Assignment 0 - QIQC

May, 2024

Here is the assignment for the first week of the course ! For the theory questions , you can either write them down, or LaTeX them. For the coding questions, you can submit either a Python file or a Jupyter Notebook (ipynb file)

The instructions to submit the assignment will be given shortly :)

1. We're going to give you some matrices, and your job is to find their eigenvalues (no calculator , and write down how you find them, because that is what we want to know)

   (a) $\begin{bmatrix} 0 & 5 & 0 & 4 \\ 5 & 0 & 4 & 0 \\ 0 & 3 & 0 & 2 \\ 3 & 0 & 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 5 & 4 \\ 0 & 0 & 3 & 2 \\ 5 & 4 & 0 & 0 \\ 3 & 2 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 25 & 20 & 20 & 16 \\ 15 & 10 & 12 & 8 \\ 15 & 12 & 10 & 8 \\ 9 & 6 & 6 & 4 \end{bmatrix}$

2. Suppose that O is a (orthogonally) diagonalizable operator with eigenvalues ± 1 Show that

   $$P_{\pm 1} = \frac{I \pm O}{2}$$

   where P_λ is the projector on the eigenspace of O associated with eigenvalue λ. (Even though this is an exercise in the reference, it's pretty nice and relevant, so here you go)

3. Let V be a *finite* dimensional inner product space, and let A be an operator on V. The following are equivalent:

   Prove that an operator that is norm preserving (for every vector $x \in V$, $|Ax| = |x|$ is equivalent to a unitary operator.

   Hint : Try using the fact that the below condition is also equivalent to the above two :

   $$\langle Ax|Ay\rangle = \langle x|y\rangle$$

   for all $x, y \in V$

4. Write code (using qiskit) to plot the following states on the Bloch Sphere

   $$|0\rangle\,, |1\rangle\,, \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

   Justify their directions, and do you notice some similarities among the first two and the last two?

5. Make a Quantum Circuit (code it up in qiskit) that prepares the state $\frac{|000\rangle+|111\rangle}{\sqrt{2}}$ from $|000\rangle$ and only with basic gates such as H, X and CNOT gates.

   Hint : First prepare the state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ and try to generalise it

   Can you generalise this further to prepare the state $\frac{|0...0\rangle+|1...1\rangle}{\sqrt{2}}$?