Automation Lab

Lab report submitted on the topic

Attitude control of Aircraft using LQR controller

for the course

EE555 Automation lab

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Chapter 1 Introduction

1.1 Attitude Control

Attitude control in aircraft refers to the process of controlling the orientation of the aircraft, which includes its pitch, roll, and yaw. In other words, it involves controlling the aircraft's attitude, or the way it is oriented with respect to the horizon, in order to achieve a specific objective.

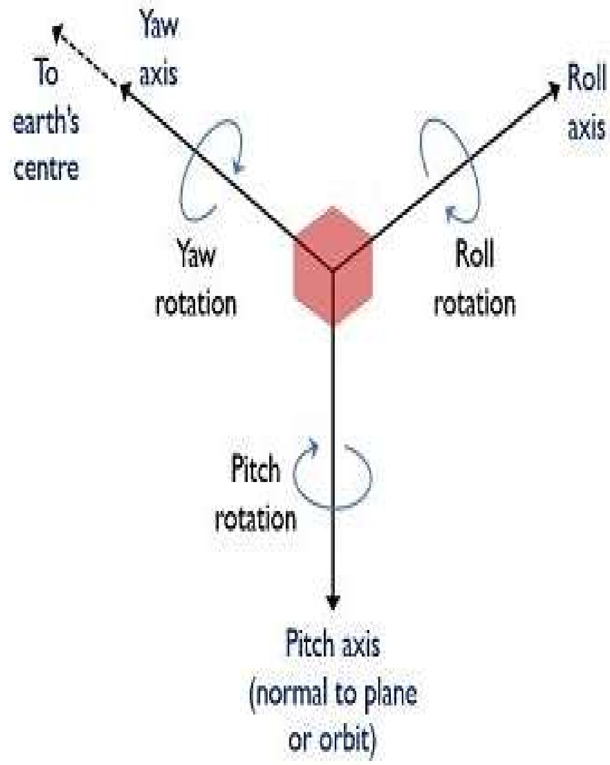


Figure 1 Axis of rotation in body reference frame

Attitude control in aircraft is typically achieved using a combination of flight control surfaces, such as ailerons, elevators, and rudder, and propulsion systems, such as engines or propellers. By adjusting these systems in a controlled manner, pilots can alter the aircraft's attitude, which affects its flight path, altitude, and speed.

Attitude control is a fundamental aspect of aircraft flying, and pilots receive extensive training on how to control and adjust the aircraft's attitude in a variety of situations, from normal flight to emergency situations. Modern aircraft also often use advanced computer systems to assist with attitude control, which can provide more precise and efficient control of the aircraft's orientation and flight path.

1.2 Brief description of feedback control and LQR

1.2.1 What is state feedback control

State feedback control is a control strategy used in control systems to achieve a desired output or response from a system by using a linear combination of the system's current state variables as feedback. State feedback control is commonly used in engineering and control systems, particularly for linear systems.

The gain matrix is a set of weights that are used to adjust the contribution of each state variable to the control input. By adjusting the gain matrix, the state feedback controller can change the way in which the system responds to changes in the input or disturbances to the system.

State feedback control is commonly used in many areas of engineering and control systems, including aerospace, automotive, and industrial control. It is particularly useful for systems that are subject to disturbances or uncertainties, as it can provide robust control that is able to compensate for these factors and maintain stable system performance.

1.2.2 What is an LQR

LQR stands for Linear Quadratic Regulator, which is a mathematical control algorithm used to design optimal control systems for linear systems. LQR is a type of feedback controller that uses a state-feedback approach to control a linear system in a way that minimizes a specified performance index. LQR is a powerful tool for designing optimal control systems for linear systems and is commonly used in aerospace, automotive, and other industries. It is particularly useful for systems that are subject to disturbances or uncertainties, as it can provide robust control that minimizes the impact of these factors on the performance of the system.

Chapter 2 Mathematical Modelling

2.1 Mathematical modelling of spacecraft

The dynamics of spacecraft can be written with the help of motion equations in x , y and z directions. The differential equations describing the dynamics can be divided into two parts –

- *One governs the translational motion*
- *The other which governs rotational motion*

  Let u ,v and w be the components of velocity vector in x , y , z direction.

  Let p ,q and r be the components of angular velocity vector in x , y and z direction.

2.2 Equation of motion

The equations of motions can be formulated as:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + m \det \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ p & q & r \\ u & v & w \end{bmatrix} = m \begin{bmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{bmatrix}$$

$$m\dot{u} + m(qw - rv) = F_x$$

$$m\dot{v} + m(ru - pw) = F_y$$

$$m\dot{v} + m(pv - qu) = F_z$$

Where F_x, F_y and F_z are components of translational forces in x, y and z directions.

The moment of inertia matrix is defined as

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \qquad \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} p_I \\ q_I \\ r_I \end{bmatrix}$$

The inertia matrix signifies that how mass is distributed over a rigid body.

Where,

$$I_{xx} = \iiint (y^2 + z^2)\,dm \qquad I_{xy} = I_{yx} = \iiint xy\,dm$$

$$I_{yy} = \iiint (x^2 + z^2)\,dm \qquad I_{xz} = I_{xz} = \iiint xz\,dm$$

$$I_{zz} = \iiint (x^2 + y^2)\,dm \qquad I_{yz} = I_{zy} = \iiint yz\,dm$$

Where H_x , H_y, H_z are represents total moment of momentum

The rotational dynamics of the aircraft about its center of mass is formulated as follows:

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \vec{\omega} \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

When I_{xy} ,I_{yz} are zero, then

$$L = I_{xx}\dot{p} - I_{xz}\dot{r} + (I_{zz} - I_{yy})qr - I_{xz}pq$$

$$M = I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - q^2)$$

$$N = I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{yy} - I_{xx})pq + I_{xz}qr$$

where,

$\boldsymbol{I_{xx}}$, $\boldsymbol{I_{yy}}$ and $\boldsymbol{I_{zz}}$ are the spacecraft moment of inertia about x, y and z axis respectively.

L, M and N are generalized disturbance moments about corresponding axis.

The force vector F is the sum of gravitational force (F_G), aerodynamic (F_A) and propulsion force (F_T).

$$\vec{F} = \vec{F}_A + \vec{F}_G + \vec{F}_T$$

We assume that propulsion induces no moment hence the moment is purely aerodynamic in nature.

The aerodynamic forces and moments are usually denoted as:

$$F_A = [X \;\; Y \;\; Z]^T \qquad \mathrm{M} = [L \;\; M \;\; N]^T$$

2.3 Aerodynamic forces and moments

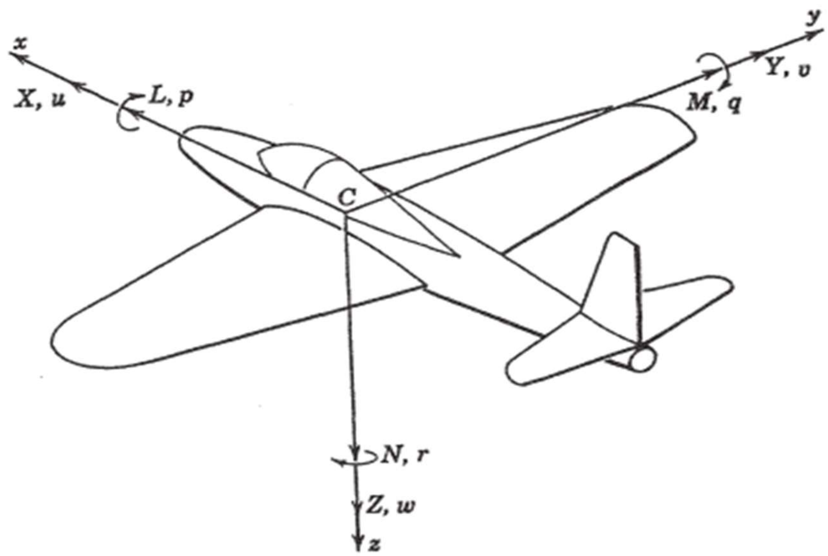


Figure 2 Aerodynamic forces and moment with respect to body fixed frame

X Axial Force- Net Force in the positive x-direction

Y Side Force- Net Force in the positive y-direction

Z Normal Force- Net Force in the positive z-direction

L Rolling Moment- Net Moment in the positive p-direction

M Pitching Moment- Net Moment in the positive q-direction

N Yawing Moment- Net Moment in the positive r-direction

2.4 Euler Angles

The term Euler Angles refers to the angles of rotation (ψ, θ, φ) needed to go from one coordinate system to another using the specific sequence of rotation

2.4.1 Yaw-Pitch-Roll

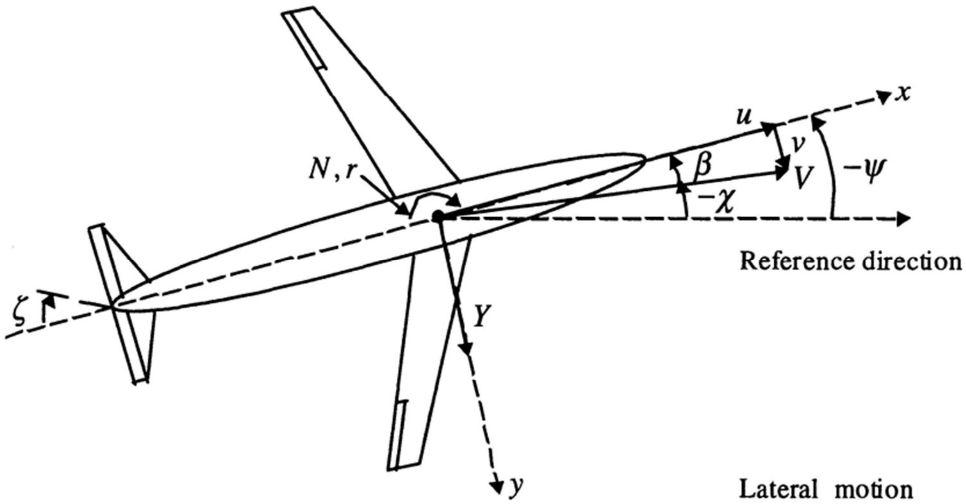


Figure 3 Yaw angle in lateral dynamics

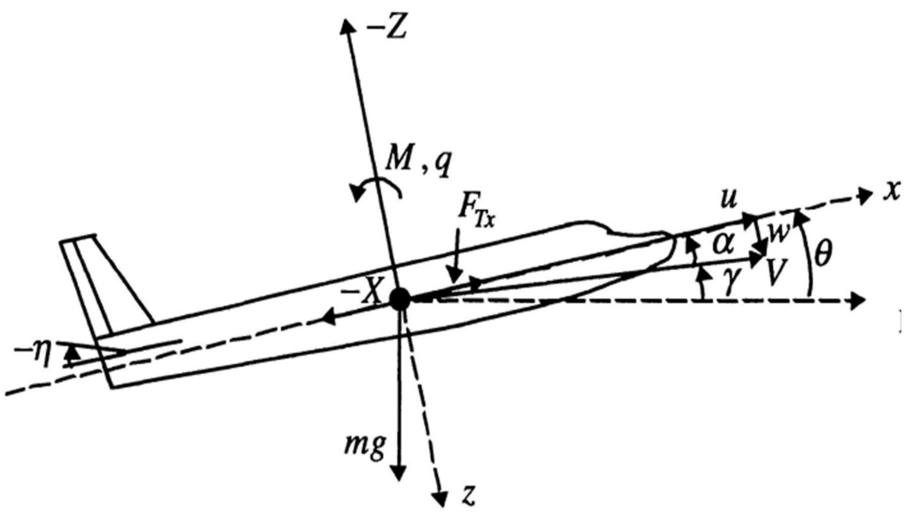


Figure 4 Pitch angle in longitudinal motion

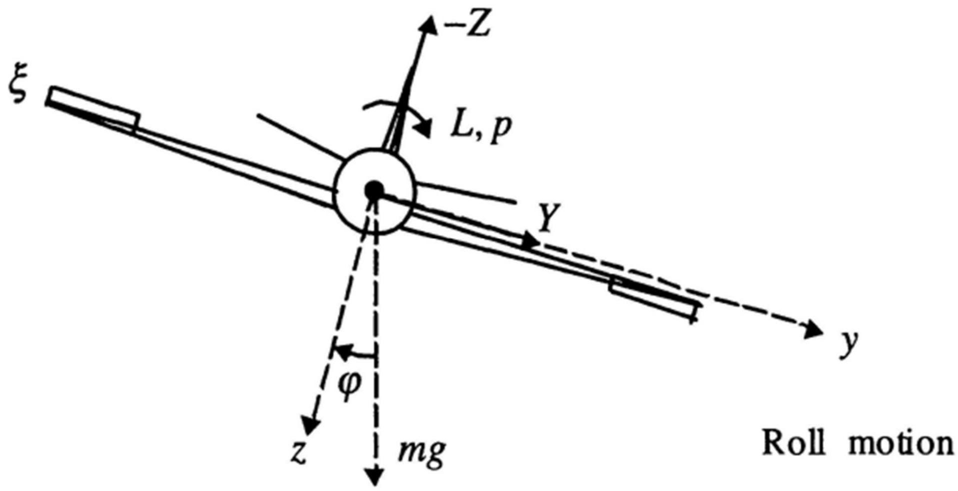


Figure 5 Roll angle in lateral motion

2.5 Relation between Euler angles and rotational motion

The orientation of the airplane can be described by three consecutive rotations, whose order is important. The angular rotations are called the Euler angles. The orientation of the body frame with respect to the fixed frame can be determined in the following manner. Imagine the airplane to be positioned so that the body axis system is parallel to the fixed frame and then apply the following rotations.

1.Rotate the x_f, y_f, z_f frame about Oz_f through the yaw angle Ψ to the frame to x_1, y_1, z_1

2.Rotate x_1, y_1, z_1 the frame about Oy_1, through the pitch angle θ bringing the frame to x_2, y_2, z_2

3. Rotate the x_2, y_2, z_2 frame about Ox_2 , through the roll angle Φ to bring the frame to x_3, y_3 , z_3, the actual orientation of the body frame relative to the fixed frame.

Remember that the order of rotation is extremely important. Having defined the Euler angles, one can determine the flight velocities components relative to the fixed reference frame.

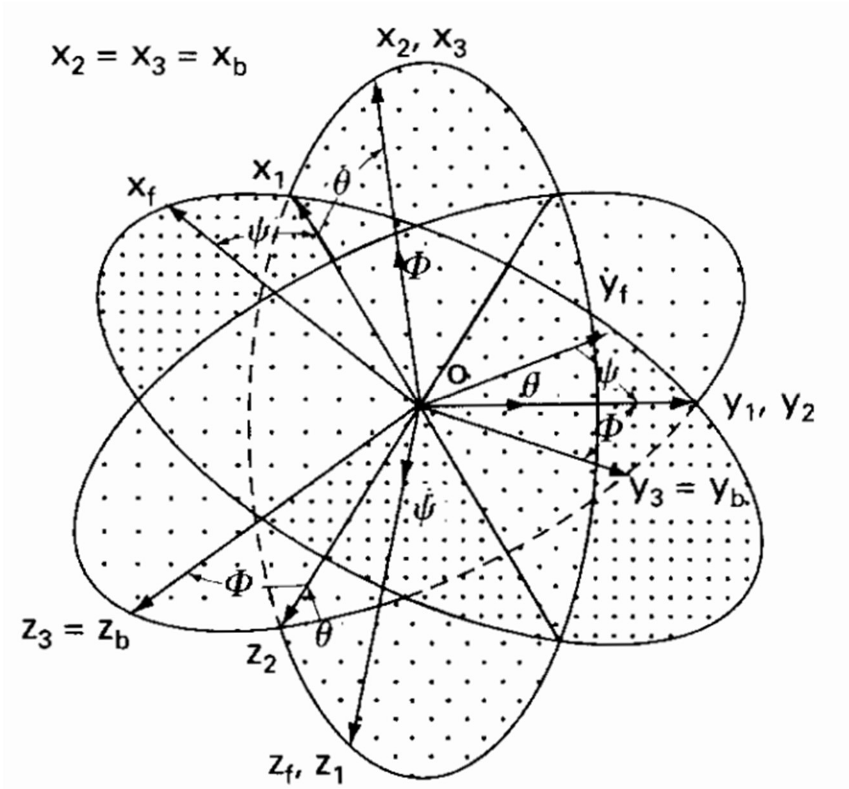


Figure 6 Relationship between body and inertial axis system

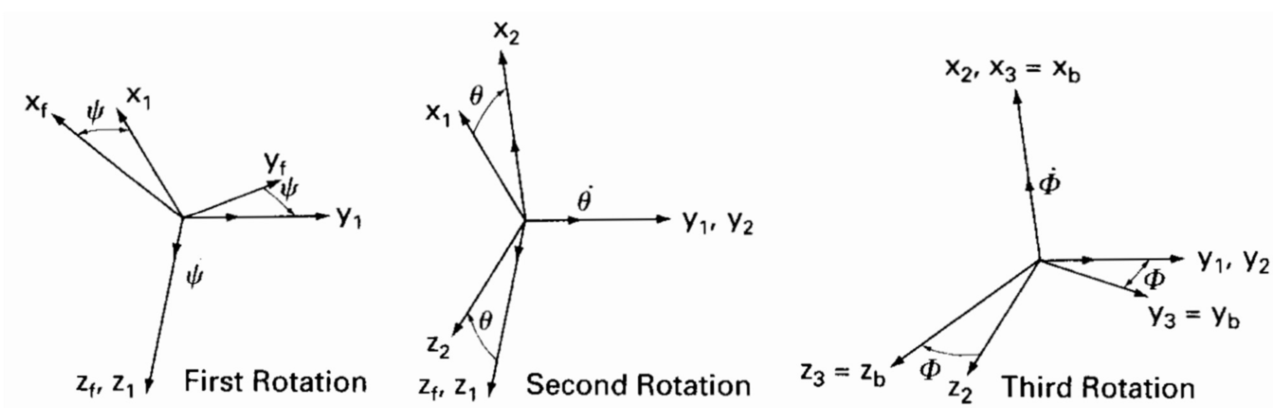


Figure 7 Orientation of body frame with respect to fixed frame

The term Euler Angles refers to the angles of rotation (ψ, θ, φ) needed to go from one coordinate system to another using the specific sequence of rotations

$$\vec{V}_{BF} = R_1(\phi)R_2(\theta)R_3(\psi)\vec{V}_I$$

The composite rotation matrix can be written as

$$R_1(\phi)R_2(\theta)R_3(\psi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This moves a vector from,

Inertial Frame ⇒ Body-Fixed Frame

To move a vector in reverse direction i.e.

Body-Fixed Frame ⇒ Inertial Frame

Rotation matrices are easily inverted, however

$$(R_1(\phi))^{-1} = R_1(-\phi)$$

$$\vec{V}_I = (R_1(\phi)R_2(\theta)R_3(\psi))^{-1}\vec{V}_{BF}$$

The rate of rotation of the Euler Angles can be found by rotating the rotation vector into the inertial frame.

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\theta & \cos\theta\cos\phi \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

This transformation can also be reversed as

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

2.6 Angle of attack and side slip angle

The Aerodynamic forces and moments primarily depend on angle of attack and sideslip angle. These two angles account for the fact that the nose is not always pointed into the wind.

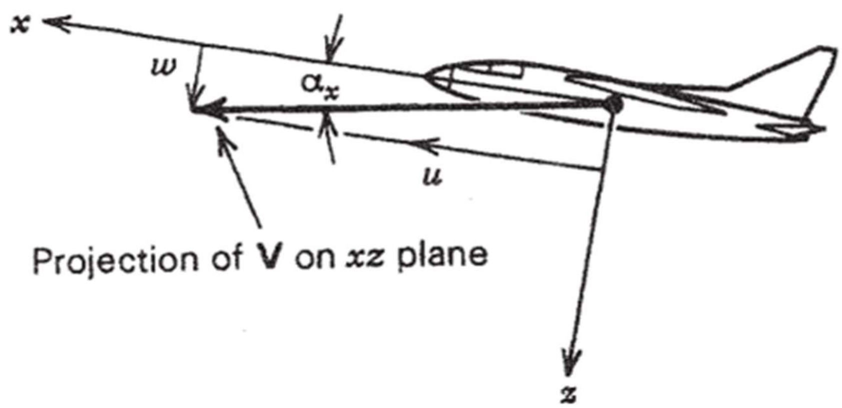


Figure 8 Angle of attack

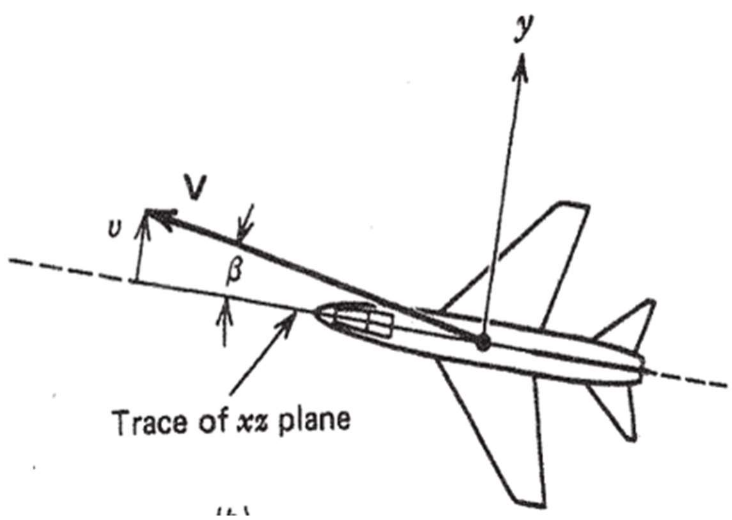


Figure 9 Side slip angle

The angle of attack is associated with the longitudinal forces and moments, while the sideslip angle is associated with the lateral-directional forces and moment. Lift, drag, and pitching moments depend on the angle of attack, whereas the side force, rolling moments and yawing moments depend on the sideslip angle.

The angle of attack and the sideslip angle are described by the following equations:

$$\alpha = \tan^{-1}\left(\frac{w}{U}\right)$$

$$\beta = \sin^{-1}\left(\frac{v}{U}\right)$$

$$U = \sqrt{u^2 + v^2 + w^2}$$

$$\beta \cong \frac{v}{U}$$

$$\Delta\dot{\alpha} = \frac{1}{(1 - Z_{\dot{w}})}\left(Z_u \Delta u + Z_w \Delta w - \theta g \sin\theta_0 \Delta\theta + (U_0 + Z_q)\Delta q + Z_{\delta_e} \delta\right.$$

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \left[A_{Lateral}\right] \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \left[B_{Lateral}\right] \begin{bmatrix} \delta_r \\ \delta_a \end{bmatrix}$$

The range of angle of attack is typically around 15 to 18 degrees and the maximum sideslip angle for aircraft is 10 degrees. As a result we can apply small angle approximations to above equations

Hence **α is directly proportional to *w*** and **β is directly proportional to *v*.**

Since sideslip angle and angle of attack are not included in our original state space equation we need above conversion to change our state space model in terms of sideslip angle and angle of attack.

Chapter 3 Linearization of equation of motion

3.1 Linearization

To linearize the equation of motion we consider some equilibrium points and substitute the variables as the sum of equilibrium points and disturbance value. To linearize the equation of motion we follow some steps as-

Step-1 Consider the collection of variables:

$X, Y, Z, p, q, r, L, M, N, u, v, w, \emptyset, \theta, \varphi.$

Step -2 Choosing equilibrium point:

$$X_0, Y_0, Z_0, p_0, q_0, r_0, L_0, M_0, N_0, u_0, v_0, w_0$$

Step -3 Substitution

$$
\begin{array}{lll}
u(t) = u_0 + \Delta u(t) & v(t) = v_0 + \Delta v(t) & w(t) = w_0 + \Delta w(t) \\
p(t) = p_0 + \Delta p(t) & q(t) = q_0 + \Delta q(t) & r(t) = r_0 + \Delta r(t) \\
X(t) = X_0 + \Delta X(t) & Y(t) = Y_0 + \Delta Y(t) & Z(t) = Z_0 + \Delta Z(t) \\
L(t) = L_0 + \Delta L(t) & M(t) = M_0 + \Delta M(t) & N(t) = N_0 + \Delta N(t)
\end{array}
$$

Step -4 Eliminate non-linear terms

$\Delta u(t)^2 = 0$ and $\Delta u(t)\Delta r(t) = 0$

For steady flight, let

$u_0 \neq 0$ $\quad$ $v_0 = 0$ $\quad$ $w_0 = 0$

$p_0 = 0$ $\quad$ $q_0 = 0$ $\quad$ $r_0 = 0$

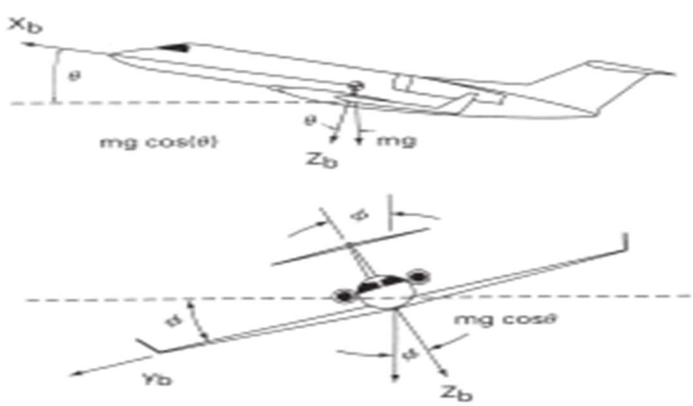
$X_0 \neq 0$ $\quad$ $Y_0 = 0$ $\quad$ $Z_0 \neq 0$

$L_0 = 0$ $\quad$ $M_0 = 0$ $\quad$ $N_0 = 0$

Other equilibrium factors

Weight: Pitching motion changes force

$$X_0 = mg\sin\theta_0$$
$$Z_0 = mg\cos\theta_0$$



We use trigonometric identities and small angle approximations (Δθ small):

$$\sin(\theta_0 + \Delta\theta) = \sin\theta_0 \cos\Delta\theta + \cos\theta_0 \sin\Delta\theta$$

$$\cong \sin\theta_0 + \Delta\theta\cos\theta_0$$

Similarly,

$$\cos(\theta_0 + \Delta\theta) = \cos\theta_0 \cos\Delta\theta - \sin\theta_0 \sin\Delta\theta$$

$$\cong \cos\theta_0 - \Delta\theta\sin\theta_0$$

Substituting into EOM, and ignoring 2nd order terms, we get

$$\Delta \dot{u} + \Delta \theta g \cos \theta_0 = \frac{\Delta X}{m}$$

$$\Delta \dot{v} - \Delta \phi g \cos \theta_0 + u_0 \Delta r = \frac{\Delta Y}{m}$$

$$\Delta \dot{w} + \Delta \theta g \sin \theta_0 - u_0 \Delta q = \frac{\Delta Z}{m}$$

$$\Delta \dot{p} = \frac{I_{zz}}{I_{xx} I_{zz} - I_{xz}^2} \Delta L + \frac{I_{xz}}{I_{xx} I_{zz} - I_{xz}^2} \Delta N$$

$$\Delta \dot{q} = \frac{\Delta M}{I_{yy}}$$

$$\Delta \dot{r} = \frac{I_{xz}}{I_{xx} I_{zz} - I_{xz}^2} \Delta L + \frac{I_{xx}}{I_{xx} I_{zz} - I_{xz}^2} \Delta N$$

<u>**Note:**</u> these are coupled with θ, ϕ

We include expressions for θ, ϕ.

$$\Delta \dot{\theta} = \Delta q$$

$$\Delta \dot{\phi} = \Delta p + \Delta r \tan \theta_0$$

$$\Delta \dot{\psi} = \Delta r \sec \theta_0$$

For steady-level flight, $\theta_0 = 0$, so we can simplify

$$\Delta \dot{\theta} = \Delta q$$

$$\Delta \dot{\phi} = \Delta p$$

$$\Delta \dot{\psi} = \Delta r$$

which is what we will mostly do.

We can also express the equations for translational motion

$$\Delta \dot{x} = \delta u \cos \theta_0 - u_0 \Delta \theta \sin \theta_0 + \Delta w \sin \theta_0$$

$$\Delta \dot{y} = u_0 \Delta \psi \cos \theta_0 + \Delta v$$

$$\Delta \dot{z} = -\delta u \sin \theta_0 - u_0 \Delta \theta \cos \theta_0 + \Delta w \cos \theta_0$$

So now we have 12 equations and 12 variables.

Since, the forces and moments depend on motion and controls:

e.g. $\Delta X(u, v, w, \cdots, \delta_e, \delta_T)$

i.e. more variables means more non-linear terms.

Out of (u, v, w, $\dot{u}$, $\dot{v}$, $\dot{w}$, p, q, r, δ_a, δ_r, δ_e, δ_T), we make the restrictive assumptions on following

$$\Delta X\left(\Delta u, \Delta w, \delta_e, \delta_T\right)$$
$$\Delta Y\left(\Delta v, \Delta p, \Delta r, \delta_r\right)$$
$$\Delta Z\left(\Delta u, \Delta w, \Delta \dot{w}, \Delta q, \delta_e, \delta_T\right)$$
$$\Delta L\left(\Delta v, \Delta p, \Delta r, \delta_r, \delta_a\right)$$
$$\Delta M\left(\Delta u, \Delta w, \Delta \dot{w}, \Delta q, \delta_e, \delta_T\right)$$
$$\Delta N\left(\Delta v, \Delta p, \Delta r, \delta_r, \delta_a\right)$$

The reason is simple here, we have following new variables.

- δ_T - Throttle control input.
- δ_e – Elevator control input.
- δ_a - Aileron control input.
- δ_r - Rudder control input.

We have notations for partial derivatives which we are using for simplification of equations in longitudinal and lateral dynamics.

$$X_u = \frac{1}{m}\frac{\partial X}{\partial u} \qquad X_w = \frac{1}{m}\frac{\partial X}{\partial w} \qquad X_{\delta_e} = \frac{1}{m}\frac{\partial X}{\partial \delta_e} \qquad X_{\delta_T} = \frac{1}{m}\frac{\partial X}{\partial \delta_T}$$

$$Y_v = \frac{1}{m}\frac{\partial Y}{\partial v} \qquad Y_p = \frac{1}{m}\frac{\partial Y}{\partial p} \qquad Y_r = \frac{1}{m}\frac{\partial Y}{\partial r} \qquad Y_{\delta_r} = \frac{1}{m}\frac{\partial Y}{\partial \delta_r} \qquad Z_{\delta_e} = \frac{1}{m}\frac{\partial Z}{\partial \delta_e}$$

$$Z_u = \frac{1}{m}\frac{\partial Z}{\partial u} \qquad Z_w = \frac{1}{m}\frac{\partial Z}{\partial w} \qquad Z_{\dot{w}} = \frac{1}{m}\frac{\partial Z}{\partial \dot{w}} \qquad Z_q = \frac{1}{m}\frac{\partial Z}{\partial q} \qquad L_{\delta_a} = \frac{1}{I_{zz}}\frac{\partial L}{\partial \delta_a}$$

$$L_v = \frac{1}{I_{zz}}\frac{\partial L}{\partial v} \qquad L_p = \frac{1}{I_{zz}}\frac{\partial L}{\partial p} \qquad L_r = \frac{1}{I_{zz}}\frac{\partial L}{\partial r} \qquad L_{\delta_r} = \frac{1}{I_{zz}}\frac{\partial L}{\partial \delta_r} \qquad M_{\delta_e} = \frac{1}{I_{yy}}\frac{\partial M}{\partial \delta_e}$$

$$M_u = \frac{1}{I_{yy}}\frac{\partial M}{\partial u} \qquad M_w = \frac{1}{I_{yy}}\frac{\partial M}{\partial w} \qquad M_{\dot{w}} = \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{w}} \qquad M_q = \frac{1}{I_{yy}}\frac{\partial M}{\partial q} \qquad N_{\delta_a} = \frac{1}{I_{xx}}\frac{\partial N}{\partial \delta_a}$$

$$N_v = \frac{1}{I_{xx}}\frac{\partial N}{\partial v} \qquad N_p = \frac{1}{I_{xx}}\frac{\partial N}{\partial p} \qquad N_r = \frac{1}{I_{xx}}\frac{\partial N}{\partial r} \qquad N_{\delta_r} = \frac{1}{I_{xx}}\frac{\partial N}{\partial \delta_r}$$

3.2 Linearization of longitudinal dynamics

Substituting into EOM, and ignoring 2nd order terms, we get

$$\Delta \dot{u} + \Delta \theta g \cos \theta_0 = \frac{\Delta X}{m}$$

$$\Delta \dot{\theta} = \Delta q$$

$$\Delta \dot{w} + \Delta \theta g \sin \theta_0 - u_0 \Delta q = \frac{\Delta Z}{m}$$

$$\Delta \dot{q} = \frac{\Delta M}{I_{yy}}$$

Now we put the following terms (which are first order derivative approximation) in above equation for linearization of longitudinal motion.

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta \delta_e + \frac{\partial X}{\partial \delta_T}\Delta \delta_T$$

$$\Delta Z = \frac{\partial Z}{\partial u}\Delta u + \frac{\partial Z}{\partial w}\Delta w + \frac{\partial Z}{\partial \dot{w}}\Delta \dot{w} + \frac{\partial Z}{\partial q}\Delta q + \frac{\partial Z}{\partial \delta_e}\Delta \delta_e + \frac{\partial Z}{\partial \delta_T}\Delta \delta_T$$

$$\Delta M = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial w}\Delta w + \frac{\partial M}{\partial \dot{w}}\Delta \dot{w} + \frac{\partial M}{\partial q}\Delta q + \frac{\partial M}{\partial \delta_e}\Delta \delta_e + \frac{\partial M}{\partial \delta_T}\Delta \delta_T$$

By substituting the above equations the set of new equations are

$$\Delta \dot{u} = X_u \Delta u + X_w \Delta w - g\cos\theta_0 \Delta\theta + X_{\delta_e}\delta_e + X_{\delta_T}\delta_T$$

$$\Delta \dot{w} = \frac{1}{(1 - Z_{\dot{w}})}(Z_u \Delta u + Z_w \Delta w - \theta g \sin\theta_0 \Delta\theta + (U_0 + Z_q)\Delta q + Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T$$

$$\Delta\dot{\theta} = \Delta q$$

$$\Delta \dot{q} = M_u \Delta u + M_w \Delta w + M_{\dot{w}}\Delta \dot{w} + M_q \Delta\dot{\theta} + M_{\delta_e}\delta_e + M_{\delta_T}\delta_T$$

By putting equation of $\Delta\dot{\theta}$ and $\Delta\dot{w}$ in equation of $\Delta\dot{q}$ we get,

$$\dot{q} = \left(M_u + \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}}\right)\Delta u + \left(M_w + \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}}\right)\Delta w - \left(\frac{M_{\dot{w}}}{1 - Z_{\dot{w}}}\theta g \sin\theta_0\right)\Delta\theta + \left(M_{\dot{w}}\left(U_0 + Z_q\right) + M_q\right)\Delta q$$

$$+ \left(M_{\dot{w}} Z_{\delta_e} + M_{\delta_e}\right)\delta_e + \left(M_{\dot{w}} Z_{\delta_T} + M_{\delta_T}\right)\delta_T$$

The state space representation of longitudinal motion is given as

$$\begin{pmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{\theta} \\ \Delta\dot{q} \end{pmatrix} = [A_{Longitudinal}] \begin{pmatrix} \Delta u \\ \Delta w \\ \Delta\theta \\ \Delta q \end{pmatrix} + [B_{Longitudinal}] \begin{pmatrix} \delta_e \\ \delta_T \end{pmatrix}$$

Now, we do not have our state space representation in terms of angle of attack (α)

We can convert it by using relation between **w** and **α**

$$\alpha = \tan^{-1}\left(\frac{w}{U}\right) \qquad\qquad \alpha \cong \frac{w}{U}$$

Hence, $\dot{w}$ equation can be modified in terms of $\dot{\alpha}$

$$\Delta\dot{\alpha} = \frac{1}{\left(1 - Z_{\dot{w}}\right)}\left(Z_u \Delta u + Z_w \Delta w - \theta g \sin\theta_0 \Delta\theta + (U_0 + Z_q)\Delta q + Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T\right)$$

Since, U is magnitude of velocity vector it will be constant and it will not create any difference in equations of $\dot{w}$ and $\dot{\alpha}$

Therefore, modified state space representation is

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\alpha} \\ \Delta\dot{\theta} \\ \Delta\dot{q} \end{bmatrix} = \left[A_{Longitudinal}\right] \begin{bmatrix} \Delta u \\ \Delta\alpha \\ \Delta\theta \\ \Delta q \end{bmatrix} + \left[B_{Longitudinal}\right] \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

Where,

$$A_{Longitudinal} = \begin{bmatrix} X_u & X_w & -g\cos\theta_0 & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{-\theta\Delta g\cos\theta_0}{1-Z_{\dot{w}}} & \dfrac{(U_0+Z_q)}{1-Z_{\dot{w}}} \\ 0 & 0 & 0 & 1 \\ \left(M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}}\right) & \left(M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}}\right) & \left(\dfrac{M_{\dot{w}}}{1-Z_{\dot{w}}}\theta g\sin\theta_0\right) & \left(M_{\dot{w}}\left(U_0+Z_q\right)+M_q\right) \end{bmatrix}$$

$$B_{Longitudinal} = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ \dfrac{Z_{\delta_e}}{\left(1-Z_{\dot{w}}\right)} & \dfrac{Z_{\delta_T}}{\left(1-Z_{\dot{w}}\right)} \\ 0 & 0 \\ \left(M_{\dot{w}}Z_{\delta_e} + M_{\delta_e}\right) & \left(M_{\dot{w}}Z_{\delta_T} + M_{\delta_T}\right) \end{bmatrix}$$

3.3 Linearization of lateral dynamics

For linearization of lateral dynamics we will use the same substitution and the same equilibrium points that we chose for longitudinal dynamics. The we get the linearized equation of lateral motion as follows-

$$\Delta\dot{v} - \Delta\phi g\cos\theta_0 + u_0\Delta r = \frac{\Delta Y}{m}$$

$$\Delta\dot{p} = \frac{I_{zz}}{I_{xx}I_{zz} - I_{xz}^2}\Delta L + \frac{I_{xz}}{I_{xx}I_{zz} - I_{xz}^2}\Delta N$$

$$\Delta\dot{r} = \frac{I_{xz}}{I_{xx}I_{zz} - I_{xz}^2}\Delta L + \frac{I_{xx}}{I_{xx}I_{zz} - I_{xz}^2}\Delta N$$

$$\Delta\dot{\phi} = \Delta p + \Delta r\tan\theta_0$$

Now we put the following terms (which are first order derivative approximation) in above equation for linearization of lateral motion.

$$\Delta Y = \frac{\partial Y}{\partial v}\Delta v + \frac{\partial Y}{\partial p}\Delta p + \frac{\partial Y}{\partial r}\Delta r + \frac{\partial Y}{\partial \delta_r}\Delta \delta_r$$

$$\Delta L = \frac{\partial L}{\partial v}\Delta v + \frac{\partial L}{\partial p}\Delta p + \frac{\partial L}{\partial r}\Delta r + \frac{\partial L}{\partial \delta_r}\Delta \delta_r + \frac{\partial L}{\partial \delta_a}\Delta \delta_a$$

$$\Delta N = \frac{\partial N}{\partial v}\Delta v + \frac{\partial N}{\partial p}\Delta p + \frac{\partial N}{\partial r}\Delta r + \frac{\partial N}{\partial \delta_r}\Delta \delta_r + \frac{\partial N}{\partial \delta_a}\Delta \delta_a$$

Then we obtain state space equations as

$$\Delta\dot{v} = \Delta\phi g\cos\theta_0 - u_0\Delta r + Y_v\Delta v + Y_p\Delta p + Y_r\Delta r + Y_{\delta_r}\delta_r$$

$$\Delta\dot{p} = \frac{I_{zz}}{I_{xx}I_{zz} - I_{xz}^2}(L_v\Delta v + L_p\Delta p + L_r\Delta r + L_{\delta_r}\delta_r + L_{\delta_a}\delta_a)$$

$$+\frac{I_{xz}I_{xx}}{I_{xx}I_{zz} - I_{xz}^2}(N_v\Delta v + N_p\Delta p + N_r\Delta r + N_{\delta_r}\delta_r + N_{\delta_a}\delta_a)$$

$$\Delta\dot{r} = \frac{I_{xz}I_{zz}}{I_{xx}I_{zz} - I_{xz}^2}(L_v\Delta v + L_p\Delta p + L_r\Delta r + L_{\delta_r}\delta_r + L_{\delta_a}\delta_a)$$

$$\Delta\dot{\phi} = \Delta p + \Delta r\tan\theta_0$$

The state space representation can be represented as

$$\begin{pmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{pmatrix} = [A_{Lateral}]\begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix} + [B_{Lateral}]\begin{pmatrix} \delta_r \\ \delta_a \end{pmatrix}$$

Similarly, here also the sideslip angle (β) is not the state directly

We can convert it by using relation between **v** and **β**

We have,

$$\beta = \sin^{-1}\left(\frac{v}{U}\right) \qquad \beta \cong \frac{v}{U}$$

Hence, $\dot{v}$ equation can be modified in terms of $\dot{\beta}$

$$\Delta\dot{\beta} = Y_v\Delta\beta + Y_p\Delta p + (Y_r - u_0)\Delta r + g\cos\theta_0\Delta\theta + Y_{\delta_r}\Delta\delta_r + Y_{\delta_a}\Delta\delta_a$$

Since, U is magnitude of velocity vector it will be constant and it will not create any difference in equations of $\dot{v}$ and $\dot{\beta}$

Therefore, modified state space representation is

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \left[A_{Lateral}\right] \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \left[B_{Lateral}\right] \begin{bmatrix} \delta_r \\ \delta_a \end{bmatrix}$$

Where,

$$A_{Lateral} = \begin{bmatrix} Y_v & Y_p & Y_r - u_0 & g\cos\theta_0 \\ \frac{L_v}{I'} + I'_{zx}I_{zz}N_v & \frac{L_p}{I'} + I'_{zx}I_{zz}N_p & \frac{L_r}{I'} + I'_{zx}I_{zz}N_r & 0 \\ \frac{N_v}{I'} + I'_{zx}I_{xx}L_v & \frac{N_p}{I'} + I'_{zx}I_{xx}L_p & \frac{N_r}{I'} + I'_{zx}I_{xx}L_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}$$

$$B_{Lateral} = \begin{bmatrix} Y_{\delta_r} & Y_{\delta_a} \\ \frac{L_{\delta_r}}{I^{'}} + I^{'}_{zx} I_{zz} N_{\delta_r} & \frac{L_{\delta_a}}{I^{'}} + I^{'}_{zx} I_{zz} N_{\delta_a} \\ \frac{L_{\delta_r}}{I^{'}} + I^{'}_{zx} I_{zz} N_{\delta_r} & \frac{L_{\delta_a}}{I^{'}} + I^{'}_{zx} I_{zz} N_{\delta_a} \\ 0 & 0 \end{bmatrix}$$

Chapter-4 LQR Control System

4.1 LQR controller design

Problems that are commonplace in the field of control is not only stabilize system, but also how output system can follow reference. If output is desired to follow reference r, then adding an integrator and defining error state x_i is output of integrator, with $\dot{x}_i$ is difference between input and output of the system.

Now take the state space model as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ u &= -Kx + k_i x_i \\ \dot{x}_i &= r - y = r - Cx \end{aligned} \qquad \ldots\ldots\ldots(1)$$

Where,

x = state vector

u = Control signal

y = output

r = reference (step function, scalar)

x_i = output of integrator

Dynamic system in Equation (1) can be written

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r \qquad \ldots.(2)$$

Design of tracking has to make system into stabilize, if x(∞), $x_i(\infty)$ and u(∞) approach constant value, then x_i = 0 ,so y(∞) = r.

In steady state Equation (2) into:

$$\begin{bmatrix} \dot{x}(\infty) \\ \dot{x}_i(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ x_i(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(\infty) \quad \text{....(3)}$$

Because r(t) is step signal, then r(∞) = r(t) = r is constant value,for t > 0. subtracting Equation (2) with (3), then we get error state:

$$\begin{bmatrix} \dot{x}_e \\ \dot{x}_{i_e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_e \\ x_{i_e} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e + \begin{bmatrix} 0 \\ I \end{bmatrix} r_e \quad \text{...(4)}$$

Where,

$$u_e(t) = -Kx_e(t) + k_i x_{i_e}(t)$$

Vector error size (6x1) can be defined into:

$$e(t) = \begin{bmatrix} x_e(t) \\ x_{i_e}(t) \end{bmatrix}$$

Then Equation (4) into:

$$\dot{e} = \hat{A}e + \hat{B}u_e$$

Where

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} K & -k_i \end{bmatrix}$$

Then final error equation can we written as

$$\dot{e} = (\hat{A} - \hat{B}\hat{K})e$$

Value of $\hat{K}$ is found with LQR method and cost functio of LQR is defined by

$$J = \frac{1}{2} \int_0^{\infty} (e^T Q e + u_e^T R u_e) dt$$

and ARE equation is

$$\hat{A}^T \hat{P} + \hat{P}\hat{A} + \hat{Q} - \hat{P}\hat{B}R^{-1}\hat{B}^T \hat{P} = 0$$

From ARE equation we get P matrix which insure the stability of the system.Now we calculate the $\hat{K}$ by solving controm law of ARE equation given as

$$u = -\hat{K}x$$

Where,

$$\hat{K} = R^{-1}B^T P$$

4.2 Simulation block diagram

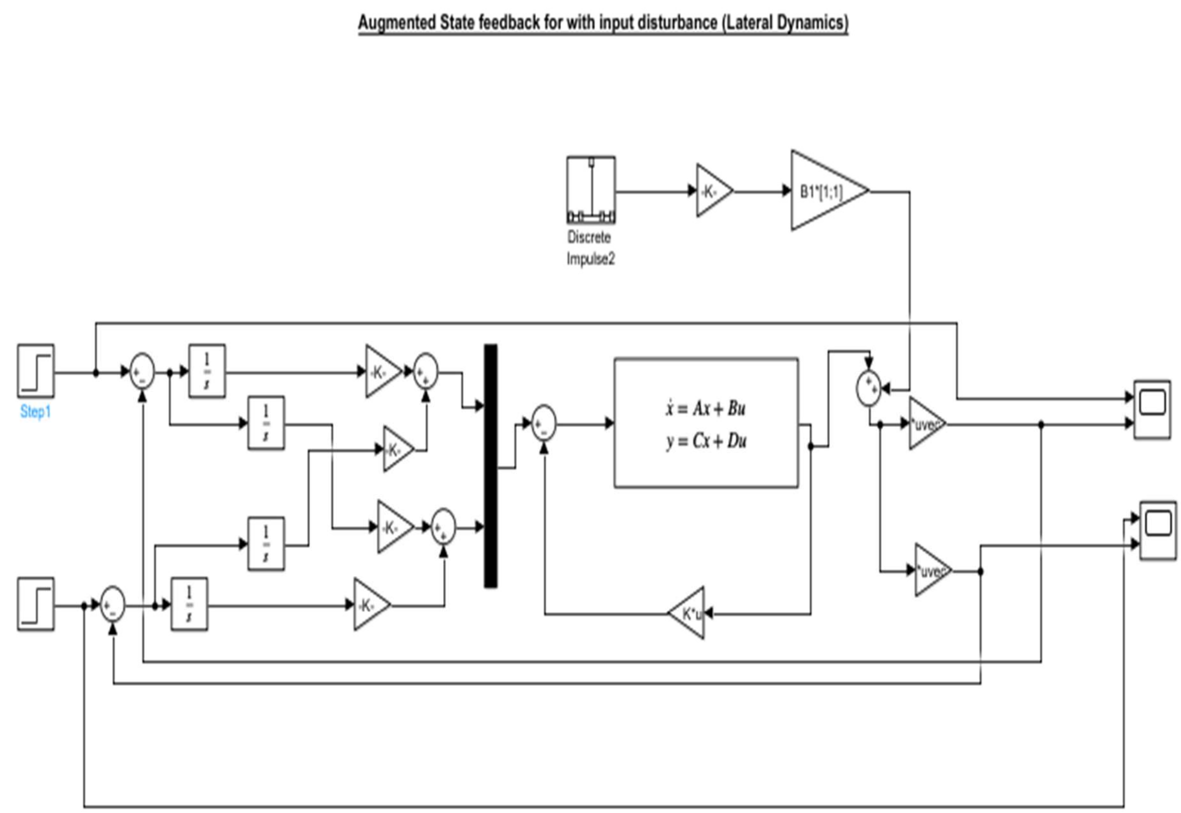


Figure 10 Simulation diagram for augmented state feedback (with disturbance)

Chapter 5 Simulation

5.1 State space of longitudinal directional motion

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.83705 & 1.7696 & -0.35236 & 0 \\ -5.9575 & -21.776 & 0.005673 & 0.8717 \\ 0 & 0 & 0 & 1 \\ 14.891 & -47.637 & -0.015802 & -7.9269 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} 0 & 3.9397 \\ -0.91092 & 0 \\ 0 & 0 \\ -30.902 & -6.9048 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

For LQR control we need Q and R matrices which set by user

$$Q_{Long} = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2050 & 0 \\ 0 & 0 & 0 & 0 & 0 & 35000 \end{bmatrix}$$

$$R_{Long} = \begin{bmatrix} 0.07 & 0 \\ 0 & 0.07 \end{bmatrix}$$

For above Q and R matrices augmented gain matrix calculeted from simulation

$$K_{long} = \begin{bmatrix} -17.13 & -10.36 & -230.73 & -26.38 \\ 119.78 & -23.71 & -13.32 & -3.011 \end{bmatrix}$$

$$K_{i_{long}} = \begin{bmatrix} -16.45 & 703.83 \\ 170.33 & 67.97 \end{bmatrix}$$

5.2 Simulation plot and result for longitudinal motion

The main target variable is

- Pitch angle (θ)

Specifications	Angle of Attack (α)			Pitch Angle (θ)		
	Without control	**State feedback control**	**LQR control**	**Without control**	**State feedback control**	**LQR control**
Rise Time (sec)	0.2834	0.0825	1.019	1.44	0.075	2.39
Settling time (sec)	7.34	0.4832	1.71	8.48	0.4625	4.52
Max. Peak Overshoot (%)	53.01%	37.60%	0.4%	18.75%	27.18%	0%
Peak time (sec)	1.60	0.1619	3.058	4.19	0.0350	10

Table 1 Simulation result of longitudinal motion

5.2.2 Simulated outputs for Pitch Angle (θ)

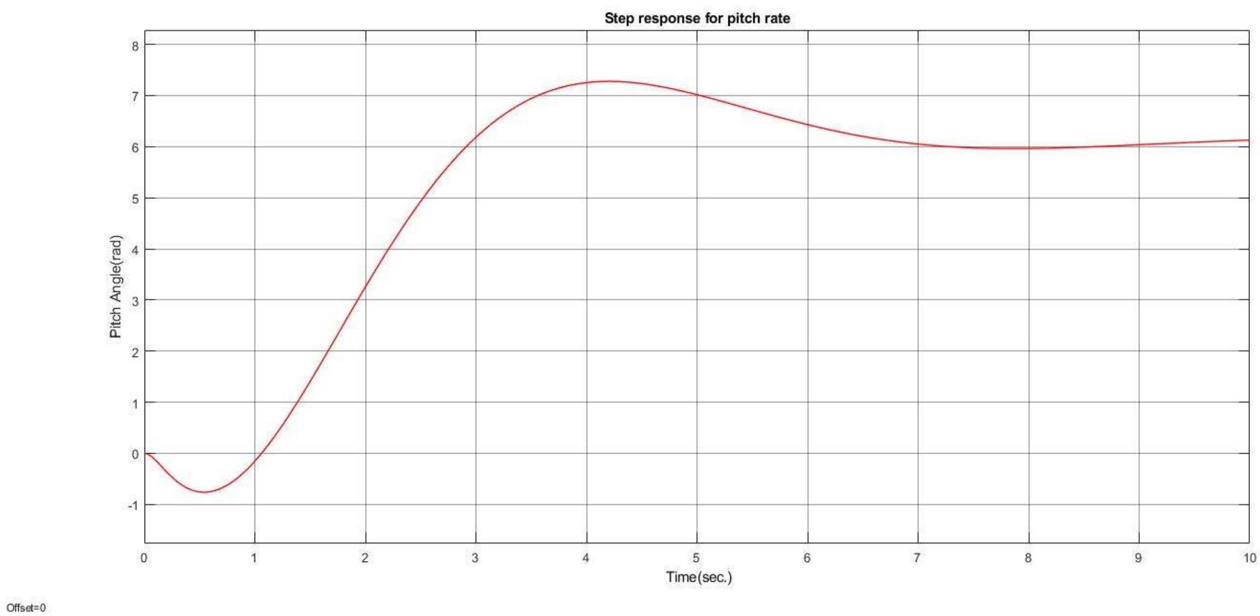


Figure 11 Step response for pitch rate

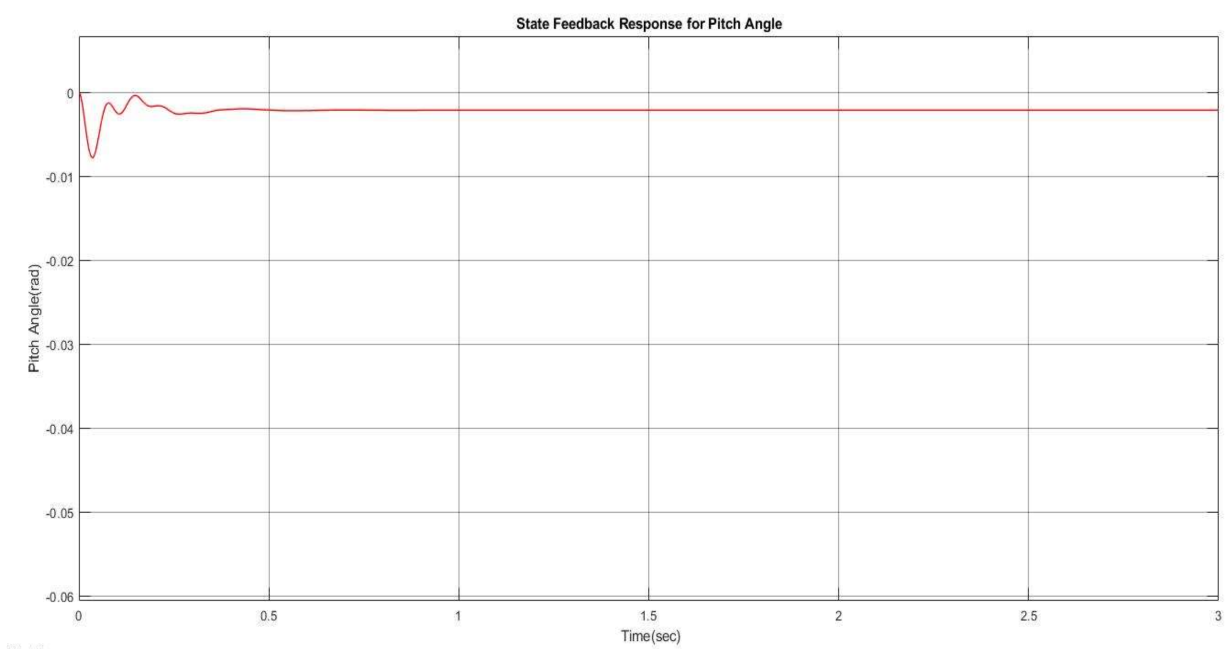


Figure 12 State feedback response for pith angle

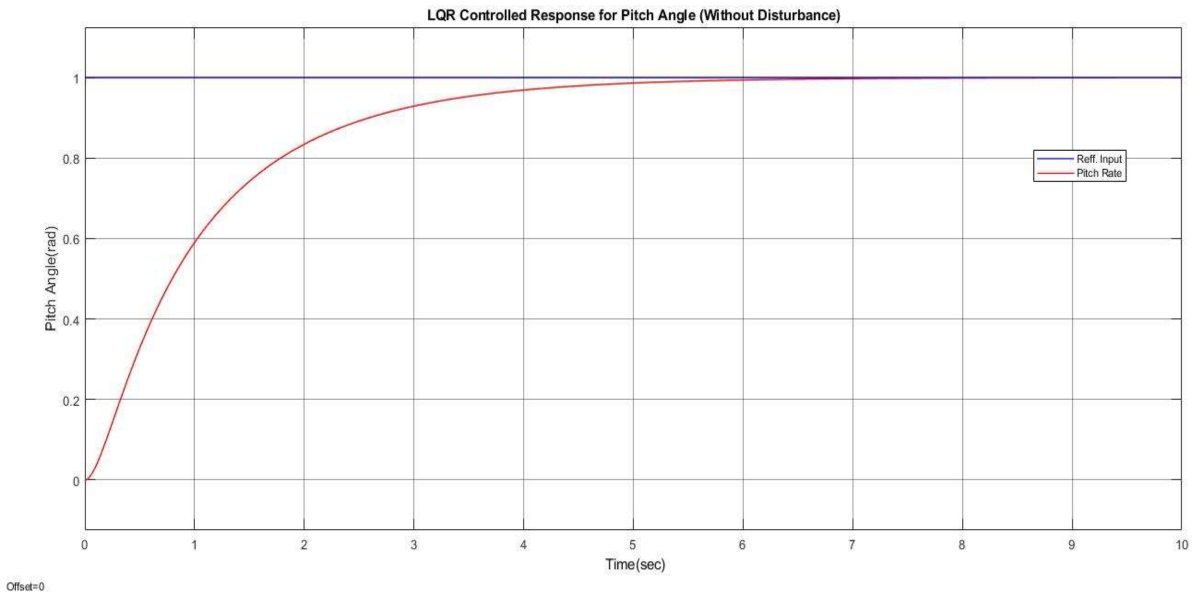


Figure 13 LQR controlled response for pitch angle (without disturbance)

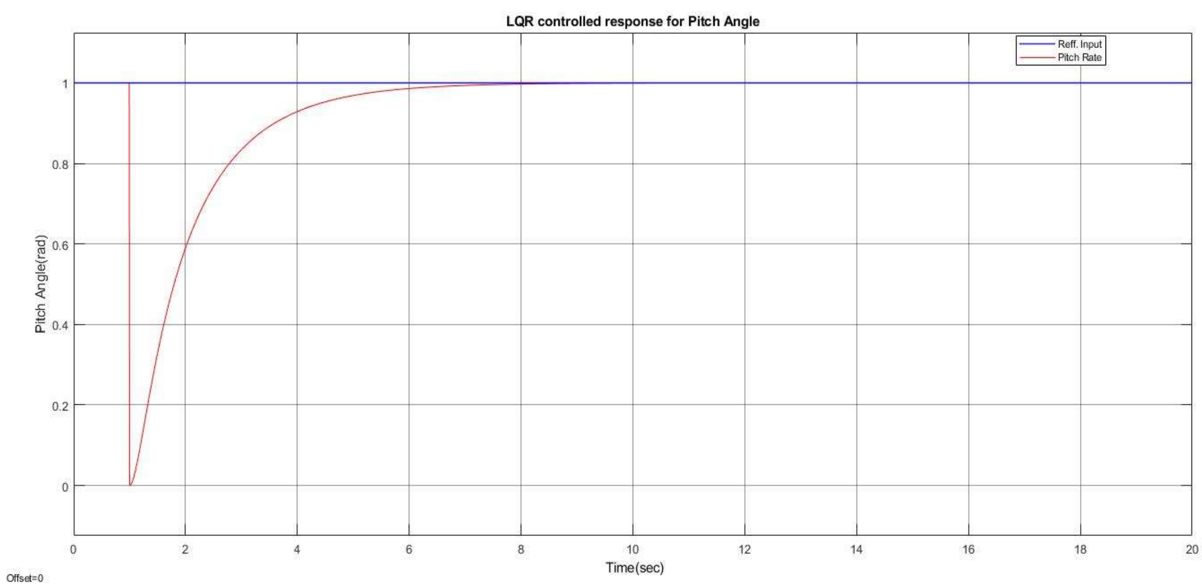


Figure 14 LQR controlled response for pitch angle (with disturbance)

5.3 State space of Lateral motion

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -2.3817 & 0 & -1.0019 & 2.1827 \\ -21.063 & -16.055 & 0.87229 & 0 \\ 24.512 & -16.651 & -3.5379 & 0 \\ 0 & 1.0026 & -0.029766 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & -0.24719 \\ -36.263 & -688.44 \\ -0.67252 & -67.983 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_R \end{bmatrix}$$

For LQR control we need Q and R matrices which set by user

$$Q_{Lateral} = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 70 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7000 \end{bmatrix}$$

$$R_{Lateral} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix}$$

For above Q and R matrices augmented gain matrix calculeted from simulation

$$K_{lat} = \begin{bmatrix} -2.888 & -0.0830 & -0.6046 & 5.6724 \\ 0.1201 & -0.4605 & -0.0264 & -64.4480 \end{bmatrix}$$

$$K_{i_{lat}} = \begin{bmatrix} 22.35 & 14.39 \\ -0.5441 & 591.43 \end{bmatrix}$$

The target variables here are

- Sideslip angle (β)
- Roll angle (ϕ)

Various time domain parameters are tabulated below:

Table 2 Simulation result for lateral motion

Specifications	**Sideslip Angle (β)**			**Roll Angle (ϕ)**		
	Without control	**State feedback control**	**LQR control**	**Without control**	**State feedback control**	**LQR control**
Rise Time (sec)	6.63	0.1982	0.1182	7.33	0.0118	0.2169
Settling time (sec)	9.91	0.7529	0.2855	9.4767	0.0236	0.4537
Max. Peak Overshoot (%)	0%	13.96%	42.73%	0%	2.04%	0.9%
Peak time (sec)	10	0.4472	0.3348	10	0.0235	0.9639

5.3.1 Simulated outputs for Sideslip angle (β)

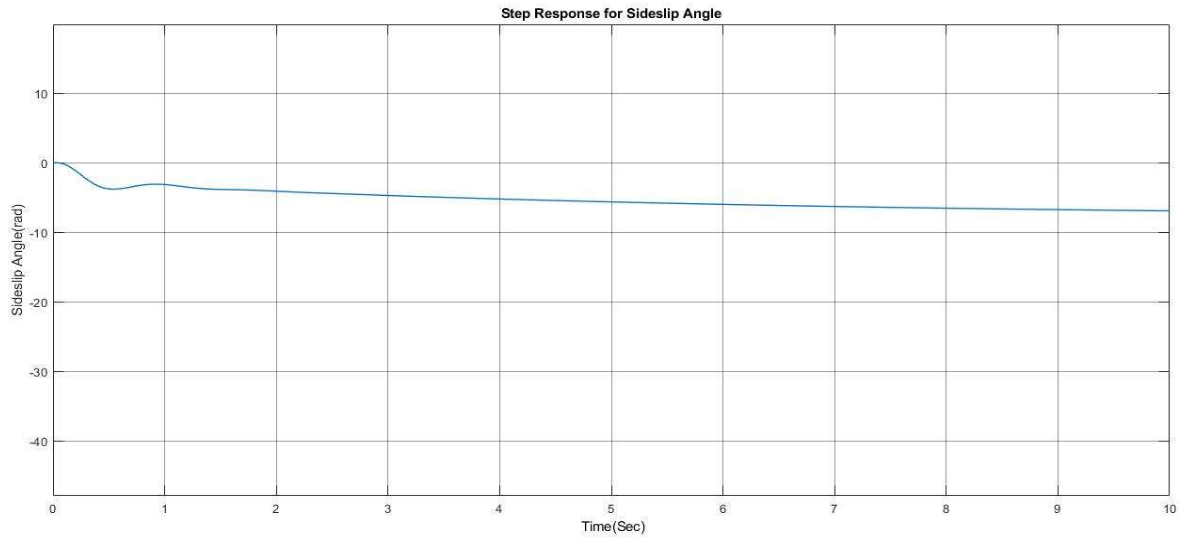


Figure 15 step response for sideslip angle

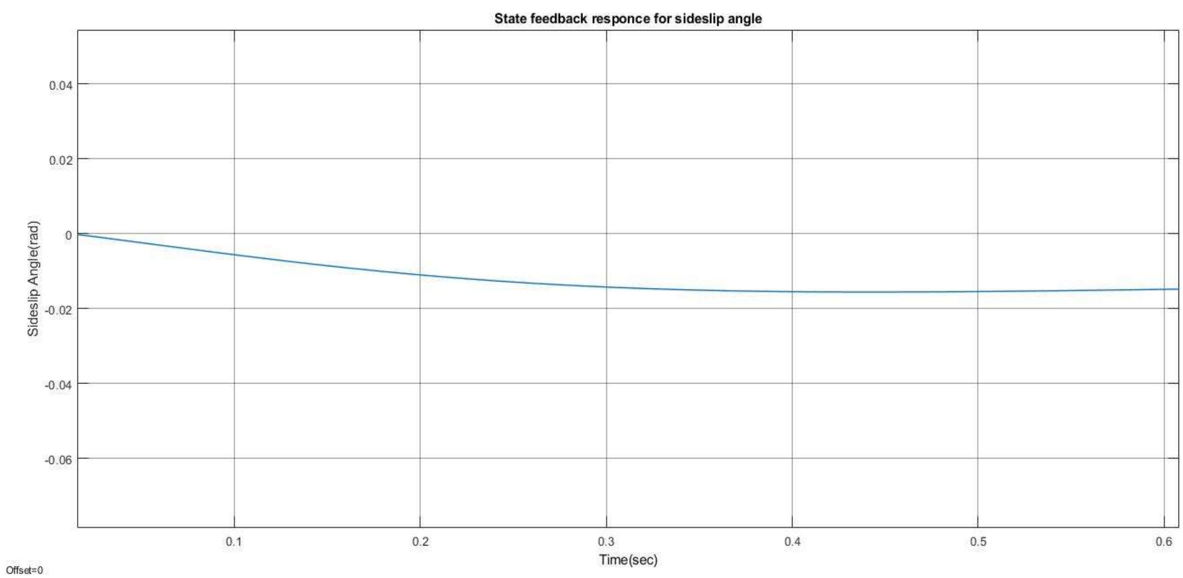


Figure 16 State feedback response for sideslip angle

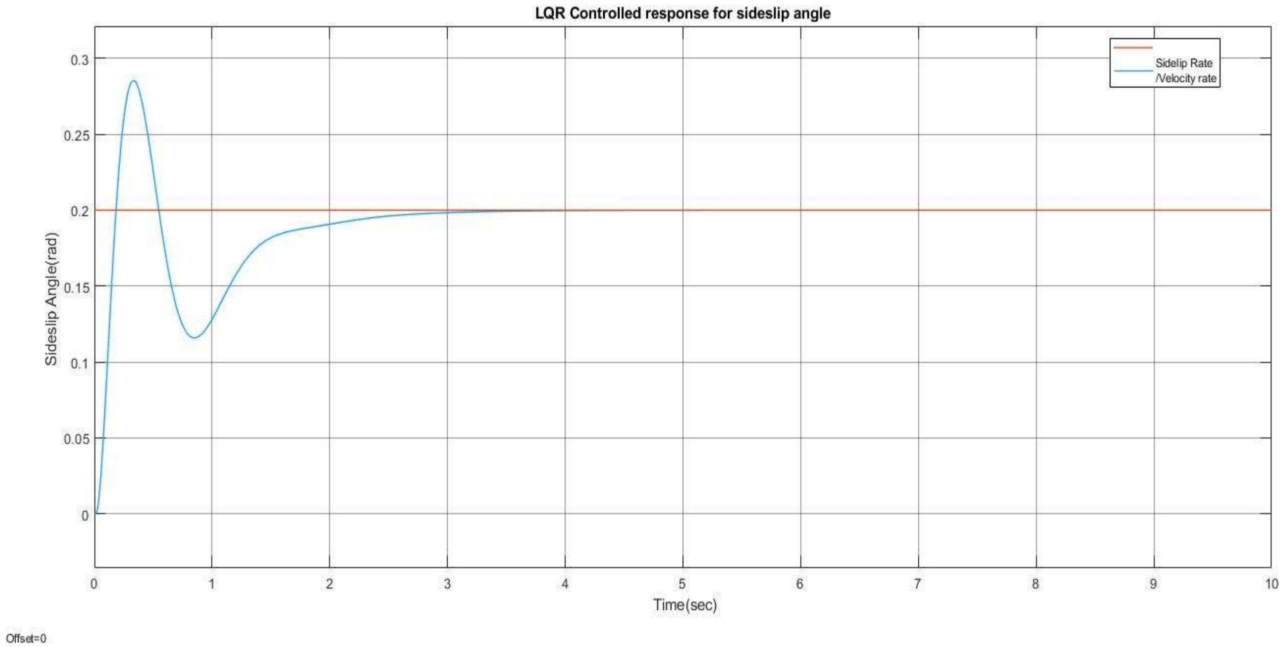


Figure 17 LQR controlled response for side slip angle

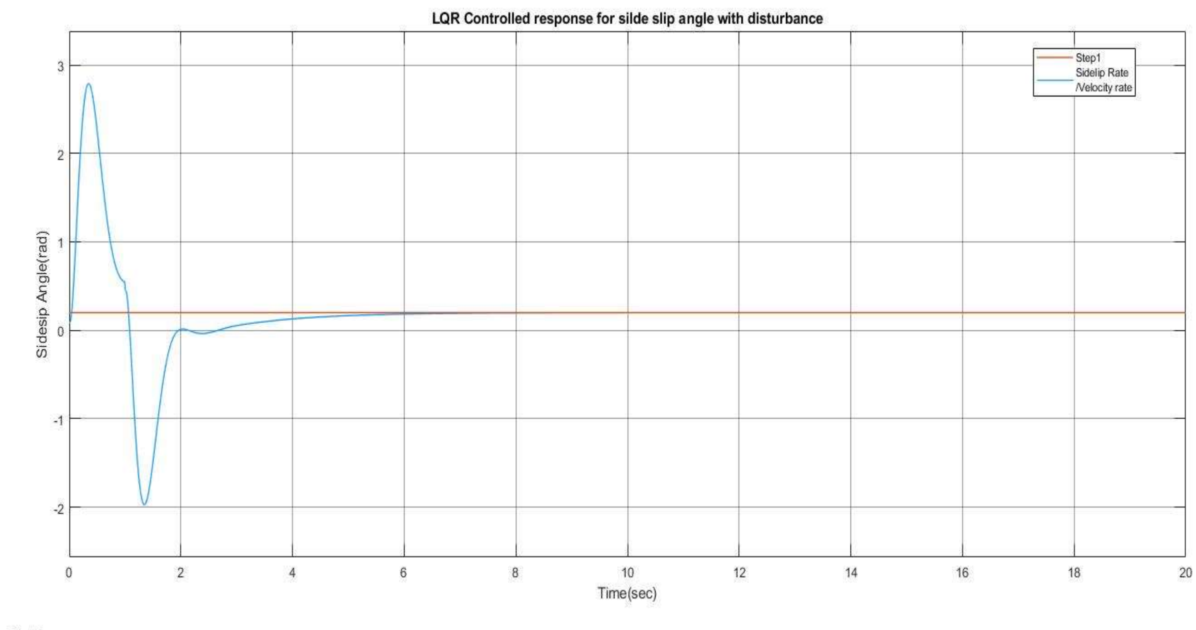


Figure 18 LQR controlled response for side slip angle (with disturbance)

5.3.2 Simulated outputs for Roll angle (ϕ)

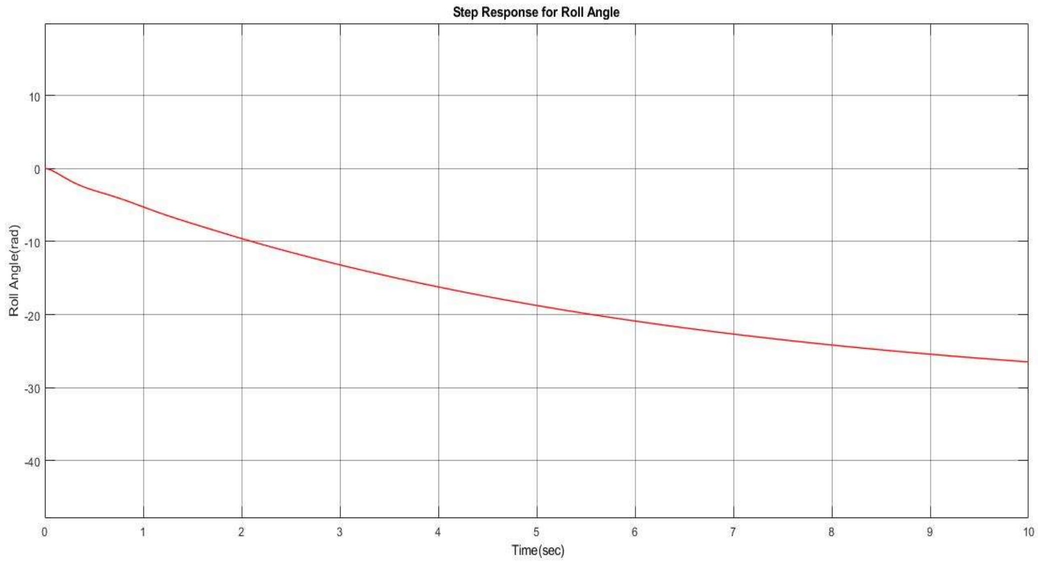


Figure 19 Step response for roll angle

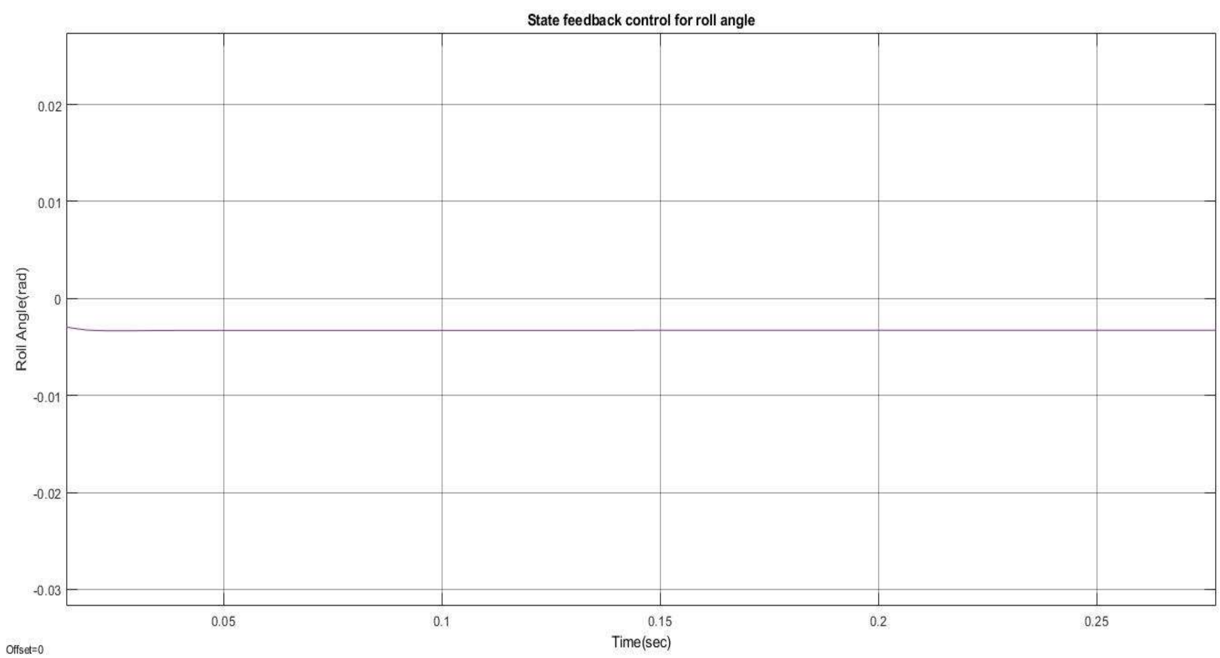


Figure 20 State feedback control for roll angle

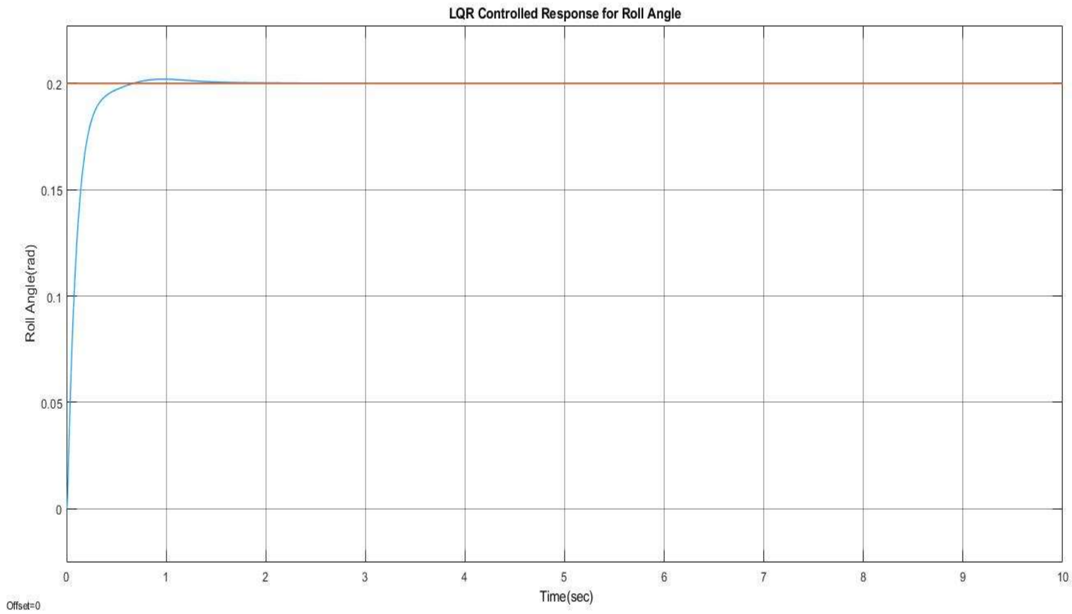


Figure 21 LQR controlled response for roll angle

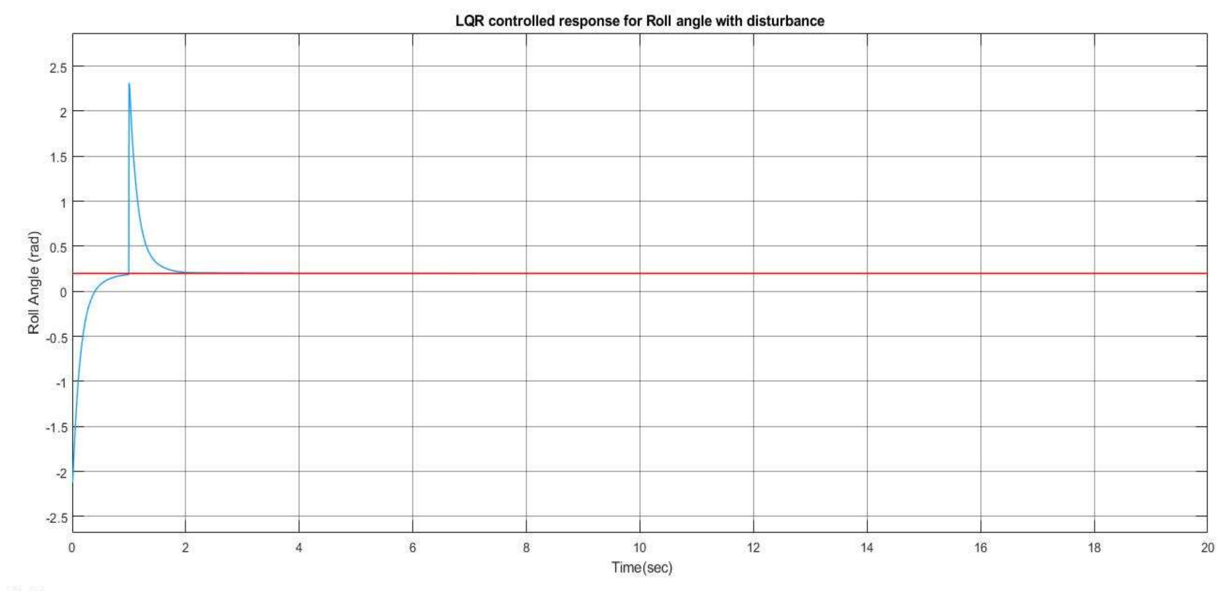


Figure 22 LQR controlled response for roll angle (with disturbance)

Chapter 6 Conclusion

In this project, we have successfully implemented state feedback control and LQR control techniques to achieve stable attitude control of an aircraft.Through extensive analysis and simulation, we have demonstrated the effectiveness of these control strategies in maintaining the desired aircraft attitude.By using state feedback technique, we were able to stabilize the system and ensure robust performance in the presence of disturbances and uncertainties.By using LQR and formulating an appropriate cost function that balances control effort and error, we obtained optimal control gains that minimized the response time and reduced the deviations from the desired attitude. We conducted comparative studies between the state feedback control and LQR control techniques. Both approaches yielded stable attitude control, but the LQR control approach showed superior performance in terms of minimizing control effort and reducing steady-state errors. These findings contribute to the advancement of aircraft control systems, and further research can focus on refining the control design, considering more complex aircraft dynamics.

References

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- Robust Control Systems (Theory and case studies) bye Uwe Mackenroth
- Spacecraft and aircraft dynamics by Matthew M. Peet Illinois University (http://control.asu.edu/Classes/MMAE441/Aircraft/441Lecture1.pdf)
- Modern Control System by Katsuhiko Ogata
- https://ieeexplore.ieee.org/document/8570323

MATLAB script for Longitudinal Dynamics

```
clear all;
close all;

% A and B matrices for Longitudinal Dynamics

AL=[-0.83705 1.7696 -0.35236 0; -5.9575 -21.766 0.0056738 0.8717; 0 0 0 1;
14.891 -47.637 -0.015802 -7.9269];
BL=[0 3.9397; -0.91092 0; 0 0; -30.902 -6.9048];
CL=[0 1 0 0;0 0 1 0];
DL=zeros(4,2);

%Eigen value of AL
disp('Eigen Values of AL are')
eig(AL)

%Controllability and Observability of a matrix
TL=ctrb(AL,BL);  %Controllability matrix

if rank(AL)==rank(TL)
    disp('System is Controllable')
else
    disp('System is not Controllable')
end

SL=obsv(AL,CL);  %Observability matrix

if rank(AL)==rank(SL)
    disp('System is Observable')
else
    disp('System is not Observable')
end
```

MATLAB script for Lateral Dynamics

```
clc;
clear all;
close all;

%A and B matrices for Lateral Dyamics

A=[-2.3817 0 -1.0019 2.1827; -21.063 -16.055 0.87229 0; 24.512 -16.651 -3.5379
0; 0 1.0026 -0.029766 0];
B=[0 -0.24719; -36.263 -688.44; -0.6725 -67.983; 0 0];
B1=[0.0869 0;4.424 1.184; 0 -1; -2.148 0.021]; %input vector for disturbance
C=[1 0 0 0; 0 0 0 1];
D=zeros(4,2);


%Eigen values of A
disp('Eigen Values of A are:');
eig(A)


%Controllability and Observability of a matrix
P=ctrb(A,B);  %Controllability matrix

if rank(A)==rank(P)
    disp('System is Controllable')
else
    disp('System is not Controllable')
end

S=obsv(A,C);  %Observability matrix

if rank(A)==rank(S)
    disp('System is Observable')
else
    disp('System is not Observable')
end


%LQR for augmented state feedback
A_hat=[A zeros(4,2);-C zeros(2,2)];
B_hat=[B;zeros(2,2)];

Q=[0.01 0 0 0 0 0; 0 0.001 0 0 0 0; 0 0 0.001 0 0 0; 0 0 0 70 0 0; 0 0 0 0 10
0;0 0 0 0 0 7000 ];
R=[.02 0; 0 .02];

[K_hat,P,CLP]=lqr(A_hat,B_hat,Q,R)

K=[K_hat(:,1) K_hat(:,2) K_hat(:,3) K_hat(:,4)];
ki=[K_hat(:,5) K_hat(:,6)];
```