Parameters Constraints on Λ CDM and XCDM using H(z)+BAO data

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Abstract

Here I reproduce the results of Cao and Ratra [1] for the spatially flat and non-flat ΛCDM and XCDM dark-energy models. Following [1], I first analyze the Old [2] and New [1] Hubble-parameter measurements, and then jointly use them with the Old [2] and New [1] BAO data to constrain the relevant cosmological parameters. The resulting parameter estimates are in close agreement with those reported in the original studies.

1 Data and Methodology

For the old H(z) and old BAO data, I use Table-I and II of [2] and for the new data, I use Table-I(accounting for correlations) and II of [1]. First, I implemented the χ^2 function and minimized it using `scipy.minimize`. This yielded the minimized value $\chi^2_{\rm min}$ and the best-fit estimates of the free parameters Ω_{k0}, w_x, $\Omega_b h^2$, $\Omega_c h^2$, and H_0, consistent with those used in the reference paper. From these free parameters one can also compute the best-fit value of the derived matter-density parameter $\Omega_m = \frac{\Omega_b h^2+\Omega_c h^2+\Omega_\nu h^2}{h^2}$ where I use the neutrino-density value $\Omega_\nu h^2 = 0.06/93.14$ as given in the paper.

I then ran `emcee` to obtain Markov chains for these free parameters. For all parameters I adopted the same priors as used in the paper. The MCMC simulations were run with 200 walkers for 5000-10,000 steps varying accross different models.

2 Comparison of Results

2.1 Plots

Below I report the plots I obtained, at first using only H(z) data and then combining with the BAO data and performing a joint analysis. I compare them with the original plots in [1].

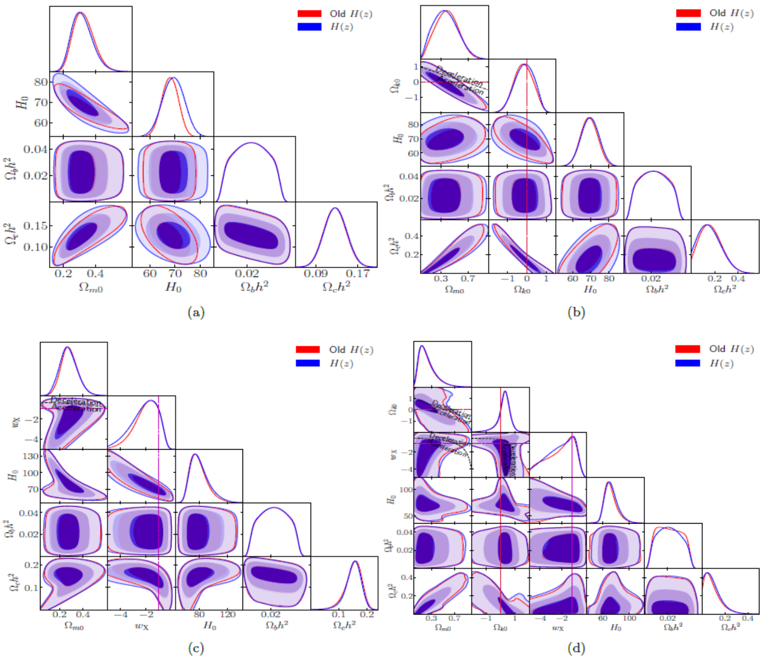
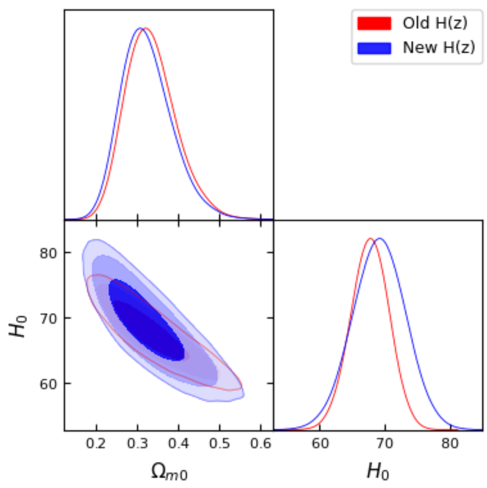
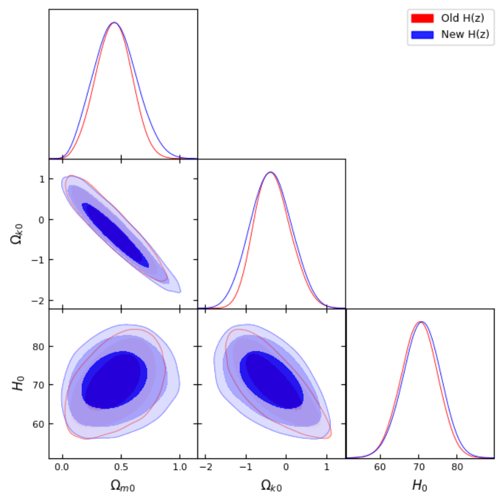


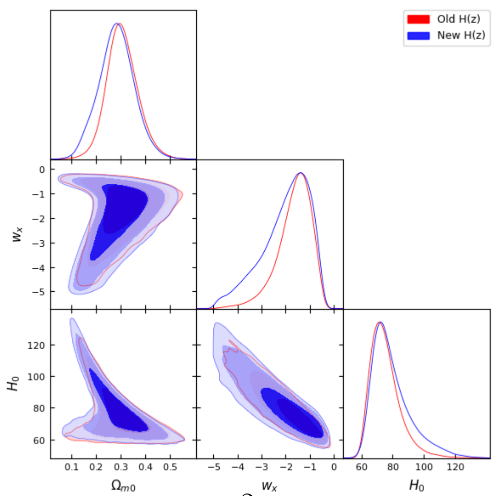
Figure 1: Original Plots in Cao and Ratra[1]. a) Flat ΛCDM, b) Non-Flat ΛCDM, c) Flat XCDM, d) Non-Flat XCDM



(a) Flat ΛCDM

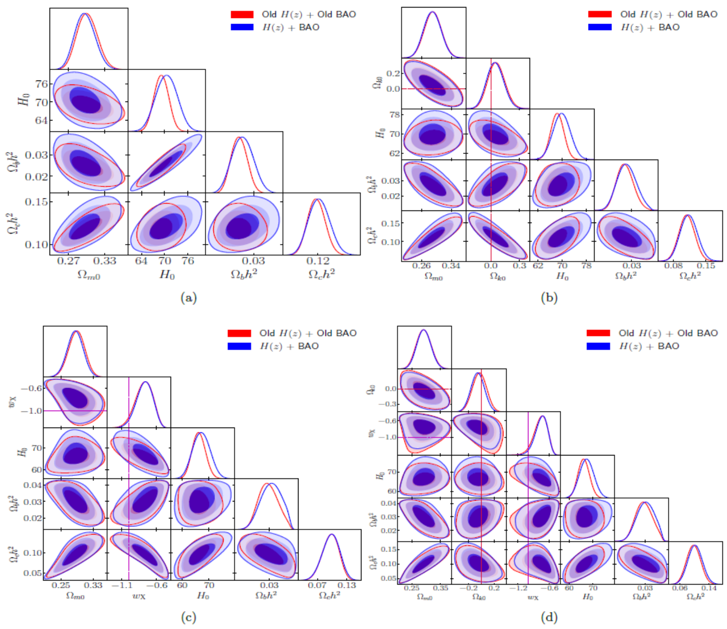


(b) Non-Flat ΛCDM

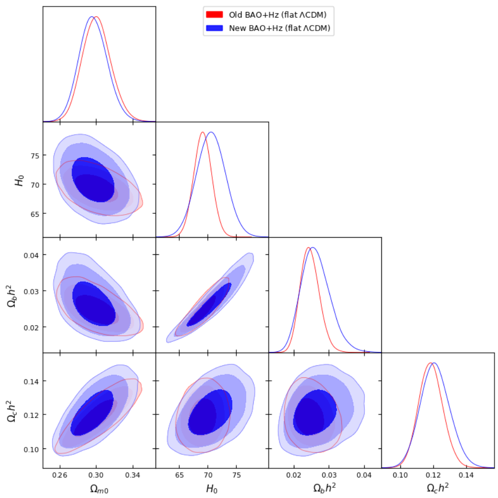


(c) Flat XCDM

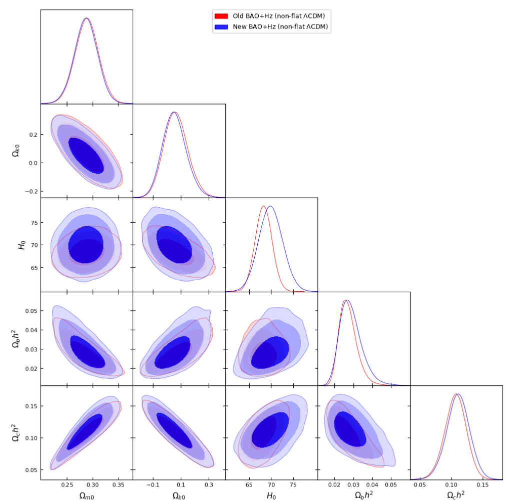
Figure 2: Plots generated using emcee and scipy.



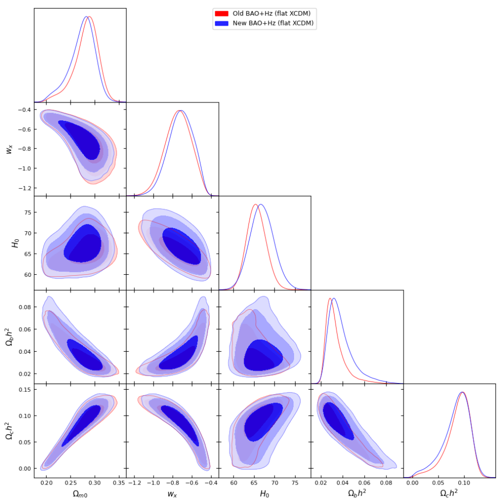
Original Plots in Cao and Ratra [1]. a) Flat ΛCDM, b) Non-Flat ΛCDM, c) Flat XCDM, d) Non-Flat XCDM



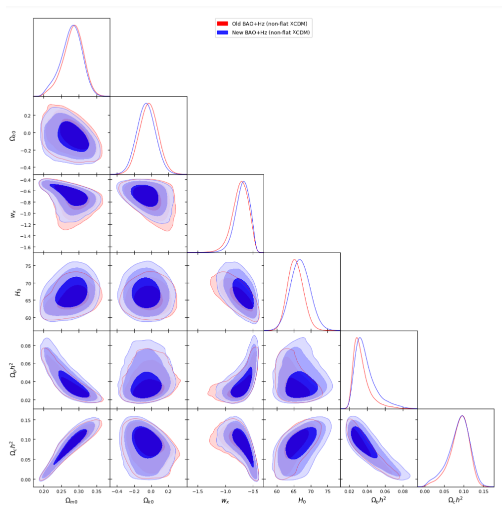
(a) Flat ΛCDM



(b) Non-Flat ΛCDM



(c) Flat XCDM



(d) Non-Flat XCDM

Figure 3: Plots generated using `emcee` and `scipy`.

Table 1: Comparison of Best-Fitting Cosmological Parameters: My Results vs. Cao and Ratra (2023)[1]

Model	**Dataset**	**My Results**						**Cao and Ratra [Source: 528]**					
		$\Omega_b h^2$	$\Omega_c h^2$	Ω_{m0}	Ω_{k0}	w_X	H_0	$\Omega_b h^2$	$\Omega_c h^2$	Ω_{m0}	Ω_{k0}	w_X	H_0
Flat ΛCDM	Old $H(z)$	–	–	0.3197	–	–	68.116	0.0273	0.1201	0.319	–	–	68.16
	$H(z)$	–	–	0.3097	–	–	69.428	0.0244	0.1181	0.301	–	–	69.43
	Old $H(z)$+Old BAO	0.024	0.118	0.299	–	–	68.953	0.0254	0.1200	0.309	–	–	68.98
	$H(z)$+BAO	0.025	0.120	0.295	–	–	70.134	0.0260	0.1200	0.297	–	–	70.12
Non-flat ΛCDM	Old $H(z)$	–	–	0.354	-0.11	–	68.93	0.0205	0.1515	0.362	-0.136	–	69.09
	$H(z)$	–	–	0.315	-0.014	–	69.5	0.0180	0.1328	0.314	-0.012	–	69.47
	Old $H(z)$+Old BAO	0.026	0.110	0.291	0.048	–	68.329	0.0260	0.1098	0.292	0.048	–	68.35
	$H(z)$+BAO	0.027	0.113	0.289	0.038	–	69.635	0.0269	0.1128	0.289	0.041	–	69.61
Flat XCDM	Old $H(z)$	–	–	0.3226	–	-1.185	70.241	0.0376	0.1236	0.321	–	-1.261	70.95
	$H(z)$	–	–	0.3158	–	-1.133	70.569	0.0106	0.1464	0.316	–	-1.140	70.63
	Old $H(z)$+Old BAO	0.030	0.094	0.288	–	-0.755	65.615	0.0951	0.0296	0.290	–	-0.754	65.79
	$H(z)$+BAO	0.033	0.092	0.281	–	-0.731	66.686	0.0938	0.0318	0.283	–	-0.734	66.67
Non-flat XCDM	Old $H(z)$	–	–	–	–	–	–	0.0223	0.0736	0.172	0.324	-2.272	75.05
	$H(z)$	–	–	–	–	–	–	0.0316	0.0530	0.378	0.151	-2.278	75.06
	Old $H(z)$+Old BAO	0.029	0.098	0.294	-0.054	-0.730	65.682	0.0289	0.0985	0.296	-0.053	-0.730	65.76
	$H(z)$+BAO	0.032	0.098	0.291	-0.081	-0.699	66.859	0.0305	0.0998	0.293	-0.084	-0.703	66.79

Table 2: Comparison of One-dimensional posterior mean parameter values and uncertainties: My Results vs. Cao and Ratra (2023)[1]

Model	**Dataset**	**My Results**						**Cao and Ratra [Source: 539]**					
		$\Omega_b h^2$	$\Omega_c h^2$	Ω_{m0}	Ω_{k0}	w_X	H_0	$\Omega_b h^2$	$\Omega_c h^2$	Ω_{m0}	Ω_{k0}	w_X	H_0
Flat ΛCDM	Old $H(z)$	–	–	$0.334^{+0.050}_{-0.070}$	–	–	67.7 ± 3.0	0.0225 ± 0.0108	0.1264 ± 0.0207	$0.328^{+0.052}_{-0.053}$	–	–	67.98 ± 3.24
	$H(z)$	–	–	$0.324^{+0.049}_{-0.071}$	–	–	69.1 ± 4.1	0.0225 ± 0.0107	0.1275 ± 0.0208	0.319 ± 0.050	–	–	69.31 ± 4.25
	Old $H(z)$+Old BAO	$0.0246^{+0.0024}_{-0.0029}$	0.1184 ± 0.0073	$0.301^{+0.014}_{-0.016}$	–	–	69.1 ± 1.6	0.0247 ± 0.0030	$0.1186^{+0.0076}_{-0.0078}$	$0.301^{+0.016}_{-0.018}$	–	–	69.14 ± 1.85
	$H(z)$+BAO	$0.0263^{+0.0035}_{-0.0045}$	$0.1211^{+0.0082}_{-0.0092}$	$0.297^{+0.015}_{-0.016}$	–	–	70.6 ± 2.5	0.0260 ± 0.0040	$0.1212^{+0.0091}_{-0.0101}$	$0.297^{+0.015}_{-0.016}$	–	–	70.49 ± 2.74
Non-flat ΛCDM	Old $H(z)$	–	–	0.44 ± 0.15	$-0.32^{+0.40}_{-0.49}$	–	70.3 ± 4.7	$0.0223^{+0.0109}_{-0.0108}$	$0.1685^{+0.0736}_{-0.1130}$	$0.390^{+0.167}_{-0.172}$	$-0.174^{+0.501}_{-0.491}$	–	$69.09^{+4.70}_{-4.67}$
	$H(z)$	–	–	$0.45^{+0.17}_{-0.19}$	-0.37 ± 0.50	–	70.9 ± 4.9	0.0222 ± 0.0108	$0.1612^{+0.0691}_{-0.1064}$	$0.374^{+0.151}_{-0.210}$	$-0.136^{+0.564}_{-0.457}$	–	$69.56^{+4.89}_{-4.88}$
	Old $H(z)$+Old BAO	$0.0274^{+0.0034}_{-0.0054}$	0.107 ± 0.017	0.289 ± 0.023	$0.064^{+0.079}_{-0.091}$	–	68.3 ± 1.9	$0.0266^{+0.0039}_{-0.0045}$	0.1088 ± 0.0166	0.291 ± 0.023	$0.059^{+0.081}_{-0.091}$	–	68.37 ± 2.10
	$H(z)$+BAO	$0.0288^{+0.0042}_{-0.0067}$	0.111 ± 0.018	0.287 ± 0.022	$0.054^{+0.077}_{-0.090}$	–	69.9 ± 2.7	$0.0275^{+0.0046}_{-0.0051}$	$0.1131^{+0.0180}_{-0.0204}$	0.289 ± 0.023	$0.047^{+0.082}_{-0.093}$	–	69.81 ± 2.80
Flat XCDM	Old $H(z)$	–	–	$0.306^{+0.057}_{-0.066}$	–	$-1.64^{+0.84}_{-0.43}$	$75.4^{+5.4}_{-11}$	0.0225 ± 0.0107	$0.1505^{+0.0337}_{-0.0617}$	$0.285^{+0.061}_{-0.117}$	–	$-1.972^{+1.164}_{-?}$	$79.55^{+7.01}_{-15.05}$
	$H(z)$	–	–	0.283 ± 0.073	–	$-1.98^{+1.3}_{-0.53}$	$79.4^{+5.9}_{-15}$	0.0225 ± 0.0108	$0.1505^{+0.0303}_{-0.0200}$	$0.278^{+0.065}_{-0.081}$	–	$-2.127^{+1.335}_{-?}$	80.96 ± 16.10
	Old $H(z)$+Old BAO	$0.0337^{+0.0032}_{-0.010}$	$0.087^{+0.028}_{-0.014}$	$0.281^{+0.029}_{-0.017}$	–	$-0.74^{+0.14}_{-0.13}$	$65.6^{+2.1}_{-2.5}$	$0.0308^{+0.0038}_{-0.0046}$	$0.0969^{+0.0178}_{-0.0174}$	0.289 ± 0.020	–	$-0.784^{+0.140}_{-0.107}$	$66.22^{+2.23}_{-2.54}$
	$H(z)$+BAO	$0.0382^{+0.0050}_{-0.013}$	$0.085^{+0.031}_{-0.016}$	$0.274^{+0.030}_{-0.019}$	–	$-0.72^{+0.15}_{-0.12}$	67.0 ± 2.9	$0.0303^{+0.0039}_{-0.0047}$	$0.0978^{+0.0181}_{-0.0175}$	0.285 ± 0.019	–	$-0.776^{+0.130}_{-0.103}$	$67.18^{+2.23}_{-2.58}$
Non-flat XCDM	Old $H(z)$	–	–	–	–	–	–	$0.0218^{+0.0072}_{-0.0083}$	0.0927 ± 0.0217	$0.228^{+0.055}_{-0.175}$	$0.241^{+0.451}_{-0.261}$	$-2.148^{+1.682}_{-0.776}$	$71.98^{+5.86}_{-?}$
	$H(z)$	–	–	–	–	–	–	0.0218 ± 0.0093	0.0927 ± 0.0217	$0.228^{+0.054}_{-0.175}$	$0.228^{+0.456}_{-0.267}$	$-2.149^{+1.673}_{-0.772}$	73.06 ± 6.61
	Old $H(z)$+Old BAO	$0.0337^{+0.0038}_{-0.011}$	$0.088^{+0.029}_{-0.018}$	$0.283^{+0.032}_{-0.025}$	-0.03 ± 0.11	$-0.73^{+0.17}_{-0.11}$	$65.6^{+2.1}_{-2.5}$	$0.0294^{+0.0047}_{-0.0045}$	0.0980 ± 0.0186	0.292 ± 0.025	-0.027 ± 0.109	$-0.770^{+0.138}_{-0.098}$	$66.13^{+2.35}_{-2.26}$
	$H(z)$+BAO	$0.0382^{+0.0053}_{-0.014}$	$0.087^{+0.033}_{-0.020}$	$0.278^{+0.034}_{-0.025}$	-0.06 ± 0.11	$-0.69^{+0.15}_{-0.10}$	67.0 ± 2.9	$0.0303^{+0.0048}_{-0.0051}$	0.1021 ± 0.0193	0.292 ± 0.024	-0.054 ± 0.103	$-0.757^{+0.135}_{-0.093}$	67.33 ± 2.96

References

[1] Shulei Cao, Bharat Ratra, "$H_0 = 69.8 \pm 1.3$ km s^{-1}Mpc^{-1}, $\Omega_{m_0} = 0.288 \pm 0.017$, and other constraints from lower-redshift, non-CMB, expansion-rate data", arXiv:2302.14203 [astro-ph.CO].

[2] Shulei Cao, Bharat Ratra, "Using lower-redshift, non-CMB, data to constrain the Hubble constant and other cosmological parameters", arXiv:2203.10825 [astro-ph.CO].