SOC Assignment 3

Sumit Kumar Adhya

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Q.1.

Ref:- `https://quantumcomputing.stackexchange.com/a/12343`
According to this reference, the number of independent stabilizer generators is n-k and the total number of stabilizers is 2^{n-k}, as if $P_1, P_2 \in S$ then $P_1 P_2 \in S$ as well.

Q.2.

The constraints required for operators to form a stabilizer group are:

1. They must belong to the pauli group $P_i \in G_n$, where G_n is the pauli group over n qubits.
2. They must stabilize the logical states, i.e. $P_i|\psi_L\rangle = (+1)|\psi_L\rangle$.
3. The stabilizers must commute with one another, i.e. $[P_i, P_j] = 0 \ \forall \ i, \ j$.

Q.3.

The types of errors stabilizer codes are designed to handle are:

1. **Single-Qubit Errors:** These are the bread and butter of stabilizer codes. They encompass:
   - **Bit-flip errors:** Where a qubit's state flips from $|0\rangle$ to $|1\rangle$ or vice versa.
   - **Phase-flip errors:** Where the qubit's phase is shifted by 180 degrees.
   - **Arbitrary single-qubit errors:** A combination of bit-flip and phase-flip errors, representing any general single-qubit unitary operation.
2. **Multiple-Qubit Errors:** While stabilizer codes are primarily designed for single-qubit errors, some codes can detect or even correct a limited number of multiple-qubit errors. However, their efficiency in handling these errors is generally lower compared to single-qubit errors.

Q.5.

The 3-qubit code can correct single-qubit bit-flip errors (X errors) and detect but can't correct single-qubit phase-flip errors (Z errors). It works by encoding one logical qubit into three physical qubits.
The discretization theorem states that any arbitrary error on a qubit can be decomposed into a combination of bit-flip (X), phase-flip (Z), and combined bit-and-phase-flip (Y) errors.
Since arbitrary errors can include phase-flip and combined errors, the 3-qubit code cannot correct these types of errors.
Moreover, a 3-qubit system has 8 possible states (2^3). To correct arbitrary errors, we need to distinguish between the original state and all possible error states. Single-qubit errors (X, Y or Z on each qubit) already account for 9 possible error states (3 errors per qubit), there's no room for the original state, so therefore, it's not possible.

Q.6.

Ref:- `https://quantumcomputing.stackexchange.com/a/4799`
In a non-degenerate code we have two logical states of the qubit and the errors map each logical state into distinct states. For an n-qubit code the set of all possible single qubit errors are:
$\{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n, Z_1, Z_2, \ldots, Z_n\}$ It means that all the states: $\{|0_L\rangle, |1_L\rangle, X_1|0_L\rangle, X_1|1_L\rangle\}$ must map to orthogonal states. So we need $2 + 2 \times (3n)$ distinct states. But for n qubits maximum number of states are 2^n. So,

$$2^n \geq 2 + 2 \times (3n)$$

. This is called the Quantum Hamming Bound. It can easily be seen that this holds true for $n \geq 5$. So the minimum number of qubits needed to correct any single qubit error is 5.