

# HCF & LCM

## HCF / GCF(Highest/Greatest Common Factor)

- HCF of two or more numbers is the greatest / largest / highest/biggest number which can divide those two or more numbers exactly.

Factors of 6 : 1, 2, 3, 6

Factors of 8 : 1, 2, 4, 8

**Common 1 & 2 Highest & Common 2**

## • LCM(Least Common Multiple)

- The LCM of two or more numbers is the smallest / lowest / least number which is exactly divisible by those two or more numbers.

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, 54,...

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, 64....

**Common 24, 48, .... Lowest & common 24**



## HCF (Factorization method)

- HCF of 54,72,126 (factorization method)

A. 21      B. 18      C. 36      D. 54

**Ans : B**



## HCF (Factorization method)

- Eg. HCF for 136, 144, 168

2	136	144	168
2	68	72	84
2	34	36	42
	17	18	21

↓ NO FURTHER COMMON FACTOR

So HCF =  $2 \times 2 \times 2 = 8$

Note : HCF is always  $\leq$  the smallest of given numbers



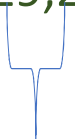
## HCF (Difference Method)

- Find HCF of 203,319

Keep smaller here



- (203, 319)
- (116, 203)
- (87, 116)
- (29, 87)
- (29, 58)
- (29, 29)



HCF = 29



## HCF (Difference Method) - (Assignment)

• HCF of 161,253 ( difference method)

A. 27      B. 18      C. 23      D. 17

**Ans : C**



# HCF

Q. Find HCF of 84,125

- (84,125)
  - (41,84)
  - (41,43)
  - (2,41)
  - (2,39)
- If nothing is common then  $HCF = 1$  and numbers are said to be co prime numbers.



## HCF & LCM

Q. Find the greatest number which can divide 284, 698 & 1618 leaving the same remainder 8 in each case?

- A. 36      B. 46      C. 56      D. 43.

Soln-

Remainder 8  $\rightarrow$  (numbers  $- 8$ ) would be exactly divisible.

$$\rightarrow 284 - 8 = 276$$

$$\rightarrow 698 - 8 = 690$$

$$\rightarrow 1618 - 8 = 1610$$

$\rightarrow$  Greatest number dividing above 3 = HCF(276, 690, 1610) (difference method)

$$\rightarrow \text{HCF} = 46$$

**Ans: B**



## HCF & LCM

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

- A. 35      B. 46      C. 56      D. 43.

**Soln:-**

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

$$\rightarrow 132 - 62 = 70$$

$$\rightarrow 237 - 132 = 105$$

$$\rightarrow 237 - 62 = 175$$

→ Greatest number dividing above 3 = HCF(70, 105, 175)

$$\rightarrow \text{HCF} = 35$$

**Ans: A**





## HCF & LCM

Q. Find the largest number such that 43,65,108 are divisible by that number and we get the remainder as 1,2,3 respectively in each case?

- A. 21                      B. 27                      C. 42                      D. 63

**Soln:**

→ (numbers – remainder) would be exactly divisible.

$$\rightarrow 43 - 1 = 42$$

$$\rightarrow 65 - 2 = 63$$

$$\rightarrow 108 - 3 = 105$$

$$\text{HCF}(42, 63, 105) = 21$$

**Ans : A**



## HCF & LCM

Q. A teacher has 25 books, 73 pens & 97 erasers. She wants to distribute them equally to maximum number of students so that after distribution she has equal number of books, pens & erasers left. What is the maximum number of students for such a distribution?

- A. 32                      B. 21                      C. 12                      D. 24

**Soln:-**

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

$$\rightarrow 73 - 25 = 48$$

$$\rightarrow 97 - 73 = 24$$

$$\rightarrow 97 - 25 = 72$$

→ Greatest number dividing above 3 = HCF(72, 48, 24)

$$\rightarrow \text{HCF} = 24$$

**Ans: D**



## HCF & LCM(Assignment)

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

- A. 35      B. 46      C. 56      D. 43.

**Ans : A**



## HCF & LCM(Assignment)

Q. Find largest number such that if 45,68 and 113 are divided by that number we get the remainder as 1,2 and 3 respectively.

- A. 21      B. 22      C. 26      D. 24

**Ans: B**



## HCF & LCM(Assignment)

Q. Find the greatest number which can divide 41, 131 & 77 leaving the same remainder in each case?

A. 28

B. 18

C. 36

D. 24

**Ans : B**



## LCM

Q. LCM for 12,24,20

A. 210

B. 180

C. 120

D. 144

**Ans : C**



## LCM

- Eg. LCM for 18, 28, 108, 105

2	18	28	108	105
2	9	14	54	105
3	9	7	27	105
3	3	7	9	35
3	1	7	3	35
5	1	7	1	35
7	1	7	1	7
Till all quotients are 1	1	1	1	1

So LCM =  $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

Note : LCM is always  $\geq$  the greatest of given nos



## LCM(Assignment)

Q. LCM for 12,24,20

A. 210

B. 180

C. 120

D. 144

**Ans : C**





## LCM (Assignment)

Q. Find LCM of 72,125

A. 9000      B. 1200      C. 1000      D. 800

**Ans : A**



## LCM (Assignment)

Find the LCM of 12, 18, and 27

A. 900

B. 120

C. 108

D. 820

**Ans : C**



## Rules to Remember

- Product of two given numbers is equal to the product of their HCF & LCM

$$A \times B = \text{HCF}(A,B) \times \text{LCM}(A,B)$$

- If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (\text{LCM of } a, b, c) + r \quad \text{e.g } 3,4,5 \text{ \& rem } 1$$



# LCM

Q. Find LCM of 147 & 231

Soln:-

- As we know,
- **HCF X LCM = product**
- Find HCF by difference method
- Put in the formula,
- $21 \times \text{LCM} = (147 \times 231)$
- 1617



# LCM

Q. Find LCM of 84 and 125

**Soln:-**

- As they are co-prime numbers the product is the LCM because  $HCF = 1$  (for co-primes)
- $HCF \times LCM = \text{product}$
- $1 \times LCM = 84 \times 125$
- $LCM = 10500$



# LCM

Q. Find the least number which when divided by 12,15,24 leaves a remainder of 5 in each case

• **Soln:**

• Find  $\text{LCM}(12,15,24) = ?$

If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (\text{LCM of a, b, c}) + r \quad \text{e.g 3,4,5 \& rem 1}$$

•  $\text{LCM} = 120$

• In an LCM problem, if remainder is common then,

**Result = LCM + common remainder**

$$= 120 + 5 = 125$$



## LCM

Q. Find the smallest number which when divided by 20,36,45 leaves a remainder 15,31 and 40 respectively.

- **Soln:**
- Find LCM(20,36,45)
- In LCM problem , if difference is common(constant) then,
- **Result = LCM – Common difference**

• 20	36	45	} 5
• 15	31	40	

- Result =  $180 - 5$   
= 175



## LCM

Q. Four numbers are in the ratio of 10: 12 : 15 : 18. If their HCF is 3, then find their LCM.

A. 420

B. 540

C. 620

D. 680

**Ans : B**





## LCM

Q. Find the least number which when divided by 5,6,7 and 8 leaves a reminder of 3 but when divided by 9 leaves no remainder.

A. 1677

B. 2523

C. 3363

D. 1683

**Ans: D**



## HCF/LCM with Decimal point

- Find HCF of 1.08, 0.36 and 0.9

- **Soln:**

1. Convert each of the decimals into like decimals.

1.08, 0.36 and 0.90

2. Write each number without decimal point.

$\text{HCF}(108, 36, 90) = 18$

3. Put decimal point after the numbers which are in like decimals.

Here it is after 2 numbers(digits)

**$\text{HCF}(1.\underline{08}, 0.\underline{36} \text{ and } 0.\underline{90}) = 0.\underline{18}$**



## Rules to Remember

- **Fractions :**

**LCM = LCM of Numerators / HCF of Denominators**

**HCF = HCF of Numerators / LCM of Denominators**

LCM of 25/12 & 35/18

LCM = 175/6

HCF of 25/12 & 35/18

HCF = 5/36



## LCM(Assignment)

Q. What is the least number which when divided by 18, 24 and 36 leaves 3 as a remainder in each case?

- A. 75
- B. 93
- C. 111
- D. 99
- E. None of the these

**Ans: A**



## HCF(Assignment)

Q. In a school of 437 boys & 342 girls it was decided to divide the girls & boys into separate classes. However it was required that each class consist of the same number of students. What would be the number of classrooms required?

A. 41 classrooms    B. 14 classrooms    C. 17 classrooms    D. 26 classrooms

**Ans : A**

Same Class Size = HCF (Boys, Girls)

$$\rightarrow \text{HCF}(437, 342) = 19$$

$$\rightarrow \text{Boys Classes} = 437/19 = 23$$

$$\rightarrow \text{Girls Classes} = 342/19 = 18$$

$$\rightarrow \text{Total Classes} = 23 + 18 = 41$$



## LCM(Assignment)

Q. Find the least number which when divided by 12,15,40 leaves a remainder of 5 in each case

- A. 120      B. 125      C. 130      D. 140

**Ans : B**



## LCM(Assignment)

Q. If the product of two numbers is 324 and their HCF is 3, then their LCM will be = ?

A. 972      B. 327      C. 321      D. 108

**Ans: D**



## LCM(Assignment)

Q. Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

- A. 40                      B. 80                      C. 120                      D. 200

**Ans: A**





## LCM(Assignment)

Q. Find the least number which when divided by 16,18,20 and 25 leaves a reminder of 4 but when divided by 7 leaves no remainder.

A. 17004

B. 18000

C. 18002

D. 18004

**Ans: D**



## HCF & LCM(Assignment)

Q. The HCF of two numbers is 8. Which one of the following can never be their LCM ?

- A. 24                      B. 48                      C. 56                      D. 60

**Ans: D**

If  $HCF = 8$  then LCM should have a factor of 8

Going by options 60 does not have a factor 8. So never be their LCM.



## HCF & LCM(Assignment)

Q. The LCM of three different numbers is 120. Which of the following cannot be their HCF?

A. 8

B. 12

C. 24

D. 35

**Ans: D**



## HCF & LCM(Assignment)

Q. HCF of 204,1190,1445

A. 17

B. 18

C. 19

D. 21

**Ans: A**



## HCF & LCM(Assignment)

Q. LCM of 22,54,108,135 and 198 is -

- A. 330
- B. 1980
- C. 5940
- D. 11880

**Ans: C**



## HCF & LCM(Assignment)

Q. Find HCF of 36 and 84

- A. 4
- B. 6
- C. 12
- D. 18

**Ans: C**



## Numbers(Assignment)

Q. The number nearest to 43582 divisible by each of 25, 50 and 75 is ?

A. 43500

B. 43550

C. 43600

D. 43650

**Ans: D**



## Numbers(Assignment)

Q. What is the smallest 5 digits number which is divisible by 12, 15, and 18?

A.10010

B. 10015

C.10020

D. 10080

**Ans: D**





## HCF & LCM Fractions(Assignment)

- Find HCF & LCM of  $\frac{5}{9}$  and  $\frac{25}{36}$
- Ans : HCF =  $\frac{5}{36}$  and LCM =  $\frac{25}{9}$



## HCF & LCM(Assignment)

Q. There are three numbers, these are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. What will be the sum of three numbers :

- A. 80      B. 82      C. 85      D. 87

**Soln:**

numbers are co primes, so there is only 1 as their common factor.  
Given that two products have the middle number in common.

So, middle number = H.C.F. of 551 and 1073 = 29;

So first number is :  $551/29 = 19$

Third number =  $1073/29 = 37$

So sum of these numbers is =  $(19 + 29 + 37) = 85$

**Ans: C**



# Properties of Square Numbers

- A square can't end with odd number of zeroes. The number of 0's of perfect square is always even and the non-zero part should also be a perfect square.

- A square can't end with 2, 3, 7 or 8.

1	2	3	4	5
6	7	8	9	0

- Square of **odd** no. is **odd** & **even** no. is **even**
- Whenever last digit of square is 6, then second last digit is always odd.
- Whenever last digit of square is 5, then second last digit is always 2.
- Whenever last digit of square is 1,4,9, then second last digit is always even.



## Properties of Square Numbers

- Square of number ending in 0 : Square of the number of tens and append two zeroes to right.

e.g.  $(130)^2 = (13^2)00 = 16900$

- Square of number ending in 5 : Multiply number of tens by next higher integer and append 25 to right.

e.g.  $105^2 = 105^2 = (10 \times 11)25 = 11025$

- Square of numbers ending in 1,9, 4 or 6

e.g. for  $(71)^2 = 70^2 + (2 \times 70 \times 1) + 1^2 = 4900 + 140 + 1 = 5041$

for  $(89)^2 = 90^2 - (2 \times 90 \times 1) + 1^2 = 8100 - 180 + 1 = 7921$



## Number system(Assignment)

Q. Find a positive number  $x$ , such that the difference between the square of this number and 21 is the same as the product of 4 times the number?

- A. 9      B. 27      C. 7      D. 13

**Ans : C**



# Progression

- Arithmetic Progression :

- If quantities increase or decrease by a common difference then they are said to be in AP e.g. 3, 5, 7, 9, 11, ....
- If  $a$  is first term,  $d$  is the common difference,  $l$  is the last term then
- General form :  $a, a+d, a+2d, a+3d, \dots, a+(n-1)d$
- $n^{\text{th}}$  term  $T_n = a + (n-1)d$  ,  $n = 1, 2, ..$
- Sum of  $n$  terms  $S_n = \frac{n}{2} [2a + (n-1)d]$

$$S_n = \frac{n}{2} (a + l)$$



# Progression

- Prove that the sum  $S_n$  of  $n$  terms of an Arithmetic Progress (A.P.) whose first term 'a' and common difference 'd' is
- $S = n/2[2a + (n - 1)d]$
- Or,  $S = n/2[a + l]$ , where  $l = \text{last term} = a + (n - 1)d$
- **Proof:**
- $a, a+d, a+2d, a+3d, \dots, a(n-2)d, a(n-1)d$ , as  $l = \text{last term}$
- $a, a+d, a+2d, a+3d, \dots, l-d, l$
- $S = a + a+d + a+2d + a+3d + \dots + l-d + l$  -----1
- Writing equation 1 in reverse order(sum remains same even if we write in reverse order)
- $S = l + l-d + l-2d + l-3d + \dots + a+d + a$  -----2
- Adding equation 1 and 2
- $2S = (a + l) + (a + l) + (a + l) + \dots + (a + l) + (a + l)$
- So for  $n$  terms,
- $2S = n(a + l)$
- $S = \frac{n}{2} (a + l)$



## Progression

Q. The sum of all two digit numbers divisible by 3 is

A. 550

B. 1550

C. 1665

D. 1680

Soln

Two digit numbers divisible by 3 are :

12, 15, 18, 21, ....., 96, 99.

This is an A.P. with  $a = 12$ ,  $d = 3$ ,  $l=99$

Let  $n$  be the number of terms.

Last term  $= a + (n-1)d$

$$99 = 12 + (n-1) \times 3$$

$$3n = 90, \quad n = 30$$

$$\text{Sum} = n/2 (a + l) = 30/2 \times (12+99)$$

$$= 1665$$

**Ans: C**





## Progression

Q. Find the sum of all natural numbers between 10 and 200 which are divisible by 7

A. 2835

B. 2865

C. 2678

D. 2646

**Soln:**

Two digit numbers divisible by 7 are :

14, 21, 28, 35, ....., , 196.

This is an A.P. with  $a = 14$ ,  $d = 7$ ,  $l=196$

Last term =  $a + (n-1)d$

$196 = 14 + (n-1) \times 7$

$196 - 14 = (n-1) \times 7$

$n-1 = 26$

$n=27$

Sum =  $n/2 (a + l)$

$= 27/2 \times (14+196)$

$= 27 \times 210 / 2$

$= 27 \times 105$

$= 2835$

OR

$$n = \frac{\text{LastTerm} - \text{FirstTerm}}{d} + 1$$

**Ans: A**



## Progression(Assignment)

Q. Find the sum of the series 3,8,13,18, .....,93

A. 912      B. 925      C. 998      D. 936

**Ans : A**



# Progression

- Geometric Progression :
- If quantities increase or decrease by a constant factor then they are said to be in GP e.g. 4, 8, 16, 32, .....
- If a is first term, r is the common ratio, then
- General form : a, ar, ar<sup>2</sup>, ar<sup>3</sup>, ....., ar<sup>n-1</sup>
- n<sup>th</sup> term  $T_n = ar^{(n-1)}$
- Sum of n terms  $S_n = \frac{a(r^n - 1)}{(r - 1)}$



## Geometric Progression of n terms :

- To prove that the sum of first n terms of the Geometric Progression whose first term 'a' and common ratio 'r' is given by-
- $S = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$  ----- 1
- Multiply both sides of this equation by r
- $Sr = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} + ar^n$  ----- 2
- - - - -
- Eq 2 - Eq 1
- $Sr - S = ar^n - a$
- $S(r - 1) = a(r^n - 1)$
- $S = \frac{a(r^n - 1)}{(r - 1)}$



## Geometric Progression

Q. Find the 10<sup>th</sup> term of the series: 4, 16, 64, 256, 1024, ....

A.  $4^{10}$       B.  $4^8$       C.  $4^9$       D. 1022480

**Soln:**

The given series is in geometric progression

Where  $a = 4$ ,  $r = 4$

$$\begin{aligned}\text{So } T_{10} &= a \times r^{(10-1)} \\ &= 4 \times 4^{(10-1)} \\ &= 4^{10}\end{aligned}$$

**Ans: A**



# Progression

- What is the difference between arithmetic progression and geometric progression?
- A sequence is a set of numbers, called terms, arranged in some particular order. An arithmetic sequence is a sequence with the difference between two consecutive terms constant. The difference is called the common difference. A geometric sequence is a sequence with the ratio between two consecutive terms constant.



