pmatrix} + 2\begin{pmatrix}1\\0\end{pmatrix}$$

$$= 0\begin{pmatrix}0\\1\end{pmatrix} + 2\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix}$$

$$= \begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix}$$



1. Let S_1 and S_2 be the sub spaces of V.
Is $S_1 \cap S_2$ a sub space of V? YES.
S_i are subspaces of V.
$\bigcap_i S_i$ is also a sub space.
$\{\underset{\sim}{0}\}$ and V are the trivial. sub spaces.

2. Let $V = \mathbb{R}^2$ and $S = \{(x,y) \mid xy \geq 0\}$
Is S a sub space of $V = \mathbb{R}^2$? NO.

$v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\quad v_2, v_3 \in S$
$v_1 = v_2 + v_3 \notin S$
$v_2 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ $\quad v_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3. $V = \mathbb{R}^3$
$$S = \left\{ (x_1, x_2, x_3) \;\middle|\; \begin{array}{l} 2x_1 - 3x_2 + \sqrt{2}\, x_3 = 0 \\ x_1 - 5x_3 = 0 \end{array} \right\}$$
Is S a sub space of $\mathbb{R}^3$?

① $\underset{\sim}{0} \in S$.
② Let $\underset{\sim}{u}$ and $\underset{\sim}{v} \in S$.
$\underset{\sim}{u} + \alpha \underset{\sim}{v} \in S$
$\forall \alpha \in \mathbb{R}$

$$\begin{bmatrix} 2 & -3 & +\sqrt{2} \\ 1 & 0 & -5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

YES.

4. Does it hold?
$$sp\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \stackrel{?}{=} sp\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

NO. $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \in$ the first span but NOT of the second.