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Relaxation rates using Binary Collision in Plasma Simulation

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Abstract

In standard particle simulation of plasmas, collisions are often dis-regarded assuming plasmas as collisionless. Under various conditions collisions among plasma particles play an important role.Therefore in this project a binary collision model for plasma particle simulation is re-examined using Monte-Carlo Method. The model conserves the number of particles, the total momentum and the total energy quasi-locally. Collision effects in spatially homogeneous plasmas are simulated.The results of measurement on various relaxation rates in velocity space are shown to agree well with test particle theory.

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1 Introduction

Computer simulations of plasmas with particle codes play important roles in the study of transport phenomena, heating process, and so on. In these simulations, the number of particles is far smaller than that in the real situations, because of the limitation of speed and capacity of the computer[1]. Secondly collisions are often discounted in standerd particle simulation of plasmas. Therefore to take into the effect of collisions, particle codes must be supplimented by collision models.

It is appropriate to introduce a method for adding the effects of binary collisions into the finite-size particle model[2], when collision effects in a plasma are studied. However, the short range Coulombic collisions are not always negligible in low-frequency simulation, and it is important to include collisional dissipation. Also, it is often desirable to have a more complete description of fusion plasmas that can cover a wide range of phenomena in both collisional and collisionless regimes.

In plasma particle simulation, the collision rate, as there are insufficient number of simulation particles, is greatly reduced by assigning the particles a finite-size shape[3], which allows us the study of collective properties of plasmas.
Collisions in a fully ionized plasma are generally thought of small-range, binary collisions, described by the Landau collision term, where the number of particles, the total momentum and the total energy are conserved locally during such collisions.

In dealing with collisions in plasma let, electrons are moving through ionized gas in plasma (homogeneous). Then electrons are making collisions with the ions. After colliding electrons will scatter in various directions. So, scattering angle will be random. If the scattering angle be random, then the ralative velocity between electrons and ions will be random. After calculating the ralative velocity we will calculate the velocity component of electrons and ions seperately.

In this report we have re-examined the model by **Takizuka** and **Abe** [Journal of Computational Physics 25, 205-219 (1977)] using Monte-Carlo collisions and verified with test particle theory.

2 Binary Collision Model

There are mainly two types of interactions between charged particles in a fully ionized plasma: Long-range collective interaction and short-range,small angle binary collisios. The reference scale to distinguish between these two types of interactions is a Debye length. In plasma particle simulations, they are usually treated in two separate, uncoupled processes, described by VlasovMaxwell equations and the Landau collision term, respectively. We have considered the velocity distribution of ion is Maxwellian type and electrons are coming in the form of a beam. The collision part, which mainly gives rise to velocity scattering, is then performed.

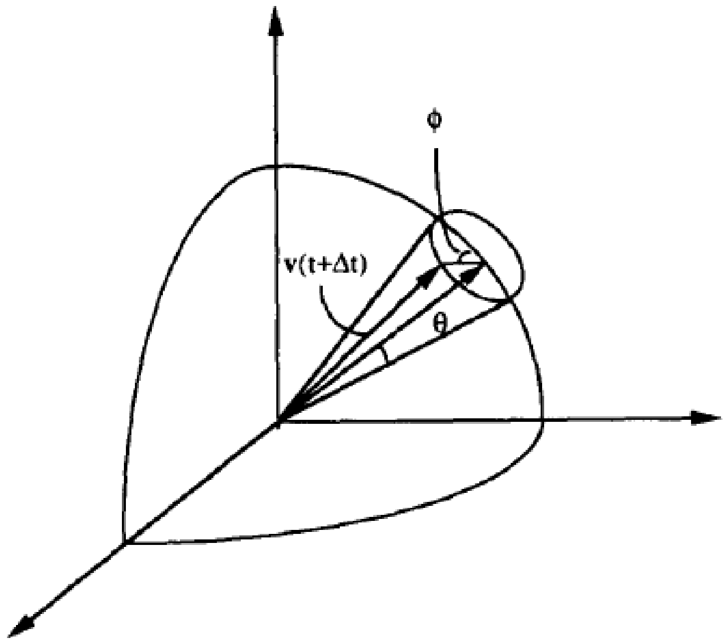

Figure 1: Diagram showing velocity space scattering. ϕ is the polar angle $(0,2\pi)$, and θ is the scattering angle, obeys Gaussian distribution

1. The simulation system is divided into anumber of spatial cells with a size such that plasma properties across each cell do not vary substantially. A typical size is the Debye length of the system. For simplicity we consider only one cell.

2. Particles in each cell are paired in a random way. In general, for a two species plasma, there may be three kinds of pairs: electron and ion, electron and electron and ion and ion.

3. Small-angle collisions are performed pair-wise.

The details of step 3 are as follows.We have denoted the velocity, mass, charge and charge density of two collision partners by v_α and v_β,m_α, and m_β, e_α, and e_β, n_α, and n_β, respectively.In the centre of mass frame(CM) system of the two partners, a collision between them may be regarded as a rotation of the relative velocity vector v=v_α-v_β through two angles, θ and ϕ, from v(t)$\rightarrow$ v(t+$\triangle$t).
Here θ and ϕ are given by

$$\theta = \sqrt{-2\nu_{\alpha\beta} \triangle t \ln(1 - R_1))} \tag{1}$$

$$\phi = 2\pi R_2 \tag{2}$$

where R_1 and R_2 are uniformly random in the interval (0,1) and

$$\nu_{\alpha\beta} = \nu_{\alpha\beta 0}\frac{3n\sqrt{\pi\upsilon_0^3/2}}{n_0\upsilon^3}, \tag{3}$$

$$\nu_{\alpha\beta 0} = \frac{n_0 e_\alpha^2 e_\beta^2 ln\lambda}{3(2\pi)^{3/2}\varepsilon_0^2 m_{\alpha\beta}^2 \upsilon_0^3} \tag{4}$$

Here $m_{\alpha\beta} = \dfrac{m_\alpha m_\beta}{m_\alpha + m_\beta}$, n=min(n_α ,n_β),n_0,υ_0 are constants,used as the reference density and velocity,respectively, and $ln\lambda$ is Coulomb Logarithm.The new velocities of two particles are

$$\upsilon_\alpha(t+\triangle t) = \upsilon_\alpha(t) + \frac{m_\beta}{m_\alpha + m_\beta}(\upsilon(t+\triangle t) - \upsilon(t)) \tag{5}$$

$$\upsilon_\beta(t+\triangle t) = \upsilon_\beta(t) + \frac{m_\alpha}{m_\alpha + m_\beta}(\upsilon(t+\triangle t) - \upsilon(t)) \tag{6}$$

It is clear that the momentum and energy as well as the number of particles are not changed during collisions,and thus conserved pairwise.
So we can understand that $\nu_{\alpha\beta}$ characterizes an angular relaxation rate of particles α (or β), known as the test particles, due to collisions with a medium of particles β(or α), known as the field particles.

- **Relaxation Rates:**

Various relaxation rates can be calculated analytically by test particle theory, in which the test particle (denoted by α) is assumed to have no effects on the field medium (denoted by β), whose velocity distribution is a Maxwellian distribution with a temperature T_β. As a result of collision with the medium, the test particle slows down, and diffuses in velocity space. This process is known as Relaxation, and the important relaxation rates are the slowing-down rate ν_s, the transverse velocity diffusion rate $\nu_\perp$, the parallel velocity diffusion rate $\nu_\parallel$, and the energy loss rate ν_ε, as defined in the following equations:

$$\frac{d\upsilon_\alpha}{dt} = -\nu_s\upsilon_\alpha, \tag{7}$$

$$\frac{d}{dt}(\upsilon_\alpha - <\upsilon_\alpha>)_\perp^2 = \nu_\perp\upsilon_\alpha^2, \tag{8}$$

$$\frac{d}{dt}(\upsilon_\alpha - <\upsilon_\alpha>)_\parallel^2 = \nu_\parallel\upsilon_\alpha^2, \tag{9}$$

$$\frac{d\upsilon_\alpha^2}{dt} = -\upsilon_\varepsilon\upsilon_\alpha^2. \tag{10}$$

The expressions for these relaxation rates are:

$$\nu_s = \left(1 + \frac{m_\alpha}{m_\beta}\right)\mu(x)\nu_0\left(\frac{T_\beta}{T_\alpha}\right)^{3/2}, \tag{11}$$

$$\nu_\perp = 2\left(\left(1 - \frac{1}{2x}\right)\mu(x) + \frac{d\mu(x)}{dx}\right)\nu_0\left(\frac{T_\beta}{T_\alpha}\right)^{3/2}, \tag{12}$$

$$\nu_\parallel = \frac{\mu(x)}{x}\nu_0\left(\frac{T_\beta}{T_\alpha}\right)^{3/2}, \tag{13}$$

$$\nu_\varepsilon = 2\left(\frac{m_\alpha}{m_\beta}\mu(x) - \frac{d\mu(x)}{dx}\right)\nu_0\left(\frac{T_\beta}{T_\alpha}\right)^{3/2}. \tag{14}$$

where, $\varepsilon_\alpha = \dfrac{1}{2} m_\alpha v_\alpha^2$, and $x = \dfrac{m_\beta \varepsilon_\alpha}{m_\alpha T_\beta}$, and $\nu_0 = \frac{n_\beta e_\alpha^2 e_\beta^2 ln\lambda}{8\sqrt{2}\pi\epsilon_0^2 m_{\alpha\beta}^2 T_\beta^{3/2}}$ and $\mu(x) = (\frac{4}{\pi})^{1/2} \int_0^x e^{-\zeta} \zeta^{1/2} d\zeta$

In measuring these rates, the average over a group of test particles with the same initial velocity are taken to improve the statistics. We have used $m_\alpha/m_\beta = 0.01$, $T_\alpha/T_\beta = 1$, $\nu_0 \bigtriangleup t$=0.0001.

- **Thermal Relaxation:**

Now we have examined the case of thermal relaxation where the initial temperature of one species (say electrons) is different than that of another species (say ions).These species with higher temperature will lose energy and vice-varsa to come to a common equilibrium. The equilibration of the two temperatures is approximately expressed by

$$\frac{d}{dt}(T_i - T_e) = -2\nu_{eq}(T_i - T_e) \tag{15}$$

where $\nu_{eq} = \frac{8}{3}\sqrt{\pi}(1 + \frac{m_e T_i}{m_i T_e})^{-3/2}\nu_0$ and $\nu_0 = \dfrac{n_e e^4 ln\lambda}{8\pi\sqrt{2}\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$

A simulation was carried out to study this process using eq 15. We have taken the parameters $m_e/m_i = 0.01$, Te(0)/Ti(0)=2 in this case.Results are shown in Fig:6

Next we have considered the case where the initial distribution with a difference between the longitudinal and transverse temperatures($T_\parallel(0)$ and $T_\perp(0)$) relaxes to an isotropic Maxwellian. This process is known as **thermal isotropization**, which can be expressed by

$$\frac{d}{dt}(T_\parallel - T_\perp) = -\nu(T_\parallel - T_\perp) \tag{16}$$

The relaxation rate ν here is given as $\nu = (8/5\sqrt{2\pi})\nu_0$, where ν_0 is calculated using $T = 1/3(T_\parallel + 2T_\perp)$.Eq. 16 is solved and results are shown in Fig: 7.

3 Discussions and Results

I have verified, how the **relaxation rates** i.e. Slowing-down rate ν_s, the transverse velocity diffusion rate $\nu_\perp$, the parallel velocity diffusion rate $\nu_\|$, and the energy loss rate ν_ε changes with ε_α(Energy of test particles). For that I have plotted the ν_s/ν_0 vs $\varepsilon_\alpha/T_\beta$, and $\nu_\|/\nu_0$ vs $\varepsilon_\alpha/T_\beta$, and $\nu_\perp/\nu_0$ vs $\varepsilon_\alpha/T_\beta$, and ν_ε/ν_0 vs $\varepsilon_\alpha/T_\beta$ both analytically and numerically. The numerical results are in good agreement with the analytical ones.

The results are following-

- **Relaxation rates:**

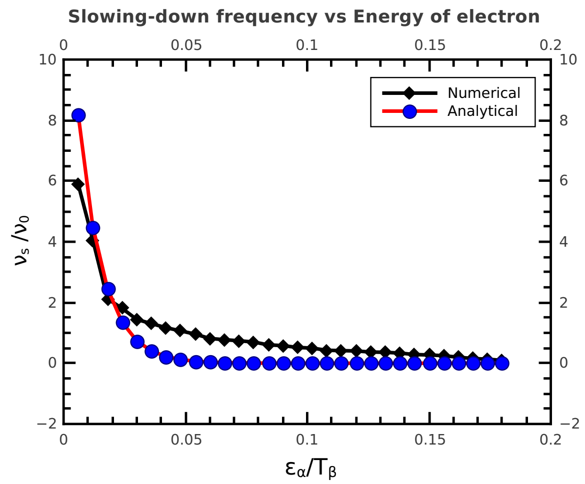

Figure 2: Slowing-down frequency rate vs Enery of electrons

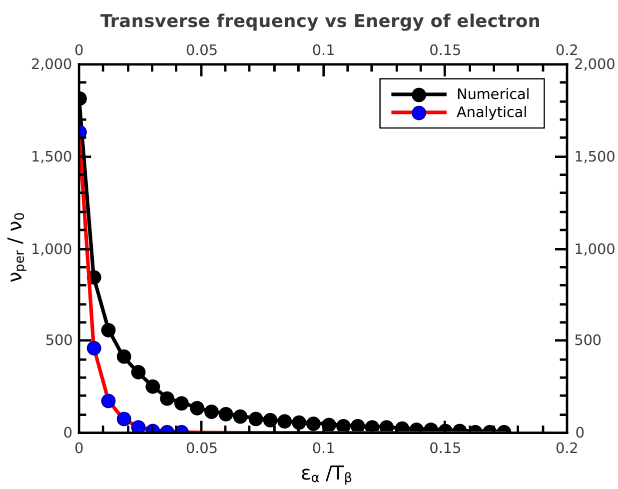

Figure 3: Transverse velocity diffusion rate vs energy of electrons

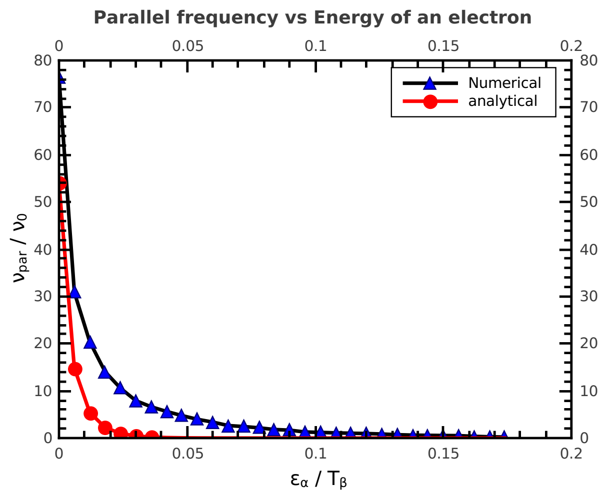

Figure 4: Parallel velocity diffusion rate vs energyof electrons

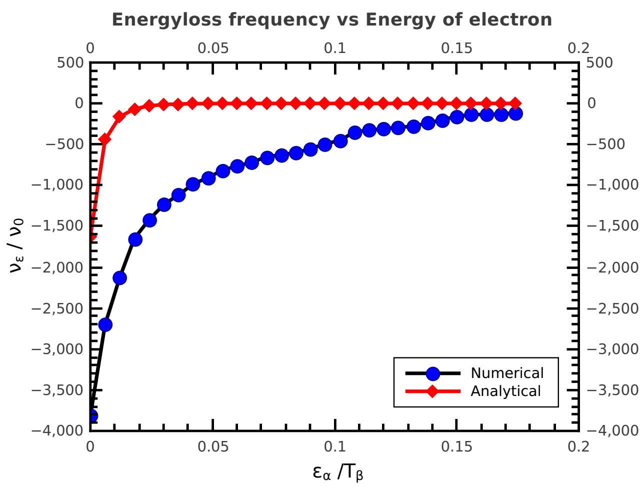

Figure 5: energyloss frequency rate vs energy of electrons

- **Thermal Relaxation:**

I have verified **thermal relaxation** rates also,and plotted $\dfrac{T_e - T_i}{T_e(0) - T_i(0)}$ vs $\nu_0 t$ and $\dfrac{T_\parallel - T_\perp}{T_\parallel(0) - T_\perp(0)}$ vs $\nu_0 t$. The numerical results are shown to agree well with the appropiate ones.

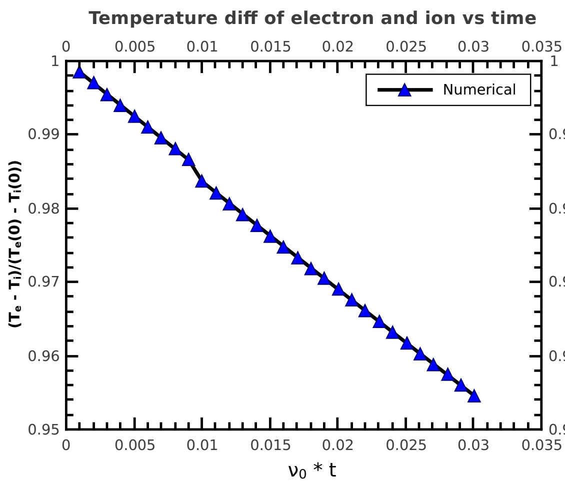

Figure 6: Difference of Electron and ion temperature showing equilibration vs time

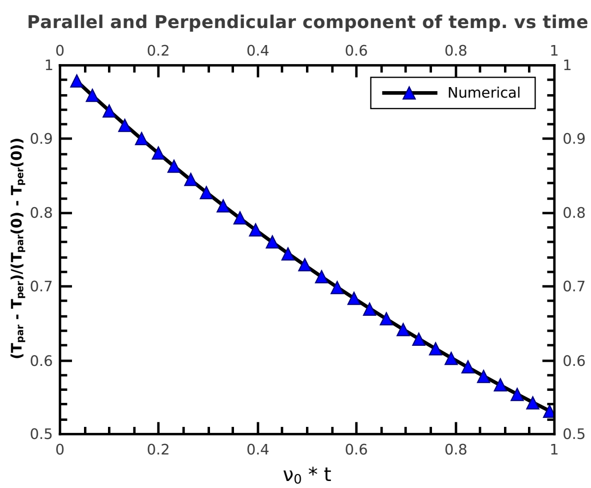

Figure 7: Difference between parallel and perpendicular temperature of electrons showing isotropization vs time

4 Conclusion

I have re-examined a binary collision model by **Takizuka** and **Abe** [JOURNAL OF COMPUTATIONAL PHYSICS 25, 205-219 (1977)] and **S.Ma et al.** [Computer Physics Communications 77 (1993) 190206], which is suitable for present-day vector computer, and tested it under various situations. The model closely describes the collisions in plasmas at the kinetic level including nonlinear effects.

For computational simplicity, I have considered only Coulombic force between the particles and thermal energy is completely responsible for the particle's motion. I have proposed that relative velocity between ion and electron is random, because it is more appropiate for the model. In deriving the collision model, I have not used the influence of magnetic field, because I have used the assumption that the particle's trajectory can be treated as Coulombic orbit without magnetic field. When the magnetic field is so strong that the cyclotron frequency is comparable to or larger than the plasma frequency, or the ratio of the electron Larmor radius to the electron Debye length is small, then some corrections are required.

5 Acknowledgement

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6 References

1. S. Ma, RD. Sydora and J.M. Dawson, Computer Physics Communications 77 (1993) 190206

2. Tomonori Takizuka and H. Abe, Journal of Computational Physics 25, 205-219 (1977)

3. K. Nanbu, Physical Review E Volume 55, Number 4 (APRIL 1997)

4. Y. Sentoku, A.J. Kemp, Journal of Computational Physics 227 (2008) 68466861

5. Bruce I. Cohen , Andris M. Dimits, David J. Strozzi, Journal of Computational Physics 234 (2013) 3343

6. David J. Larson, Journal of Computational Physics 188 (2003) 123138